


**Probabilistic Methods in
Geotechnical Engineering:
Risk and Reliability**

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Griffith University Gold Coast Campus
16-17 February 2009

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Why do probabilistic analyses?

- Society, regulations and our clients demand to know the risks quantitatively
- Reliability-based design is becoming standard practice for structural engineers
- Probabilistic analyses complement the conventional deterministic analyses in achieving a safe design, and add great value to the results by modest additional effort

Aim:

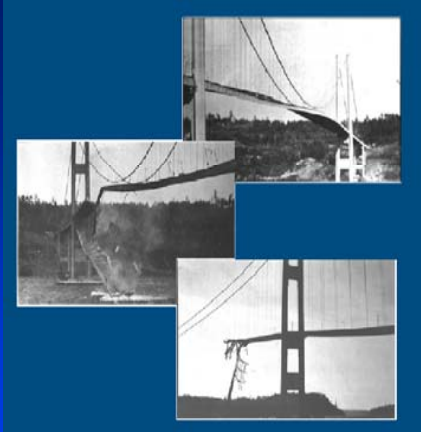
Quantify the margin against “failure”



Engineering failures result from:

- Extreme value of a single parameter
- Combination of small parameter variations
- Gross design or construction error (human factors)
- Unforeseen situations

There is no universal rule

Bridge collapse due to unforeseen dynamic behaviour in certain wind conditions, Tacoma Narrows, USA.



Example of failure in transporting construction materials due to falsely estimated load or falsely estimated weight of donkey (Ref: Michael Faber)



Living with uncertainty

In any geotechnical and geological assessment, one must deal with **uncertainties** because geo-sciences are not exact.



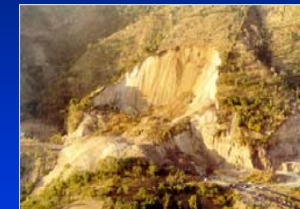
It is better to be
probably *right*...

... than to be
exactly *wrong*



Sources of Uncertainty

- Limited geo-exploration
- Measurement errors
- Spatial variability of soil and rock properties
- Limited parameter evaluation
- Limitations of calculation models



Types of uncertainty

Uncertainties associated with an engineering problem can be divided into two groups:

- ▷ aleatory (inherent)
- ▷ epistemic (lack of knowledge)



Aleatory Uncertainty

The natural randomness of a property.

The variation in a soil/rock property in the within a geological unit are aleatory uncertainties.

This type of uncertainty cannot be reduced.



Epistemic Uncertainty

The uncertainty due to lack of knowledge.

Measurement uncertainty and model uncertainty are epistemic uncertainties.

This type of uncertainty can be reduced (by increasing number of tests, improving measurement method or evaluating calculation procedure with model tests,...)



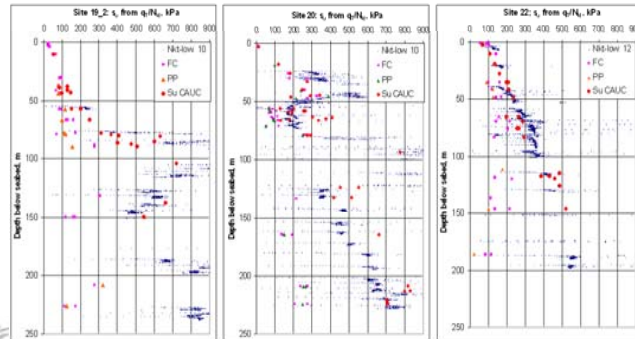
Sources of uncertainty in geomechanical parameters

Epistemic or Aleatory?

- Limited geo-exploration
- Measurement errors
- Spatial variability of soil and rock properties
- Limited parameter evaluation
- Limitations of calculation models



Soil parameters at Ormen Lange – Remoulded shear strength



Basic Concepts of Probability

Random Variables

Quantities that can take on many values

Discrete random variables - finite number of values

- Number of borings encountering peat at a site
- Date of birth

Continuous random variables - infinite number of values

- Undrained strength of a clay layer
- Unit weight of soil

Basic Concepts of Probability

Continuous Random Variables

Distribution of values described by probability density function (pdf) that satisfies the following conditions:

$$f_X(x) \geq 0$$

$$\int_{-\infty}^{\infty} f_X(x) dx = 1$$

$$P[a \leq X \leq b] = \int_a^b f_X(x) dx$$

The probability that X is between a and b is equal to the area under the pdf between a and b

Basic Concepts of Probability

Continuous Random Variables

Distribution of values can also be described by a cumulative distribution function (CDF), which is related to the pdf according to

$$F_X(x) = \int_{-\infty}^x f_X(x) dx$$

$$P[a \leq X \leq b] = F_X(b) - F_X(a)$$

Basic Concepts of Probability

Statistical Characterization of Random Variables

Distribution of values can also be characterized by statistical descriptors

$$\bar{x} = \int_{-\infty}^{\infty} x f_X(x) dx \quad \text{Mean}$$

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \bar{x})^2 f_X(x) dx \quad \text{Variance}$$

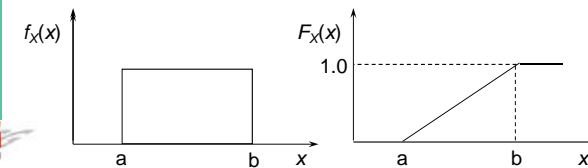
$$\sigma_x = \sqrt{\sigma_x^2} \quad \text{Standard deviation}$$

Basic Concepts of Probability

Common Probability Distributions

Uniform distribution

$$f_X(x) = \begin{cases} 0 & \text{for } x < a \\ 1/(b - a) & \text{for } a < x < b \\ 0 & \text{for } x > b \end{cases}$$

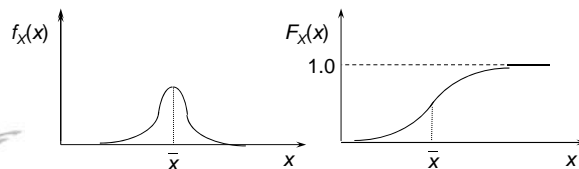


Basic Concepts of Probability

Common Probability Distributions

Normal distribution

$$f_X(x) = \frac{1}{\sqrt{2\pi} \sigma_x} \exp \left[-\frac{1}{2} \left(\frac{x - \bar{x}}{\sigma_x} \right)^2 \right]$$



Basic Concepts of Probability

Common Probability Distributions

Standard normal distribution

Mean = 0

Standard deviation = 1

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} z^2 \right]$$

Values of standard normal CDF commonly tabulated

Basic Concepts of Probability

Common Probability Distributions

Standard normal distribution

Mapping from random variable to standard normal random variable

$$Z = \frac{X - \bar{x}}{\sigma_x}$$

Compute Z, then use tabulated values of CDF



Basic Concepts of Probability

Common Probability Distributions

Example: Given a normally distributed random variable, X, with $\bar{x} = 270$ and $\sigma_x = 40$, compute the probability that $X < 300$

$$Z = \frac{X - \bar{x}}{\sigma_x} = \frac{300 - 270}{40} = 0.75$$

Looking up $Z = 0.75$ in CDF table,

$$F_Z(0.75) = 1 - F_Z(-0.75) = 0.7734$$

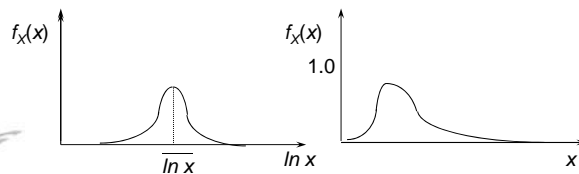


Basic Concepts of Probability

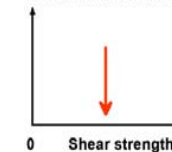
Common Probability Distributions

Lognormal distribution

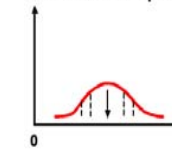
$$f_X(x) = \frac{1}{\sqrt{2\pi} \sigma_{\ln x}} \exp \left[-\frac{1}{2} \left(\frac{\ln x - \bar{\ln x}}{\sigma_{\ln x}} \right)^2 \right]$$



Deterministic description

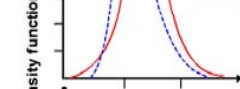


Statistical description

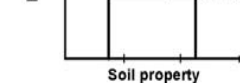


(a)

Lognormal distribution
Normal distribution

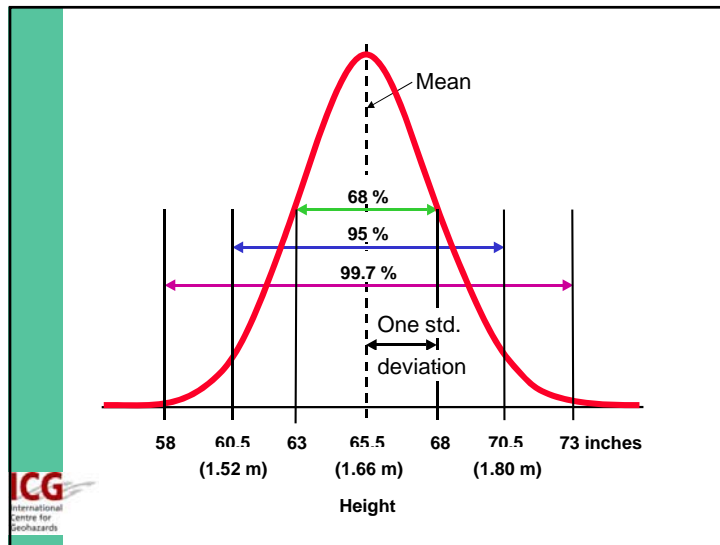


Uniform distribution



(b)





Necessary contributors to parameter evaluation

- Experience
- Expert judgement

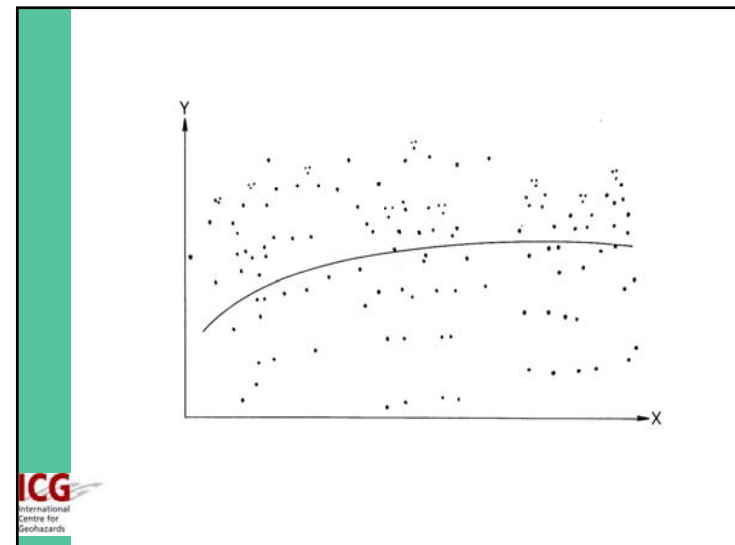
You, as the “**expert**”, are expected to evaluate how large the uncertainties are.

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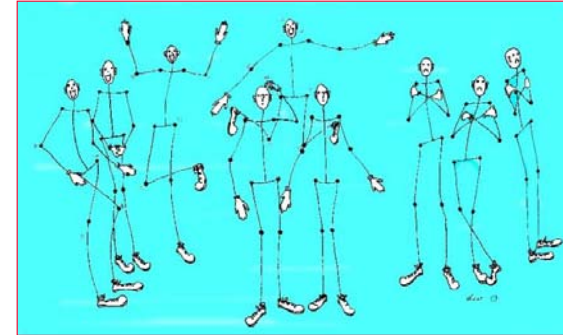
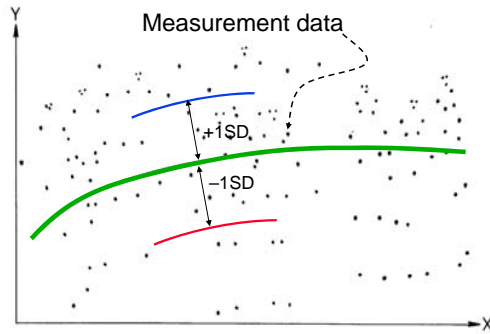
Data interpretation

Human interpretation and engineering judgment are still the most important issue in automated data processing and analysis

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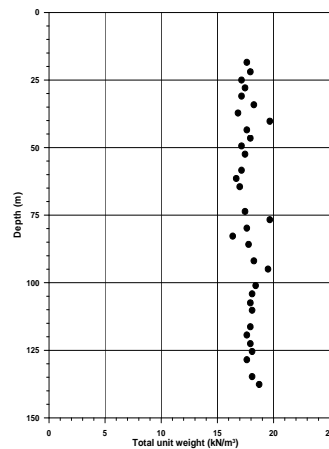
Data interpretation



Engineering judgement
gives the best
interpretation

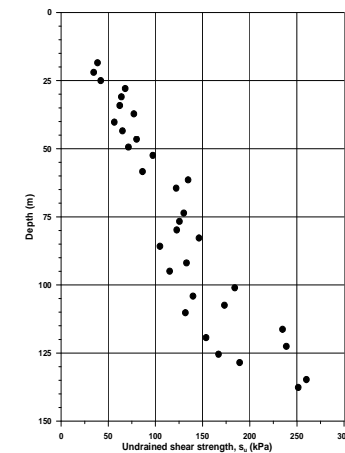
Example from an offshore site Investigation

Total unit weight vs.
Depth below seabed



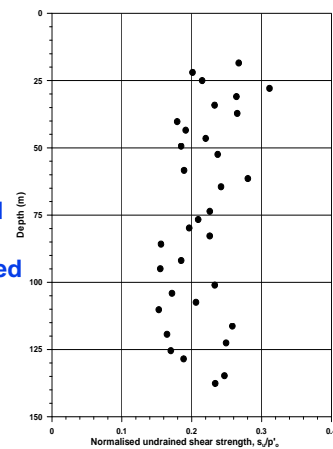
Example from an offshore site Investigation

Undrained shear
strength vs.
Depth below seabed



Example from an offshore site Investigation

Normalised undrained
shear strength (s_u/p'_o)
vs. Depth below seabed



Conventional deterministic measures of safety

Factor of Safety:

$$FS = \text{Resistance} / \text{Load}$$

$FS \geq 1 \Rightarrow$ Acceptable, safe situation

$FS < 1 \Rightarrow$ Unacceptable, unsafe situation

Conventional deterministic measures of safety

Margin of Safety:

$$M = \text{Resistance} - \text{Load}$$

$M \geq 0 \Rightarrow$ Acceptable, safe situation

$M < 0 \Rightarrow$ Unacceptable, unsafe situation

Conventional deterministic measures of safety

Factor of safety and margin of
safety are not sufficient indicators
of safety because the
uncertainties in the analysis
parameters affect the results.

