An analytical and design solution for arbitrary shape rigid spread footings subjected to biaxial loading

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ABSTRACT

An analytical and design solution has been proposed for arbitrary shape rigid footings subjected to biaxial moments. Uniform and linear soil bearing distribution patterns are adopted for the algorithm. The Visual Studio C# platform is used for the program development and has been implemented into the software package Mat3D. The proposed algorithm can be applied to determine (1) the neutral axis position and soil bearing pressure values at each of the footing vertex, based on the static equilibrium criteria; (2) the stability and the sliding check of the rigid footing; (3) Soil bearing pressure, one-way and two-way shear design and ultimate moment design of the footing. A sample case is presented to show the accuracy and consistency of this algorithm.

Keywords: Arbitrary shape spread footing; Biaxial Bending; Analytical solution; Footing design program.

1 INTRODUCTION

Since the 1960s, engineers and researchers have been trying to find effective solutions for the cumbersome problem of rigid spread footing subjected to biaxial eccentricity. Czerniak (1963) proposed an early version of computer-based program for the footing soil bearing calculation using the geometric iteration algorithm, which can handle the rectangular shape footing with linear soil distribution. Bowles (1977), Jarquio (1983), Kramrisch (1985), Rodriguez-Gutierrez and Ochoa (2012) have also developed and published numerical methods and algorithms regarding the analysis and design of rigid footings with regular or irregular shapes (trapezoidal, circular, annular, octagonal). However, there is no unified method for arbitrary shape footing available in the technical literature, particularly when a biaxial bending loading case is applied. Moreover, the ACI code is only applicable for rectangular spread footing design, which leaves confusion and obstacles for engineering practice.

Therefore, this research aims to propose an analytical solution that can be adopted for preliminary footing design and dimensioning. This paper is organized in two sections to briefly introduce: (1) the theoretical background and derivation process of the algorithm, including the equilibrium conditions, iteration algorithms, footing analysis and design methods. (2) A step-by-step analysis for a sample case, which shows the effectiveness of this algorithm in the analysis of isolated rigid footings resting on soil.

2 ANALYTICAL SOLUTION

2.1 A discrete method for arbitrary shape footing

To perform the analysis of arbitrary shape footings, the proposed method discretizes the footing exterior boundaries and interior opening edges (if any) into linear segments. In this way, the complicated footing geometry is divided into trapezoids areas between each vertex and becomes more feasible to get integrated into the computational algorithm.

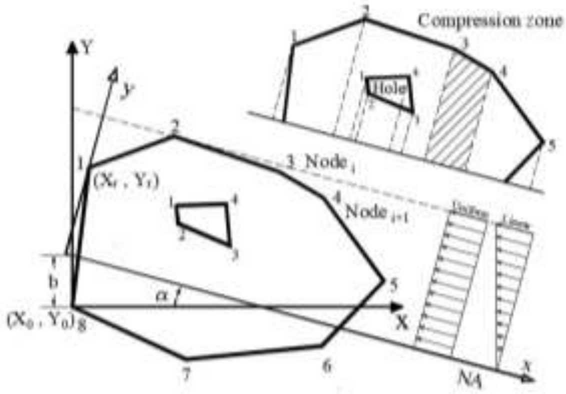

Fig. 1. Footing Plan View and Typical Neutral Axis Location

As shown in Fig.1, the coordinates of each footing vertex are input clock-wisely in the Global system of axes X-Y, where the origin (X_o,Y_o) can locate at any random point (here the origin locates at the bottom left footing vertex for convenience). It needs to be mentioned that the openings' boundaries are input counter-clock-wisely, to obtain a negative output for axial forces and biaxial moments using the same algorithm. After all the geometry info is provided, the algorithm calculates the global centroid coordinates of the whole footing area, about which the final output of biaxial moments is calculated for the static equilibrium check.

2.2 Coordinate Transfer, Loading Calculation and Iteration Method

To start the algorithm, an initial neutral axis needs to be set. In this paper, location of the neutral axis is defined based on two variations: angle **α** and interception distance **b**. The neutral axis is considered as the x-axis of the local coordinate system x-y, while the local y-axis is a perpendicular line that passes the exterior footing vertex (X_r,Y_r) with a minimum local x value. For each iteration steps of the algorithm, the calculation procedures of axial loads and biaxial moments are performed based on a specific local system of x-y, considering that the soil distribution pattern is perpendicular to the neutral axis (local x-axis) and for non-uniform distribution patterns, it is simpler to consider only one-way varying pressure distribution than two-way variations. Once the local outputs are obtained, the program transfers them back to the global system for the equilibrium check. Eq.1 shows the transformation between the global and local coordinates; Fig.2 shows the loading integration of each discrete section.

