Creep after unloading: A comparison and validation of constitutive models

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ABSTRACT

This paper evaluates a range of elasto-viscoplastic models with a particular focus on the unloading behavior. A strain rate approach is found to allow for an adequate evaluation and comparison between an empirical hypothesis A model and constitutive hypothesis B models. A simplified strain rate-based approach is proposed to predict creep after unloading based on simple field observations.

Keywords: creep, isotache, preloading, OCR, elastic-viscoplastic, strain rate dependence

1 INTRODUCTION

The prediction of creep of normally consolidated clays has been widely studied over the past decades but remains highly controversial. Two hypotheses, A (creep occurs only after primary compression) and B (creep occurs together with primary compression), representing two extreme views on creep were proposed by Ladd et al. (1977). To date, there is no general agreement on the validity of either hypothesis A or B. These diverging assumptions lead to fundamentally different types of numerical models which have significant effects on the modelled soil behavior after preloading. The lack of good quality long-term consolidation experiments further aggravates the uncertainty related to the applicability of either hypothesis for different stress paths and clay microstructures. In this paper, three general groups of models are evaluated: 1) empirical models, 2) isotache models, 3) more flexible strain rate models with a variation of the strain rate dependence.

2 MODELLING UNLOADING CREEP

2.1 Empirical model of Feng (1991)

Feng (1991) carried out a large range of oedometer tests with long term unloading and devised an empirical model fitting the results. Once the soil is unloaded, primary swelling occurs till t_{pr}, followed by a longer period of secondary swelling. Feng (1991) identified a point in time t_l where creep reappears. The creep strains develop gradually, leading to a non-linear relation in (e-log(t)). The tangent slope of the creep strains over time, C'_α, is observed to increase gradually over time, dependent on the applied stress ratio (R'_s=OCR-1). To predict the creep settlements after preloading, Feng (1991) proposed an empirical model with a set of design charts using the time of creep reappearance t_l and a set of secant slopes of the creep strains over time, C''_α which depend on the total time t and R'_s:

$$\varepsilon_{Feng(1991)}(t) = C''_\alpha\left(\frac{t}{t_l}, R'_s\right)\cdot\log\left(\frac{t}{t_l(t_{pr}, R'_s)}\right) \quad (1)$$

The model of Feng (1991) is a hypothesis A model: it assumes that all observed creep strains are directly scaled by the time for primary rebound, t_{pr}, following an H^2 scaling law.

The design curves for soft clays and silts as given in Terzaghi et al. (1996) are used for a reinterpretation of the data. From the formula, creep reappearance curves in (e-log(t)) are calculated. From these, the strain rate $\dot{e}$ ($\dot{e} = \Delta e/\Delta t$) can be calculated to achieve a better evaluation of the model. The resulting $(\mathrm{e}, \log(\dot{e}))$ curves are given in figure 1. The highly non-linear creep reappearance curves, characterized by a variable secant slope C''_α, linearizes almost perfectly in $(\mathrm{e}, \log(\dot{e}))$. Linear regression lines can be constructed through $(\mathrm{e}, \log(\dot{e}))$, characterized by the initial strain rate at reappearance of creep $\dot{\varepsilon}_{\mathrm{initial}}$ and its slope $\hat{C}_\alpha$. For a given OCR, the strain rate to strain relation is therefore constant in the empirical model, without explicitly assuming it. By varying the OCR, several regression lines can be derived: as the OCR increases, $\dot{\varepsilon}_{\mathrm{initial}}$ and $\hat{C}_\alpha$ are reduced. This will lead to a reduction in creep settlements after preloading. Given the good fit of the Feng (1991) model within a strain rate framework, and since the model is a direct fit on laboratory tests, the results are used as a benchmark of the behavior that the models should produce. If these best-fit parameters are normalized, $\hat{C}_\alpha/C_{\alpha,\mathrm{NC}}$ and $\dot{\varepsilon}_{\mathrm{initial}}/\dot{\varepsilon}_{\mathrm{NC},0}$, they follow a unique relation, as given in figure 2 (including other models discussed below).

