Overview of continuous tilting of the Tower of Pisa and its mechanism

A. Mochizuki[1], G. Tanyrbergenova[1], A. Zhussupbekov[1]

[1] Department of Design of buildings and structures, L.N. Gumilyov Eurasian National University, 2, Satpayev Str., 010008, Kazakhstan.

ABSTRACT

This paper aimed to discuss on a mechanism of continuous tilting of the Leaning Tower of Pisa over 700years. First, "*a check index of tower's settlement*" was introduced in order to verify estimated settlement of the tower, in which the boundary conditions of the pedestal of the tower's plinth, the ground level and the water table were taken into consideration. It was found that calculated settlement of consolidation of layers was out of a range of *the index*, then, local failures of the subsoils was assumed from records of the tower's tilting, and settlements of 3.5m and 3.8m were obtained which satisfied with *the check index*. A mechanism of tilting of the tower to the north side first, then the direction turned to the south side was proposed. In the mechanism, effect of the secondary consolidation of the layers did not taken into consideration.

Keywords: historical monument, Pisa Tower, settlement, failure mechanism, consolidation

1 INTRODUCTION

History of the Leaning Tower of Pisa has been widely known. A committee for investigation of the tilting tower and stabilization measures for the continuous tilting of the tower was established by the Italian government in 1965, when inclination of the tower, θ, reached to 5.3degrees (Fig. 1).

Fig. 2 shows relationships of tilting of the tower to increased load associated with construction of the tower. Initially, it tilted to the north side until the year 1273. Then, the inclination of the tower changed drastically to the south side after the second stage of construction of 5th floor to the 7th floor. Inclination of the tower was measured to be 1.86degrees to the south side when the bell chamber was mounted on the 7th floor in 1370, the tilting angle of which was not small enough to be negligible. In a period after 1370, not a small rate of increase of tower's tilting to the south side continued and total of the angle reached to 5.47degress in 1990.

After active investigations, *an under excavation method* of soil underneath the north part of foundation was carried out at the site in 1999. The countermeasure succeeded to recover -0.5degrees of the leaning of tower in the year 2001.

With regard to mechanism of continuous tower's tilt, many research papers have been published to date. It is said that the first investigation record was presented by Terzhaghi, in which the tower's tilt was caused by an unequal settlement of subsoils under the tower's foundation. After the report, most researchers on the tower's tilt seemed to follow his conclusion.

James et al. (1977) presented a paper, in which effect of secondary compression of the subsoils was taken into consideration, and total settlement of 2.45m was concluded. In recent years, a code of *PLAXIS* with a numerical model of *DSS-creep* was used by Vermeer (2002), and settlement of 3.89m at the year 2000 was calculated (see Fig. 3). Following the analyses, *FE*-analyses with a creep model to simulate behavior of the tower were carried out by Bai et al. (2008), Papadopoulou et al. (2017), etc.

In these analyses of settlement of subsoil under the tower, secondary compression of subsoils played an important roll. However, it cannot be understandable that continuous tilting of the tower was caused over 700 years by an effect of secondary compression of the

Fig. 1 Photo of the Tower of Pisa (1890, θ=*5.24*°, Library, Congress, Italy)

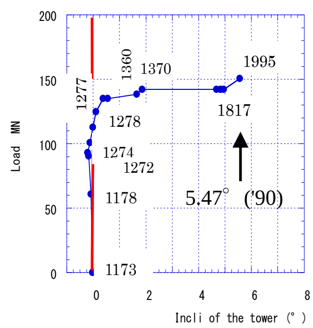

Fig. 2 Relationships of inclination of the tower to the load (Jamiolkowski, 1997)

subsoils, as a difference between the north side subsoils and those of the south side was only a composition of subsoils in *Horizon A* of thickness of 11m.

It is also questionable that such a heavy tower could be constructed with no local failure of layers under the tower's shallow foundation as it was constructed in an area of soft soils in a flood plan (see Fig. 4). Trial calculations of bearing capacity of the subsoil under the foundation showed that strength of the layers were not sufficient to support the tower's load safely.

