



COMPUTATIONAL GEOTECHNICS COURSE

CONSOLIDATION

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CONSOLIDATION: OUTLINE

- Introduction
 - Basic theory of groundwater flow
 - Permeability
 - Confined and unconfined problems
 - Finite element formulation for consolidation
 - Mechanical problem
 - Hydraulic problem
 - Global equations
 - Time step
 - Boundary conditions
-

TYPES OF ANALYSIS

■ Drained

- Loading/Construction/ excavation: very slow (in relation to the soil permeability)

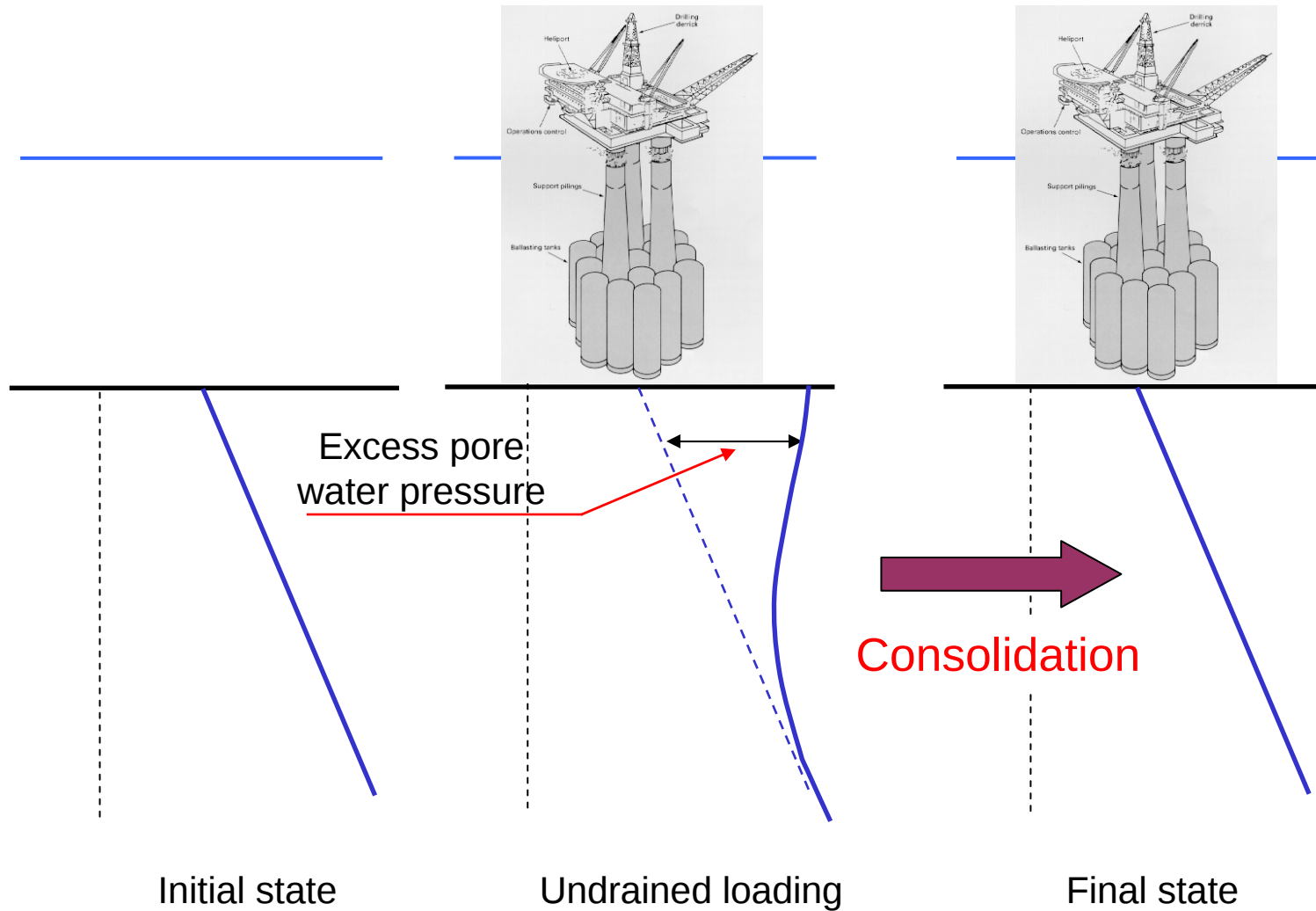
■ Undrained

- Loading/Construction/ excavation: very fast (in relation to the soil permeability)

■ Intermediate cases: **consolidation** analysis

- Both mechanical and hydraulic (flow) problems interact
 - More complex computations: coupled analysis
-

EXAMPLE



OTHER EXAMPLES



1973



1984

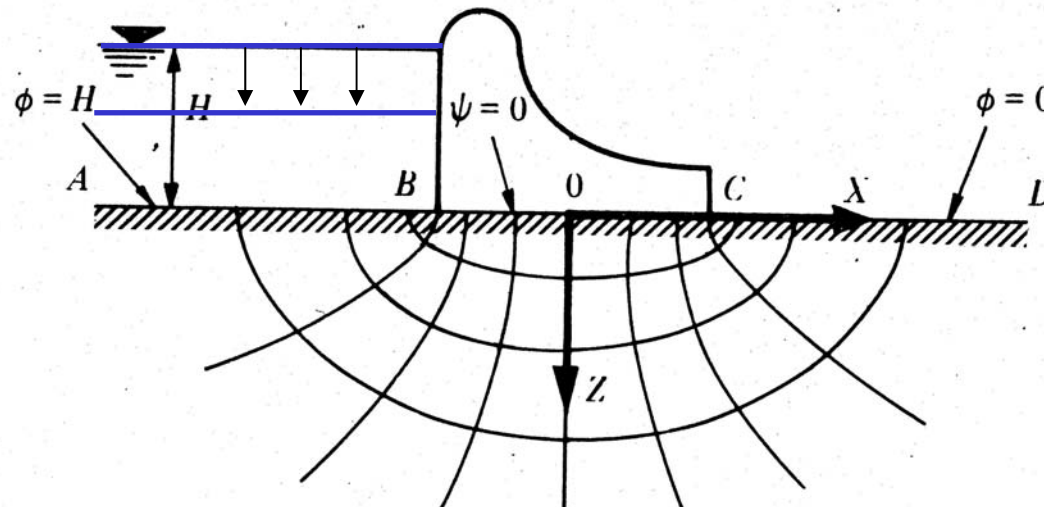
Ekofisk tank

OTHER EXAMPLES

- Construction at intermediate rates



- Change of hydraulic conditions



BASIC THEORY OF GROUNDWATER FLOW

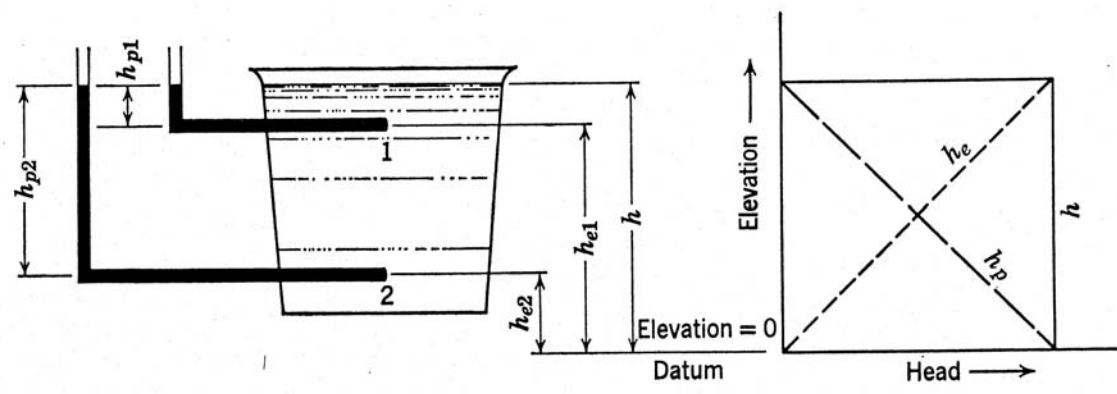
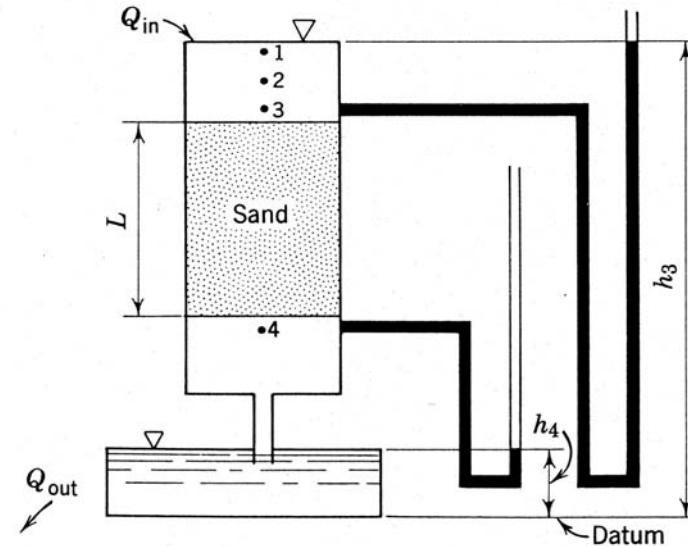
● Darcy's law

$$q = -k \frac{\Delta h}{L} = -k i$$

$$q = -k \frac{\partial h}{\partial y} = -k \frac{\partial \phi}{\partial y}$$

$$h = \phi = y + \frac{p}{\gamma_w}$$

q : flow $h = \phi$: total head
 y : vertical coordinate
 p : water pressure



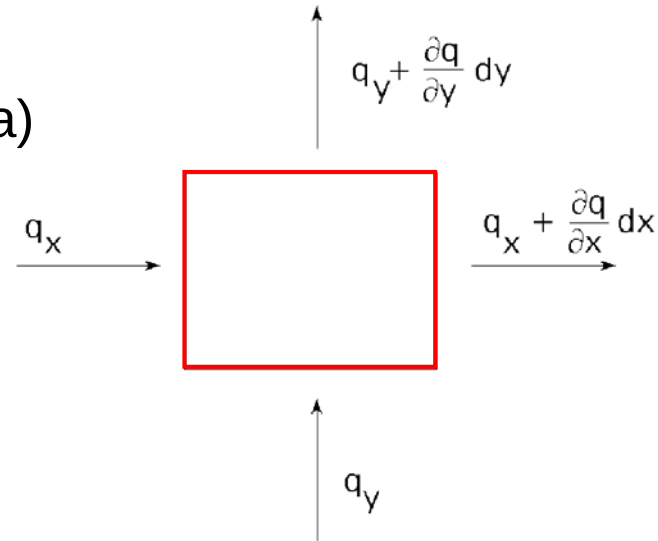
BASIC THEORY OF GROUNDWATER FLOW

- Permeability often anisotropic

$$q_x = -k_x \frac{\partial \phi}{\partial x} \quad q_y = -k_y \frac{\partial \phi}{\partial y}$$

- Equation of continuity for steady state
(no sources/sinks in an elementary area)

