

Rational Assessment of Modulus of Subgrade Reaction

Harry G. Poulos¹

¹*Coffey Geotechnics, and the University of Sydney*
E-mail: harry_poulos@coffey.com

ABSTRACT: The concept of modulus of subgrade reaction has been employed within the engineering world for almost 150 years. It has been especially embraced by structural engineers who have found it convenient to represent the behaviour of the ground supporting their structures by elastic springs. Despite the best efforts of the geotechnical profession to dissuade our structural colleagues from using this flawed concept in foundation design, requests to provide a modulus of subgrade reaction continue almost unabated. Given this situation, a suitable response is to provide such values via a rational process of evaluation, rather than by empirical correlations which have little theoretical basis and which may not be applicable to the foundation being considered.

This paper sets out an approach to the estimation of values of modulus of subgrade reaction for various types of foundation. The key points made are that the modulus of subgrade reaction (k) is not a fundamental soil property, but varies with the foundation type, foundation dimension, and type of loading. k can be related to the Young's modulus of the supporting soil and to the foundation dimensions, but for pile groups, account must be taken of the reduction in k because of group effects arising from pile-soil-pile interaction. It is also emphasized that careful distinction must be made between the modulus of subgrade reaction, k , and the spring stiffness K .

Keywords: Foundation, lateral loading, modulus of subgrade reaction, piles, pile group, settlement.

1. INTRODUCTION

Despite the best efforts of the geotechnical profession to dissuade our structural colleagues from using the flawed concept of the modulus of subgrade reaction in raft and slab design, we still have frequent requests to provide them with such a value. Given that this concept is unlikely to disappear in the near future, this paper aims to set out a logical procedure by which relevant values of the equivalent modulus of subgrade reaction can be assessed. Consideration will be given to shallow foundations, axially loaded piles, and laterally loaded piles. Some examples of the application of this approach are then presented.

2. BASIC APPROACH

The modulus of subgrade reaction, k , for a foundation resting on or in soil or rock is defined as follows:

$$k = p/S \quad (1)$$

where p = foundation pressure, and S = foundation deflection.

The units are typically MN/m^3 or kN/m^3 .

Different values of k will apply for vertical and lateral loading, and k will also vary with the type of foundation and its dimensions. Initial consideration will be given to vertical loading on a raft or spread foundation, with the corresponding modulus of subgrade reaction being denoted as k_v . Subsequently, pile foundations subjected to vertical and lateral loading will be considered.

Use can be made of elastic theory to compute k_v . For example, assuming that, as shown in Figure 1, the foundation can be idealized as an equivalent circular footing of equivalent diameter d resting on a uniform layer of thickness h , elastic theory gives the following general form of the expression for the settlement S of the footing when subjected to uniform vertical load p :

$$S = pBI / E_s \quad (2)$$

where I = displacement influence factor, B = foundation dimension, E_s = Young's modulus of soil.

The displacement influence factor I depends on the shape of the foundation, the thickness of the soil profile and the Poisson's ratio of the soil or rock.

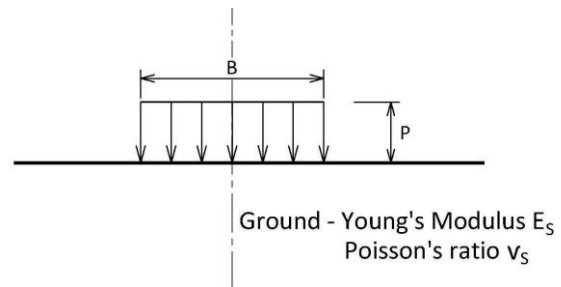


Figure 1. Loaded area on ground surface

The modulus of subgrade reaction for vertical loading, k_v , can then be obtained from eq. 2, as follows:

$$k_v = E_s / BI \quad (3)$$

The displacement influence factor can be obtained in the following ways:

1. From existing charts, such as those in Poulos and Davis (1974) and Mayne and Poulos (1999). Such charts usually require the soil profile to be simplified to have either an equivalent uniform Young's modulus with depth (see Section 3.1.1), or one that increases linearly with depth (see Sections 3.1.2 and 3.1.3).
2. From a numerical analysis, such as the program FLEA (Small, 1984). This approach is very versatile as it can take account of such factors as soil layering, foundation shape and the presence of a slab or raft beneath the loaded area.

These options are discussed in more detail below.

