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Simple analytical model for ultimate bearing capacity estimation of historic masonry structure foundation

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ABSTRACT

The Angkor ruins, a World Heritage in Cambodia, contains many masonry structures in danger of collapse due to geotechnical problems, such as uneven settlement of the foundation. In order to select suitable restoration methods, stability evaluation considering the interaction between the masonry stones and the foundation ground is required. In particular, the bearing capacity characteristics of the foundation which consists of man-made soil mound and masonry stones, so-called ‘platform’, should be reflected in the restoration design. In the past studies, the failure mechanisms of the platform structure have been investigated with the numerical analysis. However, a practical design method applicable in the actual restoration projects has not been established yet. In the this paper, a simplified model to estimate the ultimate bearing capacity of the platform structure is proposed based on the limit equilibrium method and the failure mechanism revealed in the past study. Though the implementation of the proposed method is quite simple, the estimated bearing capacity agrees quantitatively with the results of the detailed numerical analysis.

Keywords: foundation; bearing capacity; masonry structure

1 INTRODUCTION

The Angkor ruins, a World Heritage in Cambodia, includes many masonry structures in danger of collapse due to geotechnical problems, such as uneven settlement of the foundation ground (see Photo 1). In order to select suitable restoration methods, stability evaluation considering the mechanical interaction between the masonry stones (discontinua) and the foundation ground (continuum) is required (JSA 2005). Particularly, the bearing capacity characteristics of the foundation which consists of man-made soil mound and masonry stones (Fig. 1), so-called ‘platform’, should be reflected in the design. For this purpose, Hashimoto et al. (2017) performed a numerical simulation of the bearing capacity problem of the platform structure with a discontinuum based method and revealed its failure mechanisms. However, any practical design method applicable in the actual restoration projects has not been established.

Therefore, in this study, a simple analytical model to estimate the bearing capacity of the platform is newly formulated based on the limit equilibrium method incorporating the failure mechanisms revealed in the past study. The implementation procedure of the proposed method is explained for a sample problem, and its validity is examined by comparing with the results of the detailed numerical analysis.

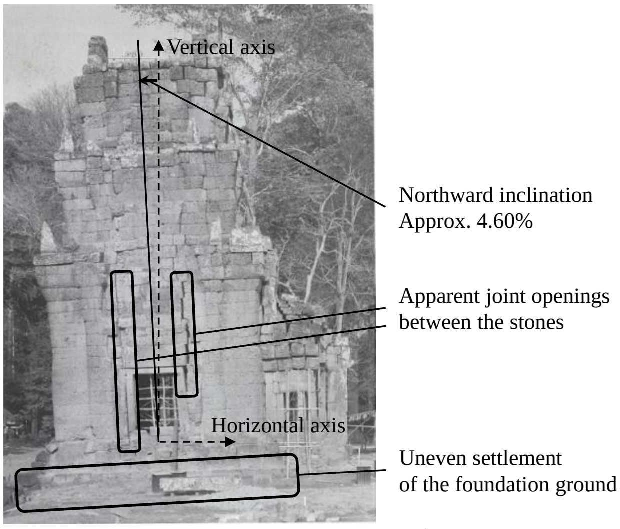

Photo 1. A damaged structure in Angkor ruins (after JSA 2005).

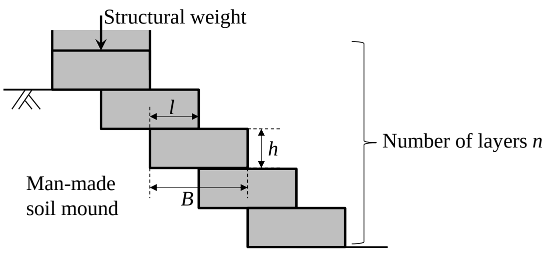

Fig. 1. Definition of platform structure (weight of upper structure is sustained by stones and man-made mound).

