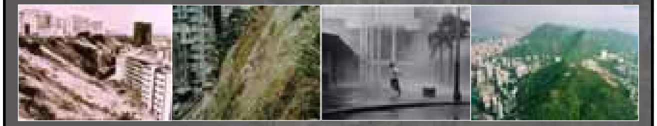
# Session 4
# Stability of Slopes and Laboratory Tests



[3:30 – 5:00 pm, 18 February 2009]

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## Objective of this session:

- Fundamental Soil Mechanics Principles
- Laboratory tests for measurement of soil parameters
- Methods of stability analysis of soil slopes

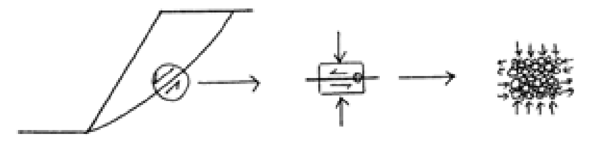
## What are the factors controlling the stability of a slope?

- Geology (material/mass fabric) - mode of failure
- Materials (soil matrix/mass) - shear strength, permeability, response to infiltration
- Environmental factors (groundwater) - main/perched groundwater table, infiltration
- Geometry – loading, stress, etc.
- External loadings – as above

## The Materials

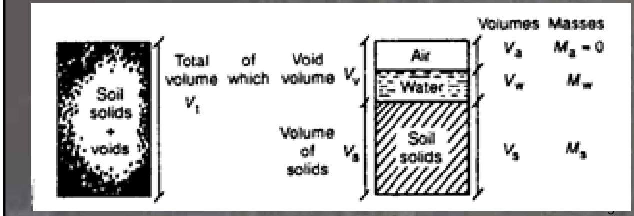
Soil / Weathered Rock – An assembly of Particles

- In macroscopic scale, soil can be looked as a continuum. In microscopic scale, all soils are assembles of particles of different sizes and shapes. The interaction of the soil grains affects the soil mass behaviour.



## Phases

- The voids between soil grains are filled with air and water, the conditions of the infill affects the behaviour of soils.
- Solid, air, pore fluid - the phase diagram
- Calculations of void ratio, moisture content, degree of saturation, dry density, etc.

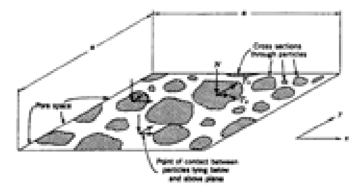


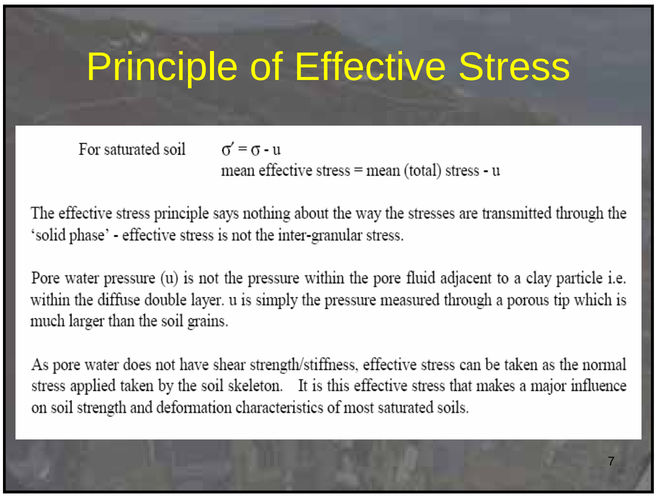
## Stress for a Particulate System

- stress : macroscopic stress - force / total area - continuum

$$\sigma = \frac{\Sigma N}{a \times a} \qquad \tau_x = \frac{\Sigma T_x}{a \times a} \qquad \tau_y = \frac{\Sigma T_y}{a \times a}$$

- contact stress
- stress in dry soil - force in the soil skeleton per unit area of soil (which includes voids)





## Principle of Effective Stress

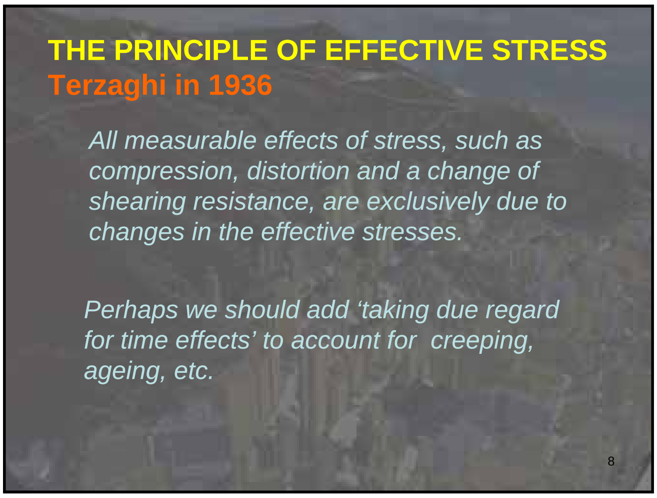
For saturated soil $\sigma' = \sigma - u$

mean effective stress = mean (total) stress - u

The effective stress principle says nothing about the way the stresses are transmitted through the 'solid phase' - effective stress is not the inter-granular stress.

Pore water pressure (u) is not the pressure within the pore fluid adjacent to a clay particle i.e. within the diffuse double layer. u is simply the pressure measured through a porous tip which is much larger than the soil grains.

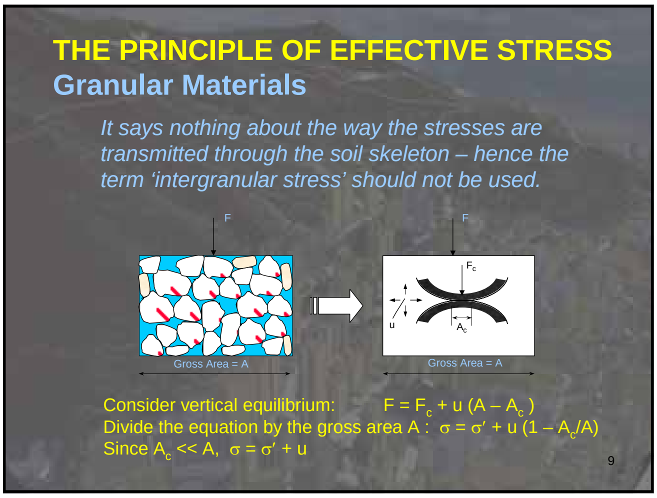
As pore water does not have shear strength/stiffness, effective stress can be taken as the normal stress applied taken by the soil skeleton. It is this effective stress that makes a major influence on soil strength and deformation characteristics of most saturated soils.



## THE PRINCIPLE OF EFFECTIVE STRESS
**Terzaghi in 1936**

*All measurable effects of stress, such as compression, distortion and a change of shearing resistance, are exclusively due to changes in the effective stresses.*

*Perhaps we should add 'taking due regard for time effects' to account for creeping, ageing, etc.*



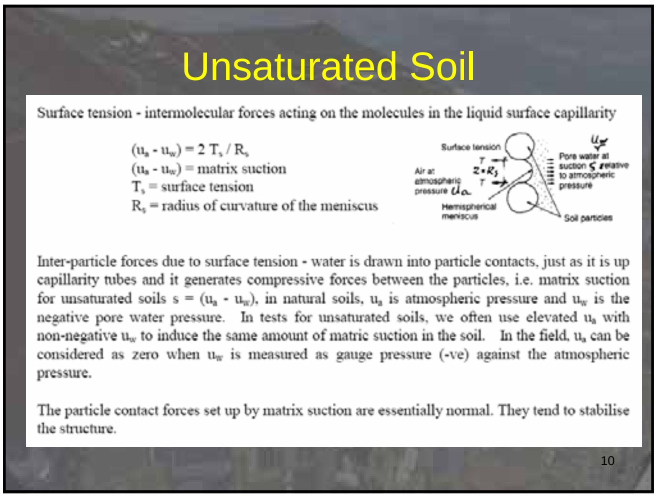
## THE PRINCIPLE OF EFFECTIVE STRESS
**Granular Materials**

*It says nothing about the way the stresses are transmitted through the soil skeleton – hence the term 'intergranular stress' should not be used.*

Consider vertical equilibrium: $F = F_c + u\,(A - A_c)$

Divide the equation by the gross area A : $\sigma = \sigma' + u\,(1 - A_c/A)$

Since $A_c << A$, $\sigma = \sigma' + u$



## Unsaturated Soil

Surface tension - intermolecular forces acting on the molecules in the liquid surface capillarity

$(u_a - u_w) = 2\,T_s / R_s$

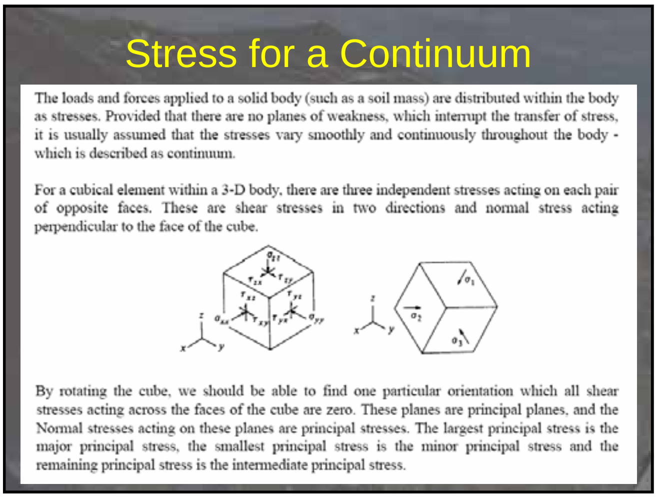
$(u_a - u_w)$ = matrix suction

$T_s$ = surface tension

$R_s$ = radius of curvature of the meniscus

Inter-particle forces due to surface tension - water is drawn into particle contacts, just as it is up capillarity tubes and it generates compressive forces between the particles, i.e. matrix suction for unsaturated soils $s = (u_a - u_w)$, in natural soils, $u_a$ is atmospheric pressure and $u_w$ is the negative pore water pressure. In tests for unsaturated soils, we often use elevated $u_a$ with non-negative $u_w$ to induce the same amount of matric suction in the soil. In the field, $u_a$ can be considered as zero when $u_w$ is measured as gauge pressure (-ve) against the atmospheric pressure.

The particle contact forces set up by matrix suction are essentially normal. They tend to stabilise the structure.

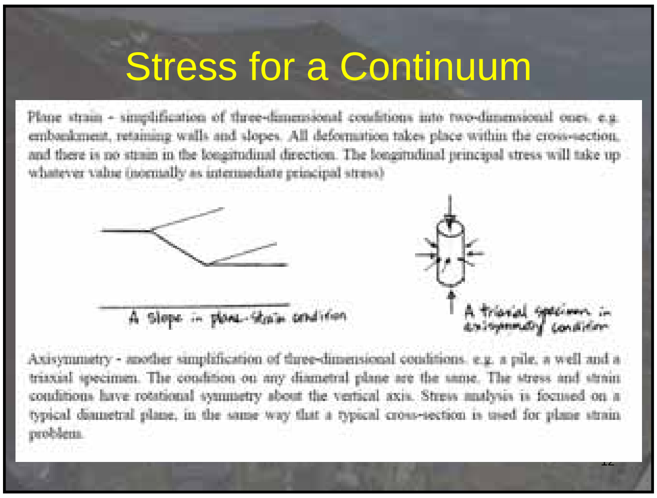


## Stress for a Continuum

The loads and forces applied to a solid body (such as a soil mass) are distributed within the body as stresses. Provided that there are no planes of weakness, which interrupt the transfer of stress, it is usually assumed that the stresses vary smoothly and continuously throughout the body - which is described as continuum.

For a cubical element within a 3-D body, there are three independent stresses acting on each pair of opposite faces. These are shear stresses in two directions and normal stress acting perpendicular to the face of the cube.

By rotating the cube, we should be able to find one particular orientation which all shear stresses acting across the faces of the cube are zero. These planes are principal planes, and the Normal stresses acting on these planes are principal stresses. The largest principal stress is the major principal stress, the smallest principal stress is the minor principal stress and the remaining principal stress is the intermediate principal stress.



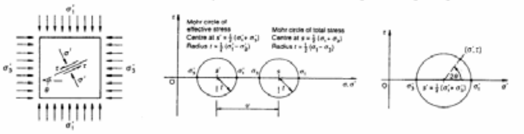
## Stress for a Continuum

Plane strain - simplification of three-dimensional conditions into two-dimensional ones. e.g. embankment, retaining walls and slopes. All deformation takes place within the cross-section, and there is no strain in the longitudinal direction. The longitudinal principal stress will take up whatever value (normally as intermediate principal stress)

Axisymmetry - another simplification of three-dimensional conditions. e.g. a pile, a well and a triaxial specimen. The condition on any diametral plane are the same. The stress and strain conditions have rotational symmetry about the vertical axis. Stress analysis is focused on a typical diametral plane, in the same way that a typical cross-section is used for plane strain problem.

# Mohr Circles – Maximum Shear

The normal and shear stresses acting on an imaginary cut within a typical cross-sectional or diametral plane will depend on the orientation of the cut with respect to the major and minor principal stress directions. if the cut is perpendicular to either the major or the minor principal stress, the shear stress acting in the direction of the cut will be zero. The magnitude of the shear stress increases as the cut is rotated away from the direction of the planes of principal stress.



The stress state within a plane containing the major and minor principal stresses is most conveniently represented by means of Mohr circle construction. The circle may be plotted for either total or effective stress.

The Mohr circle passes through the points representing the major and the minor principal stresses at ($\sigma_1$, 0) and ($\sigma_3$, 0). The centre of the total stress circle is at ($\sigma_1$+$\sigma_3$)/2,0. The average of major and minor principal total stresses ($\sigma_1$+$\sigma_3$)/2 is given the symbol s. The centre of the effective stress circle is at ($\sigma'_1$+$\sigma'_3$)/2,0. The average major and minor principal effective stresses ($\sigma'_1$+$\sigma'_3$)/2 is given the symbol s'. Recalling that $\sigma' = \sigma - u$, the centres of the circles for effective and total stress are separated by a distance u along the normal stress axis.

# Mohr Circles – Maximum Shear

The radius of the circle is ($\sigma_1$-$\sigma_3$)/2 for total stress and is ($\sigma'_1$-$\sigma'_3$)/2 for effective stress. These are identical, because pore water pressure u cancels out in the equations. ($\sigma_1$-$\sigma_3$)/2 is equal to the maximum shear stress, and is given the symbol t.

