Validity and reliability of the Krumbein-Sloss chart for the estimation of shape-dependent geomechanical properties in sands

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ABSTRACT

The Krumbein-Sloss chart has widely been used as a conventional method with its simplicity and subjectivity in evaluating particle shape parameters. However, its applicability in estimating roundness and sphericity has not yet been validated. We perform a questionnaire study of four sand samples from 2D microscopic images to estimate shape parameters from the chart, and compare it with the theoretically computed values. The results of chart estimation show that the roundness and sphericity tend to be overestimated and underestimated than the computed values, respectively by questionnaires. By the chart-based estimation, we verified the regression model about shape parameters with index void ratio in previous literature, and also proposed a new regression equation to correlate the critical state friction angle. It indicates that the Krumbein-Sloss chart based estimation for quantifying the shape parameters shows statistically meaningful and can be used for correlating geomechanical property as well as predicting shape parameter.

Keywords: Particle shape, Krumbein-Sloss chart, Image analysis, Friction angle, Void ratio

1 INTRODUCTION

The irregularity of particle shape affects the soil packing and inter-particle contact. Previous studies revealed that the particle-interlocking and mobilization determine the shear resistance which increases with non-sphericity and angularity (Altuhafi et al., 2016; Suh et al., 2017; Yang and Luo, 2015). Also, the irregularly shaped particle tends to increase compressibility and the range of index void ratio (e_{min}-e_{max}) (Cavarreta et al., 2010; Cho et al., 2006). Therefore, particle shape descriptors have been proposed to quantify a relationship between geomechanical properties and shape parameters such as bulk form, angularity, and roughness depending on observation scale (Barret, 1980; Krumbein and Sloss, 1951). Both roundness and sphericity are measured readily and simultaneously computable with visual inspection by semi-quantitative charts consisting of a set of reference particle images proposed in the 1950s (Krumbein and Sloss, 1951). Despite the convenience of the Krumbein-Sloss chart, the estimation of shape parameters from the chart needs to be validated because one chooses a shape parameter only judging by subjective matter. The statistical analysis using the chart by the previous study (Hryciw et al., 2016) observed that substantial discrepancy between the computed shape parameters and estimated values. This implies that geomechanical properties dependent particle shape still could contain errors when predicting with the Krumbein-Sloss chart. Therefore, we analyze any biases and discrepancy in the Krumbein-Sloss chart method by the questionnaire study. We also validate and propose the equations to correlate the index void ratios and critical state friction angle with shape parameters from the chart to assess the reliability and applicability of the Krumbein-Sloss chart.

2 ESTIMATION OF PARTICLE SHAPE

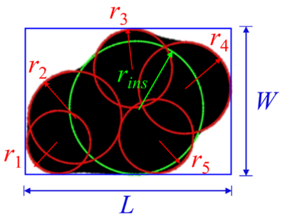
2.1 Image-based computation of particle shape

Roundness (R) is proposed by Wadell (1932) and is a common definition as the ratio of the mean of the radius of inscribed circles located in the corner of particles (r_i) to the largest inscribed circle (r_{ins}). Sphericity (SP) denotes the ratio of width to length with the concept of the minimum bounding box which delineates the longest and shortest dimensions of a particle, and this is the one of most widely used shape parameters because of its simple definition.

Fig. 1 shows that a schematic example of computing roundness and sphericity in a 2D particle image. The gray-colored reference images of the Krumbein-Sloss chart are relocated as black-colored with the minimum bounding box and inscribed circles by image processing method as shown in Fig. 2. This discrepancy indicates that there is an intrinsic problem possibly to cause error when the Krumbein-Sloss chart is used. Nevertheless, when the computation based on a 2D particle image is not available, the prediction by the Krumbein-Sloss chart is the best approximation.

Four sand types are selected for analyzing in this study (K4, Toyoura, ASTM graded, and Ottawa 20-30 sand). A total 1000 images are chosen for each sand type and 2D images are acquired from a digital microscopic camera (MDX300, ×50 magnification, Lanoptik Technologies Ltd.). Each sand particle image is binarized by Otsu's thresholding and flood-fill algorithm (Lee et al., 2017; Otsu, 1979). Then 30 images are randomly extracted for each sand type and used for image-based computation and chart based questionnaire study.

