Analytical solution for the consolidation of unsaturated soil considering thermal changes

Lin-Shuang Zhao[1] and W.-H. Zhou[1]

[1] Department of Civil and Environmental Engineering, University of Macau, Avenida da Universidade, 999078, Macau, China.

ABSTRACT

In this paper, we present an analytical solution for the consolidation of unsaturated soil, subject to thermal changes. The solution was obtained in a straightforward manner, based on Dakshanamurthy and Fredlund's consolidation theory and considering both the temperature change and an additional instantaneous loading. The dissipation of the excess pore pressure was examined and the consolidation features were presented. The study shows that the thermal change, caused by the energy pile's thermal exchange system, can have a degree of influence on the behavior of the unsaturated soil. The temperature change results in the oscillation of the excess pore pressures around the consolidation curves for the case with an additional loading. A greater temperature change could bring about more sweeping changes in the excess pore air and water pressures.

Keywords: Analytical solution; thermal change; consolidation; unsaturated soil; energy pile.

1 INTRODUCTION

In modern times, thermal changes are common because of human activities. Such a change (also known as a temperature change) may have some influence on the surrounding environment. From the geotechnical perspective, it could change the soil properties as a result of the continuous temperature change. The energy pile (Laloui et al., 2006; Amatya et al., 2012), which is an environmentally friendly technique of geothermal application, has become popular in recent years. Taking advantage of the temperature difference above and below the ground surface, the energy pile can cool a building by bringing the energy underground in summer while warming it by absorbing the thermal energy from underground in winter. As regards the foundation soil surrounding the energy piles, it commonly remains in an unsaturated state at the inland regions, and the piles may have some influence on the performance of the unsaturated soil.

Several researchers (Zhou and Zhao, 2014; Ho et al., 2018) have investigated the characteristics of water and air flows in unsaturated soil. A range of scenarios have been put forward to approximate the real practice such as different initial conditions, boundary conditions, dependent loading conditions, two-dimensional cases, installation with prefabricated vertical drains and so on. However, a discrepancy may occur when applying these models to study the consolidation feature of unsaturated soil that has been subjected to temperature change. Dakshanamurthy and Fredlund (1981) proposed a mathematical model for considering the consolidation of unsaturated soil with a temperature gradient. Alsherif and McCartney (2015), for their part, investigated the thermal behavior of unsaturated silt and found that heating after suction application in the silt results in a greater peak shear strength. Ng et al. (2016) then presented the volume change behavior of an unsaturated sample during a cyclic heating and cooling process. Most recently, Ho et al. (2018) proposed an analytical solution to analyze the influence of time-dependent diurnal temperature variation on the consolidation of unsaturated soil. These studies have shown that the thermal gradient has significant influence on the behavior of unsaturated soil. However, the authors concerned have almost exclusively concentrated on a certain aspect such as shear strength, volume change, and so on. The innovative solution, proposed by Ho et al. (2018), is suitable for a case with the thermal source above the ground surface and is not applicable to an energy pile thermal exchange system.

In this study, the spotlight is thrown on the variations of the excess pore pressures in an unsaturated soil stratum. Considering the thermal influencing factor, an analytical solution has been obtained, followed by another analytical solution associated with an additional loading. The calculated solution has been verified and the dissipation features of the excess pore pressures were presented and discussed.

2 MATHEMATICAL MODELLING

The behavior of an unsaturated soil stratum (Fig. 1) becomes complex when this stratum is subjected to an additional loading or a temperature change. To simplify the analysis process, some assumptions, relative to the consolidation theory of unsaturated soil, are:

1) Air and water phases are assumed to be continuous during the dissipation process;

2) Coefficients of conductivity and volume change, with respect to the water and air phases, are constant;
3) Vapor pressure gradients and the dissolution of air in the water are not considered;
4) Soil particles are not compressible;
5) Temperature change in the entire unsaturated soil is uniform.

