

## Modeling packing density of granular mixtures: An artificial intelligence approach

Siavash Manafi Khajeh Pasha<sup>1</sup>, H. Hazarika<sup>2</sup>, S.P.G. Madabhushi<sup>3</sup>, and N. Yoshimoto<sup>4</sup><sup>1,2</sup> Department of Civil Engineering, Kyushu University, Fukuoka, Japan<sup>3</sup> Schofield Centre, University of Cambridge, Cambridge, UK<sup>4</sup> Department of Civil and Environmental Engineering, Yamaguchi University, Ube, Japan

## ABSTRACT

The minimum and maximum packing density of soil-Scrap Tire Derived Materials are often estimated based on limited laboratory test results or to some extent, an empirical correlation. However precise modeling of void ratio characteristics of such materials is complex and usually involves many parameters might be beyond the capability of most of common physically based engineering methods. To solve this issue, Artificial Neural Network (ANN) method is used for simulating maximum and minimum packing density of Gravel-Tire Chips mixtures (GTCM). In this study, a series of maximum and minimum void ratio tests were conducted on GTCM with different fraction of gravel in mixture ( $GF = V_G/V_T$ ) at different mean particle size ratio of tire chips to gravel ( $D_{50,R}/D_{50,G}$ ). The outcome revealed that the ANN model is able to precisely predict void ratios of binary mixtures.

**Keywords:** Artificial Neural Network (ANN); Gravel-Tire Chips Mixture (GTCM); maximum and minimum void ratios

## 1 INTRODUCTION

Maximum and minimum void ratios are key characteristics which affect volume change tendency, the hydraulic conductivity and mechanical behavior of the geomaterials. On the other hand, tons of waste tires are generated and scrapped in Japan annually. More than 60% of waste tires are being used as fuel for energy production purposes in different industries. Reusing waste tires by converting them into valuable products such as Scrap Tire Derived Materials (STDMS) would decrease the emission of CO<sub>2</sub> to atmosphere and reduce the risk of climate change induced geo-disasters.

Recently STDMS standalone or mixed with soil as Tire Derived Geomaterials (TDGM) are being adopted in several civil engineering applications. These materials are non-dilative geomaterials and have unique physical and mechanical properties such as low bulk density, high hydraulic conductivity (Hazarika et al. 2008). Gravel tire chips mixtures (GTCM) is introduced to solve drawbacks of using sand-tire chips mixture in geotechnical applications such as low hydraulic conductivity of sand (e.g. Hazarika and Abdullah 2016; Pasha et al. 2019).

Research on mechanical behavior of soils, over last few decades, has revealed that physical properties and stress-strain behavior are in correlation with nature of the soil (such as the mineralogy) as well as physical state of soil which can be described by relative density (Hsiao et al. 2015). Humphres (1957) who was the first to propose a method for estimating maximum packing density of soil with different sizes. An empirical method was introduced by Lade et al. (1998) to predict the minimum void ratio of binary mixture of spheres with different size

of particles based on results of a series of minimum void ratio tests on sand-silt mixture; However, it has been shown that this method is only valid for the higher small to large mean particle size ratios ( $D_{50, Large} \gg D_{50, Small}$ ) (Chang et al. 2015).

Predicting packing density of STDMS-soil mixture is a complicated task because there are large numbers of factors affecting void ratio characteristics of mixture. Although extensive experimental and numerical studies (e.g. Ng et al. 2017) on packing density of granular mixtures with different size and shape particles were presented so far, But applicability of existing mathematical methods for predicting void ratios and examining physical characteristics of soil-tire chips mixtures were not yet examined.

Some studies have adopted equivalent void ratio by assuming the volume of tire solids as part of the total volume of voids due to low stiffness of tire chips (Feng and Sutter 2000). However, applicability of the proposed correlations has not yet been examined and it is believed to be valid for small amount of tire chips inclusion in soil-tire chips solid matrix. In recent years Artificial Intelligence (AI) technique has been implemented in several fields of geotechnical engineering to predict behavior of sophisticated systems. (Park et al. 2018; Shahin 2014).

