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Discretization error in random finite element analysis of active lateral force in spatially variable clay

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ABSTRACT

In the random finite element method (RFEM), there arises an inevitable issue of discretization error from two sources: error due to finite element (FE) discretization and error due to random field (RF) discretization. The present paper investigates the contributions of these two errors to the total discretization error. The active lateral force on a retaining wall with spatially variable clay is considered. Two separate meshes are defined, FE mesh and RF mesh, and they are generated at different resolutions. Spatial averaging (SA) method is adopted for discretization of random fields. The active lateral force is then simulated for different combinations of FE mesh and RF mesh. It is found that the two errors tend to accumulate for the SA method, with a dominant contribution from the RF discretization error. This study also provides suggestions for allowable mesh sizes to control the discretization error in the current retaining wall problem.

Keywords: Random finite element method; discretization error; random field; spatial variability; shear strength

1 INTRODUCTION

The random finite element method (RFEM) (e.g., Fenton et al. 2005) is increasingly used in geotechnical engineering. The RFEM must meet a number of challenges for its implementation. A major challenge is the discretization error. The discretization error is the discrepancy between the true solution for the continuous mathematical model and the approximate RFEM solution. In the RFEM, this error comes from two sources at least. One is the finite element (FE) discretization, and the other is the random field (RF) discretization. The former is obvious. The latter is because a discrete random field is needed to feed soil properties into finite element analysis. Hence, the discretization error in the RFEM depends on several factors such as boundary conditions, response of interest, discretization method, element size, scale of fluctuation (SOF), and type of autocorrelation function (Fenton 1994).

In the geotechnical community, the authors are aware of two studies that have been fully devoted to the issue of discretization error for spatially variable undrained shear strength: Ching and Phoon (2013) and Huang and Griffiths (2015). They both considered a similar problem (soil column subjected to axial compressive loading). To control the discretization error, they reported a dimensionless ratio of the element size to the SOF, where SOF is the distance over which property values are significantly correlated. However, their recommended ratios of (element size)/SOF are very different. In fact, Ching and Phoon's (2013) recommendation is almost ten times smaller than Huang and Griffiths' (2015) recommendation. Apart from this difference, what is less understood in both studies, and in past studies, is how the two components of the discretization error (FE discretization error and RF discretization error) interact with each other and affect the solution. Insights from this can help to reduce the discretization error in a more effective way. The present paper therefore aims to address the following questions: (1) do the two components of the discretization error accumulate or compensate?, and (2) which component is larger? The present paper also provides suggestions for the allowable (element size)/SOF to achieve a certain error tolerance in the retaining wall problem. Note that the conclusions of the paper may no longer be valid if, for example, a different autocorrelation function or a different discretization method is adopted.

2 METHODOLOGY

2.1 Random field model

This paper models spatial variability of soil as random fields. The random field is assumed to be second-order stationary, which needs three parameters: (1) mean, (2) variance, and (3) autocorrelation function. The first two are constant everywhere. The third, which defines the correlation between two points, is a function of their separation distance rather than their absolute positions. One of the common autocorrelation functions used in the geotechnical engineering literature is the single exponential model. In two dimensions, it defines the correlation between two points with separation distance of Δx and Δz as follows:

$$\rho(\Delta x,\Delta z) = \exp\left(-2\frac{|\Delta x|}{\delta_x} - 2\frac{|\Delta z|}{\delta_z}\right) \tag{1}$$

where δ_x and δ_z are, respectively, SOFs in the x and z directions. In the present study, $\delta_x = \delta_z = \delta$.

The Fourier series method (FSM) (Jha & Ching 2013) is employed to simulate a two-dimensional lognormal random field. Moreover, among different random field discretization methods, the spatial averaging (SA) method is adopted. In SA, the field value for an element is represented by the spatial average of the field over the element.

