COMPUTATIONAL GEOTECHNICS COURSE

CONSOLIDATION

Antonio Gens

Technical University of Catalunya, Barcelona

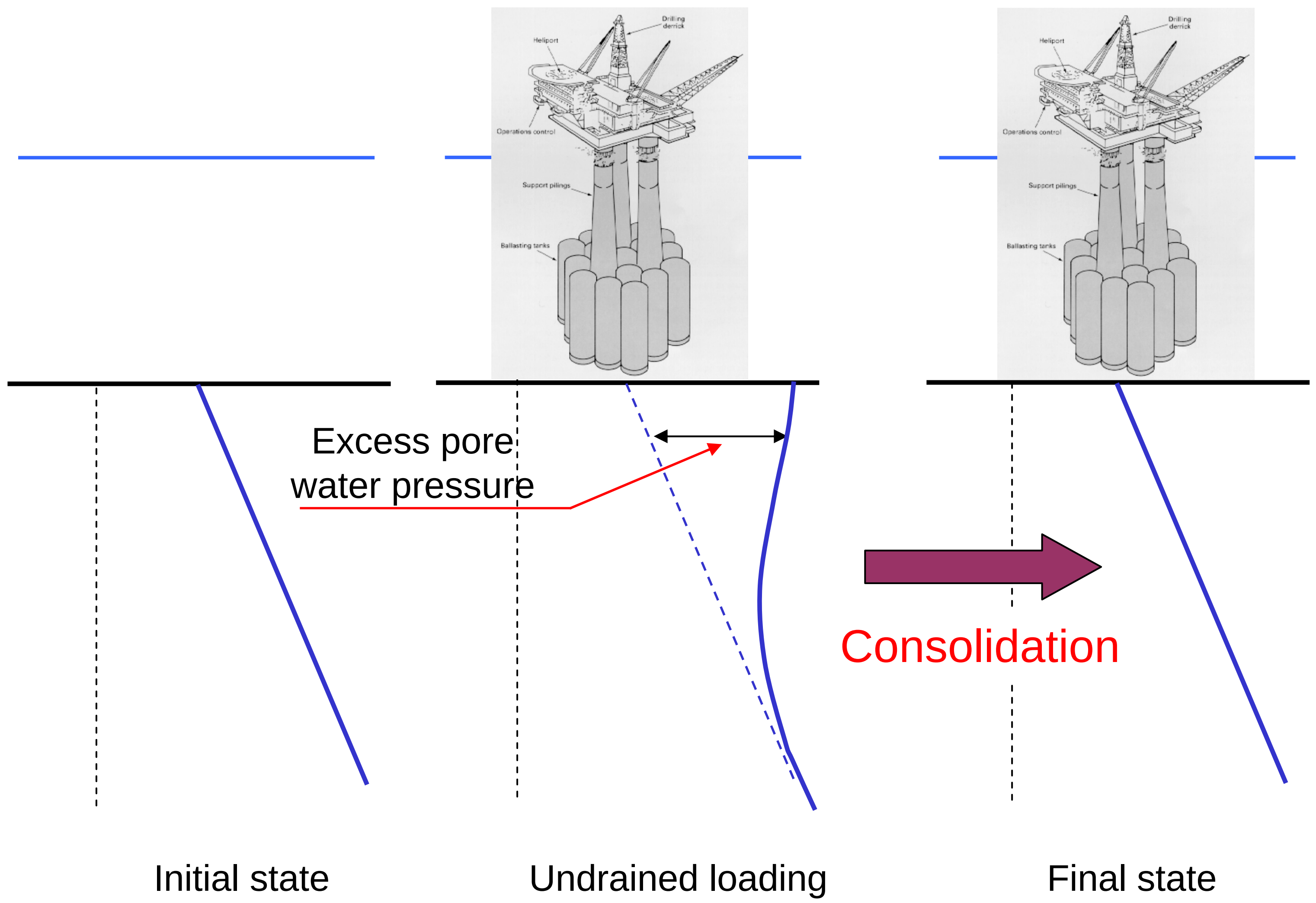
CONSOLIDATION: OUTLINE

- Introduction
- Basic theory of groundwater flow
- Permeability
- Confined and unconfined problems
- Finite element formulation for consolidation
  - Mechanical problem
  - Hydraulic problem
  - Global equations
  - Time step
- Boundary conditions

TYPES OF ANALYSIS

- Drained
  - Loading/Construction/ excavation: very slow (in relation to the soil permeability)
- Undrained
  - Loading/Construction/ excavation: very fast (in relation to the soil permeability)
- Intermediate cases: consolidation analysis
  - Both mechanical and hydraulic (flow) problems interact
  - More complex computations: coupled analysis

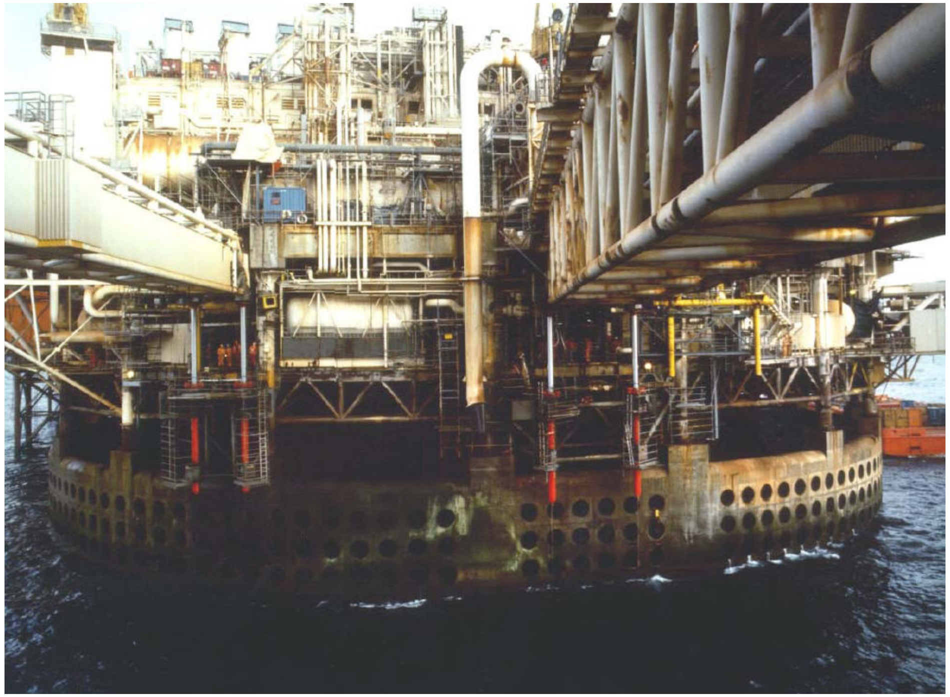
EXAMPLE



Excess pore
water pressure
Consolidation
Initial state
Undrained loading
Final state

OTHER EXAMPLES

1973



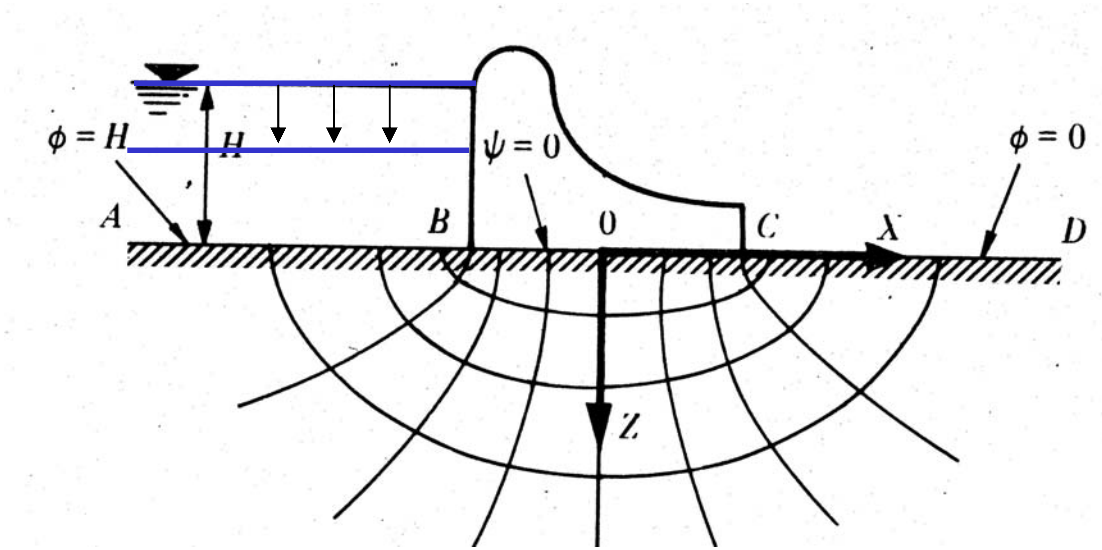
1984

Ekofisk tank

OTHER EXAMPLES

- Construction at intermediate rates

- Change of hydraulic conditions



BASIC THEORY OF GROUNDWATER FLOW

- Darcy´s law

$$q = -k\frac{\Delta h}{L} = -k\,i$$

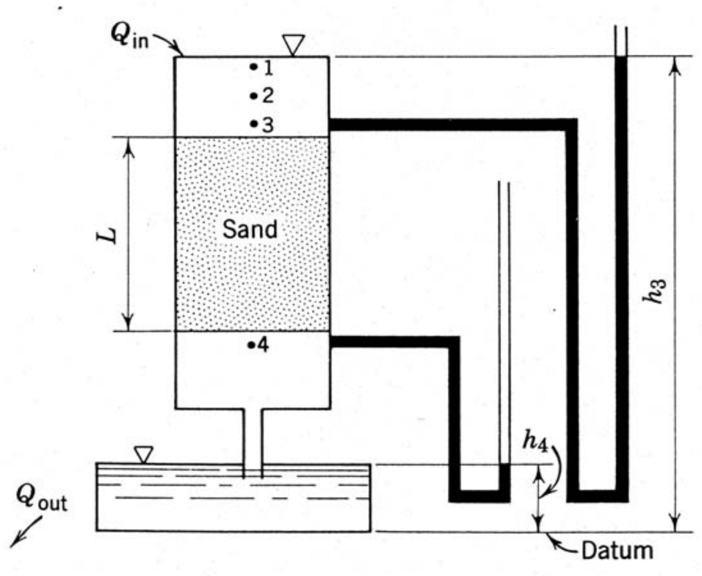
$$q = -k\frac{\partial h}{dy} = -k\frac{\partial \phi}{dy}$$