The objective of this study is to introduce new model using Artificial Neural Network (ANN) approach that can simulate complex void ratio characteristics of binary mixture taking into account important features of granular materials. In addition, we aim to examine the void ratio characteristics of GTCM. Particularly, the study highlights the effect of gravel fraction, mean

particle size ratio of tire chips to gravel ( $D_{50,R}/D_{50,G}$ ) on maximum and minimum void ratio of GTCM.

## 2 MATERIAL PROPERTIES AND TESTING PROGRAM

Two sets of gravel (G1 and G2) were examined in this study. Both sets of gravel were carefully selected to have relatively close mean grain size ( $D_{50,G1} \approx D_{50,G2} = 4.3$  mm) and coefficient of uniformity  $U_c \approx 2.17$ . Both G1 and G2 gravels are classified as poorly graded (GP) according to JGS-0051. In order to investigate the influence of particle size ratio ( $D_{50,R}/D_{50,G}$ ), in addition to the impact of relative content ratio of tire chips on void ratio characteristics of GTCM, 3 sets of tire chips (TCH1, TCH2 and TCH3) were used in this study. TCH1 has uniformity coefficient of  $U_c \approx 2.2$  and particle size ratio of ( $D_{50,R}/D_{50,G}$ )  $\approx 1.2$ . The materials TCH2 and TCH3 have a similar uniformity coefficient of  $U_c \approx 1.4$  but different particle size ratios of ( $D_{50,R}/D_{50,G}$ ) = 0.35 and ( $D_{50,R}/D_{50,G}$ ) = 3.35 respectively. Particle size distributions of gravel and tire chips is plotted in Fig. 1.

## 3 EXPERIMENT RESULTS AND DISCUSSION

A series of vibratory test were conducted on Gravel and GTCM mixtures according to JGS-0161 standard. Particle size ratio of tire chips to gravel ( $D_{50,R}/D_{50,G}$ ) varies between 0.35 and 3.35, GF varies between 0 to 100 percent. Fig. 2 shows the variation of maximum and minimum void ratios versus gravel fraction for different values GF(%) and ( $D_{50,R}/D_{50,G}$ ). The following relationship has been established to define maximum and minimum void ratio of GTCM.

$$e_{max,GTCM}, e_{min,GTCM} = X_0 + X_1/(1 + X_2(\Psi)^2 + X_3(\Psi)^4 + X_4(\Psi)^6) \quad (1)$$

$$\Psi = (GF(\%) - X_5)/X_6 \quad (2)$$

$$X_i = f(D_{50,R}/D_{50,G}) = \theta_0 + \theta_1(D_{50,R}/D_{50,G}) + \theta_2(D_{50,R}/D_{50,G})^2; i=1:6 \quad (3)$$

Where  $e_{max,GTCM}$  and  $e_{min,GTCM}$  are maximum and minimum void ratios of GTCM.  $X_i$  particle size ratio function.  $\theta_0, \theta_1, \theta_2$  are calibration parameters for  $D_{50,G} \approx 4.3$  mm and  $0.35 < (D_{50,R}/D_{50,G}) < 3.35$ .

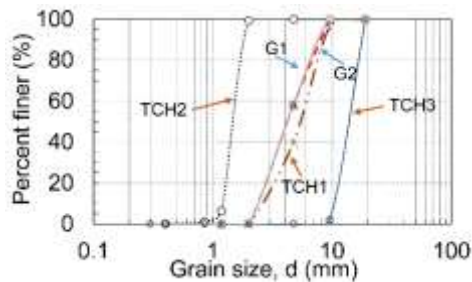


Fig. 1. Particle size distribution of Gravels and tire chips  
All Calibration parameters are presented in table. For the G2TCH2 mixture with ( $D_{50,R}/D_{50,G}$ ) = 0.35, the minimum value of both maximum and minimum void ratios is achieved when  $GF \approx 80\%$ . This finding is in