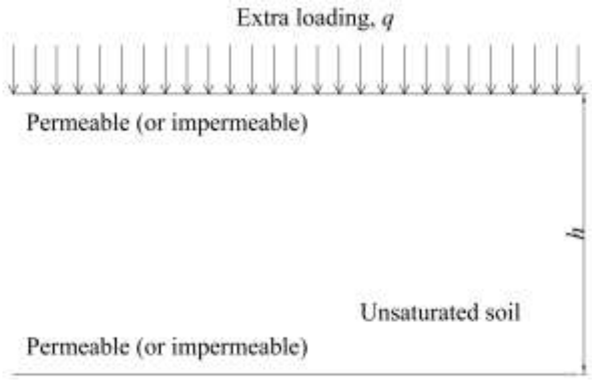

Fig. 1. Profile of an unsaturated soil strata

According to Dakshanamurthy and Fredlund's theory (1981), the excess pore water pressure, u_w, and air pressure, u_a, under K_0 loading, can be expressed as:

$$\frac{\partial u_w}{\partial t} - C_w \frac{\partial u_a}{\partial t} - C_v^w \frac{\partial^2 u_w}{\partial z^2} = C_\sigma^w \frac{\partial q}{\partial t} \tag{1}$$

$$\frac{\partial u_a}{\partial t} - C_a \frac{\partial u_w}{\partial t} - C_v^a \frac{\partial^2 u_a}{\partial z^2} - C_\theta \frac{\partial \theta}{\partial t} = C_\sigma^a \frac{\partial q}{\partial t} \tag{2}$$

where $C_w = \dfrac{1 - m_2^w / m_{1k}^w}{m_2^w / m_{1k}^w}$, $C_v^w = \dfrac{k_w}{\gamma_w m_2^w}$,

$$C_\theta = \frac{1}{\theta} \frac{(1 - S_r) n (u_a + u_{atm})}{\left(1 - m_2^a / m_{1k}^a\right)(u_a + u_{atm}) m_{1k}^a + (1 - S_r) n},$$

$$C_a = \frac{m_2^a / m_{1k}^a}{1 - m_2^a / m_{1k}^a - n(1 - S_r) / \left[(u_a + u_{atm}) m_{1k}^a\right]},$$

$$C_v^a = \frac{R\theta}{\kappa} \frac{k_a}{g\left[\left(1 - m_2^a / m_{1k}^a\right)(u_a + u_{atm}) m_{1k}^a + (1 - S_r) n\right]}.$$

m_2^w, m_{1k}^w, and m_2^a, m_{1k}^a are the coefficients of water and air volume change with respect to a change in matrix suction $d(u_a - u_w)$, under K_0 loading. γ_w is the unit weight of water. k_w and k_a are the hydraulic conductivity of the water and air phases, respectively. S_r is the degree of saturation and n is the porosity of soil element. R is the universal gas constant. u_{atm} is the atmosphere pressure. g is the acceleration of gravity. θ is the temperature. κ is the molecular mass of air. q is the extra loading.

In this study, the upper boundary is presumed to be permeable and is denoted as:

$$u_a(0,t) = 0, \quad u_w(0,t) = 0 \tag{3}$$

The lower boundary can be permeable or impermeable and is written as, respectively:

$$u_a(h,t) = 0, \quad u_w(h,t) = 0 \tag{4}$$

$$\partial u_a(h,t) / \partial z = 0, \quad \partial u_w(h,t) / \partial z = 0 \tag{5}$$

It is assumed that an instantaneous additional loading generates constant initial excess pore pressures along the entire unsaturated soil strata, which are expressed as:

$$u_a(z,0) = u_a^i, \quad u_w(z,0) = u_w^i \tag{6}$$

The subsurface temperature is assumed to be 25 °C. The daily absolute temperature variation can be written using a sine function, as per:

$$\theta = (273.16 + 25) + A \cdot \sin(\omega t) \tag{7}$$

where ω = 0.727×10^{-4} *rad/s* and A is the amplitude of the temperature change with the range of [4, 16.5] °C.

3 ANALYTICAL SOLUTION

The governing equations for the water and air phases in Eqs. (1) and (2) can be separated into the following two sets of the partial differential equation system:

$$\begin{cases} \dfrac{\partial u_w}{\partial t} - C_w \dfrac{\partial u_a}{\partial t} - C_v^w \dfrac{\partial^2 u_w}{\partial z^2} = C_\theta^w \dfrac{\partial \theta}{\partial t} \\ \dfrac{\partial u_a}{\partial t} - C_a \dfrac{\partial u_w}{\partial t} - C_v^a \dfrac{\partial^2 u_a}{\partial z^2} = C_\theta \dfrac{\partial \theta}{\partial t} \end{cases} \tag{8}$$

$$\begin{cases} \dfrac{\partial u_w}{\partial t} - C_w \dfrac{\partial u_a}{\partial t} - C_v^w \dfrac{\partial^2 u_w}{\partial z^2} = C_\sigma^w \dfrac{\partial q}{\partial t} \\ \dfrac{\partial u_a}{\partial t} - C_a \dfrac{\partial u_w}{\partial t} - C_v^a \dfrac{\partial^2 u_a}{\partial z^2} = C_\sigma^a \dfrac{\partial q}{\partial t} \end{cases} \tag{9}$$

where C_θ^w is an introduced nominal parameter and equals zero. Because the items $C_\theta \partial\theta / \partial t$ and $C_\sigma^a \partial q / \partial t$ are linear combination in Eq. (2). The combination of the solution in Eq. (8) and that in Eq. (9) is the solution of Eqs. (1) and (2). The observer may see that Eqs. (8) and (9) have the same forms. Only one set of partial differential equations needs to be solved, which makes the derivation process much simpler. Based on the solutions in Zhou et al. (2014), the analytical solution for the homogeneous form of Eqs. (8) and (9) in normalized form can be easily obtained.