Fig. 3a shows the examples of particle images for questionnaire in this study. Then the distribution of shape parameters of sand particles by image computation is shown in Fig. 3b, including each shape parameter for the total of 120 particles (hollow symbol) and the average shape parameter of each sand type (solid symbol) with ± 1 standard deviation.



$$R = \frac{\frac{1}{n}\sum_{i=1}^{n} r_i}{r_{ins}} \qquad SP = \frac{W}{L}$$

Fig. 1. Definition of roundness and sphericity in 2D particle analysis.

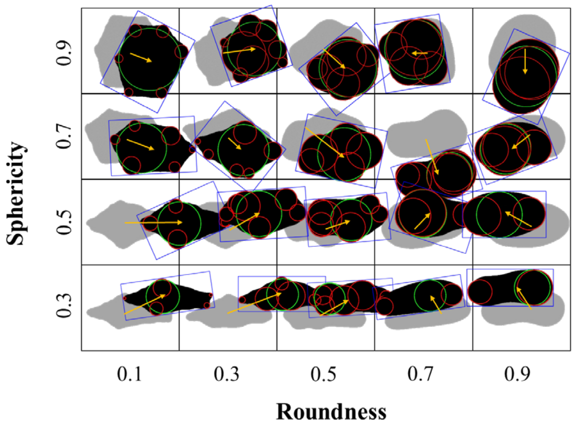


Fig. 2. Discrepancy of shape parameters (roundness R and sphericity *SP*) between reference images from the Krumbein-Sloss chart (gray-colored particle) and image-based computed images (black-colored particle).

2.2 Chart-based estimation of particle shape

In order to verify the applicability of the Krumbein-Sloss chart, the first questionnaire study is conducted by 71 undergraduate students majoring in Civil and Environmental Engineering of Yonsei University. The students are instructed to record the shape parameters of four sand types within 15 minute. Note that no further information is provided other than the definition of the shape parameters and how the Krumbein-Sloss chart is used. To confirm of validity of first questionnaire, a supplementary questionnaire study is performed with 29 graduate students majoring in geotechnical engineering of Yonsei University.

(a)

(b)

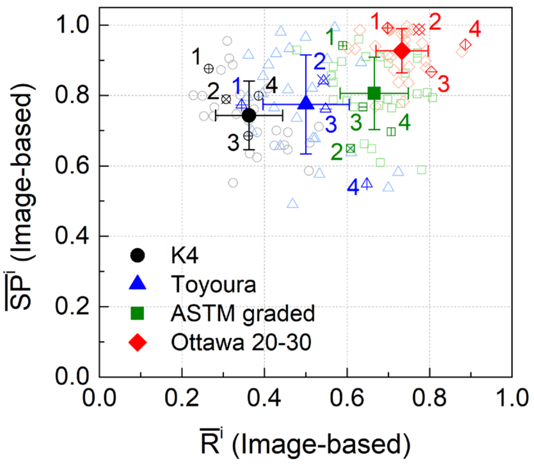


Fig. 3. (a) Examples of four type sand particle images. (b) Computation of image-based shape parameters on four type sand particles for questionnaire study.

3 RESULTS AND DISCUSSION

3.1 Statistical analysis

We assume the theoretically computed shape parameters are sufficiently accurate as a reference. Previous study revealed that the Krumbein-Sloss chart-based estimation tend to underestimate both roundness and sphericity by 0.10 and 0.05 on average (Hryciw et al., 2016). However, despite two questionnaires of different subjects in this study, Figs 4a and 4b show that the chart-based estimates are overestimated in roundness and underestimated in sphericity consistently.

