Workshop at Griffith University
Fundamental Soil Mechanics Principles (Part 1)

Basic Theoretical Concept (with emphasis on slope engineering)

What are the factors controlling the stability of a slope?

Geology (material/mass fabric) - mode of failure

Materials (soil matrix/mass) - shear strength, permeability, response to infiltration

Environmental factors (**groundwater**) - main/perched groundwater table, infiltration

Geometry – loading, stress, etc.

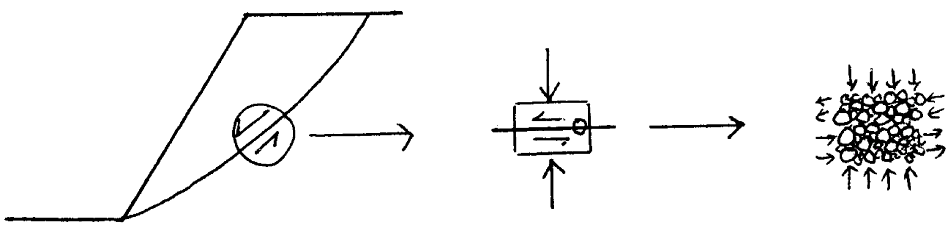
External loadings – as above

THE MATERIALS

Soil / Weathered Rock

An assembly of Particles

In macroscopic scale, soil can be looked as a continuum. In microscopic scale, all soils are assembles of particles of different sizes and shapes. The interaction of the soil grains affects the soil mass behaviour.

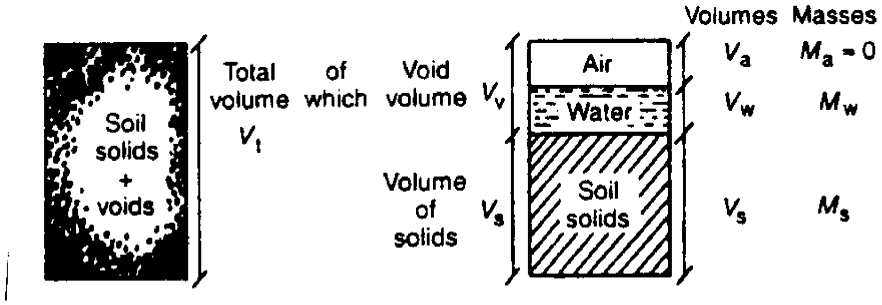


Phases

The voids between soil grains are filled with air and water, the conditions of the infill affects the behaviour of soils.

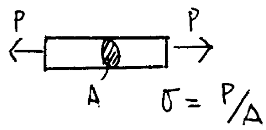
Solid, air, pore fluid - the phase diagram

Calculations of void ratio, moisture content, degree of saturation, dry density, etc.



Stress for a particulate system

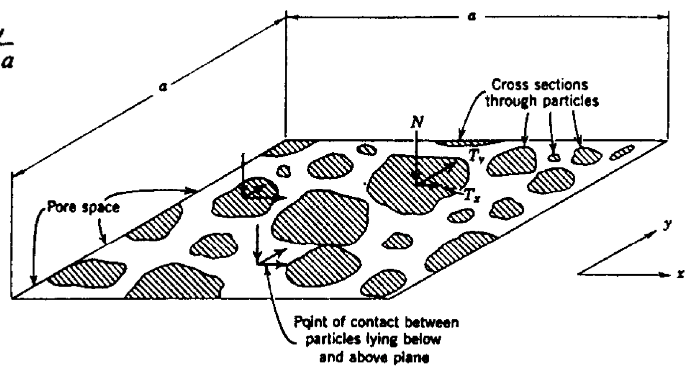
stress : macroscopic stress - force / total area - continuum



contact stress

stress in dry soil - force in the soil skeleton per unit area of soil (which includes voids)

$$\sigma = \frac{\Sigma N}{a \times a} \qquad \tau_x = \frac{\Sigma T_x}{a \times a} \qquad \tau_y = \frac{\Sigma T_y}{a \times a}$$



Principle of effective stress

For saturated soil $\sigma' = \sigma - u$

mean effective stress = mean (total) stress - u

The effective stress principle says nothing about the way the stresses are transmitted through the 'solid phase' - effective stress is not the inter-granular stress.

