Improvement in coupled DEM-LBM model for internal erosion of geomaterials with a broad particle size distribution

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ABSTRACT

This study presents an improvement in coupled particle-fluid numerical simulation model for the internal erosion of granular soils with a broad particle size distribution in order to investigate the mechanisms of the motion of fine soil particles inside the matrix of coarse soil particles from a microscopic point of view. In the model, the discrete element method (DEM) and the lattice Boltzmann method (LBM) are employed for the soil particles and for the seepage flow, respectively. On the basis of the coupled DEM-LBM model, two types of coupling schemes are newly combined. The coarse soil particles are simulated by the microscale-coupled method where the fluid flow is modeled at smaller scales than the soil particle diameter, while the fine soil particles are simulated by the macroscale-coupled method where the fluid flow is modeled at larger scales than the soil particle diameter. 3-D simulations of suffusion are performed, and the applicability of the proposed model to the internal erosion of granular soils is validated.

Keywords: internal erosion; granular materials; coupled particle-fluid simulation; DEM; LBM

1 INTRODUCTION

The internal erosion of soils is described as the process in which the soil particles are transported by the hydrodynamic force of the seepage flow. It is well known that internal erosion often contributes to serious geohazards, such as the breaching of levees, the collapse of dam bodies, and sinkholes in roads (Foster et al. 2000). Bonelli (2013) reported that internal erosion is classified into four types according to the physical process: backward piping erosion, concentrated leak erosion, suffusion, and contact erosion. Among these types of erosion, suffusion and contact erosion occur when both coarse soil particles and fine soil particles exist in the same ground. In other words, these types of internal erosion are associated with a broad particle size distribution of the soils.

In this study, the focus is placed on the internal erosion of granular soils with a broad particle size distribution, and a coupled particle-fluid numerical simulation model dedicated to such a phenomenon is proposed. It is important to develop a numerical scheme because it is difficult to experimentally observe the motion of fine soil particles inside a matrix of coarse soil particles. In the proposed model, the discrete element method (DEM) and the lattice Boltzmann method (LBM) are employed for the soil particles and for the seepage flow, respectively. On the basis of the coupled DEM-LBM model, which has been investigated in our previous research (Fukumoto et al. 2017), two types of coupling schemes are newly combined. The coarse soil particles are simulated by the microscale-coupled method where the fluid flow is modeled at smaller scales than the soil particle diameter, while the fine soil particles are simulated by the macroscale-coupled method where the fluid flow is modeled at larger scales than the soil particle diameter. 3-D simulations of suffusion are performed, and the applicability of the proposed model to the internal erosion of granular soils is validated.

2 OUTLINE OF SIMULATION MODEL

2.1 LBM for Seepage flow

The seepage flow is simulated by employing the LBM. The LBM is one of the computational fluid dynamics (CFD) methods and an alternative to the Navier-Stokes (N-S) equations. The LBM does not solve the N-S equations directly, but solves the kinetic gas theory, which can simulate the fluid flows by tracking the evolution of the density distribution of the virtual fluid particles. Since the relevant literature is affluent (e.g., Qian et al. 1995), only the essential points of the method are outlined below.

The solution of the LBM is governed by the following kinetic equation:

$$f_\alpha\left(\mathbf{x}+\mathbf{c}_\alpha\delta_t, t+\delta_t\right)-f_\alpha(\mathbf{x},t) = \Omega_\alpha(\mathbf{x},t), \tag{1}$$

where f_α is the α-th component of the density distribution function and $\mathbf{c}_\alpha$ is the α-th component of the discrete velocity. The value for $\mathbf{x}$ is the position of the LB node which is being calculated, t is the time, and δ_t is the discrete time. f_α defines the proportion of virtual fluid particles which move with velocity $\mathbf{c}_\alpha$ in the α-th direction of the LB node located at position $\mathbf{x}$ at time t. The right-hand term for Ω_α indicates the α-th component of the collision operator. α is the number of discrete velocities and depends on the choice of the model for the velocity moment. Considering precision and numerical efficiency, the D3Q19 model for three dimensions are usually used.