2.2 Finite element model

The backfill for the retaining wall is a rectangular area of size $L_x \times L_z$ = 16 m × 8 m, restrained by a wall of height H = 5 m (Fig. 1). Only cohesive soil is considered. A more comprehensive study should consider frictional materials as well. The conclusions of the paper may not be applicable to the frictional materials. The undrained shear strength (s_u) is simulated as a lognormal random field with a mean value = 20 kN/m^2 and variance = 6^2 (kN/m^2)^2. The soil unit weight is 20 kN/m^3, the Young's modulus is 100 MN/m^2, and the Poisson ratio is 0.3. The active lateral force, P_a, is calculated as follows. The wall nodes are incrementally displaced in the horizontal direction until the mobilized reaction force on the wall reaches a minimum value, and P_a is taken to be the minimum value (Fenton et al. 2005).

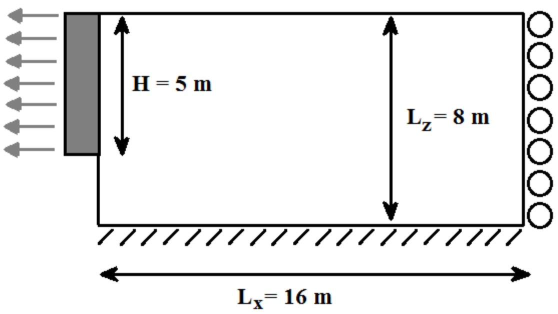

Fig. 1. The FEM model for the retaining wall problem.

2.3 Random finite element mesh layout

In practice, RFEM often adopts an identical mesh to discretize the FE and RF models. This way of discretization, hereafter called the conventional RFEM, mixes the two components of the discretization error together. As stated earlier, the total discretization error in the RFEM has at least two components. One pertains to FE and the other to RF. To decompose the discretization error into these two components, the present paper defines two separate meshes: FE mesh and RF mesh. The FE mesh consists of square elements with a side length of l_{FE}, while the RF mesh consists of square elements with a side length of l_{RF}. Both meshes are generated at different resolutions. As shown in Figure 2a, 39 different combinations of FE and RF mesh resolutions are considered. Note that the dashed line in Figure 2a represents the conventional RFEM in which $l_{FE} = l_{RF} = l$. As an illustration, the case with l_{FE}/H = 1/5 and l_{RF}/H = 1/1.25 is shown in Figure 2b. In finite element analysis, it is common to refine the mesh only in regions with a steep stress gradient. This saves the computation time without sacrificing accuracy. However, in the present study, the mesh is uniformly refined over the entire domain for sake of simplicity.

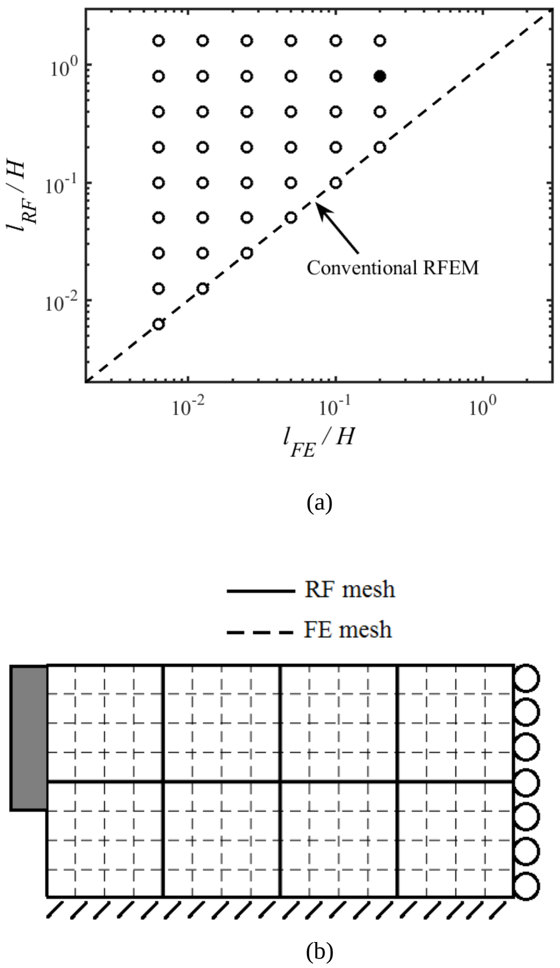

Fig. 2. (a) Different combinations of FE and RF mesh resolutions (b) The case with l_{FE}/H = 1/5 and l_{RF}/H = 1/1.25 (filled circle)