With the assumption that the temperature change in the whole soil domain is uniform, a temperature change in soil results in an increment (or decrement) in excess pore pressures. Borrowing the analogy of Olson's theory (1977), for an increment of $d\eta$, the instantaneous change in X should be $dX = d\eta$. If we assume that the instantaneous change occurs at T_a, then the variable dX at time T after a period of T–T_a, with the double drainage condition, becomes:

$$dX = \sum_{m=0}^{m=\infty} \left(\frac{4\beta \cdot d\eta}{M}\right) \sin(M\overline{z}) \exp\left(-M^2 Y (T - T_a)\right) \tag{10}$$

Similarly, for the single drainage condition, the variable dX becomes:

$$dX = \sum_{m=0}^{m=\infty}\left(\frac{2\beta \cdot d\eta}{K}\right)\sin\left(K\overline{z}\right)\exp\left(-K^2Y\left(T-T_a\right)\right) \quad (11)$$

The solution of X can be calculated through integrating in the range of $[T_a, T]$ as follows

Double drainage condition ($M = (2m+1)\pi$):

$$X = \sum_{m=0}^{m=\infty}\frac{4}{M}\frac{\sin\left(M\overline{z}\right)\beta A_T\varpi}{M^4Y^2+\varpi^2}\begin{bmatrix}M^2Y\cos\left(\varpi T\right)+\omega\sin\left(\varpi T\right)\\ -M^2Y\exp\left(-M^2YT\right)\end{bmatrix} \quad (12)$$

Single drainage condition ($K = (2m+1)\pi/2$):

$$X = \sum_{m=0}^{m=\infty}\frac{2}{K}\frac{\sin\left(K\overline{z}\right)\beta A_T\varpi}{K^4Y^2+\varpi^2}\begin{bmatrix}K^2Y\cos\left(\varpi T\right)+\varpi\sin\left(\varpi T\right)\\ -K^2Y\exp\left(-K^2YT\right)\end{bmatrix} \quad (13)$$

After obtaining the formulas of X, the excess pore water and air pressures can be calculated as:

$$u_a = \frac{\alpha_{21}X_2(\overline{z},T)-X_1(\overline{z},T)}{\alpha_{12}\alpha_{21}-1} \quad (14)$$

$$u_w = \frac{\alpha_{12}X_1(\overline{z},T)-X_2(\overline{z},T)}{\alpha_{12}\alpha_{21}-1} \quad (15)$$

where α_{21} and α_{12} can be written as (Zhou et al., 2014):

$$\alpha_{12} = \frac{W_a}{Q_2-A_a}, \quad \alpha_{21} = \frac{A_w}{Q_1-W_w} \quad (16)$$

After obtaining the analytical solutions for the excess pore air and water pressures in Eqs. (14)–(15), various scenarios, such as temperature change, additional loading, and a combination of the two, can be investigated by amalgamating the solutions of Eqs. (8) and (9).

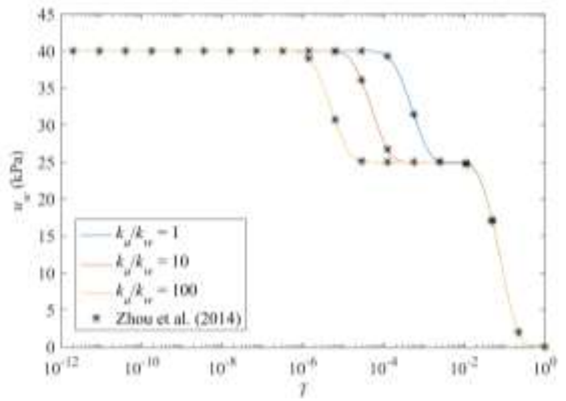

Fig. 2 Dissipation of excess pore water pressure compared with the analytical solution in literature.

