Stability analysis of spatially variable embankment using random limit equilibrium method

Szu-Wei Lee[1]

[1] Department of Civil Engineering, National Taiwan University, Taipei, Taiwan.

ABSTRACT

For an earth embankment, the failure probability is sometimes to evaluate its safety status. The evaluation of the failure probability can be computationally intensive because it may require numerous stability evaluations. The situation aggravates if a three-dimensional (3D) random finite element method (RFEM) is adopted to evaluate the stability of a long embankment with a spatially variable shear strength. To reduce the computational efforts, this study proposes the use of a 3D random limit equilibrium method (RLEM) in place of the time-consuming 3D RFEM. The results show that the safety factor estimates as well as the slide lengths and volumes obtained by the 3D RLEM are consistent with those obtained by the 3D RFEM for a wide range of embankment geometries, soil properties including constant and spatial variability parameters; moreover, the 3D RLEM only requires few seconds to complete one simulation. This study highlights the use of the 3D RLEM can effectively obtain the safety factor of an earth embankment and reduce the computational efforts.

Keywords: random finite element method; random limit equilibrium method; stability analysis; safety factor; spatial variability.

1 INTRODUCTION

For an earth embankment, the failure probability is sometimes used to quantify the safety status. Vanmarcke (1977a) proposed an approximate solution for the failure probability of a spatially variable earth embankment. The approximate solution is based on some assumptions, e.g., the critical slip surface is a cylinder extended from the two-dimensional (2D) critical slip curve obtained in a conventional 2D stability analysis, and the embankment is simplified as an in-series system with segments of equal lengths. The failure probability can then be approximately expressed as an analytical form.

A more rigorous method of evaluating the failure probability of a spatially variable embankment is to conduct random finite element method (RFEM) (e.g., Hicks and Spencer 2010; Hicks et al. 2014; Li et al. 2015; Xiao et al. 2016; Xiao et al. 2017). The method can seek out the critical zone without assuming the shape of the critical slip surface (Griffiths and Fenton 2004). It also does not simplify the embankment as an in-series system. Monte Carlo simulation-based (MCS-based) RFEM has been adopted to estimate the failure probability of a spatially variable embankment and to simulate the slide lengths and volumes (e.g., Hicks and Samy 2002; Hicks and Spencer 2010; Hicks et al. 2014; Li et al. 2015; Xiao et al. 2016; Xiao et al. 2017). The main challenge for MCS-based RFEM is the computational cost (Cho 2009), especially when the failure probability is small (e.g., $P_F < 10^{-3}$).

There are two possible ways of reducing the computational cost for a MCS-based method: (a) reduce the computational time for the stability analysis; (b) reduce the number of random samples. The current paper focuses on item (a). Item (b) is pursued elsewhere using the subset simulation (Au and Beck 2001). A three-dimensional random limit equilibrium method (3D RLEM) is adopted in this paper to reduce the computational time. A bowl-shaped slip surface is observed in some RFEM studies (e.g., Griffiths et al. 2009; Hicks and Spencer 2010; Hicks et al. 2014; Ji and Chan 2014; Xiao et al. 2016), and this bowl-shaped slip surface is adopted in the 3D RLEM. The 3D RLEM approximates the bowl-shaped slip surface by a cylindrical surface with two power-curve ends. To verify the accuracy of the 3D RLEM, the safety factors as well as the slide lengths and volumes determined by the 3D RLEM and 3D RFEM are compared for a wide range of embankment geometries as well as spatial variability settings.

