Muscale modelling of plant root-soil interaction

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ABSTRACT

Plant roots thicken under increasing mechanical impedance. Thicker plant roots are known to penetrate into compacted soil more easily. This scale effect during the growth of plant roots is studied with a mechanical model. Root penetration into soil is considered as a cavity expansion problem. In order to consider the scale effect, an elastoplastic constitutive equation with strain gradient is used. The model outcome sheds some light on the scale effect in the interaction between plant root and soil.

Keywords: Plant root, cavity expansion, scale effect, strain gradient

1. INTRODUCTION

An important factor controlling the growth and yield of plants is the ability of the roots to penetrate through soil to acquire water and nutrients, which in turn has substantial influence on the development of shoots and yields. Modern agriculture gives rise to increasing compaction of soil so-called hardpan by machines. Hardpan is a layer of compacted soil close to the ground surface. It is desirable to cultivate improved crop species with strong roots to penetrate through the hardpan for better yield.

A root grows mainly from its tip. As cells are born near the tip by repeated cell divisions, a young root elongates and leaves behind cells that differentiate and become the primary roots of the plant. Three areas of the young root are traditionally recognized, i.e. the area of birth, elongation and maturation (see Figure 1). Roots grow by a process of cell division in the apical meristem just behind the tip, and cell expansion in a zone just behind the apex. The cell expansion (growth) is mainly driven by the turgor pressure inside the cells. The cell wall is extremely thin and flexible membrane with a thickness of about 1 μm. From birth to maturation, some cells elongate some 1000 times of their initial size within relatively short time. Turgor pressure is the force generated by water pushing outward on the cell membrane, and presents the mechanical energy for cell growth. The turgor pressure can be as high as 1 MPa. It is amazing how the filigree cell wall can withstand such high pressure and manage to grow rapidly at the same time. The cell materials and the growing mechanism are not yet fully understood and a subject of recent research ([1],[2]).

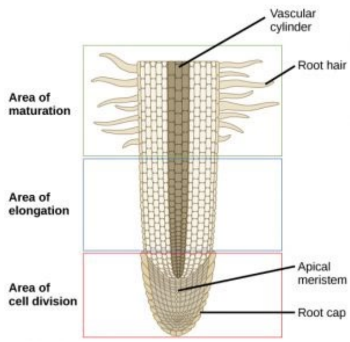


Figure 1. Anatomy of plant root

The growth of plant roots into compacted soil presents a problem of complex interaction between plant root and the surrounding soil. Plant roots are known to thicken under increasing mechanical impedance. Obviously it is beneficial for plants to have develop thicker root for strong soil. Figure 2 shows the cross sections of cotton root growing under different soil impedance. Under increasing soil strength plant roots usually become shorter and thicker. This observation has been made on various plant species and soils.

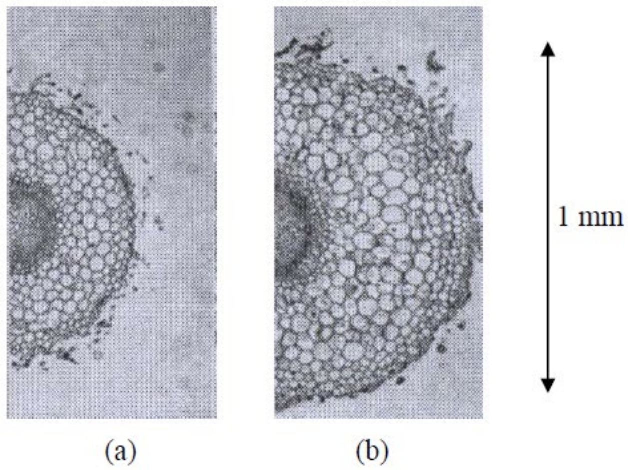


Figure 2. Cross section of cotton root under different soil densities: (a) 1.0 Mg/m3, (b) 1,5 Mg/m3. (Iijima and Kato, 2007)