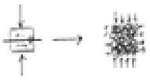
The stresses acting on an imaginary cut at an angle θ anticlockwise from the plane on which the major principal stress acts are found by drawing a line through the centre of the Mohr circle to the circumference, which makes an angle 2θ with the normal stress axis. The stress state on the cut is given by the point where this diameter meets the circumference of the circle.

The Mohr circle of stress shows that, unless the major and minor principal stresses are equal, there must be some shear stress acting within the plane under consideration. The maximum shear stress within the plane is equal to the radius of the Mohr circle ($\sigma_1$-$\sigma_3$)/2. It occurs at angles of ±45° to the planes on which the major and minor principal stresses act.

# Material Behaviour (Shear Strength)

Stress-strain behaviour (elastic / yield - strain softening / hardening) will be covered separately.

Shear resistance of assembly of soil particles – comprising two components: frictional resistance (critical state friction) between soil particles and interlocking of soil particles (represented by dilation angle, which depends on effective pressure and relative density).



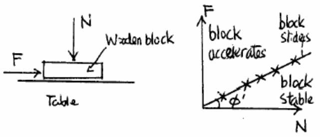
# Material Behaviour (Shear Strength)

Wooden block analogy - frictional resistance between soil grains

Imagine a wooden block on a wooden table. Normal load N is kept constant and a sideway force F is applied until the block starts to slide. F increases with larger N. If we plot F vs N

$F = N\mu$

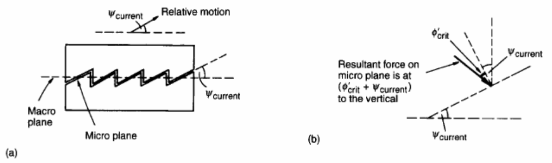
$F = N\tan\phi$ as $\mu = \tan\phi$



# Material Behaviour (Shear Strength)

Sawtooth analogy - interlocking and dilation between soil grains

The other component in shear resistance is interlocking of soil particles. Imagine we have a macroscopic shear plane through a soil element (e.g. in a shear box), shearing resistance mobilised on the (marco) shear plane can be imagined as sliding along a series of saw teeth.



# Material Behaviour (Shear Strength)

If the angle of friction along the mico planes of the saw teeth is $\phi'$ (or actually $\phi'_{crit}$), the current angle of shearing resistance of the macro plane is given by

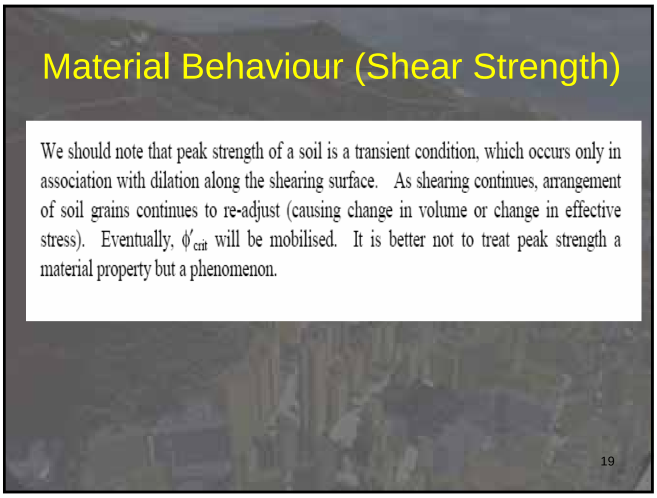
$$\phi'_{current} = \phi'_{crit} + \psi_{current}$$

where $\psi_{current}$ is the angle of the saw teeth as well as the angle of dilation – i.e. the amount of upward movement of the upper half of the shear box.

Results of tests on granular soils indicate that under plane-strain conditions

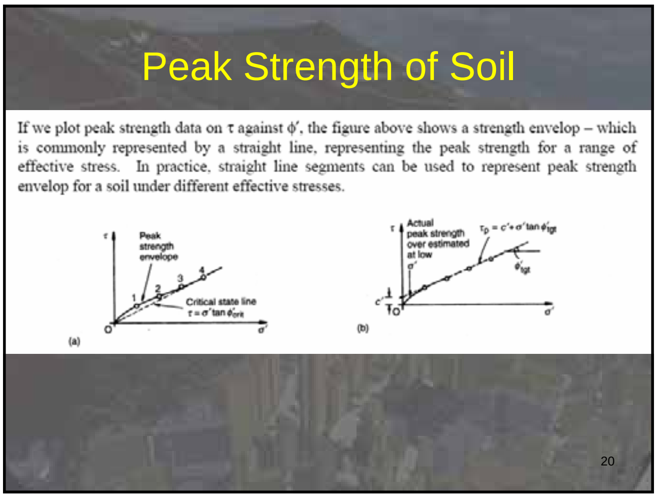
$$\phi'_{peak} = \phi'_{crit} + 0.8\,\psi_{max}$$

The maximum angle of dilation, $\psi_{max}$, is a function of stress level and density of the material.



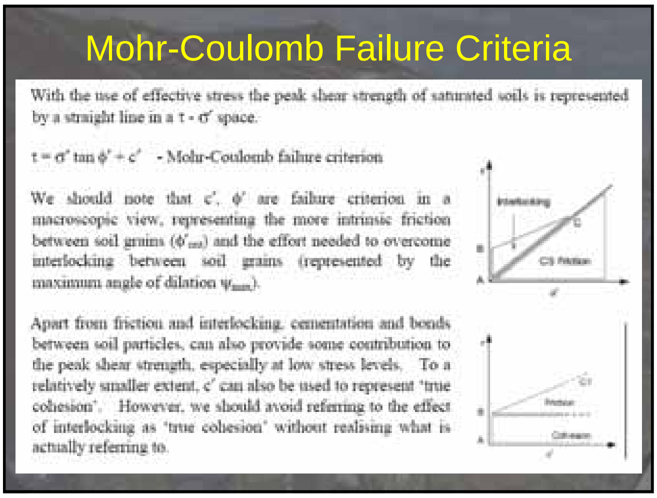
## Material Behaviour (Shear Strength)

We should note that peak strength of a soil is a transient condition, which occurs only in association with dilation along the shearing surface. As shearing continues, arrangement of soil grains continues to re-adjust (causing change in volume or change in effective stress). Eventually, $\phi'_{crit}$ will be mobilised. It is better not to treat peak strength a material property but a phenomenon.



## Peak Strength of Soil

If we plot peak strength data on $\tau$ against $\phi'$, the figure above shows a strength envelop – which is commonly represented by a straight line, representing the peak strength for a range of effective stress. In practice, straight line segments can be used to represent peak strength envelop for a soil under different effective stresses.



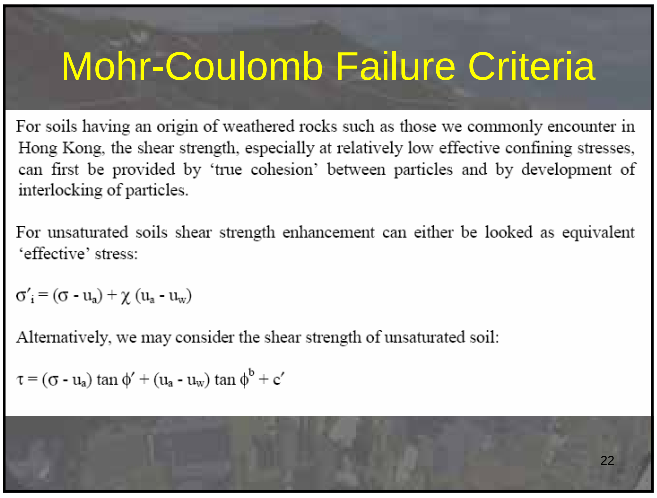
## Mohr-Coulomb Failure Criteria

With the use of effective stress the peak shear strength of saturated soils is represented by a straight line in a $\tau$ - $\sigma'$ space.

$\tau = \sigma' \tan\phi' + c'$ - Mohr-Coulomb failure criterion

We should note that $c'$, $\phi'$ are failure criterion in a macroscopic view, representing the more intrinsic friction between soil grains ($\phi'_{\mu}$) and the effort needed to overcome interlocking between soil grains (represented by the maximum angle of dilation $\psi_{max}$).

Apart from friction and interlocking, cementation and bonds between soil particles, can also provide some contribution to the peak shear strength, especially at low stress levels. To a relatively smaller extent, $c'$ can also be used to represent 'true cohesion'. However, we should avoid referring to the effect of interlocking as 'true cohesion' without realising what is actually referring to.



## Mohr-Coulomb Failure Criteria

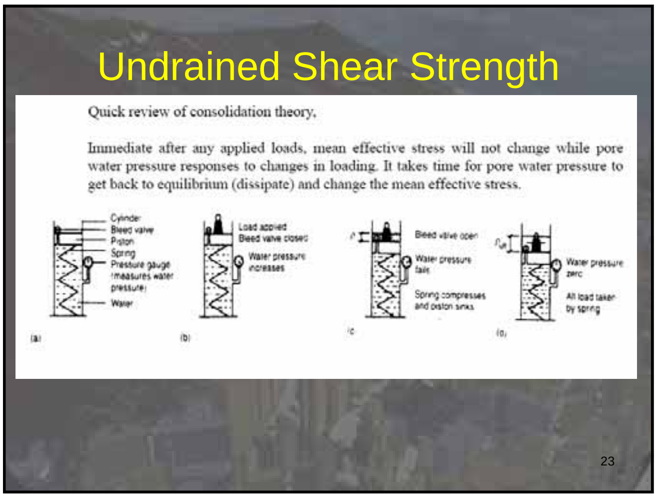
For soils having an origin of weathered rocks such as those we commonly encounter in Hong Kong, the shear strength, especially at relatively low effective confining stresses, can first be provided by 'true cohesion' between particles and by development of interlocking of particles.

For unsaturated soils shear strength enhancement can either be looked as equivalent 'effective' stress:

$\sigma'_i = (\sigma - u_a) + \chi\,(u_a - u_w)$

Alternatively, we may consider the shear strength of unsaturated soil:

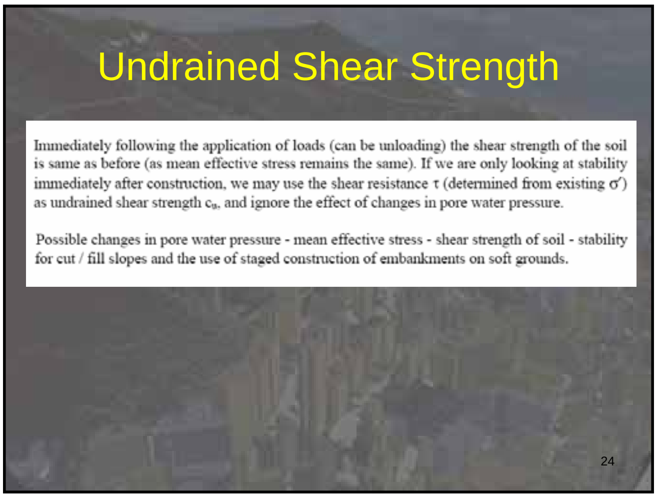
$\tau = (\sigma - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b + c'$



## Undrained Shear Strength

Quick review of consolidation theory,

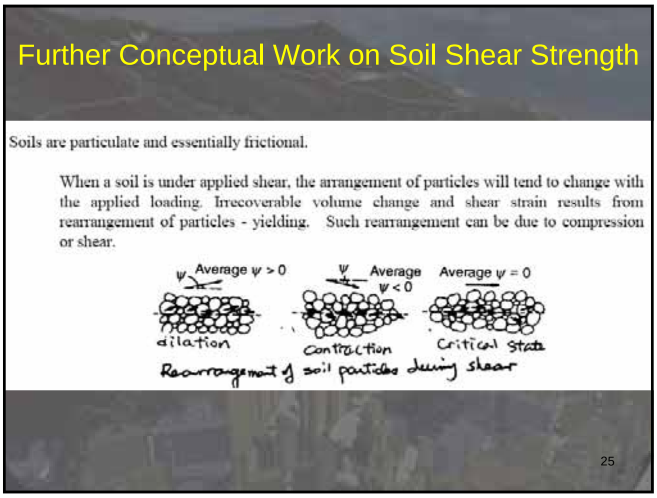
Immediate after any applied loads, mean effective stress will not change while pore water pressure responses to changes in loading. It takes time for pore water pressure to get back to equilibrium (dissipate) and change the mean effective stress.



## Undrained Shear Strength

Immediately following the application of loads (can be unloading) the shear strength of the soil is same as before (as mean effective stress remains the same). If we are only looking at stability immediately after construction, we may use the shear resistance $\tau$ (determined from existing $\sigma'$) as undrained shear strength $c_u$, and ignore the effect of changes in pore water pressure.

Possible changes in pore water pressure - mean effective stress - shear strength of soil - stability for cut / fill slopes and the use of staged construction of embankments on soft grounds.

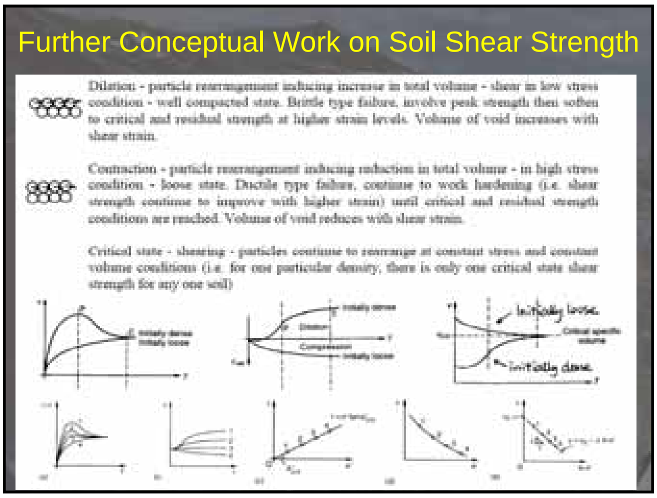


## Further Conceptual Work on Soil Shear Strength

Soils are particulate and essentially frictional.

When a soil is under applied shear, the arrangement of particles will tend to change with the applied loading. Irrecoverable volume change and shear strain results from rearrangement of particles - yielding. Such rearrangement can be due to compression or shear.