2 3D RANDOM FIELD

Spatial variabilities of soil properties are commonly modeled by random fields (Vanmarcke 1977b). Among random field models, zero-mean stationary (or statistically homogeneous) random fields are widely and practically used due to their simplicity. A 3D stationary random field for shear strength is denoted by $\tau_f(x,y,z)$, where x = horizontal coordinate, y = longitudinal coordinate, and z = depth coordinate. The auto-correlation function for $\tau_f(x,y,z)$ is defined as the correlation between two locations (Δx, Δy, Δz)

apart:

$$\rho(\Delta x,\Delta y,\Delta z) = \rho(\tau_f(x,y,z),\tau_f(x+\Delta x,y+\Delta y,z+\Delta z)) = \frac{CV(\tau_f(x,y,z),\tau_f(x+\Delta x,y+\Delta y,z+\Delta z))}{\sqrt{Var(\tau_f(x,y,z))}\cdot\sqrt{Var(\tau_f(x+\Delta x,y+\Delta y,z+\Delta z))}} \quad (1)$$

where Var(.) denotes variance; CV(.,.) denotes covariance. A popular auto-correlation model is the single exponential (SExp) model (Vanmarcke 1977b) that is adopted in this paper:

$$\rho(\Delta x,\Delta y,\Delta z)=\exp(-2|\Delta x|/\delta_x - 2|\Delta y|/\delta_y - 2|\Delta z|/\delta_z) \quad (2)$$

where δ_x, δ_y, and δ_z are, respectively, the scales of fluctuation (SOFs) in the x, y, and z directions.

A zero-mean stationary Gaussian random field simulation can be generated using Fourier serious method (FSM) (Jha and Ching 2013). A 3D zero-mean stationary Gaussian random field is adopted in this paper with the simulation space ($L_x \times L_y \times L_z$). The 3D zero-mean stationary Gaussian random field W(x,y,z) can be expressed as the following expansion:

$$W(x,y,z) = \mathrm{Re}\left[\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{o=-\infty}^{\infty}(a_{mno}+ib_{mno}) \exp\left(\frac{i2m\pi x}{L_x}+\frac{i2n\pi y}{L_y}+\frac{i2o\pi z}{L_z}\right)\right] \quad (3)$$

where Re[.] denotes the real part of a complex number, and a_{mno} and b_{mno} are independent zero-mean Gaussian random variables.

3 STABILITY ANALYSIS USING 3D RLEM

A 3D homogeneous slope stability analysis using the 3D LEM, which involves various assumptions about the simplified geometry as well as the locations and shapes of the sliding mass, is developed over the past decades (Chen and Chameau 1983; Leshchinsky and Huang 1992; Lam and Fredlund 1993; Huang et al. 2002; Cheng and Yip 2007).

3D RLEM has been adopted in probabilistic slope stability analysis in the past decades (Li and Lamb 1987; Malkawi et al. 2000; El-Ramly et al. 2002; Cho 2007; Cho 2009; Li et al. 2016). The use of the 3D RLEM is adopted to estimate the safety factor of a spatial variability embankment in this paper. The bowl-shaped slip surface of the 3D RLEM mainly refers to Gens et al. (1988) and is composed of (a) a cylindrical surface and (b) two power-curve ends. Figure 1 shows the geometry and parameter details for item (a) and (b).

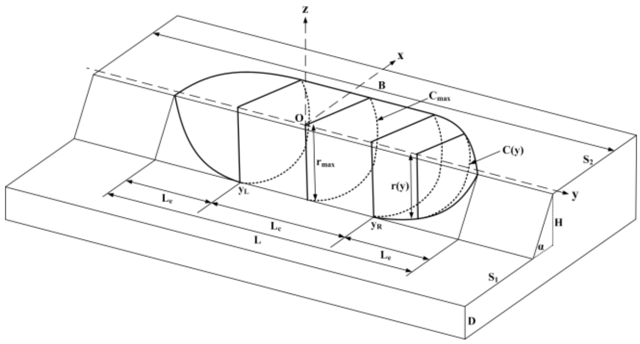

Fig. 1. The geometry and the parameter details of the cylindrical and power curve part.

The safety factor estimate of a potential failure mass is expressed as:

$$F_T = \frac{(c_u C_{max} \times L_c + R_c) r_{max} + \int_0^{L_e} R_{slip}(y)dy + \int_{L-L_e}^{L} R_{slip}(y)dy}{\int_0^L D_{slip}(y)dy} \quad (4)$$

where where c_u denotes the mean undrained shear strength; C_{max} denotes the maximum failure curve; R_c denotes the overall resistance of the cylindrical part with spatial variability; r_{max} denotes the maximum radius of the failure curve with center at O; R_{slip} (y) denotes a curve resistance function for an unit thickness at a particular y; D_{slip} (y) denotes a driving moment function for an unit thickness at a particular y. D_{slip} has an analytical solution that is derived in Taylor (1937) and it is also shown in Gens et al. (1988).