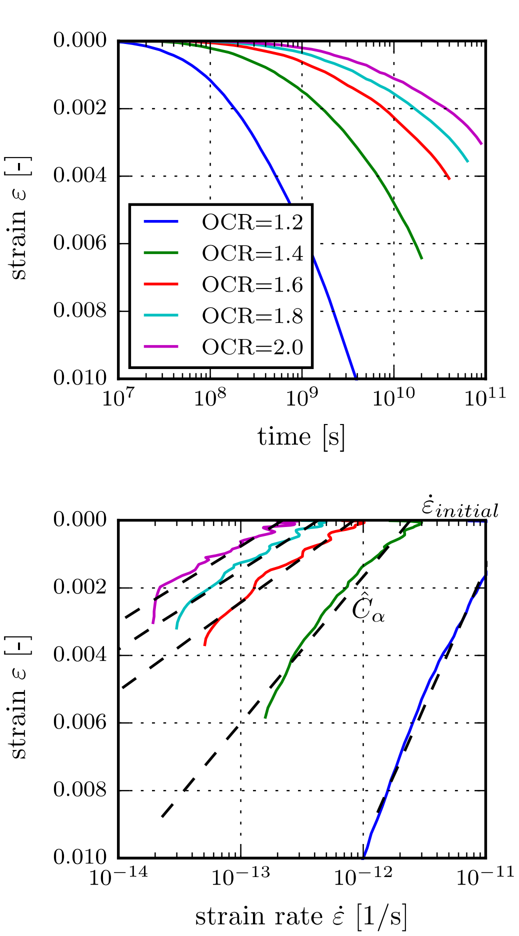

Fig. 1. Strain rate interpretation of the empirical model of Feng (1991).

2.2 Isotache model of Feng (2016)

The isotache approach is arguably the most widespread constitutive approach following hypothesis B. It assumes a fixed relationship between the strain, strain rate and effective stress, Leroueil (2006):

$$R(\varepsilon, \dot{\varepsilon}, \sigma) = 0$$

Yin and Graham (1989) proposed an implementation of the isotache model that uses equivalent time lines which represent lines of equal strain rate in (σ’,e). In the original implementation, the time lines are equally spaced which implies a linear reduction of the strain rate over time and, at least in a normally consolidated compression curve, linear creep strains in (e-log(t)).

Since the development of the isotache model of Yin and Graham (1989), many incremental steps were taken to develop it further. Feng (2016) presents the last step in this process with a focus on swelling of clays, a swelling limit and a creep limit. The model uses a set of equivalent time lines associated with creep, with a variable spacing, becoming smaller towards the creep limit, and a set of time lines for swelling up to the swelling limit. In between the swelling and creep limit, there is neutral zone of stress states (σ’,e), where the strain rate is zero: no creep, nor swelling would occur.

Once soil is unloaded, the effective stress reduces and the soil crosses equivalent time lines towards the creep limit. If the soil is unloaded before this limit is reached, the soil will exhibit reappearance of creep. If the soil is unloaded further to the neutral zone, no creep, nor swelling occurs. With further unloading, (i.e. higher OCR), the soil will enter the swelling region, exhibiting primary and thereafter a secondary swelling. The creep and swelling behavior are non-linear in this case, due to the unequally spaced time lines: with progressing creep, the rate of creep reduces at an increasingly fast rate up to the creep limit.

The standard isotache model follows a fixed strain rate reduction over time, $\hat{C}_\alpha / C_{\alpha,NC} = 1$. The reduction in creep deformation upon unloading is exclusively controlled by a reduction of $\dot{\varepsilon}_{initial}/\dot{\varepsilon}_{NC,0}(OCR)$. As the soil is unloaded, the stress path crosses equally spaced isotaches. The strain rate reduction is controlled by the well-known expression:

$$\frac{\dot{\varepsilon}_{initial}}{\dot{\varepsilon}_{NC,0}(OCR)} = \left(\frac{1}{OCR}\right)^{\frac{C_c - C_r}{C_\alpha}} \tag{2}$$

The strain rate reduction matches the empirical model of Feng (1991) well for low preloading, but starts diverging significantly at OCR = 1.4 and beyond. The initial strain rate implied by the isotache model is significantly lower than the observations, while $\hat{C}_\alpha$ is observed to be reduced in the empirical model.

