Describing temperature and confining stress effects on frozen soils within THM framework

Satoshi Nishimura[1] and J. Wang[2]

[1] Faculty of Engineering, Hokkaido University, Kita 13 Nishi 8, Kita-ku, Sapporo, 060-8628, Japan.
[2] State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University,
8 Donghu South Road, Wuhan 430072, PR China.

ABSTRACT

The engineering responses of frozen soils are rationally described by considering thermo-hyrdo-mechanical (THM) coupled processes that are characterised by partial existence of ice in the pore spaces. This study proposes a framework in which a Critical State-based elastoplastic mechanical description of the soil skeleton at its bare minimum is combined with thermo-hydraulic equations to explain key features observed in saturated frozen soils, such as temperature- and confining stress-dependency of the strength. Different aspects of experimental results on the confining stress effects are explained by considering the phase equilibrium of water, stress/pressure sharing between the soil and pore materials with appropriate variables. The proposed framework allows formulating a full THM constitutive model of frozen soils that is seamlessly continuous to conventional unfrozen soil models.

Keywords: frozen soil; modelling; strength; temperature; THM

1 INTRODUCTION

Freezing and thawing of soils as mixture of different materials and phases involve extremely complex physical processes that abide by many equilibrium conditions. It has been long recognised that the freezing-induced (i.e. temperature-driven) mass transfer, sometimes culminating in frost heave, is only addressed by disentangling and solving each process (e.g. Shen and Ladanyi, 1987 among others). The studies of frost heave, however, have focused mostly on one-dimensional deformation and paid little attention to soil skeleton's mechanics. In contrast, modelling of frozen soils' deformation and strength conventionally assumed frozen soils to be a homogeneous elasto-(visco) plastic solid, to which the multi-phase concepts such as effective stress is irrelevant. Sophisticated mechanical models have been proposed in this approach; e.g. Lai et al. (2014). However, this 'total-stress approach' poses a barrier between frozen and unfrozen soil mechanics, making legitimate analysis of transient frozen- unfrozen boundaries difficult to perform. A new approach bridging the two streams of research is in need.

Some earlier studies were suggestive in defining soil skeletal mechanics for frozen soils within a multi-phase framework. Blanchard and Frémond (1985) suggested using the equivalent of Bishop stress in unsaturated soil mechanics in describing frozen soils by THM-coupled formulation. This approach has since been followed by several researchers. Ladanyi and Morel (1990) limited their scope on two-phase, two-material systems (i.e. soil particles and pore ice only), but successfully explained frozen sands' strength based on effective stress-based soil mechanics. Nishimura et al. (2009) went further by integrating a Critical State soil skeletal model into THM formulation by introducing a 'suction' variable, exploiting the analogy between *saturated frozen* soil and *unsaturated unfrozen* soils, in which the interfaces between the two phases (ice/liquid water and air/liquid water, respectively) in the pore engender matric suction. The model successfully simulated two-dimensional frost heave problems, while it is capable of qualitatively expressing temperature- and density- dependent strength. This is the model on which this study's view is fundamentally based on. However, to present a more flexible and robust framework, this paper tentatively discards many of the earlier model features such as frozen yield surfaces, hardening rule, etc., which are yet to be directly supported by experiments. Instead, this study focuses on the ultimate strength of frozen soils and how it is described under combined effects of temperature and confining stress.

2 CONSIDERED SYSTEM OF FROZEN SOIL AND DEFINITION OF VARIABLES

This study considers water-saturated soils, whose pores are occupied by a varying degree of ice and liquid water. The degree of liquid saturation, S_l, is defined as liquid water volume / pore volume. The effective stress, $\boldsymbol{\sigma}'$, is defined as the soil skeletal force per unit soil area. Following Ladanyi and Morel's (1990) idea, a unique relationship is assumed between the soil's effective stress and strain paths, irrespective of the pore space state (i.e. unfrozen or frozen). It follows that the effective stress-strain relationship obtained from conventional unfrozen tests is used for frozen states. A big question then is how to relate this effective stress to the total stress, $\boldsymbol{\sigma}$, in frozen soils. One approach is to assume a volume-weighed sharing of the stress between the ice and liquid water as;

