

## Mechanical performance of buried pipeline considering soil elastic modulus as a random field

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### ABSTRACT

The mechanical performance of buried pipeline is closely related to the properties of the surrounding soil, especially the elastic modulus of soil. It is well recognized that the mechanical parameters of soils exhibit spatial variability, and vary more significantly in the vertical direction than in the horizontal direction. In order to exhibit the impact brought by the spatial variability of soils to the displacement of pipeline, the random field is simulated with locally averaging method. The random field of elastic modulus is combined with the finite difference method (RFDM) for the precise analysis of the pipeline. The maximum principal stress and displacement of buried pipeline was studied in variable random fields of soil elastic modulus, with piling soil on the ground. Results show the relationship between the displacement and the maximum principal stress of the buried pipeline and the random field of soil elastic modulus under cases of different horizontal correlation distance.

**Keywords:** buried pipeline; random field theory; RFDM; correlation distance; elastic modulus

### 1 INTRODUCTION

In recent years, with the rapid development of urbanization, land has become a scarce source in many cities in China. Thus, the utilization of underground space has become an increasingly important hotspot and underground pipeline is widely used for transportation of oil and gas. According to Wang et al. (2018), there will be over 104,000 km of pipeline for natural gas transmission in China. Therefore, the safety of buried pipeline has been highly valued.

Underground pipeline, as shallow underground structure, is sensitive to the displacement of nearby soil, ground load, moving vehicle load, piling soil on the ground and so forth. In addition, the mechanical performance between buried pipeline and soil has been studied widely. Sun (2014) studied the mechanical interaction of pipeline under ground piling load. Li (2017) investigated soil stress distribution around the pipeline. Martin Magura (2016) studied the interaction between buried pipeline and soil in different loading cases. However, in traditional analyses and calculation methods, soil is considered homogeneous or is simply layered to model the differences of soil property. The variability of soil property is often neglected. In this paper, the variability of soil property is taken into account for analysis of mechanical performance of pipeline under certain loading cases via random field method.

The random field model was first established by Vanmarcke (1983) in the reliability analysis of geotechnical systems. Further, Griffiths (2004, 2009) combined RFEM and Monte-Carlo method to analyze the safety of slope engineering. Nonetheless, in the above-mentioned research, soil parameter is considered an isotropic random field model, which is quite different from the actual situation. Due to stress history, geological movement, sedimentation and so on, the variability of soil property in the vertical direction outweighs that in the horizontal direction (Xue et al.

2013). Hence, the anisotropy of soil parameters is supposed to be taken into account in a random field model.

In this paper, random field theory with locally averaging method is combined with Monte-Carlo method. A program is then developed based on MATLAB and Flac<sup>3D</sup> to analyze the mechanical performance of buried pipeline considering the anisotropy of soil.

### 2 RANDOM FIELD THEORY

#### 2.1 Three-Dimensional Random Field Theory

$X(t_1, t_2, t_3)$  is assumed as a three-dimensional continuous smooth random field with mean  $\mu$  and variance  $\sigma^2$ .  $V = T_1 T_2 T_3$  is a cuboid element of the random field, in which  $(t_1, t_2, t_3)$  is the position of the center. Therefore, the locally averaged value of the element  $V$  can be calculated as follows:

$$X_V(x, y, z) = \frac{1}{V} \int_{t_1-T_1/2}^{t_1+T_1/2} \int_{t_2-T_2/2}^{t_2+T_2/2} \int_{t_3-T_3/2}^{t_3+T_3/2} X(t_1, t_2, t_3) dt_1 dt_2 dt_3 \quad (1)$$

Thus, the mean  $E[X_V(t_1, t_2, t_3)]$  and variance  $Var[X_V]$  of the three-dimensional random field using locally averaging method can be obtained as follows:

$$E[X_V(t_1, t_2, t_3)] = \frac{1}{V} \int_{t_1-T_1/2}^{t_1+T_1/2} \int_{t_2-T_2/2}^{t_2+T_2/2} \int_{t_3-T_3/2}^{t_3+T_3/2} E[X(t_1, t_2, t_3)] dt_1 dt_2 dt_3 \quad (2)$$

$$Var[X_V] = \sigma^2 \gamma(T_1, T_2, T_3) \quad (3)$$

where,  $\gamma(T_1, T_2, T_3)$  represents the reduction function of variance.