If a creep limit is introduced, $\hat{C}_\alpha$ declines with increasing overconsolidation. This leads to isotaches that are non-linearly spaced, with a reduced spacing at greater distance from the reference time line. Considering the creep strains under constant load only, the strain rate in the model given in Feng (2016) is expressed as:

$$\dot{\varepsilon_z^c} = \frac{\psi_0^c}{V} \frac{1}{\left(1 + \frac{\psi_0^c}{\varepsilon_z^{cl} V} \cdot \ln \frac{t_0^c + t_e^c}{t_0^c}\right)^2} \cdot \frac{1}{t_0 + t_e} \tag{3}$$

with a creep parameter ψ_0^c similar to $C_{\alpha,NC}$, a creep limit strain ε_z^{cl} and the equivalent time t_e^c. Since the strain rate relation is highly non-linear, an analytical solution of $\hat{C}_\alpha$, i.e. the slope in $\log_{10}(\dot{e}) - e$, is not easily achieved. Therefore, a finite difference approximation is used to calculate $\hat{C}_\alpha$ after unloading.

$$\hat{C}_\alpha = \frac{\varepsilon_z^c(t_e + dt) - \varepsilon_z^c(t_e)}{\log_{10}\left(\frac{\dot{\varepsilon_z^c}(t_e + dt)}{\dot{\varepsilon_z^c}(t_e)}\right)} \tag{4}$$

These expressions allow to calculate both $\hat{C}_\alpha$ and $\dot{\varepsilon}_{initial}$ depending on the equivalent time t_e^c after unloading, which depends on $t_{e,NC}^c$ before unloading and the OCR. If $t_{e,NC}^c$ is higher, the stress path is closer to the creep limit and the reduction in creep will be even more significant.

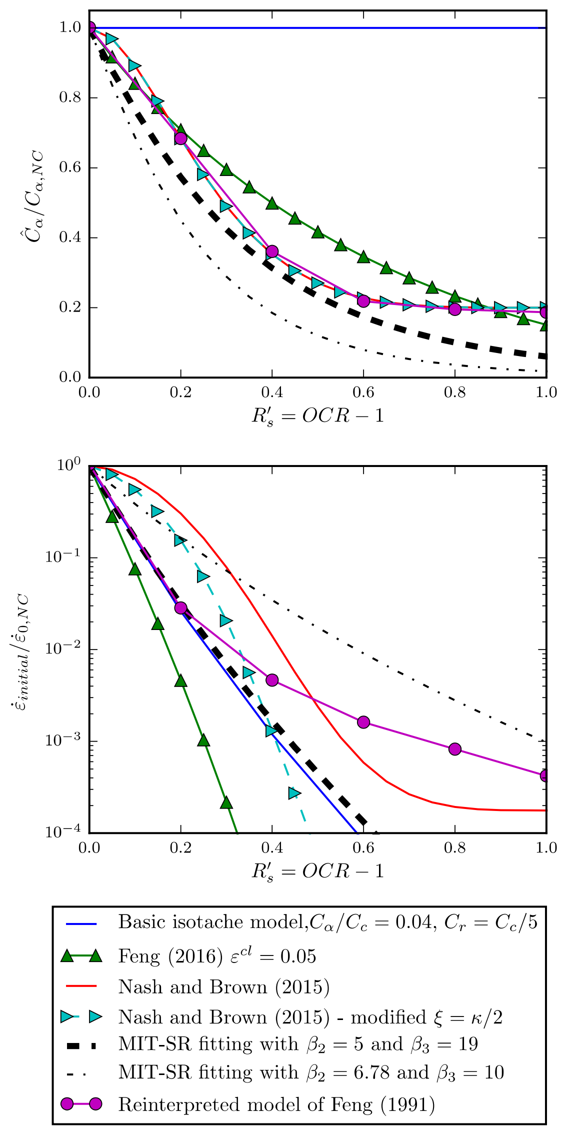

Fig. 2. Theoretical reduction of the initial strain rate $\dot{\varepsilon}_{initial}$ and strain rate evolution $\hat{C}_\alpha$ due to OCR

