Influences of porosity and permeability on convective heat transport in saturated porous media

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ABSTRACT

This paper analyses heat transfer, considering both conduction and natural convection, within a non-Darcian fluid-filled porous medium. Following Boussinesq approximation, equation of heat transfer in porous media is coupled with Darcy-Brinkman-Forchheimer momentum equation and flow continuity equation. Temperature variations within the analysis domain is investigated for a range of values of absolute permeability and porosity of the porous medium. Results show that natural convection may play a significant role in heat transport within a porous medium. For the range of parameter values used in this study, the impact of absolute permeability on temperature distribution in the medium is more prominent than that of porosity of the medium.

Keywords: Heat transfer; conduction; natural convection; non-Darcian porous media; porosity; permeability

1 INTRODUCTION

Convective heat transfer through porous media is a topic of great interest in many scientific and engineering applications such as nuclear waste repositories, cooling of mechanical and electronic devices, thermal insulation of buried power cables, shallow and deep geothermal energy systems, solar energy collectors, food processing units, to name a few (Prasad 1987, Jiménez-Islas et al. 1999, Nithiarasu et al. 2000). Nield and Bejan (2017) and (Vafai 2010) discuss natural convective heat transfer in porous media and provide a wide range of practical examples in various field of engineering and technology. Torrance et al. (1969), Beukema et al. (1983), Greenspan and Schultz (1974), Haajizadeh et al. (1984), Anderson (1986) and Prasad (1987) were among the first to investigate natural convection in cavity filled with Darcian porous media. Since then several researchers investigated natural convection in Darcian porous media through experimental, numerical and semi-analytical studies. In all these studies involving a porous cavity, commonly referred to as *cavity problems*, heat transfer happens through the liquid phase only and the solid phase remains thermally inactive. Nonetheless, many researchers also included the participation of the solid phase also in heat transfer. Anderson (1986) performed a series of experiments to investigate natural convective heat transfer in a closed enclosure with bottom wall heated at a higher temperature and one side-wall at constant low temperature. Nicolas and Nansteel (1993) conducted laboratory experiments on a water-filled cavity with bottom wall partially heated and a single vertical wall at constant cold temperature; other boundaries were at adiabatic condition. Different researchers have studied natural convection in cavity filled with Darcian porous media for different boundary conditions and non-dimensional parameters and presented result in form of isotherms and streamlines (Murthy et al. 1997, Kumar et al. 1998, Velusamy et al. 1998, Baytas and Pop 1999, Baytaş 2000, Anand and Arora 2004, Varol et al. 2008, Wu et al. 2016b, a, Cheong et al. 2018). In all these studies, inertial effect, viscus shear and viscus drag effect were neglected. From previous studies, it is evident that for a given set of conditions, induced velocity field increases with Rayleigh number and convective component of heat transfer decrease with increasing aspect ratio. Hossain and Wilson (2002) investigated unsteady laminar free convection in a rectangular cavity with linearly varying temperature on one side-wall, isothermal top and bottom walls, respectively, at low and high temperature, and with low but constant temperature (same as that of the top wall) along the other side-wall; Prandtl number and Rayleigh number were kept constant for this study. Flow characteristics were analysed for internal heat generation and porosity parameters. An increase in secondary vortex was reported for increase in internal heat generation parameter. Furthermore, in the absence of heat generation, decrease in flow rate was reported as porosity parameter decreases. In a later study, Hossain et al. (2013) investigated the effect of conduction-radiation on free convection in non-Darcian porous medium with

Forchheimer-Brinkman-Darcy drag. Negative impact of Forchheimer drag on the flow pattern and heat transfer was reported. Effect of anisotropy in non-Darcian porous medium was studied by Krishna et al. (2008) and it was found that anisotropic properties have considerable impact of convective heat transfer behaviour. An extensive review of literature on free convective heat transfer through porous media of different shape other than square cavity was conducted by Das et al. (2017). Finite volume based numerical solution was obtained for natural convection in a cubical cavity. It was found that at higher convective heat transfer component, three dimensional effect is significant (Saravanan and Nayaki 2017).

The contribution of buoyant flow towards heat transfer in saturated porous media through convection intuitively depends on soil types and hydraulic properties of porous media (Diao et al. 2004, Chen et al. 2014, Ghasemi-Fare and Basu 2018). Following this, the objective of the present research is to evaluate the effect of a range of porosity and absolute permeability values on the contribution of convective heat transfer in fluid-saturated non-Darcian porous media. Although several literature suggest that absolute permeability is a function of porosity, the presents study considers these two parameter separately and independently.

2 DEFINITION AND FORMULATION OF THE PROBLEM

Two-dimensional free convective flow within a saturated porous media is investigated. Thermal properties of fluid, except the density term in the buoyancy force calculation, is assumed to remain constant within the temperature range of investigation. Fig. (1) shows the problem geometry and boundary conditions considered in this study. Top and right wall of cavity is kept isothermal at temperature T_c and the lower wall is kept at constant temperature T_h (where $T_h > T_c$); temperature on the left wall varies linearly from T_c at the top to T_h at the bottom. In order to avoid any numerical singularity at the junction of the right and the bottom wall, a linear temperature gradient (from T_h to T_c) is applied within a short distance $H/20$. Hydraulic boundary condition for all four walls is set as *no slip* condition meaning that flow through or along these boundaries is zero.