3. ELASTIC SOLUTIONS FOR SHALLOW FOUNDATION DISPLACEMENTS

3.1. Circular foundation on a homogeneous soil

Solutions have been provided by Mayne and Poulos (1999) for the cases set out below.

3.1.1. Flexible circular foundation on a soil layer of finite depth

The solutions for this case are shown in Figure 2, where a = radius of foundation, h = layer depth, q =applied uniform loading, E_s = Young's modulus of soil layer.

It should be noted that the displacement is for the centre of the loaded area, and that the displacements away from the centre will be smaller. Thus, the modulus of subgrade reaction will NOT be a constant over the entire area, but will increase from a minimum at the centre of the circle to a maximum at the edge of the circle.

For practical purposes, it may be preferable to compute an average settlement of the loaded area, and for a circular area, the following rough approximation may be used:

$$S_{av} \approx 0.78 [S_{centre}] \quad (4)$$

where S_{av} = average settlement, and S_{centre} = settlement at centre.

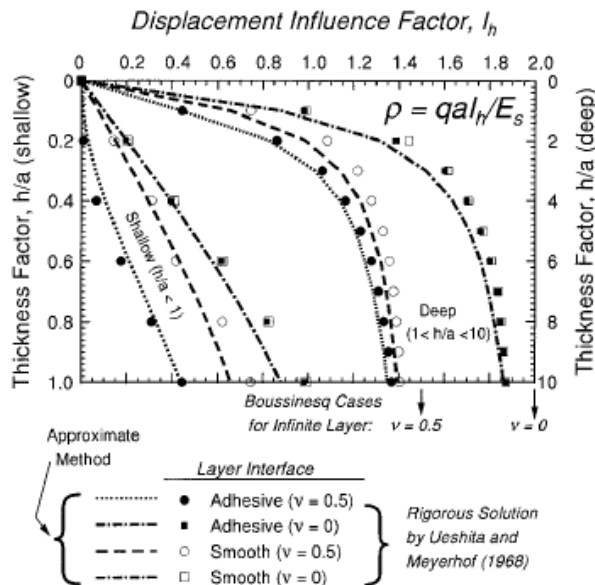


Figure 2. Displacement factors for a flexible circular footing on a finite elastic layer (Mayne & Poulos, 1999)

3.1.2. Rigid or flexible circular foundation on a deep soil layer whose modulus increases linearly with depth (a "Gibson soil")

The results for this case are plotted in Figure 3, where d =foundation diameter, q =applied uniform loading, E_0 = Young's modulus at soil surface, k_E = rate of increase of Young's modulus with depth. This figure gives both the centre settlement of a uniformly loaded (flexible) area and the settlement of a rigid footing. The average settlement of the uniformly loaded area is closely approximated by the settlement of the rigid footing.

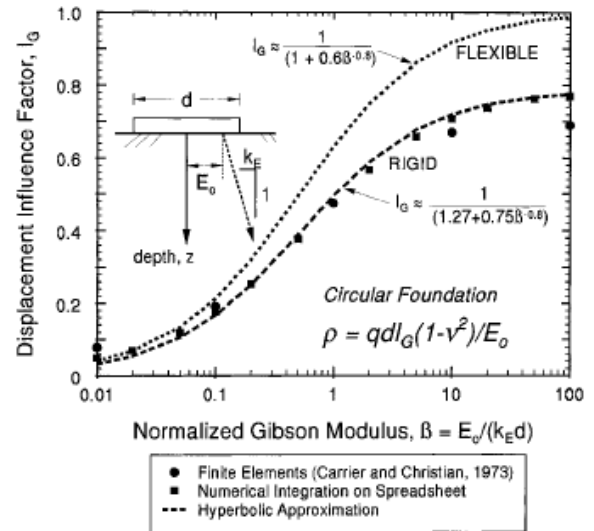


Figure 3. Influence factors for circular foundation on a deep Gibson soil ($b > 0.01$)

3.1.3. Flexible circular foundation on a "Gibson soil" of finite thickness.

Figure 4 shows this case. These results apply for the centre of a uniformly loaded (flexible) footing. Eq 4 may be used to approximate the average settlement.