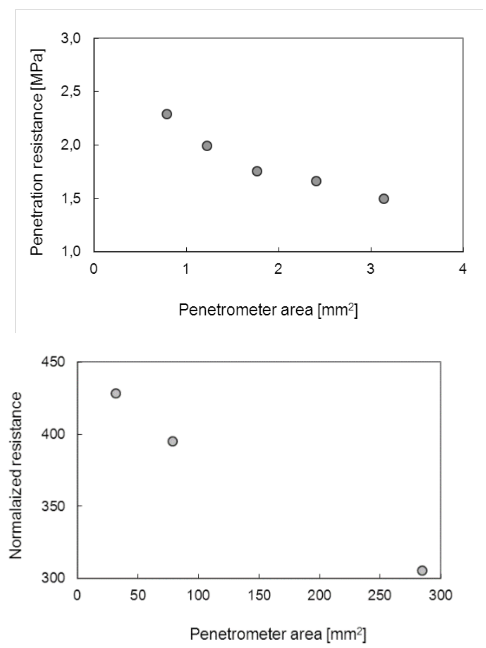
The relation between root growth and soil impedance has been a subject of intensive research ([3][4]). From the mechanical point of view, the growth of plant roots is characterized by axial penetration accompanied by radial expansion in the elongation zone. The obvious similarity between root penetration and the penetration of projectile in soil mechanics has motivated some early studies in this area. The well-known Lockhart equation presents an early attempt to understand the growing mechanism with the help of a simple mechanical model.

Root penetration was studied as cavity expansion by Farrel and Greacen [5]. Previous investigations on the penetration resistance of roots, e.g. Misra et al. ([6],[7]), were mainly based on the theory of cavity expansion by Vesic [8]. The soil surrounding the root was modelled by an elastic, perfectly plastic material. Recently, root penetration was investigated by Kirby and Bengough [9] using Critical State Model and FEM, which presents a refinement of the analytical model. These studies help better understand the relationship between the soil impedance and the turgor pressure. However, they cannot explain why plant roots choose to thicken in strong soil or why thicker roots possess better penetrability.

In geotechnical engineering, the penetration tests are used to characterize the compaction of agriculture soils. The penetrometers have typically a diameter between 10 cm and 20 cm, which are much larger than the usual root diameter, between 0.2 mm and 2 mm. Some laboratory tests with small penetrometers show clear dependence of the penetration pressure depends on the penetrometer diameter [10]. This scale dependence is elusive to the classical theory of cavity expansion, since the constitutive equation does not involve any length scale. In order to explain the scale effect in cavity expansion, the constitutive model need be endowed with a characteristic length.

2. EXPERIMENTAL EVIDENCE

Some experiments with small sized penetrometers are available from the literature. The experimental results by Whiteley and Dexter [10] and by Bolton et al. [11] are evaluated and shown in Figure 3. The tests by Bolten and his co-workers were carried out in a geotechnical centrifuge. The soil used by Whiteley and Dexter is a silty soil with a mean grain diameter of about 0. 1 mm. The specimen was prepared by remolding the soil at the water content close to its plastic limit of about 17.3%. The soil used by Bolton and his co-workers is clean sand with a mean grain diameter of about 0.9 mm. The penetration resistance in Figure 3(a) is obtained by normalizing the penetration force with the cross-section area of the penetrometers. The normalized resistance in Figure 3(b) is defined by ratio between the penetration pressure and the overburden stress.



(a)

(b)

Figure 3. Penetration tests (a) by Whiteley and Dexter [10] and (b) by Bolton et al. [11]

In both tests the penetration resistance decreases with increasing penetrometer diameter. For large diameter the penetration resistance is expected to approach a limit value, which can be obtained from the classical theory of cavity expansion. Obviously this scale dependence cannot be predicted by the classical theory, since the constitutive equation does not contain any term with length scale.

3. STRAIN GRADIENT MODEL

Until now the mechanical models based on cavity expansion use elastoplastic constitutive equation without length scale. Such models are self-similar and cannot explain the scale effect observed in plant roots and in penetration tests. In order to account for the scale effect, enhanced constitutive equations are needed by including higher order terms. The higher order terms give rise to the length scale, which can be correlated to some material properties like the mean grain diameter of soil. There are mainly three approaches to introduce higher order terms into the constitutive equation, i.e. the Cosserat theory [7], nonlocal theory [12] and strain gradient theory [13].

