

## Incremental theory of stabilizing clay soil

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### ABSTRACT

In this paper we analyze the testing of clay soils at various loading trajectories, including the load and unloading branches. It has been established that in normally compacted clay soil, the main and determining hardening factor is the change in its density, and that the angle of internal friction does not significantly change depending on the skeleton density, and therefore, has little effect on the hardening process. Based on such assumptions, the constitutive equations of hardening clayey soil are compiled. In the composed equations, i.e. a number of assumptions have been made to describe such a process in the prelimit state. In the process of active loading of clay soil, the shearing plastic process is due to the mutual slip of individual aggregates both in the pre-limit and in the limit states. The developed incremental theory of the plasticity of hardening clayey soil makes it possible to describe its nonlinear deformation right up to the moment of destruction. In contrast to the existing similar theories, the theory developed by us simultaneously takes into account dilatant hardening and the parameter of the loading trajectory, a possible change in the orientation of the slip pad.

**Keywords:** Incremental theory, clay soils, equation, plastic deformations, layer, stress.

### 1 INTRODUCTION

The test results of clay soils for strength and deformability are traditionally referred to the initial values of density-humidity. This interpretation has significant disadvantages, since it does not allow to take into account the effect of changes in the skeleton density on the deformation and strengthening characteristics in the process of its loading, which ultimately can lead to incorrect results when determining these parameters. So, for example, the angle of internal friction of clay soil, determined from the results of testing on a triaxial compression device along the crush trajectory at conventional processing of experimental data, can reach 30-35°. This value is due to the development of large (about 10–20%) volumetric deformations and stabilizing (by increasing the adhesion) [3].

If we take into account the effect of stabilizing the skeleton of the soil in the process of deformation and subsequent destruction, it can be determined the true value of the angle of internal friction of the clay soil, which will be 8-10° less than the above and will not depend on its density. The value adhesion significantly depends on the varying density of the skeleton. These circumstances are especially important to take into account where the problem arises of building the theory of plastic flow of

stabilizing clay soil under conditions of a complex stress state and at a complex loading trajectory [1].

### 2 THE PECULIARITIES OF STABILIZING CLAY SOIL

The analysis of the tests results of three types of clay soils on the devices of three-axis compression and single-plane section at complex loading trajectories, conducted by the authors, made it possible to draw the following main conclusions.

1. Based on the results of tests on a single-plane slice device, it can be argued that;

- the angle of inclination of the limiting straight line along the load branch is greater than the similar angle along the unloading branch, which is explained by the stabilizing (compaction) of the clay soil;

- the angles of inclination of the limiting straight lines along the unloading branches are practically the same, which is explained by the independence of the angle of internal friction from the density of the ground skeleton;

- the cohesion of clay soil increases in direct proportion to the normal stress, which is caused by the varying density of the ground skeleton;

- to determine the true angle of internal friction and cohesion, corresponding to a given density-moisture of clay soil, it is necessary to carried out tests for a shear on the loading branch and also to

carry out tests along the discharge branch, i.e. the shear should be done after preliminary compression by normal compactive loads and subsequent partial unloading.

2. According to the results of testing on devices of triaxial compression, it can be argued that:

- in the process of hydrostatic compression and subsequent of deviator loading of clay soils, significant volumetric deformations occur in them, constituting up to 20% or more, depending on the initial density-humidity, which leads to soil stabilizing due to an increase in cohesion;

- traditional methods for testing soil in triaxial compression devices by pre-compression and subsequent deviator loading along a crush trajectory leads to the fact that samples with the same initial density at the moment of destruction have different skeletal densities and, therefore, an increase in their shear resistance is connected not only with the increase of normal stresses, but with increasing of cohesion, too;

- the resistance of clay soils to a shift in the pre-limiting state substantially depends on the loading trajectory, since it determines the nature of changes in normal stresses and the volume of plastic deformations;

- under constant parameters of the loading trajectory, the dependency between stress intensities and shear deformations can be represented in the form of a logarithmic function, which makes it possible to more clearly fix their limiting values, namely

$$\sigma_i = \sigma_i^* [1 + (A / \sigma_i^*) \ln(\varepsilon_i / \varepsilon_i^*)], \quad 0 < \varepsilon_i \leq \varepsilon_i^*, \quad (1)$$

where  $\sigma_i^*, \varepsilon_i^*$  are the limiting values of stress intensities and shear deformation at the same time. These values are determined experimentally and depend on the average stress, the initial density, the soil moisture and the loading trajectory parameter.

$$K_\sigma = d\sigma / d\tau. \quad (2)$$

The dependence of  $\sigma_i^*$  on the average stress can be represented as

$$\sigma_i^* = c_0 + \sigma g \varphi + \sigma g \alpha_c, \quad (3)$$

where  $\alpha_c$  - is the angle of stabilizing. To describe the results of the tests on a triaxial compression device and to determine the plastic modulus  $G^p$ , the A.I.Botkin dependence can be used[1].