Rearrangement of soil particles during shear

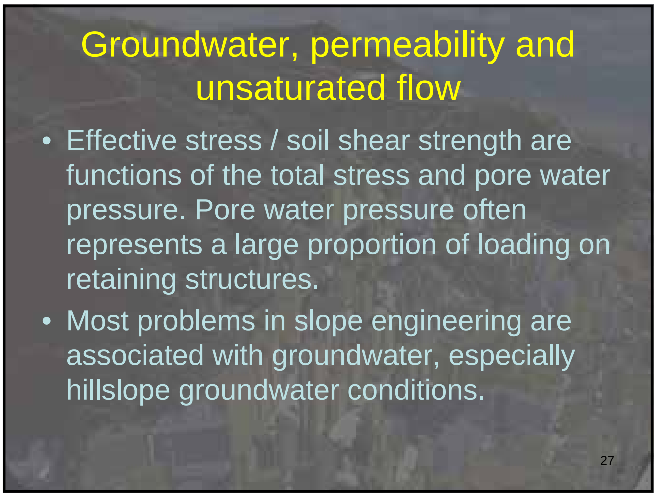


## Further Conceptual Work on Soil Shear Strength

Dilation - particle rearrangement inducing increase in total volume - shear in low stress condition - well compacted state. Brittle type failure, involve peak strength then soften to critical and residual strength at higher strain levels. Volume of void increases with shear strain.

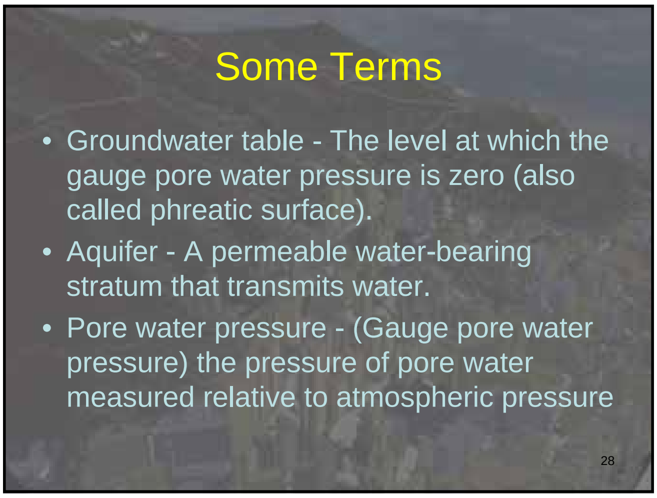
Contraction - particle rearrangement inducing reduction in total volume - in high stress condition - loose state. Ductile type failure, continue to work hardening (i.e. shear strength continue to improve with higher strain) until critical and residual strength conditions are reached. Volume of void reduces with shear strain.

Critical state - shearing - particles continue to rearrange at constant stress and constant volume conditions (i.e. for one particular density, there is only one critical state shear strength for any one soil)



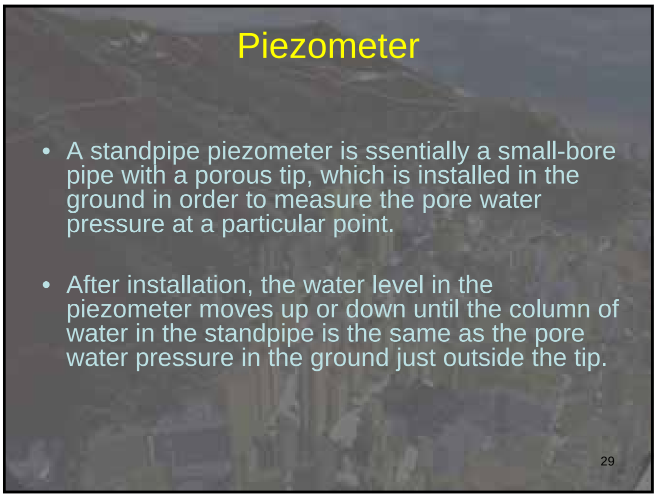
## Groundwater, permeability and unsaturated flow

- Effective stress / soil shear strength are functions of the total stress and pore water pressure. Pore water pressure often represents a large proportion of loading on retaining structures.
- Most problems in slope engineering are associated with groundwater, especially hillslope groundwater conditions.



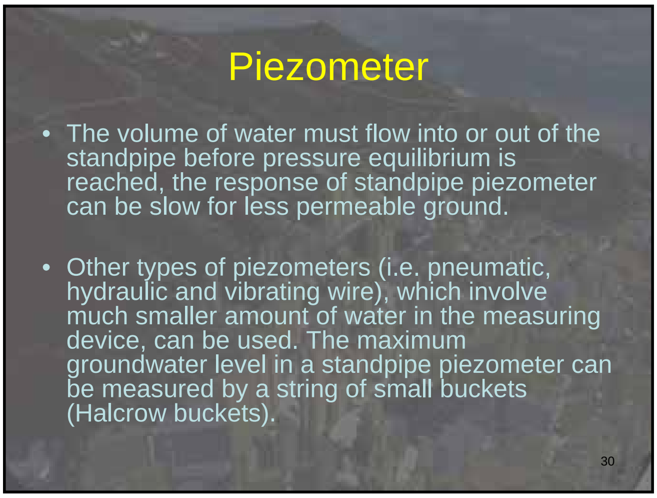
## Some Terms

- Groundwater table - The level at which the gauge pore water pressure is zero (also called phreatic surface).
- Aquifer - A permeable water-bearing stratum that transmits water.
- Pore water pressure - (Gauge pore water pressure) the pressure of pore water measured relative to atmospheric pressure



## Piezometer

- A standpipe piezometer is ssentially a small-bore pipe with a porous tip, which is installed in the ground in order to measure the pore water pressure at a particular point.
- After installation, the water level in the piezometer moves up or down until the column of water in the standpipe is the same as the pore water pressure in the ground just outside the tip.



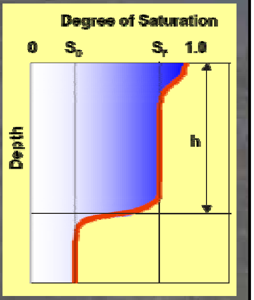
## Piezometer

- The volume of water must flow into or out of the standpipe before pressure equilibrium is reached, the response of standpipe piezometer can be slow for less permeable ground.
- Other types of piezometers (i.e. pneumatic, hydraulic and vibrating wire), which involve much smaller amount of water in the measuring device, can be used. The maximum groundwater level in a standpipe piezometer can be measured by a string of small buckets (Halcrow buckets).

# Terms

- Piezometric level - Level of groundwater pressure indicated by piezometer
- Perched water table - A localized water table exists above the main groundwater table where a local reduction in basal permeability occurs in conjunction with recharge from above or drainage from below. Perched water table may be transient, developing rapidly in response to heavy rainfall and dissipating equally quickly, or more permanent, responding to seasonal variations in rainfall level.

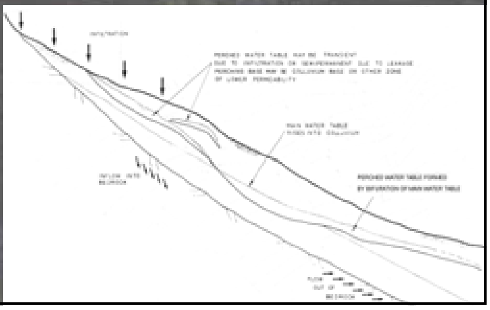
# Infiltration into Unsaturated Ground



- The ground above the phreatic surface is often unsaturated. The groundwater table may rise due to infiltration from the ground surface through the unsaturated soil into the aquifer.
- The intensity of surface infiltration is generally proportional to rainfall density but affected by the runoff characteristics of the ground.

# Hillslope Hydrogeology – A conceptual Model

- As part of the hydrogeological studies carried out by the GCO in the 1980's, a schematic hydrogeology of the hillside above Po Shan Road was established.
- The geological and hydrogeological conditions of each site are unique
- Make use this as our basic conceptual model for regional hydrogeology in hillslope areas of Hong Kong.



# Hillslope Hydrogeology – A conceptual Model

- permeability contrast in the ground give rise to a main inclined aquifer, together with areas of perched groundwater table
- The main aquifer is the colluvium/residual soil
- The 'bedrock' is relatively impermeable,
- however zones of high permeability within the top portion of the 'rockhead' due to concentrated groundwater flow, and due to fracturing of rocks close of faults or dykes.
- The source of water flow in this system is mainly from surface infiltration. However, the groundwater condition can be affected by inflow into the 'bedrock' and outflow from the 'bedrock

# Laboratory Tests on Shear Strength of Soils

# Advantages of Laboratory Testing

- Full control of the test conditions
- Greater degree of accuracy of measurements than field testing
- A test can be run under conditions, and changes in conditions can be simulated
- Tests can be carried out on soils which have been reconstituted, or processed in other ways

# Measurement and interpretation of soil shear strength from lab tests

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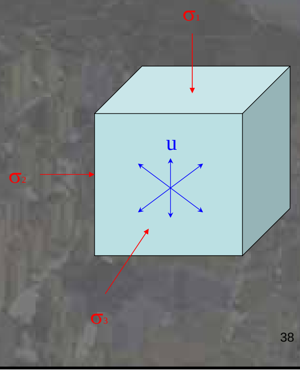
# Total Stress and Effective Stress



Total stresses : $\sigma_1 , \sigma_2 , \sigma_3$

Effective stresses :

$\sigma_1' = \sigma_1 - u$
$\sigma_2' = \sigma_2 - u$
$\sigma_3' = \sigma_3 - u$

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# Shear Strength of Soils
## General - 1

- **The following issues may cause difficulties to designers:**
  - Shear box tests vs triaxial tests
  - Drained tests vs undrained test
  - Total stress parameters vs effective stress parameters
  - CU, CD or UU
  - single or multi-stage
  - Interpretation of shear strength parameters using Mohr-Coulomb failure criterion
  - Interpretation of stress paths from triaxial tests
  - Specification of confining stresses or vertical stresses

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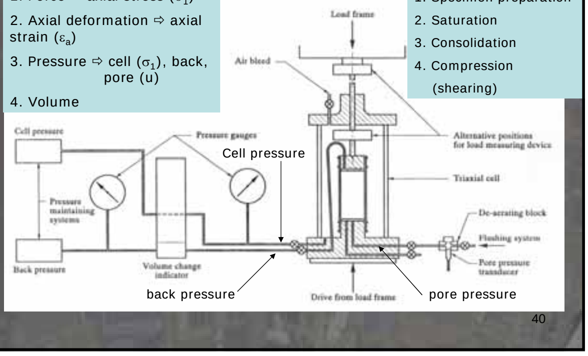
# Shear Strength of Soils
## Triaxial setup

1. Force ⇨ axial stress ($\sigma_1$)
2. Axial deformation ⇨ axial strain ($\varepsilon_a$)
3. Pressure ⇨ cell ($\sigma_1$), back, pore (u)
4. Volume

1. Specimen preparation
2. Saturation
3. Consolidation
4. Compression (shearing)



Cell pressure

back pressure

pore pressure

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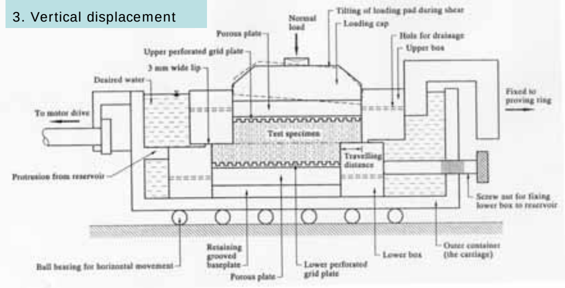
# Shear Strength of Soils
## Shear Box setup

1. Force ⇨ shear stress ($\tau$)
2. Horizontal displacement
3. Vertical displacement

Setup shown for 20mm thick specimen (Method A)



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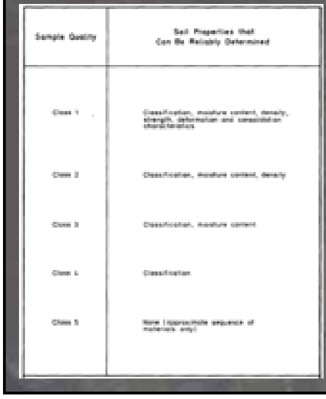
# Sample Class

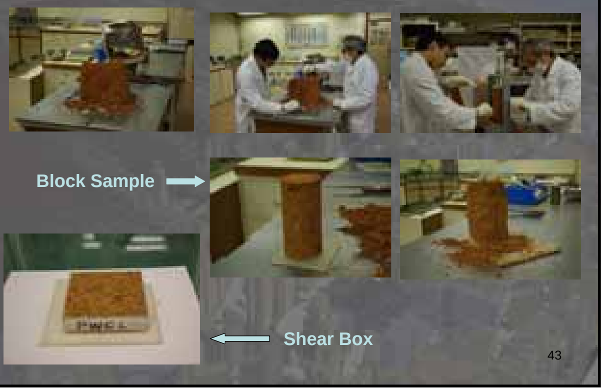


| Sample Quality | Soil Properties that Can Be Reliably Determined |
|---|---|
| Class 1 | Classification, moisture content, density, strength, deformation and consolidation characteristics |
| Class 2 | Classification, moisture content, density |
| Class 3 | Classification, moisture content |
| Class 4 | Classification |
| Class 5 | None (approximate sequence of materials only) |

| Material Type | Typical Composition of Materials | Sampling Procedure | Expected Quality Class |
|---|---|---|---|
| Soils derived from insitu rock weathering | Composition of soils varies depending on the nature of parent rock material. Soils derived from granitic rock are usually silty and clayey sands; soils derived from volcanic rock are usually sandy and clayey silts. | Block sample from dry excavation | 1 |
| | | Large diameter triple-tube core-barrel (102mm diameter cores) with retractor shoe, air-foam flush | 1 |
| | | Triple-tube core-barrel (≥ 76mm diameter cores) with retractor shoe | 1/2 |
| | | U100 sampler | 2/4 |
| | | SPT split barrel sampler with or without liner | 3/4 |
| | | Bulk samples and jar samples from dry open excavation | 3/4 |
| | | Light percussion shell and chisel for boulders | 5 |
| Colluvium | Fresh or variably decomposed rock fragments (boulders, cobbles and gravels) within a matrix of varying proportions of sand, silt and clay | The sampling procedures for soils derived from insitu rock weathering apply. | |
| Alluvial and marine deposits | The following materials can be found :<br>(a) Granular soils (sands, silty sands or sandy silts) | Piston sampler or compressed air sand sampler | 2/3 |
| | | U100 sampler (with core-catcher) | 4 |
| | | SPT split barrel sampler | 4 |
| | | Light percussion shell | 5 |
| | (b) Very soft to soft cohesive soils (sandy clays, silty clays or clays) | Piston sampler | 1 |
| | | Thin-walled sampler | 1/2 |
| | | U100 sampler | 2/3 |
| | | Delft continuous sampler | 2/3 |
| | | Light percussion clay cutter (dry boreholes) or shell (wet boreholes) | 4/5 |
| | (c) Firm to very stiff cohesive soils | Triple-tube core-barrel with retractor shoe | 1/2 |
| | | U100 sampler | 2/3 |
| | | Light percussion clay cutter | 5 |
| | (d) Cohesive and granular soils containing boulders, cobbles or gravel | The sampling procedures for soils derived from insitu rock weathering apply. | |
| Fill | Variable material, which can include compacted or uncompacted soil, rock fragments and building debris mixtures | See sampling procedures for relevant soil type and composition under "Alluvial and Marine Deposits" above. | |
| Rock | All rock types found in Hong Kong, including boulders in colluvium | Diamond core drilling with double or triple-tube core-barrel. The latter generally causes less disturbance and gives better core recovery, especially in highly fractured or jointed rocks. | N/A |

Notes : (1) The typical composition of materials should only be taken as a general guide.
(2) The quality classes are defined in Table 9.
(3) The expected quality classes given should only be taken as a guide, as sample quality is highly dependent on workmanship and on the compactness (or consistency) and grading of the soil.