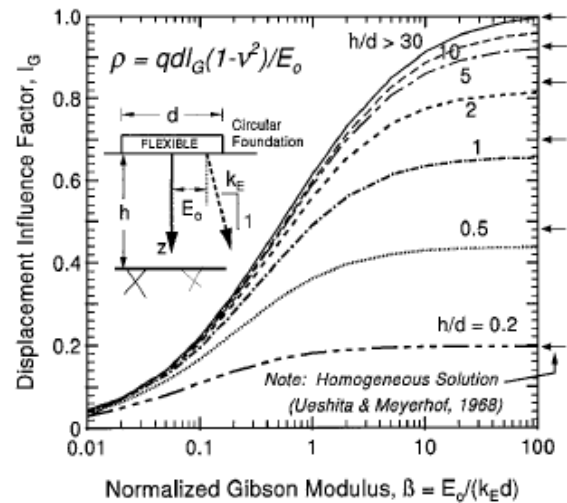


Figure 4. Influence factors for flexible foundation on a finite layer of Gibson soil

3.2. Layered soils

A layered soil profile can be transformed, approximately, into an equivalent uniform layer of the same depth, via the procedure set out in Figure 5 (Poulos, 1994). Here, E_{sbe} is the equivalent Young's modulus of the layered profile, d is the foundation diameter, h_i is the thickness of layer i , E_{si} is the Young's modulus of a layer i , and W_i is the weighting factor for layer i .

It should be noted that, when the thickness of the upper layer is greater than about $4d$, E_{sbe} can be taken as the modulus of the upper layer.

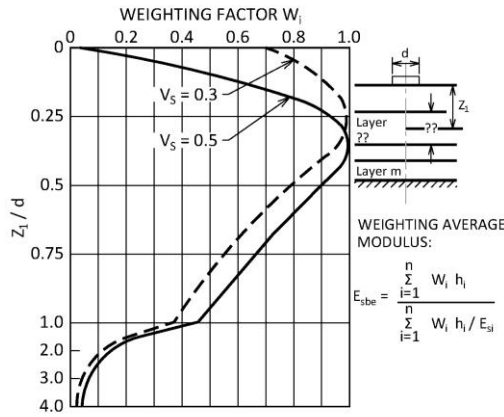


Figure 5. Weighting factor for estimation of equivalent modulus below a circular area (Poulos, 1994)

3.3. Effect of foundation shape

For footing shapes other than circular, they can generally be transformed into a circle of equal area. The equivalent diameter, d_e , of the circle is then:

$$d_e = 2.(A/\pi)^{0.5} \quad (5)$$

where A = foundation area.

3.4. Limiting value of modulus of subgrade reaction – one-dimensional compression

It should be recognised that there is an important situation that give rise to a limiting value of the modulus of subgrade reaction, and that needs to be recognised when assessing k_v .

If the loaded area is large, and/or the soil layer thickness is small (see Figure 6), then there will be essentially one-dimensional compression below the area. In this case, the value of k_v , denoted as k_{v1D} , can be calculated via the following expression:

$$k_{v1D} = D_s/h \quad (6)$$

where D_s = constrained modulus of soil, and h = soil layer thickness.

The constrained modulus D_s is related to Young's modulus E_s and Poisson's ratio v_s , and for a typical value of v_s of 0.3, $D_s = 1.35E_s$. Thus, eq. 6 can be re-expressed as follows:

$$k_{v1D} = 1.35 E_s/h \quad (7)$$

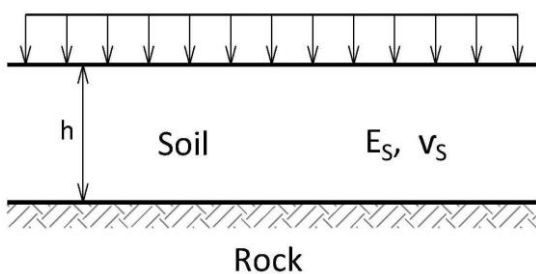


Figure 6. One-dimensional conditions

For a multi-layer soil profile, it can be readily demonstrated that the corresponding expression for k_v under one-dimensional conditions, is:

$$k_{v1D} = 1.35 / \sum (h_i/E_{si}) \quad (8)$$

where h_i = thickness of a layer i , and E_{si} = Young's modulus of layer i .

3.5. Summary

Under normal conditions, k_v can be evaluated from eq. 3, using appropriate values of the displacement influence factor I . However, a check should be carried to ensure that this value is not less than the value for one-dimensional compressional (eqs. 7 and 8).