These questions on the mechanism of tilting and continuous settlement of the tower triggered this investigation, in which the possibility of a local failure of layers under the tower's foundation was discussed.

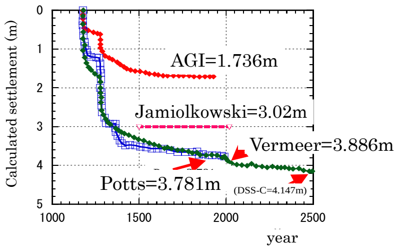

Fig. 3 Calculated settlement of the tower (Vermeer et al., 2002)

Fig. 4 Old map of Pisa in 11th century

2 CHECKING OF INDEX OF TOWER'S SETTLEMENT

Checking of index of the tower's settlement can be beneficial to evaluate a calculated settlement. However, no record of the tower's settlement was presented up to date. Therefore, *checking of index* was developed from observed boundary conditions, such as level of the water table, level of the plinth's pedestal at the time of the construction, relationships of the ground height to the pedestal at the construction, etc.

(1)*BC-1*(boundary condition); Cross sections of the subsoils and the tower were presented by Rampello et al. (1998, Fig.7) and Bai et al. (2008). From these charts, original level of the ground surface was estimated as *MSL*.+3.069m, which was a level of the ground surface at a distance of ±20m where a small heaving of the surface of *Horizon B* was recorded.

(2) *BC-2*; An observed sketch of the foundation of the north side was presented by Jamiolkwski et al (1993, see Fig. 5(b)), in which the *MSL±0m* was indicated on the pedestal of plinth. Fig. 5 (a) shows a condition of foundation when it was constructed. Thickness of the pedestal was 1.2m and that of rock fragments under the pedestal was measured as 0.4m. Level of the maximum water table was *MSL* +2.1m.

In *Case A* (see an explanatory sketch in Fig.5), the tower's foundation was assumed to be buried to a level of the top of the pedestal into the ground. *A check index* was estimated as follows: ① - depth of the ground surface to *Point A'* in Fig. 5(b) was 3.069m, ② - depth from the bottom of the foundation to a level of *Point A'* was 1.469m, ③ - settlement of the tower's center due to the tower's tilt of 5.469 degrees was calculated to be 0.937m (= the foundation's radius, 9.79m×tan (5.469 degrees). Then, a total settlement from *Point A* to *Point A'* was calculated to be 3.476m.

In *Case B*, the foundation was assumed to be buried to a level of the pedestal's bottom. A total settlement of the tower was calculated as 4.676m (=3.475m +1.2m).

A range of calculated settlements in *Case A* and *B* can be used as *a check index for the tower's settlements* which was developed from the boundary conditions of *BC-1* and *2*. It is interesting to know that the tower was not expected to settle down so much as it was, as the pedestal's height was designed only 1.2m.