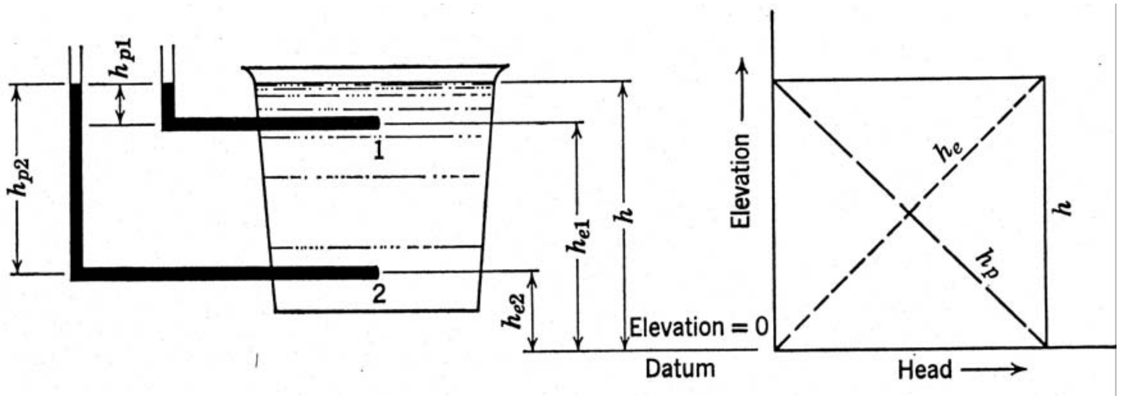
$$h = \phi = y + \frac{p}{\gamma_w}$$

q : flow $h = \phi$: total head

y: vertical coordinate

p : water pressure





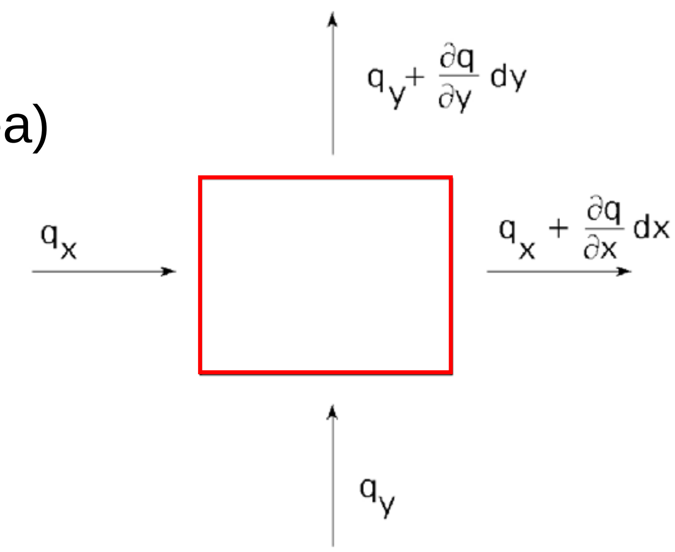
BASIC THEORY OF GROUNDWATER FLOW

- Permeability often anisotropic

$$q_x = -k_x \frac{\partial \phi}{\partial x} \qquad q_y = -k_y \frac{\partial \phi}{\partial y}$$

- Equation of continuity for steady state
(no sources/sinks in an elementary area)

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0$$



- Excess pore pressure

$$p = p_{steady} + p_{excess}$$

p : (active) water pressure

p_{steady} : steady state pore pressure

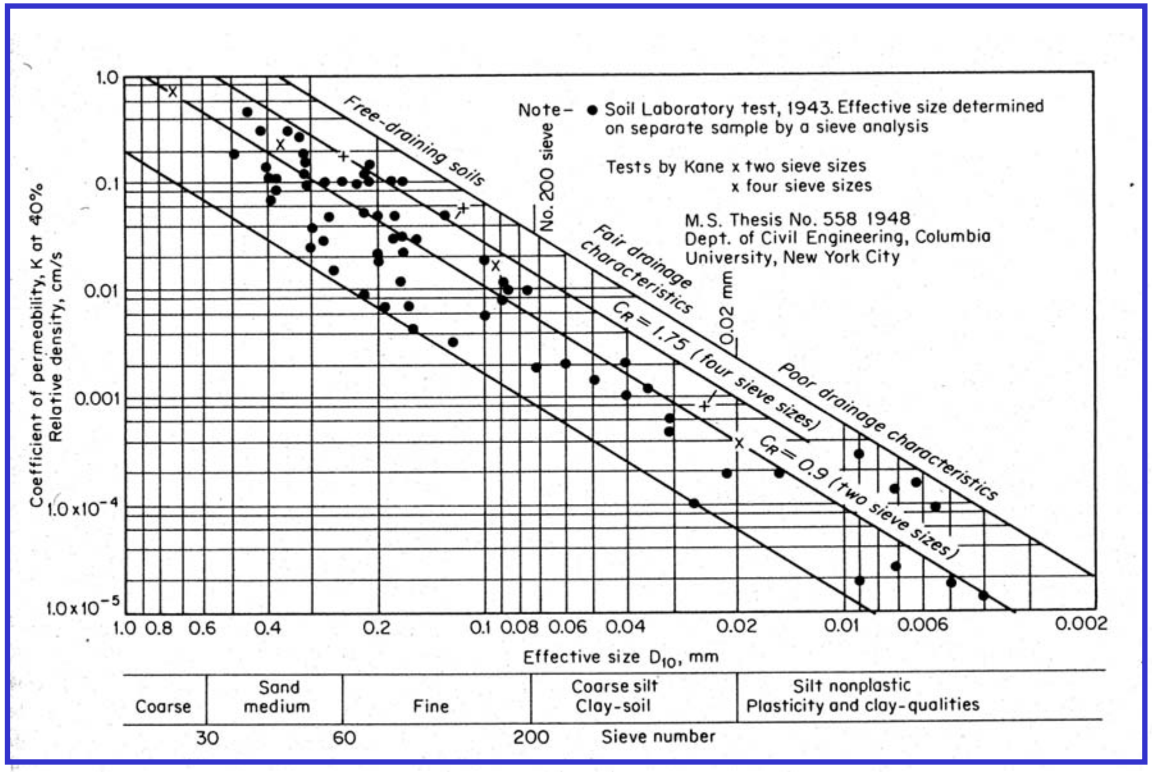
p_{excess} : excess pore pressure

PERMEABILITY

- Dependence on grain size

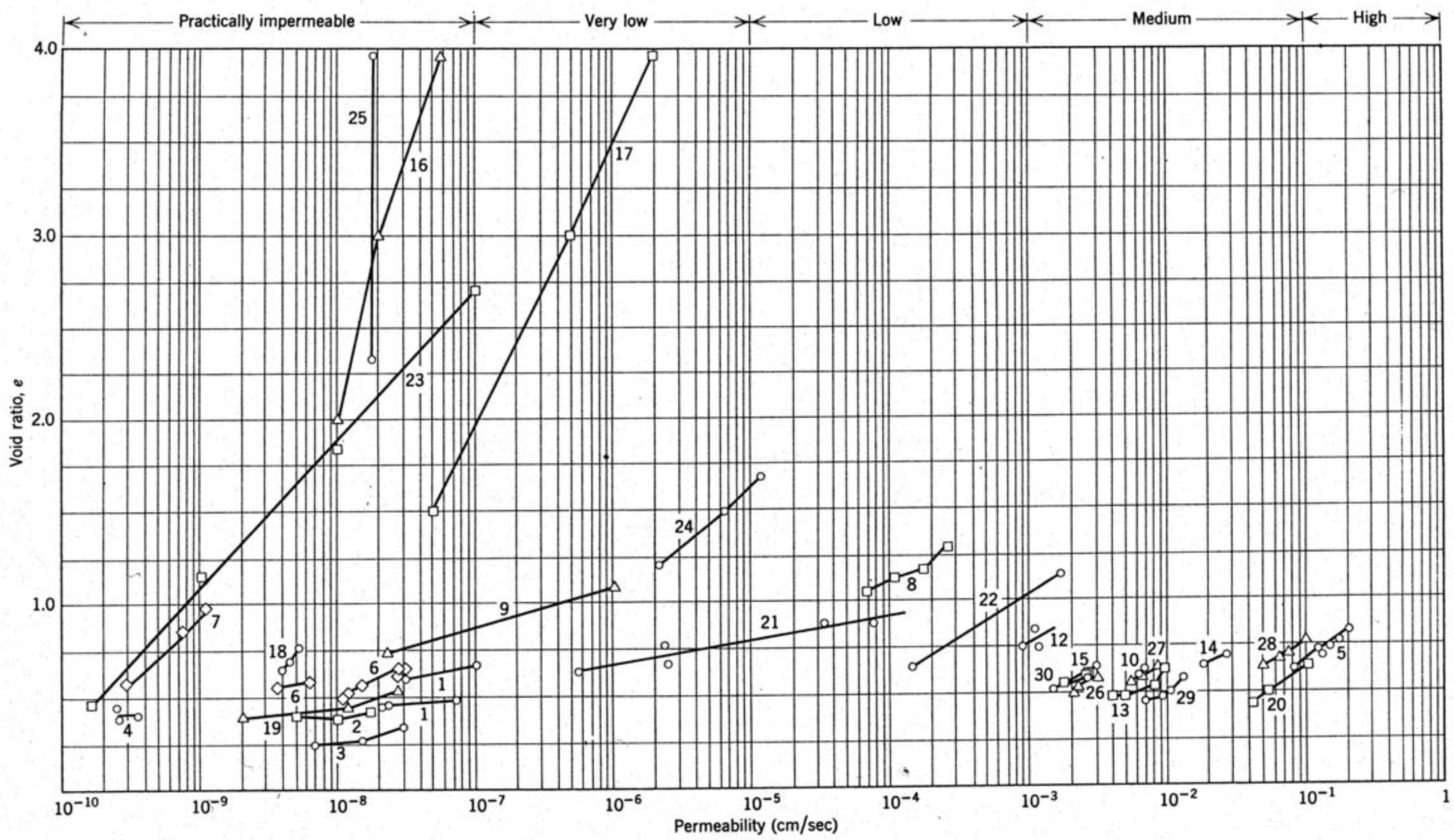
Soil	k (cm/s)
Clean gravel	> 1
Clean sand (coarse)	$1 - 10^{-2}$
Sand mixture	$10^{-2} - 5\times10^{-3}$
Fine sand	$5\times10^{-2} - 10^{-3}$
Silty sand	$2\times10^{-3} - 10^{-4}$
Silt	$5\times10^{-3} - 10^{-5}$
Clay	10^{-6} and less

Harr (1962)



PERMEABILITY

- Dependence on void ratio



Soil Identification Code

1 Compacted caliche
2 Compacted caliche
3 Silty sand
4 Sandy clay
5 Beach sand
6 Compacted Boston blue clay
7 Vicksburg buckshot clay
8 Sandy clay
10 Ottawa sand
11 Sand—Gaspee Point
12 Sand—Franklin Falls
13 Sand—Scituate
14 Sand—Plum Island
15 Sand—Fort Peck
16 Silt—Boston
17 Silt—Boston
19 Lean clay
20 Sand—Union Falls
21 Silt—North Carolina
22 Sand from dike
23 Sodium—Boston blue clay
24 Calcium kaolinite
25 Sodium montmorillonite
26–30 Sand (dam filter)

PERMEABILITY

- PLAXIS allows consideration of change of permeability with void ratio

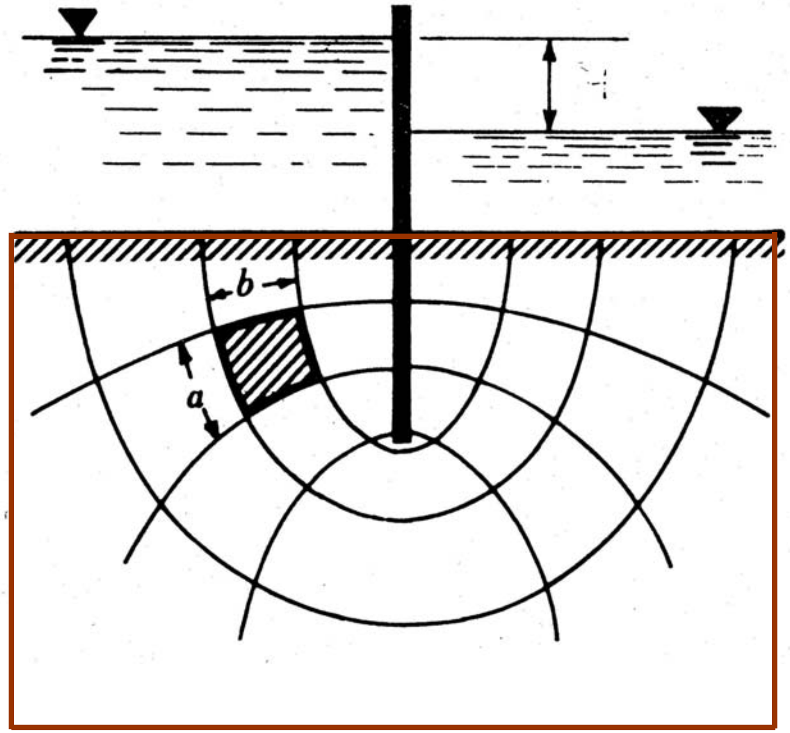
$$\log\left[\frac{k}{k_0}\right] = \frac{\Delta e}{c_k}$$