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - S_l \mathbf{p}_l - (1 - S_l)\mathbf{p}_i \tag{1}$$

where $\mathbf{p}_l$ and $\mathbf{p}_i$ are the liquid and ice pressures (the shear stress components exist in $\mathbf{p}_i$). Considering their mean principal values p_l and p_i, the thermodynamic equibrium in form of Clausius-Clapeyron equation requires;

$$p_i = (\rho_i / \rho_l) p_l - \rho_i \ln\{(T + 273.15)/273.15\} \tag{2}$$

where ρ_l and ρ_i are the density of liquid water and of ice, respectively, l the specific latent heat of fusion, T the temperature in ºC. Another approach was taken by Ladanyi and Morel (1990), who implicitly envisaged stepwise transitions between completely frozen and unfrozen states (tolerable simplification for sands, in which S_l is small). This, combined with Eq.(2), leads to;

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \max(\mathbf{p}_i, \mathbf{p}_l) \tag{3}$$

The effective stress $\boldsymbol{\sigma}'$ thus defined is in fact identical to the 'net stress' adopted by Nishimura et al. (2009) as one of the stress variables in their mechanical model.

The difference between p_i and p_l, defined as s (=p_i - p_l), is a variable akin to the matric suction in unsaturated unfrozen soils. This variable reflects the interfacial stresses between the ice and liquid water, and causes additional resistance against deformation. Nishimura et al. (2009) demonstrated that s could be a convenient proxy of temperature in modelling, as will be revisited later. This variable s can therefore be used to also describe the influence of temperature on the pore ice's strength, which naturally contributes to the frozen soil's overall strength.

3 ULTIMATE STRENGTH OF FROZEN CLAY

The authors' group performed parallel series of undrained triaxial tests on reconstituted frozen and unfrozen Kasaoka clay specimens after identical isotropic consolidation. The clay was chosen for tests as it is of high-plasticity (PL=62% and LL=26%) and of clear elasto-plastic nature. The details of the material and test procedures are found in Wang et al. (2017). Well-defined standard Critical State parameter values were obtained from the 'unfrozen' series, as shown in Table 1. In the other, 'frozen' series, quick, undrained freezing was conducted from different states on the Normal Compression Line (NCL) under confining stress. As the unfrozen water content w_u – temperature T relationship had been obtained separately by Nuclear Magnetic Resonance (NMR), the expansive volumetric strain due to the pore water phase change could be calculated. According to the Ladanyi-Morel postulate discussed earlier, the effective stress resulting in the soil skeleton after freezing can be simply calculated from the void ratio increment Δe and κ. Similarly, the mean effective principal stress p' after shearing is calculated by referring to the Critical State Line (CSL). Fig. 1 shows the CSLs for the specimens at different temperatures shown in the $q - p'$ space, where q is the deviator stress (subscript cs represents Critical State). As the q is in total stress (obviously $q \neq q'$ in frozen soils), it includes the contribution from the ice and the ice-liquid interfaces. The CSL intercept and slope, defined q_f and M_f, respectively, were found broadly proportional to $|T|$ for $T \leq 0$ºC.

Table 1. Model input parameters determine for Kasaoka clay.