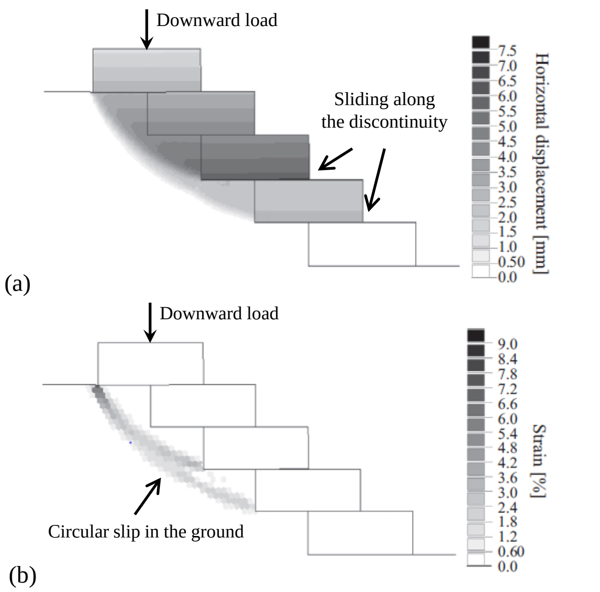

Fig. 2. Results of the loading analysis of the platform structure: (a) distribution of the horizontal displacement and (b) distribution of the deviator strain (after Hashimoto et al. 2017).

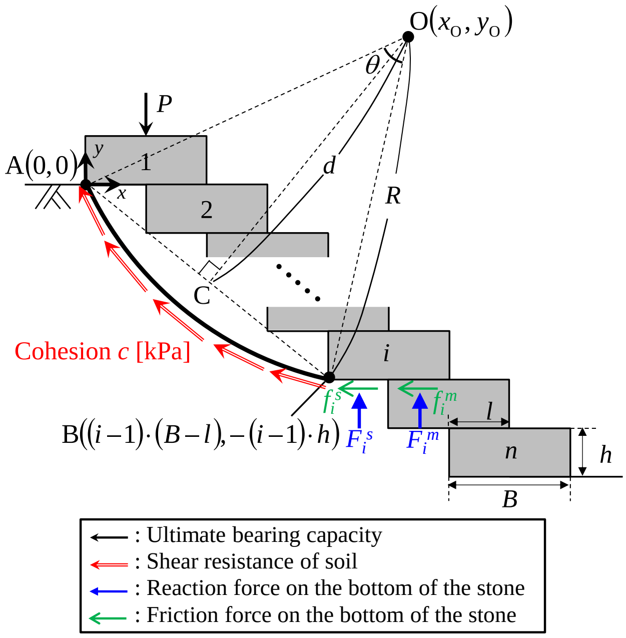

Fig. 3. Failure mechanism assumed in the simplified model for the bearing capacity estimation of the platform.

2 DEFINITION OF MASONRY PLATFORM AND ITS STRUCTURAL PARAMETERS

As mentioned above, in this study, the masonry platforms constructed by stacking the stones of equal size with a constant overlapping width, as shown in Fig. 1 are focused on. In the figure, B is the stone width, h is the stone thickness, l is the overlapping width between the stacked stones, and n is the number of layers. When B is fixed, every structural conditions can be expressed using the following three parameters: the aspect ratio of the stones h/B, the ratio of the overlapping width to the stone width l/B, and the number of steps n. Hereafter, l/B is called 'overlapping rate'.

3 FAILURE MECHANICSMS OF THE MASONRY PLATFORM

In the previous study, regarding the deformation of the platform as the mechanical interaction problem of soil (continuum) and masonry stones (discontinua), the authors (Hashimoto et al. 2017) performed the loading analysis of the masonry platform structure using a discontinuum-based numerical method, NMM-DDA (Miki et al. 2010). In the analysis, the stones were modeled as elastic body. The soil mound was assumed to be a von Mises elasto-perfectly plastic material, considering the fact that the uniaxial compressive strength estimated from the cone penetration test is the only available strength property in usual restoration projects of the Angkor. Coulomb's friction law was assumed on the material interfaces.

Fig. 2 shows the distributions of the horizontal displacement and the deviator strain after the downward loading on the top stone of the platform. From the horizontal displacement distribution, the discontinuities of the displacement, which means sliding, between the stones were observed. On the other hand, the deviator strain distribution showed that the circular slip line was induced in the ground. Considering these results, it was concluded that the masonry platform shows composite failure mechanisms of the sliding between the stones and the shear failure in the ground under vertical loading condition. This means that the bearing capacity of the platform was mobilized by both of the shear resistance in the ground and the friction strength along the stone interface.