The 3D stability analysis of a spatially variable embankment is determined using an optimization analysis, which is searching the minimum safety factor using Eq. (4) with numerous potential failure masses.

4 STABILITY ANALYSIS USING 3D RFEM

Figure 2 shows the basic model geometry and finite element mesh details, with respect to Cartesian axes x and z in two dimensions. However, a 3D embankment is extended longitudinally in the y direction with a consistent cross section. The 3D embankment geometries include the slope angle (i), slope height (H), total length (B), and foundation layer thickness (D). The 3D embankment rests on a firm base and is characterized by clay with a spatially variable c_u. Furthermore, the boundary conditions include a fixed base as well as fixed faces including the front, back, and two ends ones. Rollers are sometimes adopted to prevent the movement in the x direction on the front and back faces and in the y direction on the two ends faces. The possibility of the rollers was considered formerly, but then rejected due to a tendency for failure may be attracted to these boundaries.

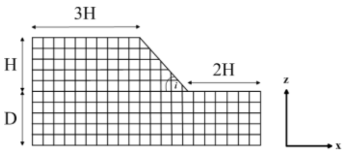

Fig. 2. Embankment geometry and finite element mesh.

The finite element mesh comprises 8-node brick elements and the element type is C3D8R in ABAQUS. The element size is 1 m deep and 1 m × 1 m in plane (the size is the average of the ones of the practical frameworks, e.g., Liu et al. 2007; Griffiths et al. 2009; Hicks and Spencer 2010; Hicks et al. 2014; Ji and Chan 2014; Xiao et al. 2016), except for the elements that have been distorted near the slope toe and crest.

The soil has been modeled as idealized linear elastic and perfectly plastic clay. The elastic component has been defined by Young's modulus, E = 50,000 kPa, and Poisson's ratio, $\nu = 0.3$; the plastic component has been defined by the Mohr – Coulomb failure criterion and a spatially varying undrained shear strength (c_u) (i.e., the friction angle, $\phi = 0$). The spatially variable c_u is modelled using a truncated normal distribution to prevent the possibility of negative values and the following statistics: depth-independent mean, μ_{cu}; coefficient of variation, COV_{cu}; vertical scale of fluctuation, δ_v; and horizontal scale of fluctuation, $\delta_h = \xi \times \delta_v$, where ξ is the degree of anisotropy of heterogeneity.

The 3D embankment model is first loaded by applying geostatic loading to generate the total stress which is originated from a soil unit weight, and static analysis in the second step. The soil unit weight is also a variable constant in each simulation. However, using a strength reduction technique can obtain the safety factor for each realization (Griffiths and Lane 1999); Figure 3 shows the load factor as a function of the maximum overall mesh dimensionless displacement. The safety factor estimate is 1.4926 in this realization.

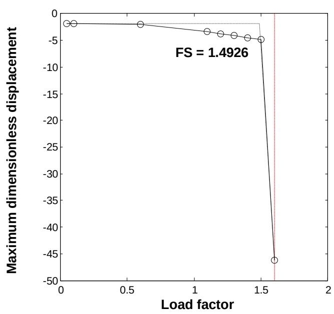

Fig. 3. The load factors versus the maximum mesh dimensionless displacements.

The stability analysis using the 3D RFEM requires intensive computational efforts. An alternative method, "random limit equilibrium method" (RLEM), is adopted to reduce the computational time.