As much, the covariance between two individual elements  $V$  and  $V'$  can be obtained from the following equation:

$$\begin{aligned} \text{Cov}[X_V, X_{V'}] \\ = \frac{1}{VV'} \frac{\sigma^2}{8} \sum_{j=0}^3 \sum_{k=0}^3 \sum_{l=0}^3 (-1)^{j+k+l} (T_{1j} T_{2k} T_{3l})^2 \gamma(T_{1j} T_{2k} T_{3l}) \end{aligned} \quad (4)$$

where,  $T_{1j}, T_{2k}, T_{3l}$  ( $j, k, l = 0 \sim 3$ ) are calculated as shown in Fig. 1.

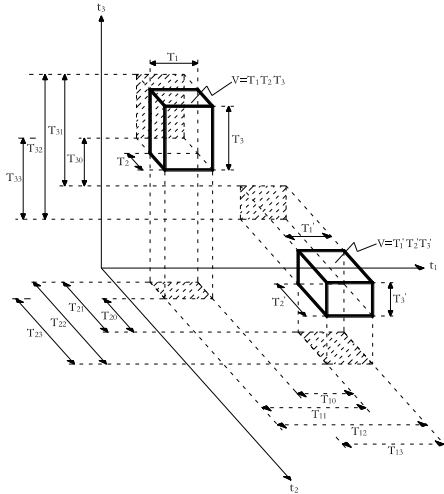


Fig. 1. The relative position parameters of  $V$  and  $V'$ .

The relationship between reduction function of variance  $\gamma(T_1, T_2, T_3)$  and the correlation function  $\rho(\tau_1, \tau_2, \tau_3)$  can be listed as follows

$$\begin{aligned} \gamma(T_{1j}, T_{2k}, T_{3l}) = \frac{1}{T_{1j} T_{2k} T_{3l}} \int_{-T_{1j}}^{T_{1j}} \int_{-T_{2k}}^{T_{2k}} \int_{-T_{3l}}^{T_{3l}} (1 - \frac{|\tau_1|}{T_{1j}}) \\ (1 - \frac{|\tau_2|}{T_{2k}}) (1 - \frac{|\tau_3|}{T_{3l}}) \rho(\tau_1, \tau_2, \tau_3) d\tau_1 d\tau_2 d\tau_3 \end{aligned} \quad (5)$$

where,  $\tau_1$ ,  $\tau_2$  and  $\tau_3$  are the coordinate difference in the direction  $t_1$ ,  $t_2$  and  $t_3$ , respectively.

Generally speaking, due to the high similarity of the geo-morphogenesis and stress history of soil in the identical horizontal level, the spatial variability of soil in the horizontal direction is considered identical. Therefore, in the random field simulation of the soil parameter, the correlation between two points at the same cross section of soil depends on the relative distance, independent of the direction. Consequently, the correlation function can be obtained as follows:

$$\rho(\tau_1, \tau_2, \tau_3) = \exp[-2(\frac{|\tau_1|}{\delta_1} + \frac{|\tau_2|}{\delta_2} + \frac{|\tau_3|}{\delta_3})] \quad (6)$$

where,  $\delta_1$ ,  $\delta_2$ ,  $\delta_3$  indicate the correlation distance.

The correlation distance  $\delta$  is of great significance in the soil random field model. Both variability and correlativity of the mechanical properties can be shown between two soil particles in the identical soil cross-section within a certain distance. As the relative distance between soil particles increases, the correlativity of the soil parameters decreases. When the relative distance exceeds a certain critical distance, the correlativity between two soil particles can be neglected. The critical distance is called the correlation distance of the soil random field model. The critical distance is obtained from the following equation.

$$\delta = \lim_{T_1 T_2 T_3 \rightarrow \infty} T_1 T_2 T_3 \gamma(T_1, T_2, T_3) \quad (7)$$

## 2.2 The Elastic modulus field

The elastic modulus field of soil is assumed to follow a lognormal distribution due to the non-negative value of elastic modulus. Thus, the parameter  $\ln(E)$  is a Gaussian random field with mean  $\mu_{\ln E}$  and variance  $\sigma_{\ln E}^2$ . The parameters of the  $\ln(E)$  Gaussian random field can be obtained from the following equations

$$\mu_{\ln E} = \ln(\mu_E) - \frac{1}{2} \sigma_{\ln E}^2 \quad (8)$$

$$\sigma_{\ln E}^2 = \ln(1 + \sigma_E^2 / \mu_E^2) \quad (9)$$

where,  $\mu_{\ln E}$  and  $\sigma_{\ln E}^2$  are the mean and variance of the  $\ln(E)$  in Gaussian random field, respectively.  $\mu_E$  indicates the mean and  $\sigma_E^2$  represents the variance of the elastic modulus  $E$  of soil medium, which can be obtained from the field measurements.