We will make use of the strain gradient theory by Vardoulakis and Aifantis [13]. The theoretical framework is briefly recapitulated below. The total strain increment is decomposed into elastic and plastic parts

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p} \tag{1}$$

The elastic strain increment can be obtained by a generalized Hooke's law, which can be written as follows

$$d\sigma_{ij} = C_{ijkl}d\varepsilon_{kl}^{e} \tag{2}$$

The plastic strain increment can be obtained with the help of the plasticity theory, which consists of the well-known procedures of yield function, flow rule and hardening parameter. To this end, let us consider the Drucker-Prager yield function

$$f(\sigma, k) = \alpha I_1 + \sqrt{J_2} - k = 0 \tag{3}$$

where $I_1 = \sigma_{ii}$ is the first invariant of stress tensor and $J_2 = s_{ij}s_{ij}/2$ is the second invariant of the stress deviator. The parameter α and k can be related to the soil strength parameters, i.e. the friction angle and cohesion. The hardening parameter k is assumed to depend on the higher order of strain gradient

$$k = k(\varepsilon_e) + \beta \nabla^2 \varepsilon_e \tag{4}$$

where $\varepsilon_e = \sqrt{2\varepsilon_{ij}\varepsilon_{ij}/3}$ is the effective plastic strain and β is a material parameter. The gradient term in (4) requires an additional boundary condition $(\nabla \varepsilon_e)_i n_i = 0$ with n_i being the outward normal to the boundary [14]. The plastic strain increment can be expressed by the following flow rule

$$d\varepsilon_{ij}^{p} = \lambda \frac{\partial f}{\partial \sigma_{ij}} \tag{5}$$

where λ is a multiplier and can be determined by taking the total differential of the yield function (consistency condition). By relating the material parameter β in (4) with the mean grain diameter of soil, the scale effect in the experiments can be modelled. The following relationship is used

$$l = \sqrt{\frac{\beta}{G}} \tag{6}$$

where G is the shear modulus of soil. The above formulation represents the simplest version of gradient theory by introducing the higher order term into the yield function while leaving the other features of plasticity theory unchanged.

4. CAVITY EXPANSION AND SOLUTION

Let us consider the cavity expansion of a thick cylinder under plane strain condition (see Figure 4). The cylinder has the inner diameter a and outer diameter b. The cylinder is subjected to a uniform internal pressure p and a uniform outer pressure p_0. For large diameter b the pressure p_0 will approach the far field pressure. The Since both the geometry and the load are axisymmetric, all field variables depend only on the radial displacement u. The axial and the circumferential displacements are zero. Moreover, there are only two non-vanishing strain components, i.e. the radial strain ε_r and the circumferential strain ε_θ.

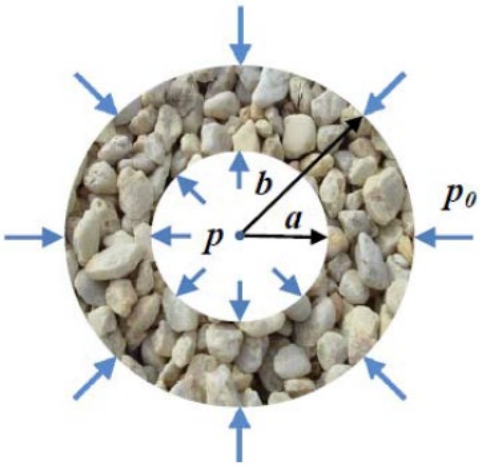


Figure 4. Cavity expansion problem

The problem formulation includes the equilibrium equation, the constitutive equation and the boundary conditions. The problem can be further simplified by assuming that the material is not compressible. This is a reasonable assumption for most agriculture soils. In this case the radial displacement can be written out $u = C/r$ with C being an integration constant. The resulting differential equation can be solved with the relaxation method. The numerical results show that the limit pressure depends on the cavity diameter a. For large cavity diameter, the solution approaches the classical solution asymptotically.

The equilibrium equation gives rise to an ordinary difference equation, which can be solved numerically. Figure 5 shows the relationship between pressure and cavity diameter. Obviously the pressure-diameter curves depend on the length scale.