## Sample Preparation



Block Sample

Shear Box

---

## Shear Strength of Soils
### Saturation

- Reasons for saturation in a triaxial test:
  - Provide reliable measurements of ***pore pressure*** (e.g. during consolidation stage and undrained shearing stage)
  - Provide reliable measurements of ***volume change*** (e.g. during consolidation stage and drained shearing stage)
- There is no saturation provision for routine shear box and oedometer tests. The specimen is soaked under a seating pressure and is ***assumed*** saturated.

---

## Shear Strength of Soils
### Saturation

- Use of ***carbon dioxide*** before the application of the above methods greatly enhance the saturation process.
- The ***direct application of back pressure*** is the recommended method in Geospec 3 and is also the most common method used in Hong Kong.
- The basic concept in saturation by using back pressure technique is to apply a sufficiently high pressure on the pore fluid to ***cause the pore air to dissolve*** completely into the pore water.
- Geospec 3 requires the B value be ≥ 0.95 ***and*** the back pressure ≥ 200 kPa as the criteria to signify the completion of the saturation stage.

---

## Shear Strength of Soils
### Saturation

- B value is defined as : $B = \frac{\delta u}{\delta \sigma_3}$

where $\delta u$ is the change in pore pressure and $\delta \sigma_3$ is the change in cell pressure

- The B value gives an indication of the degree of saturation (S) of the soil specimen but B itself is ***not*** a direct parameter to measure S.
- What is the degree of saturation of a soil for a particular B value (say B = 0.95) ?

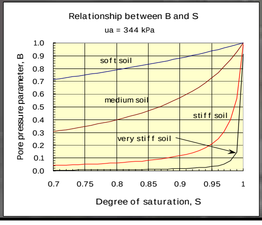
---

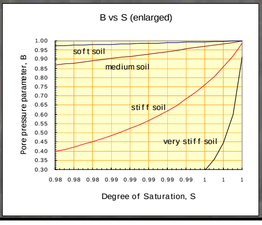
## Shear Strength of Soils
### Saturation

$$B = \frac{C_s}{C_s + nC_w S + n\frac{1-S}{u_a}}$$

$C_s$ = soil compressibility
$C_w$ = water compressibility
$n$ = soil porosity
$u_a$ = absolute air pressure

- Four different soils are used to illustrate the relationship of B and S (Black and Lee 1973):





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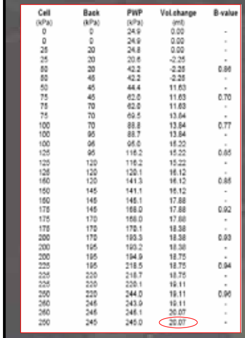
## Shear Strength of Soils
### Saturation

- The compressibility of some common Hong Kong soils back calculated from the consolidation stage of triaxial tests are given below:

| Material | CDG | CDV | Marine deposit | Fill |
|---|---|---|---|---|
| Compressibility ($m^2$ / MN) | 0.04 – 0.7 | 0.03 – 0.7 | 0.06 - 4 | 0.06 - 2 |

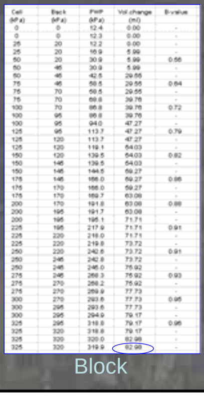
- In general, the B ≥ 0.95 criterion is good enough for most of the soils except soft soils. However, most of the soft marine or alluvial deposits are almost fully saturated when they are sent to the laboratory.
- It is also recommended to use a higher B criterion such as 0.97 for triaxial test of ***loose soil*** (such as loose fill)

# Shear Strength of Soils
## Saturation



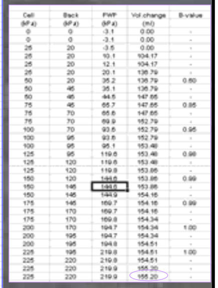
| Cell (kPa) | Back (kPa) | PWP (kPa) | Vol.change (ml) | B-value |
|---|---|---|---|---|
| 0 | 0 | 24.9 | 0.00 | - |
| 0 | 0 | 24.9 | 0.00 | - |
| 25 | 20 | 24.8 | 0.00 | - |
| 25 | 20 | 20.6 | -2.25 | - |
| 50 | 20 | 42.2 | -2.25 | 0.86 |
| 50 | 45 | 42.2 | -2.25 | - |
| 50 | 45 | 44.4 | 11.63 | - |
| 75 | 45 | 62.0 | 11.63 | 0.70 |
| 75 | 70 | 62.0 | 11.63 | - |
| 75 | 70 | 69.5 | 13.84 | - |
| 100 | 70 | 88.8 | 13.84 | 0.77 |
| 100 | 95 | 88.7 | 13.84 | - |
| 100 | 95 | 95.0 | 15.22 | - |
| 125 | 95 | 116.2 | 15.22 | 0.85 |
| 125 | 120 | 116.2 | 15.22 | - |
| 125 | 120 | 120.1 | 16.12 | - |
| 150 | 120 | 141.3 | 16.12 | 0.85 |
| 150 | 145 | 141.1 | 16.12 | - |
| 150 | 145 | 146.1 | 17.88 | - |
| 175 | 145 | 168.0 | 17.88 | 0.92 |
| 175 | 170 | 168.0 | 17.88 | - |
| 175 | 170 | 170.1 | 18.38 | - |
| 200 | 170 | 193.3 | 18.38 | 0.93 |
| 200 | 195 | 193.2 | 18.38 | - |
| 200 | 195 | 194.9 | 18.75 | - |
| 225 | 195 | 218.5 | 18.75 | 0.94 |
| 225 | 220 | 218.7 | 18.75 | - |
| 225 | 220 | 220.1 | 19.11 | - |
| 250 | 220 | 244.0 | 19.11 | 0.96 |
| 250 | 245 | 243.9 | 19.11 | - |
| 250 | 245 | 245.1 | 20.07 | - |
| 250 | 245 | 245.0 | 20.07 | - |

Mazier



| Cell (kPa) | Back (kPa) | PWP (kPa) | Vol.change (ml) | B-value |
|---|---|---|---|---|
| 0 | 0 | 12.4 | 0.00 | - |
| 0 | 0 | 12.3 | 0.00 | - |
| 25 | 20 | 12.2 | 0.00 | - |
| 25 | 20 | 16.9 | 5.99 | - |
| 50 | 20 | 30.9 | 5.99 | 0.56 |
| 50 | 45 | 30.9 | 5.99 | - |
| 50 | 45 | 42.5 | 29.55 | - |
| 75 | 45 | 58.5 | 29.55 | 0.64 |
| 75 | 70 | 58.5 | 29.55 | - |
| 75 | 70 | 69.8 | 39.76 | - |
| 100 | 70 | 86.8 | 39.76 | 0.72 |
| 100 | 95 | 86.8 | 39.76 | - |
| 100 | 95 | 94.0 | 47.27 | - |
| 125 | 95 | 113.7 | 47.27 | 0.79 |
| 125 | 120 | 113.7 | 47.27 | - |
| 125 | 120 | 119.1 | 64.03 | - |
| 150 | 120 | 139.5 | 64.03 | 0.82 |
| 150 | 145 | 139.5 | 64.03 | - |
| 150 | 145 | 144.5 | 69.27 | - |
| 175 | 145 | 166.0 | 69.27 | 0.86 |
| 175 | 170 | 166.0 | 69.27 | - |
| 175 | 170 | 169.7 | 63.08 | - |
| 200 | 170 | 191.8 | 63.08 | 0.88 |
| 200 | 195 | 191.7 | 63.08 | - |
| 200 | 195 | 195.1 | 71.71 | - |
| 225 | 195 | 217.9 | 71.71 | 0.91 |
| 225 | 220 | 218.0 | 71.71 | - |
| 225 | 220 | 219.8 | 73.72 | - |
| 250 | 220 | 242.6 | 73.72 | 0.91 |
| 250 | 245 | 242.8 | 73.72 | - |
| 250 | 245 | 246.0 | 75.92 | - |
| 275 | 245 | 268.3 | 75.92 | 0.93 |
| 275 | 270 | 268.2 | 75.92 | - |
| 275 | 270 | 269.9 | 77.73 | - |
| 300 | 270 | 293.6 | 77.73 | 0.95 |
| 300 | 295 | 293.6 | 77.73 | - |
| 300 | 295 | 294.9 | 79.17 | - |
| 325 | 295 | 318.8 | 79.17 | 0.96 |
| 325 | 320 | 318.8 | 79.17 | - |
| 325 | 320 | 320.0 | 82.98 | - |
| 325 | 320 | 319.9 | 82.98 | - |

Block



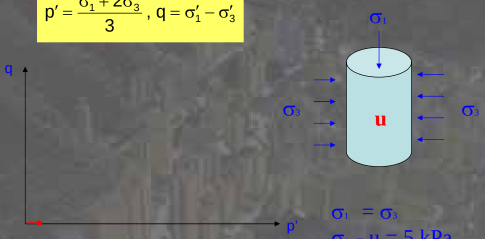
| Cell (kPa) | Back (kPa) | PWP (kPa) | Vol.change (ml) | B-value |
|---|---|---|---|---|
| 0 | 0 | -3.1 | 0.00 | - |
| 0 | 0 | -3.1 | 0.00 | - |
| 25 | 20 | -3.5 | 0.00 | - |
| 25 | 20 | 10.1 | 104.17 | - |
| 25 | 20 | 12.1 | 104.17 | - |
| 25 | 20 | 20.1 | 136.79 | - |
| 50 | 20 | 35.2 | 136.79 | 0.60 |
| 50 | 45 | 35.1 | 136.79 | - |
| 50 | 45 | 44.5 | 147.65 | - |
| 75 | 45 | 65.7 | 147.65 | 0.86 |
| 75 | 70 | 65.6 | 147.65 | - |
| 75 | 70 | 69.9 | 152.79 | - |
| 100 | 70 | 93.6 | 152.79 | 0.95 |
| 100 | 95 | 93.6 | 152.79 | - |
| 100 | 95 | 95.1 | 153.48 | - |
| 125 | 95 | 119.6 | 153.48 | 0.98 |
| 125 | 120 | 119.6 | 153.48 | - |
| 125 | 120 | 119.8 | 153.86 | - |
| 150 | 120 | 144.6 | 153.86 | 0.99 |
| 150 | 145 | 144.6 | 153.86 | - |
| 150 | 145 | 144.9 | 154.16 | - |
| 175 | 145 | 169.7 | 154.16 | 0.99 |
| 175 | 170 | 169.7 | 154.16 | - |
| 175 | 170 | 169.8 | 154.34 | - |
| 200 | 170 | 194.7 | 154.34 | 1.00 |
| 200 | 195 | 194.7 | 154.34 | - |
| 200 | 195 | 194.8 | 154.51 | - |
| 225 | 195 | 219.8 | 154.51 | 1.00 |
| 225 | 220 | 219.8 | 154.51 | - |
| 225 | 220 | 219.9 | 155.20 | - |
| 225 | 220 | 219.9 | 155.20 | - |

Remoulded with $CO_2$ flushing

---

# Shear Strength of Soils
## Saturation

$$p' = \frac{\sigma'_1 + 2\sigma'_3}{3}, \; q = \sigma'_1 - \sigma'_3$$



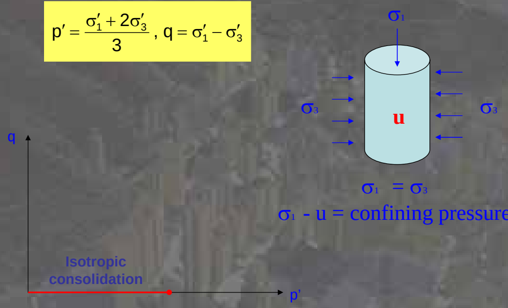
$\sigma_1 = \sigma_3$

$\sigma_1 - u = 5$ kPa

---

# Shear Strength of Soils
## Consolidation

$$p' = \frac{\sigma'_1 + 2\sigma'_3}{3}, \; q = \sigma'_1 - \sigma'_3$$



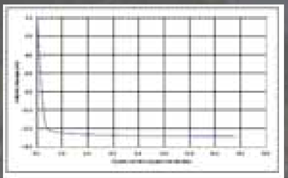
$\sigma_1 = \sigma_3$

$\sigma_1 - u =$ confining pressure

---

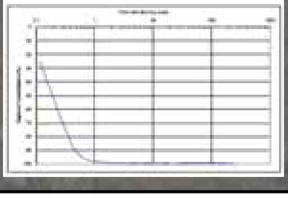
# Shear Strength of Soils
## Consolidation

Pore water pressure



→ Excess PWP fully dissipated

Volume change



→ Approaching zero

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# Shear Strength of Soils
## Shearing