	Parameter	Symbol	
Soil skeleton	Critical State parameter	M	1.08
	1+e at p'=1 kPa on NCL	N	3.07
	1+e at p'=1 kPa on CSL	Γ	2.98
	Compression index (v.s. ln p')	λ	0.21
	Swelling index (v.s.ln p')	κ	0.048
Soil -water	v-G model parameter 1 (MPa)	P	0.10
	v-G model parameter 2	m	0.22
	Constant to define q_{ult} - p'_{ult}	a_q	0.135
	Constant to define q_{ult} - p'_{ult} (1/MPa)	a_M	0.00017
Water	Spefic heat of fusion (kJ/kg)	l	334
	Density of liquid water: Ref. state (kg/m^3)	ρ_l	1000
	Density of ice water: Ref. state (kg/m^3)	ρ_i	910
	Bulk modulus of liquid water (MPa)	K_l	2000
	Bulk modulus of ice water (MPa)	K_i	2000

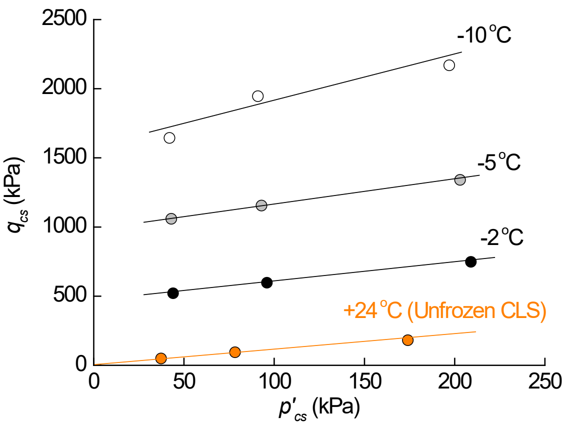

Fig. 1. Interpreted CSLs of Kasaoka clay at different temperatures in q - p' space (Wang et al., 2017)

4 MODELLING SOIL FREEZING AND CONFINING STRESS EFFECTS ON STRENGTH

4.1 Confining stress effects on frozen soil strength

The literature reports mixed findings on the confining stress effect on frozen soils' (mainly compressive) strength. The strength – confining stress envelopes have been reported in a variety of forms. In the authors' view, part of the reasons for this, among others, is that the 'confining stress' has different effects before and after freezing occurs, as frozen soils are undraining material under normal conditions. Confining stress changes after freezing lead to only marginal changes of density and effective stress. This fact is not necessarily given consideration clearly in many studies. It is clear from Fig. 1 that the increased confining stress, and hence the increased density/effective stress before freezing, has positive effects on the ultimate (CS) strength. On the other hand, Fig. 2 shows the strength of Kasaoka clay frozen from p'=400kPa with the back pressure of 200kPa, and subjected to total stress decreases from 600kPa to 400kPa and 200kPa. The strength is clearly insensitive to the total stress changes in this example. However, pressure-melting and a strength decrease at very high total stresses has also been commonly reported (e.g. Chamberlain et al., 1972). These two aspects at different stress levels must be logically explained.

4.2 Formulation of undrained isotropic freezing

It will be shown here that considering fundamental THM equations and adopting the CSLs shown in Fig. 2 along with the stress variables introduced in Section 2 rationally expresses freezing processes and the reviewed confining stress effects on frozen soils' strength. An undrained condition under isotropic stress states is assumed here to be consistent with the experiments on Kasaoka clay, but similar discussions holds even when volumetric changes are calculated as a result of seepage. The mass conservation requires;

$$e = e_0\{\rho_{l0}S_{l0} + \rho_{i0}(1-S_{l0})\}/\{\rho_l S_l + \rho_i(1-S_l)\} \quad (4)$$

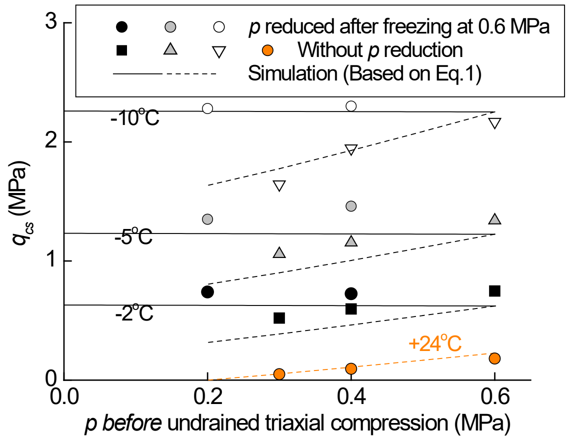

Fig. 2. Ultimate strength of frozen Kasaoka clay subjected to total stress decreases after freezing.