## 3 MODELING

### 3.1 Modeling procedure

According to the random field theory, a program to simulate the space variability of elastic modulus of surrounding soils is developed via MATLAB language. The procedure of modeling is listed as follows:

- (1) A numerical model of the buried pipeline and surrounding soils is established via Flac<sup>3D</sup> software.
- (2) A random theory is established using the numerical model.
- (3) A covariance matrix is generated according to the element information, correlation distance, correlation function and standard deviation.
- (4) A random vector of elastic modulus is generated considering the mean and variance matrix of the random theory model.
- (5) The random vector is mapped as material property to the numerical model elements one by one according to the position relationship.

(6) The numerical calculation is proceeded with mapped elastic modulus to obtain displacement and maximum principal stress of pipeline.

Repeat step (3) ~ (6) until Monte-Carlo simulation is completed.

### 3.2 Project profile

The numerical model is developed based on a buried pipeline in Guangdong Province, China. The burial depth of the pipeline is 3m and the length is 50m. The depth of the underlying stratum is 8m. Additionally, there are piling soils on the ground surface right above the middle of the pipeline. The height of soil pile is 1m, and the horizontal size is 10m×5m. A schematic view of the model is shown in Fig. 2.

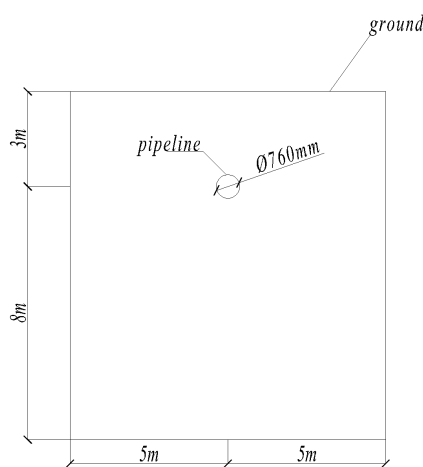


Fig. 2. A schematic view of the model.

### 3.3 FEM Model

Flac<sup>3D</sup> software is used to simulate the interaction between the pipeline and soil as shown in Fig. 3. Soil is simulated with brick and rad cylinder elements while the pipeline is simulated with structural shell element. Additionally, the frictional interaction between the pipeline and soil is neglected for simplicity. Soil is considered to follow Mohr-Coulomb constitutive model while an elastic behavior is assumed for pipeline. Piling soil is also considered to follow the constitutive relation of a homogeneous Mohr-Coulomb model. Furthermore, the in-tube pressure is set to 7MPa to simulate the gas or oil transmission in the pipeline. The material parameters of soil and pipeline are listed in Table 1. The displacement is constraint in the normal direction in four side section and in all directions in the bottom section. In order to balance the calculation efficiency and accuracy, the mesh size of ground soil near the pipeline is 0.33m×2.5m×0.33m and that of ground soil far from the pipeline and of the piling soil is 0.33m×2.5m×1m.

### 3.3 Random field model

The random field of soil elastic modulus is generated based on the locally averaging method.

Considering that the variability of soil property in the horizontal direction is more remarkable than that in the vertical direction, the random field model mainly focuses on the change of the horizontal correlation distance. The vertical correlation distance  $\delta_v$  is set to 1.0m, and the horizontal correlation distance  $\delta_h$  is considered as 2m, 5m, 8m and 10m for case 1 to 4, respectively. The homogenous field is taken as case 5. The contour plot of the elastic modulus of soils for case 4 and 5 are shown in Fig. 4.