2.3 Destructuration effects in an isotache framework, Nash and Brown (2015)

If the soil behaves in accordance with a pure isotache model, the strain, strain rate and stress relations are fixed regardless of the stress path or stress history. This leads to some limitations in case of load responses that affect the isotaches: den Haan (1994) pointed out that, during unloading, soil structure is progressively disturbed, such that virgin behavior is regained only well past the preconsolidation pressure. The reload behavior would therefore be associated with distortion of the isotaches: high rate isotache lines associated with the NC soils are bent towards the OC void ratio, but the isotaches are more closely spaced. Nash and Brown (2015) proposed a model with modified isotaches to cater for this behavior. The model is an extension of the model of Yin and Graham (1989) and manipulates the isotaches in two ways. Firstly, the creep parameter is not a constant but a variable, leading to a smaller spacing of isotaches below the yield stress. Secondly, the Reference Time Line is changed to a bilinear system following κ before the preconsolidation pressure and λ afterwards.

Upon unloading, the isotaches are bent, leading to a higher initial strain rate $\dot{\varepsilon}_{initial}$ compared to the classical isotache model, while also reducing $\hat{C}_\alpha$. The transformation of isotaches is achieved by the introduction of a variable parameter ψ, similar to $\hat{C}_\alpha$ which allows a perfect fit. $\dot{\varepsilon}_{initial}$ can be calculated by considering the expression for the isotaches in the unloading zone:

$$e = e_0 - \kappa \cdot \ ln\left(\frac{\sigma'}{\sigma'_{z0}}\right) - \psi(OCR) \cdot \ ln\left(1 + \frac{t_e}{t_0}\right) \quad (5)$$

a given void ratio, t_e can be calculated. During unloading, we know that $e_{OC,initial} = e_{NC} + \kappa \ \ln(OCR)$ and e_{NC} is directly related to $t_{e,NC}$. This ultimately leads to an expression for the equivalent time after unloading, $t_{e,OC}$:

$$t_{e,oc} = \left[\exp\frac{\psi_{NC} ln\left(1 + \frac{t_{e,NC}}{t_0}\right) + (\kappa - \xi)\ln OCR}{\psi(OCR)} - 1\right] t_0 \quad (6)$$

with $\kappa = \xi$. The initial strain rate after unloading is then calculated as $\dot{\varepsilon}_{initial} = \frac{\psi(OCR)}{V} \cdot \frac{1}{t_0 + t_{e,OC}}$.

In the model of Nash and Brown (2015), the elastic response is parallel to the isotaches below the yield stress. This leads to a reduction in $\dot{\varepsilon}_{initial}$ controlled by $\hat{C}_\alpha$, and much less controlled by the stress path travelling through lower rate isotaches. It could be useful to allow for a bit more flexibility. Therefore, a modified model is introduced here and evaluated. This model a separate parameter for the elastic response, $\xi \leq \kappa$. The modified model allows for an improved fit of $\dot{\varepsilon}_{initial}$. However, the model remains very sensitive to the initial condition, $t_{e,NC}$, and as such there is no unique relation $\dot{\varepsilon}_{initial}/\dot{\varepsilon}_{NC,0}\,(OCR)$ according to this model. As the preload period increases, and therefore $t_{e,NC}$ increases, $\dot{\varepsilon}_{NC,0}$ will be reduced but at the same time, the ratio $\dot{\varepsilon}_{initial}/\dot{\varepsilon}_{NC,0}\,(OCR)$ will be reduced as well, for the same OCR. This is similar to the model described in Feng (2016). An increased preload period therefore improves the effect of preloading significantly in those models, beyond what is predicted in the empirical model of Feng (1991).