5 COMPARISON RESULTS BETWEEN 3D RFEM AND 3D RLEM

The 3D RLEM is adopted to reduce the computational time of the 3D stability analysis. The comparison results are shown in Figure 4, 5, and 6, to verify the accuracy of the 3D RLEM (random field models in RFEM and RLEM are identical) for the safety factor estimates as well as the slide lengths and volumes. The geometries, soil properties including constant and spatial variability parameters for each simulation are selected randomly in the practical ranges in Table 1. The results show the 3D stability analyses using these two approaches are fairly consistent.

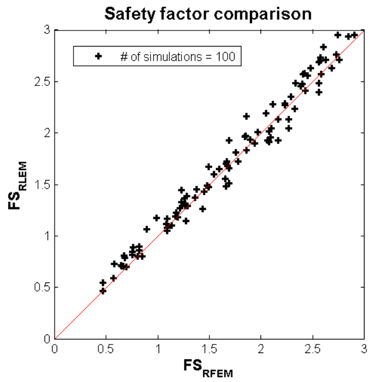

Fig. 4. The comparison result for safety factors estimates.

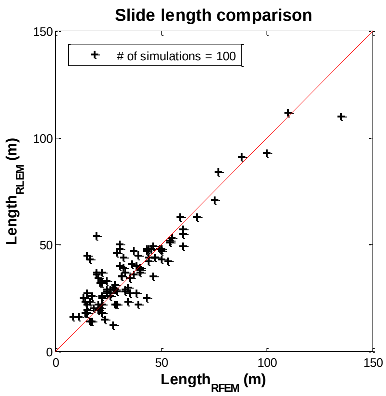

Fig. 5. The comparison result for slide lengths.

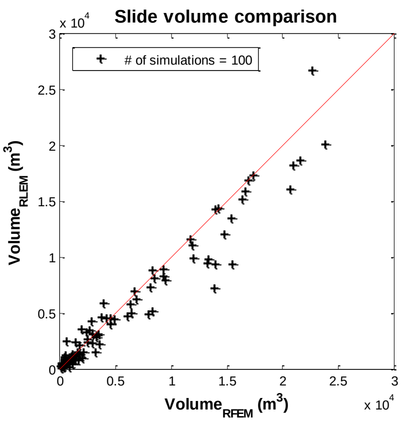

Fig. 6. The comparison result for slide volumes.

Table 1. The ranges of the geometries and soil properties.

Parameter	Size
Length (m)	20 - 5000
Unit weight (kN/m^3)	14 - 20
Vertical SOF (m)	0.1 - 10
Ratio of horizontal SOF to vertical SOF	1 - 150
Mean c_u (kN/m^2)	20 - 100
COV of c_u	0.1 – 0.5
Slope angle (º)	10 - 40
Slope height (m)	1.2 - 15
Ratio of foundation layer thickness to slope height	2 - 6

6 CONCLUSION

The 3D stability analysis of a spatially variable embankment has been analyzed by both the 3D RFEM and 3D RLEM. The use of the 3D RLEM approach can reduce the computational time. The accuracy of the 3D RLEM is also verified by the 3D RFEM with identical random field models.

The fairly consistent results may be possibly attributed to two factors. First, the 3D RFEM is able to seek out the critical zone through a spatially variable embankment; the critical zone is typically approximated as a bowl-shaped surface observed from the practical frameworks of the 3D RFEM. The bowl-shaped surface in the 3D RLEM framework is assumed as a cylindrical surface with two power-curve ends, which may be approximately consistent with the critical zone in the 3D RFEM. Second, Fig. 4 shows a minimization principle in the 3D RLEM approach is reasonable. Although it is not considering the mechanical mechanism, the minimization principle in the 3D RLEM still shows a great overall agreement with the 3D RFEM.

The 3D stability analysis using the 3D RFEM is shown to offer many advantages; however, the approach requires computational efforts, particularly for a long embankment with a fine mesh. Hence, this investigation has highlighted reducing the computational time to evaluate the safety factor as well as slide length and volume of a spatially variable embankment is feasible.

REFERENCES

Au, S. K., and Beck, J. L. (2001). Estimation of small failure probabilities in high dimensions by subset simulation. *Probabilistic engineering mechanics*, *16*(4), 263-277.