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0$$



- Excess pore pressure

$$p = p_{steady} + p_{excess}$$

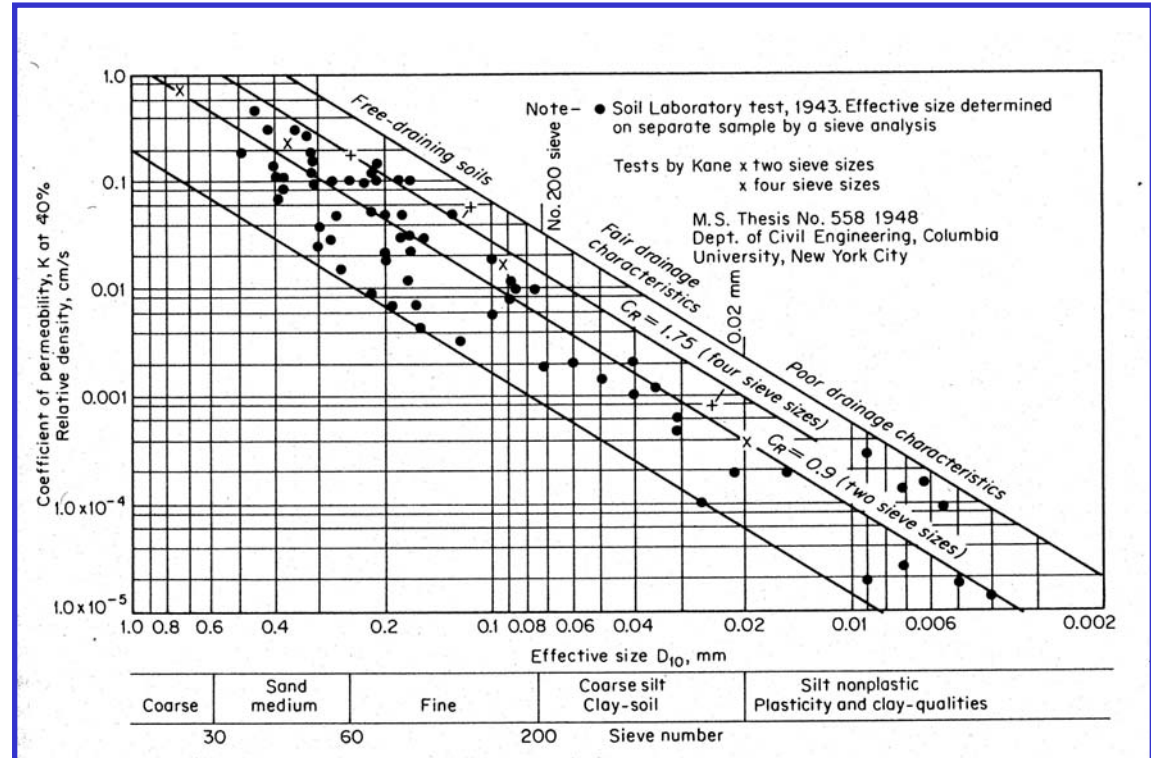
p : (active) water pressure
 p_{steady} : steady state pore pressure
 p_{excess} : excess pore pressure

PERMEABILITY

- Dependence on grain size

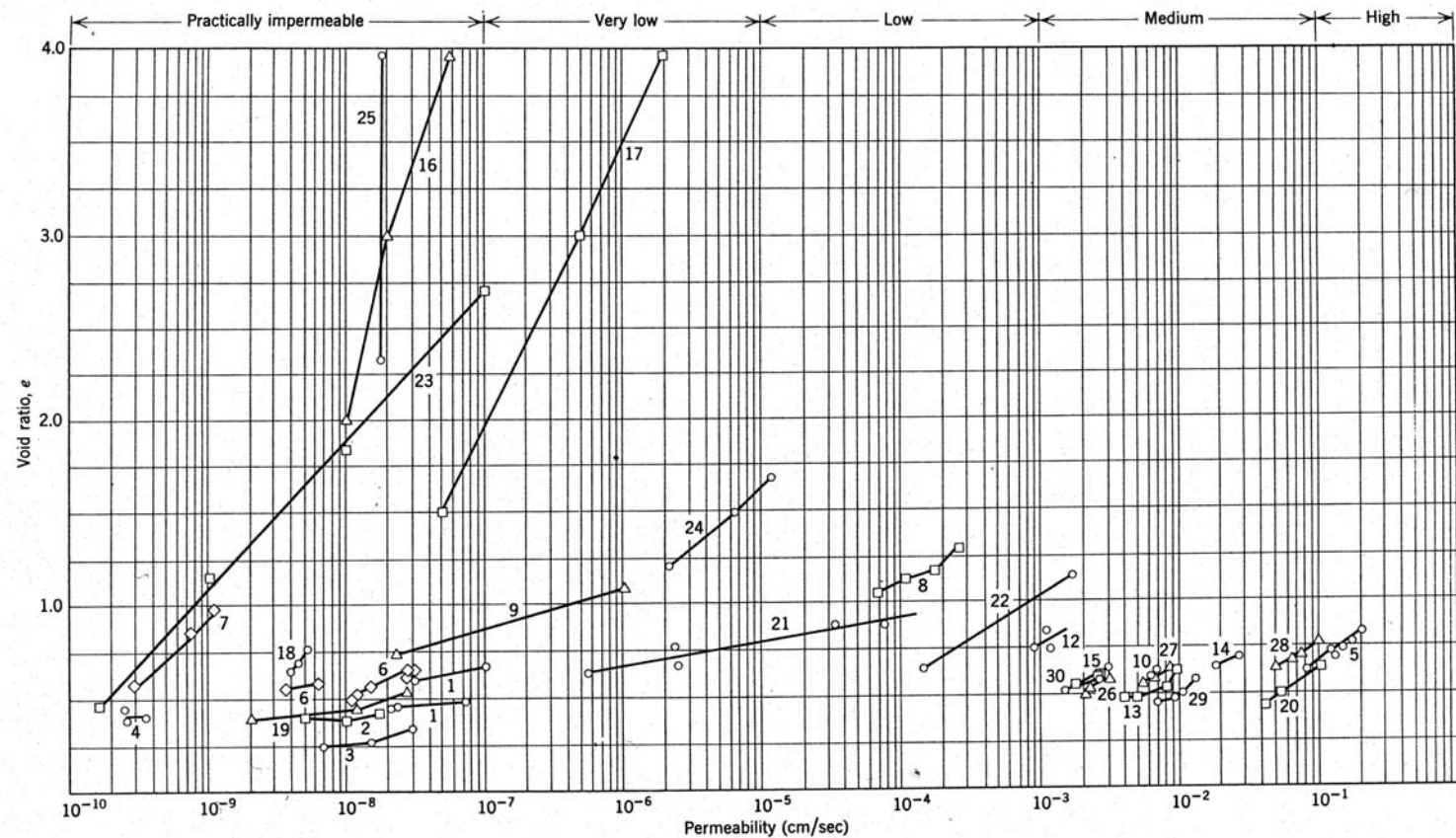
Soil	k (cm/s)
Clean gravel	> 1
Clean sand (coarse)	$1 - 10^{-2}$
Sand mixture	$10^{-2} - 5 \times 10^{-3}$
Fine sand	$5 \times 10^{-2} - 10^{-3}$
Silty sand	$2 \times 10^{-3} - 10^{-4}$
Silt	$5 \times 10^{-3} - 10^{-5}$
Clay	10^{-6} and less

Harr (1962)



PERMEABILITY

● Dependence on void ratio



Soil Identification Code

- | | | |
|------------------------------|------------------------|----------------------------|
| 1 Compacted caliche | 10 Ottawa sand | 19 Lean clay |
| 2 Compacted caliche | 11 Sand—Gaspee Point | 20 Sand—Union Falls |
| 3 Silty sand | 12 Sand—Franklin Falls | 21 Silt—North Carolina |
| 4 Sandy clay | 13 Sand—Scituate | 22 Sand from dike |
| 5 Beach sand | 14 Sand—Plum Island | 23 Sodium—Boston blue clay |
| 6 Compacted Boston blue clay | 15 Sand—Fort Peck | 24 Calcium kaolinite |
| 7 Vicksburg buckshot clay | 16 Silt—Boston | 25 Sodium montmorillonite |
| 8 Sandy clay | 17 Silt—Boston | 26–30 Sand (dam filter) |

PERMEABILITY

- PLAXIS allows consideration of change of permeability with void ratio

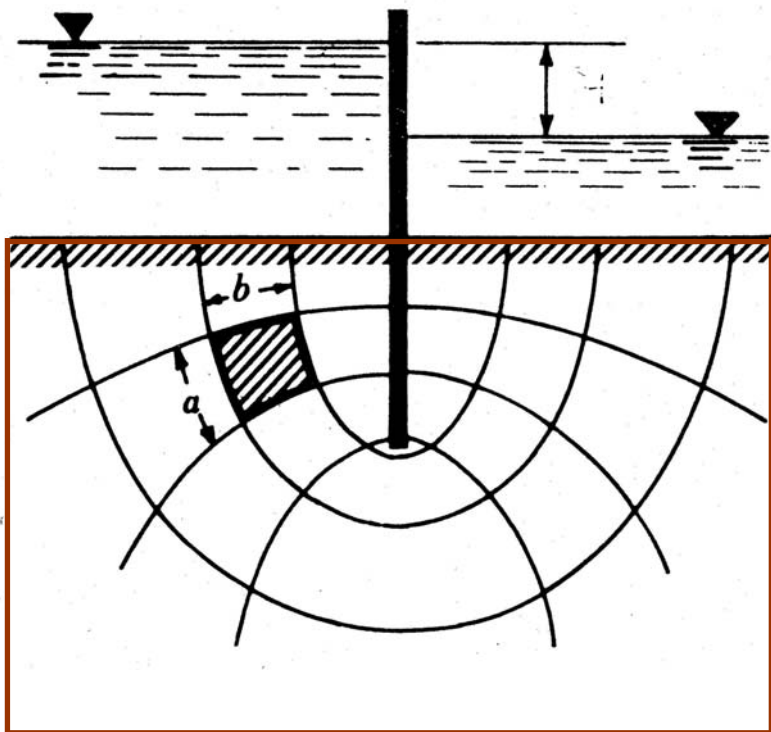
$$\log \left[\frac{k}{k_0} \right] = \frac{\Delta e}{c_k}$$

Default value for c_k is 10^{15}

- There may be large contrasts of permeability between different materials in the same problem
 - ❖ Too much permeability contrast may cause numerical difficulties
 - ❖ The ratio between the highest and lowest permeability value should not exceed 10^5
 - ❖ To simulate an almost impermeable material (e.g. concrete), a value lower by a factor 1000 is sufficient

