Stress Components in Unsaturated Soils

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ABSTRACT: In 1959, Bishop published his classic paper on "effective stress" in unsaturated soils, combining together the components of net stress and suction into a single stress variable using the empirical χ factor. Jennings and Burland (1962) criticised this approach, identifying limitations in the use of an "effective stress". In particular, the approach failed to explain volumetric collapse of soil when wetted. Burland followed up this criticism in 1965 by pointing out that the role of suction was two-fold; not only did it increase the contact stress between particles but the presence of air-water interfaces (menisci) also had a stabilising effect. He suggested that we should not try to combine the stress variables into a single "effective stress" but that we should treat the two stress components of net stress and suction differently. This approach was adopted by Fredlund and Morgenstern (1977) and led to the widely used ϕ^b approach. Thus, in critiquing the Bishop approach, John Burland had a highly significant input into the direction of unsaturated soil research and practice over a number of decades. In this paper, Burland's contribution to the debate on stress components in unsaturated soils will be examined through reinterpretation of a set of experimental data from triaxial testing of an unsaturated lateritic gravel. The results confirm that attempting to combine net stress and suction into a single variable is not sufficient for interpreting unsaturated soil behaviour.

KEYWORDS: Unsaturated soil, Effective stress, Suction

1. INTRODUCTION

In 1959, Bishop published his classic paper on "effective stress" to explain the shear behaviour of unsaturated soils (Bishop, 1959). Bishop suggested combining together the components of net stress (σ-u_a) and suction (u_a - u_w) into a single "effective" stress using the empirical χ factor. This was expressed as:

$$\sigma' = (\sigma - u_a) + \chi\,(u_a - u_w) \qquad (1)$$

where	σ'	was	the effective stress
	σ		the total stress
	u_a		the pore air pressure
	u_w		the pore water pressure
and	χ		a factor related to the degree of saturation

The χ variable was an empirical factor that varied between 0 and 1 as a function of degree of saturation, with χ=1 coinciding with full saturation. If χ=1 the equation reduces to the effective stress equation for saturated soils, so this provided a simple transition between saturated and unsaturated conditions.

Shear data from tests on unsaturated soil in which suctions were measured were first published by Bishop et al. (1960) and later by Bishop and Blight (1963). The ideas proposed by Bishop were used to interpret the data, and explained the results obtained. The same effective stress concept was extended to volume change behaviour by Blight (1965).

This paper considers the criticisms by John Burland of the "effective stress" approach to interpreting unsaturated soil behaviour. Burland's contribution to the debate on stress components in unsaturated soils will be examined through reinterpretation of a set of experimental data from tests carried out on an unsaturated lateritic gravel. This is an appropriate set of data to analyse for this purpose, as Burland had an influence on the interpretation of the original data set (Toll, 1988).

2. CRITICISMS OF THE "EFFECTIVE" STRESS APPROACH

Jennings and Burland (1962) criticised the approach proposed by Bishop, identifying limitations in the use of an "effective stress". In particular, the approach failed to explain volumetric collapse of soil when wetted. During wetting, the "effective stress" was reducing which should produce an increase in volume, but instead a volume decrease was observed.

Burland followed up this criticism in 1965 by pointing out that the role of suction was two-fold; not only did it increase the contact stress between particles (the tensile stress in the water pulling particles together) but the presence of air-water interfaces (menisci) also had a stabilising effect, so that the "water bridge" between particles had to be relocated or broken to allowing shearing to take place. He suggested that we should not try to combine the stress variables into a single "effective stress" but that we should treat the two stress components of net stress and suction differently. This approach was adopted by Fredlund and Morgenstern (1977) and led to the widely used ϕ^b approach. Therefore, John Burland had a highly significant input into the direction of unsaturated soil research and practice over a number of decades.

Blight (1967) also pointed out that there were the difficulties in evaluating the parameter χ as different values were obtained depending on the way the data were interpreted. Khalili and Khabbaz (1998) proposed that χ could be expressed as a function of suction (related to the air entry value of the soil) rather than degree of saturation as follows:

$$\chi = \begin{cases} 1 & \text{for } s \le s_e \\ (s_e/s)^{\gamma} & \text{for } s \ge s_e \end{cases} \qquad (2)$$

where s is suction, s_e is air entry value and γ = 0.55.