4 DERIVATION OF LIMIT EQUILIBRIUM EQUATION FOR THE MASONRY PLATFORMS

Based on the complex failure mechanisms described in the last chapter, the limit equilibrium equation for the bearing capacity estimation is derived. Now, for a platform with n layers of the stones, a failure mechanism consisting of a sliding beneath the i-th stone from the top and a connected circular slip in the ground is assumed as shown in Fig. 3. The stones and the ground are modeled as a rigid body and a rigid-plastic material with von Mises failure criterion, respectively. The forces acting on the sliding domain, including the stones, are defined as the ultimate load P, the shear resistance force of the soil along the slip circle, the normal reaction force F_i^s and the friction force f_i^s from the ground beneath the i-th stone, and the normal reaction force F_i^m and the friction force f_i^m from the masonry stone beneath the i-th stone. Considering the equilibrium of the rotational moment by these forces around the center of the slip circle $O(x_O, y_O)$, the following equation can be derived:

$$
\begin{aligned}
&P\cdot\left(x_{\mathrm{O}}-\frac{B}{2}\right)\\
&=cR^2\theta+F_i^s\cdot\left[x_{\mathrm{O}}-\left\{(i-1)(B-l)+\frac{B-l}{2}\right\}\right]\\
&\quad+F_i^m\cdot\left[x_{\mathrm{O}}-\left\{i(B-l)+\frac{l}{2}\right\}\right]\\
&\quad+f_i^s\cdot\left\{y_{\mathrm{O}}+(i-1)h\right\}+f_i^m\cdot\left\{y_{\mathrm{O}}+(i-1)h\right\}
\end{aligned}
\tag{1}
$$

where R is the radius of the slip circle, and θ is the central angle of the arc. Since the vertical load acting on each stone is transmitted from upper layers partially distributing to the ground, F_i^s and F_i^m are assumed as

$$
F_i^s=P\cdot(1-r)\cdot\prod_{j=1}^{i-1}r_j,\ \text{and}\ \ F_i^m=P\cdot\prod_{j=1}^{i}r_j,
\tag{2}
$$

respectively. Here, r_i is the load distribution rate of the load acting on the i-th stone onto the (i+1)-th stone (see, Fig. 4a). Although r_i would depend on the difference of the stiffness between the stones and the ground, in this study, r_i is assumed equal to the overlapping rate l/B (Fig. 4b). Thus, Eq. (2) is rewritten as follows:

$$
F_i^s=P\cdot\left(1-\frac{l}{B}\right)\left(\frac{l}{B}\right)^{i-1},\ \text{and}\ \ F_i^m=P\cdot r^i=P\cdot\left(\frac{l}{B}\right)^{i}.
\tag{3}
$$

Additionally, the friction force along the bottom of the sliding stone should have reached the friction strength, f_i^s and f_i^m are represented as following equations assuming Coulomb's friction law.

$$
\begin{aligned}
&f_i^s=F_i^s\cdot\tan\phi_{ms}=P\cdot\left(1-\frac{l}{B}\right)\left(\frac{l}{B}\right)^{i-1}\cdot\tan\phi_{ms},\\
&\text{and}\ \ f_i^m=F_i^m\cdot\tan\phi_{mm}=P\cdot\left(\frac{l}{B}\right)^{i}\cdot\tan\phi_{mm}
\end{aligned}
\tag{4}
$$

where f_{ms} and f_{mm} are the friction angle of the stone-soil and the stone-stone interfaces, respectively. After substituting Eqs. (3) and (4) into Eq. (1), the ultimate load will be derived as follows by solving the limit equilibrium equation for P.