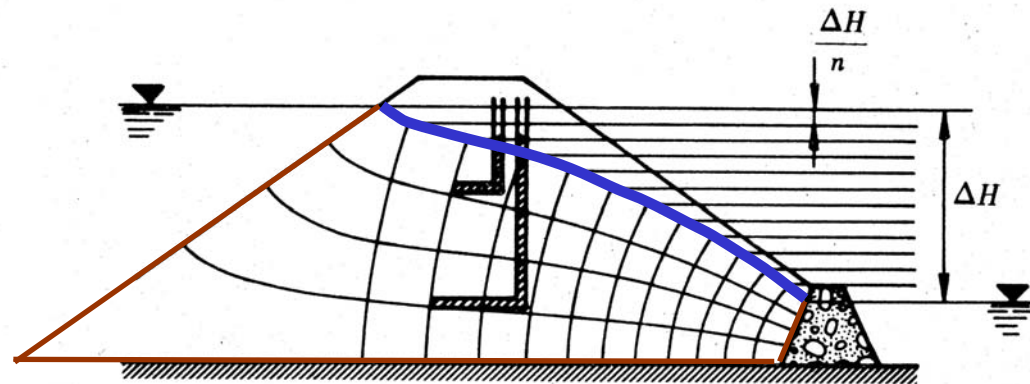
TYPES OF FLOW PROBLEMS

- Confined flow



Domain defined

- Unconfined flow



Domain undefined

TRANSITION SATURATED/UNSATURATED

$$q_x = -K^r k_x \frac{\partial \phi}{\partial x}$$

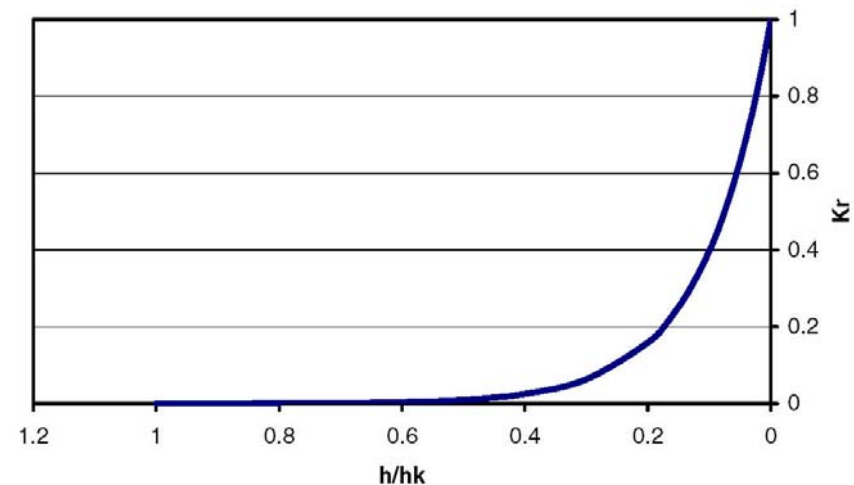
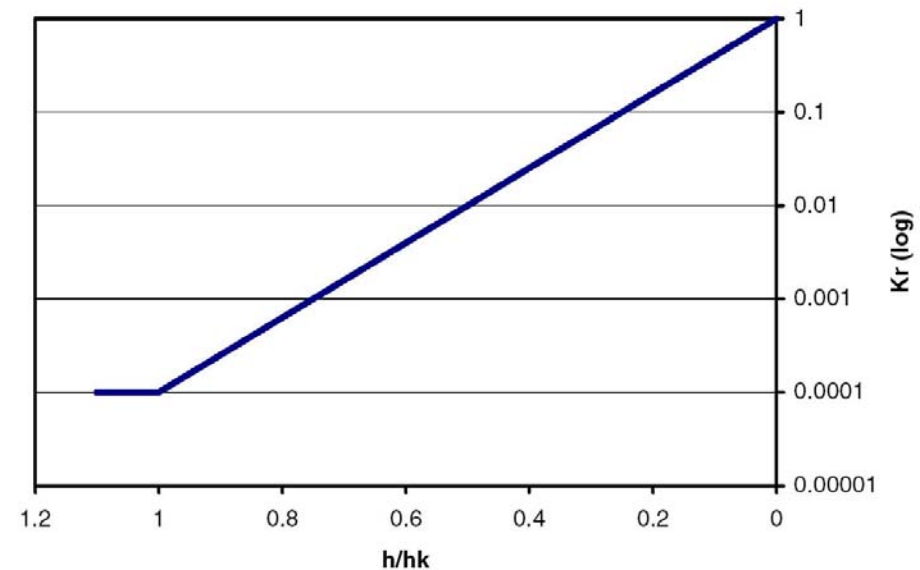
$$q_y = -K^r k_y \frac{\partial \phi}{\partial y}$$

$K^r = 1$ saturated zone

$K^r = 10^{-4}$ unsaturated zone

$$K^r = 10^{-4h/h_k} \quad \log(K^r) = -\frac{4h}{h_k}$$

$h_k = 0.7\text{m}$ (PLAXIS)



FINITE ELEMENT FORMULATION FOR CONSOLIDATION (1)

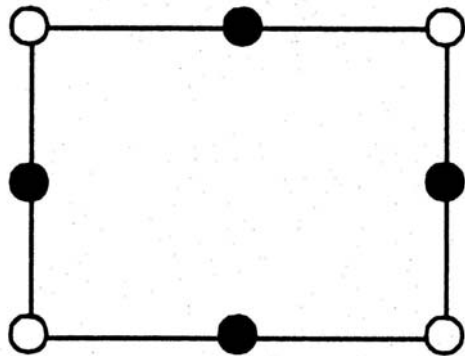
- Effective stresses $\underline{\sigma} = \underline{\sigma}' + \underline{m}(p_{steady} + p_{excess})$

$$\underline{\sigma} = (\sigma_{xx} \ \sigma_{yy} \ \sigma_{zz} \ \sigma_{xy} \ \sigma_{yz} \ \sigma_{zx})^T \quad \text{and:} \quad \underline{m} = (1 \ 1 \ 1 \ 0 \ 0 \ 0)^T$$

- Constitutive law $\underline{\dot{\sigma}}' = \underline{\underline{M}} \underline{\dot{\varepsilon}}$

$$\underline{\varepsilon} = (\varepsilon_{xx} \ \varepsilon_{yy} \ \varepsilon_{zz} \ \gamma_{xy} \ \gamma_{yz} \ \gamma_{zx})^T$$

- Discretization



$$\underline{u} = \underline{\underline{N}} \underline{v}$$

$$\underline{p} = \underline{\underline{N}} \underline{p_n}$$

$$\underline{\varepsilon} = \underline{\underline{B}} \underline{v}$$

- ❖ In terms of excess pore pressure
- ❖ same shape functions for displacements and pore pressures

FINITE ELEMENT FORMULATION FOR CONSOLIDATION (2)

- Mechanical problem: equilibrium equation

$$\int \underline{\underline{B}}^T d\underline{\underline{\sigma}} dV = \int \underline{\underline{N}}^T d\underline{\underline{f}} dV + \int \underline{\underline{N}}^T d\underline{\underline{t}} ds + \underline{\underline{r}}_0$$

$$\underline{\underline{r}}_0 = \int \underline{\underline{N}}^T \underline{\underline{f}}_0 dV + \int \underline{\underline{N}}^T \underline{\underline{t}}_0 ds - \int \underline{\underline{B}}^T \underline{\underline{\sigma}}_0 dV$$

$$\underline{\underline{K}} d\underline{\underline{v}} + \underline{\underline{L}} d\underline{\underline{p}}_n = d\underline{\underline{f}}_n$$

$$\underline{\underline{K}} = \int \underline{\underline{B}}^T \underline{\underline{M}} \underline{\underline{B}} dV \quad \text{Stiffness matrix}$$

$$\underline{\underline{L}} = \int \underline{\underline{B}}^T \underline{\underline{m}} \underline{\underline{N}} dV \quad \text{Coupling matrix}$$

$$d\underline{\underline{f}}_n = \int \underline{\underline{N}}^T d\underline{\underline{f}} dV + \int \underline{\underline{N}}^T d\underline{\underline{t}} ds \quad \text{Incremental load vector}$$

FINITE ELEMENT FORMULATION FOR CONSOLIDATION (3)

- Hydraulic (flow) problem: continuity equation

$$\nabla^T \underline{R} \nabla p / \gamma_w + \underline{m}^T \left(\frac{\partial \underline{\varepsilon}}{\partial t} \right) - \frac{n}{K_w} \frac{\partial p}{\partial t} = 0 \quad \underline{R} = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$$

$$-\underline{H} \underline{p}_n + \underline{L}^T \frac{d \underline{v}}{dt} - \underline{S} \frac{d \underline{p}_n}{dt} = \underline{q}$$

$$\underline{H} = \int (\nabla \underline{N})^T \underline{R} \nabla \underline{N} / \gamma_w dV \quad \text{Flow matrix}$$

$$\underline{L} = \int \underline{B}^T \underline{m} \underline{N} dV \quad \text{Coupling matrix}$$

$$\underline{S} = \int \frac{n}{K_w} \underline{N}^T \underline{N} dV \quad \text{Water compressibility matrix}$$

FINITE ELEMENT FORMULATION FOR CONSOLIDATION (4)

- Global system of equations

$$\begin{bmatrix} \underline{\underline{K}} & \underline{\underline{L}} \\ \underline{\underline{L}}^T & -\underline{\underline{S}} \end{bmatrix} \begin{bmatrix} \frac{d \underline{v}}{d t} \\ \frac{d \underline{p}_n}{d t} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \underline{\underline{H}} \end{bmatrix} \begin{bmatrix} \underline{v} \\ \underline{p}_n \end{bmatrix} + \begin{bmatrix} \frac{d \underline{f}_n}{d t} \\ \underline{q}_n \end{bmatrix}$$

- Step-by-step integration procedure

$$\begin{bmatrix} \underline{\underline{K}} & \underline{\underline{L}} \\ \underline{\underline{L}}^T & -\underline{\underline{S}}^* \end{bmatrix} \begin{bmatrix} \underline{\Delta v} \\ \underline{\Delta p}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \Delta t \underline{\underline{H}} \end{bmatrix} \begin{bmatrix} \underline{v}_0 \\ \underline{p}_{n0} \end{bmatrix} + \begin{bmatrix} \underline{\Delta f}_n \\ \Delta t \underline{q}_n^* \end{bmatrix}$$

$$\underline{\underline{S}}^* = \alpha \Delta t \underline{\underline{H}} + \underline{\underline{S}}$$

$$\underline{q}_n^* = \underline{q}_{n0} + \alpha \Delta \underline{q}_n$$

$0 < \alpha < 1$; Generally, $\alpha = 1$ (fully implicit)

FINITE ELEMENT FORMULATION FOR CONSOLIDATION (5)