3 RESULTS FOR A FIXED SOF

In this section, the discretization error is studied for a fixed value of SOF. The results for various SOFs will be presented in the next section. A fixed SOF (δ = 1m) is considered. Two thousand Monte Carlo simulations are performed for each of the 39 combinations of FE and RF mesh resolution. In each random field realization, s_u values are assigned to the RF mesh using the SA method, and P_a value is simulated. Figure 3 shows the mean of P_a, E(P_a), normalized by the mean of

$P_{a,s}$, $E(P_{a,s})$. The subscript s denotes the case where s_u is treated as a single random variable (homogeneous s_u).

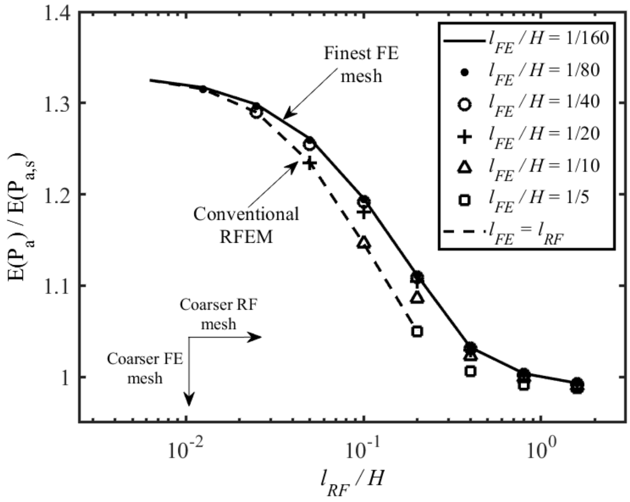

Fig. 3. The normalized mean P_a for δ = 1 m.

3.1 FE discretization error for a fixed RF mesh

In Figure 3, the effects of FE and RF discretization errors are mixed. To focus on the FE discretization error, the results for a fixed RF mesh but a variable FE mesh are shown in Figure 4 (l_{RF}/H is fixed at 1/2.5). Although not shown, similar trends as in Figure 4 are observed for other fixed values of l_{RF}. It is evident that a coarser FE mesh tends to make the model soil mass "stronger" than that produced by the finest FE mesh. This is manifested by a decrease in $E(P_a)$ with a coarser FE mesh. This observation is consistent with the general understanding in FE community that a coarse mesh tends to behave overly strong.

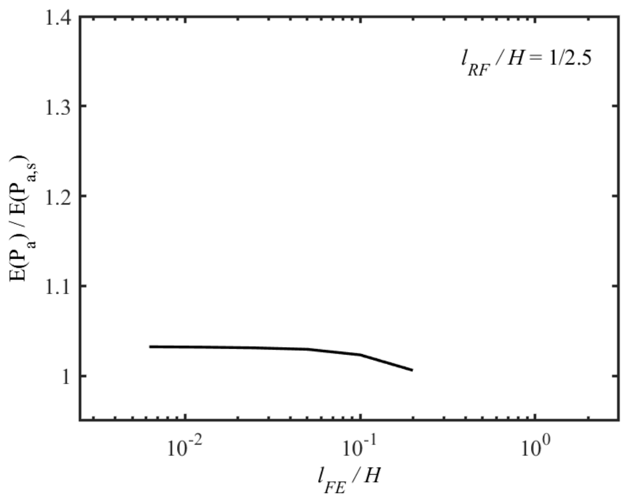

Fig. 4. The effect of the FE discretization error (l_{RF}/H = 1/2.5).