4. The Use of Program 'FLEA'

The program FLEA (Small, 1984) facilitates the assessment of k_v by taking account of all the factors mentioned in Section 3, i.e. the size and shape of the loaded area, the layering of the soil profile, and the dispersion of pressure through a slab, if present. The only limitation of this program is that the loaded area is assumed to be flexible, i.e. the applied pressure is uniform across the loaded area.

5. EFFECTS OF EXCAVATION

In many cases, excavation, for example for a basement, will be carried out prior to construction of a building or structure. In this case, there are at least two important issues to consider when assessing the modulus of subgrade reaction and the consequent foundation behaviour:

1. Because of the unloading arising from excavation, soils that were in a normally consolidated or lightly overconsolidated state will be subjected to recompression upon the application of the building load, and thus will tend to be stiffer than under initial loading. Once the previous vertical stress has been reinstated, the soil will again exhibit the initial loading stiffness. Thus, the overall soil behaviour will tend to be more stiff than if there was no excavation.
2. If the excavation extends below the water table, there will be a resulting hydrostatic uplift on the base of the foundation, which will reduce the net loading on the foundation.

Considering first a single soil layer, the effects of excavation and soil unloading can be estimated approximately as set out below.

The foundation settlement in the layer can be estimated via the following expression:

$$S = I.B \{ \Delta\sigma_{ex} / E_{sr} + (p - \Delta\sigma_{ex}) / E_s \} \quad (9)$$

where I = displacement influence factor, B = foundation dimension (e.g. diameter), $\Delta\sigma_{ex}$ = average stress change due to excavation, p = applied pressure on the foundation, E_{sr} = Young's modulus of soil, for recompression, E_s = Young's modulus of soil, for initial compression.

This can be expressed as follows:

$$S = I.B.p / E_{seq} \quad (10)$$

where E_{seq} = equivalent Young's modulus of soil.

E_{seq} is given by the following expression:

$$E_{seq} = E_s / [1 - \Delta\sigma_{ex}/p (1-1/\eta)] \quad (11)$$

where $\eta = E_{sr} / E_s =$ ratio of recompression to initial loading modulus values.

η is typically 5-10 for soft clays, but close to 1 for very stiff soils and rocks. Thus, for a single soil layer, the corresponding value of modulus of subgrade reaction, k_{vex} , taking into account the effects of unloading and reloading, can be simply related to the normal value without excavation, k_v , as:

$$k_{vex} = k_v / [1 - \Delta\sigma_{ex} / p (1-1/\eta)] \quad (12)$$

For multiple soil layers, eq. (11) has to be applied to each layer if FLEA is used, then these modified Young's modulus values are input into the program. If a hand calculation is carried out, then an equivalent profile can be developed via Figure 5.

6. WARNING!!

It appears to have been common practice to use the following simple expression to estimate the modulus of subgrade reaction:

$$k_v = 1.4E_s/d \quad (13)$$

This expression has been derived from the elastic solution for a rigid footing on a semi-infinite homogeneous mass with Poisson's ratio of 0.3, and is only correct if:

1. The soil stiffness is uniform with depth;
2. The diameter or width, d , of the foundation loading is not very large in relation to the overall depth of the compressible soil profile;
3. The load is not applied to, or through, a concrete slab;
4. There is no excavation involved.

In other cases, it is possible that the use of eq 13 will give a conservative (and sometimes extremely conservative) assessment of k_v . Such conservatism can have significant cost implications for clients.

It is therefore strongly recommended that the approach outlined in this paper be followed to avoid the provision of misleading values of k_v .

It should also be noted that k_v , if it is to be used, should be applied only to assess structural actions (moments and shears) in a raft or slab. When applied to estimate settlements, the modulus of subgrade reaction can give misleading estimates of the distribution of settlement across a foundation.

7. Laterally Loaded Footings

For a surface or near-surface footing subjected to lateral load, the modulus of subgrade reaction will be different from that for vertical loading, and may be derived via the elastic solutions that are summarized in Chapters 3 and 4 of Poulos and Davis (1974).

8. PILE FOUNDATIONS

8.1. Introduction

A number of structural analysis programs have the facility to represent the soil adjacent to piles as a series of springs. Such programs can then be used to assess the effects of the interaction between the foundation and the superstructure on both the foundation performance and the actions within the structural elements. The key to successful application of this simplified concept to soil behaviour simulation is to select appropriate values

of subgrade reaction modulus, taking account of the following factors:

1. The type of loading, i.e. axial or lateral;
2. The effects of interaction among the piles within the group; such effects will tend to "soften" the soil springs.