- Time step
 - ❖ Automatic time stepping is required
 - ❖ Critical time step

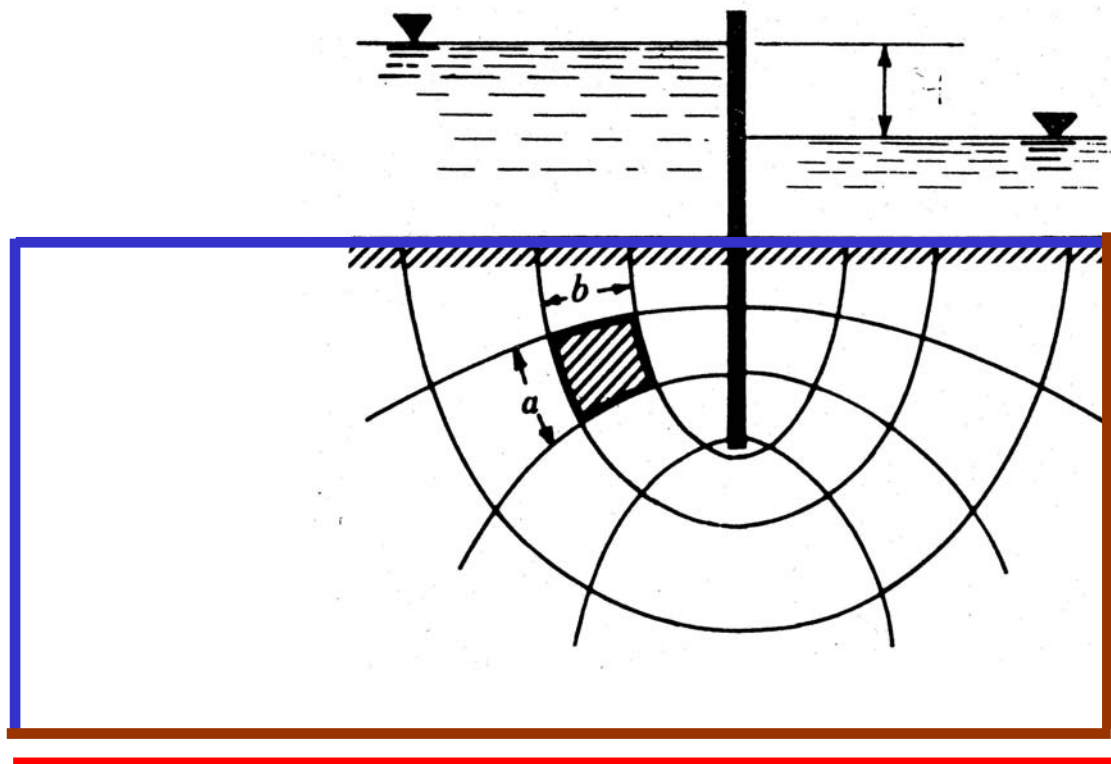
$$\Delta t_{critical} = \frac{H^2 \gamma_w (1 - 2\nu)(1 + \nu)}{80 k_y E(1 - \nu)} \quad (15\text{-node triangles})$$

$$\Delta t_{critical} = \frac{H^2 \gamma_w (1 - 2\nu)(1 + \nu)}{40 k_y E(1 - \nu)} \quad (6\text{-node triangles})$$

- Consolidation analysis
 - ❖ Prescribed time
 - ❖ Maximum excess pore pressure

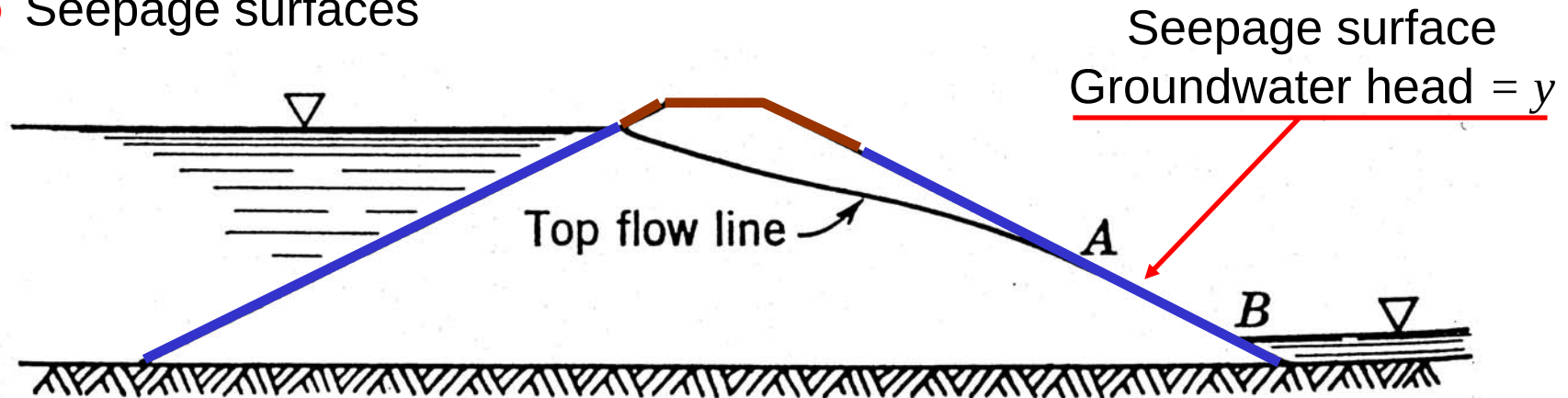
BOUNDARY CONDITIONS

- Flow boundary conditions
 - ❖ Prescribed groundwater head 
 - ❖ Closed flow boundary 
 - ❖ Closed consolidation boundaries 

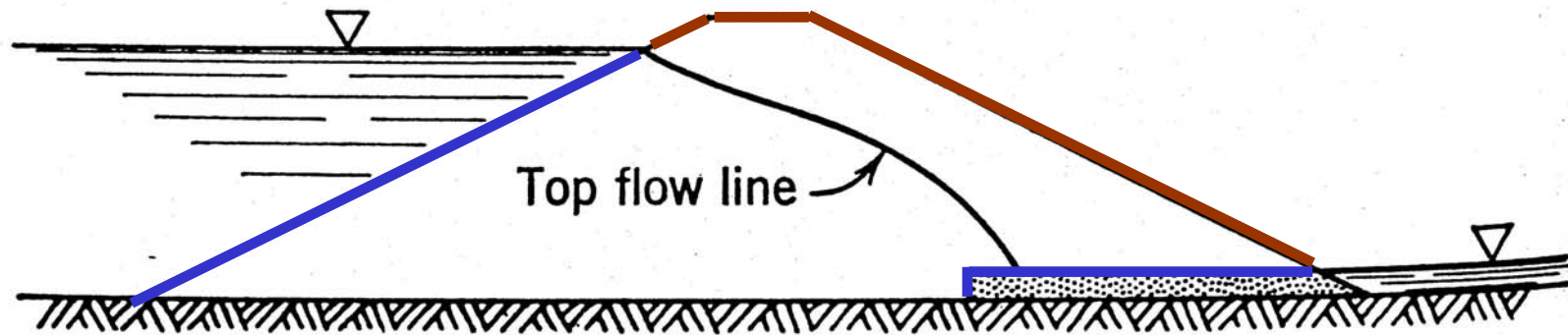


BOUNDARY CONDITIONS

- Seepage surfaces



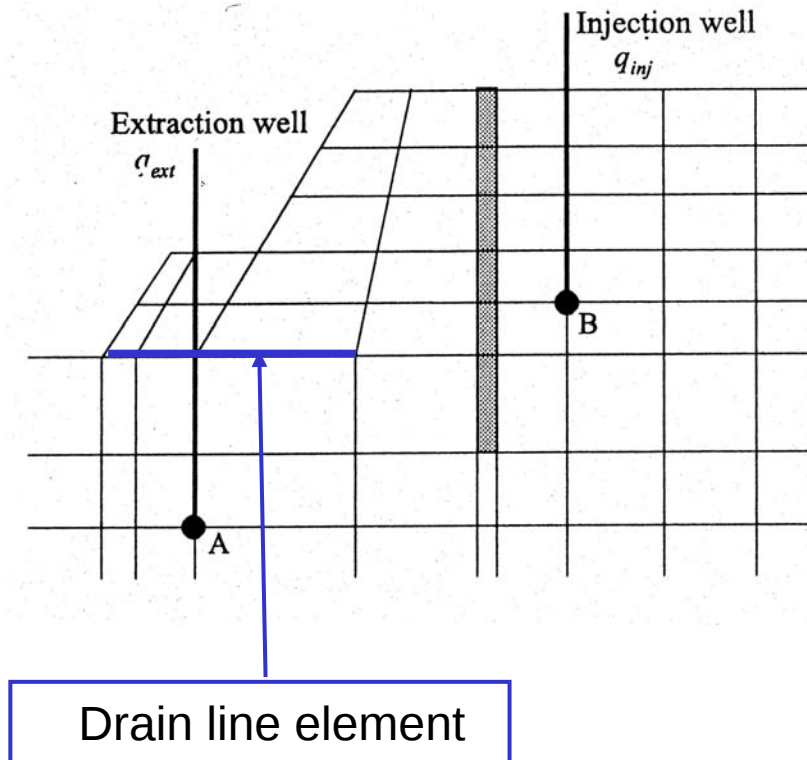
(a)



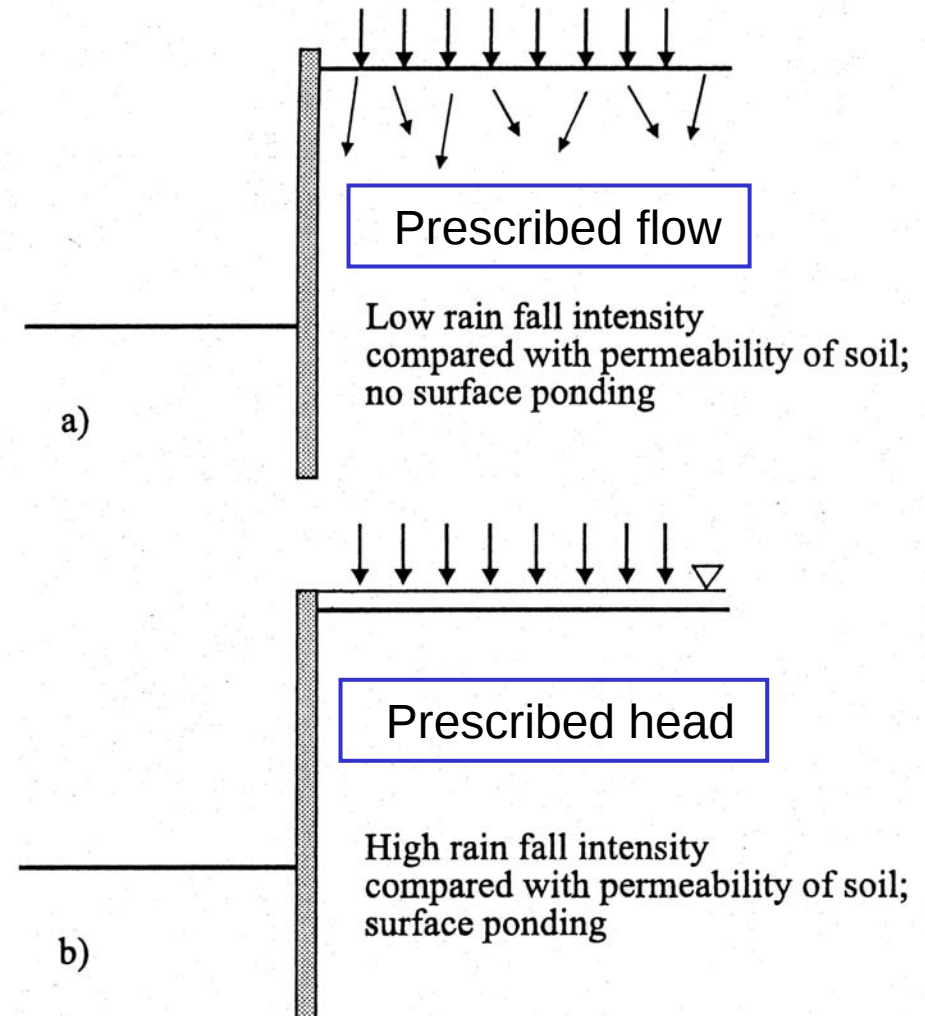
(b)