Default value for c_k is 10^{15}

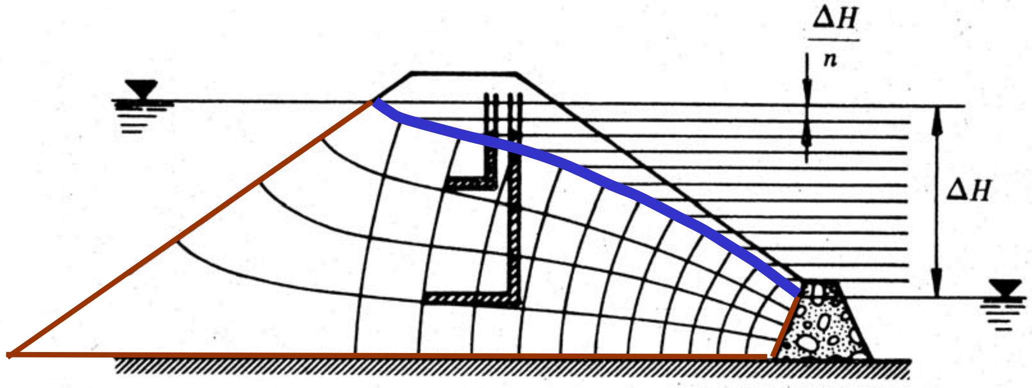
- There may be large contrasts of permeability between different materials in the same problem
  - Too much permeability contrast may cause numerical difficulties
  - The ratio between the highest and lowest permeability value should not exceed 10^5
  - To simulate an almost impermeable material (e.g. concrete), a value lower by a factor 1000 is sufficient

TYPES OF FLOW PROBLEMS

- Confined flow
- Unconfined flow



Domain defined



Domain undefined

TRANSITION SATURATED/UNSATURATED

$$q_x = -K^r k_x \frac{\partial \phi}{\partial x}$$

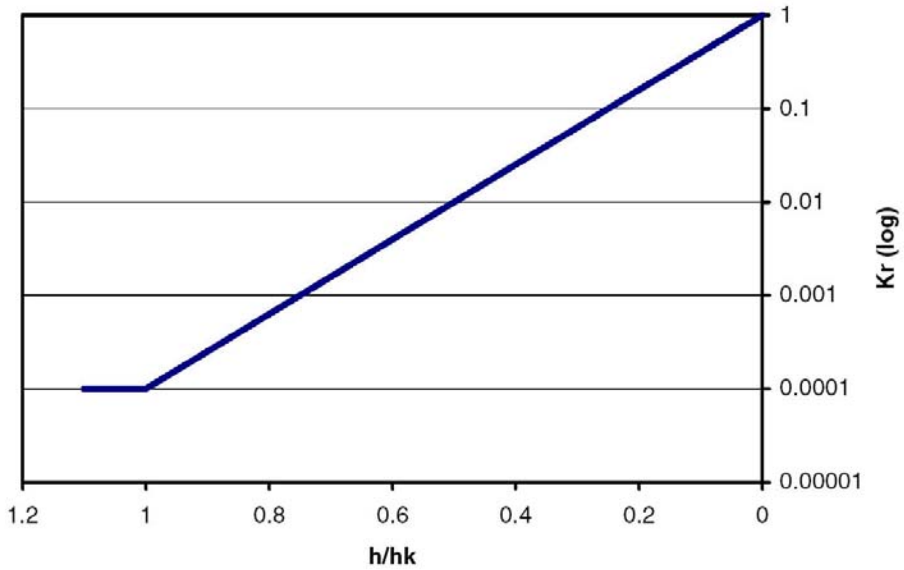
$$q_y = -K^r k_y \frac{\partial \phi}{\partial y}$$

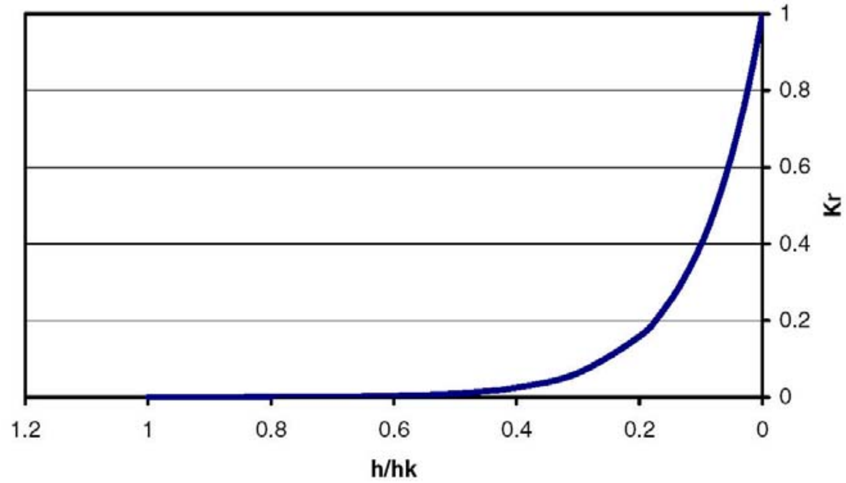
$K^r = 1$ saturated zone

$K^r = 10^{-4}$ unsaturated zone

$K^r = 10^{-4h/h_k}$ $\log(K^r) = -\frac{4h}{h_k}$

$h_k = 0.7\text{m (PLAXIS)}$





FINITE ELEMENT FORMULATION FOR CONSOLIDATION (1)

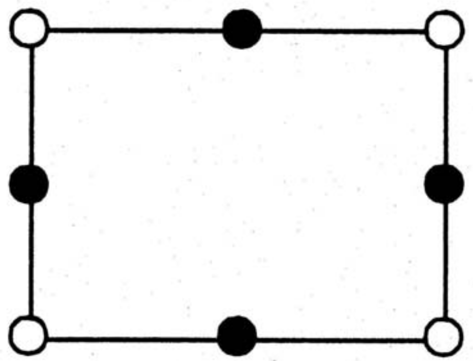
- Effective stresses $\underline{\sigma} = \underline{\sigma}' + \underline{m}(p_{steady} + p_{excess})$

$$\underline{\sigma} = \left(\sigma_{xx}\ \sigma_{yy}\ \sigma_{zz}\ \sigma_{xy}\ \sigma_{yz}\ \sigma_{zx} \right)^T \quad \text{and:} \quad \underline{m} = \left(1\ 1\ 1\ 0\ 0\ 0 \right)^T$$

- Constitutive law $\underline{\dot{\sigma}}' = \underline{\underline{M}}\,\underline{\dot{\varepsilon}}$

$$\underline{\varepsilon} = \left(\varepsilon_{xx}\ \varepsilon_{yy}\ \varepsilon_{zz}\ \gamma_{xy}\ \gamma_{yz}\ \gamma_{zx} \right)^T$$

- Discretization



$$\underline{u} = \underline{\underline{N}}\,\underline{v} \qquad \underline{p} = \underline{\underline{N}}\,\underline{p_n} \qquad \underline{\varepsilon} = \underline{\underline{B}}\,\underline{v}$$

- In terms of excess pore pressure
- same shape functions for displacements and pore pressures

FINITE ELEMENT FORMULATION FOR CONSOLIDATION (2)

- Mechanical problem: equilibrium equation

$$\int \underline{\underline{B}}^T d\underline{\sigma}\, dV = \int \underline{\underline{N}}^T d\underline{f}\, dV + \int \underline{\underline{N}}^T d\underline{t}\, ds + \underline{r}_0$$

$$\underline{r}_0 = \int \underline{\underline{N}}^T \underline{f}_0\, dV + \int \underline{\underline{N}}^T \underline{t}_0\, ds - \int \underline{\underline{B}}^T \underline{\sigma}_0\, dV$$

$$\underline{\underline{K}}\, d\underline{v} + \underline{\underline{L}}\, d\underline{p}_n = d\underline{f}_n$$

$$\underline{\underline{K}} = \int \underline{\underline{B}}^T \underline{\underline{M}}\, \underline{\underline{B}}\, dV$$ Stiffness matrix

$$\underline{\underline{L}} = \int \underline{\underline{B}}^T \underline{m}\, \underline{\underline{N}}\, dV$$ Coupling matrix

$$d\underline{f}_n = \int \underline{\underline{N}}^T d\underline{f}\, dV + \int \underline{\underline{N}}^T d\underline{t}\, ds$$ Incremental load vector

FINITE ELEMENT FORMULATION FOR CONSOLIDATION (3)

- Hydraulic (flow) problem: continuity equation

$$\nabla^T \underline{R} \nabla p / \gamma_w + \underline{m}^T \frac{\partial \varepsilon}{\partial t} - \frac{n}{K_w} \frac{\partial p}{\partial t} = 0 \qquad \underline{\underline{R}} = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$$

$$- \underline{\underline{H}} \, \underline{p}_n + \underline{\underline{L}}^T \frac{d \underline{v}}{d t} - \underline{\underline{S}} \frac{d \underline{p}_n}{d t} = \underline{q}$$

$\underline{\underline{H}} = \int (\nabla \underline{\underline{N}})^T \underline{\underline{R}} \nabla \underline{\underline{N}} / \gamma_w \, dV$ Flow matrix

$\underline{\underline{L}} = \int \underline{\underline{B}}^T \underline{m} \, \underline{\underline{N}} \, dV$ Coupling matrix

$\underline{\underline{S}} = \int \frac{n}{K_w} \underline{\underline{N}}^T \underline{\underline{N}} \, dV$ Water compressibility matrix

FINITE ELEMENT FORMULATION FOR CONSOLIDATION (4)

- Global system of equations

$$\begin{bmatrix} \underline{\underline{K}} & \underline{\underline{L}} \\ \underline{\underline{L}}^T & -\underline{\underline{S}} \end{bmatrix} \begin{bmatrix} \dfrac{d\,\underline{v}}{d\,t} \\ \dfrac{d\,\underline{p}_n}{d\,t} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \underline{\underline{H}} \end{bmatrix} \begin{bmatrix} \underline{v} \\ \underline{p}_n \end{bmatrix} + \begin{bmatrix} \dfrac{d\,\underline{f}_n}{d\,t} \\ \underline{q}_n \end{bmatrix}$$