4 VERIFICATION

In this part, the proposed analytical solution is first verified by comparing it with one analytical solution in the literature. Then, the case of thermal change caused by the installed energy pile system, as well as the case of thermal change combined with an additional loading, are considered to investigate the consolidation process of an unsaturated soil strata. The properties of said strata are listed as follows. The height of the strata is h = 10 m. The parameters related to the volume change are $m_{1k}^s = -2.5\times10^{-4}$ kPa^{-1}, $m_2^s = -1.0\times10^{-4}$ kPa^{-1}, $m_{1k}^w = -0.5\times10^{-4}$ kPa^{-1}, and $m_2^w = -2.0\times10^{-4}$ kPa^{-1}. Moreover, these parameters fulfill the relationships of $m_{1k}^s = m_{1k}^w + m_{1k}^a$ and $m_2^s = m_2^w + m_2^a$. The hydraulic conductivity for the water phase is $k_w = 10^{-10}$ m/s, and the initial porosity and saturation degree are n = 0.50 and S_r = 0.80, respectively.

Fig. 2 presents the dissipation process of excess pore water pressure with a constant temperature. The consolidation curves, with a different ratio of k_a/k_w, are presented and compared with the analytical solutions of Zhou et al. (2014). They match each other very well, proving the accuracy of this study.

5 CASE STUDIES

The daily temperature variation in the atmosphere can cause temperature oscillation in the unsaturated soil as a result of the energy pile thermal exchange system. As such, the excess pore pressures vary simultaneously. Because this study concentrated on the behavior of unsaturated soil, under the assumption that the installation of energy piles cannot change the drainage path of the unsaturated soil strata, the piles are not shown in this figure (Fig. 1). Because of the installed energy piles, the presumption was that the temperature of the entire unsaturated soil strata would vary simultaneously.

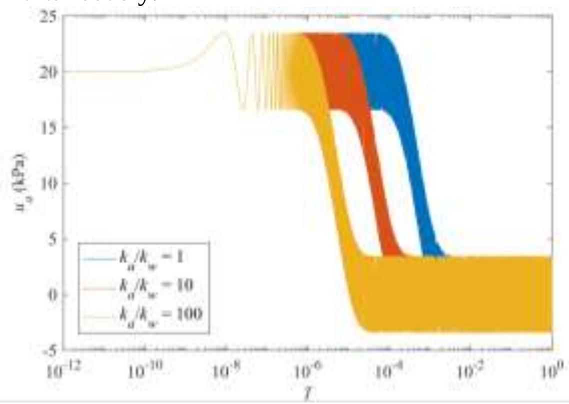

Fig. 3 Variation of excess pore air pressure versus time

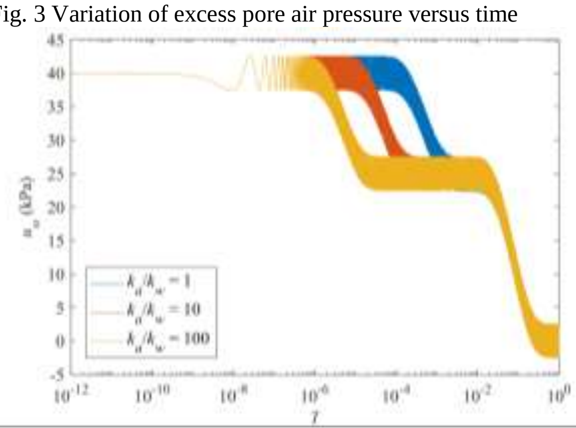

Fig. 4 Variation of excess pore water pressure versus time

Figs. 3 and 4 present the dissipation process of excess pore water and pore air pressures, considering the additional loading as well as the variation of temperature under double drainage condition. Because of the additional loading, the researchers assumed that the initial excess pore water and pore air pressures are 40 kPa and 20 kPa respectively. Both the air and water phases dissipate gradually with the passing of time and vary cyclically in a small range. The effect of the ratio of k_a/k_w has also been considered. The solutions show that a large ratio of k_a/k_w can accelerate the consolidation process of the air phase and the first stage of the water phase. However, this ratio has little influence on the oscillation of the excess pore pressures as a result of the temperature change.

Considering the additional loading associated with the temperature change, Figs. 5 and 6 further illustrate the variations of excess pore water and pore air pressure versus the time factor T at different depths of the unsaturated soil strata. The soil stratum was subjected to the single drainage condition, and the excess pore pressures at the depth ratios of 0.2, 0.5, and 0.8 were taken into account. The writer found that the dissipation of excess pore pressures, close to the permeable boundary, changed faster than that of those close to the impermeable boundary.