BOUNDARY CONDITIONS

- Sources and sinks

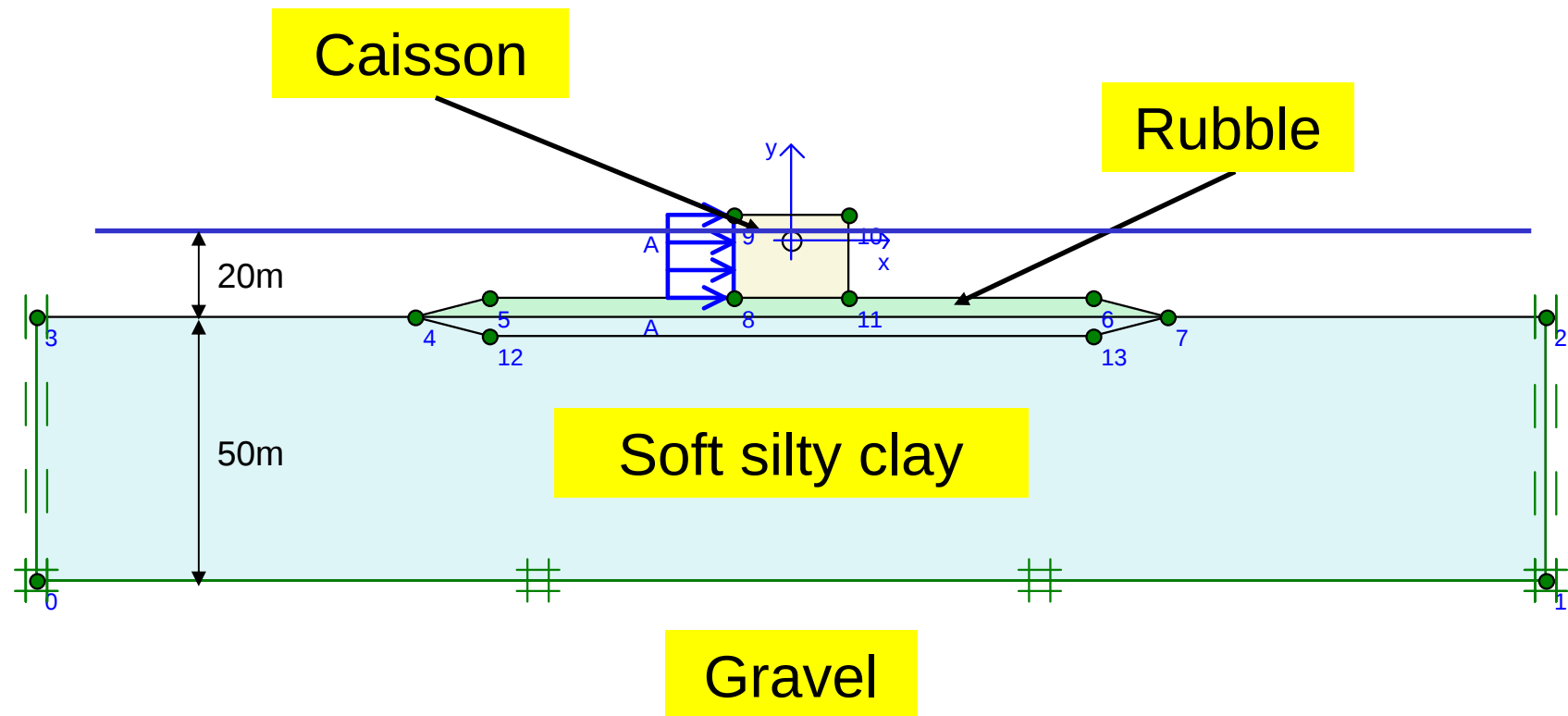


- Infiltration



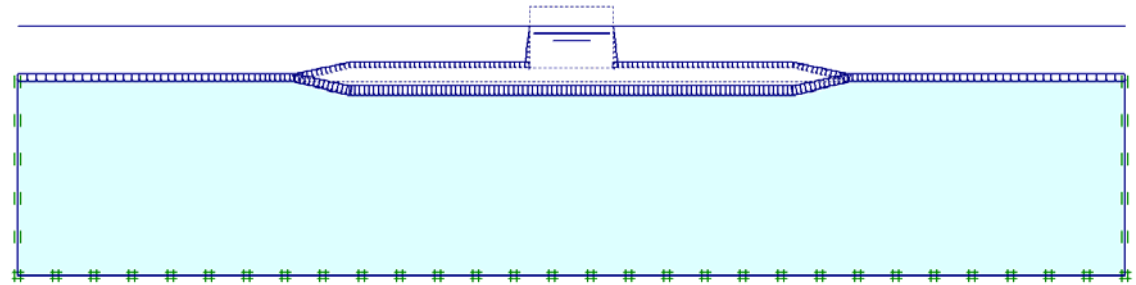
Potts & Zdravkovic (1999)

Barcelona breakwater

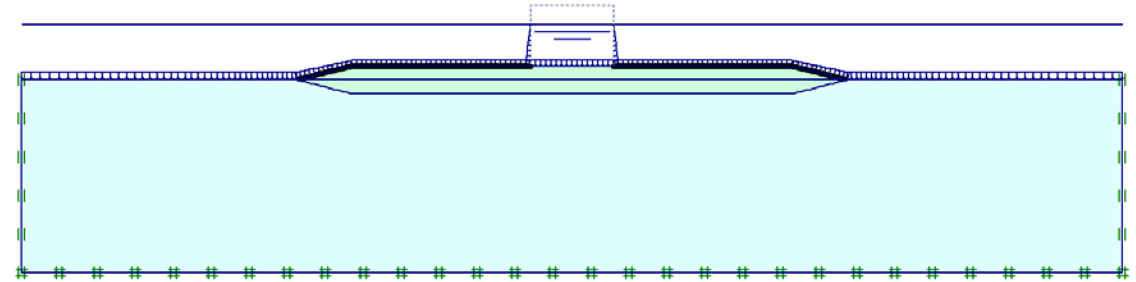


Barcelona breakwater: stages (1)

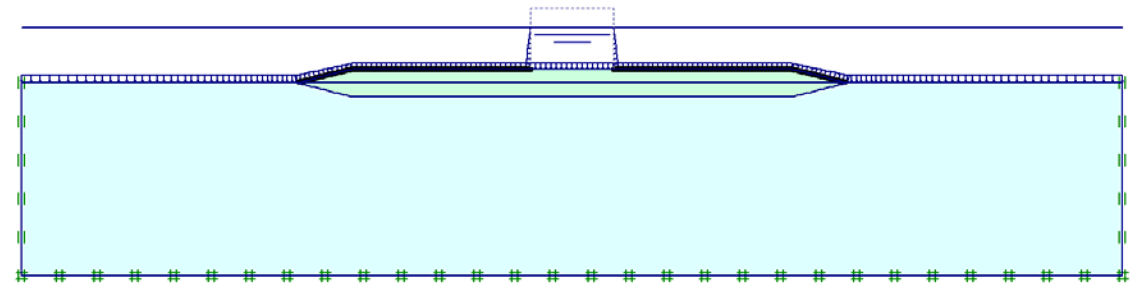
Dredging



Bench
construction

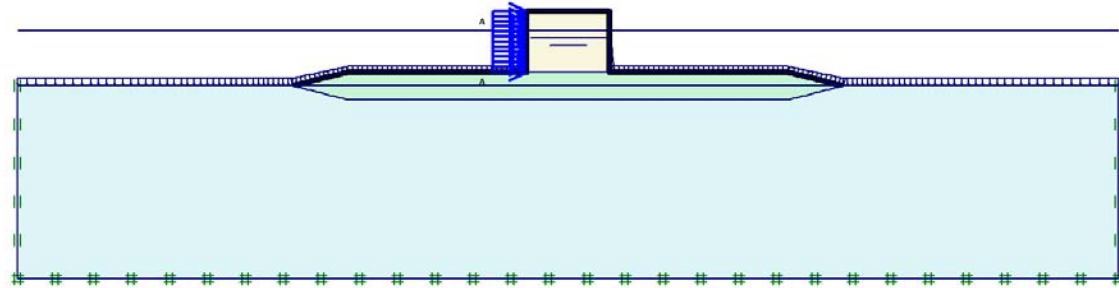


Consolidation

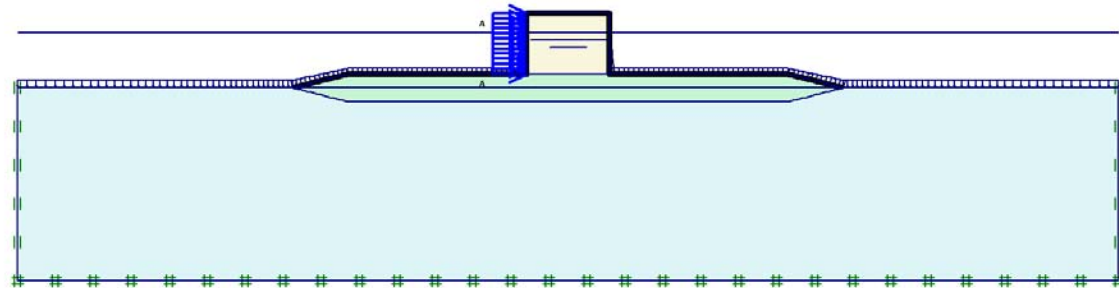


Barcelona breakwater stages (2)