- Step-by-step integration procedure

$$\begin{bmatrix} \underline{\underline{K}} & \underline{\underline{L}} \\ \underline{\underline{L}}^T & -\underline{\underline{S}}^* \end{bmatrix} \begin{bmatrix} \Delta\underline{v} \\ \Delta\underline{p}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \Delta t\,\underline{\underline{H}} \end{bmatrix} \begin{bmatrix} \underline{v}_0 \\ \underline{p}_{n0} \end{bmatrix} + \begin{bmatrix} \Delta\,\underline{f}_n \\ \Delta t\,\underline{q}_n^* \end{bmatrix}$$

$$\underline{\underline{S}}^* = \alpha\ \Delta t\ \underline{\underline{H}} + \underline{\underline{S}} \qquad\qquad \underline{q}_n^* = \underline{q}_{n0} + \alpha\ \Delta\,\underline{q}_n$$

$0 < \alpha < 1$; Generally, $\alpha = 1$ (fully implicit)

FINITE ELEMENT FORMULATION FOR CONSOLIDATION (5)

- Time step
  - Automatic time stepping is required
  - Critical time step

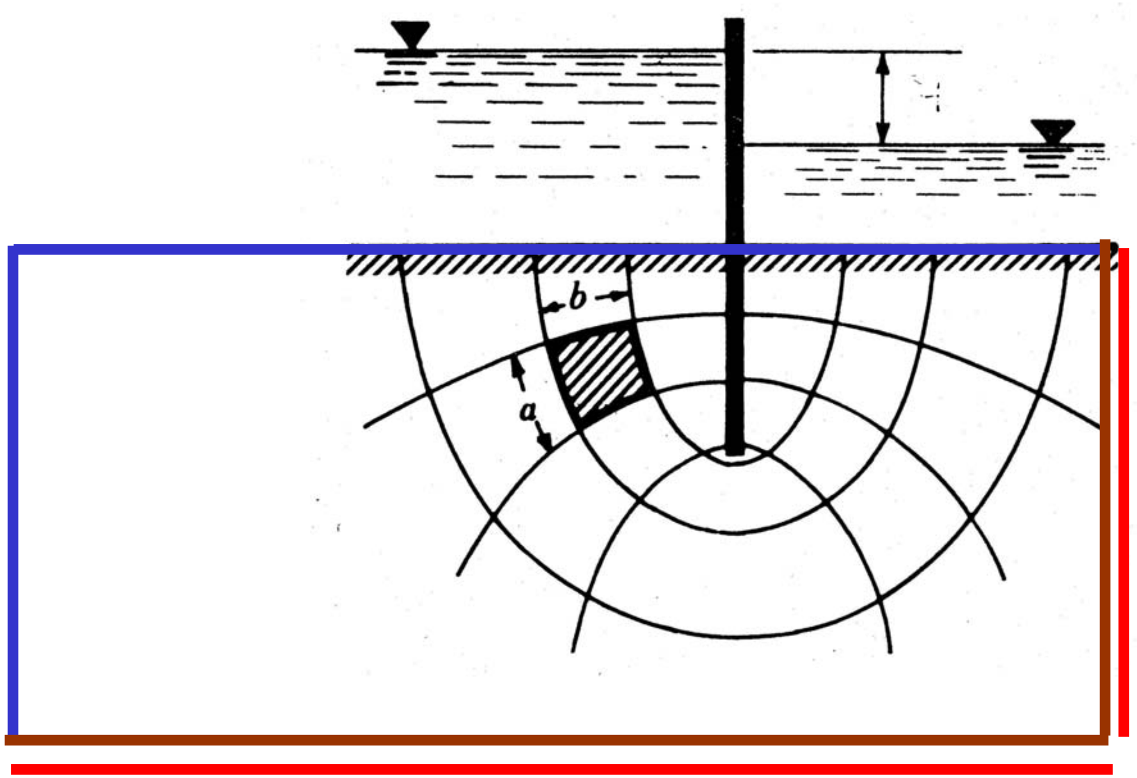
$$\Delta t_{critical} = \frac{H^2 \gamma_w (1-2\nu)(1+\nu)}{80\ k_y E(1-\nu)} \qquad \text{(15-node triangles)}$$

$$\Delta t_{critical} = \frac{H^2 \gamma_w (1-2\nu)(1+\nu)}{40\ k_y E(1-\nu)} \qquad \text{(6-node triangles)}$$

- Consolidation analysis
  - Prescribed time
  - Maximum excess pore pressure

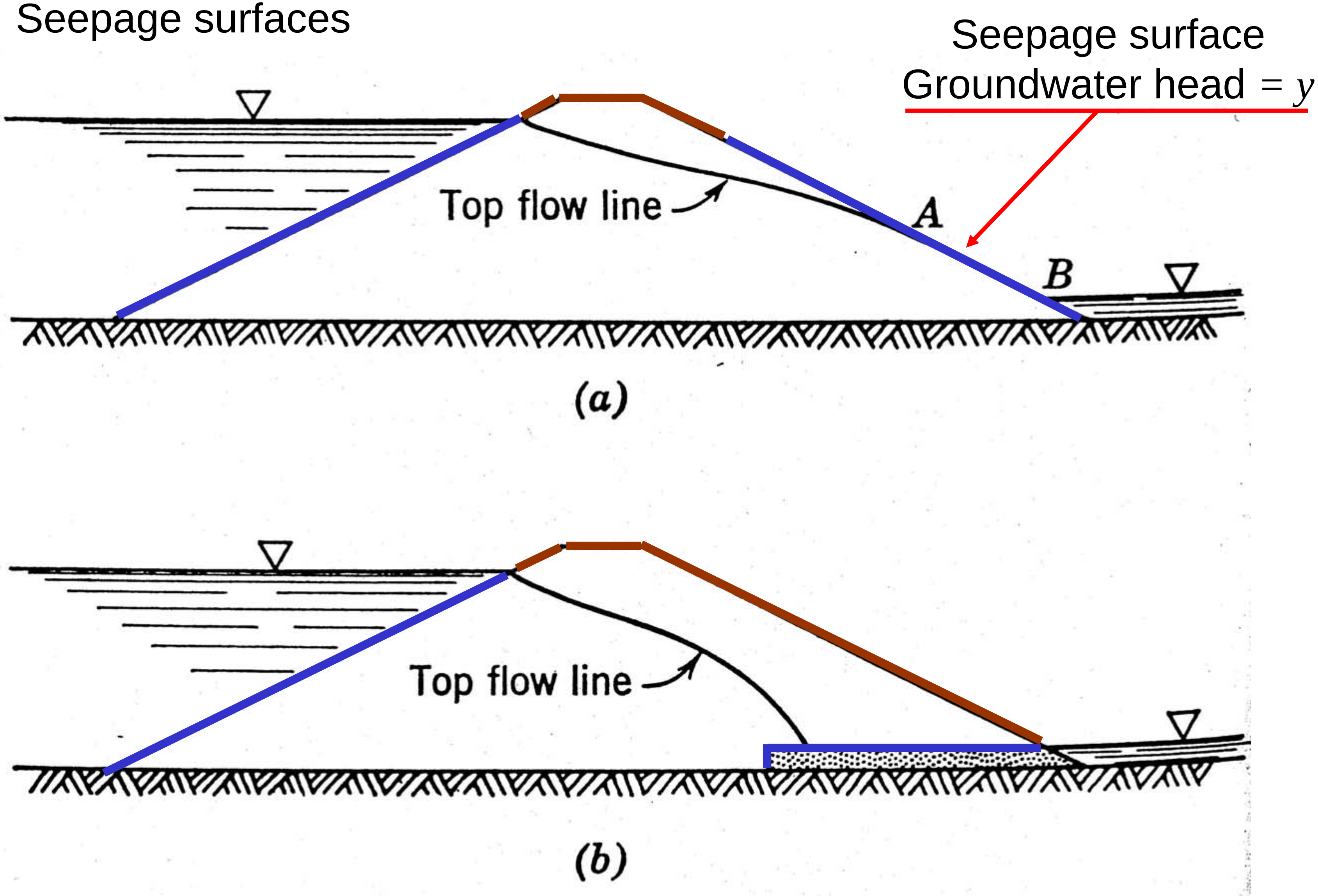
BOUNDARY CONDITIONS

- Flow boundary conditions
  - Prescribed groundwater head
  - Closed flow boundary
  - Closed consolidation boundaries