The discussion below considers both axial and lateral loading, for single piles and for piles within a group.

8.2. Axially loaded piles – single piles

From Fleming et al (2009), the following relationship can be derived for the modulus of subgrade reaction along the shaft, k_s , for a typical Poisson's ratio of 0.3:

$$k_s \approx 0.6E_{sv} / d \quad (14)$$

where E_{sv} = Young's modulus of the soil along the shaft for vertical loading, d = pile shaft diameter.

For the pile base, the corresponding modulus of subgrade reaction, k_b , is:

$$k_b \approx 1.4E_{sb}/d_b \quad (15)$$

where E_{sb} = average soil modulus below pile base, for vertical loading, d_b = pile base diameter.

8.3. Axially loaded piles – a pile within a group

Pile-soil-pile interaction within a group will tend to reduce the equivalent stiffness of the soil and hence the modulus of subgrade reaction. A simple approach to take group effects into account is to multiply the single pile values in eqs 14 and 15 by a group reduction factor, R_G , so that the group values of modulus of subgrade reaction, k_{sG} for the shaft and k_{bG} for the base, can be approximated as:

$$k_{sG} \approx R_G.k_s \quad (16a)$$

$$k_{bG} \approx R_G.k_b \quad (16b)$$

R_G can be approximated as follows (Poulos, 1989):

$$R_G \approx n^{-w} \quad (17)$$

where n = number of piles in group, w = exponent which depends on the soil profile and the average spacing of piles within the group (relative to diameter).

As pointed out by Fleming et al (2009), R_G will in fact be different for the shaft and the base of the pile, but given the nature of the approximations involved, the use of a single value of R_G appears to be adequate for the present purposes.

As a first approximation, the following guidelines are suggested for pile groups with an average spacing of 3 to 5 pile shaft diameters:

- Friction piles in soil profile with constant E_{sv} with depth: $w \approx 0.5$
- Friction piles in soil profile with linearly increasing E_{sv} with depth: $w \approx 0.33$
- End bearing piles founded on or in a hard bearing layer: $w \approx 0.25$.

More accurate values of R_G can be obtained via a computer analysis of the pile group behaviour, for example, via programs such as DEFPIG (Poulos, 1990), REPUTE (Geocentix, 2014) or PIGLET (Randolph, 2005).

8.4. Laterally loaded piles – single piles

A reasonable method for evaluating the modulus of subgrade reaction for lateral loading, k_h , is to equate the solutions for deflection of a rigid fixed head pile from elastic continuum theory and from subgrade reaction theory. On this basis, the following approximate relationships can be obtained:

$$k_h \approx X_1 \cdot E_{sh} / d \quad (18)$$

where X_1 is typically 0.8 to 1.0, depending on the length to diameter ratio of the pile, E_{sh} = Young's modulus of soil, for lateral loading, d = pile diameter, or width in the direction of loading.

For lateral loading, it is common practice to adopt Young's modulus values which are less than those for vertical loading, because of the greater strain levels in the soil under lateral loading. A reduction factor of 0.7 is commonly applied to E_{sv} values to obtain E_{sh} . A more complex expression has been derived by Vesic (1961) in which the relative stiffness of the pile and soil is included. The factor X_1 in eq 17 can then be reduced to the following expression:

$$X_1 \approx 0.92 (E_{sh} / E_p)^{1/12} \quad (19)$$

where E_p = Young's modulus of pile.

The use of Eq 19 will generally give a lower value of k_h than that based on the recommendations in eq 18.

8.5. Laterally loaded piles – pile groups

As for axial loading, pile-soil-pile interaction will tend to "soften" the lateral response of piles and hence reduce the modulus of subgrade reaction as compared with a single isolated pile. This reduction can be approximated in terms of a lateral group reduction factor, R_{Gh} . Thus, for a laterally loaded pile in a group environment, the modulus of subgrade reaction, k_{hG} , is related to the value for a single isolated pile, k_h , as follows:

$$k_{hG} \approx R_{Gh} \cdot k_h \quad (20)$$

Following Poulos (2001), R_{Gh} can be approximated as follows:

$$R_{Gh} \approx n^{-w_l} \quad (21)$$

where n = number of piles within group, w_l = lateral group exponent, which depends on the nature of the soil profile, the effective lateral length, L_c , of the pile and the typical pile spacing, s . For a uniform soil mass, values of w_l are plotted in Figure 7, for three values of s/d , as a function of the ratio of critical pile length, L_c , divided by pile diameter d , where:

$$L_c/d = 2.09 (E_p/E_s)^{0.25} \quad (22)$$

and where E_p = Young's modulus of pile, E_s = Young's modulus of soil.