They suggested that expressing χ in this way allowed a unique value of χ to be defined. However, Loret and Khalili (2000) recognised that an effective stress alone was insufficient for complete modelling and the effective stress had to be combined with suction to fully explain behaviour (as will be explored further in Section 4).

3. THE EXTENDED MOHR-COULOMB APPROACH

Following Burland's (1965) suggestion that net stress and suction should be treated as separate in their effect, Fredlund and Morgenstern (1977) put forward a framework for unsaturated soil behaviour based on independent stress state variables. Later, Fredlund et al. (1978) and Fredlund (1979) set out concepts for the shear strength of unsaturated soil, extending the Mohr-Coulomb failure condition to unsaturated soils to include suction, giving the shear strength equation as:

$$\tau = c' + (\sigma - u_a)\tan\phi^a + (u_a - u_w)\tan\phi^b \qquad (3)$$

where τ was the shear strength
c' the effective cohesion
ϕ^a the angle of friction for changes in net stress (σ - u_a)
and ϕ^b the angle of friction for changes in matrix suction (u_a - u_w)

The separation of net stress and suction, as suggested by Burland, is recognised in the Fredlund approach by having two angles of friction relating to the two components of stress. Additional test data, interpreted using these concepts, were presented by Ho and Fredlund (1982). This approach has subsequently been widely used in interpreting shear strength data on unsaturated soils.

Fredlund et al. (1978) suggested that ϕ^a could be assumed to be equal to the effective stress angle of friction measured in saturated conditions (ϕ'). This would suggest that ϕ^a was constant for all values of matrix suction. The use of a linear relationship between τ and u_a - u_w (i.e. a constant value of ϕ^b) was shown to be in error by Escario and Saez (1986). This non-linearity was confirmed by Fredlund et al. (1987), who assumed ϕ^b varied as a function of suction. Below the air entry value (when the soil remains saturated) ϕ^b is equal to ϕ', but at higher suctions the value of ϕ^b reduces and may fall to zero at high suctions. Data presented by Delage et al. (1987) showed that both ϕ^a and ϕ^b could vary. Toll (2000) pointed out that ϕ^a was dependent on the fabric of the soil and could not be assumed to be equal to ϕ'.

4. THE COMBINED STRESS APPROACH

It is generally recognised that two stress components are needed to fully explain unsaturated soil behaviour, as Burland argued in 1965. An effective stress approach which reduces the stresses to a single component is generally seen to be insufficient. However, rather than use the stress variables of net stress (σ - u_a) and the matric suction (u_a – u_w) (as proposed by Fredlund and Morgenstern (1977)), some authors have used a combined stress (like Bishop's "effective stress") but with the additional variable of suction (Loret and Khalili, 2000; Alonso et al. 2010). It has to be recognised that the combined stress is not an "effective" stress in the sense that the soil behaviour is not exclusively controlled by this stress component, since the soil behaviour is also dependent on the suction component.

The variable used combines the stress contributions from the pore air and pore water phases, usually using degree of saturation as a weighting factor to represent the proportions of each phase in the soil pores. This stress component (variously described as *Bishop's stress* (Bolzon et al. 1996) or *average soil skeleton stress* (Jommi, 2000)) is similar to Bishop's formulation but uses degree of saturation, S_r, in place of the empirical factor χ and is defined as:

$$\sigma^* = (\sigma - u_a) + S_r\,(u_a - u_w) \quad (4)$$

This stress component was used in conjunction with suction by Gallipoli et al. (2003) and Wheeler et al. (2003). Murray (2002) argued that rather than weighting the stress components over the void space (using S_r), they should be weighted with respect to the total volume, which leads to the formulation:

$$\sigma^* = (\sigma - u_a) + \frac{v_w}{v}\,(u_a - u_w) \quad (5)$$

where v_w is the specific water volume [$1 + S_r\,(v - 1)$]
and v the specific volume.