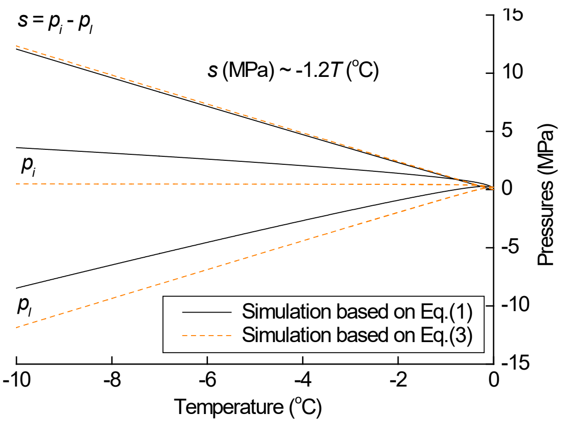

Fig. 3. Simulated freezing process: Pressure and suction changes.

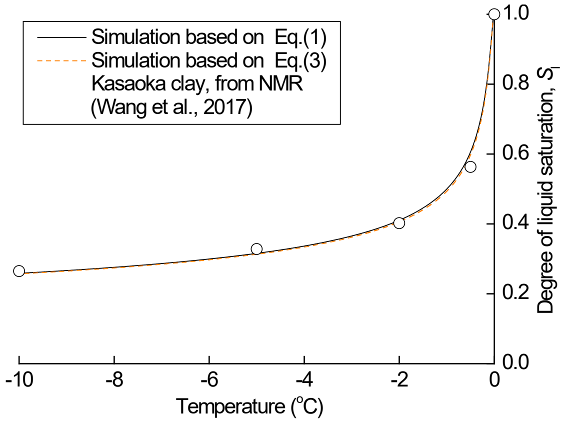

Fig. 4. Simulated freezing process: S_l reduction upon freezing.

where the subscripts 0 denotes an initial state. The van Genuchten (v-G) model is adopted here to express S_l as a function of the suction;

$$S_l = [1+\{(p_i - p_l)/P\}^{(1-m)/m}]^{-m} \quad (5)$$

where P and m are material constants. As the undrained freezing involves void expansion due to water phase changes (Eq.(3)), this leads to p' reduction along the unloading $e - \log p'$ line;

$$p' = p'_0 \exp\{-(e-e_0)/\kappa\} \quad (6)$$

There are five equations ((1) or (2), and (3)-(6)) for five unknown variables (S_l, e, p_i, p_l and p') and one controlling variable, T, in simulating freezing/thawing. Figs. 3 and 4 show calculated changes of p_i, p_l and s, and S_l changes against T, respectively. With appropriate input values determined from the triaxial tests and the NMR (Table 1), the experimentally obtained S_l is successfully modelled. Eqs.(1) and (3) are not making any significant difference in s and S_l, and other results shown later, while the different assumptions led to moderately different p_i and p_l.

4.3 Formulation of undrained confining stress changes and its effects on ultimate strength

Undrained confining stress changes after the freezing process involves partial sharing of the stress increments between the pore ice, pore liquid water and soil skeleton. From the compatibility;

$$\varepsilon = S_l n \varepsilon_l + (1 - S_l) n \varepsilon_i \tag{7}$$

where ε is the soil skeletal strain and ε_i and ε_l are strains of the ice and liquid water. Defining the bulk moduli for these phases as K_{ss}, K_i and K_l, and the porosity as n;

$$\Delta p' = \frac{nK_{ss}\rho_l}{\rho_i - \rho_l}\left[\frac{S_l}{K_l} + \frac{\rho_i}{\rho_l}\frac{(1-S_l)}{K_i}\right]\Delta s \tag{8}$$