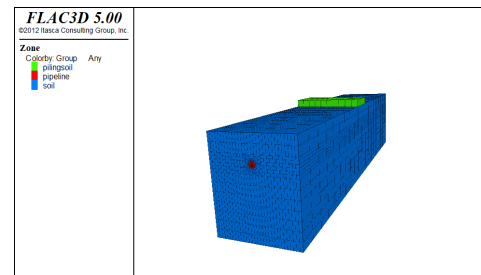
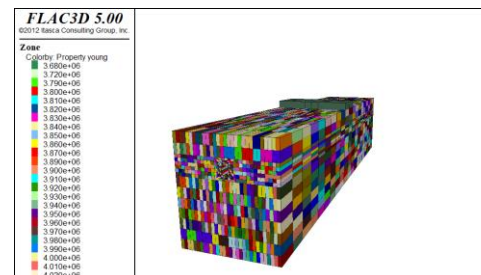


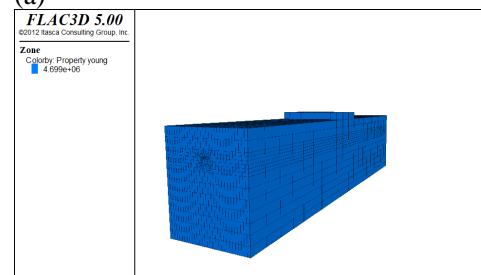
Fig. 3. Numerical model.

Table 1. Material parameters of soil and structure.

Parameter	Soil	Pipeline
Density/(kg · m <sup>-3</sup> )	1850	7850
Poisson ratio	0.25	0.3
Elastic modulus/GPa	4.7(mean)	207
Internal friction angle/°	26.5	
Cohesion stress/kPa	11	
Yield stress/MPa		441



(a)



(b)

Fig. 4. Contour plots of soil elastic modulus considering  $\delta_h = 10m$  (a) and homogenous field (b).

## 4 RESULTS

### 4.1 Maximum principal stress of buried pipeline

Contour plot of the max principal stress of pipeline for case 4 are shown in Fig. 5, where stress  $\delta_v$  is assumed

positive for tension and negative for compression. The means and standard deviations of the maximum principal stress under each case are listed in Table 2.

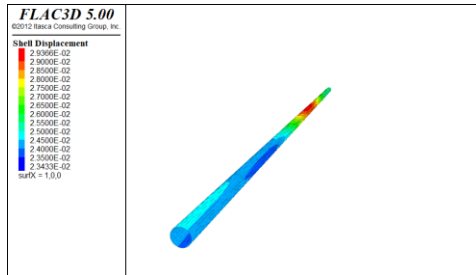


Fig. 5. Contour plot of the calculated maximum principal stress of pipeline in one of the random samples used in Monte-Carlo simulation when  $\delta_h = 10\text{m}$ .

Table 2. Mean and standard deviation of the maximum principal stress under different cases ( $\delta_v=1\text{m}$ , number of samples in Monte-Carlo simulation=100).

$\delta_h$	Mean/MPa	Standard deviation/MPa
2m	128.29	0.1004
5m	128.30	0.1189
8m	128.26	0.1152
10m	128.25	0.1152
homogenous	128.43	

#### 4.2 Maximum displacement of buried pipeline

Contour plot of the maximum displacement of buried pipeline for case 4 is shown in Fig. 6. The means and standard deviations of the maximum displacement under each case are listed in Table 3.

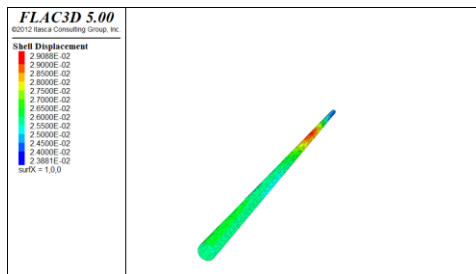


Fig. 6. Contour plot of the calculated maximum displacement of pipeline in one of the random samples used in Monte-Carlo simulation when  $\delta_h = 10\text{m}$ .

Table 3. Mean and standard deviation of the max displacement under different cases ( $\delta_v=1\text{m}$ , number of samples in Monte-Carlo simulation=100).

$\delta_h$	Mean/mm	Standard deviation/mm
2m	29.241	0.547
5m	29.224	0.928
8m	29.410	1.059
10m	29.234	1.134
homogenous	30.855	

## 5 CONCLUSION AND EXPECTATION

(1) The top and bottom of buried pipeline are critical section in terms of displacement and principal

stress.

(2) The section of pipeline which locates right under the ground load is a critical part in the design of pipeline.

(3) The correlation distance of elastic modulus of the homogenous soil stratum exerts a negligible influence on the mean and standard deviation of displacement and maximum principal stress in the buried pipeline.

(4) It deserves to be discussed how the correlation distance of soil properties in multi stratum influences the displacement and maximum principal stress in the buried pipeline.

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