$$
\begin{aligned}
x_i &= X_i \cos\alpha + Y_i \sin\alpha - X_r \\
y_i &= -X_i \sin\alpha + Y_i \cos\alpha - b\cos\alpha
\end{aligned} \qquad (1)
$$

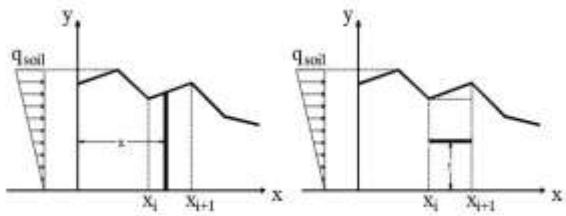

Fig. 2. (a) Integration along x-axis (b) Integration along y-axis

For each iteration step, expressions of axial force (P) and biaxial moments (M_x, M_y) are functions of neutral axis variations (**α**, **b**) and soil variations (**q_{soil}**). By using Cramer's rule, a set of three linear equations (Eq.2) could be solved and residuals through every iteration can be evaluated (Yen, 1991)

$$
\begin{aligned}
p\ (\alpha, b, q_{soil}) &= P\ (\alpha, b, q_{soil}) - P_{applied} \\
m_x(\alpha, b, q_{soil}) &= M_x(\alpha, b, q_{soil}) - M_{applied} \\
m_y(\alpha, b, q_{soil}) &= M_y(\alpha, b, q_{soil}) - M_{applied}
\end{aligned} \qquad (2)
$$

It is observed that the convergence process highly depends on the initial values, especially the **α** value. For example, if M_x and M_y are both positive, a positive initial α cannot converge due to the wrong location of the compression zone (Fig.3). In the algorithm, an added-on function has been implemented to solve this issue, by automatically judging the compression zone location and generating initial variations based on the input biaxial information. The add-on highly optimizes the stability and efficiency of the program.

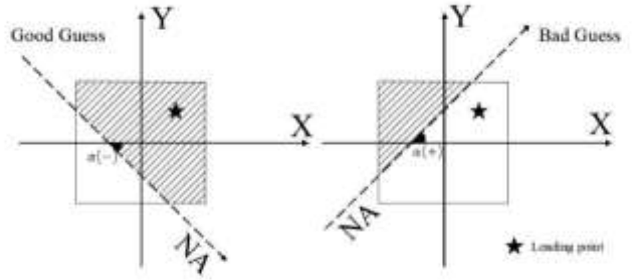

Fig. 3. Initial Value Setting

2.3 Stability Check

The biggest challenge for stability analysis comes from the footing geometry. Unlike regular rectangular shape footing, where the overturning moment and the resisting moment about each edge can be easily obtained, the irregular shape footing does not always follow the orthogonal coordinate. Moreover, the stability factor of the footing edge may not be the governing case since the neutral axis's location remains uncertain. Thus, the overturning check needs to be performed for the footing vertexes as well to find the most critical case. The overturning moment and resisting moment are decoupled into orthogonal coordinates, to precisely show the overturning ratio of inclined boundaries and vertexes, as shown in Fig.4.

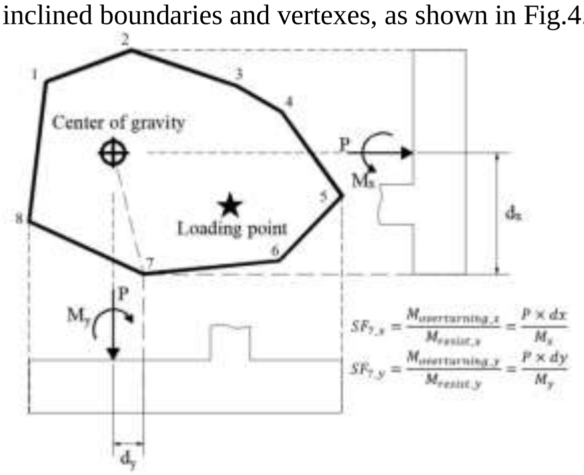

Fig. 4. Equivalent Loading for Stability Check

2.4 Sliding Check

Sliding analysis calculates the ratio of resisting sliding force from the soil and the applied equivalent horizontal force caused by the biaxial moment. The two different sources of the resisting forces are categorized as: (1) force from the footing bottom interface in contact with soil, where a friction force is developed against the applied force; (2) passive resistance force from the side surface of the footing that is opposite to the direction of the applied force, where cohesive resistance force is generated according to Coulomb earth pressure theory. For different types of soil or granular, the two resistance forces' calculation may vary. The proposed algorithm summarized the resistance force and decouple them into global system of X-Y, where they can be compared to the applied load

V_x and V_y and obtain the sliding ratio.