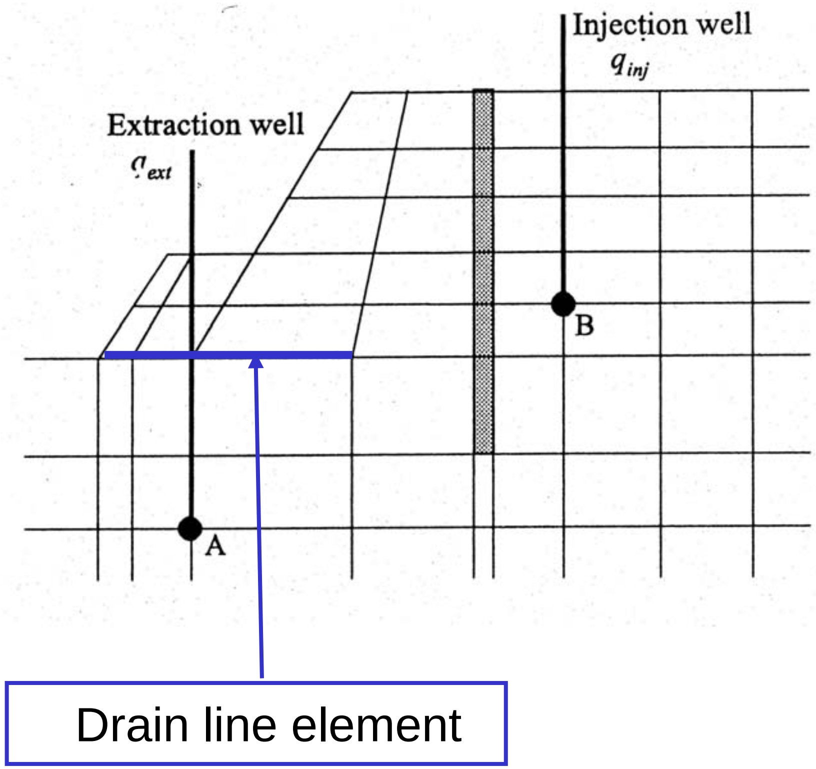
BOUNDARY CONDITIONS

- Seepage surfaces

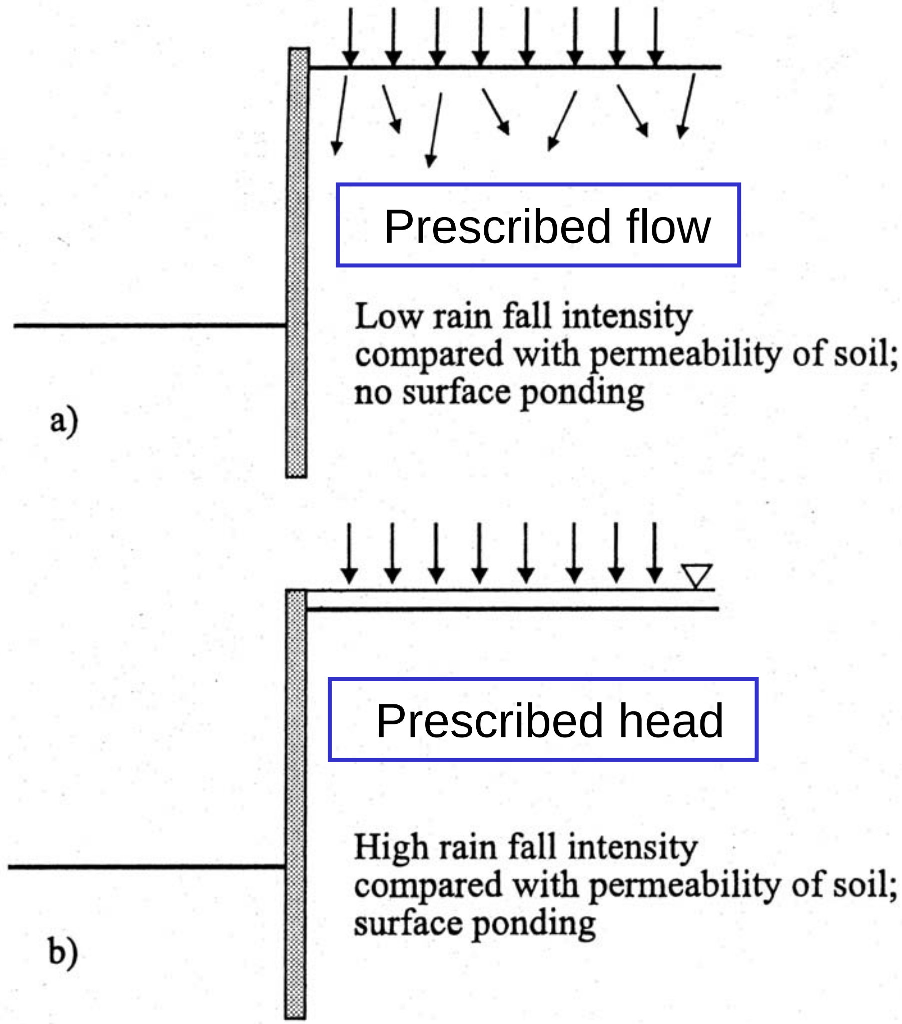


BOUNDARY CONDITIONS

- Sources and sinks

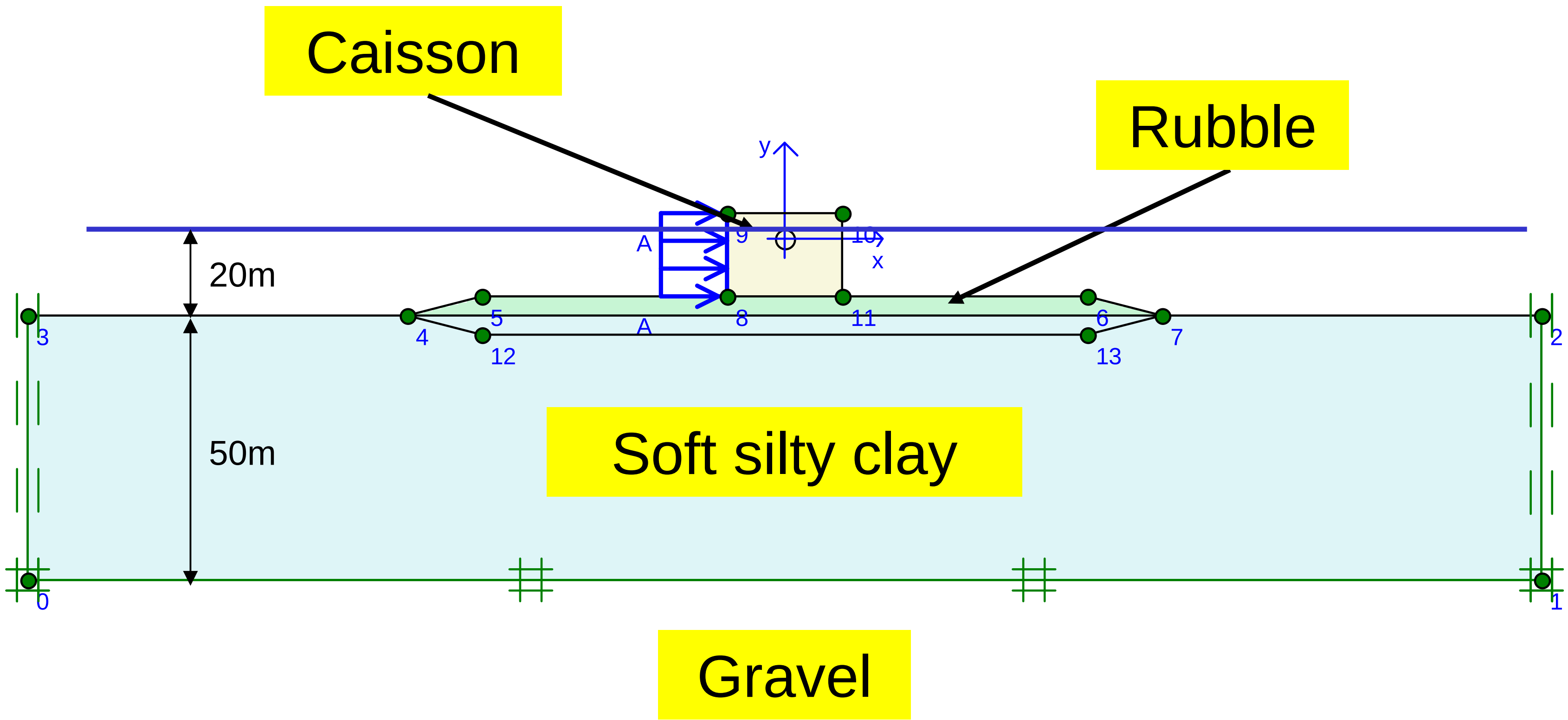


- Infiltration



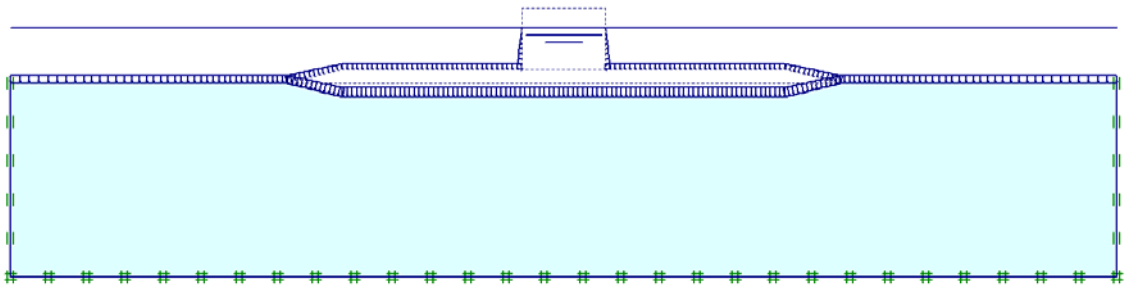
Potts & Zdravkovic (1999)

Barcelona breakwater

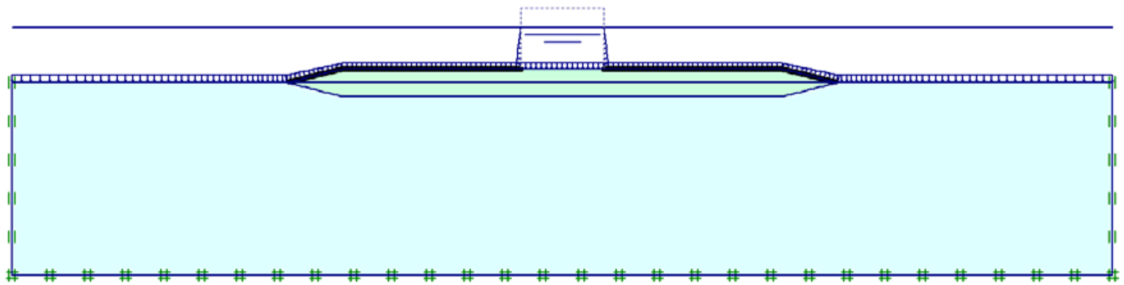


Barcelona breakwater: stages (1)

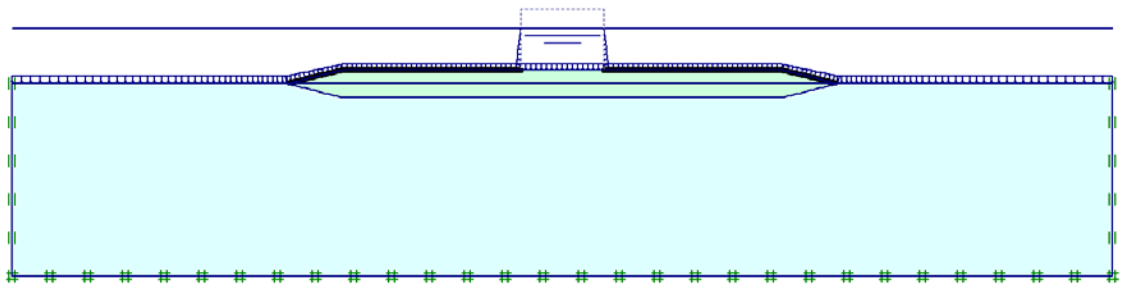
Dredging



Bench construction

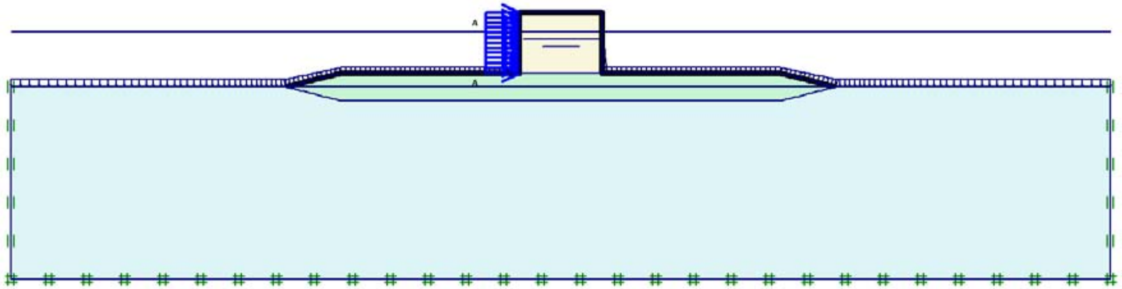


Consolidation

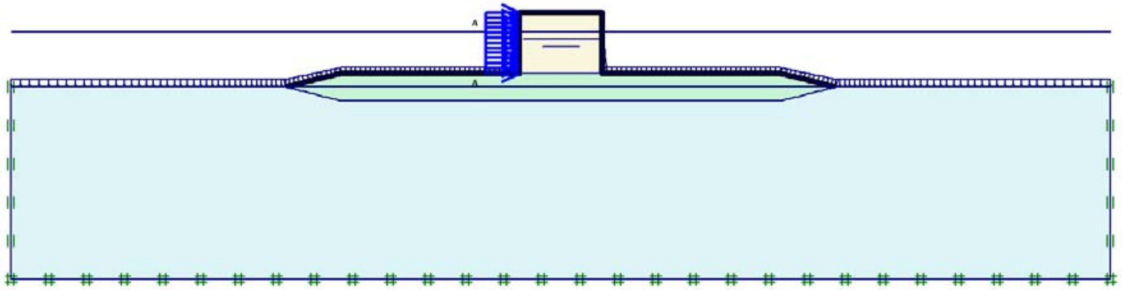


Barcelona breakwater stages (2)