In the D3Q19 model, on the other hand, the discrete velocities are defined as $\mathbf{c}_\alpha$ = (0, 0, 0), ($\pm c$, 0, 0), (0, $\pm c$, 0), (0, 0, $\pm c$), ($\pm c$, $\pm c$, 0), ($\pm c$, 0, $\pm c$), and (0, $\pm c$, $\pm c$) for α = 0-18. For the single relaxation time (SRT) model (Qian et al. 1995), Ω_α is given as

$$\Omega_\alpha(\mathbf{x},t) = -\tau^{-1}\left(f_\alpha(\mathbf{x},t) - f_\alpha^{eq}(\mathbf{x},t)\right) , \quad (2)$$

where τ is a relaxation time coefficient and $\mathbf{f}_\alpha^{eq}$ is the α-th component of the equilibrium distribution function. Functions $\mathbf{f}_\alpha^{eq}$ are calculated by

$$f_\alpha^{eq} = \omega_\alpha \rho \left\{1 + \frac{3(\mathbf{c}_\alpha \cdot \mathbf{u})}{c^2} + \frac{9(\mathbf{c}_\alpha \cdot \mathbf{u})^2}{2c^4} - \frac{3(\mathbf{u}\cdot\mathbf{u})}{2c^2}\right\}, \quad (3)$$

where ω_α = 4/9 for α = 0, ω_α = 1/9 for $\alpha \in$ [1, 4], and ω_α = 1/36 for $\alpha \in$ [5, 8] in the D2Q9 model, and ω_α = 1/3 for α = 0, ω_α = 1/18 for $\alpha \in$ [1, 6], and ω_α = 1/36 for $\alpha \in$ [7, 18] in the D3Q19 model. The fluid velocity at the LB node is given by vector $\mathbf{u}$, which is determined by $\rho\mathbf{u} = \sum_\alpha f_\alpha c_\alpha$ and where density ρ is given by $\rho = \sum_\alpha f_\alpha$. Pressure p is easily computed as a function of density using the speed of sound, c_s: $p = c_s^2\rho$, where c_s is defined as $c_s^2 = c^2/3$. In the LBM, the velocity and the pressure are available in local forms according to the above equations. Then, relaxation parameter τ is directly related to dynamic viscosity ν, as follows: $\nu = c^2\delta_t (\tau - 0.5)$. The value for τ also affects the numerical stability, and it is often chosen in the range of 0.5 to 1.0.

2.2 Coupling method for coarse soil particles

In order to perform the coupled particle-fluid simulations within the framework of the LBM, the partially saturated lattice Boltzmann model (Noble and Torczynski 1998) is used. The model allows for the momentum transfer inside the solid phase. Fig. 1 (a) presents a conceptual description of the coupled DEM-LBM for coarse soil particles, where the space of the grid for fluid is smaller than the diameter of the soil particles (Fukumoto et al. 2017; Okada et al. 2017). In this approach, the collision operator in the SRT model, as described in Eq. (2), is reformulated to account for additional parameter B and additional term Λ_α.

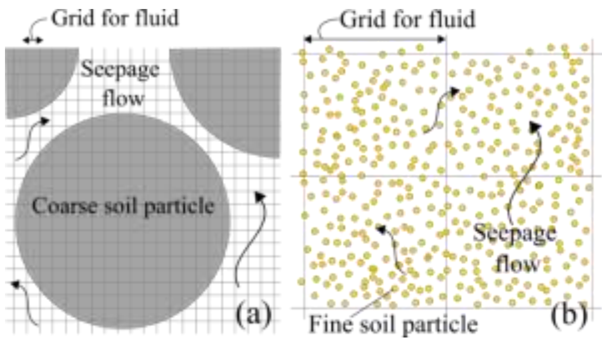

Fig. 1. Conceptual description of numerical model: (a) Coupling scheme for coarse particles and (b) Coupling scheme for fine particles.

$$\Omega_\alpha(\mathbf{x},t) = -\tau^{-1}\left(1 - B(\mathbf{x},t)\right)\left(f_\alpha(\mathbf{x},t) - f_\alpha^{eq}(\mathbf{x},t)\right) + B(\mathbf{x},t)\Lambda_\alpha ,$$

where B is given by

$$B(\mathbf{x},t) = \frac{\chi(\mathbf{x},t)(\tau - 0.5)}{\left(1-\chi(\mathbf{x},t)\right) + (\tau - 0.5)} . \quad (5)$$

Parameter $\chi(\mathbf{x}, t)$ is the volume fraction of the solid phase at each LB node. The value for χ varies between 0 for a completely fluid phase and 1 for a completely solid phase. Then, the value for Λ_α is calculated as follows:

$$\Lambda_\alpha = f_{-\alpha}(\mathbf{x},t) - f_{-\alpha}^{eq}(\rho,\mathbf{u}) + f_\alpha^{eq}(\rho,\mathbf{u}_p) - f_\alpha(\mathbf{x},t) \quad (6)$$

where $\mathbf{u}_p$ is the velocity of the soil particles including both the translation and the rotation motion of the corresponding soil particles. Notation $-\alpha$ is the opposite direction of α. According to the above equations, when $|\mathbf{u}_p| = 0$ and $\chi = 1$, the bounce-back rule, i.e., the non-slip condition at the solid-fluid boundary, is obtained.