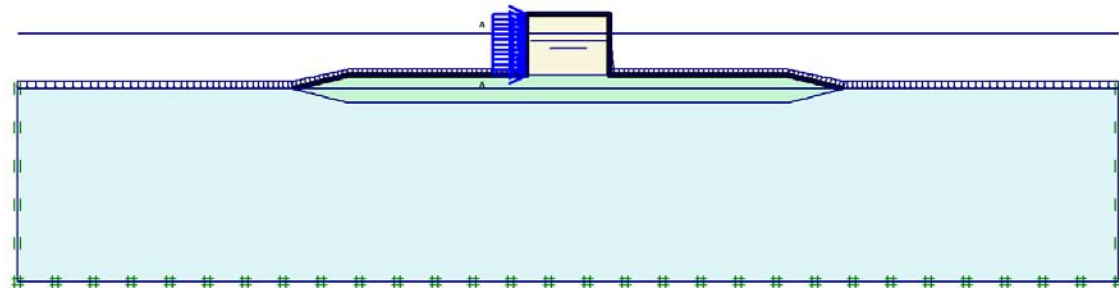
Caisson
construction



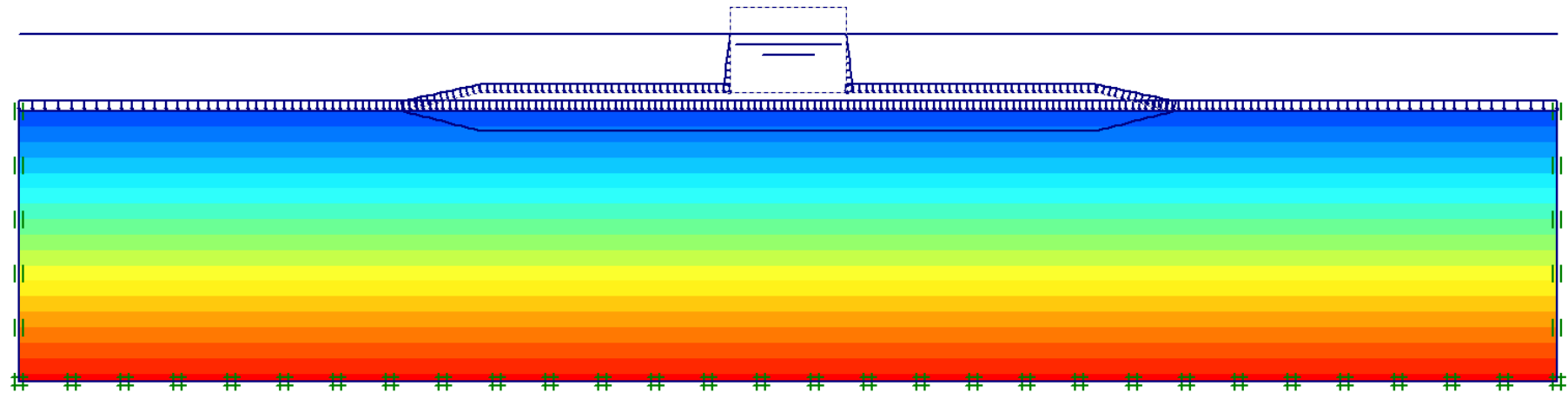
Consolidation



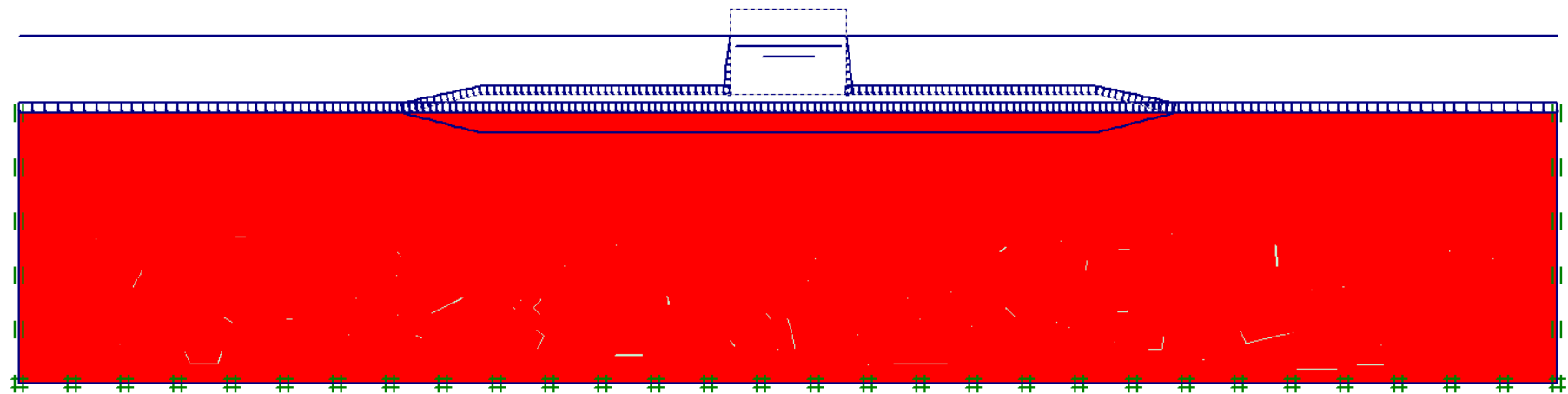
Storm loading



Initial pore pressures

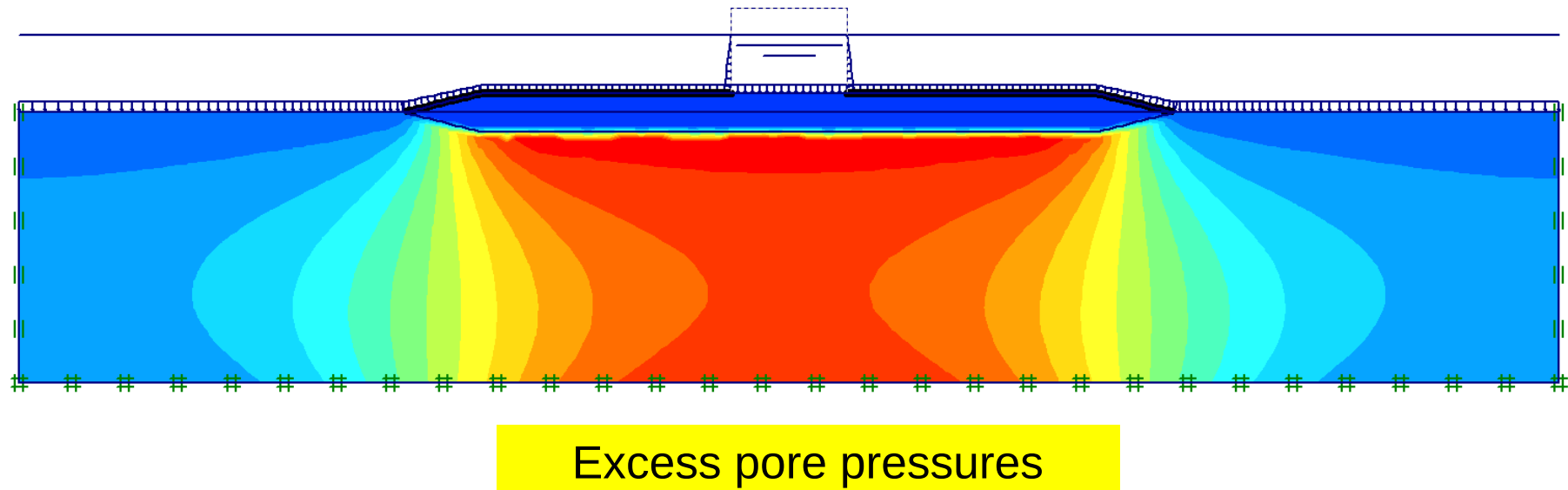
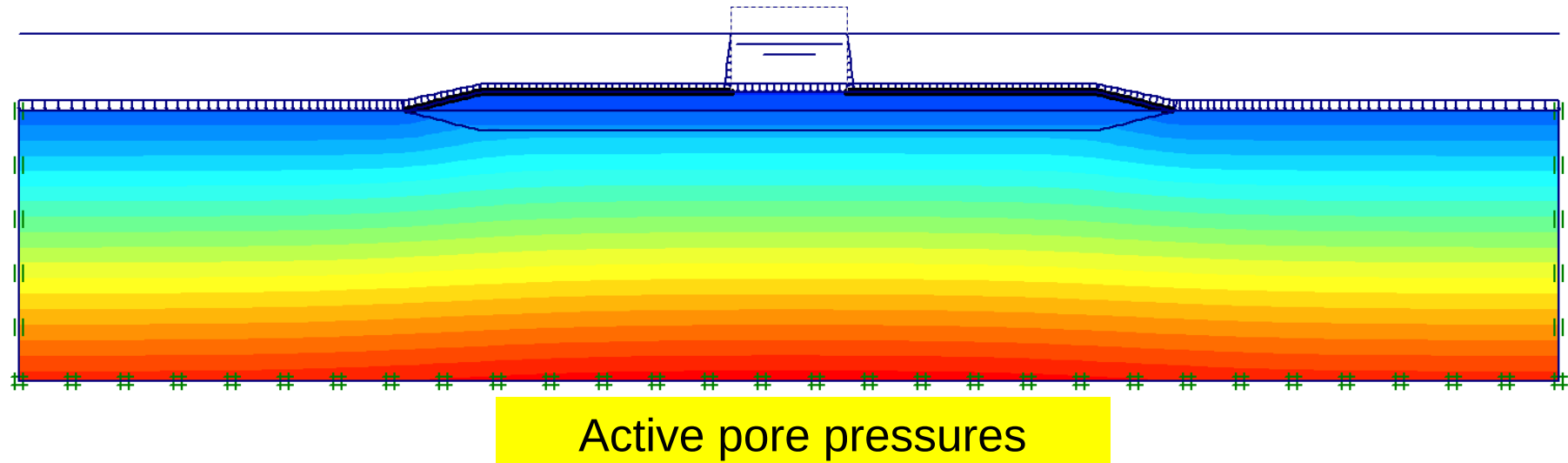