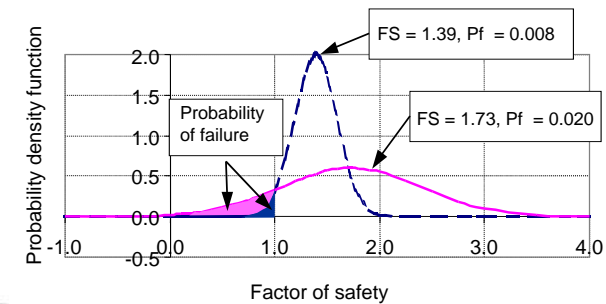
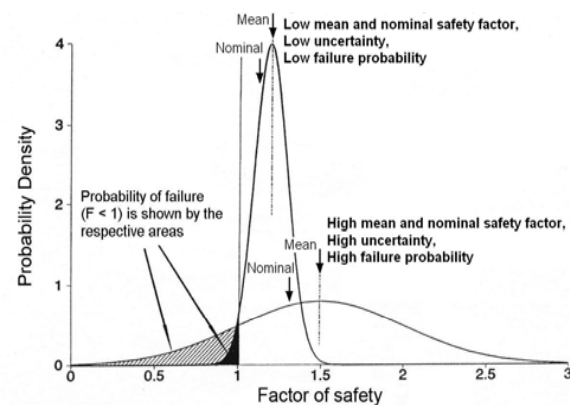
Probabilistic measures of safety

- Reliability index, β
- Probability of failure, P_f

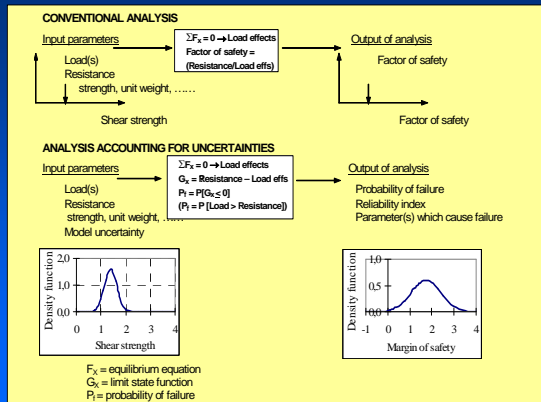
P_f and β include information about the uncertainty in load and resistance

Results of reliability/uncertainty-based analysis

- Probability of failure
- Reliability index and most probable combination of parameters causing failure
- Sensitivity of results to any change in the uncertain parameters



Deterministic vs. Probabilistic Analyses



Reliability and risk in geological and geotechnical evaluations

- **WHY** do risk analysis?
- **HOW** to do risk analysis?

Terminology

- Probability
- Uncertainty
- Hazard
- Risk
- Consequence
- Failure
- Vulnerability
-

Terminology: Danger (threat)

Danger (Threat): The natural phenomenon that could lead to damage, described in terms of its geometry, mechanical and other characteristics. The danger can be an existing one (such as a creeping slope) or a potential one (such as a rockfall). The characterisation of a danger or threat does not include any forecasting.

Terminology: Hazard & Risk

Hazard: Probability that a particular danger (threat) occurs within a given period of time.

Risk: Measure of the probability and severity of an adverse effect to life, health, property, or the environment.

Quantitatively, **Risk = Hazard x Potential Worth of Loss**. This can be also expressed as “Probability of an adverse event times the consequences if the event occurs”.



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Terminology: Hazard & Risk

Quantitatively:

$Risk = Hazard \times Consequence$, or

$Risk = Hazard \times Potential\ Worth\ of\ Loss$

Loss could be:

- Loss of human life
- Economic loss
- Loss of reputation



Often we are not consistent, and mix up “risk” and “hazard”



Conventional Factor of Safety

Criterion: Load < Strength / FS

Factor of safety (FS) accounts for

- Variations in loads & materials
- Inaccuracies in design equations and modelling approximations
- Construction effects etc.

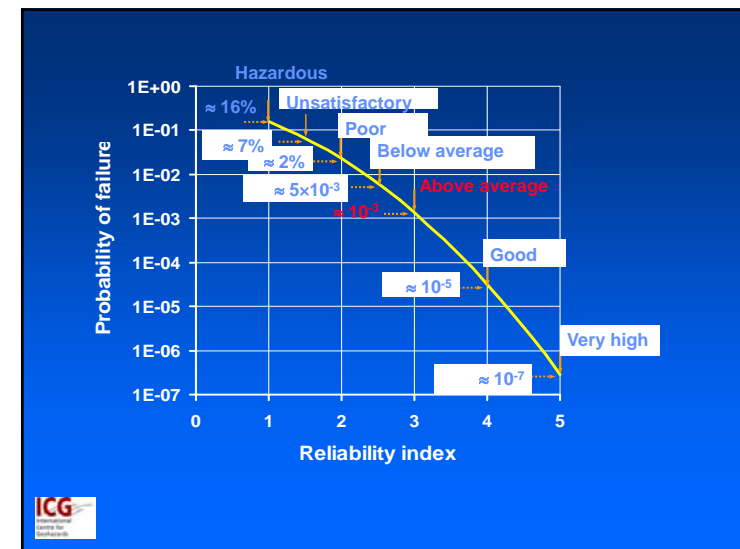
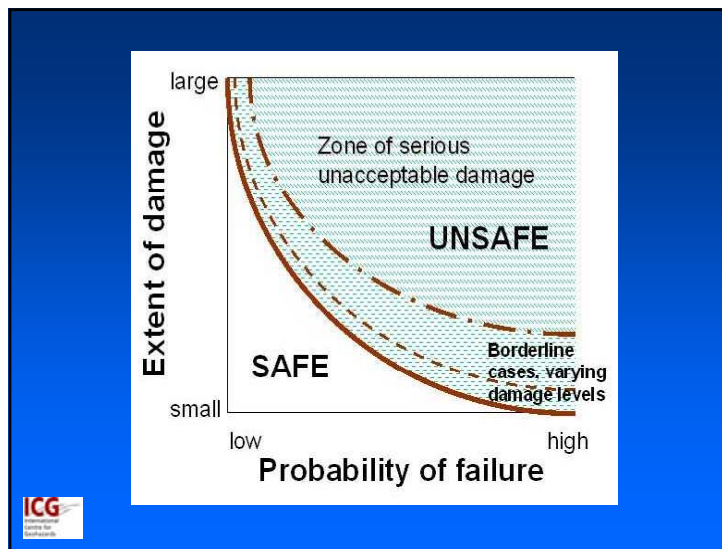
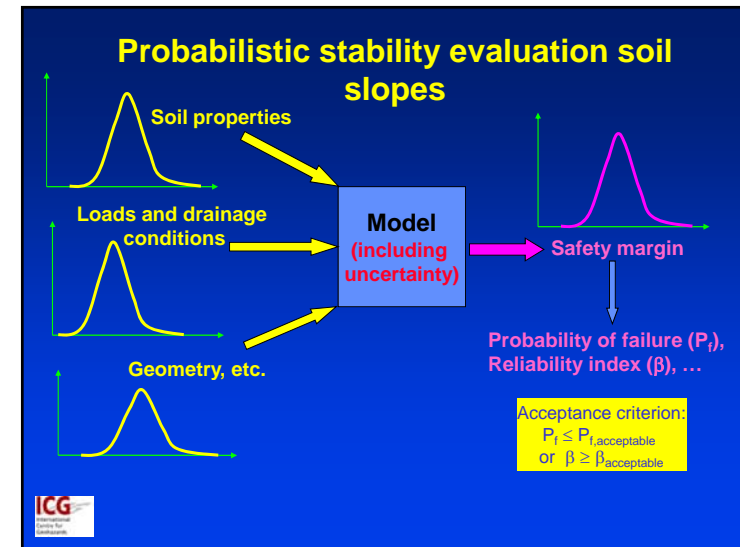
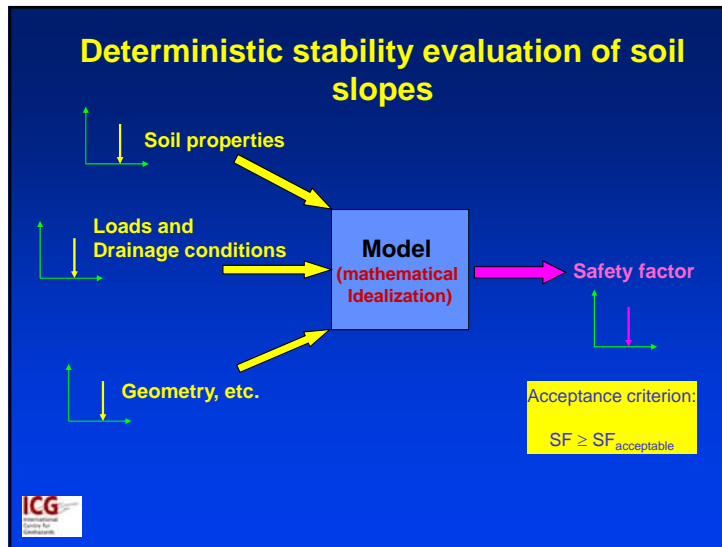
UNCERTAINTIES IMPLICITLY RECOGNIZED

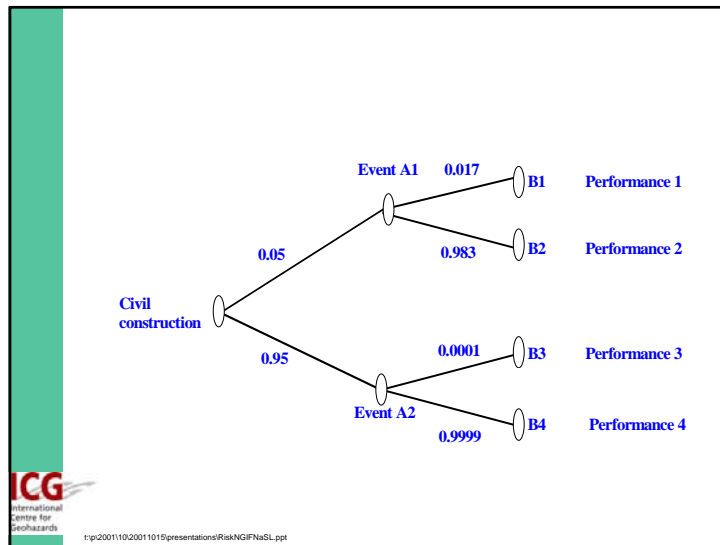


Reliability-Based Design

- Reliability analysis is the consistent evaluation of **probability of failure** using probability theory
- **Reliability-based design** (RBD) is any methodology that uses reliability analysis, explicitly or otherwise
- RBD requires access to **tools for doing reliability analysis** and a conscious choice of **acceptable probability of failure**







Risk Analysis of Dams

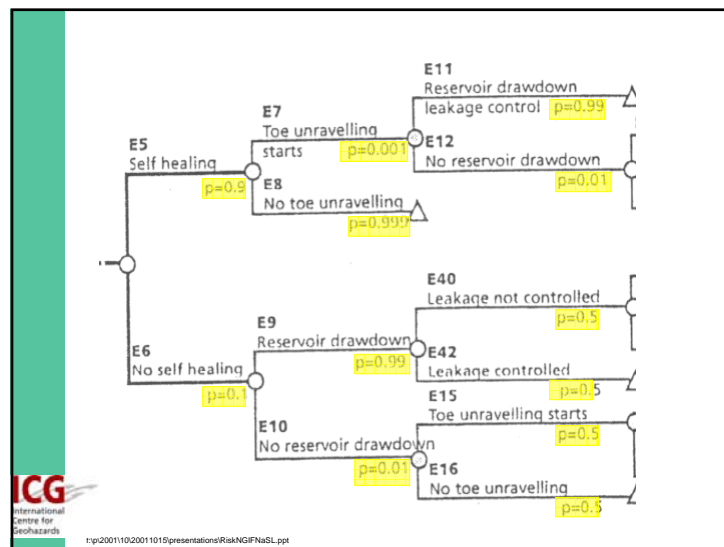
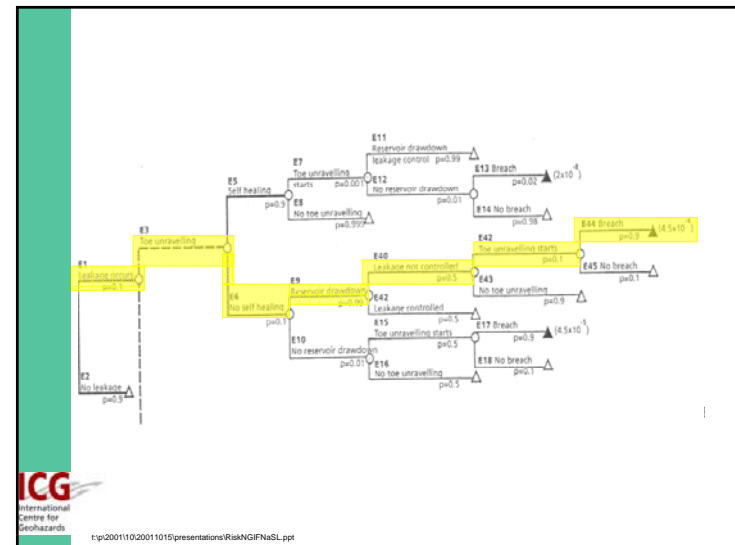
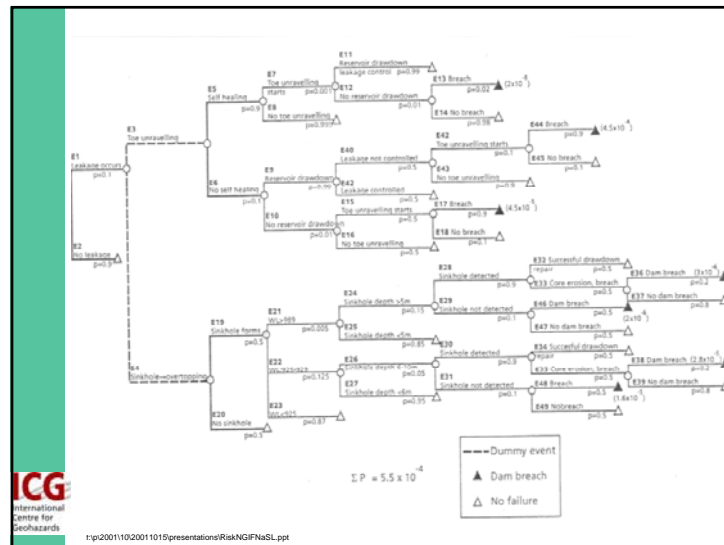
- focus on safety and reliability of existing dams
- establish a diagnosis or set priorities among possible failure modes, to act as support in decision-making on issues related to dam safety modifications

Probabilistic analysis is systematic application of engineering judgement

- 1) Dam site inspection and document review
- 2) Failure mode screening (defining all failure modes)
- 3) Construction of event tree, listing failure (events and their interrelationship)
- 4) Probability assessment of reach event (often subjective)
- 5) Failure probability from product of probability of each event along any one branch of the event tree
- 6) Iteration