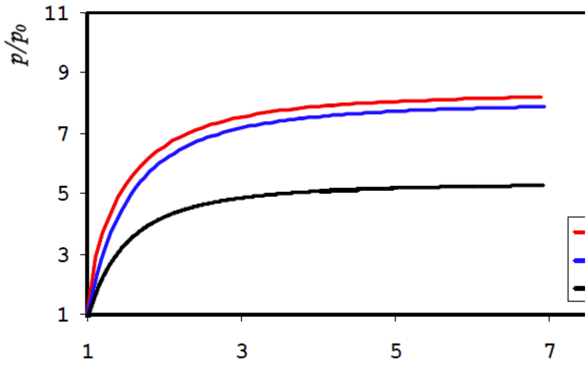


Figure 5. Pressure-expansion response with the cavity radius for cohesive-frictional soil, $l = 0.02$ mm

The relationship between the limit pressure and the cavity diameter is given in Figure 6 for both classical and gradient theories. The limit pressure shows size dependence for small size cavity. For large diameter (> 1 mm), the limit pressure approaches a limit value, which is given by the classical theory. Obviously, this scale dependence cannot be predicted by the classical theory, since the constitutive equation does not contain any term with length scale.

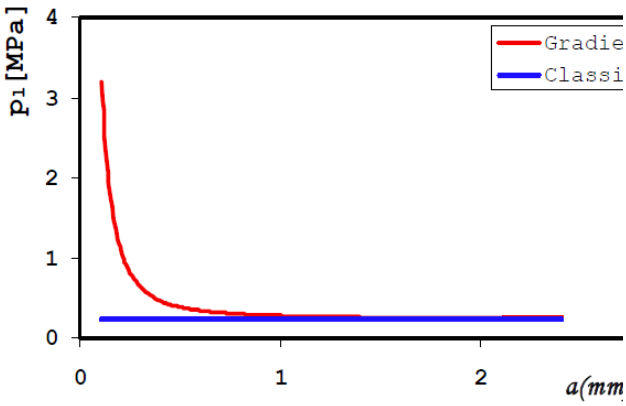


Figure 6. Relationship between normalized limit pressure with cavity radius

Smaller cavities give rise to higher limit pressure. Such size dependence is also observed in other problems, e.g. twisting and bending of tiny beams. Smaller structures are usually stronger than larger ones. One explanation is the large number of defects in a larger structure.

5. CONCLUSIONS

That growing roots in strong soil become thicker is a well confirmed observation. However, this phenomenon is rather difficult to grasp and has continued to defy a consistent theoretical explanation. Imagine that you try to push a rod into the ground and feel increasing resistance. Will you change for a thicker rod? Our theoretical model sheds some light on this intricate phenomenon.

According to the classical theory of cavity expansion, an increase in root diameter does not bring any benefit since the penetration force is proportional to the cross section of the root. It would be more economic for the root to increase the penetration force rather than the diameter. There are two conjectures as to why roots thicken under increasing impedance. Farrel and Greacen [5] suggested that the larger radial expansion of thicker root may cause cracking near the root tip, which in turn makes root penetration easier. Logsdon et al. [15] showed that thicker roots possess higher strength against buckling. However, none of these conjectures is verified by experiments.

The driving force is known to be provided by the turgor pressure in the root cells. When faced with increasing mechanical impedance, plant roots may choose either to increase the turgor pressure while keeping the root diameter unchanged or to increase the root diameter while keeping the turgor pressure unchanged. Plant root may of course increase both the diameter and the turgor pressure simultaneously. Note that the turgor pressure (about 1 MPa) is not unlimited and is mainly dictated by the behaviour of the cell walls. Therefore, it makes sense to increase the root diameter instead of the turgor pressure. Some experiments in plant science indicate that the turgor pressure does not show remarkable change when roots are mechanically impeded. For constant turgor pressure, the penetration force will be proportional to the cross section area of the roots. However, root growth is enhanced by the reduced penetration resistance from the soil. Finally, it should be noticed that the interaction between root and soil is not purely mechanical. In general, compaction changes not only the mechanical strength of soil but also its water potential, which requires a hydro-mechanically coupled analysis.

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