3.2 RF discretization error for a fixed FE mesh

To focus on the RF discretization error, the results for a fixed FE mesh (finest FE mesh) but a variable RF mesh are shown in Figure 5. It is evident that a coarser RF mesh tends to make the model soil mass stronger than that produced by the finest RF mesh. This is because weak spots are suppressed by SA. This behavior is manifested by a decrease in $E(P_a)$ with a coarser RF mesh.

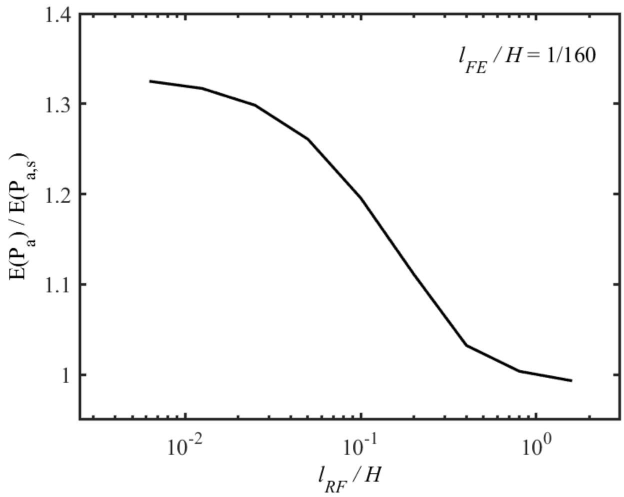

Fig. 5. The effect of the RF discretization error (l_{FE} is the finest).

3.3 Discretization error for the conventional RFEM

Recall that the conventional RFEM adopts equal FE and RF meshes (l_{FE} = l_{RF} = l), so a coarser FE mesh implies that the RF mesh is coarser as well. The total discretization error for the conventional RFEM can be decomposed into FE and RF discretization errors, as described below. Consider the solid line (finest FE mesh) and dashed line (conventional RFEM) in Figure 3. These lines are reproduced in Figure 6 for ease of illustration. Three points are marked in Figure 6: Points A (conventional RFEM), B (finest FE mesh), and C (finest FE and RF mesh). It is clear that the vertical difference between A and C is the total discretization error for the conventional RFEM. Note that A and B are with the same l_{RF}, but B is with l_{FE} = the finest l_{FE}. It is then clear that the vertical difference between A and B is the FE discretization error for the conventional RFEM. Note that B and C are with the same l_{FE}, but C is with l_{RF} = the finest l_{RF}. It is then clear that the vertical difference between B and C is the RF discretization error for the conventional RFEM. As a result, the FE and RF discretization errors can be separated.

Figure 7 shows how the FE and RF discretization errors vary with mesh size in the conventional RFEM (l_{FE} = l_{RF} = l). It can be seen that the RF discretization error is larger than the FE discretization error. More importantly, the FE and RF discretization errors are both negative, so the total discretization error becomes even larger: the FE and RF discretization errors "accumulate".

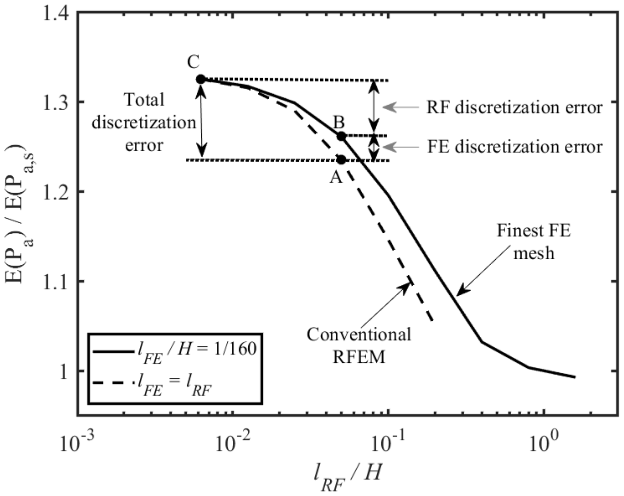

Fig. 6. The decomposition of the total discretization error for the conventional RFEM into the FE and RF discretization errors.

