Coupled MPM/DEM multiscale modelling geotechnical problems involving large deformation

Weijian Liang[1] and J. Zhao[1*]

[1] Department of Civil and Environmental Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong. * Email: jzhao@ust.hk.

ABSTRACT

Accurate modeling of many geotechnical problems needs the consideration of large deformation soil behavior. The Material Point Method (MPM) has gained increasing popularity over many other conventional numerical methods such as Finite Element Method (FEM) in continuum modeling of large deformation problems. In this study, we present a novel coupling scheme combining MPM and DEM (Discrete Element Method) for multiscale modeling of large deformation geomechanics problems. A computational multiscale scheme based on hierarchical coupling of MPM and DEM is proposed, based on the similar concept of FEM-DEM coupling proposed previously by the authors (Guo and Zhao, 2014, 2016a, 2016b). In this scheme, the MPM is employed to treat a typical boundary value problem in geotechnical problem that may experience large deformations, and the DEM is used to derive the nonlinear material response required by MPM for each of its material points. The proposed coupling framework helps avoid phenomenological constitutive assumptions in typical MPM, while inherits its advantageous features in tackling large deformation problems over the use of FEM (e.g., no need for re-meshing to avoid highly distorted mesh in FEM). It offers the capability of direct micro-macro linking for us to understand complicated behavioral changes of granular media over all deformation levels, from the initial elastic stage en route to the large deformation regime before failure. Demonstrative examples, including the cyclic loading of granular media and the footing foundation problem, are shown to highlight the advantages of the new MPM-DEM framework.

Keywords: hierarchical multiscale approach; MPM; DEM; large deformation

1 INTRODUCTION

It is challenging to model a wide range of geotechnical problems involving large deformations, including foundation settlement, pile installation, slope instability and landslides. Modeling the large deformation by the standard Finite element method (FEM) may suffer well-known issues of severe mesh distortion and computational stability (Zhang et al., 2013). Popular remedies for FEM-based approaches include the adaptive mesh and re-meshing, but may create further issues such as fluctuations appearing in load-deformation relation (Hu and Randolph, 1998). The Material Point Method (MPM) (Sulsky et al., 1994, 1995; Bardenhagen and Kober, 2004; Nairn, 2003; Soga et al., 2016) has gained increasing popularity recently over many other conventional numerical methods such as FEM in treating large deformation problems. MPM is indeed a combination of Lagrangian-Eulerian method. It discretizes a continuum domain by a collection of Lagrangian material points (particles) that carry essential state variables, and solves their movement based on a background Eulerian mesh using Eulerian approaches (Bardenhagen et al., 2000). Such combination significantly extends its capabilities in modeling the large deformation of history-dependent materials such as soil. Being a continuum-based approach, however, MPM still needs the assumption of constitutive models to describe the mechanical behavior of modeled material, and its predictions depend crucially on the adopted constitutive models. Since the responses of granular materials (e.g., sand) are highly non-linear, the constitutive model for accurate description of soil behavior may become extraordinarily complex and frequently phenomenological.

In this study, we present an innovative, physically-based multiscale framework to model geotechnical problems involving large deformation. The proposed computational framework is based on a hierarchical coupling of MPM and DEM, following the same concept of FEM/DEM coupling proposed previously by the authors (Guo and Zhao, 2014, 2016a, 2016b). Within the multiscale framework, the MPM is employed to tackle typical geotechnical problems that may experience large deformations, while the DEM is used to derive the nonlinear material response required by MPM for each of its material points. The proposed coupling framework circumvents phenomenological constitutive assumptions in the conventional MPM, while inherits its advantage on tackling large deformation problems over the use of FEM (e.g., no need for re-meshing to avoid highly distorted mesh in

FEM). It offers the capability of direct micro-macro linking for us to understand complicated behavioral changes of granular media over all deformation levels, from the initial elastic stage en route to the large deformation regime before failure. Demonstrative examples, including a cyclic simple shear and a footing foundation problem, are shown to highlight the advantages of the newly proposed MPM-DEM framework.