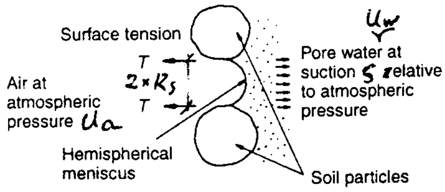
Pore water pressure (u) is not the pressure within the pore fluid adjacent to a clay particle i.e. within the diffuse double layer. u is simply the pressure measured through a porous tip which is much larger than the soil grains.

As pore water does not have shear strength/stiffness, effective stress can be taken as the normal stress applied taken by the soil skeleton. It is this effective stress that makes a major influence on soil strength and deformation characteristics of most saturated soils.

Unsaturated Soils

Surface tension - intermolecular forces acting on the molecules in the liquid surface capillarity

$(u_a - u_w) = 2\ T_s / R_s$
$(u_a - u_w)$ = matrix suction
T_s = surface tension
R_s = radius of curvature of the meniscus



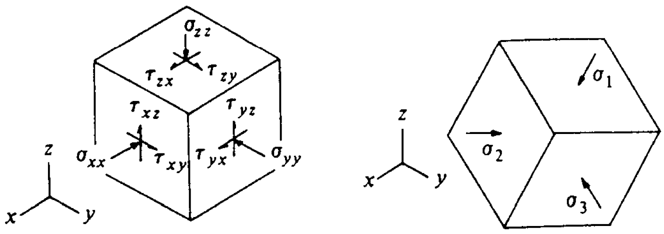
Inter-particle forces due to surface tension - water is drawn into particle contacts, just as it is up capillarity tubes and it generates compressive forces between the particles, i.e. matrix suction for unsaturated soils $s = (u_a - u_w)$, in natural soils, u_a is atmospheric pressure and u_w is the negative pore water pressure. In tests for unsaturated soils, we often use elevated u_a with non-negative u_w to induce the same amount of matric suction in the soil. In the field, u_a can be considered as zero when u_w is measured as gauge pressure (-ve) against the atmospheric pressure.

The particle contact forces set up by matrix suction are essentially normal. They tend to stabilise the structure.

Stress for a continuum

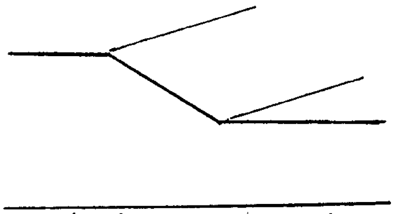
The loads and forces applied to a solid body (such as a soil mass) are distributed within the body as stresses. Provided that there are no planes of weakness, which interrupt the transfer of stress, it is usually assumed that the stresses vary smoothly and continuously throughout the body - which is described as continuum.

For a cubical element within a 3-D body, there are three independent stresses acting on each pair of opposite faces. These are shear stresses in two directions and normal stress acting perpendicular to the face of the cube.

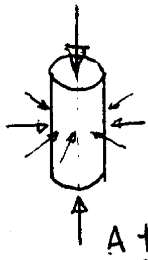


By rotating the cube, we should be able to find one particular orientation which all shear stresses acting across the faces of the cube are zero. These planes are principal planes, and the Normal stresses acting on these planes are principal stresses. The largest principal stress is the major principal stress, the smallest principal stress is the minor principal stress and the remaining principal stress is the intermediate principal stress.

Plane strain - simplification of three-dimensional conditions into two-dimensional ones. e.g. embankment, retaining walls and slopes. All deformation takes place within the cross-section, and there is no strain in the longitudinal direction. The longitudinal principal stress will take up whatever value (normally as intermediate principal stress)