Descriptors of uncertainty

- | | |
|-------|---|
| 0.001 | Virtually impossible , due to known physical conditions or process that can be described and specified with almost complete confidence |
| 0.01 | Very unlikely , although the possibility cannot be ruled out on the basis of physical or other reasons |
| 0.10 | Unlikely , but it could happen |
| 0.50 | Completely uncertain , with no reason to believe that one possibility is more or less likely than the other |
| 0.90 | Likely , but it may not happen |
| 0.99 | Very likely , but not completely certain |
| 0.999 | Virtually certain due to know physical conditions or process that can be described and specified with almost complete confidence |

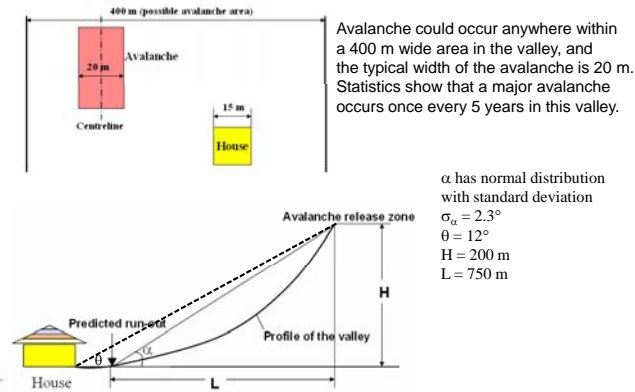


Case study of Viddalsvatn dam in Norway

<u>Loading</u>	<u>Annual probability of failure</u>
Flood	1.2×10^{-6}
Earthquake	1.1×10^{-5}
Internal erosion	5.5×10^{-4}

- The total annual probability of failure for all modes is the sum of the three components, or 5.6×10^{-4}
- The results represent a **relative order of magnitude** for the different scenarios

Example: Event tree construction



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Risk/uncertainty-based analysis

The approach is effectively
a systematic application of
engineering judgement

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Risk analysis

Pros (for)

- Encourages to scrutinize problem as a whole
- Helps communication
- Encourages gathering, compilation and organisation of data for systematic examination of problem
- Identifies the optimum among alternative solutions
- Emphasizes where decisions have to be made
- Provides a framework for contingency planning and continued evaluation

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Risk analysis

Cons (against)

- More complex calculation (?)
- Need to include judgement
- Uncertainties can be too large to enable a good basis for decision-taking
- Not always possible to have explicit formulation of a thought process
- Danger of leaving consideration that cannot be quantified out of the process
- Does not account for human error

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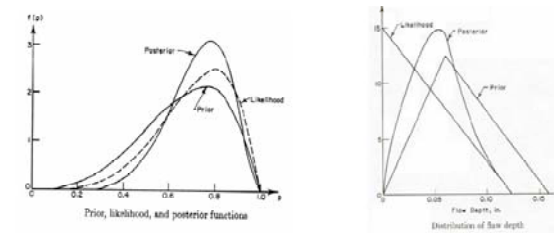
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Risk/uncertainty-based analysis

It is possible to use whatever **data** are available, to supplement them with **judgement** and to do a few simple **calculations** to get an idea of the uncertainty and the combined effects of possible variation in parameters.

Bayesian Updating

Bayesian updating is a powerful technique for combining subjective judgement and data from different sources.



Posterior distribution = Prior x Likelihood x normalising factor

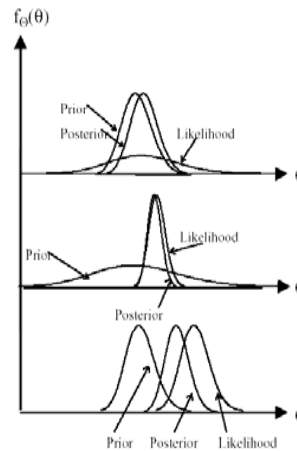
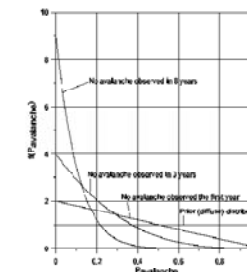


Illustration of updating of uncertainty models.

Bayesian updating – Example application to annual probability of avalanche



No avalanche in n years:
 $f(P_{\text{annual}}) = (n+1)(1 - P_{\text{annual}})^n$

Probability distribution for annual avalanche occurrence after 0, 1, 3, and 8 years of observation of no avalanche

Bayesian updating – Some useful equations (assuming normal distribution)

- **Prior estimates:**

Mean = μ_1 , Stand. Dev. = σ_1

- **Likelihood estimates:**

Mean = μ_2 , Stand. Dev. = σ_2

- **Posterior estimates (updated estimates):**

$$\mu_{\text{updated}} = (\mu_1 / \sigma_1^2 + \mu_2 / \sigma_2^2) / (1 / \sigma_1^2 + 1 / \sigma_2^2)$$

$$\sigma_{\text{updated}}^2 = (\sigma_1^2 \cdot \sigma_2^2) / (\sigma_1^2 + \sigma_2^2)$$

Quantitative Risk Assessment (QRA) – Theory and applications

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Natural threats

- Flood
- Earthquake
- Tsunami
- Soil- and rockslide
- Snow avalanche
- Wind and storm



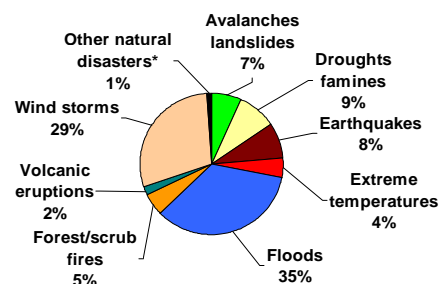
Landslides are the natural threat which occur most frequently (compared to other natural threats like flood, earthquake, cyclone and volcano).

Europe is the continent with the next highest fatalities caused by landslides (after America) and with the highest economic consequences.



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Global incidence of natural disasters (1991-2000)

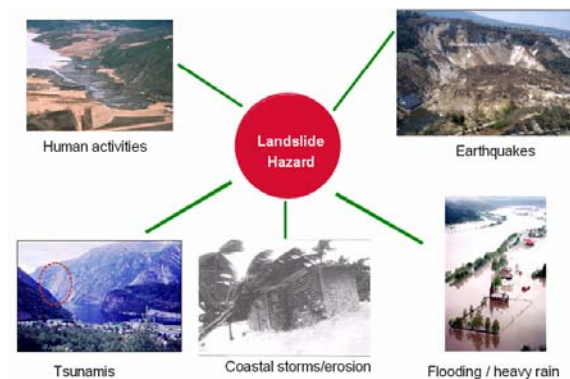


Much of damage and casualties attributed to earthquakes and floods are caused by the landslides triggered by these events.

Sources: OFDA/CRED international disaster database
& 2001 IFRC World disaster report

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Correlation with other types of natural threats



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Socio-economic consequences of natural disasters in Europe



European
statistics
1900-2000

45 floods
1700 landslides
32 earthquakes

Source: EMDAT/CRED international disaster database

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Examples of major landslides



El Salvador – Las Colinas
January 2001
~ 600 casualties



Nicaragua – Casita Volcano slide
October 1998
~2500 casualties

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The 5-6 May 1998 mudflows in Sarno ridge area in Campania, Italy



Residents and fatalities of the affected municipalities

Municipality	Residents	Fatalities
Sarno (SA)	31,509	137
Siano (SA)	9,265	5
Bracigliano (SA)	5,105	6
Quindici (AV)	3,023	11
S. Felice a Cancelli (CE)	16,771	1
TOTAL	65,673	160



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New York City Slide



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Landslide problems in Denmark!



The most recent landslide at Møns Klint, which occurred in January 2007. 100 000 m³ chalk from the cliff section known as St. Taler collapsed into the sea.

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Rule of thumb in slope stability evaluation*

* Karstein Lied, NGI

All slopes that look unstable

... will eventually fail.

All slopes that look stable

... will also eventually fail.

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DEFINITIONS

(Based on Glossary of TC32 of the ISSMGE)

Danger (Threat): Natural phenomenon that could lead to damage. Described by geometry, mechanical and other characteristics. Can be an existing one, or a potential one, such as a rockfall. Characterisation of threat involves no forecasting.

Hazard: Probability that a particular danger (threat) occurs within a given period of time.

Risk: Measure of the probability and severity of an adverse effect to life, health, property, or the environment.

Risk = Hazard × Potential Worth of Loss

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Definition of Risk (from an engineer's viewpoint)

Risk = Hazard x Consequence

$$R = H \cdot V \cdot U$$

H = Hazard (temporal probability of a threat)

V = Vulnerability of element(s) at risk

U = Utility of the consequence to the element(s) at risk



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Quantitative Risk Assessment (QRA) of landslides or slope failures

QRA refers to the assessment of threat, hazard, risk and countermeasures in terms of **numbers**. It addresses the following questions:

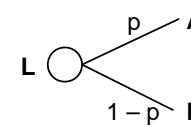
- (1) What can cause harm? → landslide threat identification
- (2) How often? → frequency of failure occurrence (hazard)
- (3) What can go wrong? → consequence of failure
- (4) How bad? → severity of failure consequence
- (5) So what? → acceptability of landslide risk
- (6) What should be done? → landslide risk management

QRA is an important element in **Decision Making Under Uncertainty**

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Decision Theory

- A calculus for decision-making under uncertainty
- Set of primitive outcomes
- Subjective degrees of belief (probabilities)
- Lotteries: uncertain outcomes



With probability p , outcome **A** occurs.

With probability $1 - p$, Outcome **B** occurs.

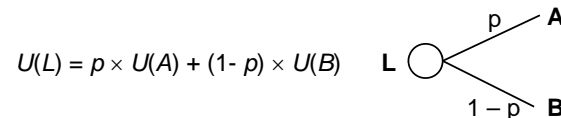
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Decision Theory – Utility function

If certain assumptions are satisfied, then there exists U (a real valued function) such that:

- If $A > B$, then $U(A) > U(B)$
- If $A \approx B$, then $U(A) = U(B)$

Utility of a lottery = **expected utility** of the outcomes



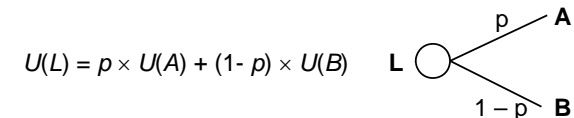
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Utility of a lottery = **expected utility** of the outcomes



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Survey Question 1

Which alternative would you prefer:

- A. A sure gain of \$240
- B. A 25% chance of winning \$1000 and a 75% chance of winning nothing

85% prefer option A to option B

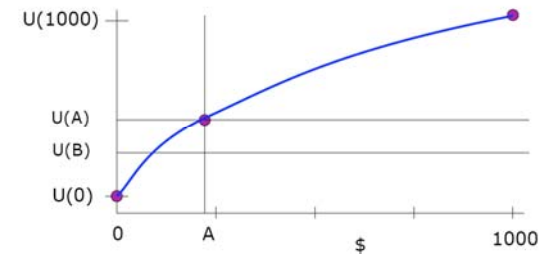
- $U(B) = .25 U(\$1000) + .75 U(\$0)$
- $U(A) = U(\$240)$
- $U(A) > U(B)$

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Utility of Money

- $U(B) = .25 U(\$1000) + .75 U(\$0)$
- $U(A) = U(\$240)$
- $U(A) > U(B)$

concave utility function
risk averse

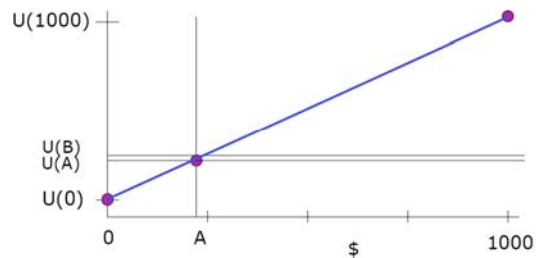


NGI ICG

Risk neutrality

- $U(B) = .25 U(\$1000) + .75 U(\$0) = U(\$250)$
- $U(A) = U(\$240)$

linear utility function
risk neutral



NGI ICG

Survey Question 2

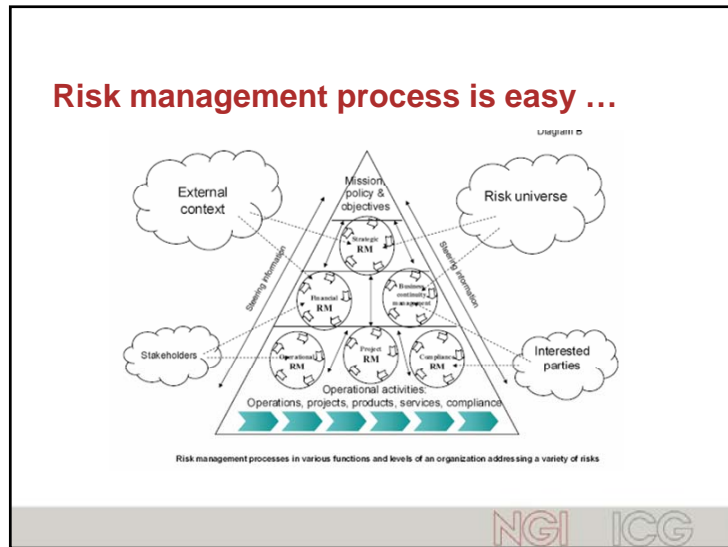
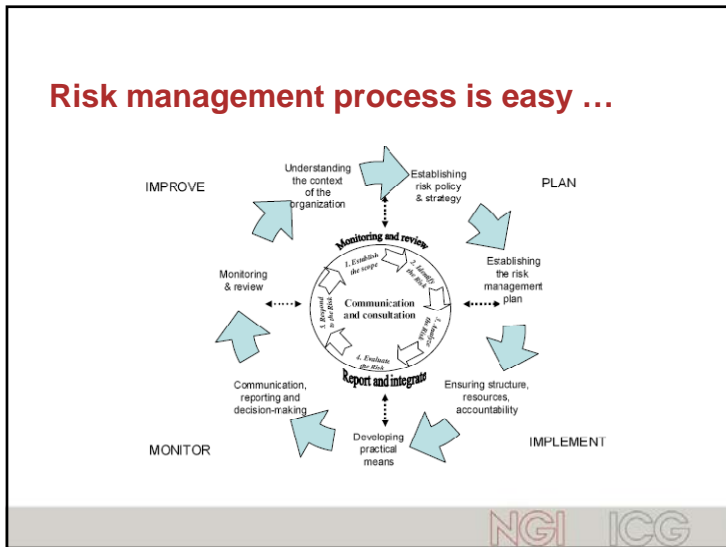
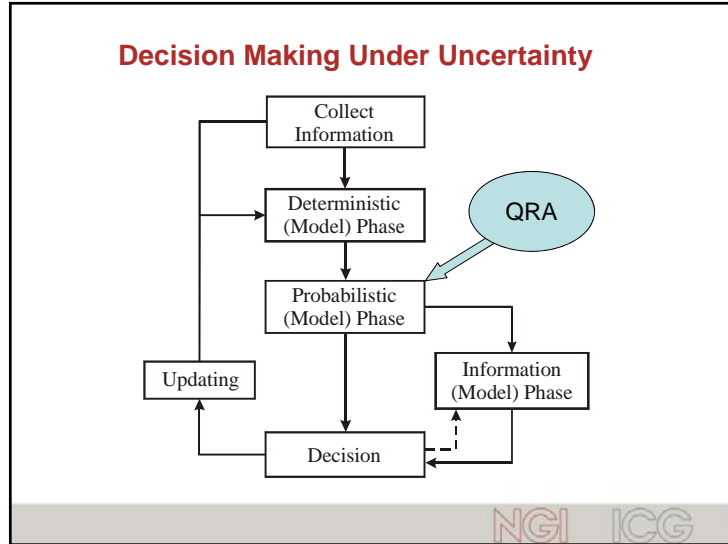
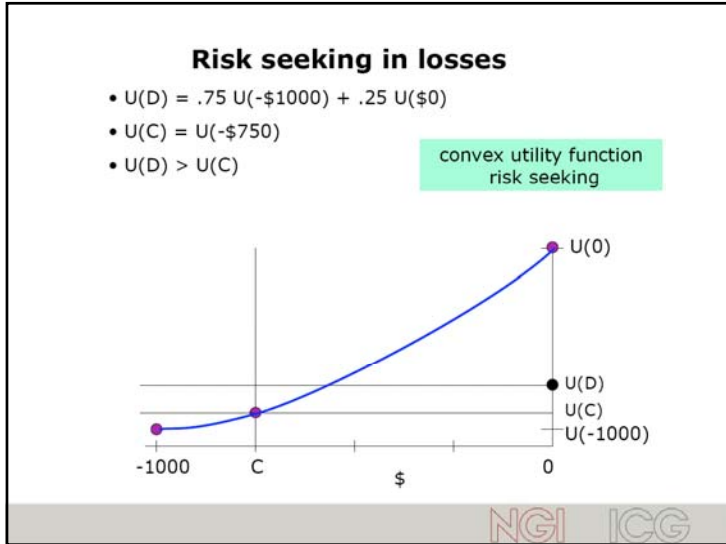
Which alternative would you prefer:

- C. A sure loss of \$750
- D. A 75% chance of losing \$1000 and a 25% chance of losing nothing

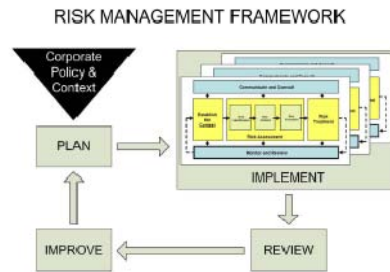
91% prefer option D to option C

- $U(D) = .75 U(-\$1000) + .25 U(\$0)$
- $U(C) = U(-\$750)$
- $U(D) > U(C)$

NGI ICG

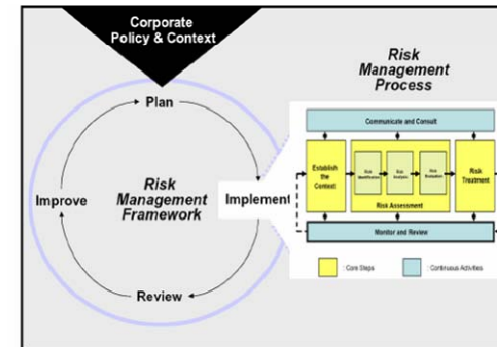


Risk management process is easy ...



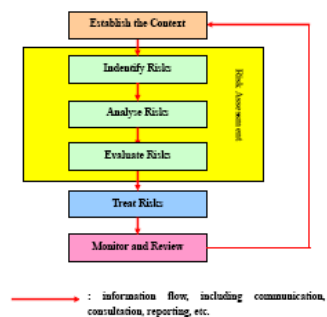
NGI ICG

Risk management process is easy ...



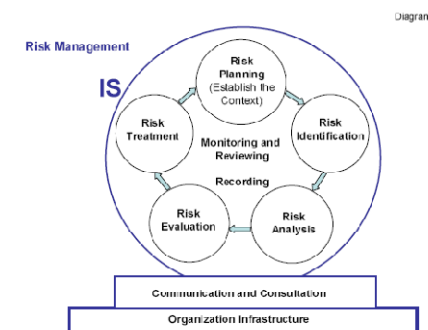
NGI ICG

Risk management process is easy ...



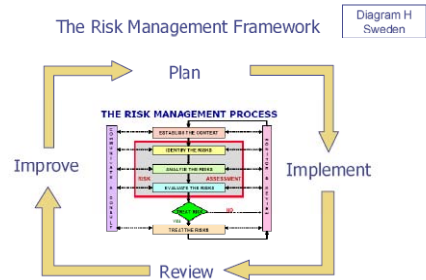
NGI ICG

Risk management process is easy ...



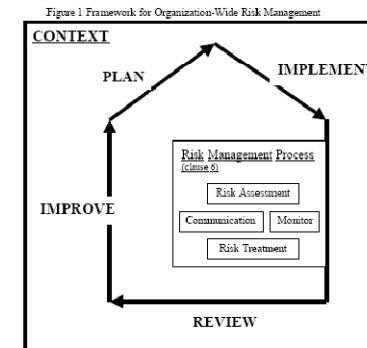
NGI ICG

Risk management process is easy ...



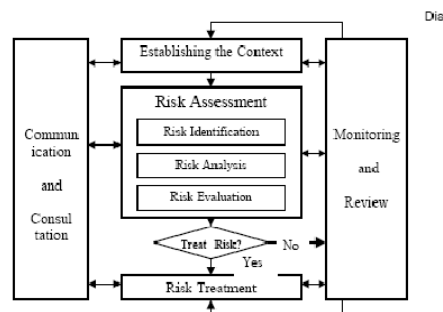
NGI ICG

Risk management process is easy ...



NGI ICG

Risk management process is easy ...



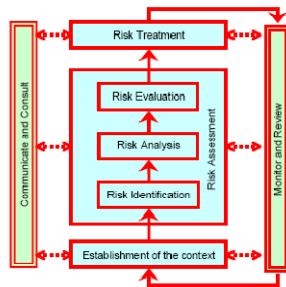
NGI ICG

Risk management process is easy ...



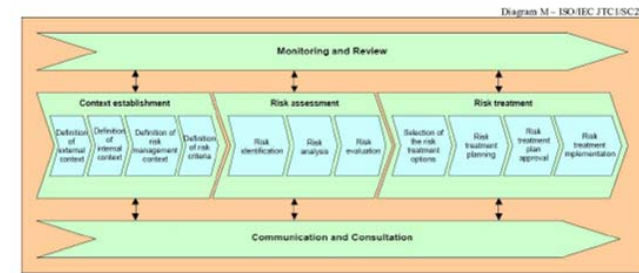
NGI ICG

Risk management process is easy ...



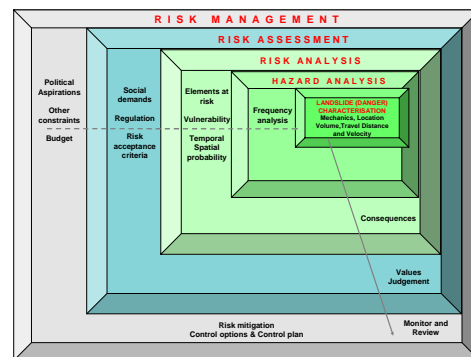
NGI ICG

Risk management process is easy ...



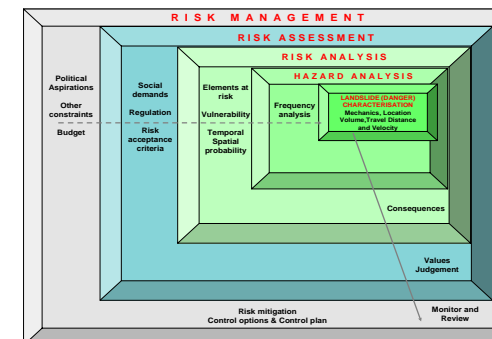
NGI ICG

Risk management process is easy ...



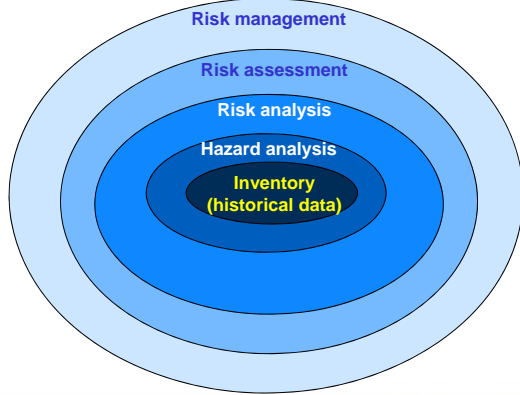
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Landslide risk management framework (JTC1 experts)



NGI ICG

Landslide risk management framework (NGI)



NGI ICG

Computation of Hazard

- Heuristic methods
- Statistical methods
- Probabilistic methods
 - Reliability analyses
 - Monte Carlo Simulations

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Example of heuristic/statistical approach

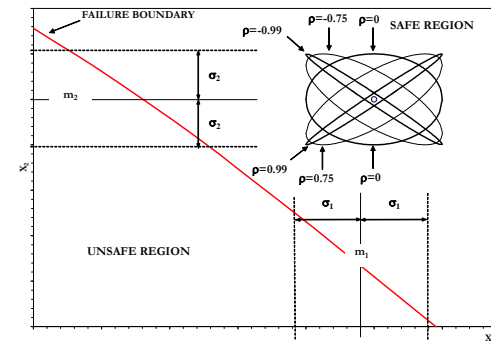
New York State Rockfall Hazard Rating Procedure

$$\text{Relative Hazard} = \text{GF} \times \text{SF} \times \text{HEF}$$

- GF = Geologic Factor
 = Sum of Seven Subjectively Assessed Indicators:
 Fractures, Bedding Planes, Block Size, Rock Friction,
 Water/Ice, Rock Fall History, Backslope
- SF = Section Factor
 Ditch and Slope Geometry (Largely Deterministic)
- HEF = Human Exposure Factor
 Probability of Being Hit by Falling Rock or Hitting
 Rock Lying on Road (Objective or Subjective Probabilistic
 Assessment)

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Probabilistic methods: Reliability Analysis



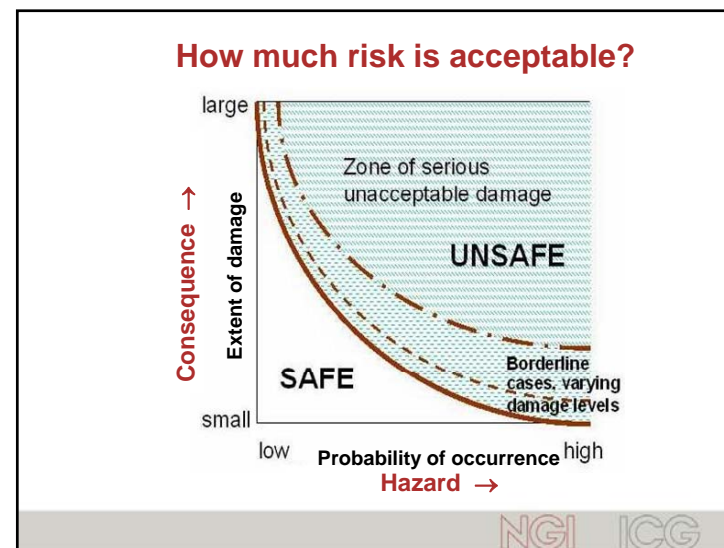
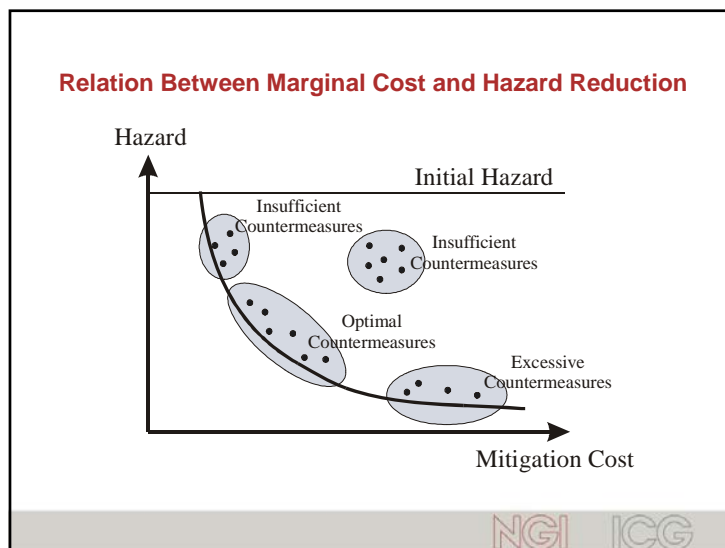
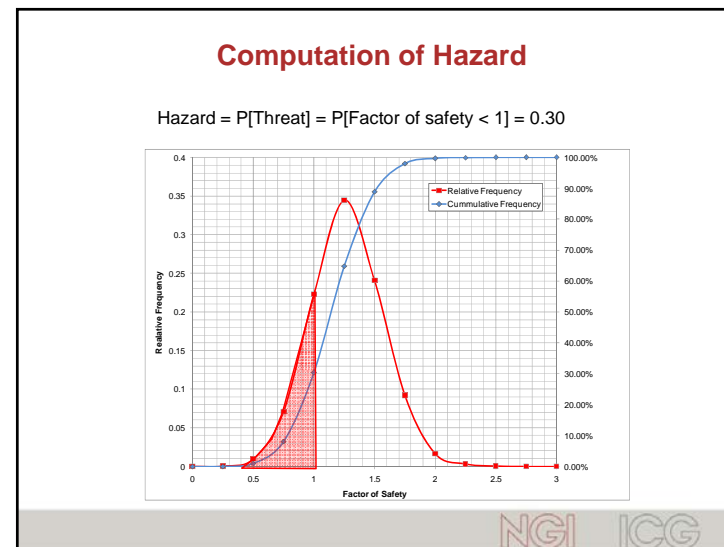
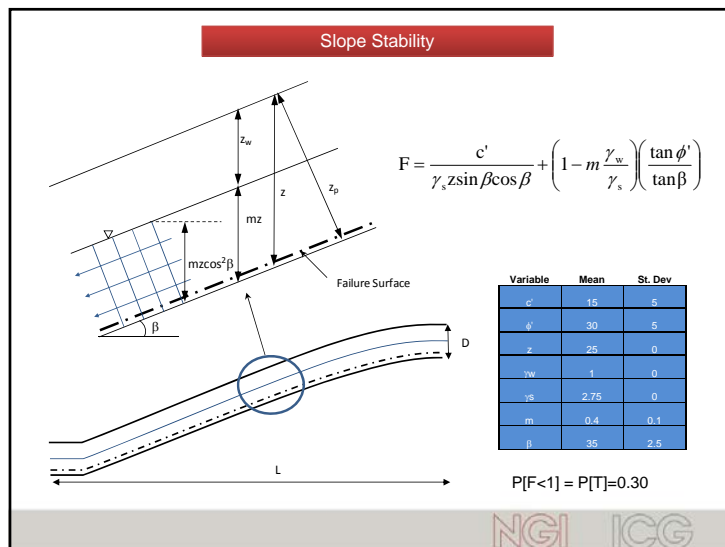
β = Reliability Index

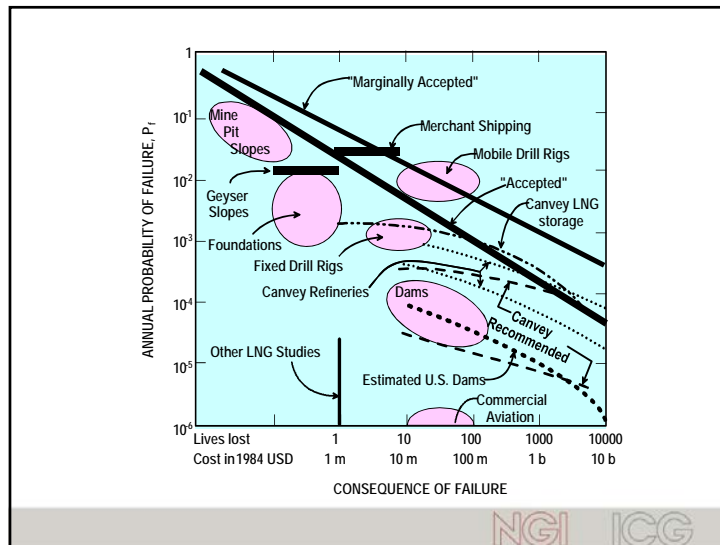
Single variable:

$$\beta = \frac{E[X] - X^*}{\sigma[X]}$$

Multiple variables:
$$\beta = \min_{\lambda \in \Lambda} \sqrt{(X - E[X])^T \sum_x^{-1} (X - E[X])}$$

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How much risk are we willing to accept?