More accurate values of R_{Gh} can be obtained via a computer analysis of the pile group behaviour, for example, via the programs PIGLET, DEFPIG or REPUTE.

9. SPRING STIFFNESS VALUES

It is not uncommon for the structural engineer to request values of the spring stiffness for the foundation elements, rather than the modulus of subgrade reaction. It is critical to distinguish between these two values.

If the foundation width or diameter is d , and an elemental length of the foundation of ΔL is considered, then the spring stiffness K for that element can be calculated from the relevant modulus of subgrade reaction, k , as follows:

$$K = k \cdot d \cdot \Delta L \quad (23)$$

K will then have the units of stiffness (force per unit length, for example MN/m), whereas k has the units of force per length cubed, for example MPa/m or MN/m³.

For the overall foundation, or for individual piles with a group, the spring stiffness can be computed as the ratio of the applied load to the deflection. In general, different values will be obtained for each pile within a group, and for each component of load (i.e. vertical, lateral and moment).

It has been found that the most reliable approach to estimating the stiffness of individual piles within a group is to carry out analyses in which an equal load (generally equal to the working load) is applied simultaneously to each pile within the group. The resulting deflection of each pile can then be used to compute the pile head stiffness. Vertical, lateral and moment loads should be considered separately. This procedure avoids unrealistic computed stiffness values that can arise under some combinations of vertical, lateral and moment loading.

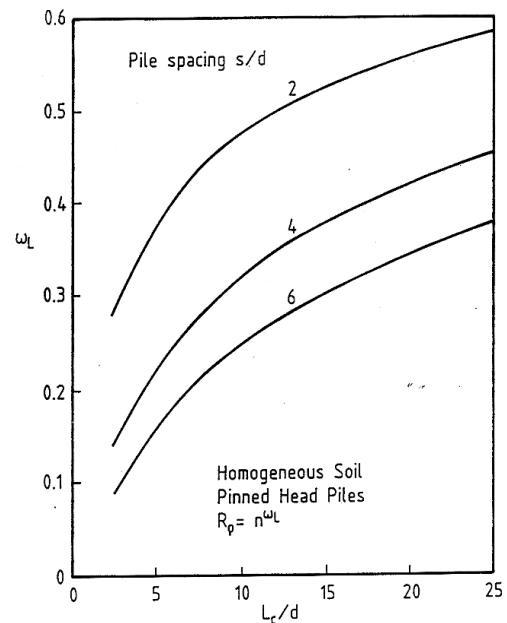


Figure 7 Values of lateral group exponent w_l (after Poulos, 2001)

10. EXAMPLES

To illustrate the application of the approach described herein, three numerical cases will be considered below.

10.1. Case 1 – Uniform loading directly on the ground

This case is illustrated in Figure 8. A circular foundation of diameter 20m is located directly on a layered soil profile with the properties shown in the figure. For the evaluation of k_v , a unit pressure ($p=1.0$ MPa) is applied to the loaded area.

The FLEA analysis gives the following result for the central deflection S_c of the foundation:

$$S_c = 1.069 \text{ m.}$$

Thus, the modulus of subgrade reaction for the centre of the foundation is:

$$k_v = p/S_c = 1.0/1.069 = \mathbf{0.94 \text{ MPa/m.}}$$

The same problem has been evaluated by hand calculations, using Figure 5 to obtain an equivalent modulus for the layered profile, Figure 2 to obtain the displacement influence factor I (denoted as I_h in this figure), and then eq 3 to evaluate k_v . In these calculations, the equivalent modulus was calculated as 12.6 MPa, while interpolation from Figure 2 gave a value of I of 1.25. From eq 3, k_v was found to be **1.01 MPa/m**, which was similar to, but somewhat larger than, the value of 0.94 computed from FLEA.