A slope in plane-strain condition

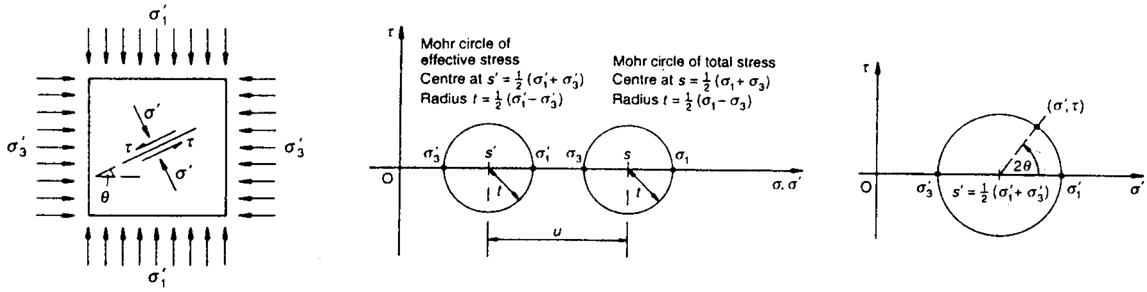


A triaxial specimen in axisymmetry condition

Axisymmetry - another simplification of three-dimensional conditions. e.g. a pile, a well and a triaxial specimen. The condition on any diametral plane are the same. The stress and strain conditions have rotational symmetry about the vertical axis. Stress analysis is focused on a typical diametral plane, in the same way that a typical cross-section is used for plane strain problem.

Mohr circles - maximum shear

The normal and shear stresses acting on an imaginary cut within a typical cross-sectional or diametral plane will depend on the orientation of the cut with respect to the major and minor principal stress directions. if the cut is perpendicular to either the major or the minor principal stress, the shear stress acting in the direction of the cut will be zero. The magnitude of the shear stress increases as the cut is rotated away from the direction of the planes of principal stress.



The stress state within a plane containing the major and minor principal stresses is most conveniently represented by means of Mohr circle construction. The circle may be plotted for either total or effective stress.

The Mohr circle passes through the points representing the major and the minor principal stresses at (σ_1, 0) and (σ_3, 0). The centre of the total stress circle is at $(\sigma_1+\sigma_3)/2,0$. The average of major and minor principal total stresses $(\sigma_1+\sigma_3)/2$ is given the symbol s. The centre of the effective stress circle is at $(\sigma'_1+\sigma'_3)/2,0$. The average major and minor principal effective stresses $(\sigma'_1+\sigma'_3)/2$ is given the symbol s'. Recalling that $\sigma' = \sigma - u$, the centres of the circles for effective and total stress are separated by a distance u along the normal stress axis.

The radius of the circle is $(\sigma_1-\sigma_3)/2$ for total stress and is $(\sigma'_1-\sigma'_3)/2$ for effective stress. These are identical, because pore water pressure u cancels out in the equations. $(\sigma_1-\sigma_3)/2$ is equal to the maximum shear stress, and is given the symbol t.

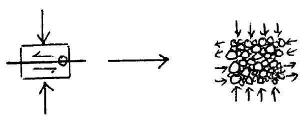
The stresses acting on an imaginary cut at an angle θ anticlockwise from the plane on which the major principal stress acts are found by drawing a line through the centre of the Mohr circle to the circumference, which makes an angle 2θ with the normal stress axis. The stress state on the cut is given by the point where this diameter meets the circumference of the circle.

The Mohr circle of stress shows that, unless the major and minor principal stresses are equal, there must be some shear stress acting within the plane under consideration. The maximum shear stress within the plane is equal to the radius of the Mohr circle $(\sigma_1-\sigma_3)/2$. It occurs at angles of ±45° to the planes on which the major and minor principal stresses act.

Material Behaviour (Shear Strength)

Stress-strain behaviour (elastic / yield - strain softening / hardening) will be covered separately.

Shear resistance of assembly of soil particles – comprising two components: frictional resistance (critical state friction) between soil particles and interlocking of soil particles (represented by dilation angle, which depends on effective pressure and relative density).

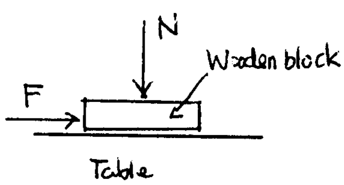


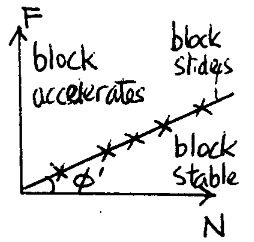
Wooden block analogy - frictional resistance between soil grains

Imagine a wooden block on a wooden table. Normal load N is kept constant and a sideway force F is applied until the block starts to slide. F increases with larger N. If we plot F vs N

