Theory of loosening earth pressure on a shallow tunnel in unsaturated ground

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ABSTRACT

A theory for estimating the loosening earth pressure on a trapdoor in unsaturated soil is proposed in this paper. The proposed theory is a limit equilibrium theory, in which a soil–water characteristic curve, effective stress for unsaturated soils proposed by Bishop and the Mohr-Coulomb criteria are combined. The theory is validated by comparing with Terzaghi's theory under fully-saturated conditions. We further investigate the loosening earth pressure under varied groundwater level. It is revealed that total loosening earth pressure decreases and effective loosening earth pressure increases when the groundwater level is lowered.

Keywords: loosening earth pressure; shallow tunnel; unsaturated ground; the degree of saturation; soil water characteristic curve; limit equilibrium theory

1 INTRODUCTION

Fluctuation of groundwater level varies earth pressure around a tunnel and may affect the stability of the tunnel. However, loosening earth pressure–vertical earth pressure in the ground above a tunnel after excavation–is usually evaluated assuming fully-dried or fully-saturated condition (Terzaghi, 1943) in practical engineering (e.g. Zhang et al., 2016; Chen and Peng, 2018). Meanwhile, coupled numerical methods for unsaturated soils tend to require much time and evaluating the loosening earth pressure on a tunnel in unsaturated ground is needed especially from the practical viewpoint.

Theories for unsaturated soil have been gradually established over the past decades. The effective stress tensor for unsaturated soil (Bishop, 1959) has been applied usually as it can uniquely describe the critical state stress ratio regardless of degree of saturation. Several soil-water characteristic curve models have been proposed to describe variation in water content or degree of saturation with suction (e.g. van Genuchten,1980; Fredlund and Xing, 1994). Owing to the existing theories on unsaturated soils, a limit equilibrium theory for evaluating loosening earth pressure on a trapdoor in unsaturated ground is proposed and validated in this study.

2 THEORY FOR LOOSENING EARTH PRESSURE IN UNSATURATED GROUND

We herein consider a shallow trapdoor problem in unsaturated ground with groundwater depth, H_w, as shown in Figure 1. A trapdoor of a

z: vertical downward coordinate;
x: horizontal coordinate;
Other symbols are indicated in text.

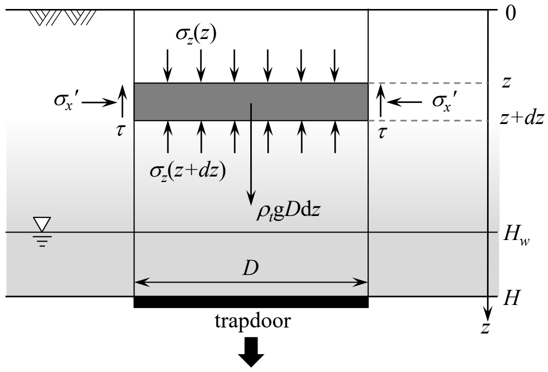

Fig. 1. Trapdoor problem in unsaturated soil.

width of D, locating at the depth of H, simulates stress release during a tunnel excavation by its vertical-downward descent. The ground is assumed to be uniform, sandy ground having solid density of ρ_s and dry density of ρ_d (void ratio, $e = \frac{\rho_s}{\rho_d} - 1$).

2.1 Failure mode, equilibrium and boundary condition

We assume two failure surfaces developed vertically upward from both ends of the trapdoor as shown in Fig. 1. For this failure mode, an equation of equilibrium of vertical forces acting on a small soil element above the trapdoor is given. Applying Taylor's expansion, the equilibrium equation reduces to:

$$\frac{d\sigma_z}{dz} = \rho_t g - \frac{2}{D}\tau \qquad (1)$$

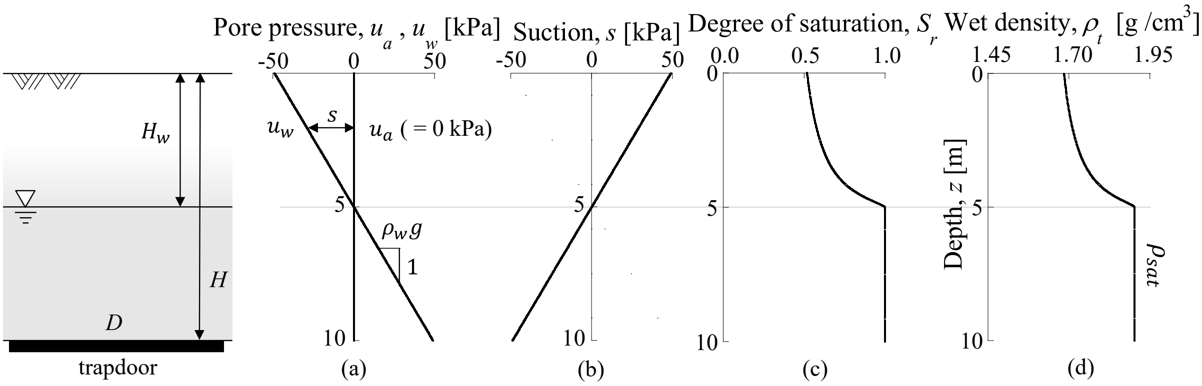

Fig. 2. Vertical distributions of pore pressure, suction, the degree of saturation and wet density (H_w = 5 m).

where $\sigma_z(z)$ is total vertical stress; τ is shear resistance; ρ_t is wet density of the soil; g is gravitational acceleration. As we consider traction free condition at the surface of the ground, boundary condition is given as $\sigma_z(0) = 0$.

2.2 Pore pressures and suction

Pore air pressure, $u_a(z)$, and pore water pressure, $u_w(z)$, are both assumed to be under static pressure conditions. u_a is assumed to be zero (atmospheric pressure) at any depth as the density of air is negligible, and u_w has a linear distribution $\left(u_w = \rho_w g(z - H_w)\right)$ in vertical direction with a slope of $\rho_w g$ (ρ_w is density of water) (Pufahl et al., 1983; Ng and Menzies, 2007). Suction, $s(z)$, is given as the difference between u_a and u_w.

$$s = u_a - u_w = \rho_w g(H_w - z) \tag{2}$$

2.3 Degree of saturation and wet density

Degree of saturation, $S_r(z)$, is given as a function of suction, s, by a classical soil-water characteristic curve (van Genuchten, 1980):

$$S_r = S_{min} + (S_{max} - S_{min})\{1 + \langle \alpha s\rangle^n\}^{-m} \tag{3}$$

where $\langle \quad \rangle$ denotes Macaulay brackets, S_{max} and S_{min} are the maximum and minimum degrees of saturation, respectively, and α, m and n are material parameters. Vertical distribution of S_r is given by combining Eqs. (2) and (3). Wet density, ρ_t, varies with S_r as follows.

$$\rho_t = \frac{\rho_s + eS_r\rho_w}{1 + e} \tag{4}$$

where ρ_w is the density of water. From Eqs. (2) to (4), S_r and ρ_t are given as functions of depth z, respectively. Vertical distributions of u_w, u_a, s, S_r and ρ_t are shown in Fig. 2.

2.4 Effective stress and shear resistance

Effective stress proposed by Bishop (1959) is usually applied for unsaturated soils, as it can uniquely arrange critical state stress ratio of unsaturated soils regardless of S_r as shown in Fig. 3. Using this, σ_z in Eq. (1) is given as:

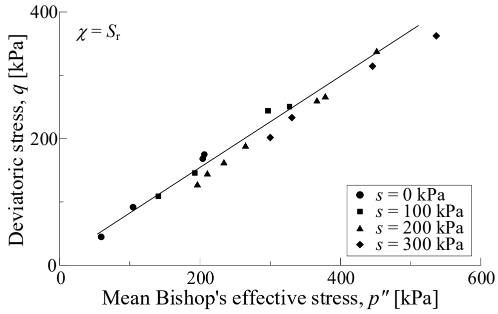

Fig. 3. Relation between mean stress, p'', and deviator stress, q, of Bishop's effective stress (replotted from Sivakumar, 1993).

$$\sigma_z = \sigma_z' + (1 - \chi)u_a + \chi u_w \tag{5}$$

where χ is effective stress parameter, for which S_r is usually applied for simplicity (Schrefler, 1984; Lu and Likos, 2006), and σ_z' is Bishop's vertical effective stress.