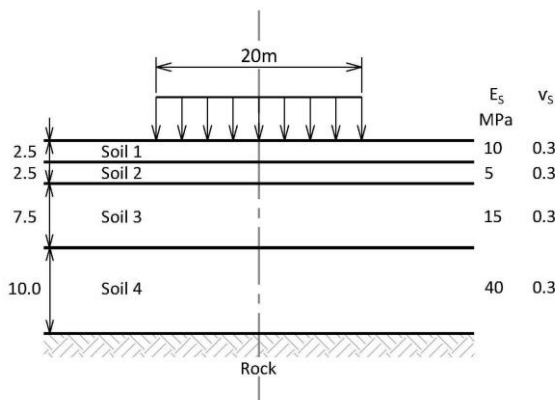


Figure 8. Example 1: loaded area on the surface of a layered soil profile

Had the simplified expression (eq 13) been applied in this case, with a modulus of 10 MPa used (i.e. the modulus for the layer directly below the foundation), the value of k_v would have been computed to be $1.4 \cdot 10 / 20 = \mathbf{0.70 \text{ MPa/m}}$, about 25% less than the value derived from the FLEA analysis.

For the case of one dimensional compression, which would give a lower limit value, the use of eq 8 leads to a value of $k_{v1D} = 0.90 \text{ MPa/m}$, which is slightly less than the values obtained from FLEA and the hand calculation method. It would thus appear that, in this case, the geometry for this example is approaching that of a one-dimensional situation.

10.2. Case 2 – foundation within an excavation

This case is illustrated in Figure 9. The soil profile is similar to that for Case 1, but now it is assumed that a 5m deep excavation will be made and the foundation will be constructed at the base of the excavation.

The following assumptions are made:

- The water table is located 2.5m below the surface;
- The unit weight of the upper two layers is 20 kN/m^3
- The ratio of the reload to unload moduli is 5 for the upper three layers, and 2.5 for the lower (and stiffer) layer.
- The final applied loading on the foundation will be 150 kPa.

The first step is to estimate revised values of the “operative” modulus which allow for the effects of the excavation and the subsequent reloading of the soil profile.

Assuming that the unit weight of the upper two layers is 20 kN/m^3 , the stress relief due to excavation will be $\Delta\sigma_{ex} = 2.5 \times 20 + 2.3 \times (20 - 10) = 75 \text{ kPa}$.

Assuming that $\Delta\sigma_{ex}$ is reasonably constant with depth, and that the ratio of reload to initial loading modulus, η , is 5 for Soil 3 and 2.5 for Soil 4, application of eq 10 gives the following equivalent values of Young’s modulus:

$$\text{Soil 3: } E_{seq} = 15.0 / [1 - \{75/125(1-1/5)\}] = 28.8 \text{ MPa}$$

$$\text{Soil 4: } E_{seq} = 40.0 / [1 - \{75/125(1-1/2.5)\}] = 62.5 \text{ MPa}$$

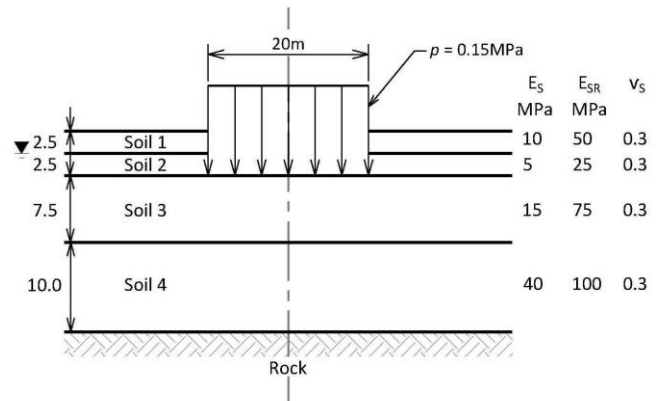


Figure 9. Case 2: Foundation within an excavation

For the two-layer system consisting of Soils 3 and 4, the program FLEA gives a central deflection of 0.302 m/MPa applied pressure. Thus, the modulus of subgrade reaction is:

$$k_v = 1/0.302 = \mathbf{3.31 \text{ MPa/m.}}$$

Using hand calculation methods with Figures 2 and 5, and assuming a rough footing and a soil Poisson’s ratio of 0.3, the computed central deflection is 0.279 m/MPa , thus giving:

$$k_v = 1/0.279 = \mathbf{3.58 \text{ MPa/m.}}$$

This is similar to, but slightly larger than, the value obtained from the FLEA analysis.

It should be noted that, comparing Cases 1 and 2, the effect of the excavation in Case 2 is to increase k_v by a factor in excess of 3. This arises both because the upper two soft layers are removed, and also because the Young’s modulus of the lower two layers is increased due to the effects of the larger recompression modulus over a part of the range of foundation loading.