2.5 Shear Design and Moment Design

Compared to the shear and flexural design of regular shape footing, this research's challenge is in finding the direction at which the critical shear stress and moment stress sections are located.

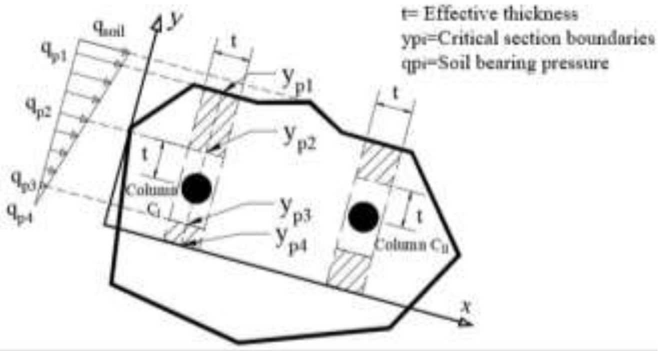

Fig. 5. Design for Arbitrary Shape Footing

After establishing a specific neutral axis that satisfies the applied load, the critical direction of each column/wall is perpendicular to the neutral axis as shown in Fig.5. It can be seen that for each loading zone, there are two cases to be checked: one towards the footing edge in the compression direction and the other one towards the neutral axis (or the opposite side of the footing edges if the whole footing is under compression). For one-way shear and ultimate moment check, the ACI method can be applied using the described critical direction; for the two-way shear check, since the shape of the footing does not affect the design procedure, it is not discussed here in this paper.

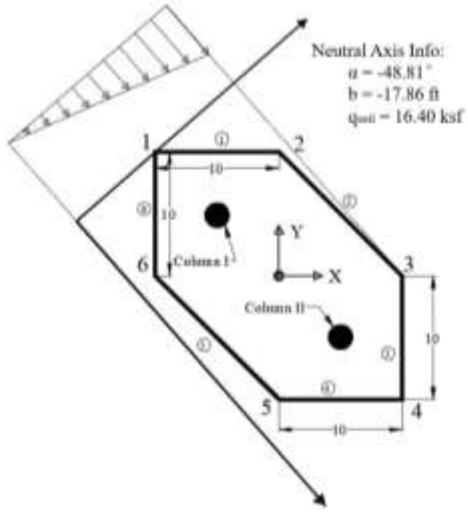

Fig. 6 Diamond Shape Footing

3 COMPREHENSIVE EXAMPLES

The full analysis of a diamond-shape footing with two biaxial loaded columns that subjected to linear soil distribution pattern is performed using the developed program (Fig. 6). The origin of the global coordinates is set at the centroid of the footing and the final neutral axis location is also shown, indicating that the whole footing is under compression with the given loading case. Table 1 shows the columns' information and the total equivalent loading status. With positive eccentricities in both directions, the algorithm automatically assigns α=-89°, b=0' and $q_{soil}=5P/A_{footing}$ =50 ksf as the initial values. Each initial values will run 100 loops to check the convergence before it is switched to the next initial guess with $\alpha_{i+1}=\alpha_i+1°$. For this case, the program shows that it takes 22 guesses to reach a proper initial value of α= -67° and the final maximum soil bearing pressure output is q_{soil}=16.40 ksf. Detailed iteration information is shown in Table 2.

Table 1. Columns Information and Equivalent Loading

Unit: ft., kip	Column I	Column II	Equivalent Loading
Location	(-4,4)	(4,-4)	$e_x = 1$
Height	10	10	$e_y = 0.67$
P	1500	1500	P = 3000
M_x	2000	0	$M_x = 2000$
M_y	0	3000	$M_y = 3000$

Table 2. Critical Variations in each iteration step (Unit: ft, kip)

Iteration	α°	B	q_{soil}	P	M_x	M_y
Initial	-67.000	0.00	50.00	2926	-1573	16411
1	-62.458	-26.07	-34.81	-6016	-264	-9089
2	-69.365	-32.04	16.40	2690	-764	5027
3	-47.828	-3.66	16.40	1531	3159	4295
4	-48.946	-21.69	6.64	1303	711	1082
5	-48.576	-10.99	16.40	2422	2652	3876
6	-48.727	-16.25	16.38	2889	2124	3156
7	-48.809	-17.77	16.40	2994	2006	3008
8	-48.811	-17.82	16.39	2999	2000	3000
Last	**-48.814**	**-17.86**	**16.40**	**3000**	**2000**	**3000**

As mentioned before, the overturning check is decoupled into x and y directions for each vertex and edge of the footing. Table 3 presents the detailed results.