Kohgo et al. (1993) proposed an alternative form of the combined stress, but this requires the definition of a further material parameter. Alternative formulations for stress variables have also been put forward by Karube and his co-workers (Karube and Kato, 1994; Karube and Kawai, 2001) which try to separate the role of the pore water into *bulk* water and *meniscus* water. However, a water retention (soil-water characteristic) curve for the "driest soil" is needed to determine the components.

There seem to be some advantages in using a combined stress approach to account for the different phases within the soil, but also to include suction to represent the effect of surface tension in providing shear resistance. However, it is likely that the simple formulations for combined stress based on phase relations which require no further information will be most realistic (Bolzon et al. 1996; Jommi, 2000; Murray, 2002).

While it might seem that a combined stress approach disagrees with Burland's (1965) proposal to separate the effects of net stress and suction, this is not the case. These combined stress models still recognise the need to include suction as an additional variable, thus recognising that suction acts differently to the "soil skeleton stress" that can be thought of conceptually as the stress acting at particle contacts (from the combination of net stress and suction). However, as Burland noted, suction has an additional role as a "bonding" agent, as was recognised in Gallipoli et al.'s (2003) approach.

5. A CRITICAL STATE APPROACH

The Critical State concept (Schofield and Wroth, 1968) is well established as a useful framework within which saturated soil behaviour can be interpreted. It provides the coupling between volumetric behaviour and shear behaviour that is essential to understand how a soil will behave. Major strides in understanding unsaturated soil behaviour have been made by developing elasto-plastic frameworks of behaviour as originally proposed in conceptual form by Alonso, Gens and Hight (1987). This was developed more fully by Alonso, Gens and Josa (1990) into what has become known as the Barcelona Basic Model (BBM). Variations on this approach have been put forward by Wheeler and Sivakumar (1995), Delage and Graham (1995), Cui and Delage (1996) among others.

The critical state concept is central to understanding shear strength within an elasto-plastic model. In saturated soils, the critical state can be expressed through the deviator stress, q, the mean effective stress, p' and the specific volume, v. At the critical state these variables are related through three critical state parameters, M, Γ and λ as set out in Eq. (6) and (7).

$$q = M\,p' \quad (6)$$

$$v = \Gamma - \lambda \ln p' \quad (7)$$

However, unsaturated soils have an additional phase (the air phase) and it is no longer possible to interpret their behaviour through effective stresses, nor to assume that water content and volume are linked. For unsaturated soils, the stress state can be represented by the net stress (σ - u_a) and the matric suction (u_a – u_w) (Fredlund and Morgenstern, 1977). In addition to specific volume (or void ratio), the phase state of the soil has to be represented by an additional variable; this can be either gravimetric water content (w), volumetric water content (θ) or degree of saturation (S_r).

Toll (1990) proposed that the critical state for unsaturated soils could be expressed in terms of q, p - u_a, u_a - u_w, v and S_r. The unsaturated critical state requires five parameters, M_a, M_b, Γ_{ab}, λ_a and λ_b as set out in Eq. (8) and (9).

$$q = M_a\,(p - u_a) + M_b\,(u_a - u_w) \quad (8)$$

$$v = \Gamma_{ab} - \lambda_a \ln (p - u_a) - \lambda_b \ln (u_a - u_w) \quad (9)$$

Toll and Ali Rahman (2017) note that for comparison with the "effective stress" approach (Bishop, 1959; Khalili and Khabbaz, 1998) the Critical State would be given by:

$$q = M\,[(p - u_a) + \chi\,(u_a - u_w)] \quad (10)$$

$$q = M\,(p - u_a) + \chi M\,(u_a - u_w) \quad (11)$$

Therefore, for comparison with Toll's approach, the "effective stress" approach implies:

$$M_a = M \quad (12)$$

$$M_b = \chi M \quad (13)$$

For comparison with the Barcelona Basic Model (Alonso et al. 1990), the BBM assumes that the contribution from net stress is constant and equal to the saturated critical state stress ratio, M. In the BBM the contribution from matric suction is represented as a decrease in the intercept of the Critical State Line (CSL) on the $p - u_a$ axis defined by a parameter k. Therefore, the relationships in the BBM are:

$$M_a = M \quad (14)$$

$$M_b = kM \quad (15)$$

The major difference in the Toll (1990) approach is that M_a and M_b change with degree of saturation or fabric of the soil. The Khalili and Khabbaz approach assumes that M (and hence M_a) is a constant but that χ, and hence M_b, varies as a function of suction. The BBM approach assumes that M and k are constants (implying M_a and M_b are both constant).

Toll (1990), Toll and Ong (2003) and Toll and Ali Rahman (2017) have shown that the critical state stress ratios M_a and M_b do not have constant values for unsaturated conditions. The variations in the parameters can be expressed as functions of the degree of saturation, S_r. It was argued by Toll (1990) that it was not degree of saturation *per se* that affected these parameters, but rather the fabric of the soil. Other research has also identified variations in the critical state parameters. The results of tests on compacted kaolin (Zakaria et al. 1995; Wheeler and Sivakumar, 1995) showed that the slope of the Critical State Line in q, p-u_a space (i.e. M_a) increased as suction increased. This would be consistent with an increase in M_a with decreasing degree of saturation (since degree of saturation will reduce as suction increases). It is also consistent with other observations that the friction angle for changes in net stress (i.e. ϕ^a) increased with increasing suction (Escario and Saez, 1986) although these observations were not made for critical state conditions. However, Delage et al. (1987) and Maâtouk et al. (1995) found that for silts, ϕ^a decreased with increased suction. Ng et al. (2000) found that M_a seemed to be constant and independent of the value of suction for two loosely compacted volcanic fills. Nevertheless, Ng et al.'s values of M_b did show a clear variation with degree of saturation, similar to that shown by Toll (1990).

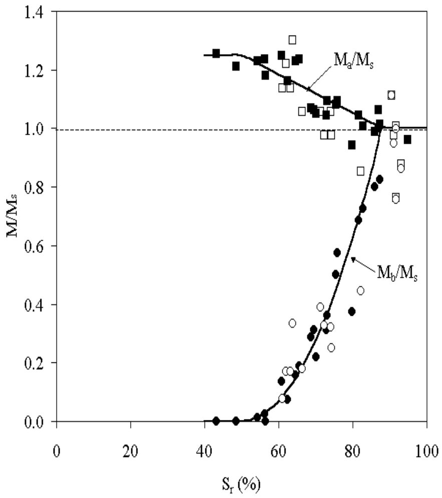

Figure 1 Normalised functions for critical state stress ratios related to degree of saturation, S_r [Black symbols for Kiunyu Gravel (Toll, 1990): Open symbols for Jurong soil (Toll and Ong, 2003)].

Toll and Ong (2003) showed that the values of M_a and M_b could be normalised with respect to M_s (the critical state stress ratio for saturated conditions) to allow comparisons between different soil types. The values of M_a/M_s and M_b/M_s are plotted against degree of saturation in Figure 2. It can be seen that the experimental data for a lateritic gravel (Kiunyu gravel) (Toll, 1990) and a residual sandy clay (Jurong soil) (Toll and Ong, 2003) show the same form of relationship with degree of saturation. This suggests that the form of these functions may be common to a range of soil types.

At lower degrees of saturation (dry of optimum water content), M_a has greater values than M_s (the stress ratio in saturated conditions). This was explained by Toll (1990; 2000) as being due to the presence of aggregations at lower degrees of saturation, causing the soil to behave in a coarser fashion than would be justified by the grading. In unsaturated conditions the aggregated fabric can be maintained during shear because the suction gives strength to the aggregations. In a saturated soil, the aggregations would be broken down during shear and would not be expected to affect the critical state parameters.