agreement with the values reported by (Li et al. 2016) on sand-rubber with particle size ratio of ( $D_{50,R}/D_{50,S}$ ) = 0.25. For the GTCM with particle size ratio of ( $D_{50,R}/D_{50,G}$ ) = 1.2,  $e_{max,GTCM}$  and  $e_{min,GTCM}$  of GTCM decreases with increasing gravel fraction. Similar trends have been reported by (Li et al. 2016). Theoretical value of the minimum void ratio ( $e_{min,T}$ ) of GTCM can be calculated from the void ratio of large particles ( $e_1$ ) and small particles ( $e_2$ ) as is shown in Fig.3 (Lade et al. 1998).

$$e_{min,T} = (n_1 n_2 / ((n_1/e_1) + n_1(n_2/e_2))) \quad (4)$$

Although the optimum gravel fraction value where the minimum void ratio is achieved is in good agreement with that of obtained from experimental result, but the theoretical minimum value of void ratio is lower than that of experimental value. Because theoretically, the particles were assumed to be sphere with large to small particle size ratio  $D_{50,L}/D_{50,S} \gg 7$ . Furthermore, the deformation of particles within the binary mixture was disregarded in calculation of theoretical value of minimum void ratio.

## 3 ARTIFICIAL NEURAL NETWORK MODEL

Artificial Neural Networks (ANN) are one of Artificial Intelligence (AI) techniques that inspired by function of human biological nervous system and mimics brain problem solving process. The concept of ANN is first introduced by (McCulloch and Pitts 1943) but first training algorithm for a feed-forward multilayer perceptron is introduced in 1986.

Table 1. Calibrated parameters of maximum and minimum void ratio of GTCM

Plot	$X_0$	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$
$e_{min}$	$\theta_0$	1.39	-0.86	7.18	-15.47	46.53	74.58
	$\theta_1$	-0.48	0.52	-8.15	25.32	-111.1	34.93
	$\theta_2$	0.11	-0.12	2.17	-10.10	61.79	-11.57
$e_{max}$	$\theta_0$	17.35	-2.94	-5.32	7.2	-8.32	1319.72
	$\theta_1$	-19.86	4.43	6.88	-10.88	22.4	-1406.68
	$\theta_2$	5.32	-1.9	-1.64	3.2	-6.4	317.42

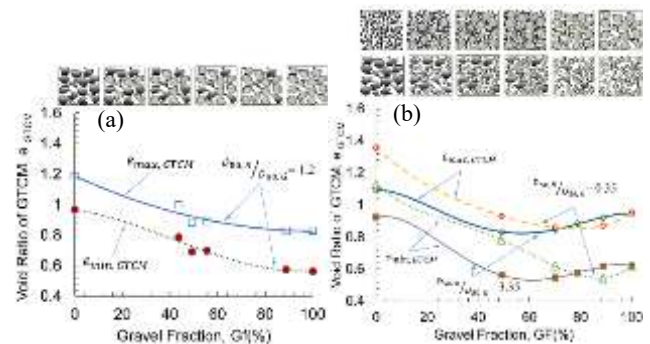


Fig 2. Minimum and maximum void ratio versus GF(%); (a) ( $D_{50,R}/D_{50,G}$ ) = 1.2 (b) ( $D_{50,R}/D_{50,G}$ ) = 0.35 and 3.35.

In this study, the feed-forward ANN in which information moves forward from the input layer directly towards output layer through any hidden layers was selected. Back propagation learning algorithm (BPP)

which uses the gradient decent laws and is suitable for prediction problems, is implemented. A total number of 72 minimum and maximum void ratio tests are used as the database for building ANN model. The goal of the model is to predict maximum and minimum void ratio of GTCM binary mixture for the given gravel fraction and Particle size ratio of tire chips to gravel ( $D_{50,R}/D_{50,G}$ ). The architecture of the network is determined as follows and is illustrated in Fig. 4.