Depends on whether the situation is voluntary or imposed.



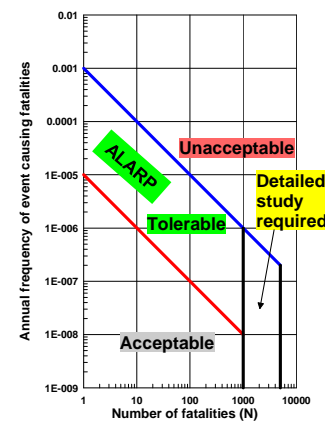
NGI ICG

Acceptable / Tolerable Risk

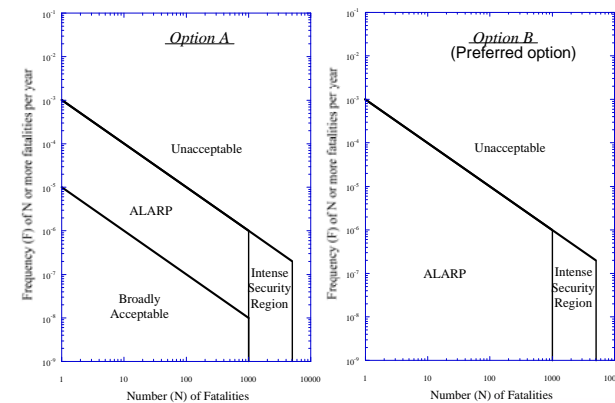
Criteria of Hong Kong Geotechnical Engineering Office

Societal: F - N Charts (Ho et al., 2000)

ALARP = As Low As Reasonably Practical



Consideration of Life Losses

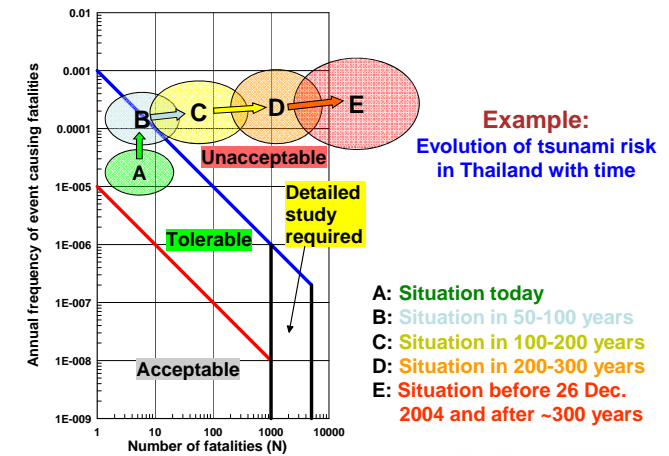


Tsunami risk mitigation strategy in Thailand



Thailand – Aftermath of 26 December 2004 tsunami

NGI ICG

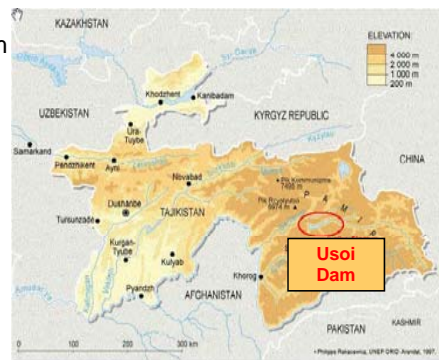


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Example: Usoi Dam on Lake Sarez in Tajikistan

Usoi Dam is a 600m high landslide dam.

It is the largest dam in the world!



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Usoi dam and Lake Sarez



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Usoi dam



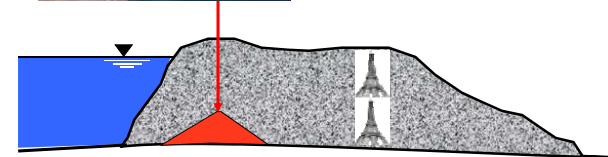
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How big is Usoi dam?



Bennett dam, 183 m
One of the largest dams in
North America

- Eiffel tower in Paris



Horizontal scale of Usoi Dam is compressed

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Lake Sarez

Length, ~ 60 km
Maximum depth: 500 m
Maximum width: 3.3 km
Average width: 1.3 km
Volume: ~ 17 km³
Elevation 3260 – 3265 m



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The threat and consequences

- The 600 m high Usoi dam is the largest dam in the world.
- Lake Sarez behind the dam currently holds 17 cubic-kilometers of water.
- If the dam were to fail, the resulting flood would be a catastrophe of inconceivable dimensions!
- Flood waters would flow down the Bartang valley to the Panj River valley and end up in the Aral Sea.

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Valleys downstream



Bartang valley



Panj valley between Tajikistan and Afghanistan

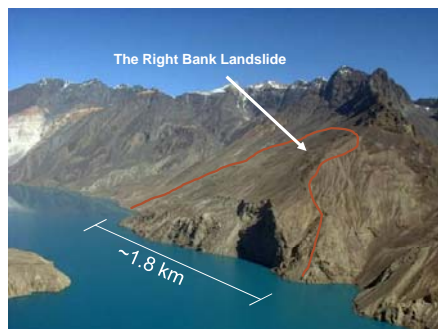
NGI ICG

Disaster scenarios at Lake Sarez



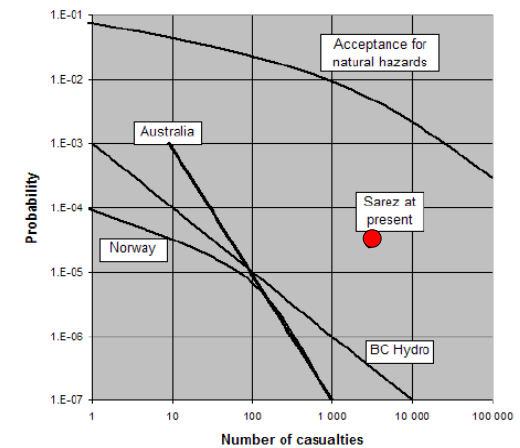
NGI ICG

Right bank active landslide



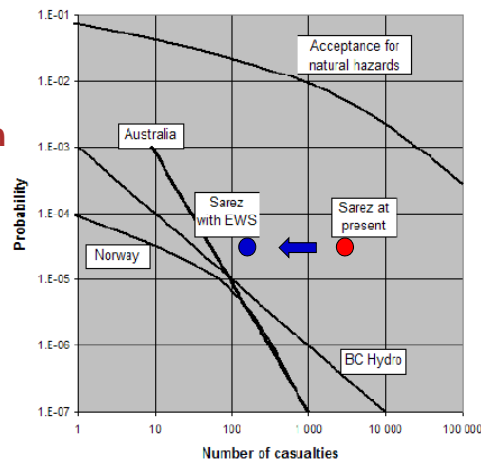
NGI ICG

No mitigation measures



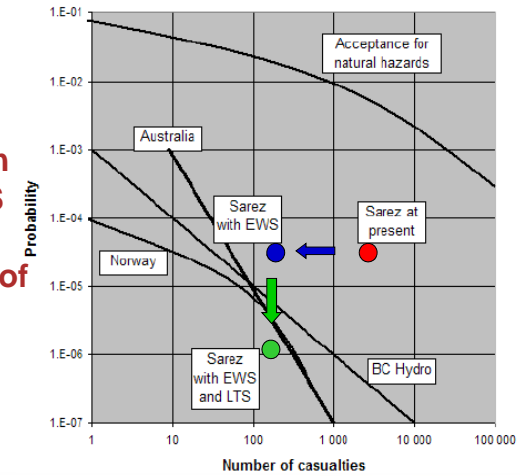
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Mitigation with early warning system (EWS)



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Mitigation with EWS and lowering of reservoir



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Example: "Slope Safety" programme in Hong Kong



NGI ICG

"Slope Safety" programme in Hong Kong

Quotes from <http://hkss.cedd.gov.hk/hkss/eng/studies/qra/>

The use of **QRA** technique in evaluating and managing landslide risk is gradually becoming recognized by the geotechnical practitioners in Hong Kong.

Using the technique of **QRA**, it was shown that the overall **landslide risk** arising from old substandard man-made slopes in Hong Kong **had been reduced to less than 50% of the 1977 level by 2000**, through the Government's Landslip Preventive Measures (LPM) Programme.

NGI ICG

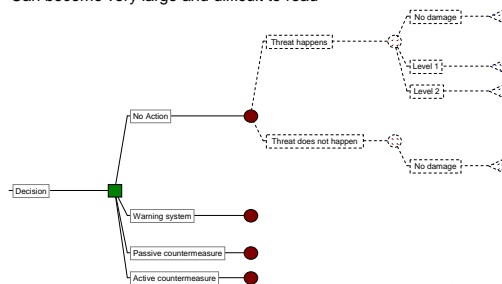
Decision Tree Tool

Advantages:

- Easy to understand and interpret: show in detail all different scenarios and paths

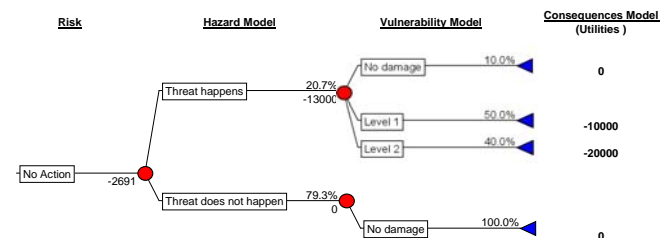
Disadvantage:

- Can become very large and difficult to read



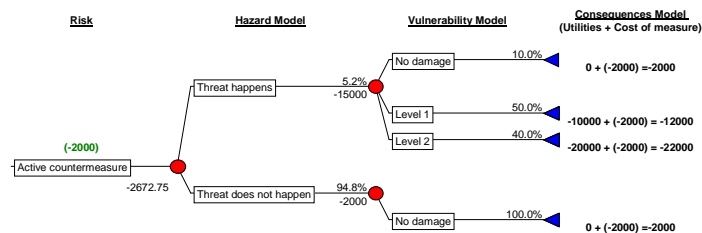
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Example: Decision Tree – No Action



NGI ICG

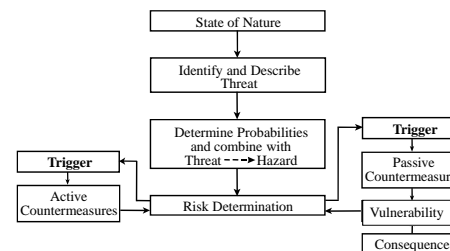
Example: Decision Tree – Active Countermeasure



Reduction in hazard $r = 0.25$, $P'(T) = r \times P(T)$

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Risk Decision Cycle for Natural Threats with Warning System



"Trigger" indicates the triggering of countermeasures by the Warning System

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Swiss - Avalanche Warning System (WSL/SLF)

- Meteorological forecast
- Automatic wind and snow stations
- Local observers (80)
- Reports on actual avalanche occurrences
- SNOBACK model

Bulletin (updated daily at 5 p.m. & 8 a.m.)
(Accessible by Telephone & Radio)

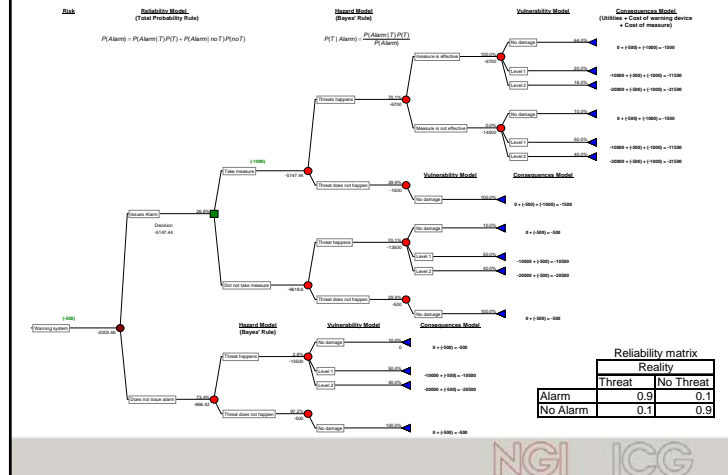
Local and Regional Safety Experts



Automatic Measurement Station

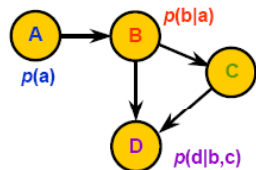
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Decision Tree – Warning System



Flow Chart Models (e.g. Bayesian Network) – Chain Rule

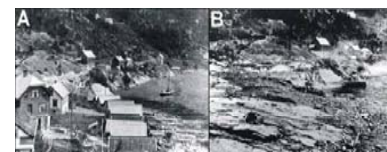
Compact and graphical representation of a joint distribution (based on simplifying assumptions)



Chain Rule (with independency assumptions):

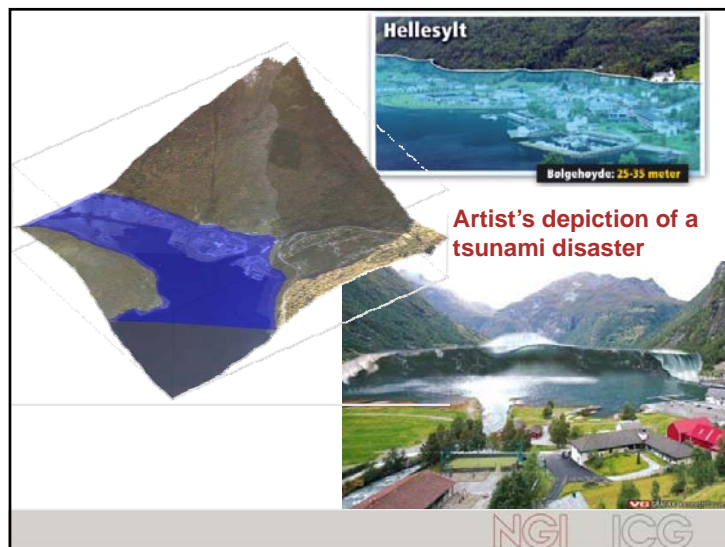
$$p(a,b,c,d) = p(a) p(b|a) p(c|b) p(d|b,c)$$

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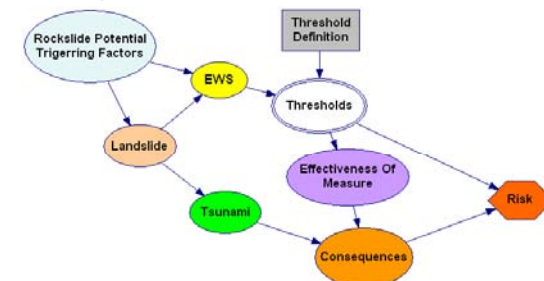
Tafjord, 1934
3 million m³ rock mass dropped into the fjord
The tsunami reached 62m above sea level
More than 40 people were killed

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Flow chart model for Åknes Rockslide (with Early Warning System)

- Elements defined into nodes
- Influences defined as arcs
- Non-cyclic network



CONCLUDING REMARKS

- Landslides will happen.
- Landslide risk management involves decision-making under uncertainty.
- The uncertainty has to be reflected in:
 - Predictions of Hazard and Risk
 - Countermeasures - Active, Passive or Warnings
- Quantitative Risk Assessment (QRA) is a useful tool when one is confronted with decision-making under uncertainty.
- The optimal solution on the basis of QRA is not necessarily the most appropriate solution.