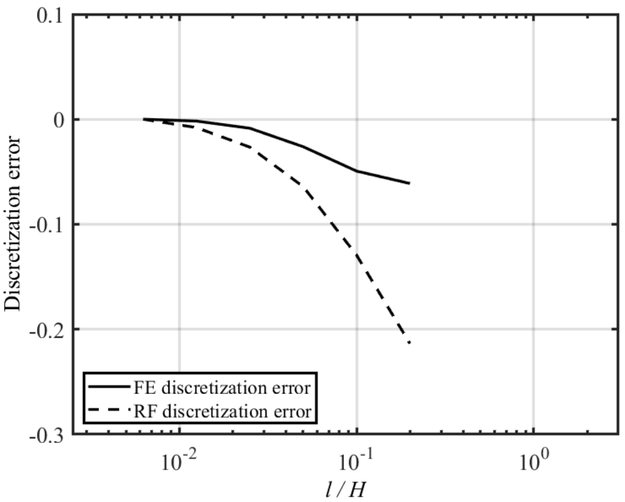

Fig. 7. The variation of the FE and RF discretization errors in the conventional RFEM (l_{FE}=l_{RF}=l)

4 RESULTS FOR VARIOUS SOFS

This section studies the discretization error for various SOFs. The analyses in the previous section are repeated for SOF/(height of wall), δ/H, = {0.02, 0.1, 0.2, 0.5, 1, 2, 10, 50, 300}. Although not shown, the qualitative observations made in the previous sections still hold true for various SOFs. In the following, the allowable mesh size for the conventional RFEM considering various SOFs is investigated.

4.1 Suggestions for allowable mesh size

The allowable normalized mesh size, $(l/\delta)_{allow}$, is suggested as follows. The suggestion is based on the relative error with respect to the reference solution. The reference solution refers to the solution produced by the finest FE and RF meshes. The relative error is defined as (the mean of the conventional RFEM solution – the mean of the reference solution)/(the mean of the reference solution) or (the standard deviation of the conventional RFEM solution – the standard deviation of the reference solution)/(the standard deviation of the reference solution). $(l/\delta)_{allow}$ is selected such that the absolute value of the relative error does not exceed a certain "error tolerance" for both the relative errors in the mean and standard deviation. Two error tolerances are considered: 0.01 and 0.05. The most conservative $(l/\delta)_{allow}$ value over various SOFs is adopted for conservatism. The suggested $(l/\delta)_{allow}$ are as follows. If the error tolerance is 0.01, $(l/\delta)_{allow}$ is about 0.03. If the error tolerance is 0.05, $(l/\delta)_{allow}$ is about 0.13. Note that these suggestions may be restricted to the current numerical problem.

5 CONCLUSIONS

In this paper, the discretization error of the random finite element method (RFEM) for spatially variable undrained shear strength is decomposed into two components: error due to finite element (FE) discretization and error due to random field (RF) discretization. Two separate meshes, FE and RF meshes, are used to study the contribution of each component. Spatial averaging (SA) discretization method is considered. Based on a series of analyses on the active lateral force (P_a) on a retaining wall problem, the following conclusions are obtained: (1) The FE discretization error tends to make the model soil "stronger" than that produced by the finest FE mesh; (2) The RF discretization error tends to makes the model soil stronger than that produced by the finest RF mesh; (3) In the conventional RFEM (equal-size FE and RF meshes), the FE and RF discretization errors accumulate, and the RF discretization error is larger than the FE discretization error; (4) The allowable normalized mesh size, $(l/\delta)_{allow}$, that keeps the relative error within a certain error tolerance is as follows. If the error tolerance is 0.01, $(l/\delta)_{allow}$ is about 0.03. If the error tolerance is 0.05, $(l/\delta)_{allow}$ is about 0.13; (5) The conclusions of the paper may no longer be valid if, for example, a different discretization method or a different soil type is adopted.

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