$$K_{ss} = (1+e)p'/\kappa \ \text{ for } \ p' < p'_c \tag{9}$$

where p'_c is p' at yield. Upon yielding, κ is replaced by λ. Fig. 4 shows that s is approximately proportional to T via $s \approx -1.2T$ (MPa/°C). With this relationship and the data from Fig. 1, the CSLs are expressed as;

$$q_{cs} = M_f p'_{cs} + q_f = (M + a_M s) p'_{cs} + a_q s \tag{11}$$

where a_M and a_q are material parameters to express the convoluted effects of temperature on the ice strength and the ice-liquid interface resistance to deformation. For a frozen soil at given void ratio e, the Critical State theory dictates p'_{cs} as p' on the CSL defined by parameters in Table 1. From these relationships, the stress-paths and the ultimate (CS) strength are calculated as shown in Figs. 5 and 6, respectively. For a smaller p range, the results are shown in Fig. 2 along with the experimental data. The results indicate the strength being apparently insensitive to the stress over limited ranges, but pressure melting and hence the strength decrease at higher stress levels prevail. The latter effect (Fig. 6) is remarkably consistent with Chamberlain et al.'s (1972) data on West Lebanon Till. This pressure melting is successfully simulated by decreasing s, as shown in Fig. 5. As the figures indicate, adopting the variable s conveniently expresses the combined effects of temperature and confining stress.

5 CONCLUSIONS

This study proposed an approach to thermo-hydro-mechanical coupled formulation of frozen soil behaviour, in which the state-dependent (i.e. temperature- and confining stress-/density-dependent) ultimate strength is logically expressed within a simple framework adopting a matric suction-equivalent variable s and the Critical State theory. Undrained isotropic freezing and pre-/post-freezing stress changes are simulated as example of its capability to express the multi-physical, multi-phase processes. It was shown that the confining stress changes lead to different effects before and after freezing, and over different stress levels. The proposed framework successfully reproduced frozen soils' responses with minimum submodels. It will be relatively straightforward to include more elaborate description of frozen soil behaviour such as viscoplasticity on this platform.

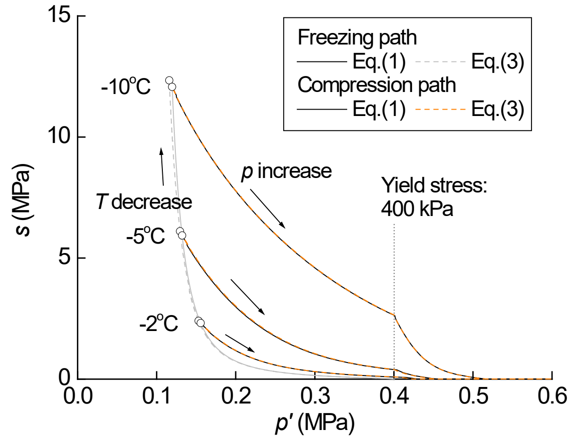

Fig. 5. Simulated paths for post-freezing isotropic compression.

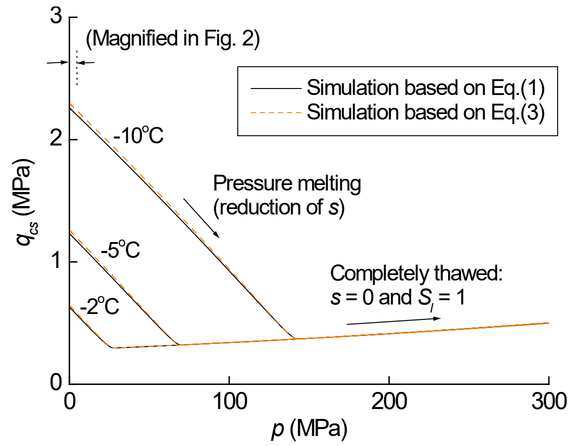

Fig. 6. Simulated post-freezing ultimate strength envelopes.

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