$F = N\mu$

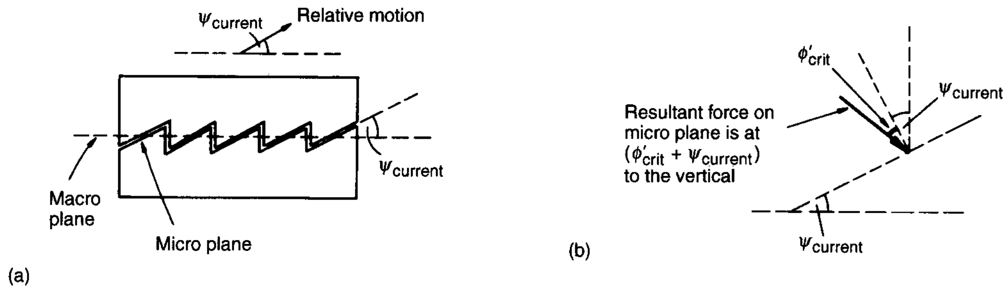
$F = N \tan\phi$ as $\mu = \tan\phi$





Sawtooth analogy - interlocking and dilation between soil grains

The other component in shear resistance is interlocking of soil particles. Imagine we have a macroscopic shear plane through a soil element (e.g. in a shear box), shearing resistance mobilised on the (marco) shear plane can be imagined as sliding along a series of saw teeth.



If the angle of friction along the mico planes of the saw teeth is ϕ' (or actually ϕ'_{crit}), the current angle of shearing resistance of the macro plane is given by

$$\phi'_{current} = \phi'_{crit} + \psi_{current}$$

where $\psi_{current}$ is the angle of the saw teeth as well as the angle of dilation – i.e. the amount of upward movement of the upper half of the shear box.

Results of tests on granular soils indicate that under plane-strain conditions

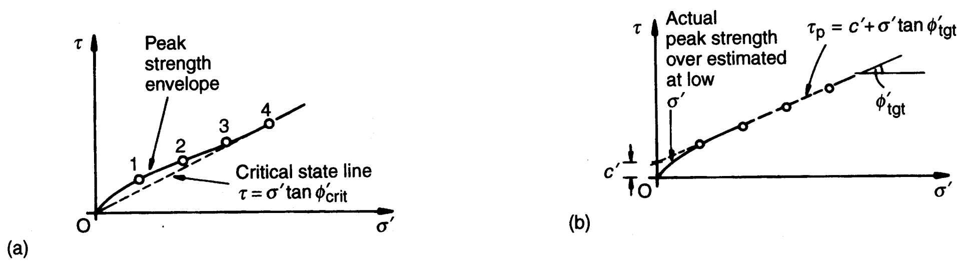
$$\phi'_{peak} = \phi'_{crit} + 0.8\ \psi_{max}$$

The maximum angle of dilation, ψ_{max}, is a function of stress level and density of the material.

We should note that peak strength of a soil is a transient condition, which occurs only in association with dilation along the shearing surface. As shearing continues, arrangement of soil grains continues to re-adjust (causing change in volume or change in effective stress). Eventually, ϕ'_{crit} will be mobilised. It is better not to treat peak strength a material property but a phenomenon.

Peak strength of soils

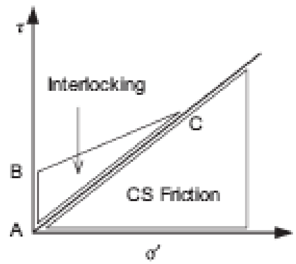
If we plot peak strength data on τ against ϕ', the figure above shows a strength envelop – which is commonly represented by a straight line, representing the peak strength for a range of effective stress. In practice, straight line segments can be used to represent peak strength envelop for a soil under different effective stresses.



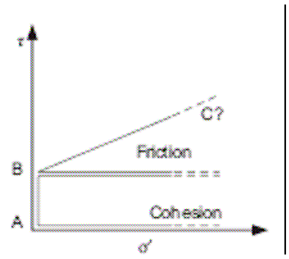
Mohr-Coulomb failure criteria

With the use of effective stress the peak shear strength of saturated soils is represented by a straight line in a τ - σ' space.

