Applicability of energy-based simplified evaluation on earthquake-induced slope displacements

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ABSTRACT

Simplified numerical analyses are conducted wherein the Newmark type slope model is shaken underneath by vertically propagating SH wave. Earthquake energy for slope sliding as energy difference between upward and downward waves is confirmed to balance with the energies associated with slope sliding. The residual slope displacements δ_r are uniquely correlated with the energy irrespective of earthquake waves, indicating that δ_r can be readily evaluated without using acceleration time-histories. The evaluation procedure has been developed using the analytical results and empirical formulas on wave energies. An example study for slope displacements of varying hypocenter distances during a M6.8 earthquake has shown a qualitative compatibility with a case history of road embankments.

Keywords: earthquake energy; slope failure; residual displacement; design chart

1 CONCEPT OF ENERGY-BASED METHOD

To evaluate seismic slope failures in terms of energy, an energy approach was previously proposed by the present author (Kokusho & Ishizawa 2007). In that method, earthquake induced slope displacement δ_r for an infinitely long slope is expressed simply as;

$$\delta_r = E_{eq} / \left[\rho g D \tan(\phi - \theta) \right] \quad (1)$$

where, E_{eq} = earthquake energy for slope displacement in unit area, ρg and D = unit weight and thickness of sliding soil, ϕ = mobilized slope friction angle including cohesion, and θ = slope angle. This equation was theoretically derived from energy balance in slope failure and demonstrated by model shaking table tests.

In applying Eq. (1) to slopes as schematically shown in Fig. 1, it is necessary to determine E_{eq} in field conditions by steps illustrated in Fig. 2. If a design earthquake motion is given at a base, the cumulative incident energy E_{IP} per unit area is formulated (Kokusho 2017);

$$E_{IP} = (\rho_s V_s)_{base} \int \left[\dot{u}(t) \right]^2 dt \quad (2)$$

where, $\dot{u}(t)$ =time history of upward-propagating particle velocity and $(\rho_s V_s)_{base}$=S-wave impedance at the base. If the earthquake motion is not available, E_{IP} may be roughly estimated at a seismological base layer corresponding to V_s=3000 m/s and ρ_s=2.7 t/m^3 as;

$$E_{IP} = E_0 / 4\pi R^2 , \quad \log E_0 = 1.5M + 1.8 \quad (3)$$

where M=earthquake magnitude, R=hypocenter distance and E_0=total energy released based on Gutenberg (1956).

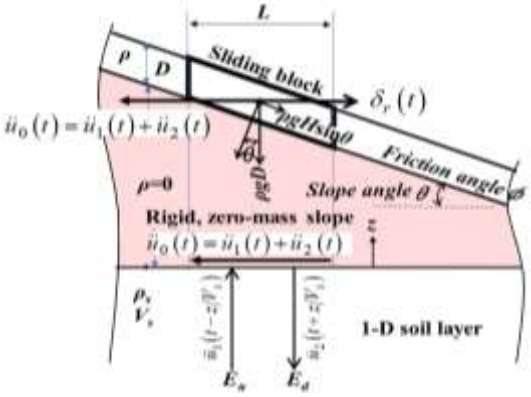

Fig.1. Infinitely long slope model with SH-wave.

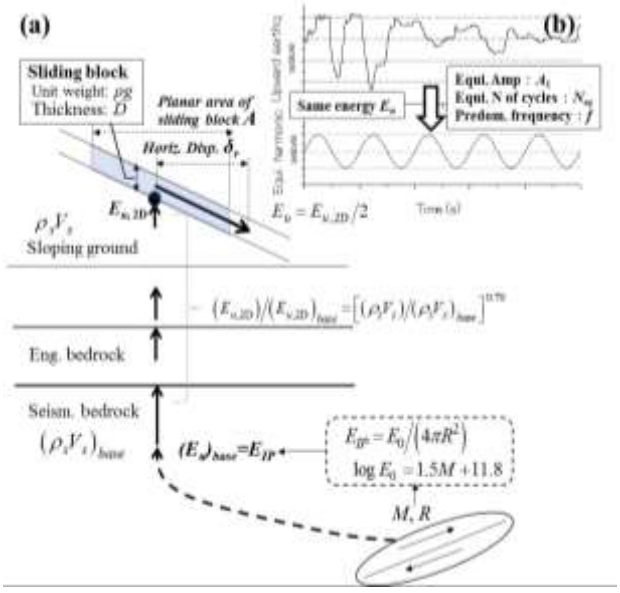

Fig. 2. Evaluation steps for input earthquake energy for energy-based slope failure evaluation.