Mohr-Coulomb criteria is assumed to be satisfied by Bishop's effective stress along the vertical failure surfaces (Fig. 1):

$$\tau = \sigma_x' \tan\phi \tag{6}$$

where σ_x' is Bishop's horizontal effective stress and ϕ is internal friction angle of soil.

2.5 Earth pressure coefficient

A coefficient of earth pressure, K, is considered for Bishop's effective stress in the ground above the trapdoor in the similar way as the classical theory in saturated ground (Terzaghi, 1943).

$$K = \frac{\sigma_x'}{\sigma_z'} \tag{7}$$

For saturated ground, K tends to be larger than earth pressure coefficient at rest and is nearly unity (Kikumoto et al., 2003).

Table 1. Physical properties and their values of loam.

ρ_s	2.65 g/cm^3	Solid density
ρ_d	1.45 g/cm^3	Dry density
ϕ	30°	Internal friction angle

Table 2. Parameters for SWCC and their values of loam.

S_{max}	1.0000	Maximum saturation
S_{min}	0.2975	Minimum saturation
α	0.246 kPa^{-1}	Parameters for the shape of SWCC
m	0.3155	
n	1.461	

2.6 Loosening earth pressure

Loosening earth pressure, $\sigma_z(z)$, is derived by solving the ordinary differential Eq. (1) with Eqs. (2) to (7) under the boundary condition that $\sigma_z(0) = 0$. For this, we applied a simple, explicit numerical scheme.

3 RESULTS AND DISCUSSIONS

The proposed theory is applied to evaluate loosening earth pressure in loamy ground with different groundwater depths. The properties of the loamy ground are given in Tables 1 and 2. Overburden height, *H*, and width of the trapdoor, *D*, are both set to be 10 m throughout this paper, as several studies (e.g. Koutsabeloulis and Griffiths, 1989) indicated that the shear bands developed from the side of a tunnel usually reach the ground surface if overburden ratio *H*/*D* is one or smaller. Parameters for the SWCC of the loamy ground are determined referring to Hodnett and Tomasella (2002).

3.1 Loosening pressure in saturated ground (comparison with Terzaghi's theory)

Loosening earth pressure in fully-saturated ground (H_w= 0.0 m) is evaluated by the proposed theory and Terzaghi's theory, respectively, and they are compared in Fig. 4. The initial stress represents the vertical earth pressure at rest, and the loosening earth pressures represents the earth pressure after lowering of the trapdoor (tunnel excavation). Loosening total and effective earth pressures are both smaller than their initial pressures, respectively. It is also seen that the total and effective loosening earth pressures calculated by the proposed theory are identical with those calculated by Terzaghi's theory, which validates the consistency of the proposed theory.

3.2 Loosening pressure in unsaturated ground

The loosening earth pressure in unsaturated ground (H_w = 5.0 m) is evaluated by the proposed theory and the results are shown in Fig. 5. Loosening effective and total earth pressures are both smaller than the corresponding initial stresses at any depth. The effective stress is larger

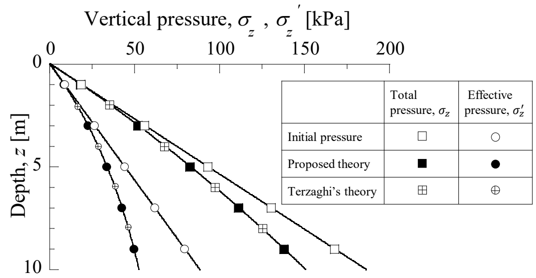

Fig. 4. Depth vs. vertical earth pressures (saturated ground; H_w = 0.0 m).

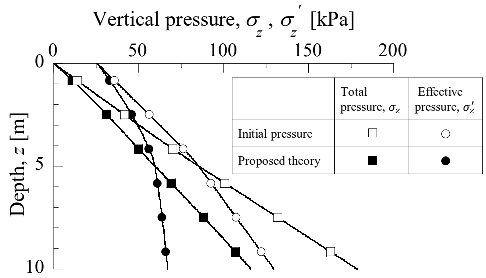

Fig. 5. Depth vs. vertical earth pressures (unsaturated ground; H_w = 5.0 m).

than the total stress in unsaturated zone above the groundwater level, which contributes the increase in shear resistance. Meanwhile the magnitude correlation is reversed in the saturated zone.