Chen, R. H., and Chameau, J. L. (1983). Three-dimensional limit equilibrium analysis of slopes. *Geotechnique*, *33*(1), 31-40.

Cheng, Y. M., and Yip, C. J. (2007). Three-Dimensional asymmetrical slope stability analysis extension of Bishop's, Janbu's, and Morgenstern–Price's techniques. *Journal of geotechnical and geoenvironmental engineering*, *133*(12), 1544-1555.

Cho, S. E. (2007). Effects of spatial variability of soil properties on slope stability. *Engineering Geology*, *92*(3-4), 97-109.

Cho, S. E. (2009). Probabilistic assessment of slope stability that considers the spatial variability of soil properties. *Journal of geotechnical and geoenvironmental engineering*, *136*(7), 975-984.

El-Ramly, H., Morgenstern, N. R., and Cruden, D. M. (2002). Probabilistic slope stability analysis for practice. *Canadian Geotechnical Journal*, *39*(3), 665-683.

Gens, A., Hutchinson, J. N., and Cavounidis, S. (1988). Three-dimensional analysis of slides in cohesive soils. *Geotechnique*, *38*(1), 1-23.

Griffiths, D. V., and Fenton, G. A. (2004). Probabilistic slope stability analysis by finite elements. *Journal of Geotechnical and Geoenvironmental Engineering*, *130*(5), 507-518.

Hicks, M. A., and Samy, K. (2002). Influence of heterogeneity on undrained clay slope stability. *Quarterly Journal of Engineering Geology and Hydrogeology*, *35*(1), 41-49.

Hicks, M. A., and Spencer, W. A. (2010). Influence of heterogeneity on the reliability and failure of a long 3D slope. *Computers and Geotechnics*, *37*(7-8), 948-955.

Hicks, M. A., Nuttall, J. D., and Chen, J. (2014). Influence of heterogeneity on 3D slope reliability and failure consequence. *Computers and Geotechnics*, *61*, 198-208.

Jha, S. K., and Ching, J. (2013). Simulating spatial averages of stationary random field using the Fourier series method. *Journal of Engineering Mechanics*, *139*(5), 594-605.

Ji, J., and Chan, C. L. (2014). Long embankment failure accounting for longitudinal spatial variation–A probabilistic study. *Computers and Geotechnics*, *61*, 50-56.

Lam, L., and Fredlund, D. G. (1993). A general limit equilibrium model for three-dimensional slope stability analysis. *Canadian geotechnical journal*, *30*(6), 905-919.

Leshchinsky, D., and Huang, C. C. (1992). Generalized three-dimensional slope-stability analysis. *Journal of geotechnical engineering*, *118*(11), 1748-1764.

Li, K. S., and Lumb, P. (1987). Probabilistic design of slopes. *Canadian Geotechnical Journal*, *24*(4), 520-535.

Li, Y. J., Hicks, M. A., and Nuttall, J. D. (2015). Comparative analyses of slope reliability in 3D. *Engineering Geology*, *196*, 12-23.

Li, D. Q., Xiao, T., Cao, Z. J., Phoon, K. K., and Zhou, C. B. (2016). Efficient and consistent reliability analysis of soil slope stability using both limit equilibrium analysis and finite element analysis. *Applied Mathematical Modelling*, *40*(9-10), 5216-5229.

Vanmarcke, E. H. (1977a). Reliability of earth slopes, *Journal of Geotechnical Engineering Division*, 103(11), 1247-1265.

Vanmarcke, E. H. (1977b). Probabilistic modeling of soil profiles, *Journal of Geotechnical Engineering Division*, 103(11), 1227-1246.

Xiao, T., Li, D. Q., Cao, Z. J., Au, S. K., and Phoon, K. K. (2016). Three-dimensional slope reliability and risk assessment using auxiliary random finite element method. *Computers and Geotechnics*, *79*, 146-158

Xiao, T., Li, D. Q., Cao, Z. J., and Tang, X. S. (2017). Full probabilistic design of slopes in spatially variable soils using simplified reliability analysis method. *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, *11*(1), 146-159.