Vertical array earthquake observation data demonstrated

that observed energies at seismological base layer E_{IP} are mostly compatible with Eq. (3). Total upward energy $E_{u,2D}$ summed in 2 horizontal directions at a slope can be evaluated using another empirical formula based on strong motion vertical array records (Kokusho & Suzuki 2012) as;

$$E_{u,2D}/E_{IP} = \left[(\rho_s V_s)/(\rho_s V_s)_{base}\right]^{0.7} \quad (4)$$

wherein $\rho_s V_s$ and $(\rho_s V_s)_{base}$ are S-wave impedance at the slope and base, respectively. The upward energy E_u in sloping direction should be $E_u = E_{u,2D}/2$ averagely for a soil mass to slide in that direction according to previous study on earthquake observation (Kokusho et al. 2014).

2 ENERGY-BASED NEWMARK SLOPE MODEL

From the upward energy E_u beneath a potential sliding block, the earthquake energy E_{eq} for slope sliding is to be determined. For that goal, "Energy-based Newmark slope model" has been developed and shaken by SH-wave as depicted in Fig. 1, wherein a shaded virtual slope body of infinite rigidity and no mass is on the top of a horizontal layer where the SH-wave propagates vertically with S-wave velocity V_s as;

$$\ddot{u}(t,z) = \ddot{u}_1(t - z/V_s) + \ddot{u}_2(t + z/V_s) \quad (5)$$

At the surface of the horizontal layer (z=0);

$$\ddot{u}(t,0) \equiv \ddot{u}_0(t) = \ddot{u}_1(t) + \ddot{u}_2(t) \quad (6)$$

Relative acceleration for the block sliding downslope in the Newmark model is expressed as (Sarma 1975);

$$\ddot{\delta}_r(t) = \left[\ddot{u}_0(t) - g\tan(\phi-\theta)\right]\cos(\phi-\theta)\cos\theta/\cos\phi \quad (7)$$

where ϕ =friction angle between the block and slope and θ = slope angle. In Eq. (7), $\ddot{\delta}(t) > 0$ only if

$$\ddot{u}_0 - g\tan(\phi-\theta) > 0 \quad (8)$$

Horizontal force equilibrium of the block coupled with the SH-wave vibration transmitted through the virtual slope body can be expressed as;

$$\rho D\left(\ddot{u}_0(t) - \ddot{\delta}_r(t)\right) + G_s\left(\partial u(t,z)/\partial z\right)\Big|_{z=0} = 0 \quad (9)$$

where $\ddot{u}_0(t) - \ddot{\delta}_r(t)$ =absolute acceleration of the block, and $G_s = \rho_s V_s^{\,2}$ =shear stiffness. Substituting Eq. (5) into Eq. (9) yields the following basic equation.

$$\rho D\left(\ddot{u}_1(t) + \ddot{u}_2(t) - \ddot{\delta}_r(t)\right) = \rho_s V_s\left[\dot{u}_1(t) - \dot{u}_2(t)\right] \quad (10)$$

Eq. (10) together with Eqs. (7) and (8) can solve the slope system shown in Fig. 1 and its stationary harmonic response for angular frequency ω can be obtained by substituting

$$\left.\begin{aligned} \ddot{u}_1(t) &= A_1 e^{i\omega t} \quad \dot{u}_1(t) = (A_1/i\omega)e^{i\omega t} \\ \ddot{u}_2(t) &= A_2 e^{i\omega t} \quad \dot{u}_2(t) = (A_2/i\omega)e^{i\omega t} \end{aligned}\right\} \quad (11)$$

into Eq. (10) as;

$$i\frac{\omega\rho D}{\rho_s V_s}\left[(A_1 + A_2)e^{i\omega t} - \ddot{\delta}_r(t)\right] = (A_1 - A_2)e^{i\omega t} \quad (12)$$

This indicates that $\alpha = \omega\rho D/(\rho_s V_s)$ serves as a key parameter controlling Eq. (10), named here as impedance ratio, wherein $\omega\rho D$ has the same dimension as $\rho_s V_s$. The nonlinear equation Eq. (10) for $\ddot{\delta}(t) \neq 0$ has to be solved by numerical methods as explained below.