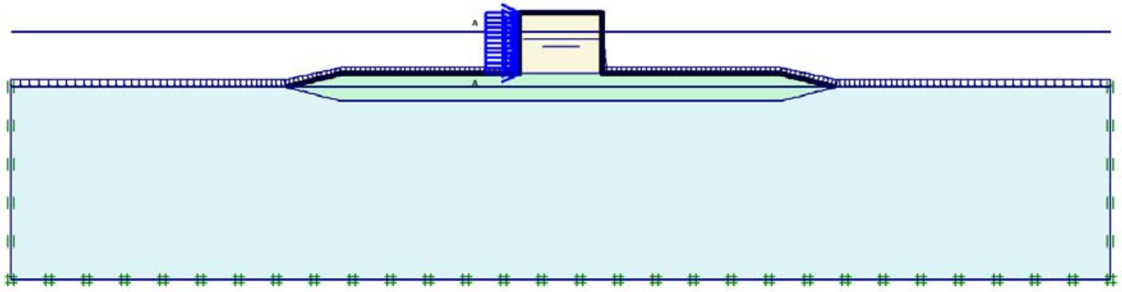
Caisson construction



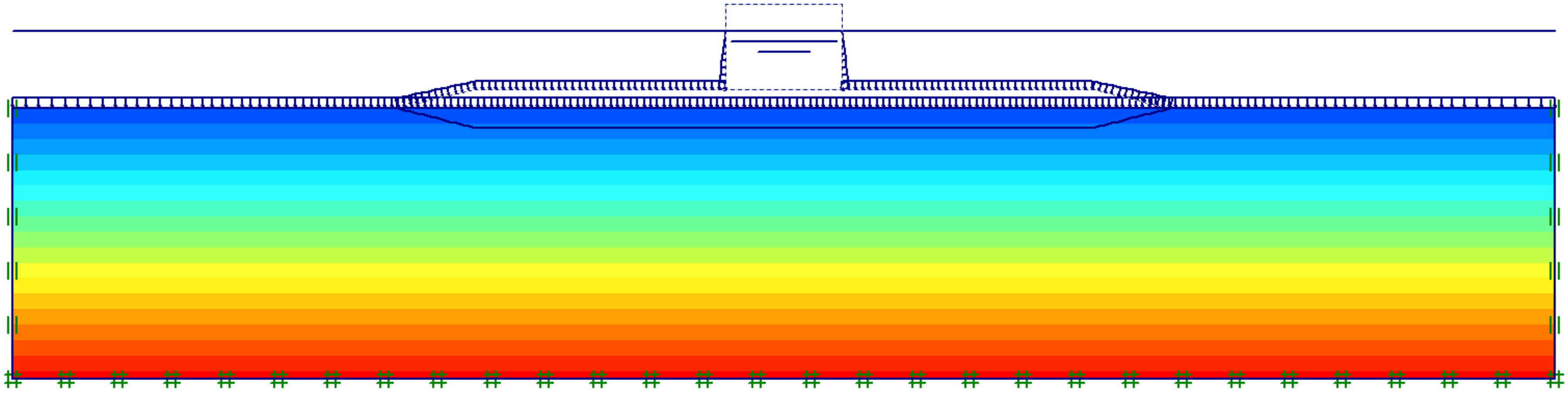
Consolidation



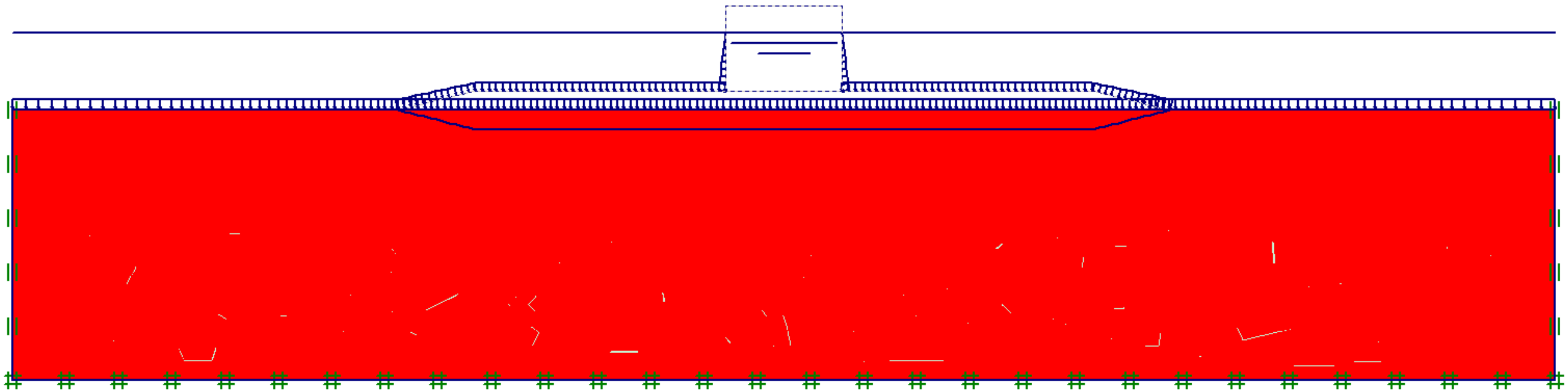
Storm loading



Initial pore pressures

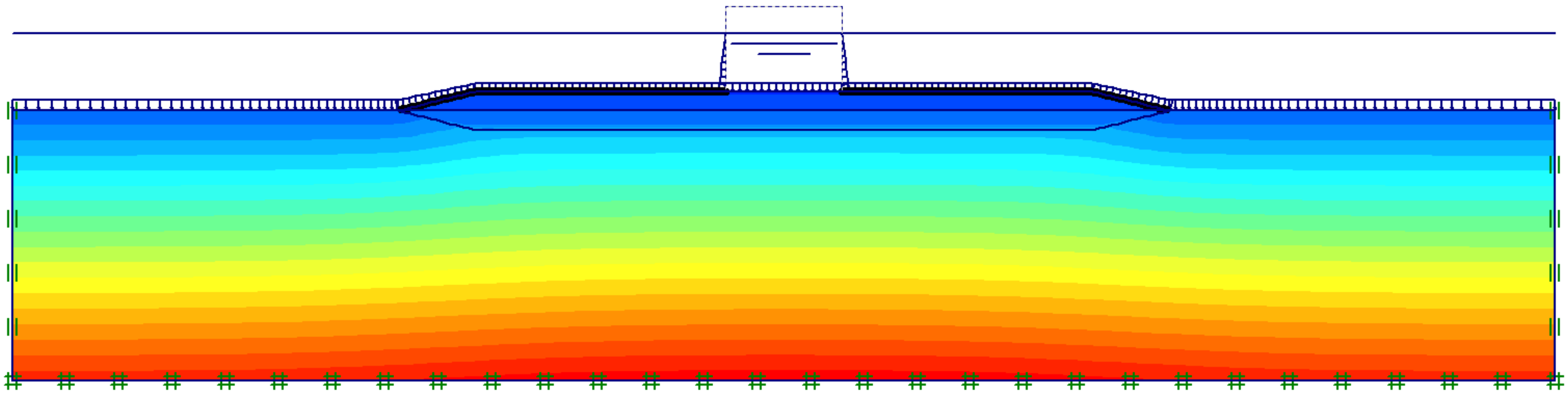


Active pore pressures

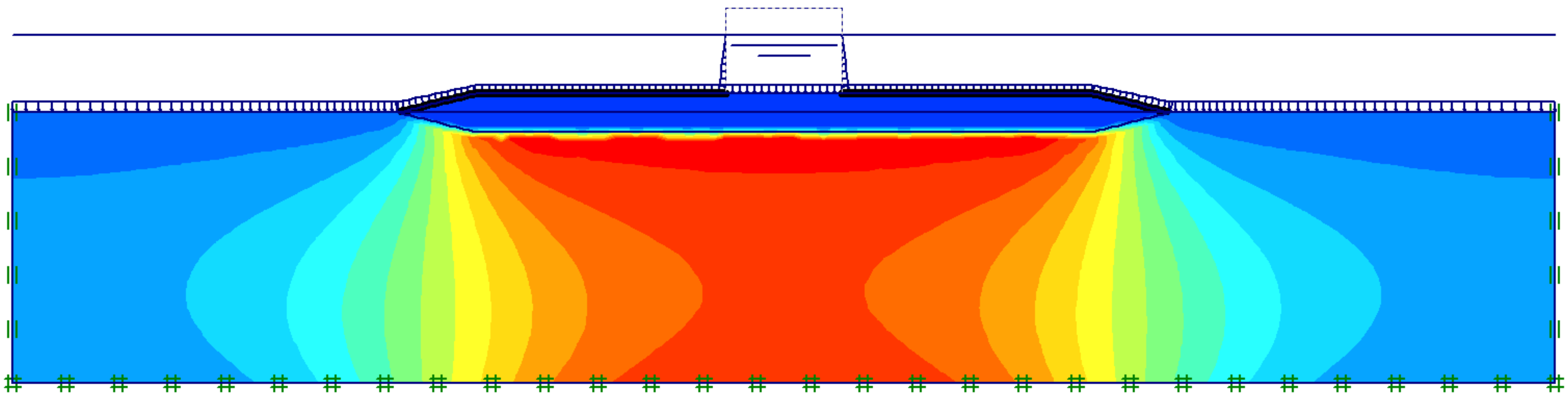


Groundwater head

Pore pressures after placing the bench



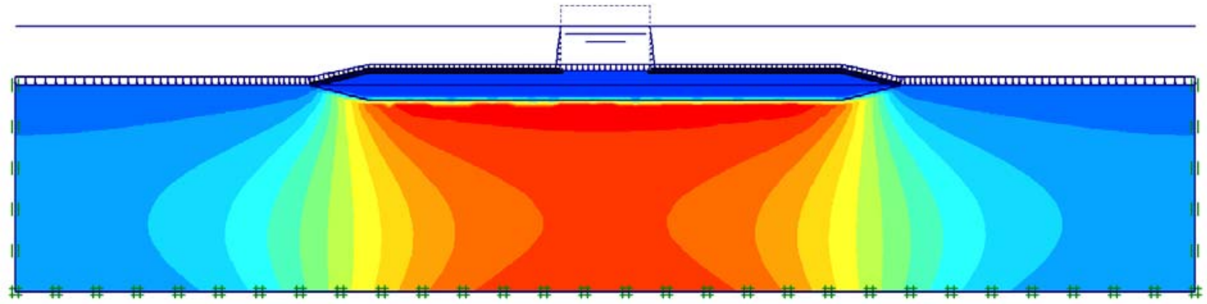
Active pore pressures



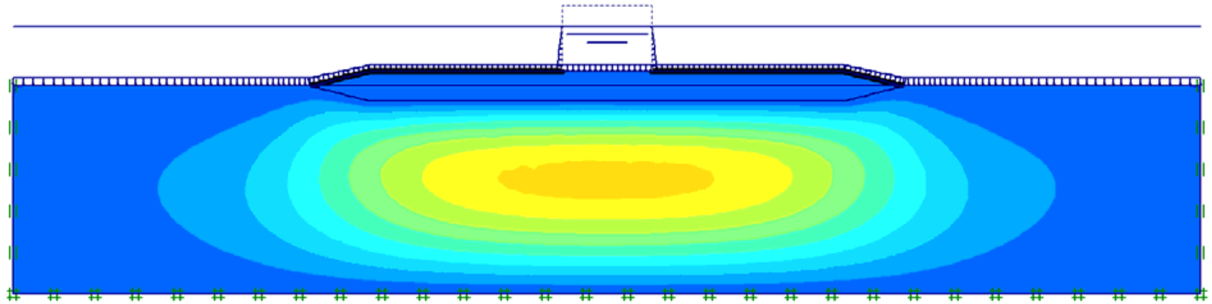
Excess pore pressures

Excess pore pressures during consolidation