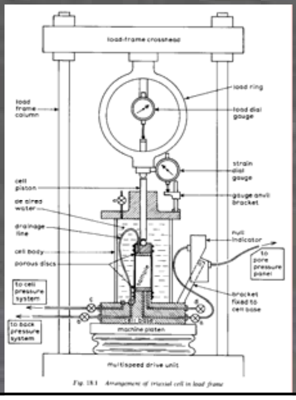


*Fig. 18.1 Arrangement of triaxial cell in load frame*

---

# Shear Strength of Soils
## Shearing

**Effective stress**

$\sigma_1' = \sigma_1 - u$

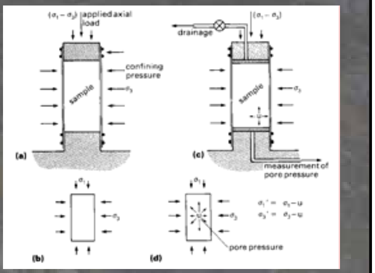
$\sigma_2' = \sigma_3'$

**Deviator stress**

$\sigma_1' - \sigma_3' = \sigma_1 - \sigma_3$

**Stress ratio**

$\sigma_1'/\sigma_3'$



# Shear Strength of Soils
## Shearing

- Geospec 3 requires the following 3 graphs be plotted for drained shearing:

  (1) $\sigma_1 - \sigma_3$ vs $\varepsilon_a$ (2) $\sigma'_1/\sigma'_3$ vs $\varepsilon_a$ (3) $\varepsilon_v$ vs $\varepsilon_a$

- Two stress path plots are also specified by Geospec 3 for CU and CD tests:

  (1) t vs s' (2) q vs p'

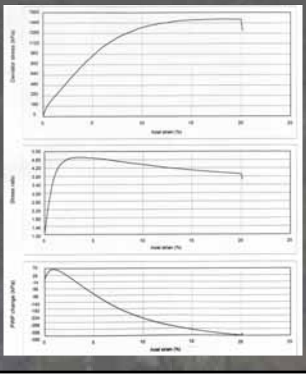
$$s' = \frac{\sigma'_1 + \sigma'_3}{2},\ t = \frac{\sigma'_1 - \sigma'_3}{2}$$

$$p' = \frac{\sigma'_1 + 2\sigma'_3}{3},\ q = \sigma'_1 - \sigma'_3$$

- Typical stress paths during shearing for a dilating and a contracting soil in CU tests are shown

---

# Shear Strength of Soils
## Shearing



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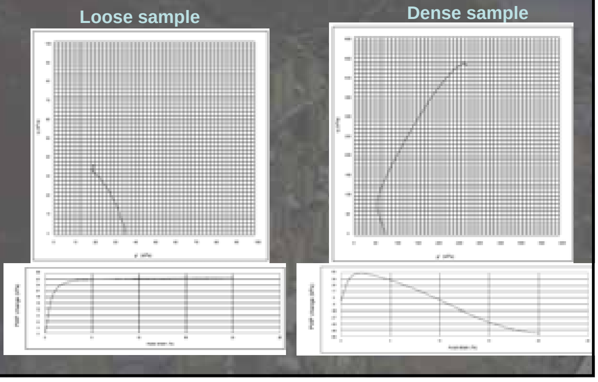
# Shear Strength of Soils
## Shearing



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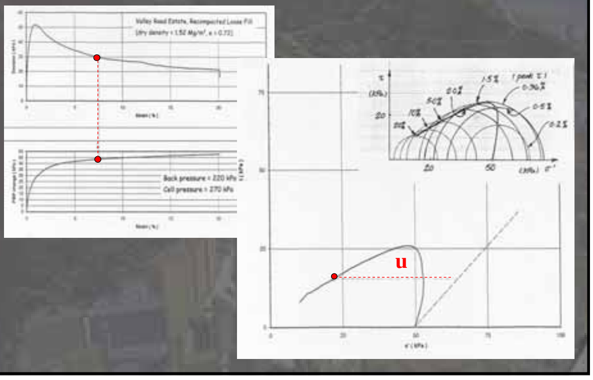
# Shear Strength of Soils
## Shearing



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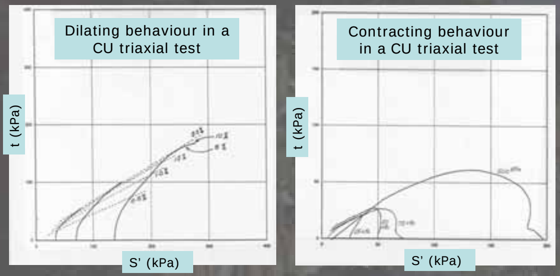
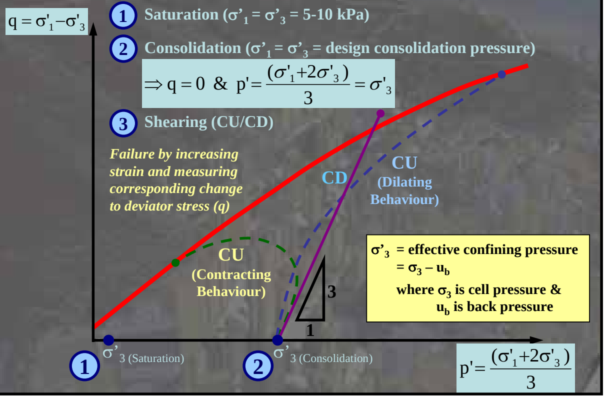
# Shear Strength of Soils
## Shearing



- For CD test, the stress path during shearing will be a straight line with a slope = 1 in s'-t plot and a slope = 3 in p'-q plot

---

# Shear Strength of Soils
## Shearing



① Saturation ($\sigma'_1 = \sigma'_3$ = 5-10 kPa)

② Consolidation ($\sigma'_1 = \sigma'_3$ = design consolidation pressure)

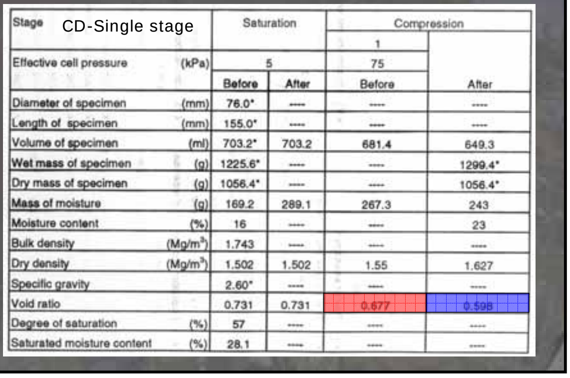
$$\Rightarrow q = 0\ \&\ p' = \frac{(\sigma'_1 + 2\sigma'_3)}{3} = \sigma'_3$$

③ Shearing (CU/CD)

*Failure by increasing strain and measuring corresponding change to deviator stress (q)*

**$\sigma'_3$ = effective confining pressure**
**$= \sigma_3 - u_b$**
**where $\sigma_3$ is cell pressure & $u_b$ is back pressure**

## Shear Strength of Soils
### Shearing

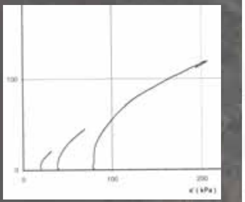


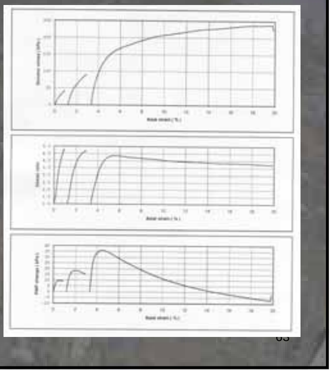
| Stage CD-Single stage | | Saturation | | Compression | |
|---|---|---|---|---|---|
| | | | | 1 | |
| Effective cell pressure | (kPa) | 5 | | 75 | |
| | | Before | After | Before | After |
| Diameter of specimen | (mm) | 76.0* | ---- | ---- | ---- |
| Length of specimen | (mm) | 155.0* | ---- | ---- | ---- |
| Volume of specimen | (ml) | 703.2* | 703.2 | 681.4 | 649.3 |
| Wet mass of specimen | (g) | 1225.6* | ---- | ---- | 1299.4* |
| Dry mass of specimen | (g) | 1056.4* | ---- | ---- | 1056.4* |
| Mass of moisture | (g) | 169.2 | 289.1 | 267.3 | 243 |
| Moisture content | (%) | 16 | ---- | ---- | 23 |
| Bulk density | (Mg/m³) | 1.743 | ---- | ---- | ---- |
| Dry density | (Mg/m³) | 1.502 | 1.502 | 1.55 | 1.627 |
| Specific gravity | | 2.60* | ---- | ---- | ---- |
| Void ratio | | 0.731 | 0.731 | 0.677 | 0.598 |
| Degree of saturation | (%) | 57 | ---- | ---- | ---- |
| Saturated moisture content | (%) | 28.1 | ---- | ---- | ---- |

## Shear Strength of Soils
### Shearing

## Shear Strength of Soils
### Shearing

**Multi-stage shearing**





**Deviator stress**

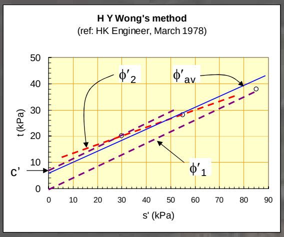
$\sigma_1' - \sigma_3' = \sigma_1 - \sigma_3$

**Stress ratio**

$\sigma_1'/\sigma_3'$

**Max.**

## Shear Strength of Soils
### Shearing



H Y Wong's method on the intrepretation of multi-stage triaxial test data

Assume at the 3rd stage of shearing, the cohesion term at failure is zero ⇨ $\phi'_1$

Using this $\phi'_1$ value and the data point obtained from the 1st stage to obtain c'

Using the first and second stage results, obtain another $\phi'_2$

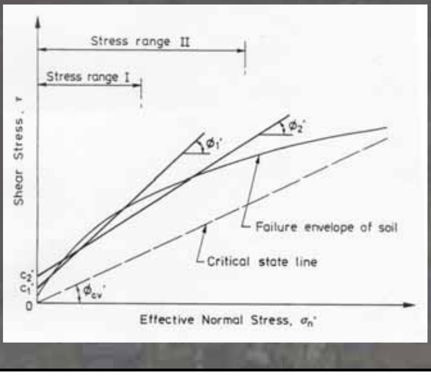
Average the two $\phi'$ values to give the most probable $\phi'_{av}$ of the soil

Using the data from the previous slide, Wong's method gives a c' = 6.4 kPa and $\phi'$ = 39.6°

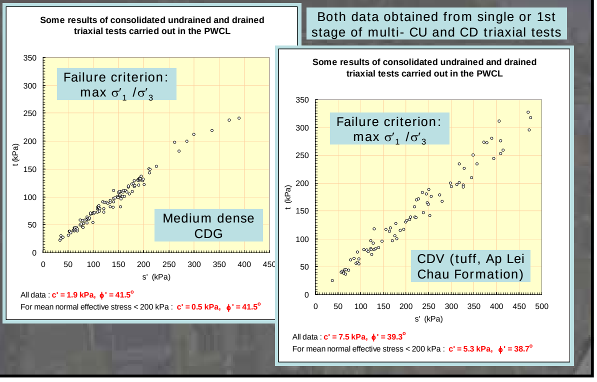
## Shear Strength of Soils
### Shearing

**Multi-stage Vs Single-stage**

## Shear Strength of Soils
### Shearing



# Shear Strength of Soils
## Shearing

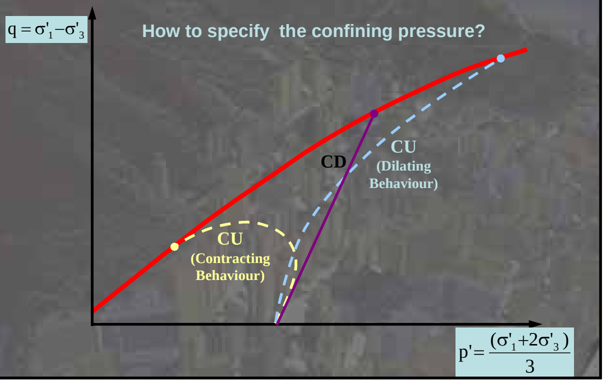


Both data obtained from single or 1st stage of multi- CU and CD triaxial tests

---

# Shear Strength of Soils
## Shearing

How to specify the confining pressure?



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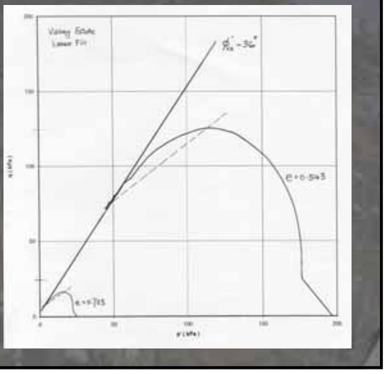
# Shear Strength of Soils
## Shearing

**Very loose fill with collapsible behaviour**



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# Shear Strength of Soils
## Shear Box test

- Geospec 3 provides three types of shear box tests:
  - Test Method 16.1 - small shear box 60mm or 100mm square (Method A with specimen thickness = 20mm)
  - Test Method 16.1 - small shear box 60mm or 100mm square (Method B with specimen thickness = 44mm)
  - Test Method 16.2 - large shear box 300mm square
- The shear box apparatus can normally be used only for carrying out ***drained*** tests.

---

# Shear Strength of Soils
## Shear Box test

- The shear box test is particularly useful in the following cases:
  - Determination of shear strength of compacted fill
  - Determination of friction between fill and geo-reinforcement
  - Determination of shear strength of clay infill
  - Determination of residual strength (multi-reversal method)

---

# Shear Strength of Soils
## Shear Box test

# Shear Strength of Soils
## Shear Box test

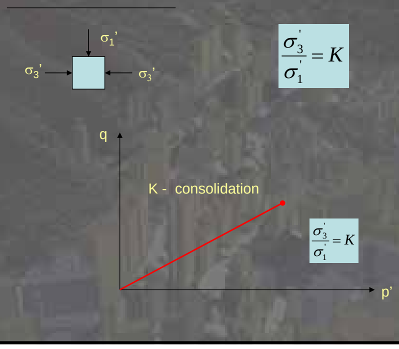
- The limitations of shear box are well known, e.g.
  - Soil specimen is constrained to fail along a predetermined plane of shear
  - Distribution of stresses on this surface is not uniform
  - No control can be exercised over drainage
  - Pore water pressures cannot be measured
- Other drawbacks include no effective saturation of the soil specimen, tilting of the top cap during shear etc.

# Shear Strength of Soils
## Miscellaneous

- Effects of cobbles and boulders on shear strength in colluvium:
  (a) no practical increase in shear strength occurs up to a coarse fraction content of about 25% (coarse fraction means particle size >60mm)
  (b) With coarse fraction varies from 25% to 60%, an increase of 4° for every 10% increase in coarse fraction is recommended in GEO report No. 23
- Refer to this Report on limitations and more discussion on the above recommendation.
- The practical difficulties in using the recommendation is the uncertainty of the % of coarse fraction on site.