2 COUPLING SCHEME

Focus here is placed on explanation of the hierarchical coupling scheme between MPM and DEM, while the respective detailed formula of MPM and DEM can be referred to Sulsky et al. (1994); Bardenhagen et al. (2000), and Cundall and Strack (1979); Oda (1982); Iwashita and Oda (1998), and will not be repeated here for concision. Detail of the MPM-DEM coupling scheme can also be found in Liang and Zhao (2018) and Zhao and Liang (2018).

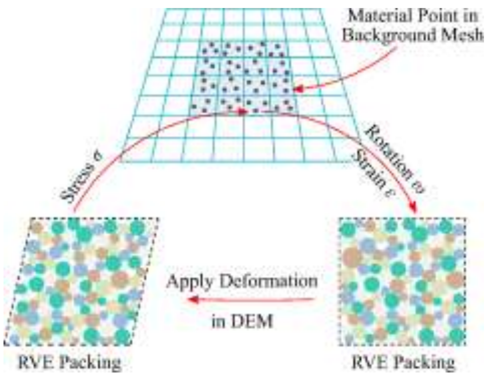

Fig.1. Illustration of the coupling scheme between MPM and DEM.

Fig. 1 shows a flowchart of proposed multiscale framework. The macroscopic continuum domain is initially discretized in MPM by a set of material points. Assemblies of granular particles are generated and assigned to material points in MPM as Representative Volume Elements (RVEs). Depending on the specific problem, assigned RVEs can be either identical or variable, representing a homogeneous or inhomogeneous soil domain. At each loading step, a typical coupling cycle comprises the following procedures: (a) MPM is firstly employed to derive the deformation for each material point in the macroscopic scale, (b) the deformation information (e.g., strain, rotation) at each material point is transferred to its corresponding RVE and serves as prescribed boundary conditions, (c) DEM is invoked to solve the granular assembly of each RVE at the corresponding boundary conditions, (d) an updated Cauchy stress is homogenized over the whole deformed RVE configuration and is transferred back to its attached material point in MPM, (e) MPM uses this updated stress to solve the momentum equation in background mesh and update the motion of each material point.

In doing so, no constitutive models as essential to the conventional continuum modeling are needed, and the history of each material point (microstructures) can be fully stored in its corresponding RVE, which is crucial for faithfully reproducing the highly nonlinear, loading-path dependent responses of granular media.

3 DEMOSTRATURED EXAMPLES

3.1 Cyclic simple shear

The first demonstrative example is the undrained (constant volume) cyclic simple shear that is controlled by a maximum shear strain $\gamma_{yx}^{max} = 1\%$. The model setup is depicted in Fig. 2, including 200 elements with 4 PPC (particle per cell). The RVEs attached to the material points are prepared using following parameters: particle number (in each RVE) $N = 400$, particle radius $r = 3{\sim}7$ mm, density $\rho = 2650\text{kg}/m^3$,Young's modulus $E = 600\text{MPa}$, dimensionless stiffness ratio $\nu = 0.8$, interparticle friction coefficient $\varphi = 28.6°$ and damping $\alpha = 0.1$. The initial void ratio (2D) of the RVE $e_0 = 0.229$.

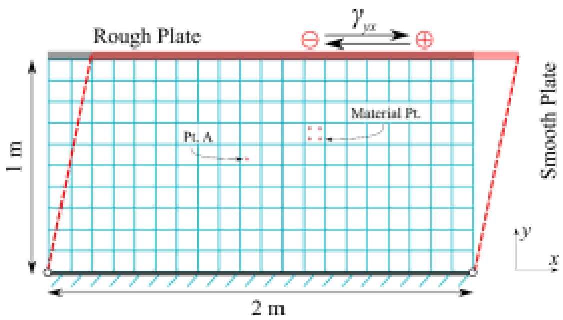

Fig.2. Model setup for cyclic simple shear. A material point marked as Pt. A is selected for meso-scale analyses.