Then, the hydrodynamic force and the hydrodynamic torque resulting from the seepage flow are needed for the motion of the soil particles. The values for hydrodynamic force $\mathbf{F}^{hyd}$ and hydrodynamic torque $\mathbf{T}^{hyd}$ are given by summing up the changes in momentum inside the solid phase.

$$\mathbf{F}^{hyd} = \frac{\delta_x^3}{\delta_t}\sum_n B_n\left(\sum_\alpha \Lambda_\alpha \mathbf{c}_\alpha\right) , \quad (7)$$

$$\mathbf{T}^{hyd} = \frac{\delta_x^3}{\delta_t}\sum_n\left\{(\mathbf{x}_n - \mathbf{x}_p) \times B_n\left(\sum_\alpha \Lambda_\alpha \mathbf{c}_\alpha\right)\right\}, \quad (8)$$

where n is the number of LB nodes belonging to the solid phase covered by a soil particle.

2.3 Coupling method for fine soil particles

When the above-mentioned microscale-coupled method is adopted for fine soil particles in the same manner as for coarse soil particles, the computational efficiency decreases because a large number of fluid grids is needed in cases where the grid space is smaller than the diameter of the soil particles. For this reason, different coupling schemes are employed in the proposed method according to the size of the soil particles. The fine soil particles are simulated by the macroscale-coupled method where the fluid flow is modeled at larger scales than the soil particle diameter (Wang et al. 2013), as shown in Fig. 1 (b). The hydrodynamic force for the fine soil particles is obtained as follows:

$$\mathbf{F}^{hyd} = \frac{\delta_x^3}{\delta_t} B_p \left(\sum_\alpha \Lambda_\alpha \mathbf{c}_\alpha \right) \quad , \tag{9}$$

where B_p is given by

$$B_p(\mathbf{x}, t) = \frac{\chi_p(\mathbf{x}, t)(\tau - 0.5)}{\left(1 - \chi_p(\mathbf{x}, t)\right) + (\tau - 0.5)} \quad . \tag{10}$$

Parameter $\chi_p(\mathbf{x}, t)$ is the volume fraction of the soil particle at each LB node. The value for B_p can be obtained in the same manner as Eq. (5). In this case, the hydrodynamic torque is assumed to be zero because the shear component of the hydrodynamic force acting on the soil particles cannot be considered.

2.4 DEM for motion of soil particles

The handling of the collision law and the motion of the coarse soil particles and the fine soil particles are presented in this subsection. The collision law for a contact force $\mathbf{F}^{con}$ and a contact torque $\mathbf{T}^{con}$ is governed by the DE method, where the contact logic is followed by the Voigt model (Cundall and Strack 1979). The normal repulsive force is assumed to be proportional to the overlap distance, and a dissipative component is set to be proportional to the relative normal velocity between particles. For the calculation of the tangential force, the Coulomb law of friction is considered in the same way as the normal force.

The equations of motion accounting for the interaction with the fluid flow are described by the following equations:

$$m_p \frac{d^2 \mathbf{x}_p}{dt^2} = \sum_{n_c} \mathbf{F}^{con} + \mathbf{F}^{hyd}, \tag{11}$$

$$I_p \frac{d^2 \boldsymbol{\theta}_p}{dt^2} = \sum_{n_c} \mathbf{T}^{con} + \mathbf{T}^{hyd}, \tag{12}$$

where m_p is the particle mass, $\mathbf{I}_p$ is the particle moment of inertia, and θ_p is the rotational displacement vector. The value for n_c is the number of contacts between soil particles.