3 NUMERICAL ANALYSIS

Time integration of Eq. (10) together with Eqs. (7), (8) was implemented for a given input harmonic motion by using Wilson's "θ-method" with "θ" =1.4 to have a stable solution. Fig. 3 exemplifies the numerical results of the slope of ϕ =35°, θ =30°, D=10 m, V_s=200 m/s, and $\rho = \rho_s$ =1.8 t/m^3. As the input wave, a 10-cycle harmonic wave of frequency f=1.0 Hz, was given wherein the amplitude increases from 0 to 100% linearly with time until 5th cycle followed by a constant amplitude A_1=2.0 m/s^2 to avoid unfavorable effects of initial conditions. The slope starts to slide at 2nd cycle when $\ddot{u}_0$ exceeds threshold (0.85 m/s^2) defined by Eq. (8), and accumulates downward displacements. The bottom frame of Fig. 3 shows time-dependent variations of the associated energies. As the difference of energy $E_{eq} = E_u - E_d$ and its sum with the gravitational energy E_{gr} accumulate with time, $E_{gr} + E_{eq}$ is observed to be almost identical with the energy dissipating between the block and slope E_{ds}. This is compatible with the theoretical energy balance already discussed in previous papers by the author (e. g. Kokusho 2017).

Fig. 4(a) shows a slope displacement δ_r versus earthquake energy E_{eq} relationship obtained as a stationary response per one cycle (in the 10th cycle of the tapered harmonic wave for different input accelerations). The calculations conducted for 3 different frequencies f=0.5~1.0 Hz tend to give a unique correlation for the displacement δ around 0.3 m or smaller though they tend to diverge with increasing δ for higher f in particular presumably due to errors in the numerical

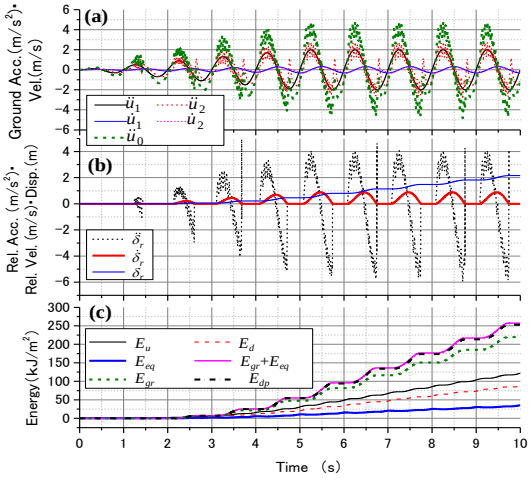

Fig. 3. Example analytical result of Energy-based Newmark slope model by a tapered harmonic wave (ϕ =35°. θ =30°)

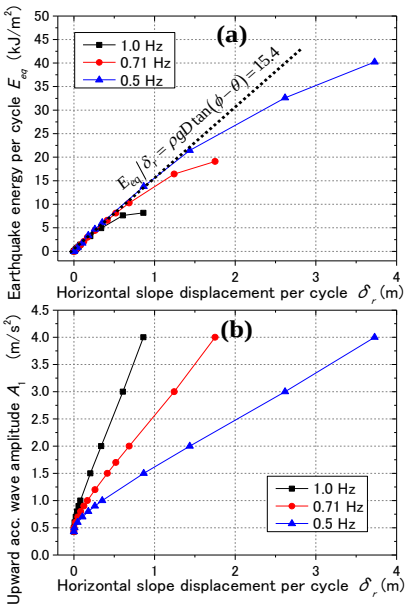

Fig. 4 Earthquake energy or upward acceleration amplitude versus horizontal slope displacement : (a) E_q～δ, (b) A_1～δ

analysis. Also note that for the small displacements the calculated results coincide with dashed straight line in the diagram representing Eq. (1) derived theoretically from a simple energy principle (Kokusho & Ishizawa 2007). In Fig. 4(b), the same calculated δ -values are plotted versus horizontal slope accelerations ($\approx 2A_1$) quite differently for three f-values, indicating that not the acceleration but the energy can serve as a unique indicator for slope displacements as already observed in previous model tests (Kokusho & Ishizawa 2007).