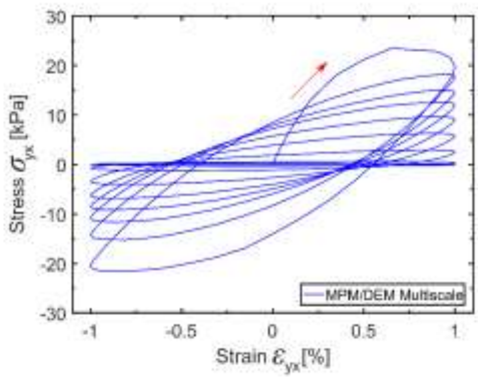

Fig.3. Shear strain versus shear stress in cyclic simple shear.

The strain-stress response obtained by our multiscale simulation is shown in Fig.3, where a hysteresis loop is clearly observed. As the number of cycle increases, the shear modulus decreases gradually, and the shear strength of the soil drops to an extremely low value after around eight cycles, entering a state which can be viewed as liquefaction. This observation is further confirmed by the force chain network of the select point as shown in Fig. 4. The contacts inside the RVE at the liquefaction state are so weak (~1N) and scattered to hardly form any effective force chain to sustain the external load, which is in great contrast to its original state wherein the force chain is strong (~100N) and dense (Fig.4a).

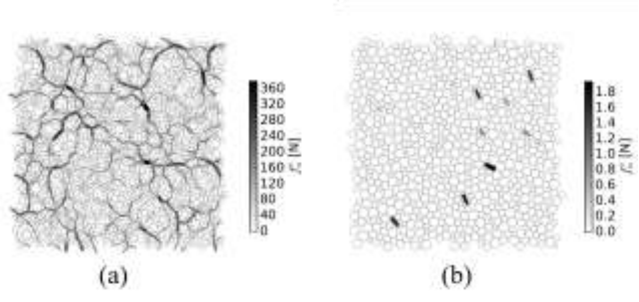

Fig.4. Force chain of selected RVE (a) before loading, (b) after loading. The position of selected RVE is shown in Fig.2.

3.2 Rigid footing

The second example treated by the proposed multiscale modeling approach is a rigid footing problem, as shown in Fig.5. The whole soil domain is discretized into 7776 elements with 1 PPC (particle per cell). A dense RVE packing is generated using the following microscopic parameters: the particle number $N = 400$, particle radius $r = 3\sim7$ mm, density $\rho = 2650\text{kg}/m^3$, Young's modulus $E = 800\text{MPa}$, dimensionless stiffness ratio $\nu = 0.5$, interparticle friction coefficient $\varphi = 23°$, rolling resistance coefficient $\beta = 1$ and damping $\alpha = 0.1$. Note that the rolling resistance is also considered in the DEM modeling to account the influence of particle shape on soil responses. The macroscopic friction angle for the RVE is estimated though a series of biaxial compression test, and the estimated value is $\varphi' = 28.1°$. The initial void ratio $e_0 = 0.187$.

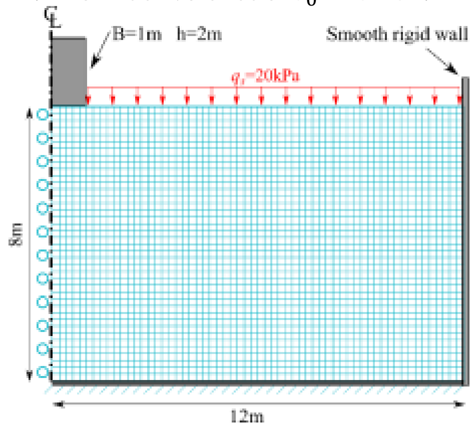

Fig.5 Model setup for rigid footing problem.