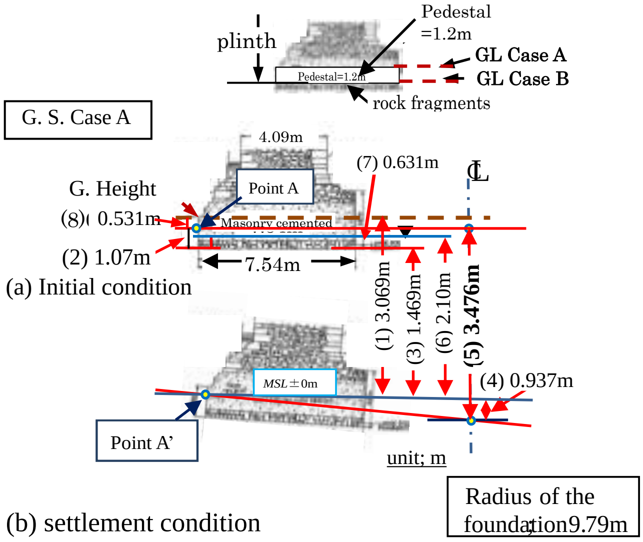

Fig. 5 Conditions of the foundation; *Case A*

3 SETTLEMENTS DUE TO CONSOLIDATION

3.1 Soil parameters and initial stress distribution

First, three horizons, *Horizon A*, *B* and *C*, of the subsoils at the site were divided into 6 sub-layers, 36 sub-layers and 12 sub-layers, respectively. Then, soil parameters of w_n, w_L, w_p, and G_s were found for each sub-layers. Next, the first approximations of void ratio of e_1,γ_{sat}, γ' and $\Sigma\sigma'$ were calculated from the parameters in order to find their initial conditions of the ground. Then, an equation, "$e=A+C_c \cdot log(\sigma')$ " was used for the second approximation of void ratio, e_2, which was compared with the prior void ratio, e_1 by an iteration method until a gap of Δe (=e_{i+1}-e_i) being sufficiently small in which a guideline, $\Delta e \leqq 0.05$, was used. Finally, distributions of $e_{initial}$,σ' and γ' for each sub-layer were found. Settlement of each sub- layers due to consolidation was calculated using an equation, "$\Delta S = (\Delta e_{load}/[1+e_{initial}]) \cdot H$". Here, H is thickness of each layer. Here, e_{load} is a void ration of a gap of e between $e_{initial}$ and e after stress loading.

3.2 Stress distribution of the tower's load

In order to calculate settlement of consolidation for each sub-layer, stress distribution of the tower's load in the ground was estimated.

Table 1 shows five variations of stress distribution in the ground. In *Case 1*, no expanding of stress distribution of the tower's load was assumed, and in *Case 2*, it was calculated using Boussinesque's formula. Calculated settlements for the north side and south side are shown in columns ② and ③ in the table. And, settlement of the tower's center are shown in column ④. No matter to say that those settlements are an overestimation as the stress distributions in the ground are overestimated.

In *Case 3 to Case 5*, stress distributions in the ground were calculated using *Kogler's first formula* of Eq.(1).

$$\sigma_z = q/[\pi \cdot (R + z \cdot \tan\theta)^2] \qquad (1)$$

Here, R is a radius of the foundation (9.79m), z is a depth from the foundation base and q is uniform loading stress. θ is a spreading angle of stress in the ground, which was assumed to be 30, 25 or 20degrees for each *Cases*.

In *Case 4* and *5*, increased stresses, $\pm\Delta q$, due to overturning moment was taken into the calculation of the each settlement, though in *Case 3*, no stress increase due to the tower's tilt was taken into the calculation. In these calculations, it was assumed that a foundation was buried at a level of *GL*. -1m into the ground in *Case 4*, and that of *GL*.-4m in *Case 5* in order to simulate settlement of -3m of the foundation.

Settlements of the tower's center (should be see in ④ of the table 1) were estimated to be 2.40m in *Case 4-1* to 3.05m in *Case 5-3*, and the ultimate angles of tilt of the tower were estimated as 6.12° to 7.13°, respectively.

Table 1 Calculated settlement of layers due to consolidation

Case	① Comment		Calculated settlement (m) ②North	③South	④S_{center}	⑤ΔS (m) (S_{south}-S_{north})	Ultimate inclination of foundation (θ')	
1	No-stres reduction		4.504	4.823	4.664	0.319		
2	Boussinesque's Eq. (center)		3.806	4.173	3.990	0.367		
3-1	Kogler's equation	θ=30°	2.398	2.649	2.524	0.251	⑦ΔS/L	⑧ θ'= $\tan^{-1}$ΔS/L
3-2		θ=25°	2.603	2.848	2.726	0.245		
3-3		θ=20°	2.847	3.102	2.975	0.255		
4-1	Kogler's equation +Moment	θ=30°	1.354	3.450	2.401	2.098	0.1072	6.12
4-2		θ=25°	1.481	3.694	2.585	2.218	0.1133	6.46
4-3		θ=20°	1.642	3.984	2.813	2.342	0.1196	6.82
5-1	Kogler's eq.+Moment +3msettelment	θ=30°	1.565	3.806	2.686	2.241	0.1145	6.53
5-2		θ=25°	1.685	4.023	2.854	2.338	0.1194	6.81
5-3		θ=20°	1.829	4.279	3.054	2.450	0.1251	7.13

It should be noted that settlement of 3.054mcoincide well with that of Jamiolkowski's calculation of 3.02m (1999). However, they violated to *the check indexes* in a range of 3.48m to 4.68m, and they were not acceptable, which showed that a tilting mechanism was not simply consolidation of layers.