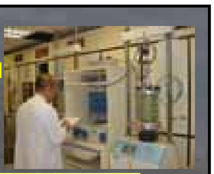
# Recent Development of Some Advanced Testing Techniques in PWCL

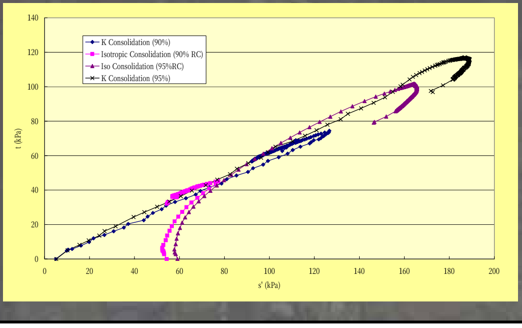
## Isotropic Confining Pressure?
K and Ko consolidation



$$\frac{\sigma_3'}{\sigma_1'} = K$$

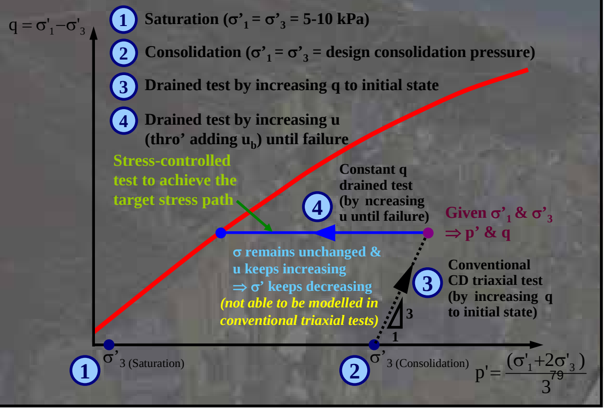
## K – consolidation using Conventional Triaxial Testing Apparatus





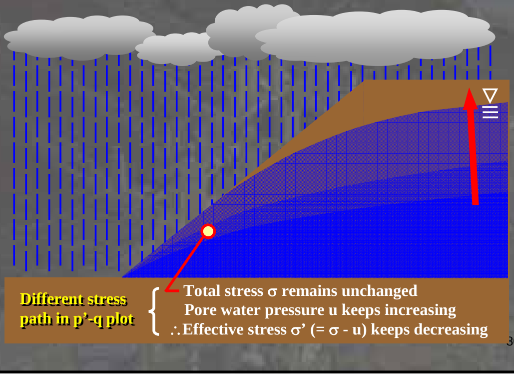
## GDS Triaxial Testing Apparatus

## In p’-q plot (Constant-q Test)



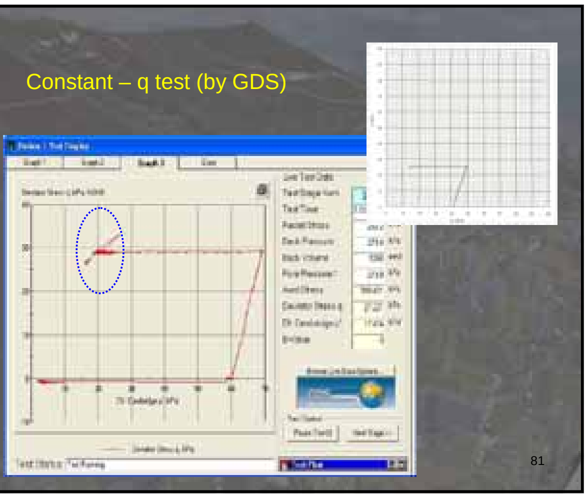
## Constant – q test

1) Slope failure induced by rainfall
2) Slope failure induced by rise in groundwater table

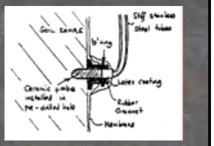


**Different stress path in p’-q plot**
Total stress σ remains unchanged
Pore water pressure u keeps increasing
∴Effective stress σ’ (= σ - u) keeps decreasing

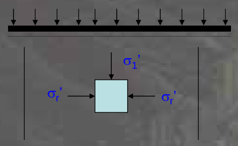
## Constant – q test (by GDS)



## Local Axial, Radial and Pore Pressure Transducers

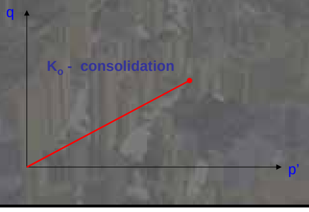


## Ko consolidation

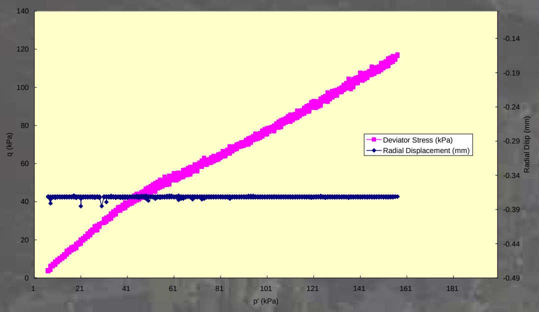


$$\frac{\sigma_3'}{\sigma_1'} = K_O$$

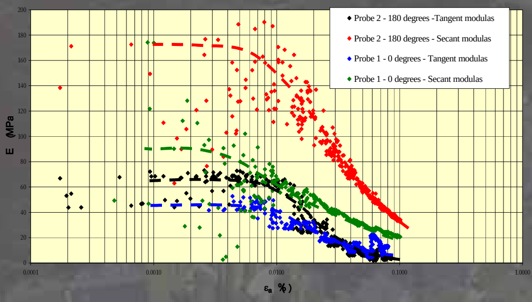
$$\Delta\varepsilon_r = 0$$



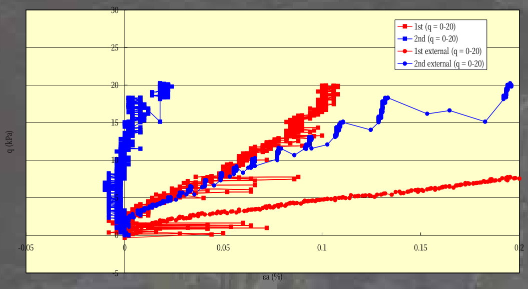
## Ko consolidation
## q value and radial displacement



## Small strain modulus



## Small strain modulus – GDS testing



## References

(a) Manual of Soil Laboratory Testing, vols. 1 to 3 – K.H. Heads

(b) Geospec 3 : Model Specification for Soil Testing

(c) An Introduction to the Mechanics of Soils and Foundations – John Atkinson

## Questions?

## BREAK

# Slope Stability Analysis

# Outline

- Fundamentals of LEA
- Selection of an appropriate method
- Determination of critical slip surface

# Slope Stability Analysis

A theoretical solution must satisfy:

- **Equilibrium**
- **Compatibility**
- **Material Constitutive Behaviour**
- **Boundary Conditions**

# Background

- Use of LEA for stability analysis is the oldest and best known numerical technique in geotechnical engineering
- The idea of dividing a potential sliding mass into slices dates back to the 1930s

  *Calculation of the stability of earth dams – by Fellenius, W. (1936). Trans. 2nd Congress on Large Dams, 4:445.*

- LEA with method of slices is now routinely used in practice

# Fundamentals

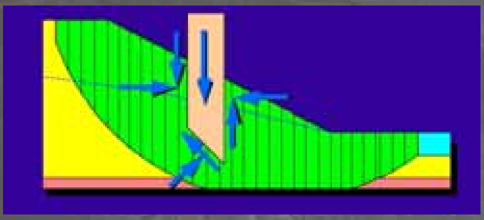
| Method of Analysis | Equilibrium | Compatibility | Constitutive Behaviour | Boundary Conditions | |
|---|---|---|---|---|---|
| | | | | Force | Disp. |
| Limit Equilibrium | S | NS | Rigid with a failure criterion | S | NS |

# Fundamentals

- Formulation in SLOPE/W is based on LEA
- Forces/moments on a free body are such that it remains stationary
- Satisfy statics

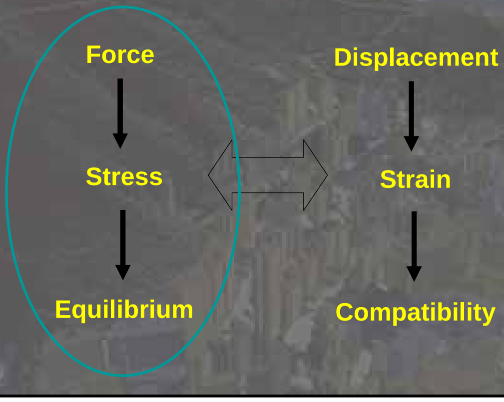
  Summation of moments, horizontal and vertical forces is zero

- FoS is constant along slip
- Some kind of assumed interslice force function required to satisfy all equations of statics



The key issue is to deal with interslice shear and normal forces

## Principle of Mechanics



Force

Stress

Equilibrium

Displacement

Strain

Compatibility

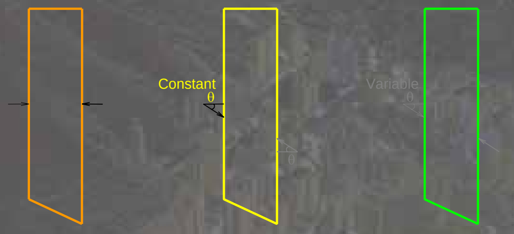
## Equation Associated with Method of Slices (*n* Slices)

| Condition | Equations |
|---|---|
| Moment equilibrium for each slice | *n* |
| Force equilibrium in two direction for each slice | 2*n* |
| Mohr-Coulomb failure criteria | *n* |
| **Total number of equations** | **4*n*** |

## Unknowns Associated with Method of Slices (*n* Slices)

| Variables | Unknowns |
|---|---|
| FoS | 1 |
| Normal force at base of each slice | *n* |
| Location of normal force | *n* |
| Shear force at base of each slice | *n* |
| Interslice force | *n* – 1 |
| Inclination of interslice force | *n* – 1 |
| Location of interslice force (line of thrust) | *n* – 1 |
| **Total number of unknowns** | **6*n* – 2** |

## Differences in Assumptions Regarding Side Forces in Common Methods of Slope Stability Analysis

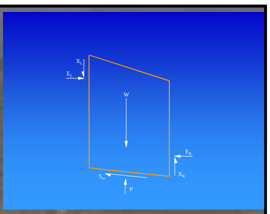


Bishop's simplified method & Janbu simplified method

Spencer's method (SLOPE/W V.6)

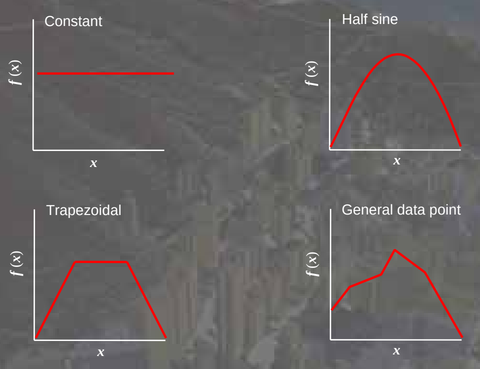
Morgenstern and Price method

## Interslice Force Equation

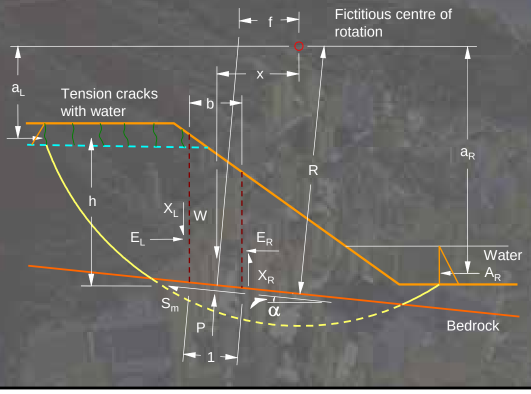


$$X = E\lambda f(x)$$

- X - interslice shear
- E - interslice normal
- λ - scaling factor (portion of function used)
- f(x) - a function

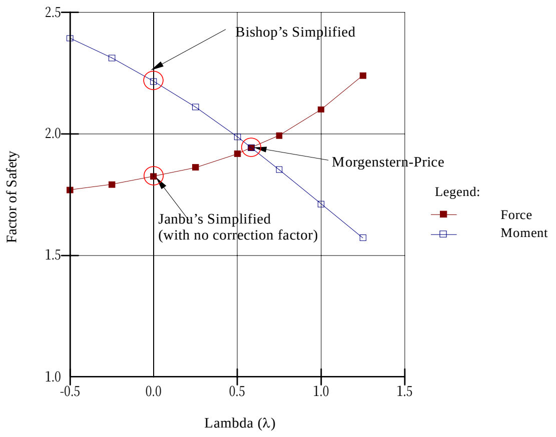


## General Limit Equilibrium (GLE)



$$F_m = \frac{\Sigma\left[c'\ell R + (P - u\ell)\,R\tan\phi'\right]}{\Sigma Wx - \Sigma Pf \pm Aa}$$

$$F_f = \frac{\Sigma\left[c'\ell\cos\alpha + (P - u\ell)\tan\phi'\cos\alpha\right]}{\Sigma P\sin\alpha \pm A}$$



## Methods of Slices

- **Ordinary or Fellenius**
- **Bishop's Simplified (BS)**
- **Janbu's Simplified (with and without correction factor)**
- **Janbu's Generalized**
- **Spencer**
- **Morgenstern and Price**
- **Generalized Limit Equilibrium**
- **Corps of Engineers 1 & 2**
- **Lowe-Karafiath**
- **Sarma**

## Elements of Statical Equilibrium Satisfied by Various Limit Equilibrium Methods

| Method | Force Equilibrium | | Moment Equilibrium |
|---|---|---|---|
| | Vertical | Horizontal | |
| Bishop's Simplified | Yes | No | Yes |
| Janbu's Simplified | Yes | Yes | No |
| Janbu's Generalized | Yes | Yes | ** |
| Spencer | Yes | Yes | Yes |
| Morgenstern-Price | Yes | Yes | Yes |
| GLE | Yes | Yes | Yes |

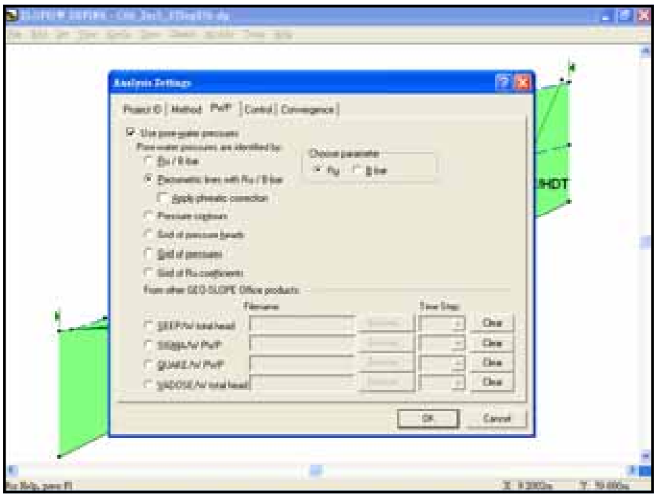
** Moment equilibrium is used to calculate interslice shear forces.