John Burland had an influence on the development of this framework of unsaturated soil behaviour. Burland took on a supervisory role during the final stages of Toll's PhD thesis (1988). He was familiar with the work of Brackley (1973; 1975) from his connections in South Africa (having undertaken his Masters studies in South Africa with Jennings at Witwatersrand University). Brackley was an early pioneer in recognising what he called "packet fabric" and what we now call "double structure" in compacted clays. It was this concept of aggregation of clay particles that helped to develop the understanding of the critical influence of fabric on unsaturated soil behaviour.

It can be seen from Figure 1 that at low degrees of saturation M_b becomes significantly lower than M_s and eventually drops to zero. This suggests that the packet fabric also affects M_b as the water phase withdraws into the packets and suction makes no contribution to the overall strength of the soil. It is also possible that this represents a similar effect to the reduction in the friction angle for suction (ϕ^b) as suction increases, as observed by Escario and Saez (1986) and Fredlund et al. (1987). This effect can be explained by a reduction in the area of water over which the suction acts (very like the χ factor used by Bishop (1959)). It has been shown by Fredlund et al. (1995) and Vanapalli et al. (1996) that the reduction of ϕ^b can be related to normalised volumetric water content or degree of saturation. This was revisited by Toll and Ong (2003).

It is clear from Figure 1 that the assumption that ϕ^a is equal to ϕ' (which implies that $M_a = M_s$) will not be correct for soils that exhibit significant fabric differences between the unsaturated and saturated states. The packet fabric in compacted soils is maintained by suction and provides the intra-packet strength that resists packet breakdown during shear. However, the same material, compacted under the same conditions but then subjected to saturation, will demonstrate a loss of the intra-packet strength and the packets will be easily broken-down during shear. Therefore, the observation that ϕ^a "appears to be essentially equal to the effective angle of internal friction obtained from shear strength tests on saturated soil specimens" (Fredlund and Rahardjo, 1983) must be viewed with caution.

6. A RE-EXPLORATION OF THE "EFFECTIVE STRESS" APPROACH

The Critical State approach outlined in Section 5 adopts the separation of net stress and suction advocated by Burland in 1965. However, there have been proposals, such as by Khalili and Khabbaz (1998), that an "effective stress" approach can be used.

The data for Kiunyu lateritic gravel, reported by Toll (1990), has firstly been reinterpreted in Figure 2 using Bishop stress (p^*) (as defined in Eq. (4) using $\chi = S_r$) to analyse the critical state points (end of test points) in deviator stress q-p space for a set of unsaturated (constant water content) triaxial tests. These are compared with results from saturated triaxial tests, plotted in terms of effective stress, p'. The degrees of saturation (S_r) for each test are marked alongside each data point. It can be seen that for higher degrees of saturation.

(S_r >75%) the data for the unsaturated tests, interpreted using Bishop stress, show good agreement with the saturated test results. The results fall within the shaded area, close to the saturated Critical State Line (CSL). However, for degrees of saturation of 40-75%, the Bishop stress interpretation shows poor agreement with the saturated CSL.

The same data set is also presented in specific volume *v-p* space in Figure 3. It can be seen that the critical state points for the unsaturated tests plot well above the CSL defined by the saturated test results. This is consistent with the concept of aggregated fabric presented in Section 5. In the unsaturated tests the packet fabric in compacted soils is maintained by suction and is not destroyed by shearing, so the specific volumes are much higher due to the presence of macro-voids between the clay packets.

Therefore, for both *q-p* space and *v-p* space, the use of Bishop stress to interpret the results does not show good agreement with saturated test results for the Critical State Line. In *q-p* space, the tests results at high saturation (S_r > 75%) give reasonable agreement, but not for lower degrees of saturation.