4 MECHANISM OF THE TOWER'S TILTING

4.1 Rate of tilting and settlement of the tower

Table 2 shows records of angles of the tower's tilt and rates of them per year, $\Delta\theta/y$, for each step. Here, to tilt to the south side of the tower is expressed in a positive rate. Fig. 6 shows relationships of $\Delta\theta/y$ to the middle year in each step in Table 2. A term from the year 1173 to 1990 was divided into three, *Term 1*, 2 and *3*, and averaged rates of them were *0.19°/y*, *0.017°/y* and *0.0023°/y*, respectively.

A trial calculation of *Degree of consolidation* (*U*) of a most thickest layer of 5.4m (*Horizon B*) in the north side to the tower's load showed that a term of four years was enough to reach over 95% of *U*, therefore, it was assumed that consolidation of the north side layers was completed for the tower's load and an increased load of overturning moment due to tower's tilt to the north side until the year 1273.

After the year 1274 in *Term 1*, the tower turned to tilt to the south side. The averaged rates in this term were extremely high which was compared with those in *Term 2* and *3*, and the rates increased year by year up to 1277. However, the increase rates in the figure could not be understandable if the movement of the tower's tilting was caused only by a consolidation phenomenon under a constant increasing load condition as the earlier rate of the tilting (or settlement) should be the higher rate under that condition of consolidation.

Therefore, a local failure of subsoils under the tower's foundation was taken into consideration of a mechanism of the continuous tilting after many trials. It is a point that a local failure mechanism will solve a big gap of the rate in the former term to that of latter term.

Table 2 Rate of increased angles for each step

① Step	②year	③ Δyear	Tilt of the tower ④θ°	⑤Δθ°	⑥ Δθ°/y
1	1173	-	0.000	-	-
2	1272	99	-0.121	-0.121	-0.0012
3	1274	2	-0.210	-0.089	-0.0445
4	1275	1	-0.006	0.204	0.2040
5	1276	1	0.108	0.114	0.1140
6	1277	1	0.379	0.271	0.2710
7	1278	1	0.552	0.173	0.1730
8	1360	82	1.655	1.103	0.0135
9	1370	10	1.858	0.203	0.0203
10	1550	180	4.684	2.826	0.0157
11	1758	208	4.831	0.147	0.0007
12	1817	59	5.103	0.272	0.0046
13	1859	42	5.167	0.064	0.0015
14	1990	131	5.469	0.302	0.0023

A ratio of settlement due to a local failure to that of consolidation was different in each term. It should be noted that layers of the north side did not settle so much due to consolidation after the year 1275 as working stress on the foundation increased due to overturning moment of the tower to the north side until 1273, then the stress due to tilting decreased due to change of titling direction to the south side (though the tower's load of the 5th floor to 7th floor was increased).

Table 3 shows calculated total settlements of "*a proposed mechanism of consolidation and local failure*". In the calculation, settlements of the north side in *Case 3-1 to 3* in Table 1 were adopted as a base settlement of the tower. Calculated settlements of 3.54m in *Case 6-2* and 3.78m in *Case 6-3* were acceptable as a range of *the check indexes* was 3.475m to 4.675m. It is interesting that the calculated settlement of the tower in *Case 6-3* coincided well with that of 3.781m of Potts' calculation (1993, see Fig. 3).

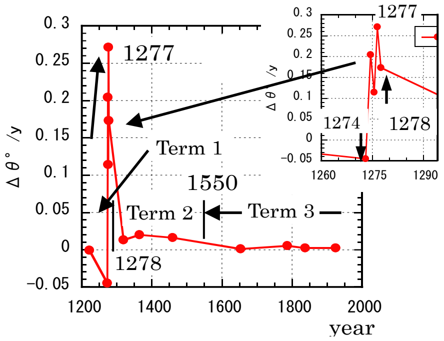

Fig. 6 Rate:

Table 3. Calculated total settlement due to a mechanism of consolidation and local failure.