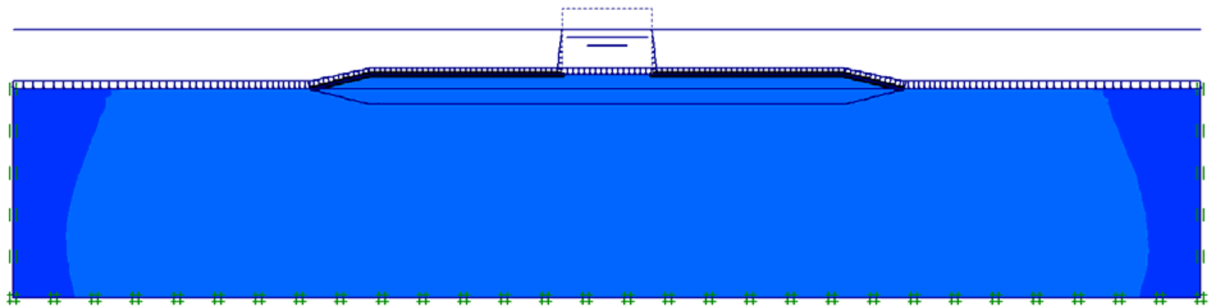
Initial



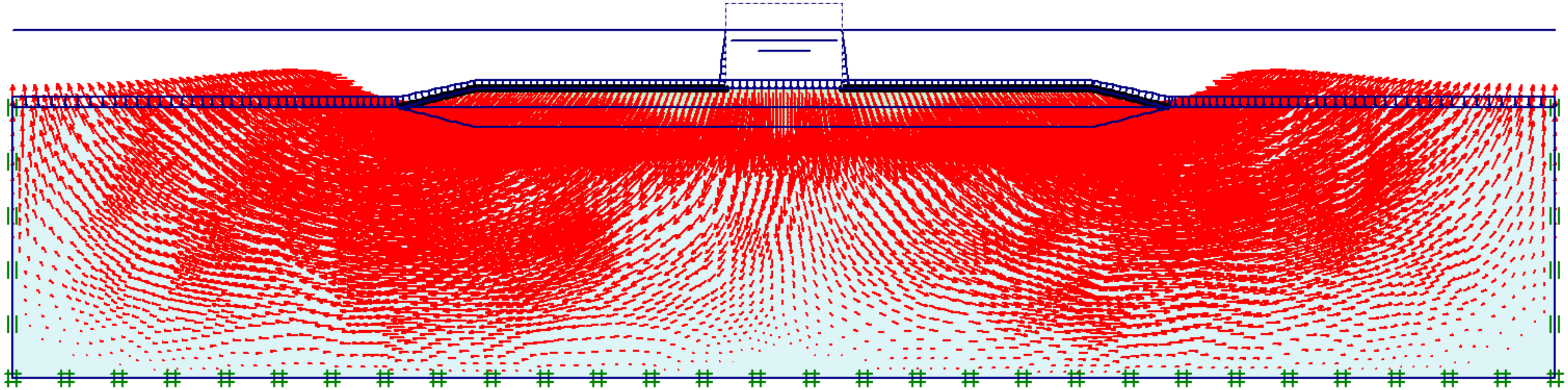
After 30 days



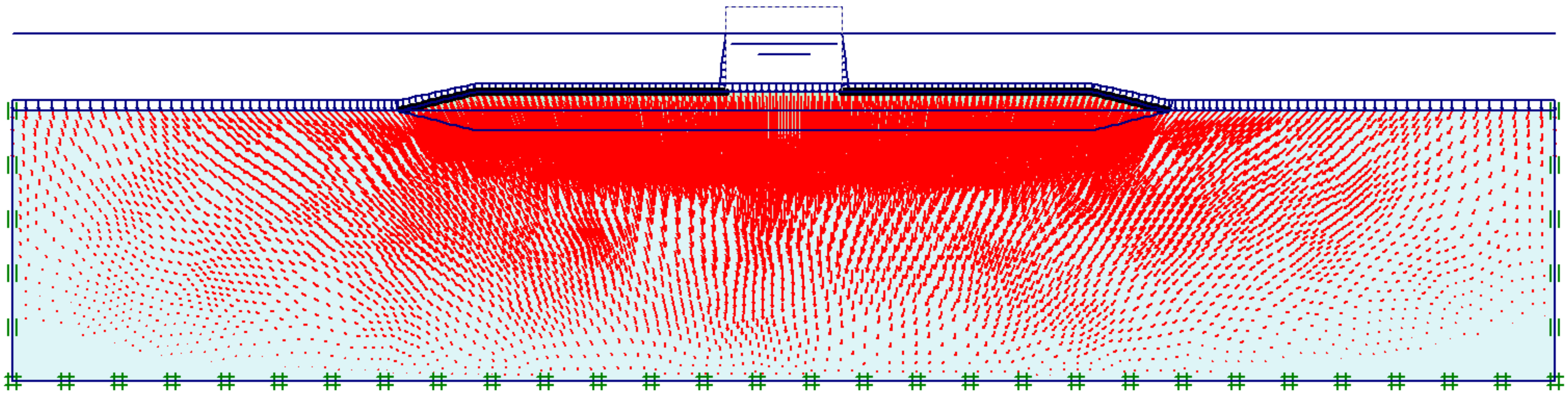
Final



Displacements during construction and consolidation

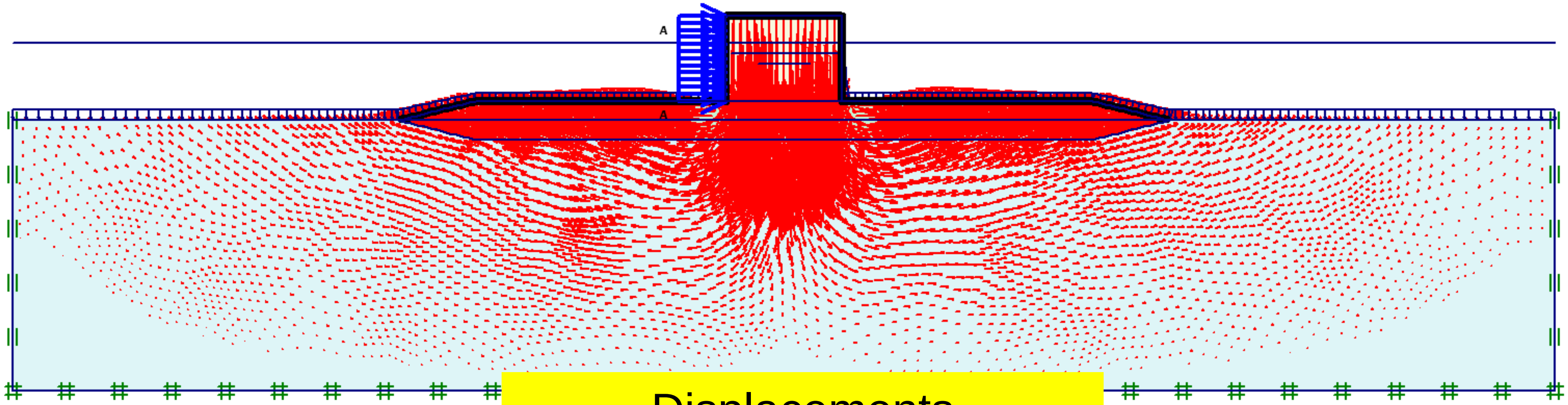


Bench construction

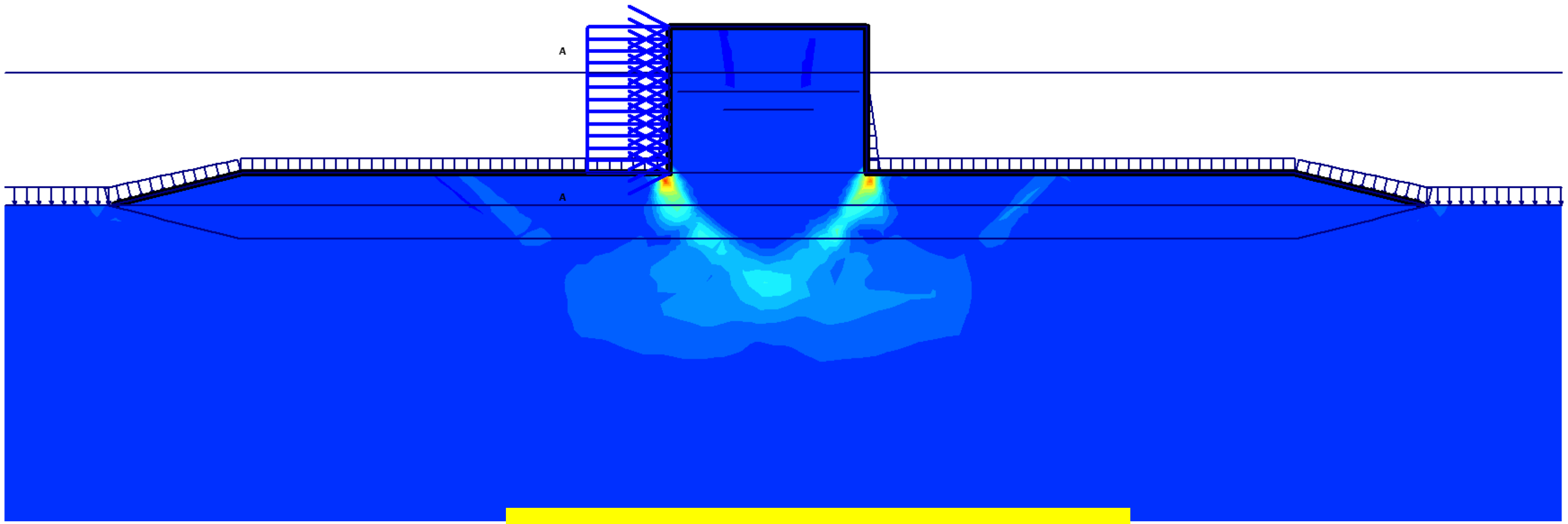


Consolidation

Caisson construction



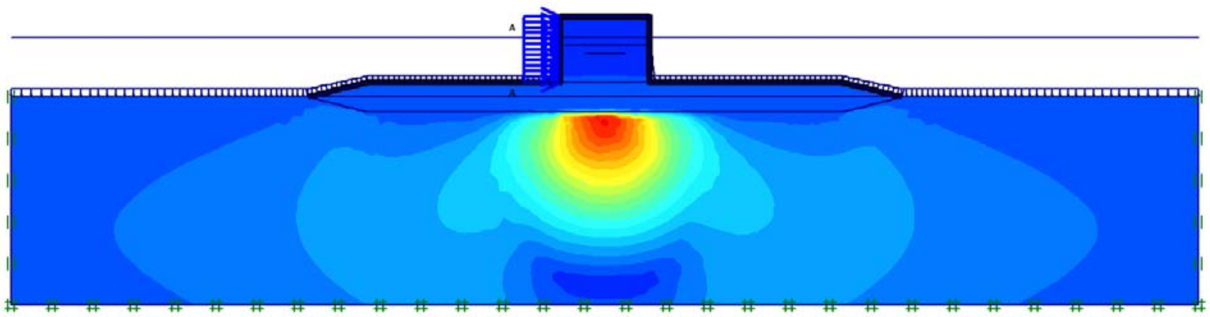
Displacements



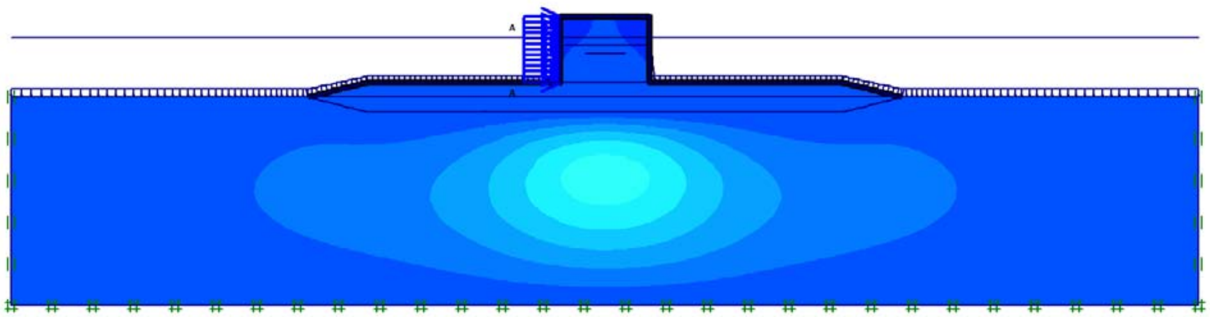
Incremental shear strains

Excess pore pressures during consolidation (caisson)