$\tau = \sigma' \tan \phi' + c'$ - Mohr-Coulomb failure criterion



We should note that c', ϕ' are failure criterion in a macroscopic view, representing the more intrinsic friction between soil grains (ϕ'_{crit}) and the effort needed to overcome interlocking between soil grains (represented by the maximum angle of dilation ψ_{max}).



Apart from friction and interlocking, cementation and bonds between soil particles, can also provide some contribution to the peak shear strength, especially at low stress levels. To a relatively smaller extent, c' can also be used to represent 'true cohesion'. However, we should avoid referring to the effect of interlocking as 'true cohesion' without realising what is actually referring to.

For soils having an origin of weathered rocks such as those we commonly encounter in Hong Kong, the shear strength, especially at relatively low effective confining stresses, can first be provided by 'true cohesion' between particles and by development of interlocking of particles.

For unsaturated soils shear strength enhancement can either be looked as equivalent 'effective' stress:

$\sigma'_i = (\sigma - u_a) + \chi (u_a - u_w)$

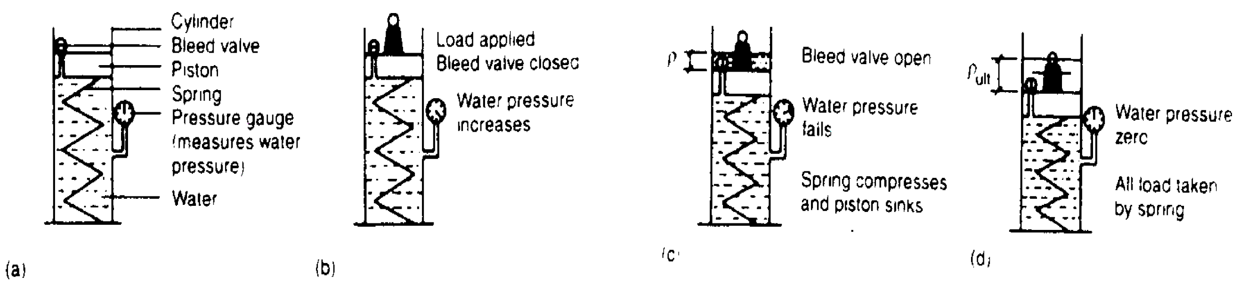
Alternatively, we may consider the shear strength of unsaturated soil:

$\tau = (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b + c'$

Undrained shear strength

Quick review of consolidation theory,

Immediate after any applied loads, mean effective stress will not change while pore water pressure responses to changes in loading. It takes time for pore water pressure to get back to equilibrium (dissipate) and change the mean effective stress.



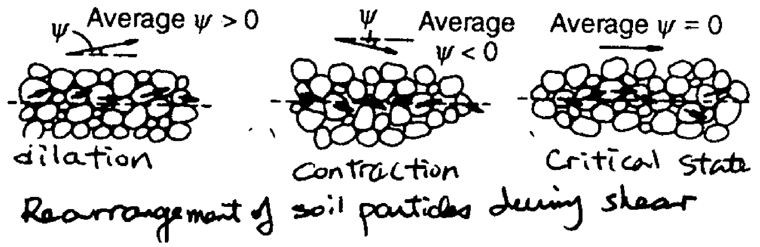
Immediately following the application of loads (can be unloading) the shear strength of the soil is same as before (as mean effective stress remains the same). If we are only looking at stability immediately after construction, we may use the shear resistance τ (determined from existing σ') as undrained shear strength c_u, and ignore the effect of changes in pore water pressure.

Possible changes in pore water pressure - mean effective stress - shear strength of soil - stability for cut / fill slopes and the use of staged construction of embankments on soft grounds.

Further conceptual work on soil shear strength

Soils are particulate and essentially frictional.

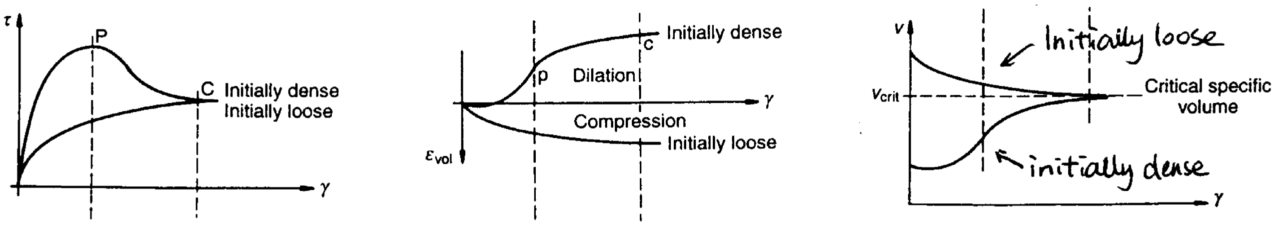
When a soil is under applied shear, the arrangement of particles will tend to change with the applied loading. Irrecoverable volume change and shear strain results from rearrangement of particles - yielding. Such rearrangement can be due to compression or shear.