3.3 Effect of the movement of groundwater level on loosening earth pressure

The groundwater level is varied in the calculation using the proposed theory in order to discuss its effect on loosening earth pressure.

Fig. 6 shows the vertical distributions of degree of saturation and total and effective loosening earth pressures for groundwater depth H_w of 0.0, 2.5, 5.0, 7.5 and 10.0 m. As discussed with Fig. 5, the effective stress is larger than total stress at any depth in the unsaturated zone due to the effect of suction. It is also known that the difference of groundwater depth, H_w, has significant influence on loosening earth pressure: the total loosening earth pressure is smaller at any depth in larger H_w case, while the effective earth pressure is larger. One of the reasons is that: in case of larger H_w (deeper groundwater level), saturation degree, S_r, is smaller as shown in Fig. 6 (a), wet density is also smaller and overburden pressure (initial total earth pressure) for larger H_w case is smaller. Another reason is that, in case of larger H_w, effective stress is larger as suction is larger and shear resistance exerted to reduce the vertical pressure on the trapdoor becomes larger.

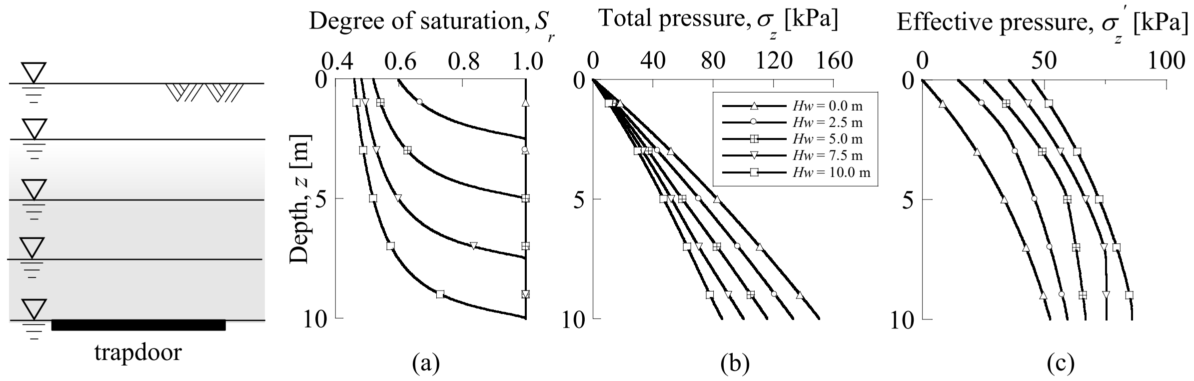

Fig. 6. The curve of (a) distribution of degree of saturation, (b) total earth pressure and (c) effective earth pressure versus depth with different depth of groundwater level (overburden height-H is 10 m).

4 CONCLUSIONS

A simple theory for estimating the loosening earth pressure in shallow trapdoor problems in unsaturated ground has been proposed in this paper. The proposed theory is developed based on limit–equilibrium theory combining a soil–water characteristic curve and effective stress for unsaturated soils (Bishop, 1954). The validity of the theory was first checked by comparing calculation results with those by Terzaghi's theory under fully-saturated condition and could be successfully confirmed.

Furthermore, the loosening earth pressure in unsaturated ground with different groundwater levels were investigated to investigate the effect of the groundwater level. It is demonstrated that groundwater depth, H_w has significant effect on the loosening earth pressure. The total loosening earth pressure in unsaturated ground is smaller than that in saturated ground, while the effective loosening earth pressure is larger. Shear resistance in unsaturated ground becomes larger due to the effect of suction on the effective confining pressure. It is also revealed that the magnitude correlations of total and effective pressure for different groundwater depth are opposite in any depth, and are reversed at the groundwater level.

It is well-known that moderately-wet beach sands under an unsaturated condition can sustain a steep excavated surface (such as tunnel wall) rather than a fully saturated or dry state due to capillary forces. The proposed theory could successfully describe such mechanism of unsaturated ground.

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