10.3. Case 3 – group of 30 piles

Figure 10 shows a group of 30 piles within a layered soil profile consisting of stiff clays extending to considerable depth. Values of the modulus of subgrade reaction will be developed for both axial and lateral loading.

(1) Axial Loading

Considering first a single isolated pile, the modulus of subgrade reaction for the shaft is given by eq 13. Thus, from the surface to a depth of 12m, $k_s \approx 0.6 \times 30 / 0.8 = 22.5 \text{ MPa/m}$.

From 12m to the pile base at 20m, $k_s \approx 0.6 \times 100 / 0.8 = 75 \text{ MPa/m}$.
For the pile base, from eq 14, $k_b \approx 1.4 \times 100 / 0.8 = 175 \text{ MPa/m}$.

Considering now the group effects via eq 16, a value of w of between 0.25 (for a hard stratum) and 0.5 (for a uniform stratum) would be appropriate and a value of 0.375 is therefore chosen. The corresponding group reduction factor from eq 17 is thus $30^{-0.375} = 0.279$.

Accordingly, the values of k_{sG} and k_{bG} for the piles in a group environment would be, from eqs 16a and 16b:

$k_{sG} \approx 0.279 \times 22.5 = 6.3 \text{ MPa/m}$ for 0 to 12m depth,
 $0.279 \times 75 = 20.9 \text{ MPa/m}$ for 12 to 20m depth, and
 $0.279 \times 175 = 48.8 \text{ MPa/m}$ for the base.

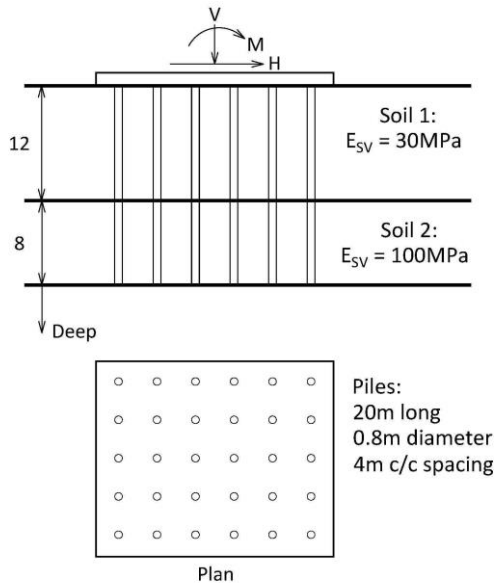


Figure 10. Case 3: 30-Pile group

(2) Lateral Loading

A reduction factor of 0.7 will be applied to E_{sv} values to obtain Young's modulus for lateral loading, E_{sh} .

For a single isolated pile, from eq 18, taking $X_1 = 0.9$, the values of horizontal modulus of subgrade reaction would be as follows:

0-12m: $k_h \approx 0.9 \times (0.7 \times 30) / 0.8 = 23.6 \text{ MPa/m}$

12-20m: $k_h \approx 0.9 \times (0.7 \times 100) / 0.8 = 78.7 \text{ MPa/m}$.

Allowing now for group effects, and adopting the case of constant stiffness with depth (rather than a linearly increasing stiffness with depth), L_c/d is about 12.8 m (for $E_p = 30000 \text{ MPa}$), and for $s/d = 5$, $w_l = 0.3$ from Figure 7. Thus, from eq 21, $R_{Gh} \approx 30^{-0.3} = 0.36$.

Therefore, for the piles in a group environment,

0-12m: $k_{hG} = 0.36 \times 23.6 = 8.5 \text{ MPa/m}$

12-20m: $k_{hG} = 0.36 \times 78.7 = 28.3 \text{ MPa/m}$.

11. CONCLUSIONS

This paper has set out an approach to the rational estimation of values of modulus of subgrade reaction. The key points made are as follows:

- (1) The modulus of subgrade reaction (k) is not a fundamental soil property, but varies with the foundation dimension.
- (2) k is different for different types of foundation and for different types of loading applied to the same foundation.

- (3) K can be related to the Young's modulus of the supporting soil and to the foundation dimensions.
- (4) For pile groups, account must be taken of the reduction in k because of group effects arising from pile-soil-pile interaction.
- (5) Careful distinction must be made between the modulus of subgrade reaction, k , and the spring stiffness K . K is often best obtained via a foundation analysis, as the ratio of the applied load to the computed displacement of the foundation.

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