$$
P=\frac{cR\theta^2}{g}
\tag{5}
$$

$$
\begin{aligned}
g=&\left\{1-\left(\frac{l}{B}\right)^{i-1}\right\}\left(x_{\mathrm{O}}-\frac{B}{2}\right)+\left(\frac{l}{B}\right)^{i-1}(i-1)(B-l)\\
&-\left(\frac{l}{B}\right)^{i-1}\left\{\left(1-\frac{l}{B}\right)\tan\phi_{ms}+\frac{l}{B}\tan\phi_{mm}\right\}\left\{y_{\mathrm{O}}+(i-1)h\right\}
\end{aligned}
\tag{6}
$$

As shown above, the bearing capacity formula for the masonry platform structures has been proposed based on the limit equilibrium method. However, to estimate the ultimate bearing capacity using Eqs. (5) and (6), the parameters i, x_{O}, y_{O}, R and θ, which locates the slip circle, must be determined. The determination method of these parameters are described in the next chapter.

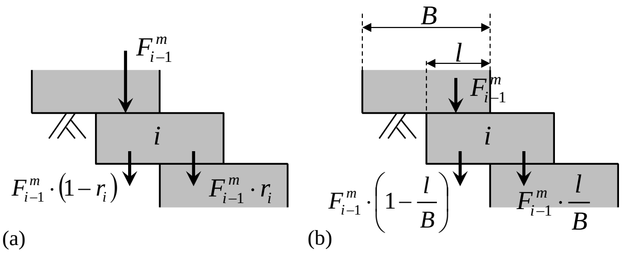

Fig. 4. Load distribution rate: (a) the definition of r_i and (b) the expression of r_i with the overlapping rate l/B.

5 DETERMINATION SCHEME OF FAILURE POSITION AND ULTIMATE LOAD

In general, when a structure reaches failure state, the failure mechanisms that show the minimum ultimate load will appear. Hence, to estimate the bearing capacity of the masonry platforms with Eqs. (5) and (6), the parameter set that minimizes the left hand side of Eq. (5) should be found. As described above, there are five parameters i, x_{O}, y_{O}, R and θ that locate the failure line of the platform. However, in fact, R and θ are the dependent variables that will be automatically determined if i, x_{O}, and y_{O} are fixed. In addition, considering the fact that the center of the slip circle must be located on the perpendicular bisector of the line segment connecting the endpoints of the arc (line AB in Fig. 3), x_{O} and y_{O} can be parameterized with the distance d between the point C (midpoint of AB) and the point O (also see Fig. 3) as follows.

$$
x_{\mathrm{O}}(i,d)=\frac{(i-1)\cdot(B-l)}{2}+\frac{h}{\sqrt{(B-l)^2+h^2}}d
\tag{7}
$$

$$
y_{\mathrm{O}}(i,d)=-\frac{(i-1)\cdot h}{2}+\frac{B-l}{\sqrt{(B-l)^2+h^2}}d
\tag{8}
$$

Consequently, R and θ are parameterized with i and d as following equations.

$$
R=R(i,d)=\sqrt{\frac{(i-1)^2}{4}\left\{(B-l)^2+h^2\right\}+d^2}
\tag{9}
$$

$$
\theta=\theta(i,d)=2\arctan\left(\frac{(i-1)\sqrt{(B-l)^2+h^2}}{2d}\right)
\tag{10}
$$

Since Eq. (6) is the function of i and d considering Eqs. (7) and (8), finally, Eq. (5) can also be parameterized with i and d. Therefore, by seeking the parameter set (i, d) that minimizes Eq. (5), the ultimate bearing capacity of the masonry platforms can be obtained. The important feature of this scheme is that the possible values of i are limited to the integers from 2 to n. Thus, after determining d that minimizes Eq. (5) for each i, the true ultimate load can be easily estimated by finding i value showing the minimum value of P.