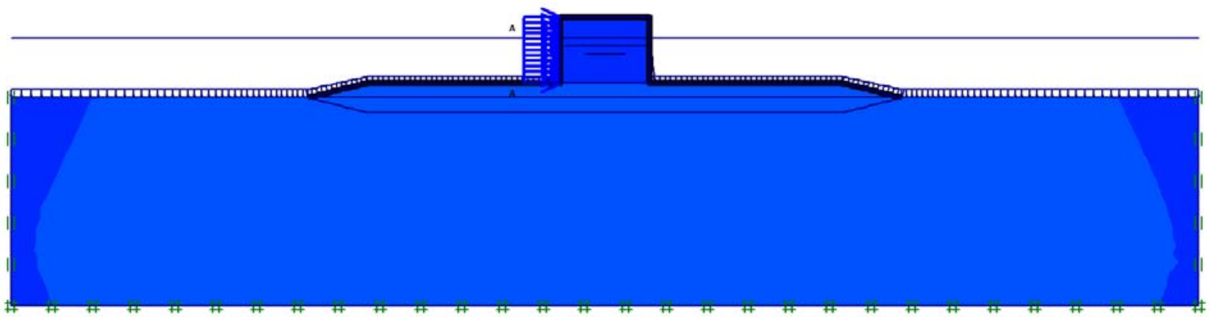
Initial



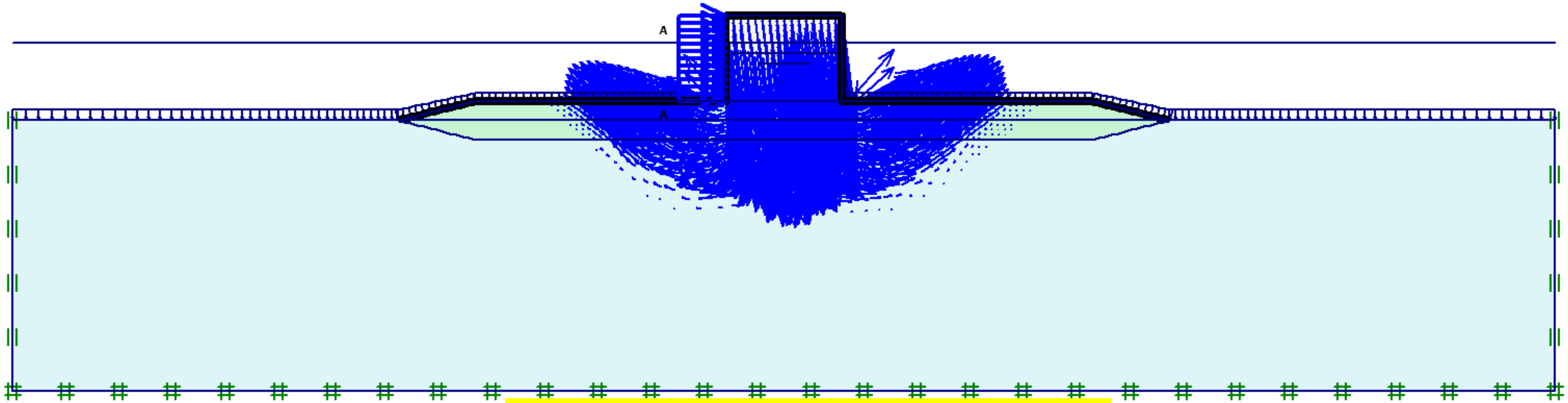
After 30 days



Final



Failure (factor of safety)



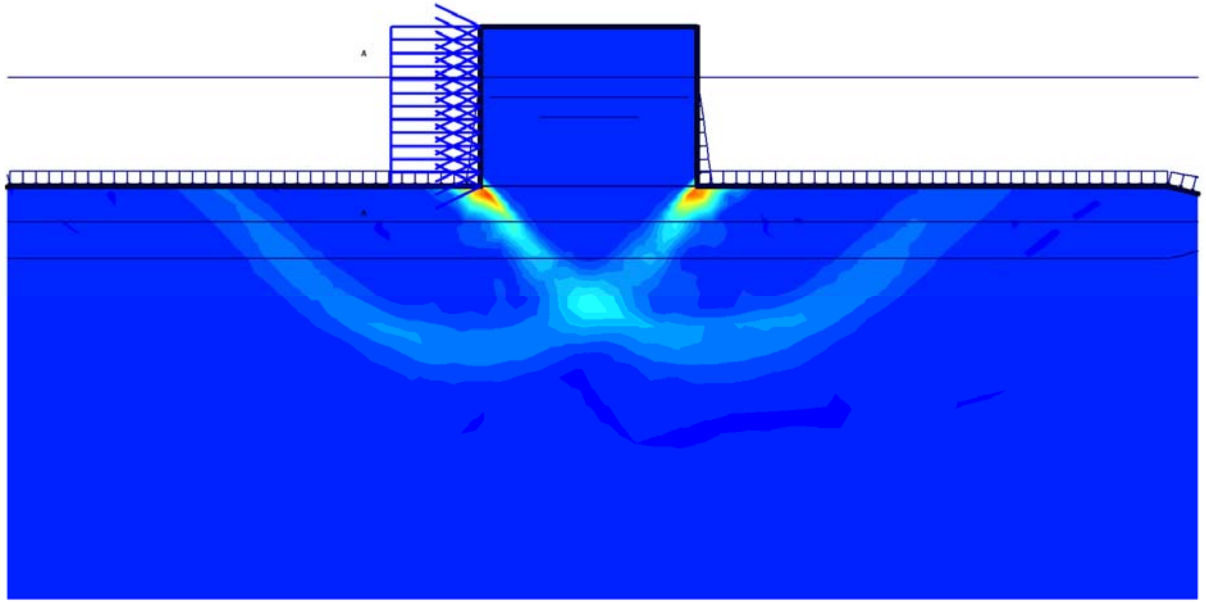
Incremental displacements

Factors of safety

After construction
FS=1.06

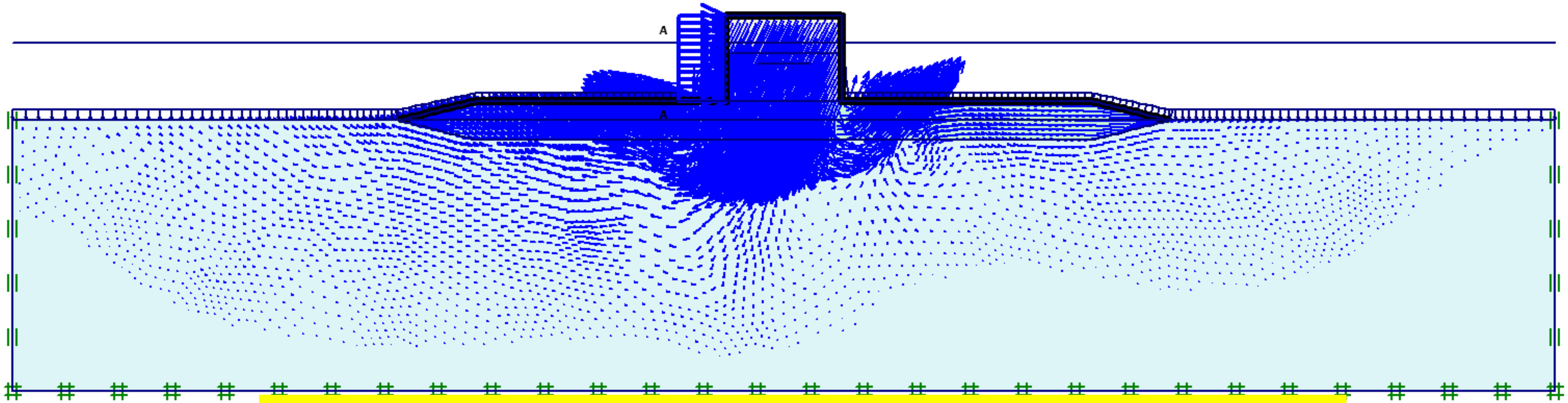
After 30 days
FS=1.60

End of consolidation
F=1.74

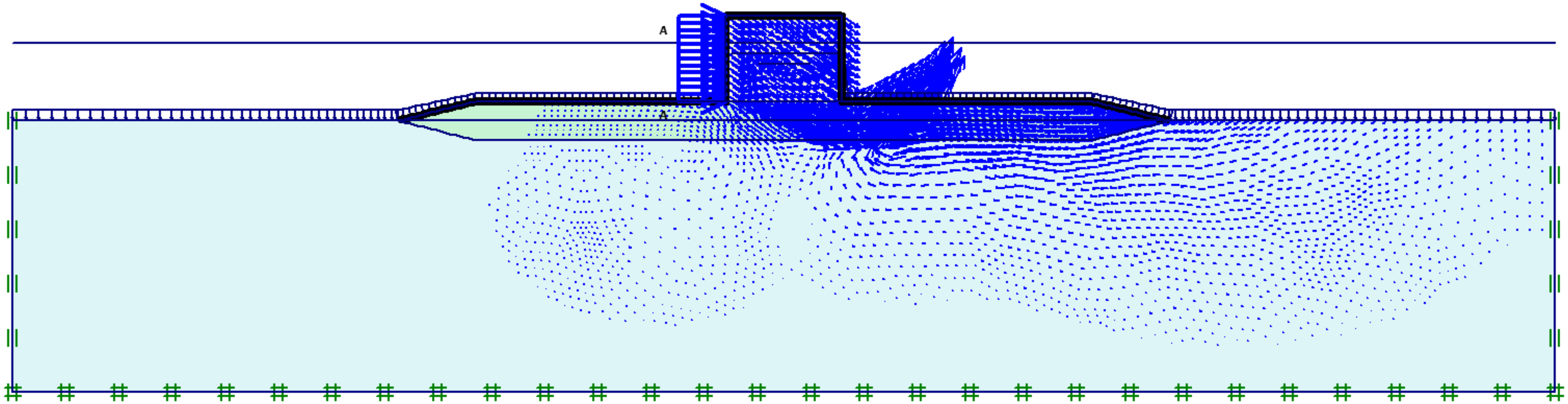


Incremental shear strains

Horizontal storm load



After construction, horizontal pressure = 0.7 kPa



After consolidation, horizontal pressure = 178.2 kPa

Barcelona breakwater

Barcelona breakwater

Barcelona breakwater

Quay failure 1-01-2007