2.4 Internal strain rate approach: MIT-SR model

The controversy of hypothesis A versus B has led to the development of more flexible models to cater for behavior compatible with either one of the hypotheses. Yuan and Whittle (2018) developed the MIT-SR model, an elasto-viscoplastic model for clays that can behave within hypothesis A or B by selecting a rate sensitivity parameter β. The model uses an internal strain rate which can be controlled based on the loading history.

Based on the findings of Yuan et al. (2015), a distinct function to control the strain rate for unloading was introduced with an associated parameter β_3. Thanks to this modification, the model is able to behave generally as an isotache model, but with a more flexible unloading function which allows a modification of the isotache lines.

The MIT-SR model introduces two parameters, β_2 and β_3 to cater directly for changes in $\hat{C}_\alpha$ and $\dot{\varepsilon}_{initial}$ respectively. This is done by an internal strain rate approach: during unloading, the internal strain rate R_a is changed by a unique constitutive relation:

$$\frac{\dot{R}_a}{R_a} = (\beta_3 - 1) \cdot \frac{\dot{\sigma}'_v}{\sigma_v} \tag{8}$$

The visco-plastic strain rate is calculated from this internal strain rate, depending on the current stress state versus an equivalent preconsolidation pressure as:

$$\dot{\varepsilon}_{vp} = R_a \cdot \sqrt{\frac{\sigma'_v}{\sigma'_{pe}}} \exp\left[\frac{1-\left(\frac{\sigma'_{pe}}{\sigma'_v}\right)^{\beta_2}}{2\beta_2}\right] \tag{9}$$

The evolution of $\hat{C}_\alpha$ is determined by β_2 and can be approximated by, Yuan (2016):

$$\frac{\hat{C}_\alpha}{C_\alpha}(OCR) = \frac{2}{OCR^{\beta_2}+1} \tag{10}$$

The effect on the strain rate after unloading is two-fold: first there is a direct reduction due to the change in internal strain rate controlled by β_3 which has the most obvious and direct effect. A secondary effect from unloading in the MIT-SR model can be expected due to the β_2, which will lead to a slightly stronger reduction of the strain rate. This implies a reduction of:

$$\frac{\dot{\varepsilon}_{initial}}{\dot{\varepsilon}_{NC,0}}(OCR) = OCR^{-\beta_3} \exp\left[\frac{1-OCR^{\beta_2}}{2\beta_2}\right] \tag{11}$$

While the MIT-SR model allows for a non-linear strain rate effect, the relations $\hat{C}_\alpha/C_{\alpha,NC}(OCR)$ and $\dot{\varepsilon}_{initial}/\dot{\varepsilon}_{NC,0}(OCR)$ are constant with increasing preloading time. This is different from the isotache models discussed above and allows for a more direct comparison and fit to the empirical model of Feng (1991). As shown in figure 2, $\hat{C}_\alpha/C_{\alpha,NC}(OCR)$ can be modelled by the MIT-SR model over the full OCR range, while $\dot{\varepsilon}_{initial}/\dot{\varepsilon}_{NC,0}(OCR)$ can only be fitted well to the empirical model for low OCR (<1.3).

5 PRACTICAL ESTIMATION OF UNLOADING CREEP

The evaluation of EVP models shows a significant amount of variation in unloading behavior of the different models. Thanks to the consistency of the reinterpreted empirical model of Feng (1991), the models can be evaluated against a common basis. From this evaluation, it is found that the MIT-SR model allows for the closest approximation of the empirical results, especially for low OCR.

A direct approach can be used to adequately predict the unloading creep by using the reinterpreted empirical model proposed in this paper. Thanks to the relations $\hat{C}_\alpha/C_{\alpha,NC}(OCR)$ and $\dot{\varepsilon}_{initial}/\dot{\varepsilon}_{NC,0}(OCR)$, the soil behavior after unloading can be evaluated based on both lab-based measurements but also based on field measurements of $C_{\alpha,NC}$ and $\dot{\varepsilon}_{NC,0}$ during loading.