Following the stated assumptions, heat transfer equation consisting convection and conduction within the porous media can be expressed as

$$\left(\rho C_\rho\right)_{eff}\frac{\partial T}{\partial t}+\rho_f C_{\rho_f}\vec{v}\cdot\hat{\nabla}T=k_{eff}\hat{\nabla}^2T \qquad (1)$$

$$\left(\rho C_p\right)_{eff}=(1-\varepsilon)\rho_s C_{\rho s}+\varepsilon\rho_f C_{\rho f} \qquad (2)$$

$$k_{eff}=(1-\varepsilon)k_s+\varepsilon\rho_f k_f \qquad (3)$$

where ρ (kg/m^3) is mass density, C_ρ (J.kg.K^{-1}) is specific heat capacity, k (Wm^{-1}K^{-1}) is thermal conductivity, T (K) is temperature , t(s) is time and $\vec{v}$(ms^{-1}) is velocity vector and subscripts s and f are for soil and pore fluid respectively .

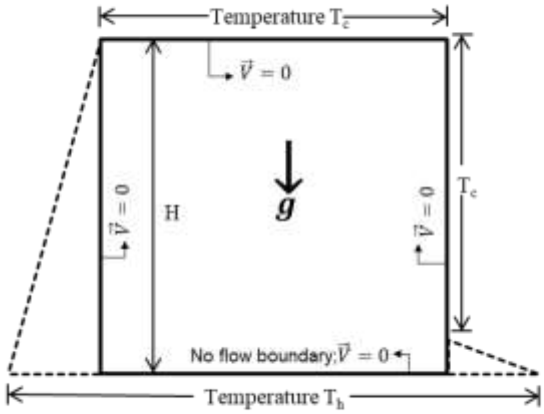

Fig. 1. Problem geometry and boundary conditions

Velocity vector $\vec{v}$ associated with convective heat transfer is calculated from Darcy-Brinkman-Forchheimer momentum equation (Equation 4) that also accounts for viscous drag and inertial forces. The momentum equation is solved simultaneously with continuity equation presented in Equation (7).

$$\begin{aligned}\frac{\rho_0}{\epsilon}\left[\frac{\partial\vec{v}}{\partial t}+\left(\vec{v}.\hat{\nabla}\right)\frac{\vec{v}}{\varepsilon}\right]&\\ =&-\hat{\nabla}P\\ &+\hat{\nabla}\left[\frac{\mu}{\varepsilon}\left\{\hat{\nabla}\vec{v}+\left(\hat{\nabla}\vec{v}\right)^T\right.\right.\\ &\left.\left.-\frac{2}{3}\left(\widehat{\nabla.\vec{v}}\right)I\right\}\right]-\frac{\mu}{K}\vec{v}\\ &-\frac{C_F\rho_0\epsilon}{K^{\frac{1}{2}}}|\vec{v}|\vec{v}+F_B\end{aligned} \qquad (4)$$

where ϵ is porosity of the porous media, μ(Pa.s) is dynamic viscosity of pore fluid, K(m^2) is absolute permeability of porous media, C_F is Forchheimer coefficient (Equation 8) and F_B is the body force. Following the Boussinesq's approximation, F_B is affected by temperature variations.

$$F_B=\begin{Bmatrix}0\\ \rho g\end{Bmatrix} \qquad (5)$$

and

$$\rho=\rho_0\big(1-\beta(T-T_c)\big) \qquad (6)$$

$$\hat{\nabla}\vec{v}=0 \qquad (7)$$

$$C_F=\frac{b}{\sqrt{a}\varepsilon^{3/2}} \qquad (8)$$

where $\beta(K^{-1})$ is coefficient of volumetric thermal expansion of pore fluid. The coefficients *a* and *b* are Ergun Constant (*a* = 215 and *b* = 1.92 for natural convection (Su and Davidson 2015).

A commercially available finite element (FE) software COMSOL Multiphysics™ (Comsol 2014) is used to obtain temperature field within the cavity through coupled solutions of Equations (1) , (4) and (7). Input parameters used in the FE analyses are listed in table (1).