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First-Order, Second Moment First- and Second-Order Reliability Methods Monte Carlo Simulation System reliability

Farrokh Nadim

International Centre for Geohazards,
Norwegian Geotechnical Institute

Griffith University Gold Coast Campus
16-17 February 2009



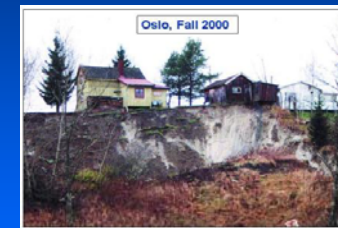
First-order, second moment approximation (FOSM)

Problem:

Y is a function several
random variables X_i :

$$Y = G(X_i)$$

What are the mean value
 μ_Y and standard deviation
 σ_Y of Y ?



Reliability Analysis Methods

- **First-Order Second-Moment (FOSM) approach**
 - Uses only mean and standard deviation of random variables (i.e. ignores the distribution functions)
 - No need for special software or add-ons
 - Additional assumptions must be made to estimate probability of failure
 - Reliability index not uniquely defined, depends on safety format used



First-order, second moment approximation (FOSM)

$Y = G(X_i)$, Taylor series expansion at point X^* :

$$\mu_Y \approx G(x_i^*) + \sum_{i=1}^n (\mu_{X_i} - x_i^*) \cdot \left. \frac{\partial G}{\partial X_i} \right|_*$$

$$\sigma_Y^2 \approx \sum_{i=1}^n \left(\left. \frac{\partial G}{\partial X_i} \right|_* \right)^2 \cdot (\sigma_{X_i})^2$$

Choose X^* = mean value of X_i :

$$\mu_Y \approx G(\mu_{X_i})$$



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First-order, second moment approximation (FOSM)

Approximate estimate of $\partial G(X_i)/\partial X_i$:

$$\partial G(X_i)/\partial X_i \approx \{G(X_i + \Delta X_i) - G(X_i - \Delta X_i)\} / 2\Delta X_i$$

Practical suggestion:

Choose $\Delta X_i = 0.1 \sigma_{X_i}$

First-order, second moment approximation (FOSM)

$$Y = G(X_i)$$

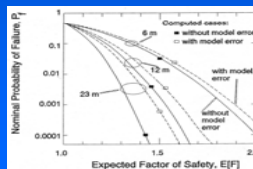
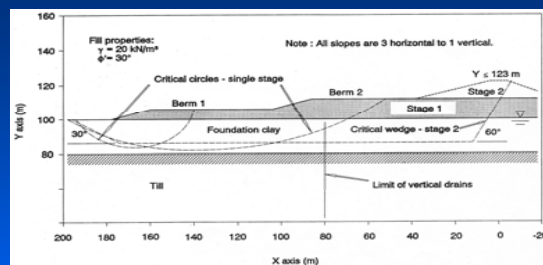
Taylor series expansion at mean value of all variables, and neglecting higher order terms:

Mean value: $\mu_Y \approx G(\mu_{X_i})$

Standard dev.: $\sigma_Y^2 \approx \sum (\partial G(X_i)/\partial X_i)^2 \cdot \sigma_{X_i}^2 | \mu_{X_i}$

NOTE: The FOSM method does not use the probability distribution functions.

Example of FOSM Approach



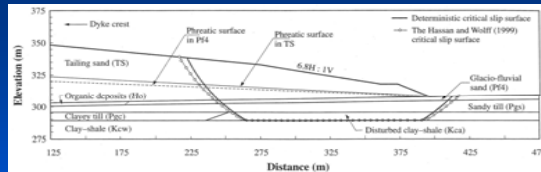
Staged construction of an embankment

Reliability Analysis Methods

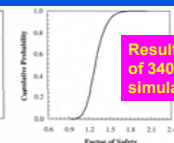
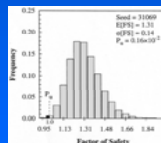
• Monte Carlo Simulation

- General method, can be applied to any problem for which a physical model exists
- Need special software or add-ons
- Could be computationally intensive when probability of failure is low
- Modern commercial slope stability software include option for simple Monte Carlo simulation

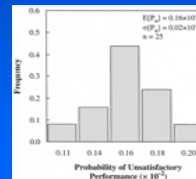
Example of Monte Carlo Simulation



Tailings dyke, Monte Carlo simulation performed using @Risk and a spreadsheet model for stability



Results of 34000 simulations



FORM (and SORM) approximation

- First- and second-order reliability methods (FORM & SORM) are the most popular approach in structural reliability analyses
- Need special software or very good programming skills
- Very efficient when probability of failure is low
- Reliability index and probability of failure independent of safety format used
- Valuable additional information (sensitivity factors and most likely combination of variables leading to failure)

FORM and SORM

In the first- and second-order reliability methods (**FORM & SORM**), a limit state function (performance function) $g(\mathbf{X})$, is defined such that $g(\mathbf{X}) \geq 0$ means that performance is **acceptable** and $g(\mathbf{X}) < 0$ means **failure**.

\mathbf{X} is a vector of **basic random variables** including soil properties, load effects, geometry parameters and modelling uncertainty.

FORM & SORM (cont.)

If the joint probability density function $F_{\mathbf{X}}(\mathbf{X})$ is known, then the probability of failure P_f is given by

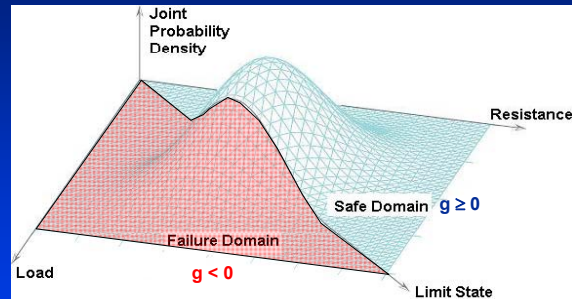
$$P_f = \int_L F_{\mathbf{X}}(\mathbf{X}) \cdot d\mathbf{X}$$



where L is the domain of \mathbf{X} where $g(\mathbf{X}) < 0$.

In general the above integral **cannot be solved analytically**.

Limit State Function



FORM Approximation

1. Transform the general random vector into a standard Gaussian vector:

The general case is approximated to an ideal situation where \mathbf{X} is a vector of **independent, standard Gaussian variables** (with zero mean and unit standard deviation).

FORM Approximation

2. Locate the point of maximum probability density (most likely failure point or **design point**) within the failure domain.
3. Linearize $g(\mathbf{X})$ at the **design point**, and find the distance β from the origin to this point.



FORM Approximation

4. Estimate the probability of failure as

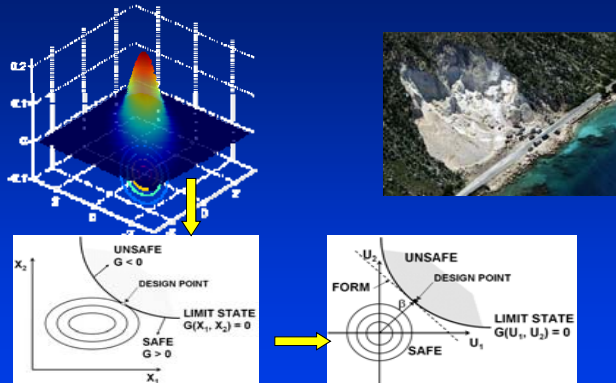
$$P_f \approx \Phi(-\beta)$$

where $\Phi(\cdot)$ is the standard Gaussian cumulative distribution function.

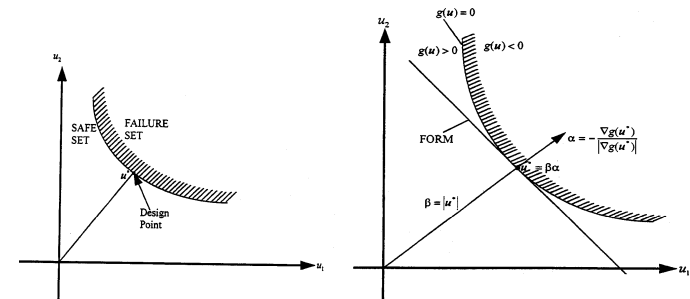
$$P_f = P[g(\mathbf{X}) < 0] \approx P[\alpha_i U_i - \beta < 0] = \Phi(-\beta)$$

α_i : Sensitivity factors, β : Reliability index

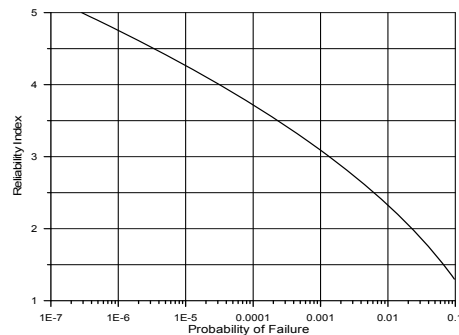
FORM approximation (summary)



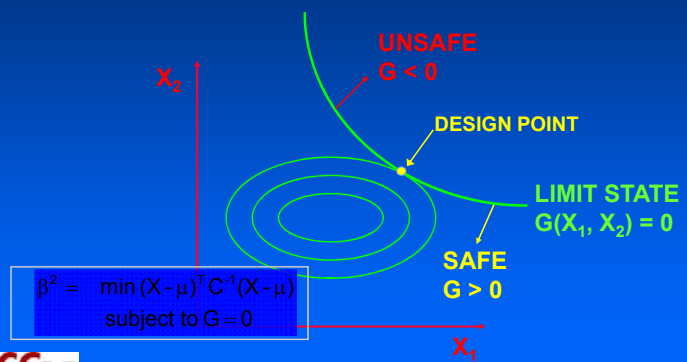
The FORM approximation



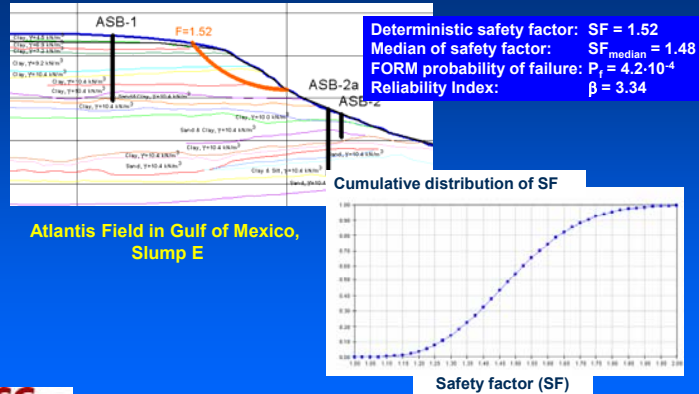
Reliability index β vs. Probability of failure P_f



First-Order Reliability Method Using EXCEL™

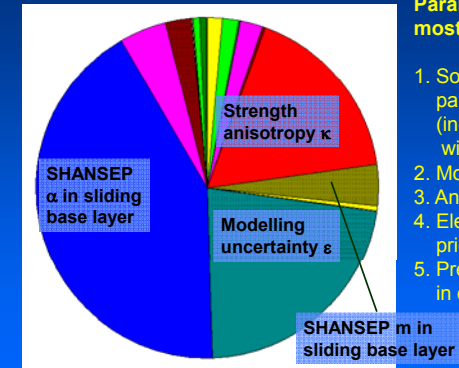


Example of FORM analysis



Atlantis Field in Gulf of Mexico,
Slump E

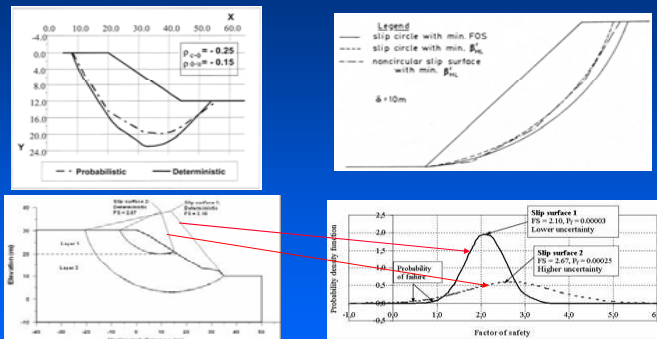
Sensitivity factors for random variables



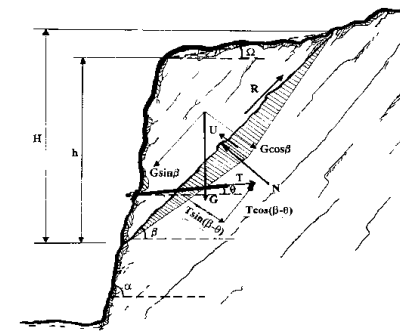
Parameters contributing
most to total uncertainty:

1. Soil shear strength parameters α and m (increasing importance with depth)
2. Modelling uncertainty
3. Anisotropy parameter
4. Elevation of seabed prior to previous slide
5. Preconsolidation stress in deep layers