Hypothesis A versus B is still highly disputed, and the effect of consolidation time on unloading is even more uncertain since virtually all published tests are based on a 20mm drainage path. Based on the Feng (1991) model, an increase in the drainage path would result in an increase of t_l which leads to a reduction in the strain rate (as captured by other models) but also an increased delay before creep reappears. The delay in creep response t_l could be only apparent, caused by the reduced strain rate in e-log(t) curves, or t_l could not scale according to an H^2 law. The current unloading data on 20mm samples does not provide a definite answer on the scaling effect.

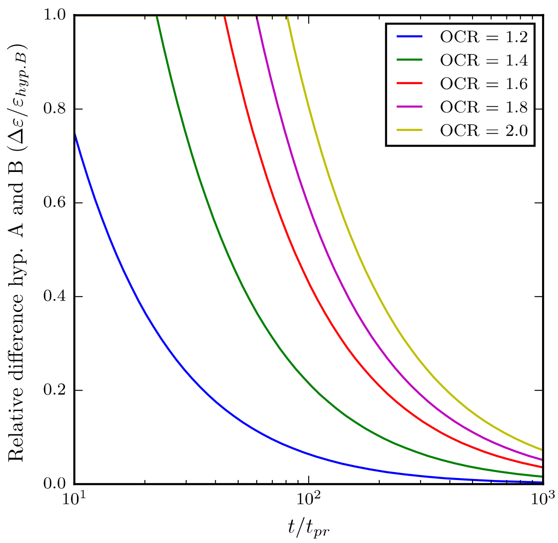

Fig. 3. Relative difference between hypothesis A and B for a fixed strain rate relation derived from the empirical model of Feng (1991)

This practical approach allows to cater for both hypothesis A or B predictions: the strain rate relation is integrated over time, starting at either t_l (hyp. A) or the primary rebound period t_{pr} (hyp. B). The resulting difference between hyp. A and B, following the same empirical strain rate relation is normalized by t_{pr} and can therefore be scaled to any drainage path, figure 3. It can be seen that the effect of the hypothesis is significant for high OCR at short time periods t/t_{pr} and becomes much less relevant at higher design times. Once the relevant creep period is 1000 times larger than t_{pr}, the difference is negligible.

While the original model was highly dependent on the rebound time t_{pr}, the alternative approach requires either the preloading strain rate or the preloading time. Both can be measured more directly. Since the design curves are reduced from an infinite set for the secant slope $C_\alpha^{"}$ curves to two fixed relations, the calculation process is also simplified. This reinterpreted strain rate approach allows for two additional considerations: firstly, the positive effect of preloading beyond t_p for quickly consolidating soil layers and secondly the approach can be made compatible with both hypothesis A or B.

6 CONCLUSION

A range of recent constitutive models are evaluated and compared to model creep behavior after unloading. While the models are fitted with parameters to render near-identical compression behavior during loading, the unloading creep behavior varied widely depending on the OCR applied.

A strain versus strain rate reinterpretation of the empirical model of Feng (1991) indicates that the relation $(\log(\dot{e}), e)$ is linear after unloading. This allows for a unification of the modelling: rather than a large set of design graphs, it was found that two relations in terms of $(\log(\dot{e}), e)$: $\hat{C}_\alpha/C_{\alpha,NC}(OCR)$ and $\dot{\varepsilon}_{initial}/\dot{\varepsilon}_{NC,0}(OCR)$ can represent the behavior adequately, in contrast to the $(\log(t), e)$-space where the behaviour is highly non-linear.

This reinforces the basic premise of the isotache models, that strain rate-based models are more adequate by avoiding the problem of the time origin. However, unloading behavior is characterized by changes in the isotache relation which are not fully consistent with the loading behavior. It is therefore necessary to utilize more flexible, non-linear strain rate-based models to fully describe the unloading creep behavior. A simplified method based on the empirical model is proposed.

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