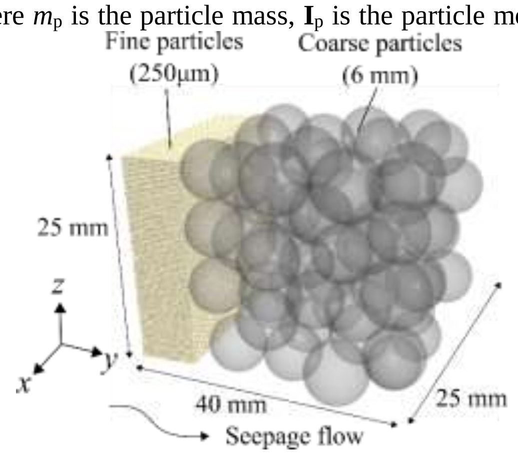

Fig. 2. Initial configuration of 3-D simulation model.

Eqs. (11) and (12) are solved by using the leap-frog algorithm as an explicit time integration method. The time difference for the time integration of the DEM, δ_t', is usually smaller than the value for δ_t, which is the time step for the LBM. Until the fluid flow is newly updated, the hydrodynamic force and the hydrodynamic torque acting on the soil particles are assumed to be the same values.

All of these calculations are performed on the graphic processing unit. The parallelized algorithm for the DEM (Nishiura and Sakaguchi 2011) and that for the LBM (Wang and Aoki 2011) are incorporated into our in-house code.

3 3D APPLICATION OF DEM-LBM MODEL

The three-dimensional simulation of suffusion is performed. The configuration of the analytical model is presented in Fig. 2, where the translucent grey particles indicate the coarse particles and the yellow particles indicate the fine particles. In the packing process of the coarse particles, only the calculation of the DEM was performed in the gravity field. The number of coarse particles is 63 and their diameter is 6 mm. On the other hand, the number of fine particles is 98,000 and their diameter is 250 μm.

The density of the soil particles, ρ_s, is 2500 kg/m^2 and the sliding friction between the immersed soil particles is 0.3. The contact springs are assumed to be linear and k_n/k_t = 4 (k_n = 5.0×10^5 N/m and k_t = 1.25×10^5 N/m), where k_n is the normal spring constant and k_t is the shear spring constant, respectively. The values for the normal and the tangential viscous damping, for inhibiting the numerical oscillation, are determined.

To generate the upward seepage flow resulting from the difference in pressure, the Zhou and He pressure boundary condition (Zhou and He 1997) is used in the boundaries of the yz plane. Four other boundaries in the

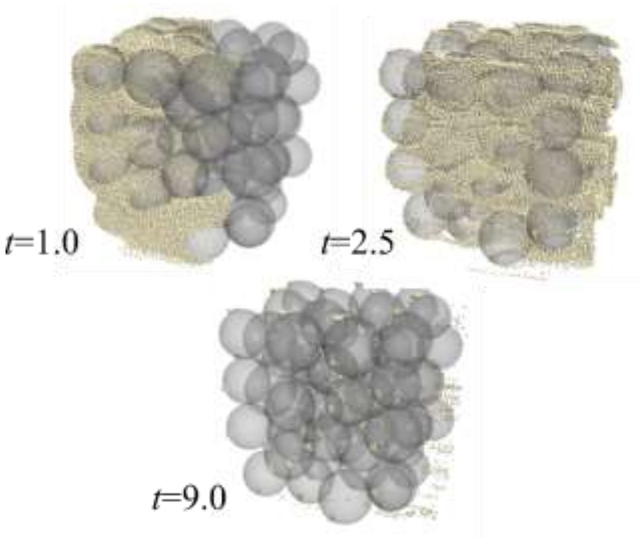

Fig. 3. Snapshots of 3-D simulation at t = 1.0, 2.5, and 9.0 s.

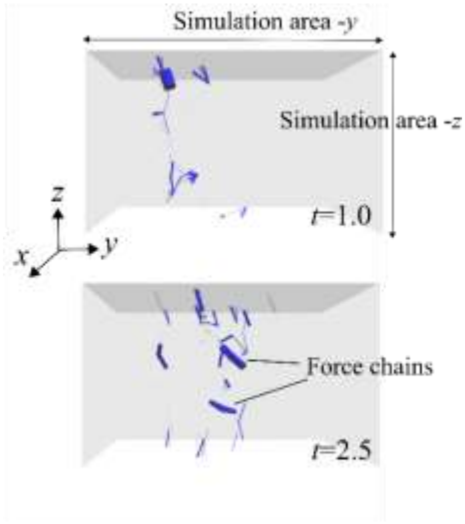

Fig. 4. Force chains of 3-D simulation at t = 1.0, 2.5s.