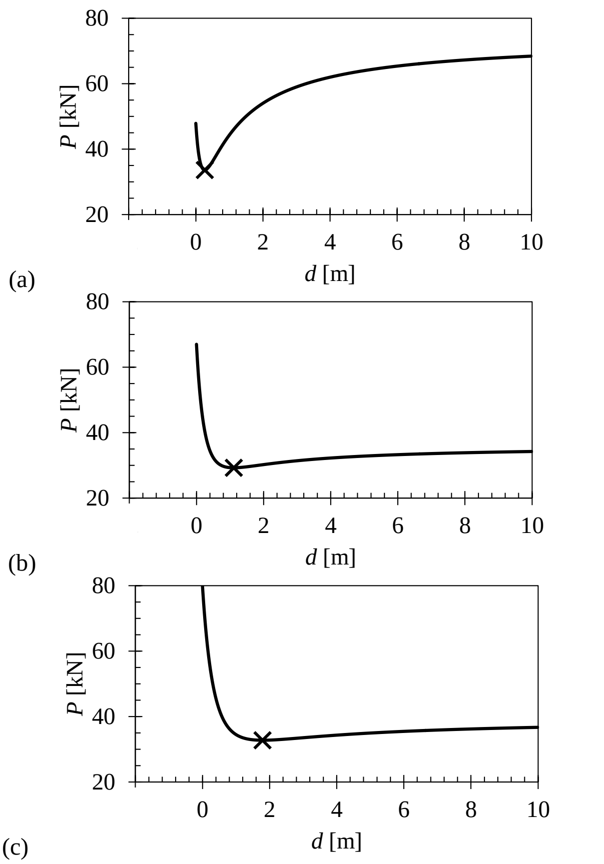

Fig. 5. *d*-*P* relationship and the minimum *P* (cross mark) estimated with Eq. (5): (a) $i = 2$, (b) $i = 3$ and (c) $i = 4$.

Table 1. Minimum value of *P* and corresponding *d* for each layers.

i	*d* [m]	Minimum value of *P* [kN]
2	0.265	33.64
3	1.106	29.26
4	1.794	32.77

6 VALIDATION OF THE PROPOSED METHOD

In this chapter, an implementation example of the proposed method is introduced below. Hereafter, a platform with the following conditions is assumed: B = 1.0 m, h/B = 0.40, l/B = 0.50, n = 4, the cohesion c = 10 kPa, and the surface friction angle $\phi_{ms} = \phi_{mm} = 30°$.

Firstly, the relationship between *d* and *P* represented by Eq. (5)-(10) is calculated for i = 2, 3, 4 (without the top stone), as shown in Fig. 5. The cross marks in the figures show the minimum values of *P* for each *i* listed in Table 1. The table shows that the ultimate load estimated by Eq. (5) is minimized when i = 3 and d = 1.106 m. Therefore, the platform assumed here is predicted to fail at the third layer from the top, and the ultimate bearing capacity is estimated to be 29.26 kN. As shown above, the proposed method enables simple and definite predictions of the bearing capacity and the failure mechanisms of the platform structures.

To check the validation of the proposed method, the estimated ultimate bearing capacity is compared with the computed results with the numerical method by

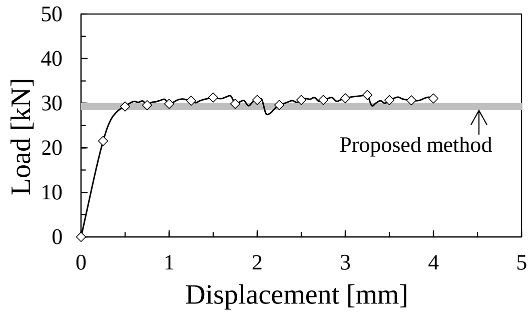

Fig. 6. Comparison of the ultimate bearing capacity estimated with the numerical simulation and the proposed method.

Hashimoto et al. (2017). Fig. 6 shows the computed load-displacement curve of the numerical analysis, and the ultimate load estimated with the simplified method is also indicated. As shown in the figure, the results by the simplified method quantitatively agrees with the bearing capacity by the numerical method, and the proposed method is successfully validated.

7 CONCLUSION

In this study, for the rational conservation of the Angkor monuments, a simple analytical model to estimate the ultimate bearing capacity of the foundation so-called ‘platform’ was proposed. The proposed method is constructed in a completely novel approach that incorporates the complex mechanisms of the ground failure and the sliding between stones.

The validity of the proposed method was examined comparing with the detailed numerical simulation. The predicted ultimate bearing capacity with the proposed method agreed quantitatively with that computed with the detailed numerical method, regardless of the simple implementation procedures. Although the further validation study for other structural and ground conditions is required, the proposed method would be quite useful in the practical restoration design process.

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