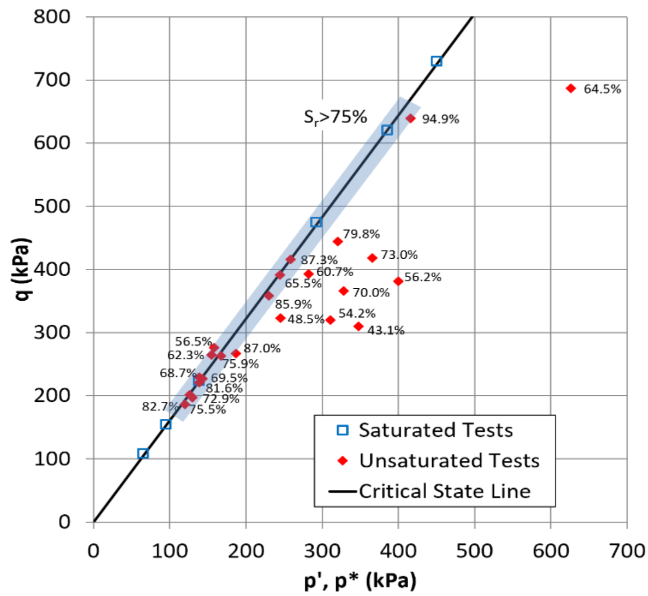

Figure 2 Comparison of Critical State Line in *q-p* space, using Bishop stress (based on $\chi = S_r$) for unsaturated soil tests.

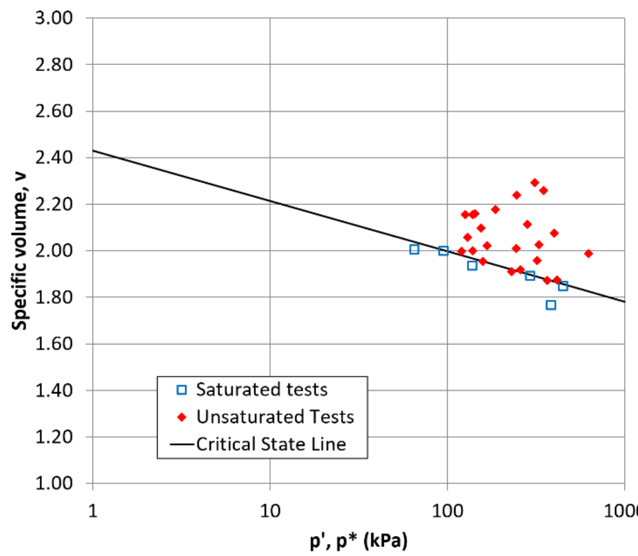

Figure 3 Comparison of Critical State Line in *v-p* space, using Bishop stress (based on $\chi = S_r$) for unsaturated soil tests.

The same set of results are now interpreted using Khalili and Khabbaz's (1998) χ factor (as defined in Eq. 2). The results in *q-p* space are shown in Figure 4 and in *v-p* space in Figure 5.

It can be seen from Figure 4 that Khalili and Khabbaz's χ factor gives an improved interpretation of the *q-p* behaviour. The results for the unsaturated soil tests are clustered around the CSL defined from the saturated tests. This is even true for the lower degree of saturation tests (S_r < 75%) where the agreement was poor for the Bishop stress comparison shown in Figure 2. This suggests that an "effective stress" approach can give a realistic interpretation for the shear stress aspects of Critical State.

However, the interpretation of the volumetric behaviour as shown by the *v-p* plot in Figure 5 shows barely any improvement compared to the Bishop stress plot in Figure 3. This means that a simple "effective stress" approach, based purely on applying a χ factor, cannot give a full interpretation of unsaturated soil behaviour. This finding is consistent with the observations by Bishop and Blight (1963) and upholds the observation by Jennings and Burland (1962) and Burland (1965) that a simple "effective stress" approach, attempting to combine net stress and suction into a single variable will not be successful for interpreting unsaturated soil behaviour.