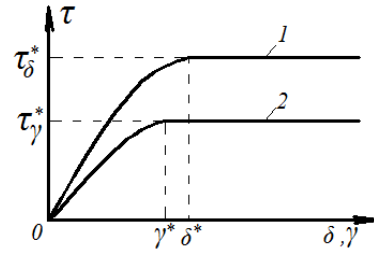


Figure 1. Dependence graphics of linear sliding  $\delta$  (1) and angular deformation  $\gamma$  (2) at a step loading with tangential stresses  $\tau$  in single-plane shear and skewing devices.

$$\sigma_i = \sigma_i^* F(\varepsilon_i), \quad (4)$$

$$\text{where } F(\varepsilon_i) = 1 + (A / \sigma_i^*) \ln(\varepsilon_i / \varepsilon_i^*)$$

$$\text{and finally } G^p = \sigma_i^* F(\varepsilon_i) / \varepsilon_i. \quad (5)$$

Volumetric plastic deformations can be determined based on the incremental ratios of the form

$$d\varepsilon_v^p = \frac{\partial \varepsilon_v^p}{\partial \sigma} d\sigma + \frac{\partial \varepsilon_v^p}{\partial \tau} d\tau \quad (6)$$

for the given loading trajectories or for the well-known values of the incremental modulus of volumetric plastic deformations depending on the loading trajectory

We will consider the plastic shear deformations of the stabilizing clay soil in the pre-limiting (by strength) condition, due to the relative slip of the soil aggregates accompanied by compaction and stabilizing within the framework of the incremental theory of plastic flow, that is, we will try to find a relationship between the regularity of plastic deformation of the skeleton and the contact resistance of the soil shear (Figure 1).

The need to use the results of soil testing for shear when describing plastic deformations in the prelimit state was noted by M.N.Goldstein on the assumption that there is a dependency between the contact resistance to shear and the value of plastic shear deformations. This problem is considered to be the main one in the modern theory of plasticity of soils, where the main issue is to determine the orientation of sliding areas in the process of plastic deformation of the soil.

To describe this process, we will consider a shift at a certain site, the normal of which  $\nu$  ( $l, m, n$ ) is oriented in a definite way in the stress space  $\sigma_{ij}$  (where  $l, m, n$  are guides cosines). At this site, the tangent  $\tau_\nu$  and normal  $\sigma_\nu$  stresses, as well as the deformations  $\gamma_\nu$  and  $\varepsilon_\nu$ , are associated with the well-known dependences of the stress and strain components [2].

In the process of active loading of clay soil, plastic deformations are due to the mutual sliding of individual soil aggregates. Inherein, the sliding process in the pre-limiting state under the influence of the next load step is of a damped nature due to stabilizing (compaction) and reorientation of the particles. In the limiting state, this process has a undamped character due to the absence of stabilizing. Moreover, in the limiting state in densed and over-compacted clays, softening (decompaction) and a change in the orientation of the particles are possible, leading to a decrease in shear resistance. Relative slip of soil aggregates in the prelimit state is possible under the condition

$$\tau_v > \sigma_v \tan \varphi + c(t) \quad (7)$$

$$\text{or} \quad d\tau_v > d\sigma_v \tan \varphi + dc(t)$$

where  $\varphi$  - is the angle of internal friction of the soil;  $c(t)$  - time-varying cohesion due to the increment of the skeleton density under the action of  $d\sigma_v$  and  $d\tau_v$ , as well as other structural changes.

Obviously, if in the process of loading there is a change in the density of the soil, then the increment of tangential stresses  $dc(t)$  will be spent on overcoming the dry friction of  $d\sigma_v \tan \varphi$  and to overcoming the cohesion increment of the clutch  $dc(t)$ . It is also obvious that the process of plastic flow in the prelimit state will enter into the damping phase as soon as the cohesion increment reaches a certain value and condition (7) will not be fulfilled.

Let us now consider the shift diagram at a given loading trajectory (Figure 2). The relationship between stresses and deformations in the elementary soil layer under consideration can be represented in the incremental form, which allows for the possibility of both elastic unloading and for the further plastic deformation.