The Undergraduate students participated in the first questionnaire study statistically analyzed that the roundness is overestimated by 0.12 on average regardless of sand type. The sphericity was underestimated by 0.05 on average. On the other hand, sphericity is more underestimated in K4 sand compared to Ottawa 20-30 sand because angular particles seem to be perceived as more non-spherical. For roundness with opposite tendency, only the Krumbein-Sloss chart

(Krumbein and Sloss, 1951) was provided in this study, whereas the previous studies (Hryciw et al., 2016; Muszynski and Vitton, 2017) used the Krumbein chart (Krumbein, 1941) for roundness estimation and the Krumbein-Sloss chart (Krumbein and Sloss, 1951) for sphericity. This difference may be attributed to the different reference images in the chart, which depends on the visual comparison. The Krumbein chart (Krumbein, 1941) also allows more accurate roundness judgement by providing a total of 9 steps in roundness for each 0.1 unit. However, in this study, only the Krumbein-Sloss chart (Krumbein and Sloss, 1951) is used because it is possible to estimate both roundness and sphericity at the same time. An additional questionnaire is conducted to graduate students to confirm the validity of the first questionnaire results from undergraduate students and to investigate the influence of individual expertise. The observation from the results of supplementary study is similar but slightly more accurate to the previous trend; overestimated roundness by 0.10 and underestimated sphericity by 0.06 on average.

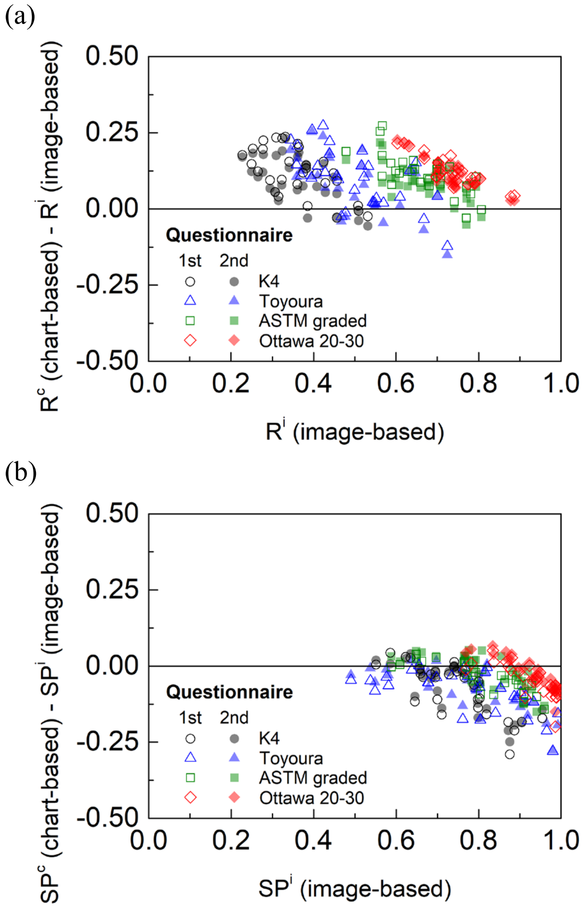


Fig. 4. Comparison between the Krumbein-Sloss chart-based and image-based computation of shape parameter estimation. (a) Overestimated roundness (R^c) and (b) Underestimated sphericity (SP^c).

Although the particle angularity and bulk form are independent at different scales (Hayakawa and Oguchi, 2005), the previous study (Suh et al., 2017) depicts that both roundness and sphericity are somewhat inter-correlated each other. We therefore present how shape parameters based chart are correlated with index void ratio and critical state friction angle in next section.

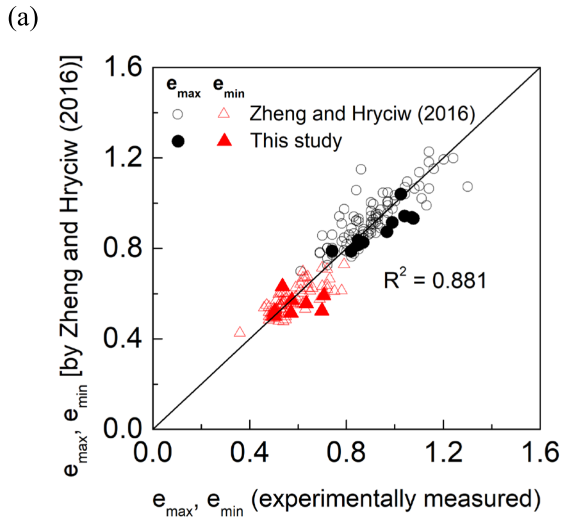
3.2 Estimation of geomechanical properties by shape parameters