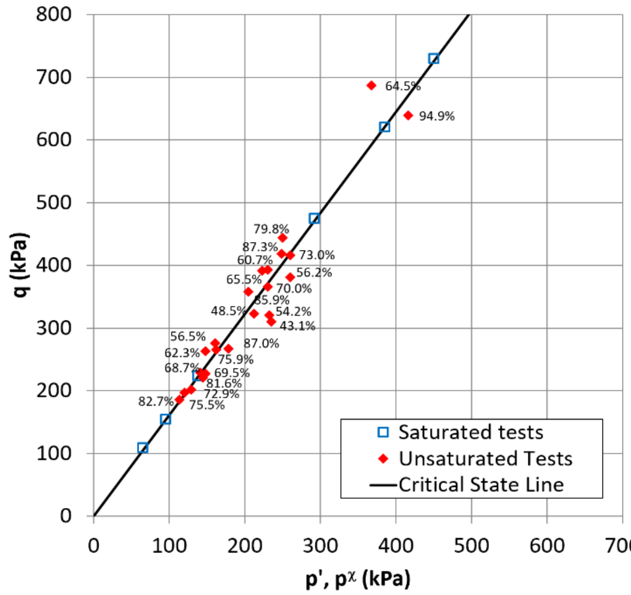

Figure 4 Comparison of Critical State Line in *q-p* space, using Khalili and Khabbaz's (1998) χ factor for unsaturated soil tests.

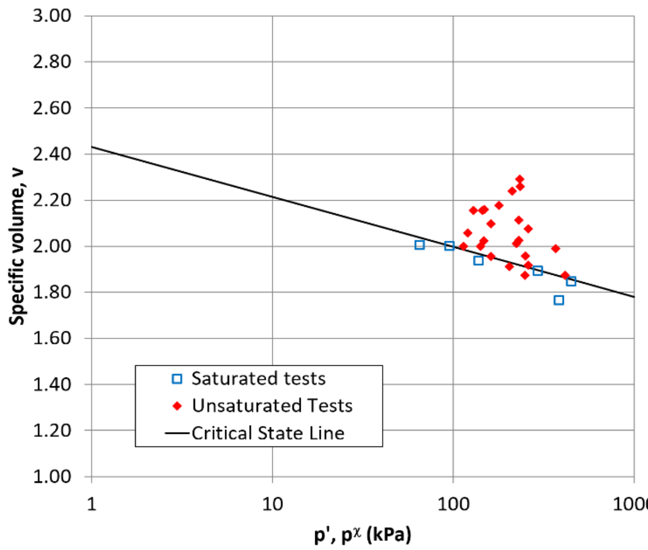

Figure 5 Comparison of Critical State Line in *v-p* space, using Khalili and Khabbaz's (1998) χ factor for unsaturated soil tests.

7. CONCLUSIONS

Jennings and Burland (1962) and Burland's (1965) critique of Bishop's (1959) classic paper on an "effective stress" approach to interpreting unsaturated soil behaviour were instrumental in recognising that the stress variables of net stress and suction act differently; they should not be combined into a single stress variable. Fredlund and Morgenstern (1977) went on to identify separate stress state variables leading to the widely used ϕ^b approach for interpreting shear strength of unsaturated soils. Thus, in critiquing Bishop's approach, John Burland had a highly significant input into the direction of unsaturated soil research and practice over a number of decades.

A fuller understanding of the behaviour of unsaturated soils can be obtained if the shear behaviour is coupled to the volumetric behaviour. A Critical State approach provides such a framework. The critical state for unsaturated soils can be expressed in terms of separate stress state variables.

A set of experimental data for an unsaturated lateritic gravel has been re-interpreted using both the Bishop stress approach (using $\chi = S_r$) and using Khalili and Khabbaz's (1998) χ factor. The use of Bishop stress to interpret the results does not show good agreement with saturated test results for the Critical State Line. In terms of shear stress, reasonable agreement is achieved for higher degrees of saturation $S_r > 75\%$ but the volumetric behaviour is in poor agreement due to the presence of an aggregated fabric. Khalili and Khabbaz's approach gives better agreement for shear stress compared to the Bishop stress approach but in volumetric terms shows barely any improvement.

This means that a simple "effective stress" approach, based purely on applying a χ factor, cannot give a full interpretation of unsaturated soil behaviour. This upholds the observation by Jennings and Burland (1962) and Burland (1965) that a simple "effective stress" approach, attempting to combine net stress and suction into a single variable, is not sufficient for fully interpreting unsaturated soil behaviour.

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