Active pore pressures



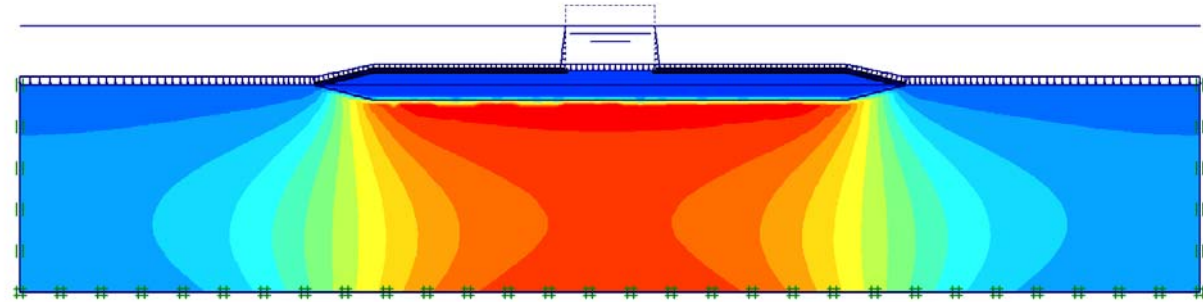
Groundwater head

Pore pressures after placing the bench

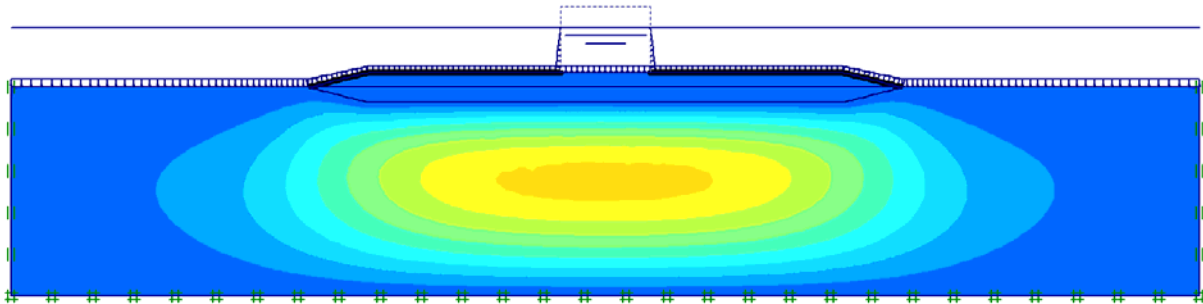


Excess pore pressures during consolidation

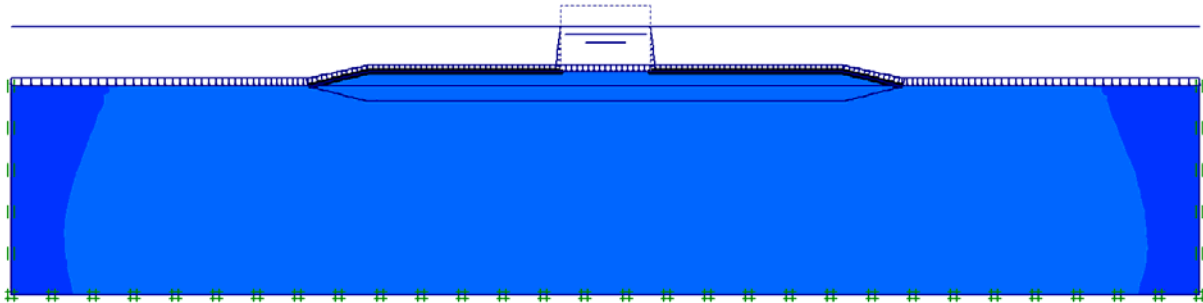
Initial



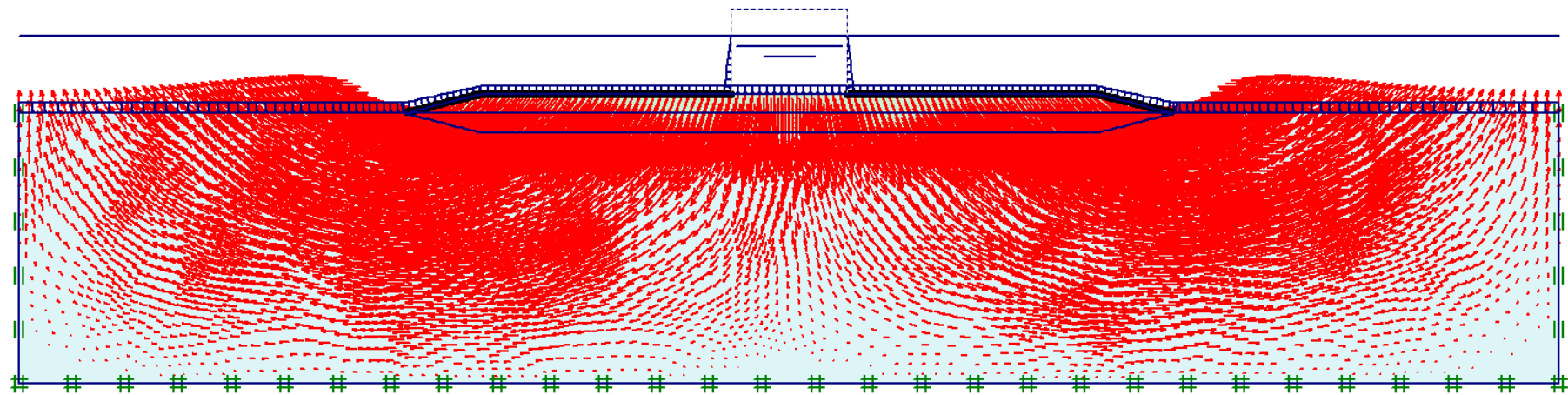
After 30 days



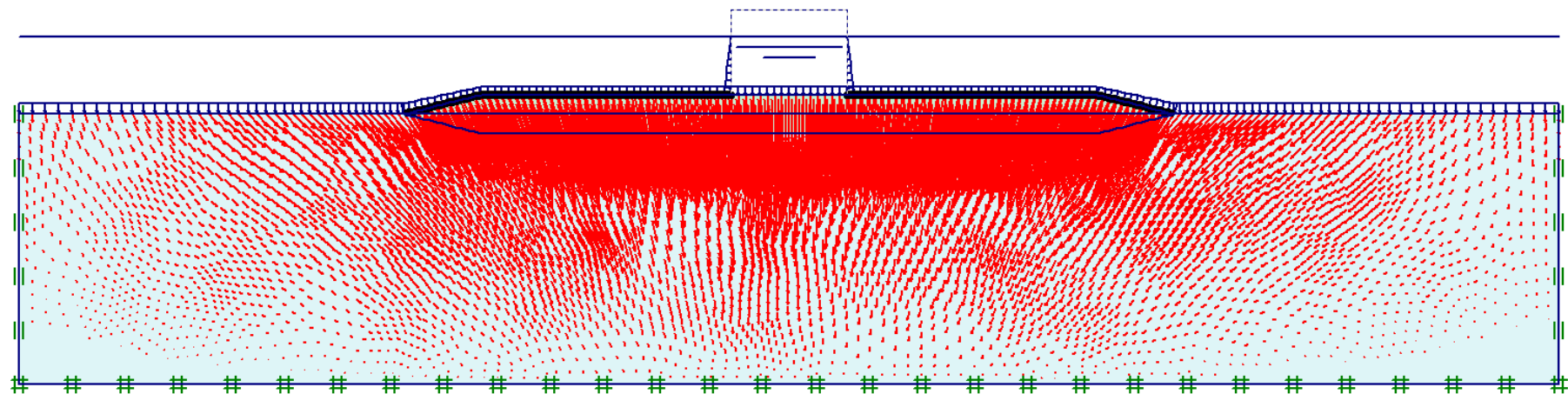
Final



Displacements during construction and consolidation

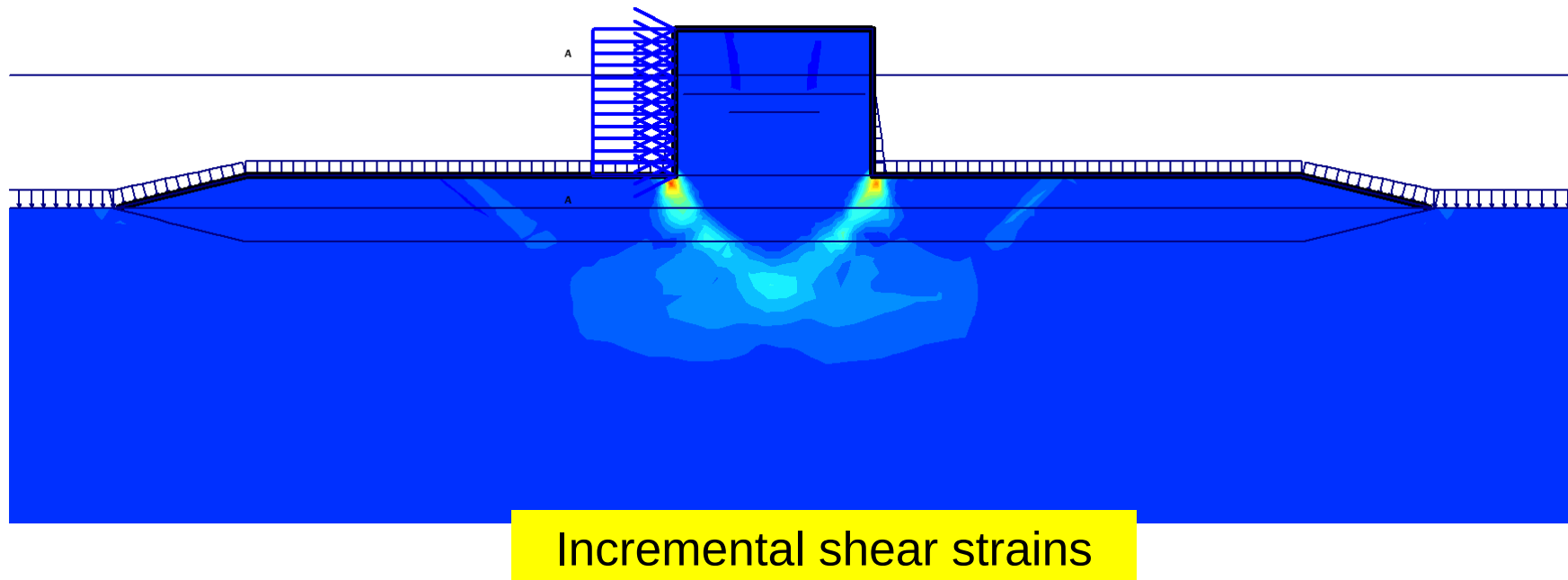
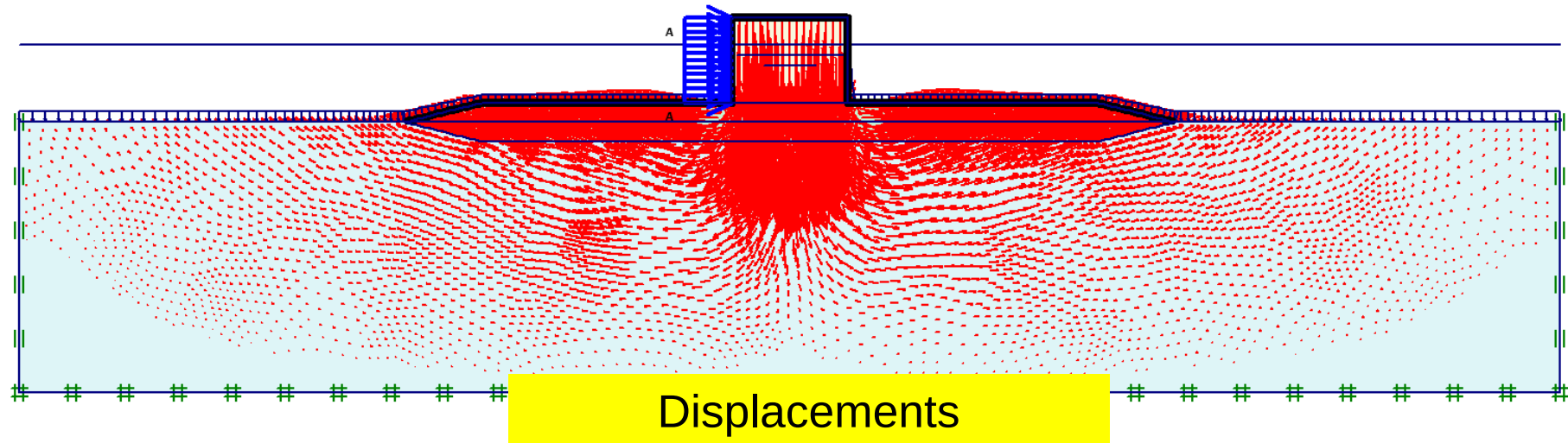


Bench construction



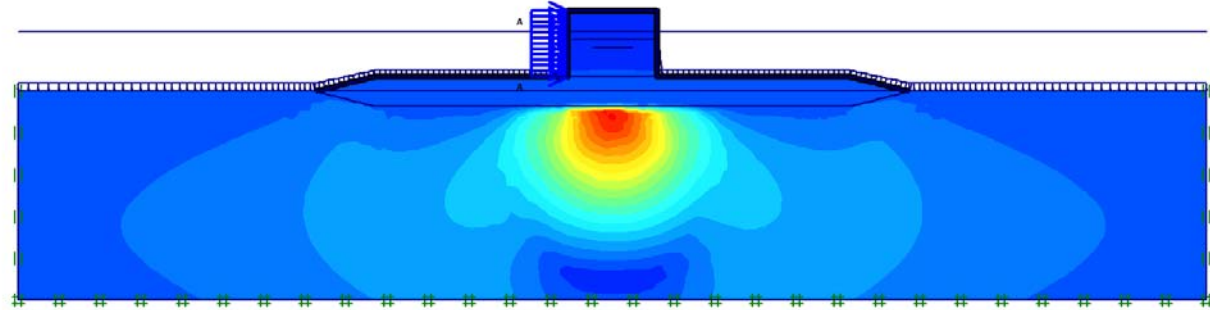
Consolidation

Caisson construction

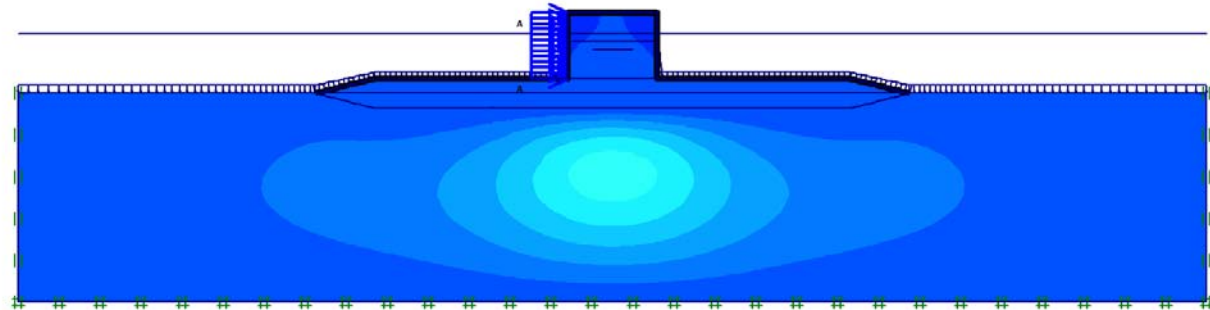


Excess pore pressures during consolidation (caisson)