A series of analyses conducted for pertinent parameters, ϕ =35º, θ =20~30º, f=0.5~1.0Hz, D=2.5~10 m, V_s=150~300 m/s, yield relationships between normalized energies $E_{eq}/E_u/\alpha$ and $E_u/N_{eq}/E_{u0}$ superposed in Fig. 5, where E_u and E_{eq} are upward and dissipated wave energies of an given earthquake,α is impedance ratio involved in Eq. (12), N_{eq}= number of cycles of equivalent harmonic motion and E_{u0} is threshold upward energy for initiating slope sliding expressed as;

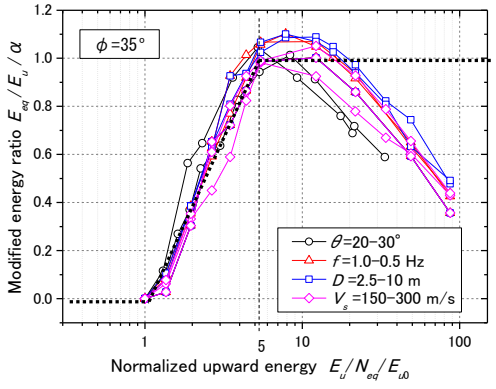

Fig. 5. $E_{eq}/E_u/\alpha$ and E_u/E_{u0} relationships numerically

$$E_{u0} = \frac{\pi \rho_s V_s}{4\omega^3} g^2 \tan^2(\phi - \theta) \tag{13}$$

for an equivalent harmonic motion with its angular frequency $\omega = 2\pi f$. Despite data dispersions due to calculation errors in the nonlinear analyses presumably, a trilinear dashed line may be drawn commonly for all the parameters considered here, yielding the following equation;

$$\left.\begin{array}{ll} E_u/N_{eq}/E_{u0} < 1.0: & E_{eq}/E_u/\alpha = 0 \\ 1.0 < E_u/N_{eq}/E_{u0} < 5.0: & E_{eq}/E_u/\alpha = 1.43\log_{10}\left(E_u/N_{eq}/E_{u0}\right) \\ 5.0 \le E_u/N_{eq}/E_{u0}: & E_{eq}/E_u/\alpha = 1.0 \end{array}\right\} \tag{14}$$

Using earthquake energy E_{eq} here, the slope residual displacement δ_r can be calculated from Eq. (1) using upward energy E_u depending on the impedance ratio α and the threshold upward energy for sliding E_{u0}.

4 EVALUATION EXAMPLE

Fig. 6(a) exemplifies a typical slope profile for conventional Newmark-type slope analysis along a circular slip surface, wherein the centroid O of soil block BCD slides to O′ of B′ C′ D′. If the sliding displacement is not so large (less than a few meters for normal engineering design), the line OO′ may be approximated parallel to the line BD with its angle θ. Hence, it may be replaced in the energy method by a slide of a soil mass (horizontal length L) on an infinitely long slope with the angle θ as depicted in Fig. 6(b), wherein the mass M and planar slip area $A = L \times 1$are the same (exposed to the same upward wave energy) and the sliding soil thickness $D = M/(\rho A)$. In some cases, the slope profile in 6(b) may be directly employed for the analysis rather than replacing the circular slip analysis.

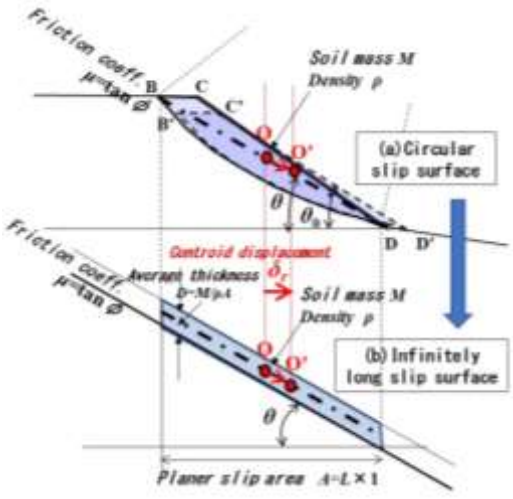

Fig. 6. Slope failures along circular surface (a) and straight surface (b) having the same soil mass and planer area