same direction as the seepage flow are subjected to the non-slip boundary condition by using the bounce-back scheme (Ziegler 1993). Lattice space δ_x is 5.0×10^{-4} m and the system size is 40×25×25 mm. Fluid density ρ_f is 1,000 kg/m^2, where the density ratio of the solid to the fluid is 2.5. The time step of the LBM for the fluid phase, δ_t, is 1.0×10^{-4} s and the time step of the DEM for the solid phase, δ_t', is 1.0×10^{-7} s. Relaxation time coefficient τ is 0.560, and the kinetic viscosity of the fluid, ν, is 5.0×10^{-5} m^2/s.

Fig. 3 presents snapshots of the results of the 3-D simulation at t = 1.0, 2.5, and 9.0 s. From the figure, the proposed model can reproduce the motion of the fine particles which are driven by the seepage flow. It is not necessary to make the artificial packing of the coarse soil particles in the 3-D case because the pores connect three-dimensionally with each other, unlike the 2-D case.

In addition to Fig. 3, the force chains between the fine soil particles at t = 1.0, 2.5 s are illustrated in Fig. 4 in order to see the details of the clogging. The blue cylinders indicate the force chains, and the diameter of the cylinder corresponds to the magnitude of the normal contact force. Such an observation, with respect to the contact networks, can be obtained when the soft sphere model such as the DEM is employed as a collision law, which allows the contacts between the soil particles to be retained. Otherwise, a hard sphere model cannot reproduce the evolution of the force chains.

It should be noted here that the number of total calculation steps is 90,000 for the LBM and 90,000,000 for the DEM. The computational time is about 100 hours for the physical time of 9 s.

4 CONCLUSION

In the present study, a coupled particle-fluid numerical simulation model for the internal erosion of granular soils with a broad particle size distribution has been proposed. The numerical model is based on the DEM-LBM and takes on different coupling schemes depending on the size of the soil particles for computational efficiency. The microscale-coupled method was employed for the coarse soil particles and the macro-coupled method was employed for the fine soil particles. As a result of the 3-D simulations of suffusion, it was demonstrated that there are future prospects in the application of the proposed model to various problems of internal erosion. In future studies, it will be necessary to compare the results of laboratory experiments with the numerical model.

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REFERENCES

Bonelli, S. (2013). Erosion in geomechanics applied to dams and levees. Wiley-ISTE.

Cundall, PA. and Strack, OD. (1979). A discrete numerical model for granular assemblies. Geotechnique, 29(1), 47-65.

Foster, M., Fell, R. and Spannagle, M. (2000). The statistics of embankment dam failures and accidents, Can. Geotech. J., 37, 1000–1024.

Fukumoto, Y., and Ohtsuka, S. (2017). 3-D direct numerical model for failure of non-cohesive granular soils with upward seepage flow. Computational Particle Mechanics, 1-12.

Nishiura, D. and Sakaguchi, H. (2011). Parallel-vector algorithms for particle simulations on shared-memory multiprocessors. J Comput Phys, 230(5), 1923-1938.

Noble, DR., Torczynski, JR. (1998). A lattice-Boltzmann method for partially saturated computational cells. Int J Mod Phys C, 9(08), 1189-1201.

Okada, H., Fukumoto, Y., Fujisawa, K., Murakami, A. (2017). Numerical simulation of soil erosion by particle-fluid coupled scheme, 15th International Conference of the International Association for Computer Methods and Advances in Geomechanics, 15.

Qian, Y. H., Succi, S. and Orszag, S. A. (1995). Recent advances in lattice Boltzmann computing, Annu. Rev. Comput. Phys., 3(195-242).

Wang, L., Zhang, B., Wang, X., Ge, W. and Li, J. (2013). Lattice Boltzmann based discrete simulation for gas–solid fluidization. Chemical engineering science, 101, 228-239.

Wang, X. and Aoki, T. (2011). Multi-GPU performance of incompressible flow computation by lattice Boltzmann method on GPU cluster. Parallel Comput, 37, 521-535.

Ziegler, DP. (1993). Boundary conditions for lattice Boltzmann simulations. J Stat Phys, 71(5-6), 1171-1177.

Zhou, Q. and He, X. (1997). On pressure and velocity boundary conditions for the lattice Boltzmann BGK model. Phys Fluids, 9(6), 1591-1598.