Input vector  $\{X\} = \{e_{\max,G}, e_{\max,R}, e_{\min,G}, e_{\min,R}, GF, (D_{50,R}/D_{50,G})\}$ ; Output  $\{Y\} = \{e_{\max,GTCM}, e_{\min,GTCM}\}$

Where  $e_{\max,G}$  and  $e_{\max,R}$  are maximum void ratio of gravel and tire chips,  $e_{\min,G}$  and  $e_{\min,R}$  are minimum void ratio of gravel and tire chips respectively. A MATLAB code is written to determine neurons (nodes) in the hidden layer by a trial and error method. In this method, training of networks starts with minimum number of nodes in the first hidden layer and sum squared error is calculated and compared with the allowable threshold error. If the error exceeds the threshold value, next neuron is added to the hidden layer. The above iterative process is repeated until the desired stopping criterion is met. The Levenberg-Marquardt algorithm was selected for training of the ANN.

The data is randomly divided into three subsets: training (70%), validation (15%), testing (15%). During the learning or training process, training data sets are used to obtain ANN parameters in each layer (Weights and biases) by minimizing the error function.

The training will stop when the error on the validation data set begins to rise. At the next stage, the testing data set is introduced to the ANN and its performance is evaluated.

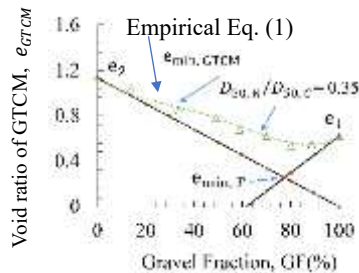


Fig. 3. Minimum void ratio of GTCM with  $D_{50,R}/D_{50,G}=0.35$

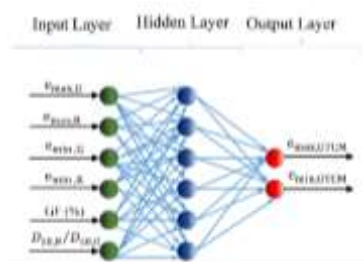


Fig. 4. Structure of ANN model

In this study, the proposed neural network model has 6 nodes in the input layer, 6 nodes in the first hidden layer 2 nodes in the output layer (Fig. 4). Once the ANN

performance is evaluated, ANN is utilized to conduct a parametric study.

Mean squared error between the target and predicted values of all outputs over all possible configurations was monitored constantly. In order to assess the performance of ANN model, the measured error during the training process is plotted with that of error measured during testing process, training will stop where the validation error starts to rise while the training errors constantly decreasing.

The performance of the optimized ANN model over training, validation and testing data sets are depicted in Fig. 5. As can be seen from Fig.5, there is a strong correlation between predicted values by ANN model and corresponding experimental values. Best validation performance of network is shown in Fig. 6.

A comparison of artificial neural network simulations with laboratory results over training data set is shown in Fig. 7. As can be seen from the plot, predicted and the measured values of void ratios on different percentage of gravel fraction are in good agreement. Fig. 8 shows the comparison between the predicted values of maximum void ratio  $e_{\max,GTCM}$  and minimum void ratio  $e_{\min,GTCM}$  of GTCM with  $D_{50,R}/D_{50,G} = 3.35$  by ANN model simulation and laboratory results. As can be seen from the plot, predicted and the measured values of void ratios on different percentage of gravel fraction are in good agreement. However significant differences between estimated values of void ratios given by equation proposed by (Anastasiadis et al. 2011) and experimental values can be observed.

$$e_{eq} = ((V_{voids} + V_{Tire})/V_{soil}) \quad (5)$$

Where  $e_{eq}$  is the equivalent void ratio and  $V_{voids}$ ,  $V_{Tire}$  and  $V_{soil}$  are volume of voids, tire chips particles and total volume of gravel and tire chips particles in the GTCM mixture. It seems that Eq. 5, overestimates values of void ratio, mainly due to considering tire chips in mixture as voids. This shows that the effect of tire chips on void ratio characteristics of GTCM mixture cannot be disregarded.