To do this, we introduce incremental moduls of changing in the shape of  $G^t, G^e, G^p$  and volume  $K^t, K^e, K^p$  where  $t, e, p$  are indices, meaning respectively tangential, elastic and plastic parameters. The first two are determined by the branch of loading and unloading, respectively, at small steps of reloading and unloading, and the third can be expressed through the first two using the well-known formulas

$$1/G^t = 1/G^e + 1/G^p; \quad 1/K^t = 1/K^e + 1/K^p. \quad (8)$$

It follows that

$$G^t = G^p(1 + G^p/G^e); \quad K^t = K^p(1 + K^p/K^e).$$

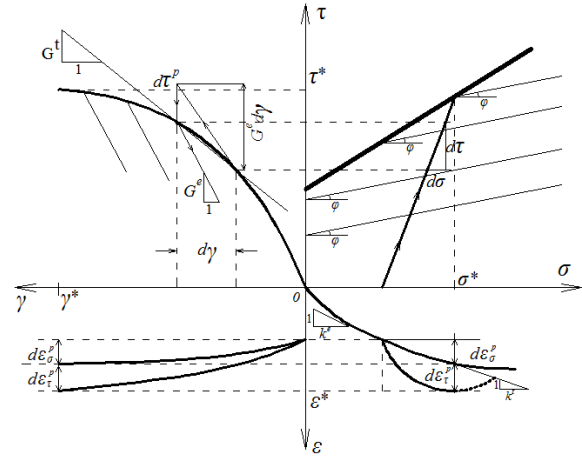


Figure 2. Shear diagram of stabilizing clay soil.

We also introduce the incremental modulus of pure shift  $g^e(\gamma)$ , whose meaning is explained below. Let us assume that in the process of reloading, the relative sliding of soil aggregates in the prelimited state occurs in accordance with the law of a viscoplastic flow, i.e.

$$\dot{\gamma}_v^p(t) = [\tau_v - \tau_v^*(t) / \eta_\gamma^p(\gamma_v^p)] \quad (9)$$

Where  $\tau_v^*(t)$  is the time-varying the strength of the soil, determined by the expression of the form

$$\tau_v^*(t) = \sigma_v \tan \varphi + c(t), \quad (10)$$

$\eta_\gamma^p$  - is a parameter of a viscous-plastic flow, depending on shear deformations  $\gamma_v$  and density  $\rho$ ;  $c(t)$  - is time-dependent cohesion due to changes in the skeleton density of the soil, etc.

With a known loading trajectory, the increment of shear and normal stresses are related by dependence (2). Then the increment of the plastic flow velocity will be determined in accordance with (9) and (10) as follows

$$\Delta \dot{\gamma}_v^p(t) = \Delta[\tau_v - \sigma_v \tan \varphi - c(t)] / \eta_\gamma^p(\dot{\gamma}_{v,p}^p). \quad (11)$$

$$\text{or} \quad \Delta \tau_v = \Delta \sigma_v \tan \varphi + \Delta c(t) + \Delta \dot{\gamma}_v^p(t) \eta_\gamma^p(\dot{\gamma}_{v,p}^p)$$

This shows that the increment of tangential stresses  $\Delta \tau_v$  is expended to overcome the increments of dry friction, the increments of cohesion changing in time and on the viscous shear resistance. Thus, the process of viscous-plastic deformation is not directly affected by the total increment,  $\Delta \tau_v$  but only by a certain part of it. It follows that the process of development of viscous-plastic deformations will significantly depend on the structural features of this and their changes, and above all from the coefficients of its shear and bulk viscosity, which in turn depend on the accumulated viscous-plastic deformation and

the rate of viscous-plastic deformations, which leads to a decrease of these coefficients.

For a certain period of time, characteristic of this type of soil and depending on the value of the step of addition, the parameter of the trajectory of the laddition and the level of acting stresses, the plastic strain rate increases to zero  $\Delta\dot{\gamma}^p \rightarrow 0$ , and the cohesion receives another  $c \rightarrow c' + \Delta c$ , increment whose value will depend on the increment of the skeleton density or increment of volumetric changes, as well as other structural changes in the soil during the shear process.

In this way, for the characteristic time  $t^*$ , plastic shear deformations  $\Delta\gamma_v^p$ , are accumulated in the ground, which are obviously associated with increments of stresses by the dependence of the form  $\Delta\gamma_v^p = (\Delta\tau_v - \Delta\sigma_v tg\varphi - \Delta c) / g^p(\gamma)$  or, going to infinitely small quantities

$$d\gamma_v^p = d(\tau_v - \sigma_v tg\varphi - c) / g^p(\gamma). \quad (12)$$

Since according to the stated theory, the accumulation of plastic deformation from the next step of the vector of addition is associated with the time  $t^*$ , characteristic for a given soil due to intrastructural changes, this theory is sometimes called the endochronic theory of plastic flow, i.e. the theory of flow, taking into account the internal time.