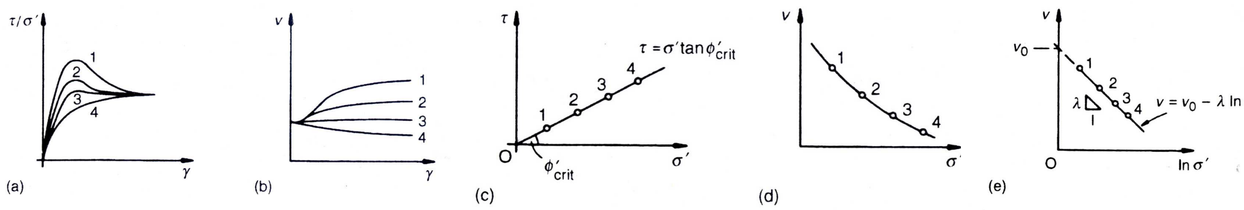


Dilation - particle rearrangement inducing increase in total volume - shear in low stress condition - well compacted state. Brittle type failure, involve peak strength then soften to critical and residual strength at higher strain levels. Volume of void increases with shear strain.

Contraction - particle rearrangement inducing reduction in total volume - in high stress condition - loose state. Ductile type failure, continue to work hardening (i.e. shear strength continue to improve with higher strain) until critical and residual strength conditions are reached. Volume of void reduces with shear strain.

Critical state - shearing - particles continue to rearrange at constant stress and constant volume conditions (i.e. for one particular density, there is only one critical state shear strength for any one soil)





Tutorial Questions

1 (a) Explain briefly contractive, dilative and critical states of soil under shear.

(b) Data obtained from three slow shearbox tests on samples of a silty sand material are given in the following table. Use them to construct a peak and critical state envelope in terms of the shear and normal stresses τ and σ' on the horizontal plane of the apparatus. Also construct a critical state line for specific volume v against $\ln(\sigma')$.

Shearbox test data

Parameters	Sample A	B	C	D
Vertical effective stress (kPa)	50	100	200	300
Peak shear stress (kPa)	57	90	156	234
Critical state shear stress (kPa)	39	78	156	234
Specific volume at end of test	2.15	2.11	2.05	2.02

2 A soil nail is installed in the silty sand material as in question 1 behind a slope. Estimate the tensile capacity (peak and residual) under a slowly applied load on a 5 m section at an average depth of 9 m. Assume that failure will occur by slippage between the grout and the surrounding soil; that soil grout interface has the same frictional properties of the soil; and that the effective stress at any depth is the same in all directions. Comment on the reliability of the calculated peak capacity of the nail. Diameter of the grouted section = 150 mm.

(a) Dry soil, pore pressure zero and unit weight of soil = 15 kN/m^3.

(b) Partially saturated soil, matrix suction = 40 kPa along the 5 m section, taking ϕ^b = critical state ϕ' and unit weight of soil = 16 kN/m^3.

(c) Saturated soil, average pore water pressure = 50 kPa and unit weight of soil = 17 kN/m^3.

3 As 2(c), what will be the pull out capacity of the 5 m section of the soil nail if the load is applied rapidly and does not allow for any dissipation of excess pore water pressure? Assume that the specific volume of the material is 2.15.

Principal Sources/References

Lambe, T.W. & Whitman, R.V. (1969). *Soil Mechanics*. John Wiley & Sons.

Powrie, W. (1997). *Soil Mechanics Concepts and Applications*. E & FN Spon.

Scholfield, A.N. (2006). Letter to editor – Interlocking, and peak and design strengths. *Géotechnique*, Vol. 56, No. 5, pp 357-358.