Table 3 (a). Stability Check – Vertexes (Unit: kip-ft.)

Vertex	Direction	Resisting	Overturning	Safety Factor against Overturning
1	x	0	33000	0
	y	30000	2000	15
2	x	0	3000	0
	y	30000	2000	15
3	x	30000	3000	10
	y	0	32000	0
4	x	30000	3000	10
	y	0	32000	0
5	x	0	3000	0
	y	0	2000	0
6	x	0	33000	0
	y	0	2000	0

Table 3 (b). Stability Check – Edges (Unit: kip-ft)

Edge	Footing Coordinate		Resisting	Overturning	Stability Factor
1	x	0	0	3000	0
	y	10	30000	2000	15
2	**x**	**5**	**15000**	**3000**	**5**
	y	**5**	**15000**	**2000**	**7.5**
3	x	10	30000	3000	10
	y	0	0	2000	0
4	x	0	0	3000	0

	y	-10	0	32000	0
5	x	-5	0	18000	0
	y	-5	0	17000	0
6	x	-10	0	33000	0
	y	0	0	2000	0

Per the ACI code, the stability factor should be larger than 1.5. In this case, since the whole footing is under compression, there is no overturning issue. The most critical case occurs in Edge 2 (highlighted), considering that the neutral axis is barely parallel to it, providing the minimum resisting moment.

Table 4. Soil Properties

Soil Type	Cohesive	Footing Thickness	4’
Cohesion Factor	1.5	Concrete Weight	150 pcf
Adhesion Factor	1.5	Soil Unit Weight	100 pcf
Soil Cover	3’		

Table 5. Sliding Check

Item	Type	X	Y
a	Cohesive force	300	300
b	Adhesive force	450	450
c=a+b	Resistance force	750	750
d	Horizontal force	300	200
Sliding factor = d:c		2.5	3.75

Table 4 shows the soil property that is used to evaluate the sliding factor of the footing. The sliding factor in x and y directions both satisfy the minimum requirement per code, as shown in Table 5.

Finally, the shear and moment checks are performed. The detailed results are listed in Tables 6~8. The shear and moment stress for both columns in two directions are outputted by the program for further thickness and reinforcement design.

Table 6. Critical Shear Section Information

(ft)	y_{p1}	y_{p2}	y_{p3}	y_{p4}
Column I	18.47	14.88	7.88	4.17
Column II	19.22	15.64	8.64	3.42
(kips)	q_{p1}	q_{p2}	q_{p3}	q_{p4}
Column I	15.70	12.66	6.70	3.55
Column II	16.35	13.29	7.34	2.91

Table 7. One-Way Shear Stress

Dir	Item	Column I	Column II
1	Soil Bearing Support	132.24	138.03
	Soil & Footing Weight	8.14	8.14
	Net Shear Force (kip)	124.09	129.89
	Net Shear Stress (psi)	**70.34**	**73.63**
2	Soil Bearing Support	44.61	69.37
	Soil & Footing Weight	8.54	13.28
	Net Shear Force (kip)	36.07	56.08
	Net Shear Stress (psi)	**20.44**	**31.79**

Allowable: $2\phi_c\lambda\sqrt{f_c'} = 2\times0.75\times1\times\sqrt{3000} = 82.16$ *psi ……***OK!**

Table 8. Ultimate Moment Stress

Dir	Item	Column I	Column II
1	Soil Support Moment	175.33	182.83
	Self-Weight Moment	10.53	10.53
	Net Moment (kip-ft)	164.79	172.29
	Net Moment per foot	**47.08**	**49.22**
2	Soil Support Moment	65.44	164.88
	Self-Weight Moment	11.58	28.01
	Net Moment (kip-ft)	53.86	136.87
	Net Moment per foot	**15.38**	**39.10**

4. CONCLUSION

A complete method of analyzing and designing biaxially loaded arbitrary shape rigid footing is developed based on loading equilibrium conditions. Uniform and linear soil bearing distribution patterns have been implemented using pure theoretical method. This also enables the future introduction of more customized patterns that may take other variations into account.

The proposed algorithm adds value in solving engineering design issues of arbitrary shape footing and allow the engineer to efficiently finish the design work. A comprehensive example is presented in this paper, showing a step-by-step analysis and design process of a diamond-shape spread footing. The example results validate and indicate the good precision and application scope of the proposed algorithm.

The developed program is based on static equilibrium and assumption of certain soil distribution patterns, future work should focus on the development of more sophisticated soil models that consider other effects like soil-structure interaction, soil properties and settlements, and so on. Also, the future development of the user interface can also be performed based on the latest Mat3D version, to satisfy the engineering requirements of real projects.

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