Deterministic and probabilistic critical failure surfaces often do not coincide



ROCK BLOCK STABILITY ANALYSIS



ROCK BLOCK STABILITY ANALYSIS

Forces acting on the block:

- total weight of the block, **G**
- force in the rock bolts, **T**
- lifting force due to pore pressure in the joint, **U**
- effective normal force on the joint plane, **N**
- shear force on the joint plane, **R**

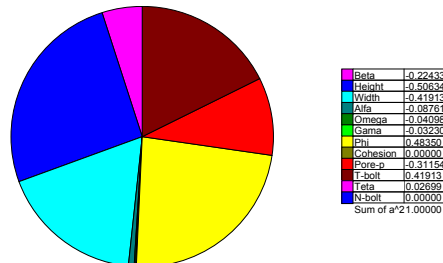
$$\text{Safety Factor: } FS = (c \cdot \text{Area} + N \cdot \tan \phi) / R$$

ROCK BLOCK STABILITY ANALYSIS

Random variable	Distribution	Mean	Standard deviation
Joint angle, β	Normal	63°	2°
Height of block, h	Lognormal	2.0 m	0.2 m
Effective width, B	Lognormal	1.0 m	0.1 m
Front face slope, α	Normal	90°	2°
Upper face slope, Ω	Lognormal	10°	1°
Unit weight of rock, $\rho \cdot g$	Normal	27.0 kN/m ³	1.0 kN/m ³
Average pore pressure on joint plane, u	Normal	5.0 kPa	0.5 kPa
Mohr-Coulomb friction angle of joint plane, ϕ	Normal	62°	3°
Capacity of one rock bolt	Lognormal	10.0 kN	1.0 kN
Rock bolt angle, θ	Normal	-15°	1°
Cohesion of joint, c	Fixed	0 kPa	-
Number of rock bolts per unit width	Fixed	2	-

ROCK BLOCK STABILITY ANALYSIS

Representative Alphas of Variables FLIM(1) [BLOKSTAB.PTJ]



Probability of failure $P_f = 1.3\%$

UNIVERSITY OF OSLO, MAY 10, 2004

RELIABILITY-BASED FOUNDATION DESIGN

(Example application of FORM)

KOK KWANG PHOON

NATIONAL UNIVERSITY OF SINGAPORE

BACKGROUND

- RELIABILITY ANALYSIS IS THE CONSISTENT EVALUATION OF DESIGN RISK USING PROBABILITY THEORY
- RELIABILITY-BASED DESIGN (RBD) IS ANY METHODOLOGY THAT USES RELIABILITY ANALYSIS, EXPLICITLY OR OTHERWISE

Reliability-Based Design

- Reliability analysis is the consistent evaluation of **probability of failure** using probability theory
- **Reliability-based design (RBD)** is any methodology that uses reliability analysis, explicitly or otherwise
- RBD requires access to **tools for doing reliability analysis** and a conscious choice of **acceptable probability of failure**

Conventional Factor of Safety (Working Stress Design)

Criterion: Load < Strength / FS $F_n < \frac{Q_n}{FS}$

Factor of safety (FS) accounts for

- Variations in loads & materials
- Inaccuracies in design equations and modelling approximations
- Construction effects etc.

UNCERTAINTIES IMPLICITLY RECOGNIZED

FS FROM PRECEDENTS & JUDGMENT

ITEM	FS
EARTHWORKS	1.3 - 1.5
RETAINING STRUCTURES	1.5 - 2.0
FOUNDATIONS	2.0 - 3.0
UPLIFT HEAVE	1.5 - 2.0
EXIT GRADIENT, PIPING	2.0 - 3.0
PILE LOAD TESTS	1.5 - 2.0

Data after Terzaghi & Peck (1948, 1967)

DRILLED SHAFT IN UNDRAINED UPLIFT

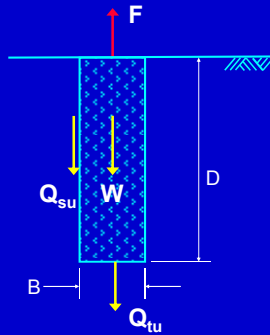
ULTIMATE LIMIT STATE (ULS)

$$Q_u = Q_{su} + Q_{tu} + W$$

$$Q_{su} = \pi B D \alpha s_u$$

$$\alpha = 0.31 + 0.17 p_a / s_u$$

$$Q_{tu} = (-\Delta u - u_i) A_{tip}$$

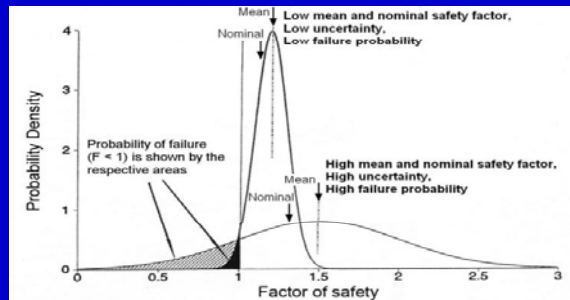


FS IS AMBIGUOUS

Design Assumption	Design Equation	Q_{ud} (kN) for FS = 3	Q_u / Q_{ud} ("actual" FS)
1	$Q_{ud} = (Q_{su} + Q_{tu} + W) / FS$	170.7	3.0
2	$Q_{ud} - W = (Q_{su} + Q_{tu}) / FS$	214.2	2.4
3	$Q_{ud} = (Q_{su} + W) / FS$	108.9	4.7
4	$Q_{ud} - W = Q_{su} / FS$	152.4	3.4
5	$Q_{ud} = W / FS$	21.8	23.5

Note: Q_{su} = side resistance = 261.8 kN, Q_{tu} = tip resistance = 184.4 kN, W = shaft weight = 65.3 kN, Q_{ud} = design uplift capacity, FS = factor of safety, Q_u = available uplift capacity = $Q_{su} + Q_{tu} + W = 511.6$ kN

Lack of clarity between FS & probability of failure



NOTE: Failure probability = Prob (safety factor < 1)

OBJECTIVE OF RBD

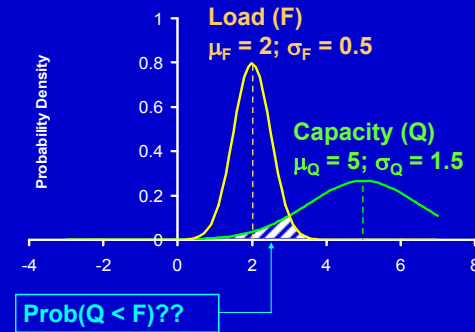
$$p_f = \text{Prob}(Q < F) < p_T$$

- Design risk quantified by probability of failure (p_f)
- Conscious choice of acceptable target failure probability (p_f)
- Same as controlling % "failures" in weighted parametric study

RELIABILITY ANALYSIS

- RENDERS UNCERTAINTY & RISK INTO PRECISE MATHEMATICAL TERMS THAT CAN BE EVALUATED CONSISTENTLY
- UNCERTAIN Q AND F MODELLED AS **RANDOM VARIABLES**
- p_f FROM SIMPLE FORMULAE OR FIRST-ORDER RELIABILITY METHOD (FORM)

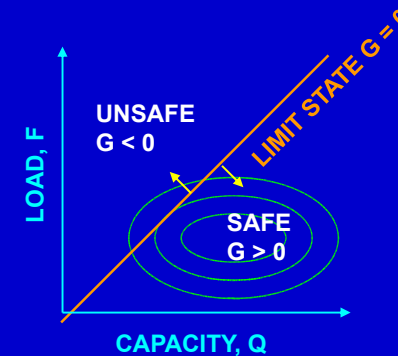
SIMPLE EXAMPLE



- ASSUME Q & F UNCORRELATED NORMAL RANDOM VARIABLES

$$\begin{aligned} p_f &= \text{Prob}(Q < F) \\ &= \text{Prob}(Q - F < 0) \\ &= \text{Prob}(G < 0) \end{aligned}$$

- **G = PERFORMANCE FUNCTION**



- FOR THIS SIMPLE CASE, $G = Q - F$ IS ANOTHER **NORMAL** RANDOM VARIABLE

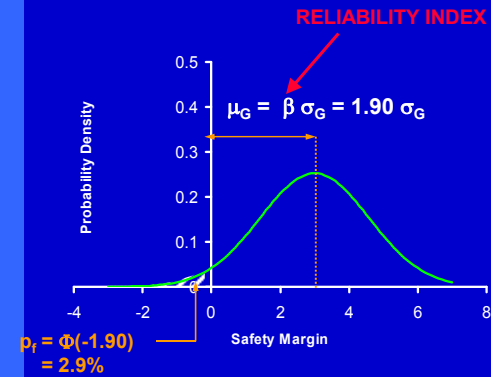
- MEAN OF G IS

$$\mu_G = \mu_Q - \mu_F$$

- VARIANCE OF G IS

$$\sigma_G^2 = \sigma_Q^2 + \sigma_F^2$$

$$\begin{aligned}\beta &= \frac{\mu_G}{\sigma_G} \\ &= \frac{\mu_Q - \mu_F}{\sqrt{\sigma_Q^2 + \sigma_F^2}} \\ &= 1.90\end{aligned}$$



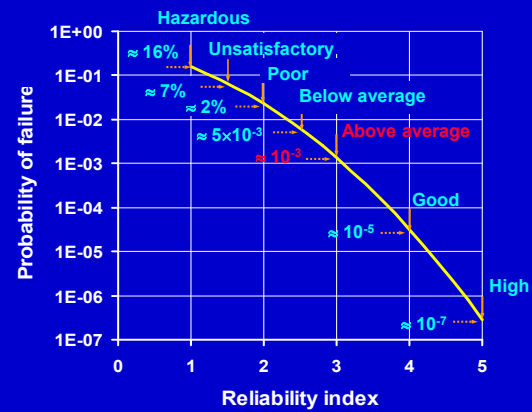
RELIABILITY INDEX

- p_f IS CUMBERSOME TO USE BECAUSE IT IS VERY SMALL
- p_f CARRIES THE NEGATIVE CONNOTATION OF “FAILURE”
- β (RELIABILITY INDEX) IS MORE CONVENIENT & PALATABLE TO USE

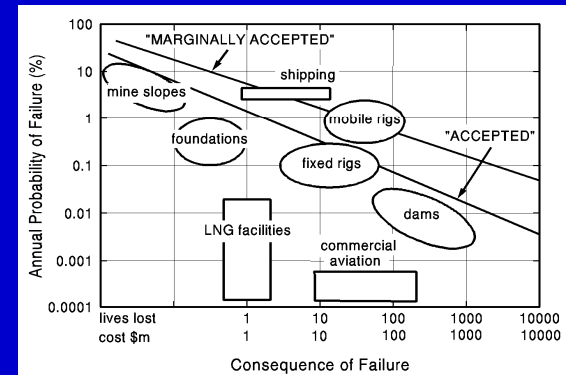
EASY TO CONVERT USING MS EXCEL™

$$\begin{aligned}p_f &= \Phi(-\beta) \\ &= \text{NORMSDIST}(-\beta)\end{aligned}$$

$$\begin{aligned}\beta &= \Phi^{-1}(1 - p_f) \\ &= \text{NORMSINV}(1 - p_f)\end{aligned}$$



Source: US Army Corps of Engineers 1997

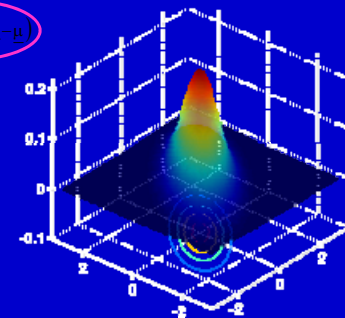


Source: Baecher (1987)

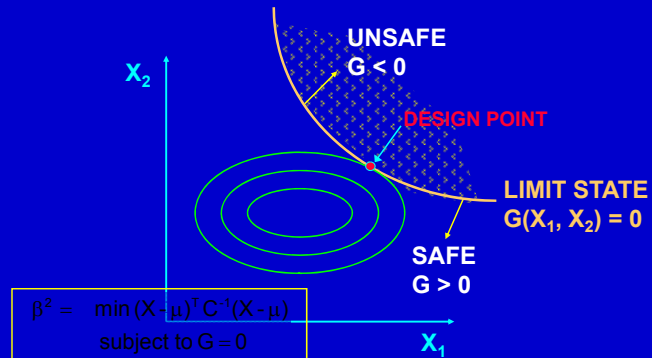
FIRST-ORDER RELIABILITY METHOD (FORM)

MULTI-VARIATE NORMAL

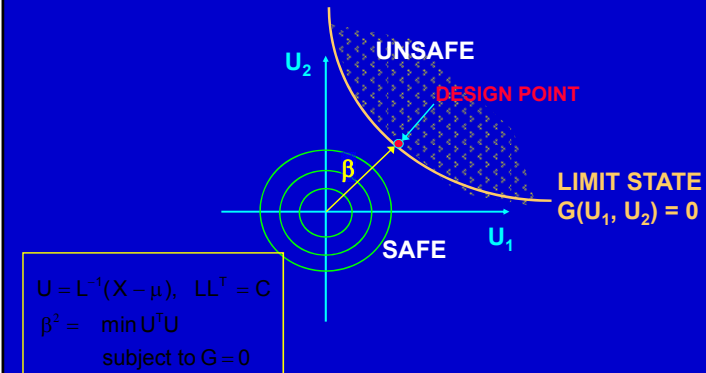
$$f(\underline{X}) = |C|^{-\frac{1}{2}} (2\pi)^{-\frac{n}{2}} e^{-\frac{1}{2}(\underline{X}-\underline{\mu})^T C^{-1}(\underline{X}-\underline{\mu})}$$



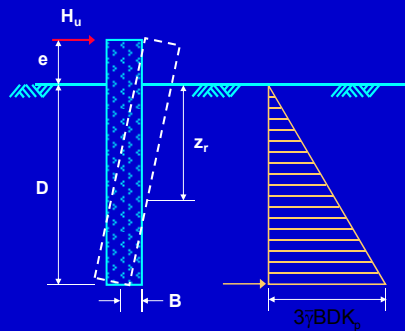
FIRST-ORDER RELIABILITY METHOD USING EXCEL™



FIRST-ORDER RELIABILITY METHOD USING EXCEL™



Example: DRILLED SHAFT LATERAL-MOMENT LOADING



SIMPLIFIED BROMS METHOD

PERFORMANCE FUNCTION

$$H_u = 0.5 \frac{\bar{\gamma} B D^3 K_p}{(e + D)} = 0.5 \frac{\bar{\gamma} B D^3 \tan^2(45^\circ + \phi/2)}{(e + D)}$$