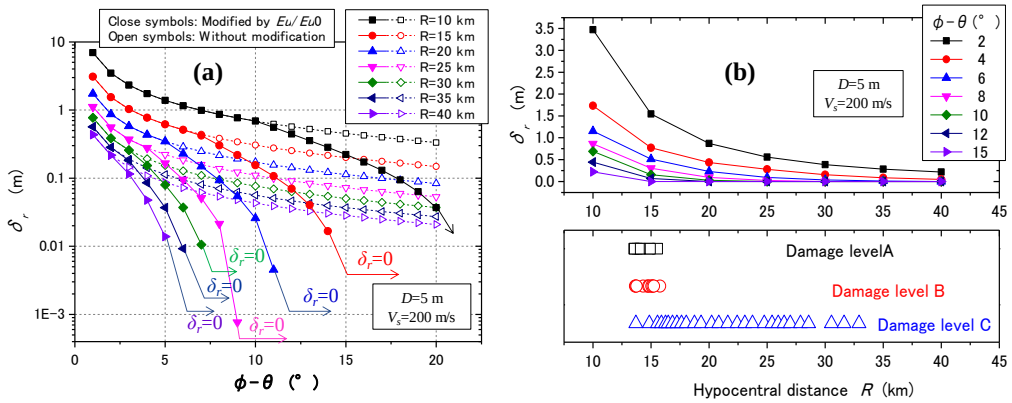

Fig. 7. Example evaluation of embankment slope displacements by M=6.8 earthquake: (a) displacements δ_r versus ϕ -θ for varying hypocenter distance R, (b) δ_r versus R compared with case history

In Fig. 7(a), slope displacements δ_r for an earthquake of the magnitude M=6.8 are plotted versus $\phi-\theta$ for varying hypocenter distance R, wherein the earthquake energy was calculated by using Eqs. (3) and (4) as E_{IP}=796～49.7 kJ/m^2 for R=10～40 km. The earthquake motion is represented by a harmonic motion of equivalent number of cycles N_{eq}=9 according to an empirical relationship (Idriss & Boulanger 2008) considering M=6.8 and of equivalent frequency f=1.51 Hz assuming PGA of M=6.8 earthquake attenuating almost inversely proportional to R for R=10～40 km (NIED 2004). This gives the wave energy beneath the slope per cycle (dividing E_{IP} by N_{eq}=9) as E_u/N_{eq}=5.00～0.313 kJ/m^2 for S-wave velocity V_s=200 m/s using Eq. (4). By comparing this E_u/N_{eq} -value with the threshold energy E_{u0} in Eq. (13), E_{eq} for slope displacements can be determined by Eq. (14), leading to the evaluation of δ_r in Eq. (1) assuming average thickness D=5.0 m corresponding to circular slip. In Fig. 7(a), the curves of open symbols are originally plotted for $E_{eq}/E_u/\alpha=1.0$. Then from these, close symbols are replotted depending on the kinks of E_u/E_{u0} at 1.0 and 5.0 as indicated in Eq. (14). Thus, co-seismic slope displacement can be evaluated by this method seamlessly from nonoccurrence (δ_r =0) to residual displacements (if occurred) without needs to design acceleration time histories.

In Fig. 7(b), the same displacements δ_r are replotted versus R and compared with case history of the Kan-etsu highway embankment during 2004 Niigataken Chuetsu earthquake (M=6.8) where the damage level was classified into A (heavy), B (medium) to C (light) in slope sliding or road settlements (Kataoka et al. 2015). Although the comparison is still very much qualitative, the energy-based method seems to be able to properly evaluate slope behavior in terms of hypocenter distance and other pertinent slope parameters.

5 SUMMARY

Energy-based Newmark method has been developed based on a series of numerical analyses where Newmark-type slope model is shaken by SH-wave propagation underneath. Occurrence/Nonoccurrence of slope failures and associated slope displacements can be evaluated directly from upward wave energy using pertinent slope parameters without using acceleration time histories. An example study by this method has indicated qualitative compatibility with embankment failure case history during a M6.8 earthquake in Japan.

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