## Characteristics of Limit Equilibrium Method of Slices

- Adopt the same definition of FoS

  FoS = Shear strength of soil / Shear stress required for equilibrium

- Independency of stress-strain relationship
- Use equations of equilibrium to calculate the average value of shear stress along the potential slip surface and the normal stress at the base of the slip
- Make assumptions to handle the problem of indeterminacy



☑ Use pore-water pressures

Pore-water pressures are identified by:

○ Ru / B-bar

● Piezometric lines with Ru / B-bar

☐ Apply phreatic correction

○ Pressure contours

○ Grid of pressure heads

○ Grid of pressures

○ Grid of Ru coefficients

Choose parameter: ● Ru ○ B-bar

From other GEO-SLOPE Office products:

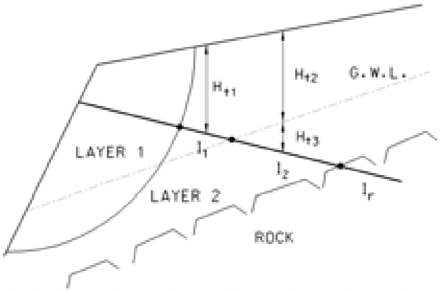
○ SEEP/W total head

○ SIGMA/W PWP

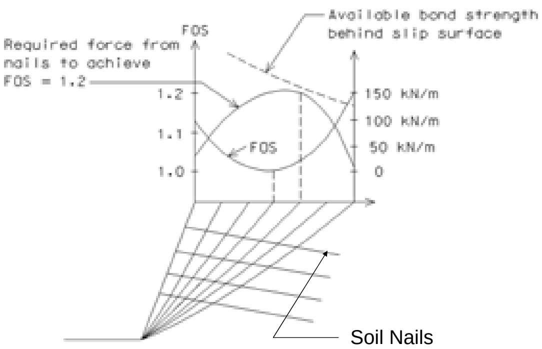
○ QUAKE/W PWP

○ VADOSE/W total head

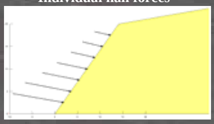
## FAILURE MODES

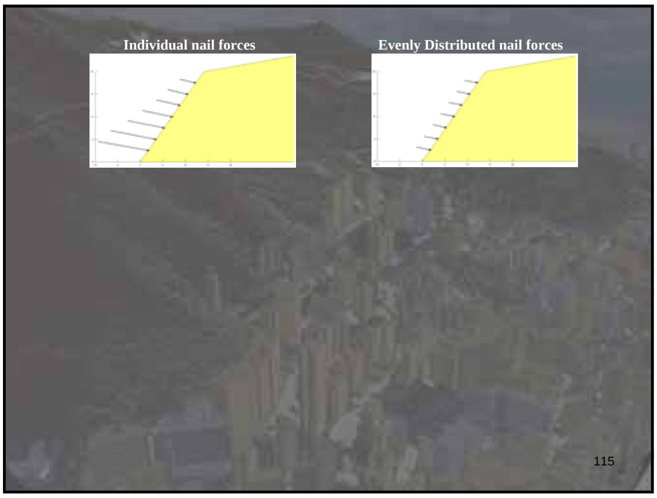


| Pull out resistance for a soil nail Ta is determined as :- | | | |
|---|---|---|---|
| in soil | Ts | = | $0.5 \times l \times [\pi \times (D/1000) \times c' + 2 \times (D/1000) \times \sigma_v' \times \tan\phi')]$ |
| in rock | Tr | = | R $\times \pi \times D/1000 \times l_r$ |
| | Ta | = | Ts + Tr OR Ta = F (maximum available force per steel bar) |
| | | | , whichever is the less |