Case	θ (degree)	1	Calculated settlement (m)		
			Settlement of North side 2	Increased settlement due to tilting	Settlement of Center
6-1	by	30	2.398	0.937	3.335
6-2	Kogler's	25	2.603	0.937	3.540
6-3	equation	20	2.847	0.937	3.784

4.2 A mechanism of the tower's tilting

A mode of *tower's rigid rotation* was proposed by Burland (2004) based on monitored records of the tower's motion in the 20th, which was concluded to be a problem of *leaning instability of the tower*. However, as the mechanism left some questions, an alternative mechanism was developed based on a model of bearing capacity failure accompany with consolidation of subsoils as follows:

(1) From the year 1173 to 1273: As layers in *Horizon A* of the tower's north side were much more sandy than those of the south side, the tower tilted to the north side first. The layers were consolidated with the tower's stress and increased stress of the overturning moment. So, the consolidation of the north side was presumed to be completed in the term;

(2) Up to the year 1278: As 5th to 7th floor of the tower were mounted. When settlement of the south side became to be greater than that of the north side, direction of the tower's tilt turned to the south side, and the layers under the tower's foundation settled drastically due to consolidation and local failures;

(3) Up to the year 1990: Due to an increase of overturning moment of the tower to the south side, an increased stress of $+\Delta q$ caused settlement of consolidation to the south side (1). When bearing capacity of layers under the foundation's toe was not sufficient to the increased stress, a local failure of layers under the foundation caused settlement (2). In a case when bearing capacity was sufficient to the stress increase, the layers were consolidated (3). Then, the settlement caused a small increase of the angle of tower's tilt and stress of the overturning moment increased (4), then, the tower tilted more (5). The tower's tilting progressed with a very slow rate. It is considered that the mechanism of 1 to 5 was repeated continuously for over 700 years.

5 CONCLUSION

A new *check index of tower's settlement* was introduced in order to verify an estimated settlement of the tower. Then, a mechanism of continuous tilting and settlement was proposed, and a possibility of local failure of layers under the tower's foundation was concluded without taking consideration of secondary compression of the layers into the calculation.

REFERENCES

Bai, J., Chan, D. and Morgenstern, J.N. (2008). Numerical analysis of time-dependent behavior for the Leaning Tower of Pisa. Soils and Foundations, 48-2, 207-220.

Burland, J. B. (2004). The Leaning Tower of Pisa Revisited, Fifth International Conf. on Case Histories in Geotechnical Engineering, 1-13.

James, K. and Michell, F. (1977). Foundation Performance of tower of Pisa, Journal of the Geotechnical Eng. division (GT3), ASCE, 227-249.

Vermeer, P. A. and Neher, H. P. (1999). A soft soil model that accounts for creep, in Bringreve, R.B.J. (Ed.), Proc. of the Int. Symposium "Beyond 2000 in Computational Geotechnics", 249-261.

Jamiolkowski, M., Lancellotta, R. and Pepe, C. (1993). Leaning Tower of Pisa – Updated Information, Proc. of Third International Conf. on Case Histories in Geotechnical Eng., SOA 3, 1319-1330.

Potts, D.M. (1993). Calibrazione di un modello geotecnico agli elementi finiti e valuzione degli effetti indotti a seguito di alcuni interventi di consolidamento della torre di Pisa, GCG computing, London, UK.

Rampello, S. and Callisto L. (1998). A study on the subsoil of the tower of Pisa based on results from standard and high-quality samples. Canadian Geotechnical Journal, 35, 1074-1092.

Papadopoulou, K. and Gazetas, G. (2017). Leaning instability of the tower of Pisa re-examined by 3D F. E. analyses, Proc. of the 19th Int. Conf. on Soil Mechanics and Geotechnical Eng., Seoul, 3024-3026