The coefficient of uniformity (C_u) is an important determinant of soil packing. The previous study (Zheng and Hryciw, 2016) proposed the regression models that combine the chart-based estimation of the particle roundness and the coefficient of uniformity with the index void ratio as follows.

$$e_{max} = R^{-0.20}\, S^{-0.25}\, C_u^{-0.10} e^{\circ}_{max} \quad (1)$$

$$e_{min} = R^{-0.15}\, S^{-0.25}\, C_u^{-0.15} e^{\circ}_{min} \quad (2)$$

where e°_{max} and e°_{min} are the reference index void ratio term from the ASTM tests for spherical and uniform glass beads (e.g., $R = S = C_u = 1.0$). In this study, we compile the questionnaire study results and supplementary chart-based estimates in literature (Cho et al., 2006; Dodds, 2003) to assess the reliability of estimated values from the Krumbein-Sloss chart using Eqs. (1) and (2). Fig. 5a shows that the prediction of index void ratio from the chart-based shape parameter is well fitted with R^2=0.881.



(b)

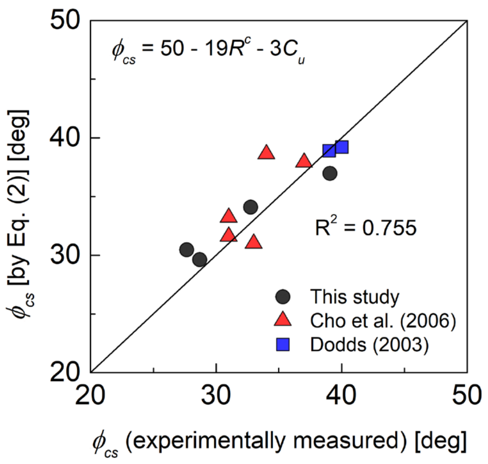


Fig. 5. (a) Prediction of index void ratio by regression model from Zheng and Hryciw (2016). (b) Correlation between critical state friction angle (ϕ_{cs}) and estimated particle roundness by the Krumbein-Sloss chart (R^c) with coefficient of uniformity (C_u).

Similarly, there are many attempts to correlate the critical state friction angle with the chart- or image-based estimates of particle roundness; ϕ_{cs} = 42-17$\cdot \overline{R^c}$ (Cho et al., 2006), ϕ_{cs} = 41.20-21.21$\cdot \overline{R^i}$ (Yang and Luo, 2015), and ϕ_{cs} = 25.02$\cdot$(1- $\overline{R^i}$)+20 (Suh et al., 2017). Nevertheless, Eqs. (1) and (2) hint that it is beneficial to consider soil packing to predict the critical state friction angle. Therefore, we newly propose the regression equation to predict the critical state friction angle (ϕ_{cs}) with the chart-based roundness (R^c) and coefficient of uniformity (C_u) in Eq. (3) and Fig. 5b.

$$\phi_{cs} = 50 - 19R^c - 3C_u \quad (3)$$

4 CONCLUSION

The questionnaire study using the Krumbein-Sloss chart is conducted to verify the reliability of the chart itself, which was used for particle shape quantification over a long period of time, and to examine its applicability. In this paper, we compare the image-based computation and chart-based estimation for four type of sands. Based on any tendency from the prediction of the shape parameter by the questionnaire results, we clarify the correlation from the chart-based estimates of the shape parameter to the geomechanical properties, which leads to the following conclusions: Reference silhouettes in the Krumbein-Sloss chart differ from the definition-based computational results, which indicate that the chart itself contains an error. According to the chart-based survey, the Krumbein-Sloss chart induces overestimated roundness (R) regardless of the sand type and underestimated sphericity (SP) especially in angular sand. Nevertheless, the chart estimates of shape parameters are closely correlated with index void ratio and critical state friction angle because it systematically generates a certain tendency.

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