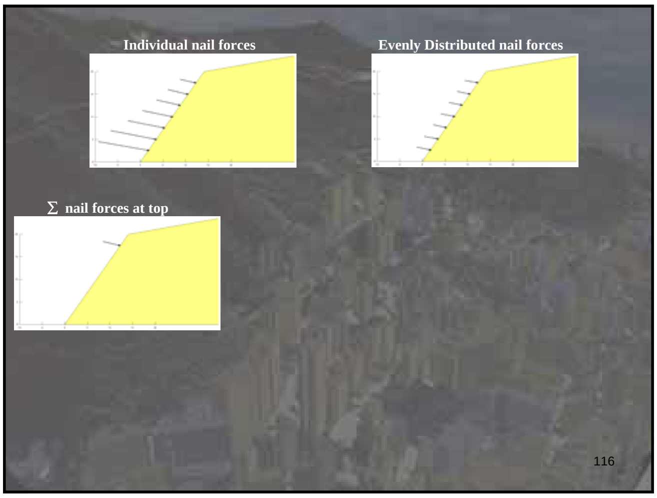


Individual nail forces



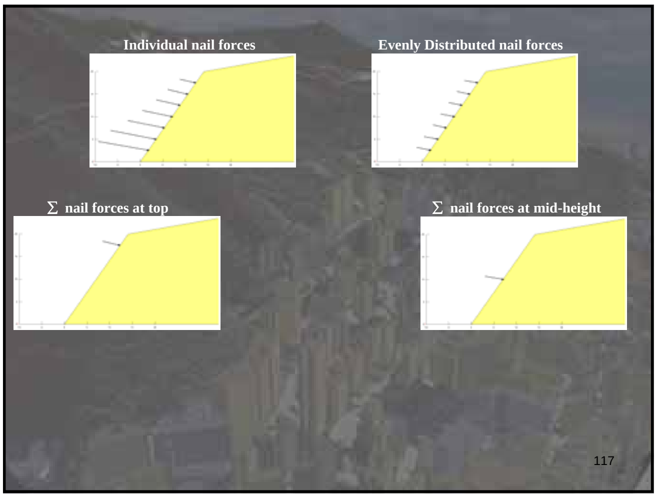

**Individual nail forces**

**Evenly Distributed nail forces**


**Individual nail forces**

**Evenly Distributed nail forces**

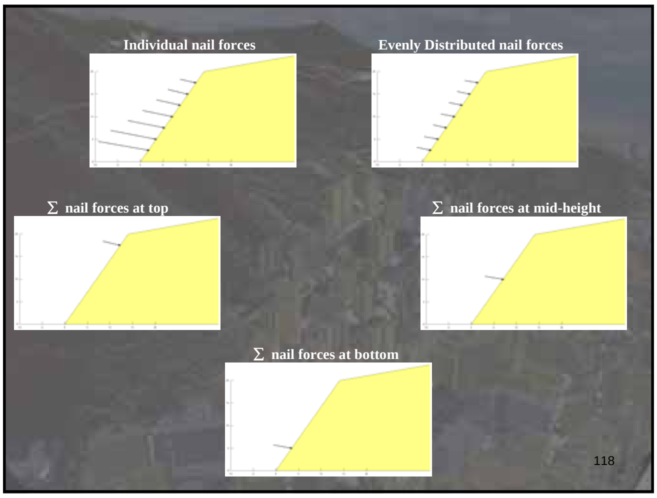
**∑ nail forces at top**


**Individual nail forces**

**Evenly Distributed nail forces**

**∑ nail forces at top**

**∑ nail forces at mid-height**

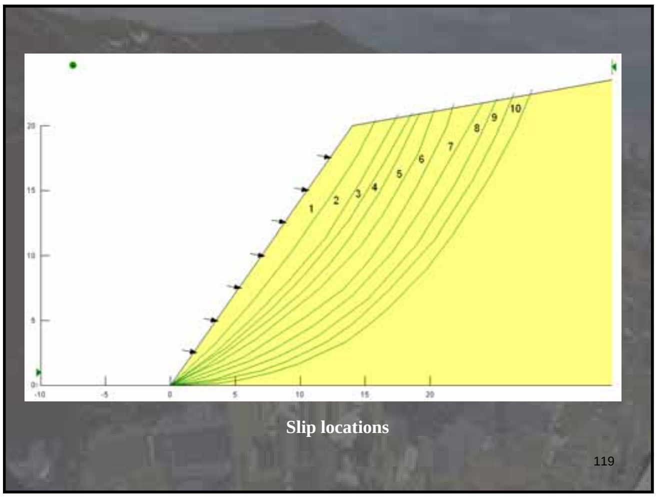

**Individual nail forces**

**Evenly Distributed nail forces**

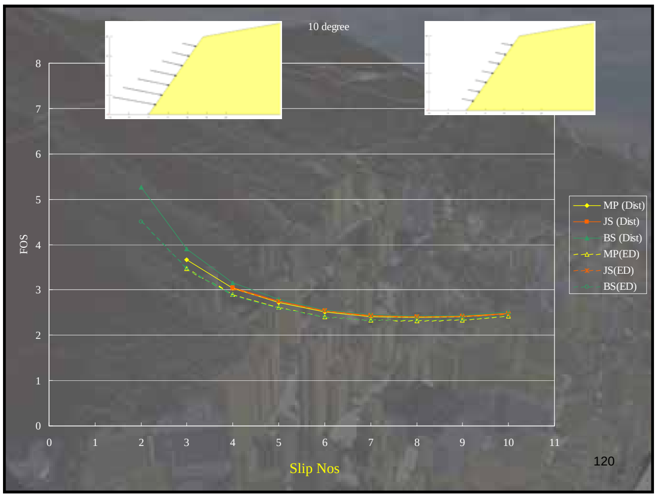
**∑ nail forces at top**

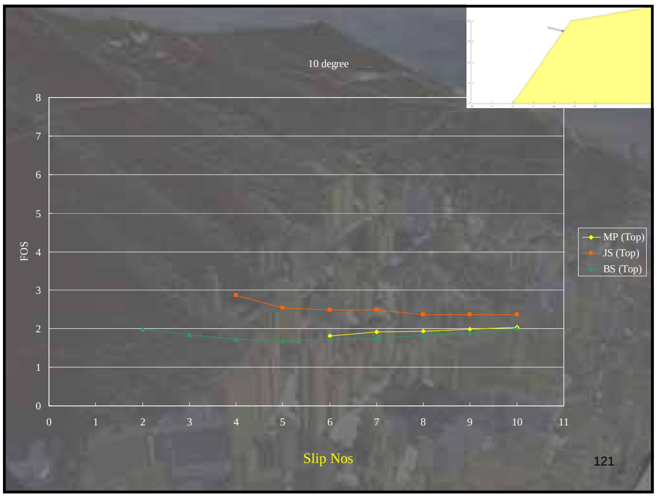
**∑ nail forces at mid-height**

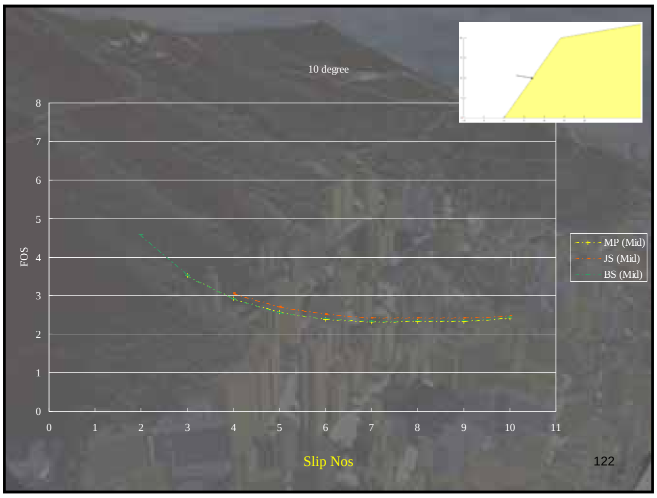
**∑ nail forces at bottom**

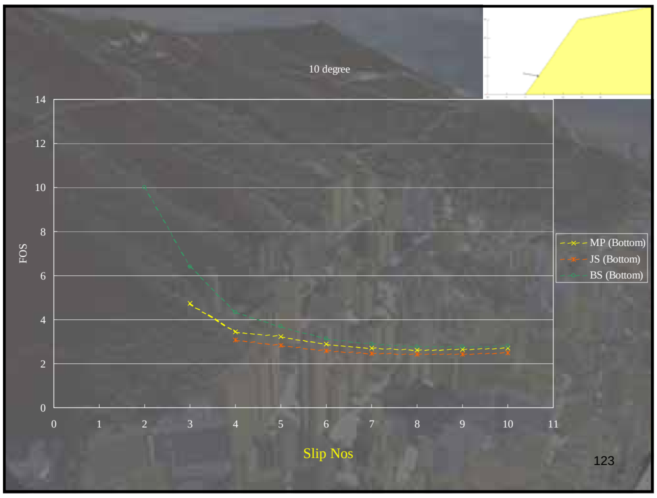


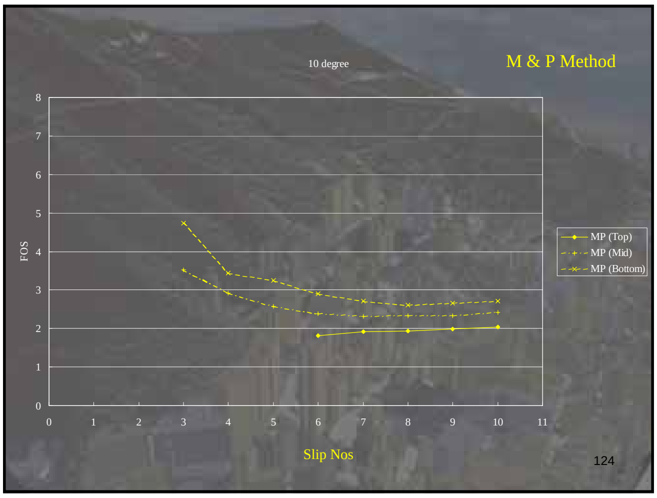
**Slip locations**

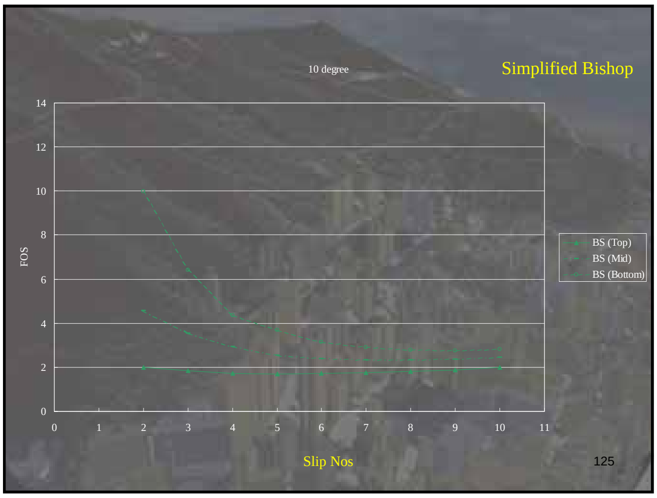

10 degree

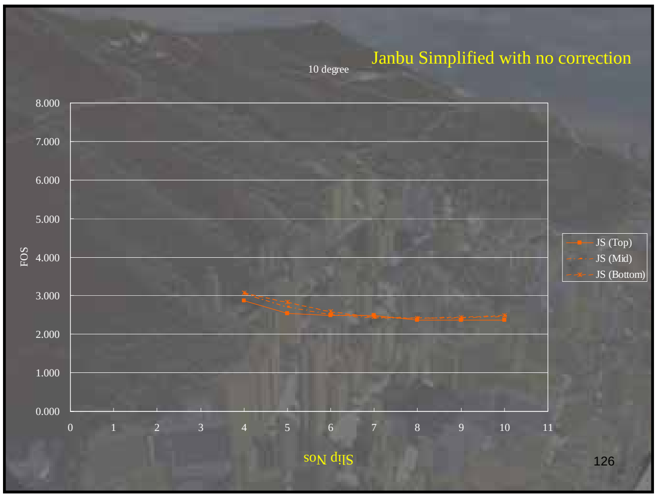


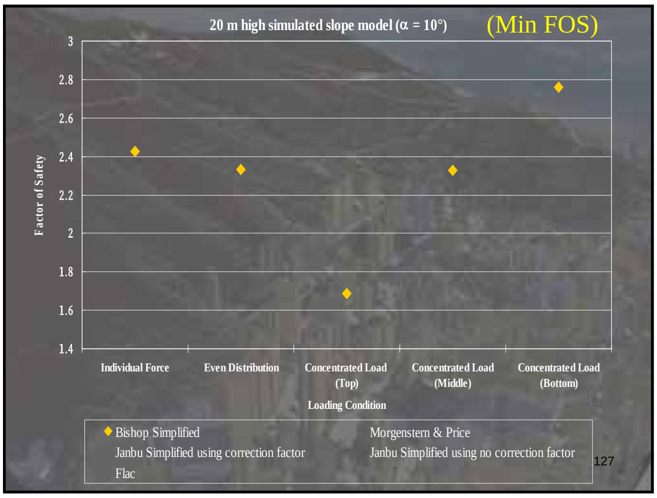


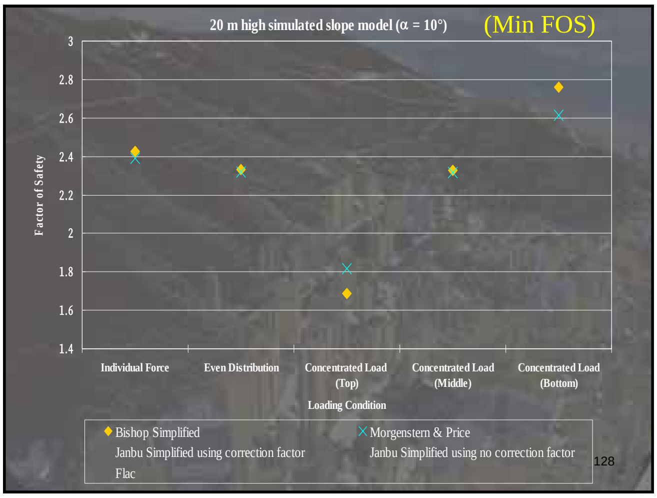



M & P Method


Simplified Bishop


Janbu Simplified with no correction


20 m high simulated slope model (α = 10°)
(Min FOS)
Factor of Safety
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
Individual Force
Even Distribution
Concentrated Load (Top)
Concentrated Load (Middle)
Concentrated Load (Bottom)
Loading Condition
Bishop Simplified
Morgenstern & Price
Janbu Simplified using correction factor
Janbu Simplified using no correction factor
Flac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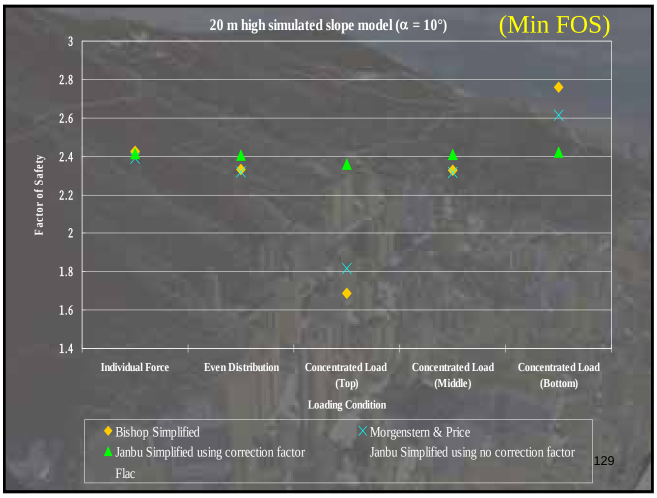
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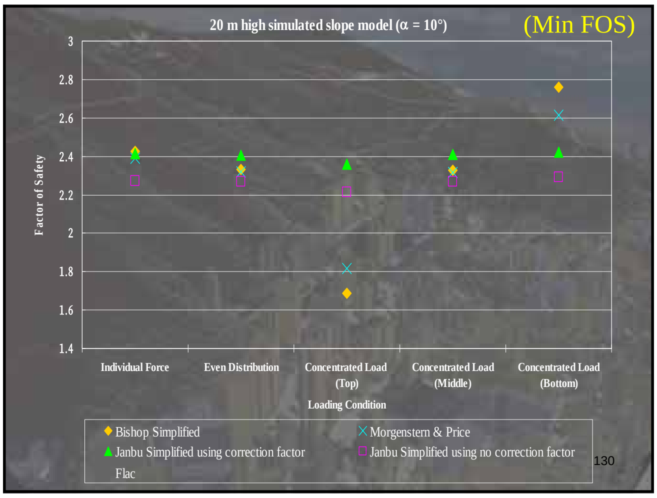
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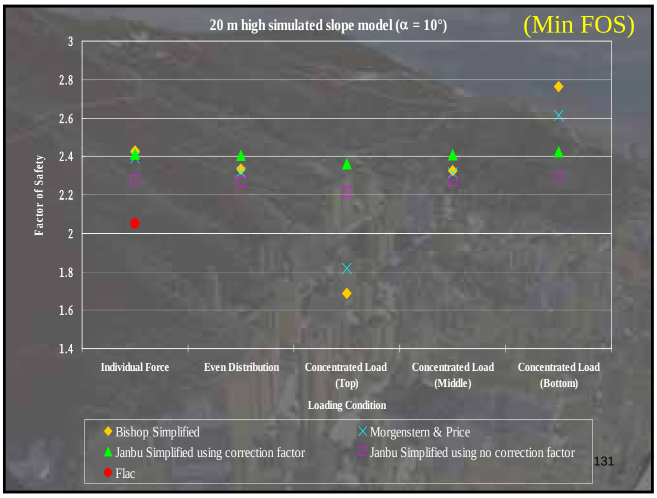
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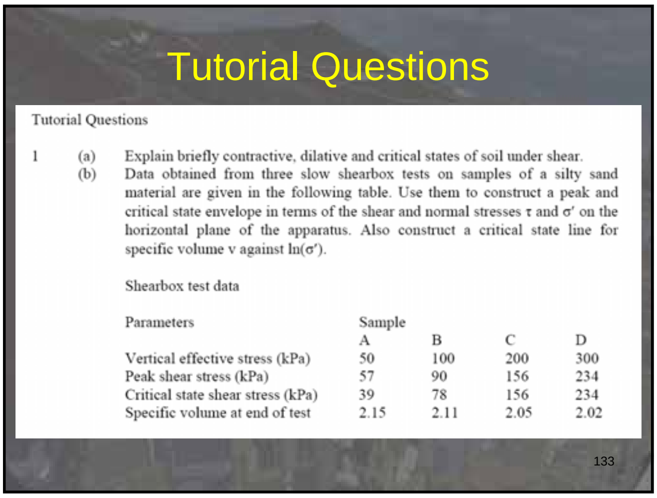
Janbu Simplified using no correction factor
Flac

**Questions?**



## Tutorial Questions

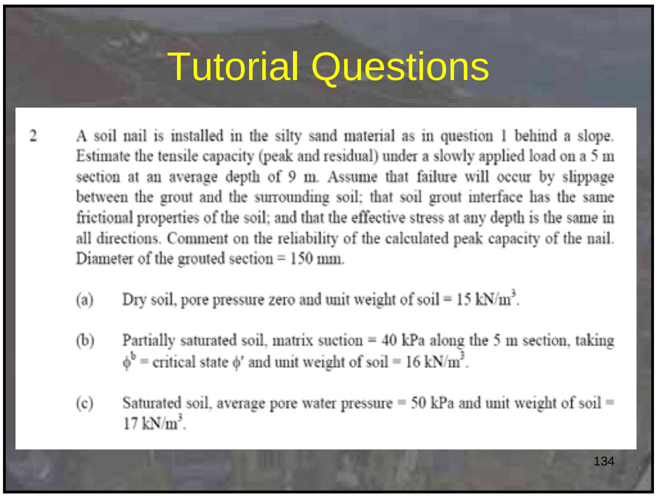
Tutorial Questions

1 (a) Explain briefly contractive, dilative and critical states of soil under shear.

(b) Data obtained from three slow shearbox tests on samples of a silty sand material are given in the following table. Use them to construct a peak and critical state envelope in terms of the shear and normal stresses $\tau$ and $\sigma'$ on the horizontal plane of the apparatus. Also construct a critical state line for specific volume v against $\ln(\sigma')$.

Shearbox test data

| Parameters | Sample A | B | C | D |
|---|---|---|---|---|
| Vertical effective stress (kPa) | 50 | 100 | 200 | 300 |
| Peak shear stress (kPa) | 57 | 90 | 156 | 234 |
| Critical state shear stress (kPa) | 39 | 78 | 156 | 234 |
| Specific volume at end of test | 2.15 | 2.11 | 2.05 | 2.02 |



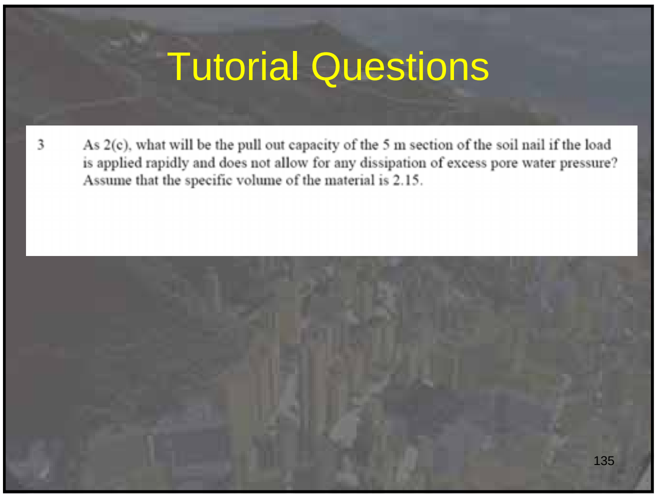
## Tutorial Questions

2 A soil nail is installed in the silty sand material as in question 1 behind a slope. Estimate the tensile capacity (peak and residual) under a slowly applied load on a 5 m section at an average depth of 9 m. Assume that failure will occur by slippage between the grout and the surrounding soil; that soil grout interface has the same frictional properties of the soil; and that the effective stress at any depth is the same in all directions. Comment on the reliability of the calculated peak capacity of the nail. Diameter of the grouted section = 150 mm.

(a) Dry soil, pore pressure zero and unit weight of soil = 15 kN/m$^3$.

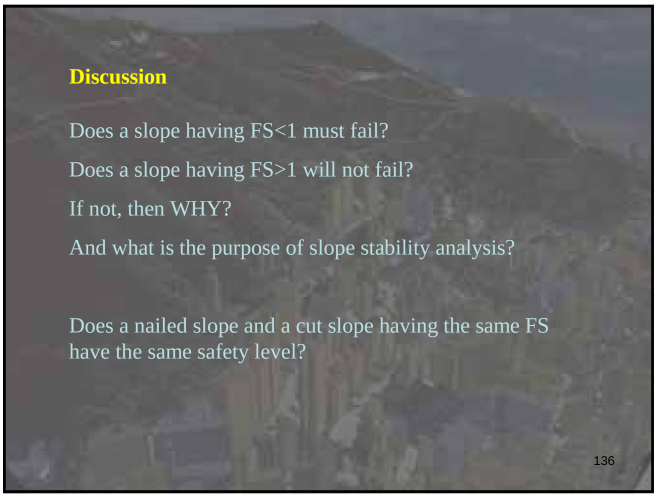
(b) Partially saturated soil, matrix suction = 40 kPa along the 5 m section, taking $\phi^b$ = critical state $\phi'$ and unit weight of soil = 16 kN/m$^3$.

(c) Saturated soil, average pore water pressure = 50 kPa and unit weight of soil = 17 kN/m$^3$.



## Tutorial Questions

3 As 2(c), what will be the pull out capacity of the 5 m section of the soil nail if the load is applied rapidly and does not allow for any dissipation of excess pore water pressure? Assume that the specific volume of the material is 2.15.



**Discussion**

Does a slope having FS<1 must fail?

Does a slope having FS>1 will not fail?

If not, then WHY?

And what is the purpose of slope stability analysis?

Does a nailed slope and a cut slope having the same FS have the same safety level?

Thank You