We assume that the cohesion increment is associated with the increment of plastic volume deformation, i.e.

$$dc = K_c d\varepsilon_v^p = K_c (d\varepsilon_{v\sigma}^p + d\varepsilon_{v\tau}^p), \quad (13)$$

where  $K_c$  is the dilatant stabilizing coefficient determined by the results of shear tests on the loading and unloading branch. Then on the basis of (2) and (6) we get

$$dc = \beta_\sigma d\sigma_v + \beta_\tau d\tau_v, \quad (14)$$

where  $\beta_\sigma = K_c / K^p$ ;  $\beta_\tau = \delta K_c / G^p$ ;  $\delta$  - is dilatancy parameter, determined by the dependence of the form:  $\delta = \partial\varepsilon_v^p / \partial\gamma_v^p$ .

Putting (14) in (12) with (2), we get

$$d\gamma_v^p = d\tau_v [1 - K_\sigma (tg\varphi + \beta_\sigma) - \beta_\tau] / g^p(\gamma) \quad (15)$$

It follows that the incremental modulus of plastic shear deformations  $G^p$  is connected with the parameters of the soil strength depending of the type

$$G^p = d\tau_v / d\gamma_v^p = g^p(\gamma) / [1 - K_\sigma (tg\varphi + \beta_\sigma) - \beta_\tau] \quad (16)$$

The parameters of  $tg\varphi$ ,  $\beta_\sigma$ ,  $\beta_\tau$  including in this equation are easily determined according to the results of standard tests, and the incremental modulus of plastic deformations  $G^p$ , can be expressed through of  $G^e$  and  $G^t$  according to the formula

$$1/G^p = (G^e - G^t) / G^e G^t$$

Therefore, it is obvious that the incremental modulus of pure shear can be determined by the formula

$$g^p(\gamma) = G^p [1 - K_\sigma (tg\varphi + \beta_\sigma) - \beta_\tau]$$

It follows from (16) that the incremental modulus of plastic shear deformations  $G^p$  depends on the loading path parameter  $K_\sigma$ , of the angle of internal friction of the soil  $\varphi$  and the stabilizing parameters  $\beta_\sigma$  and  $\beta_\tau$ . It is also seen that at  $\varphi = \beta_\sigma = \beta_\tau = 0$ ;  $G^p = g^p(\gamma)$ , i.e. the incremental modulus of pure shear  $g^p(\gamma)$  in the absence of friction and stabilizing coincides with the incremental modulus of plastic shear deformations  $G^p$ . From this it follows that in the case of such loading trajectories, when  $K_\sigma = 0$  and where in there are no stabilizing, we can assume that  $g^p(\gamma) = G^p(\gamma)$  [3].

We take as the first approximation that

$$g^p(\gamma) = g^* (\gamma_v^* / \gamma_v) \quad (17)$$

where  $g^*$  is the limiting value  $g^p$ . Then, taking into account (8), we obtain

$$d\tau_v = G^e g^* \gamma_v^* / (G^e D \gamma_v + g^* \gamma_v^*) \quad (18)$$

where  $D = 1 - K_\sigma (tg\varphi + \beta_\sigma) - \beta_\tau$ .

Integrating this equation with a constant loading path and the initial condition  $\tau_v = \gamma_v = 0$  gives

$$\tau_v = \frac{g^* \gamma_v^*}{D} \ln(1 + G^e D \gamma_v / g^* \gamma_v^*). \quad (19)$$

The limiting values of the tangential stresses  $\tau^*$  are easy to determine, assuming that  $\gamma_v = \gamma_v^*$ , i.e.

$$\tau_v^* = \frac{g^* \gamma_v^*}{D} \ln(1 + G^e D / g^*) \quad (20)$$

With the constancy of  $G^e$ ,  $tg\varphi$ ,  $\beta_\sigma$ , the shear stress limit value depends on the loading path  $K_\sigma$ .

The more  $K_\sigma$ , the more  $\tau_v^*$  and vice versa, which corresponds to the results of the experiment.



In engineering practice, there are often situations when it is necessary to predict not only the magnitudes of plastic deformations, but also their speed of development over time under the influence of constant and cyclically varying stresses. The time lag of plastic (residual) deformation is a characteristic feature of clay soils. In this case, the latency of bulk plastic deformations is insignificant as compared with shear plastic deformations and can often be neglected [3].

At the same time, it is obvious that the process of plastic deformation of the soil is accompanied by hardening due to its volumetric and structural changes, which often leads to plastic deformations that fade with time. This is especially clearly seen with stepwise loading of soil samples in devices of triaxial compression.

The proposed theory plastic flow of stabilizing soil will give the possibility to describe the deformations of soil in dependence of the load trajectory parameters  $K_\sigma$  and stabilizing and that it can be used in calculations of the stress stress-stain state of massifs clay soil.

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