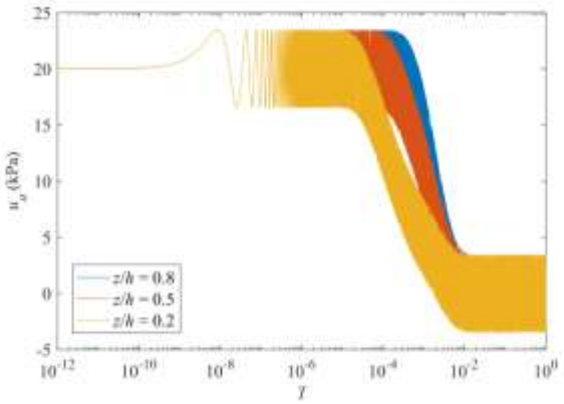

Fig. 5 Dissipation of excess pore air pressure versus time

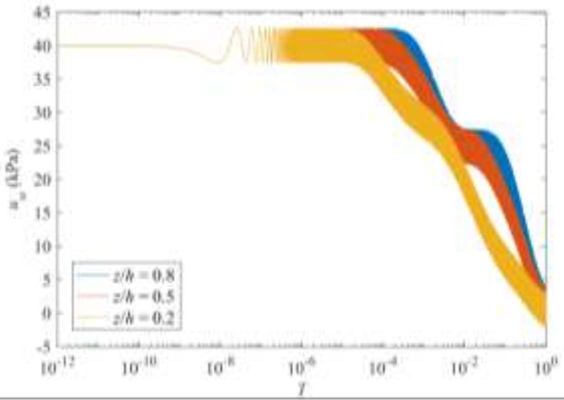

Fig. 6 Dissipation of excess pore water pressure versus time

6 CONCLUSION

In this study, the consolidation feature of an unsaturated soil strata was investigated, and an analytical solution was proposed considering the influence of thermal change. The proposed analytical solution was verified as accurate and two scenarios were investigated to show the influence of thermal change and that of thermal change combined with an additional loading. It should be noted that the temperature change in the concerned unsaturated soil is assumed to be uniform and the hydraulic conductivities for air and water phases keep constant during the dissipation process. The study reveals that the thermal change in an unsaturated soil stratum may cause the oscillation of excess pore pressures in unsaturated soil. The amplitude of the oscillation in the excess pore air pressure is greater than that in the excess pore water pressure. This is because the former is sensitive to the thermal change and the change in it drives the change in the latter. The consolidation process of unsaturated soil, under a building equipped with energy piles, can be altered because of the heating and cooling process.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support from the Science and Technology Development Fund, Macao S.A.R (FDCT) (193/2017/A3) and University of Macau Research Fund (MYRG2018-00173-FST).

REFERENCES

Laloui, L., Nuth, M., and Vulliet, L. (2006). Experimental and numerical investigations of the behaviour of a heat exchanger pile. International Journal for Numerical and Analytical Methods in Geomechanics, 30(8), 763–781.

Amatya, B.L., Soga, K., and Bourne-Webb, P.J., Amis, T., and Laloui, L. (2012). Thermo-mechanical behaviour of energy piles. Géotechnique, 62(6), 503–519.

Zhou, W.H., and Zhao, L.S. (2014). One-dimensional consolidation of unsaturated soil subjected to time-dependent loading with various initial and boundary conditions. International Journal of Geomechanics, 14(2), 291–301.

Ho, L., Fatahi, B., and Khabbaz, H. (2018). Analytical solution to one-dimensional consolidation in unsaturated soil deposit incorporating time-dependent diurnal temperature variation. International Journal of Geomechanics, 18(5), 1–13.

Dakshanamurthy, V., and Fredlund, D.G. (1981). A mathematical model for predicting moisture flow in an unsaturated soil under hydraulic and temperature gradients. Water Resources Research, 17(3), 714–722.

Alsherif, N.A., and McCartney, J.S. (2015). Thermal behaviour of unsaturated silt at high suction magnitudes. Géotechnique, 65(9), 703-716.

Ng, C.W.W., Cheng, Q., Zhou, C., and Alonso, E.E. (2016). Volume changes of an unsaturated clay during heating and cooling. Géotechnique Letter, 6(3), 192-198.

Zhou, W.H., Zhao, L.S., and Li, X.B. (2014). A simple analytical solution to one-dimensional consolidation for unsaturated soils. International Journal for Numerical and Analytical Methods in Geomechanics, 38(8), 794-810.

Olson, R.E. (1977). Consolidation under time-dependent loading. Journal of the Geotechnical Engineering Division, 103, 55–60.