In the simulation, a rough footing is penetrated into the soil with a large displacement (e.g., d/B=1.5 where d denotes the penetration depth). Fig. 6 shows the relation of normalized settlement with the normalized ultimate loads, in comparison with the analytical solution deriving from following formula (Vesić, 1973):

$$N_q = tan^2\left(\frac{\pi}{4} + \frac{\varphi'}{2}\right)e^{\pi tan\varphi'} \qquad (1)$$

where φ' is the effective friction angle of the soil. Evidently, the peak load predicted by the multiscale approach agrees well with the analytical prediction.

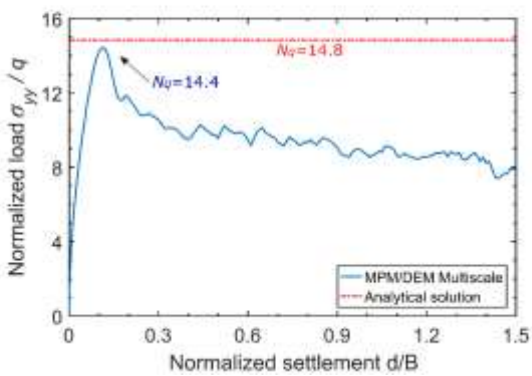

Fig.6 Comparison of the bearing capacity from current study and the analytical solution.

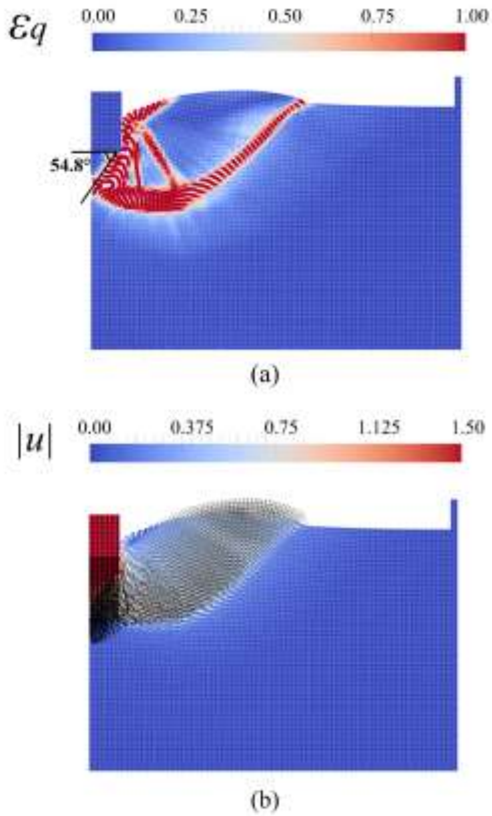

Fig. 7 Contour of (a) deviatoric strain and (b) displacement at final state *d/B*=1.5.

Fig. 7 further shows the deformation patterns in the soil after the footing penetration, in terms of contour of deviatoric strain ε_q and displacement u. From the Fig.

7(a), a clear general shear failure pattern can be observed, wherein a dominated slip surface originates from the tip of a triangular wedge immediately beneath the foundation and extends to the ground surface, intercepting several orthogonal shear bands that starting at the corner of the foundation. While the soil inside the triangular wedge is pushed downward as a rigid body, the soil mass above the major slip surface is mobilized sideward and upward, forming an appreciable heave at the ground surface, as shown in Fig. 7(b).

CONCLUSION

A novel MPM/DEM multiscale framework has been proposed for modeling geotechnical problems involving large deformation. The framework features a rigorous coupling of MPM and DEM, where the MPM is employed to tackle the large deformation at macro-scale of a practical boundary value problem, while the mechanical response needed by the MPM is obtained by DEM through a meso-scale assembly attached to each material point. Two typical geotechnical problems, cyclic simple shear and rigid footing were chosen to demonstrate the predictive capability of proposed multiscale framework in capturing complex soil behavior, such as strain localization and liquefaction, and its advantages in modeling large deformation.

ACKNOWLEDGEMENTS

The authors acknowledge financial supports from National Science Foundation of China under Project No. 51679207, Research Grants Council of Hong Kong under GRF Project N. 16210017, a Theme-based Research Project No. T22-603/15N and a Collaborative Research Fund Project No. C6012-15G.

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