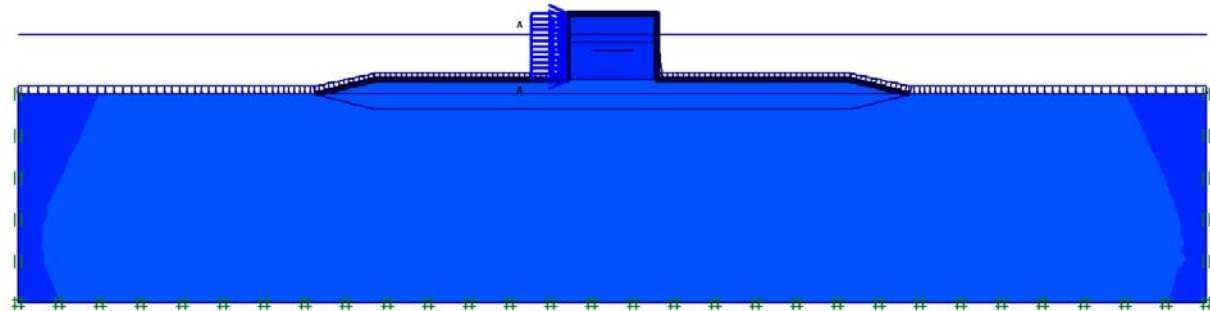
Initial



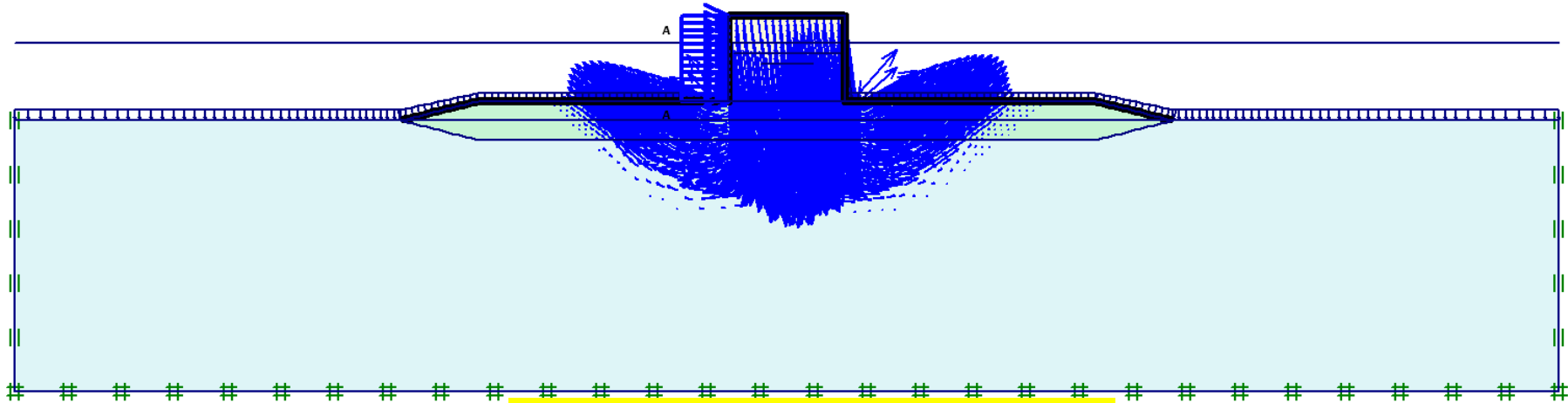
After 30 days



Final



Failure (factor of safety)



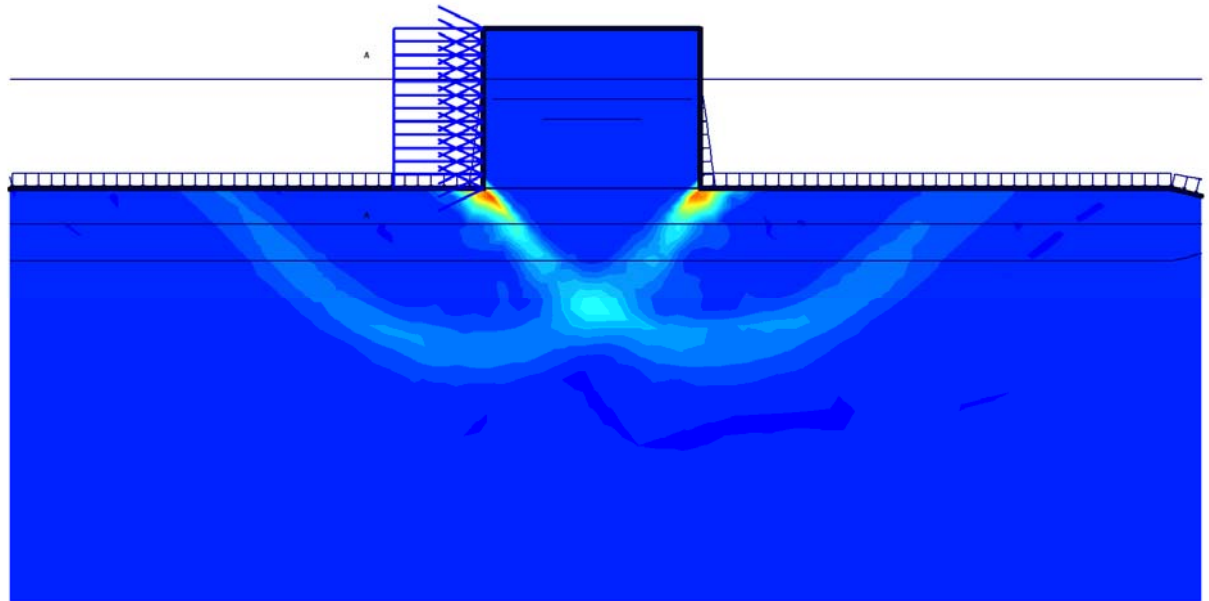
Incremental displacements

Factors of safety

After construction
FS=1.06

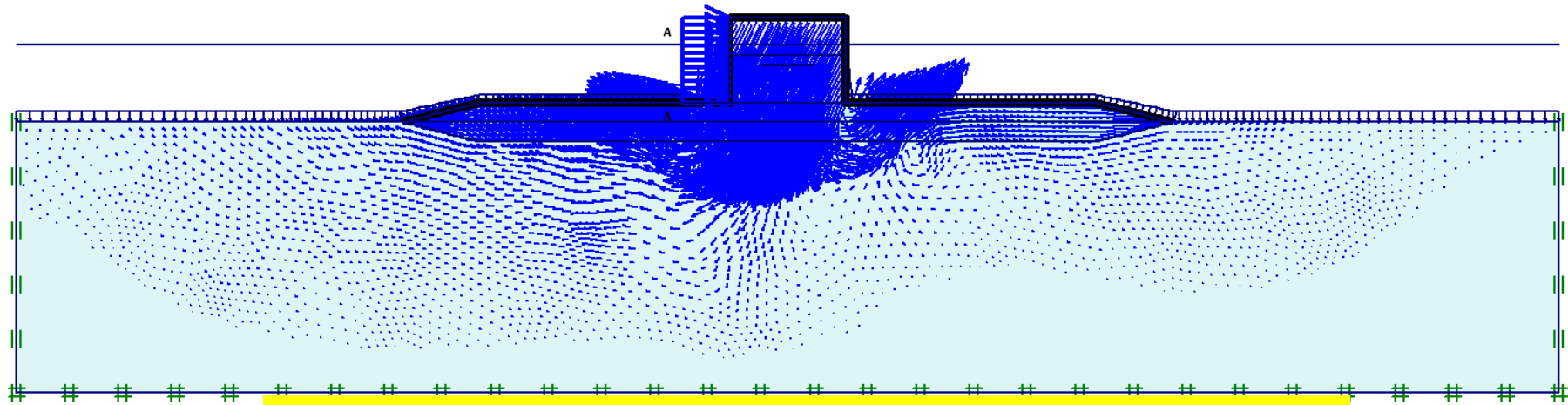
After 30 days
FS=1.60

End of consolidation
F=1.74

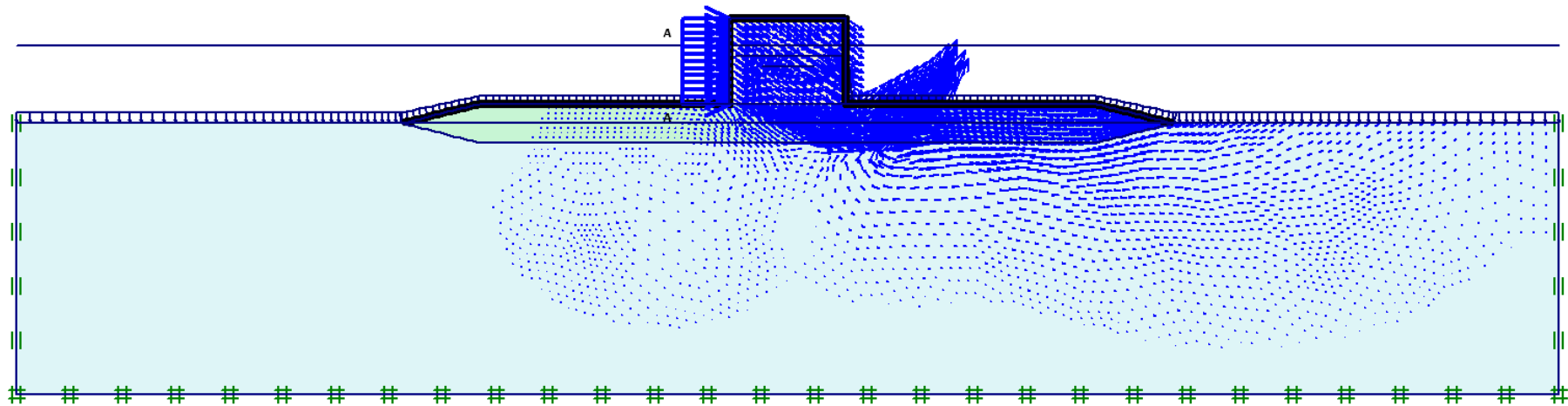


Incremental shear strains

Horizontal storm load



After construction, horizontal pressure = 0.7 kPa



After consolidation, horizontal pressure = 178.2 kPa

Barcelona breakwater



Barcelona breakwater



Barcelona breakwater



Quay failure 1-01-2007