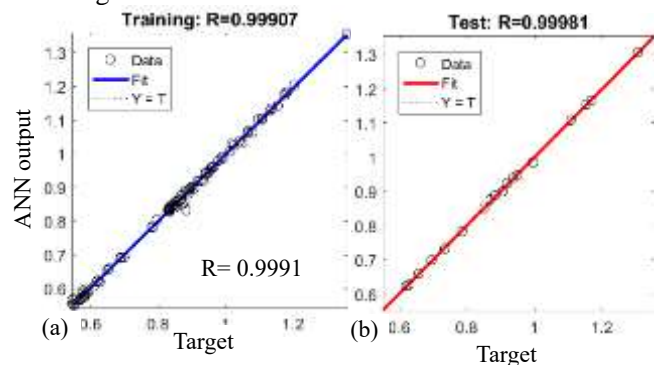


Fig. 5. Performance of the ANN Model over: (a) training; (b) Testing datasets



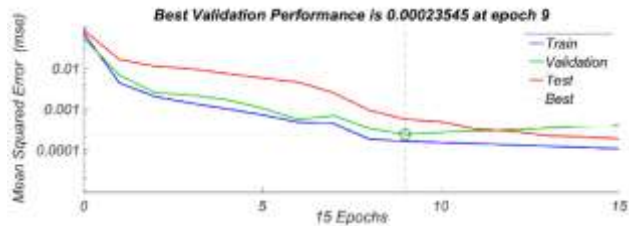


Fig. 6. Best validation performance of ANN model

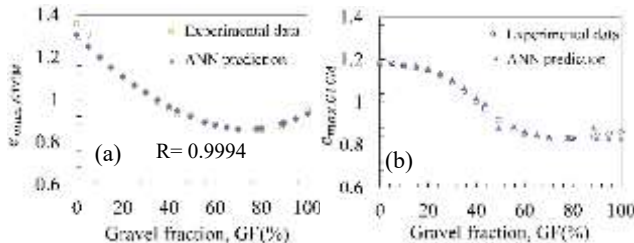


Fig. 7. Comparison of ANN simulation with experimental data:  
(a)  $D_{50,R}/D_{50,G} = 1.2$ ; (b)  $D_{50,R}/D_{50,G} = 0.35$

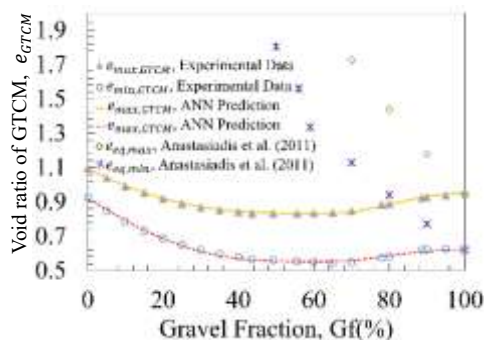


Fig.8. Comparison of ANN simulation with experimental data

#### 4 CONCLUSIONS

Void ratio characteristics of geomaterials such as gravel-tire chips mixture are key parameters that affect volume tendency, hydraulic conductivity and shear strength behavior of these materials.

In this study, a new model using Artificial Neural Network (ANN) is developed to estimate maximum and minimum void ratio of GTCM. The effect of gravel fraction (GF) and particle size ratio of tire chips to gravel ( $D_{50,R}/D_{50,G}$ ) on void ratio characteristics of GTCM is also examined. Based on the results of this study, following conclusions can be drawn:

- Theoretical model proposed by Lade et al. (1998) for prediction of void ratio was not successful in prediction of minimum void ratio of GTCM. This was mainly because of dismissing the deformation of particles in binary mixtures.
- A feed-forward artificial neural network model using MLPs with back-propagation training algorithm can be employed to model very sophisticated void ratio characteristics of gravel tire chips mixture with relatively high resolution.
- Comparison between estimated values by ANN and those obtained from equivalent void ratio equation indicated that equivalent void ratio expression

overestimates the value of maximum and minimum void ratio and has significant degree of tolerance.

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