Table 1 Input parameters used for simulation

Parameters	Value
Length scale of cavity	$H = 0.5$ m
Constant initial temperature	T_c =10°C
Constant high temperature	T_h= 15° C
Density of pore fluid	$\rho_0 = 1000$ kg/m^3
Dynamic viscosity of pore fluid	$\mu = 113 \times 10^{-5}$ Pa.s
volumetric thermal expansion coefficient of pore fluid	$\beta = 1.48 \times 10^{-4}$K^{-1}
Thermal conductivity of pore fluid	$k_f = 0.59$ W/(m.K)
Specific heat capacity of pore fluid	$C_{\rho f} = 4082$J/(kg.K)
Porosity of porous media	ε =0.3
Absolute permeability of porous media	$K = 1\times10^{-8}$ m^2
Thermal conductivity of soil skeleton	k_s=2.5 W/(m.K)
Specific heat capacity of soil skeleton	$C_{\rho s}$ = 1500 J/(kg.K)
Density of ground	2000 kg/m^3

3 RESULTS AND DISCUSSIONS

Analyses are performed for a wide range of values of absolute permeability *K* and porosity *ε*, as applicable for soils. For constant values of pore fluid properties and boundary conditions (i.e., for Pr = 7.9 and Ra = 5.5×10^9), variations of normalized temperature field *θ* (Equation 9) within the analysis domain is studied for different values of *K* and *ε*.

$$\theta = \frac{T - T_c}{T_h - T_c} \tag{9}$$

3.1 Effect of absolute permeability (*K*)

Fig. (2) shows variations, for *K* ranging from 10^{-7} m^2 to 10^{-10} m^2, of normalized temperature *θ* and temperature difference ΔT (=$T-T_c$) along the horizontal centreline (A-A′) of the domain. At low values of *K* (in the order of 10^{-9}~10^{-10} m^2), *θ* and ΔT values are indicative of mostly conductive heat transfer in the medium. However, a significant difference in medium temperature is observed as *K* value is increased up to the order of 10^{-8}~10^{-7} m^2. At higher values of *K*, thermally-induced velocity field becomes strong enough to trigger significant convective heat transfer in comparison to conductive heat transfer alone.

3.2 Effect of porosity (ε)

For a value of permeability ($K = 1\times10^{-8}$ m^2) that enables considerable convective heat transport in the porous medium, Fig. (3) demonstrates that the effect of porosity *ε* on temperature distribution along A-A′ is not prominent enough. It is well understood that *K* does not play a role in conductive heat transport, and even for a case involving pure conduction in a porous medium the role of *ε* on change in medium temperature is negligible (Fig. 3).

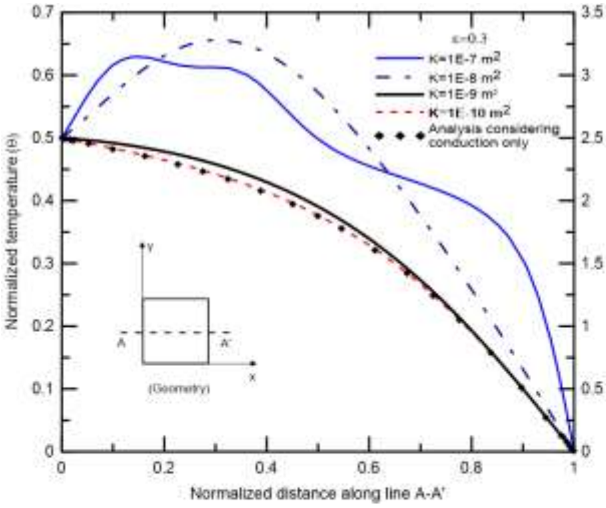

Fig. 2. Variation of temperature with absolute permeability of medium along line A-A'

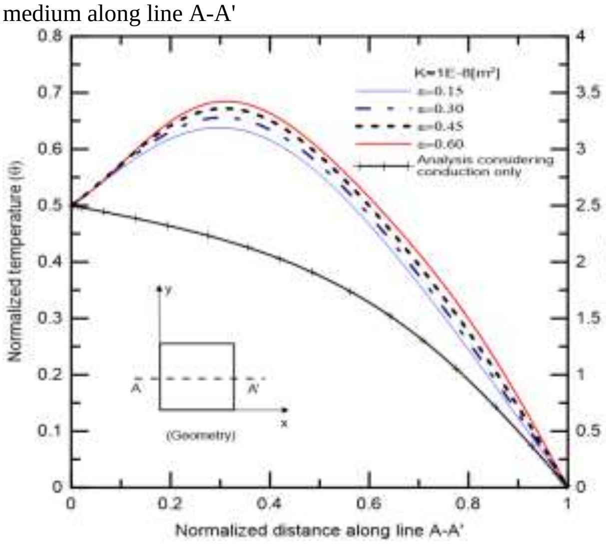

Fig 3. Variation of temperature with porosity of medium along line A-A'

4 CONCLUSION

The potential effects of absolute permeability and porosity on natural convection within a non-Dacian porous enclosure with non-isothermal walls is explored in this paper. A porous medium with non-isothermal temperature, no-flow condition at the boundaries, and with constant values of normalized heat transfer parameters (Pr=7.9 and Ra = 5.5×10^9) is considered for this study. Results from FE analyses with a wide range of permeability values (10^{-7} to 10^{-10} m^2) show that thermally-induced pore fluid flow plays a role only when K is in the order of 10^{-8}~10^{-7} m^2. As opposed to the effect of K on the contribution of natural convection towards heat transport within the porous medium, the change in temperature within the medium is quite insensitive to the change in the value of medium porosity.

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