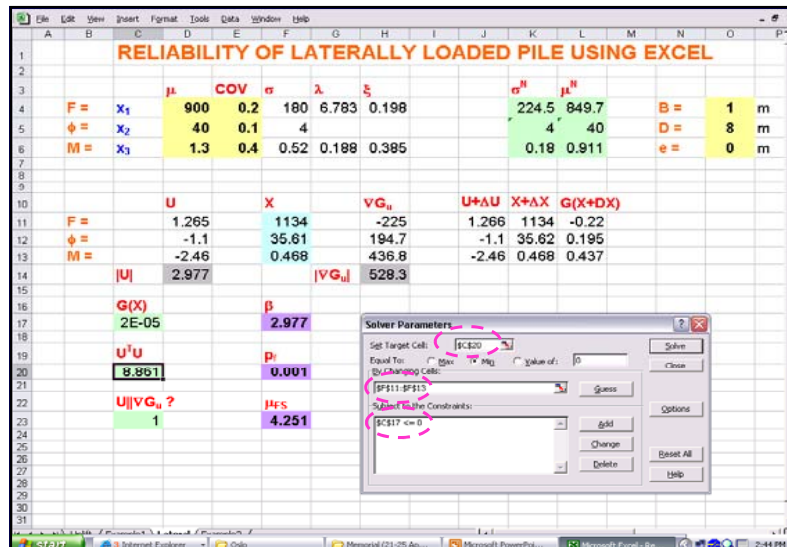
$$H_h = M H_u$$

$$G = H_h - F$$

LOGNORMAL
MEAN = 1.3
COV = 40%

NORMAL
MEAN = 35° to 45°
COV = 5% to 20%

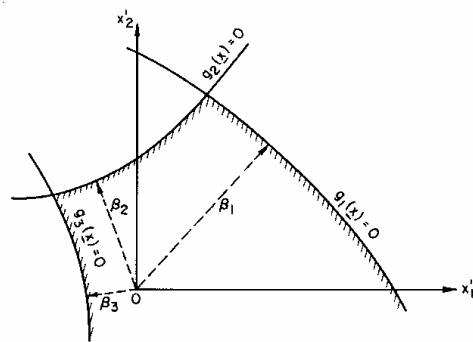
LOGNORMAL
MEAN = 0.85 NOMINAL
COV = 10% to 20%



CONCLUDING REMARKS

- RBD PROVIDES A CONSISTENT METHOD FOR CONTROLLING DESIGN RISK
- TWO KEY ITEMS NEEDED:
 - (1) TOOL FOR RELIABILITY ANALYSIS
 - (2) TARGET ACCEPTABLE FAILURE PROBABILITY
- RELIABILITY ANALYSIS CAN BE EASILY CARRIED USING EXCEL

System – Multiple failure criteria

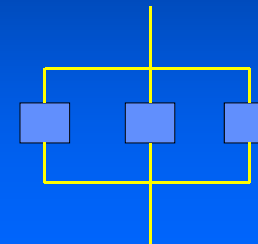


Multiple modes of failure

Systems



Series system



Parallel system

Series system



Figure 4/14

The reliability R of a series system is given by the probability that neither E_1 nor E_i nor E_n , nor any of its elements will fail. This probability is given by:

$$R = (1 - p_{f1}) (1 - p_{f2}) \dots (1 - p_{fn}) = \prod_{i=1}^n (1 - p_{fi}) \quad (4.40)$$

If all elements of a series system are *perfectly correlated*, e.g., all are produced from the same batch of material, then:

$$P_f = \max [p_{fi}] \quad (4.42)$$

Parallel system

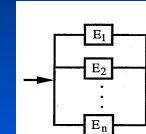
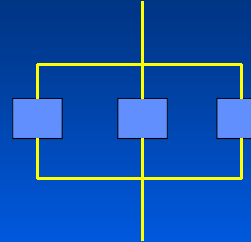


Figure 4/15

$$P_f = p_{f1} \cdot p_{f2} \dots p_{fn} = \prod_{i=1}^n p_{fi} \quad (4.44)$$

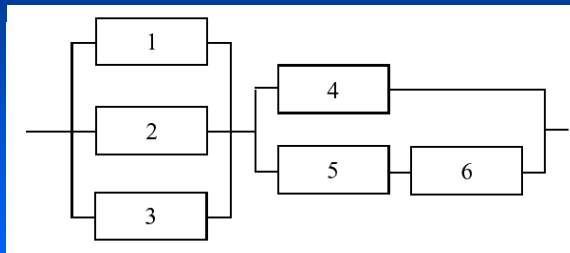
Again *statistical independence* of the elements is a condition.

$$P_F = \prod_{i=1}^n P(F_i)$$

If all elements are *completely correlated*, then:

$$P_f = \min [p_{fi}]$$

Mixed systems



Example: Oil production from Statfjord Field

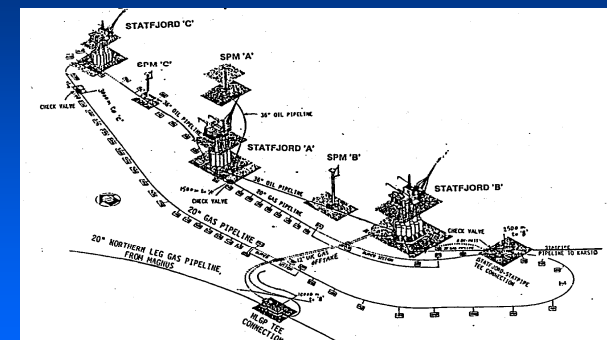
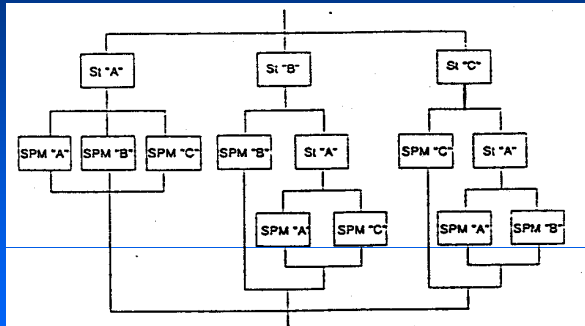


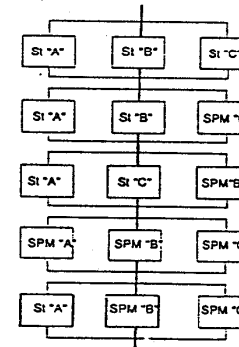
Fig. 3 Layout of platforms and pipelines at Statfjord Field

Example: Oil production from Statfjord Field



a) Representation of oil production system at the Statfjord Field

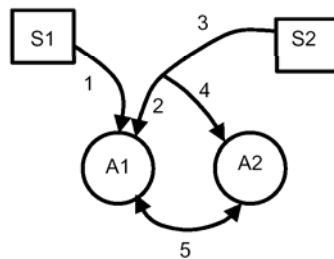
Example: Oil production from Statfjord Field



b) Minimal cut set representation of the system

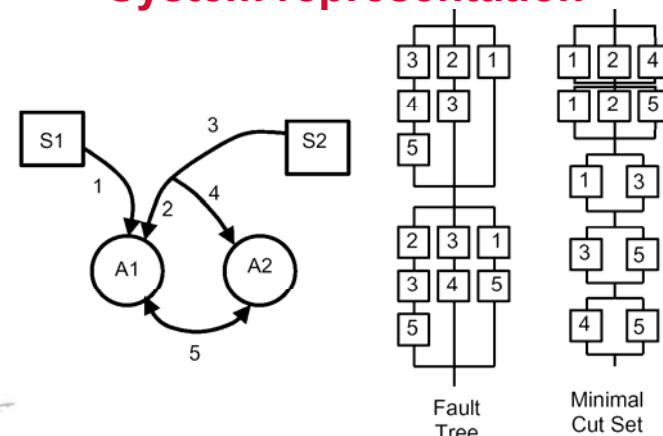
Minimal Cut Set

Example: Lifeline system



Consider the water supply system above under earthquake loading. Two source S1 and S2 supply two areas A1 and A2. The arrows indicate the direction of flow. The system is said to failure if either of the areas loses drinking water. Draw the minimal cut set for evaluating system reliability.

System representation



First-Order, Second Moment

Consider 3 springs in series with the following parameters:

Parameter	Mean value	Standard deviation
K_1 (kN/m)	25	2.5
K_2 (kN/m)	15	3
K_3 (kN/m)	30	2

Estimate the mean value of $K_{\text{equivalent}}$ and its standard deviation using FOSM

$$\frac{1}{K_{\text{equivalent}}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} \rightarrow K_{\text{equivalent}} = \frac{K_1 K_2 K_3}{K_1 K_2 + K_1 K_3 + K_2 K_3}$$

$$\mu_{K_{\text{equivalent}}} \approx (15 \times 25 \times 30) / (15 \times 25 + 25 \times 30 + 15 \times 30) = 7.143 \text{ kN/m}$$

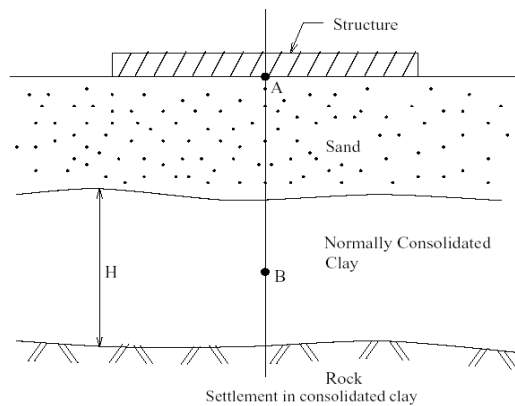
Variable X_i	$\partial K_{\text{equivalent}} / \partial X_i$	Value of $\partial K_{\text{equivalent}} / \partial X_i$ At mean value of parameters	$(\partial K_{\text{equivalent}} / \partial X_i)^2$	$(\partial K_{\text{equivalent}} / \partial X_i)^2 \cdot \sigma_{X_i}^2$
K_1	$\left(\frac{K_2 K_3}{K_1 K_2 + K_1 K_3 + K_2 K_3} \right)^2$	0.0816	0.0067	0.0419
K_2	$\left(\frac{K_1 K_3}{K_1 K_2 + K_1 K_3 + K_2 K_3} \right)^2$	0.2268	0.0514	0.4626
K_3	$\left(\frac{K_1 K_2}{K_1 K_2 + K_1 K_3 + K_2 K_3} \right)^2$	0.0567	0.0032	0.0129
$\sigma_{K_{\text{equivalent}}}^2 = \sum (\partial K_{\text{equivalent}} / \partial X_i)^2 \cdot \sigma_{X_i}^2$				0.5174

$$\sigma_{K_{\text{equivalent}}} \approx 0.72, \quad \text{CoV} = 0.72 / 7.143 = 10.1\%$$

$$\text{Sensitivity factor for } K_1 = 0.0419 / 0.5174 = 8.1\%$$

$$\text{Sensitivity factor for } K_2 = 0.4626 / 0.5174 = 89.4\%$$

$$\text{Sensitivity factor for } K_3 = 0.0129 / 0.5174 = 2.5\%$$



$$S = N \left(\frac{C_c}{1 + e_o} \right) H \log_{10} \left(\frac{p_o + \Delta p}{p_o} \right)$$

where N is the model error, C_c is the compression index, p_o is the effective pressure at B, and Δp is the increase in pressure at B.

Given the statistics (where δ is the coefficient of variation)

Variable	Mean	SD	δ
N	1.0	0.100	0.1
C_c	0.396	0.099	0.25
e_o	1.19	0.179	0.15
H	168 inches	8.40	0.05
p_o	3.72 ksf	0.186	0.05
Δp	0.50 ksf	0.100	0.20

If $Y = g(X_1, X_2, \dots, X_m)$ then first order estimates of the mean, μ_Y , and coefficient of variation, δ_Y , of Y are

$$\mu_Y = g(\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_m})$$

$$\delta_Y^2 = \sum_{j=1}^m \left(\frac{\partial g}{\partial X_j} \mu_{X_j} \right)^2 \delta_j^2 = \sum_{j=1}^m S_j^2 \delta_j^2$$

In this case $\mu_g = 1.66$.

Defining $S_j = (\partial S / \partial X_j)(\mu_{X_j} / \mu_g)$, the components contributing to the uncertainty in S can be found as follows:

X_j	μ_{X_j}	δ_j	S_j	$S_j^2 \delta_j^2$	%
N	1.0	0.10	1.0	0.01	8.4
C_c	0.396	0.25	1.0	0.0625	52.4
e_o	1.19	0.15	-0.55	0.0068	5.7
H	168	0.05	1.0	0.0025	2.1
p_o	3.72	0.05	-0.94	0.0022	1.8
Δp	0.50	0.20	0.94	0.0353	29.6

Giving $\delta_g = 0.345$.

The shear strength at the soil-rock interface follows the Mohr-Coulomb rule (cohesion c and friction angle ϕ). Considering a unit area of the slope, from equilibrium considerations we have:

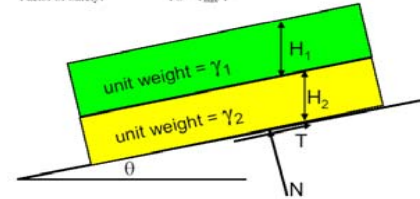
$$N = (\gamma_1 H_1 + \gamma_2 H_2) \cos^2(\theta)$$

$$T = (\gamma_1 H_1 + \gamma_2 H_2) \cos(\theta) \sin(\theta)$$

$$\text{Mohr-Coulomb criterion: } T_{\max} = N \tan(\phi) + c$$

$$FS = \tan(\phi) / \tan(\theta) + 2c / \{(\gamma_1 H_1 + \gamma_2 H_2) \sin(2\theta)\}$$

$$\text{Factor of Safety: } FS = T_{\max} / T$$



Parameter	Mean value	Standard deviation
H_1 (m)	5	0.5
γ_1 (kN/m ³)	18	1.0
H_2 (m)	5	1.0
γ_2 (kN/m ³)	18	1.0
c (kPa)	10	3
ϕ (degrees)	33	2 (= 0.0349 rad)
θ (degrees)	22	0 or 2° (Ques. 2)

$$\mu_{FS} \approx \tan(33^\circ) / \tan(22^\circ) + 2 \times 10 / \{(18 \times 5 + 18 \times 5) \times \sin(44^\circ)\} = 1.767$$

Variable X_i	$\partial FS / \partial X_i$	Value of $\partial FS / \partial X_i$ At mean value of parameters	$(\partial FS / \partial X_i)^2$	$(\partial FS / \partial X_i)^2 \sigma_{X_i}^2$
ϕ	$\frac{1}{\tan(\theta) \cos^2(\phi)}$	3.519	12.383	0.0151
c	$\frac{2}{(\gamma_1 H_1 + \gamma_2 H_2) \cdot \sin(2\theta)}$	0.016	0.0003	0.0023
θ	$\frac{-\tan(\phi)}{\sin^2(\theta)} \cdot \frac{4c \cos(2\theta)}{(\gamma_1 H_1 + \gamma_2 H_2) \sin^2(2\theta)}$	-4.959	24.592	0.0
γ_1	$\frac{-2cH_1}{(\gamma_1 H_1 + \gamma_2 H_2)^2 \sin(2\theta)}$	-0.0044	$1.974 \cdot 10^{-8}$	$1.974 \cdot 10^{-8}$
H_1	$\frac{-2c\gamma_1}{(\gamma_1 H_1 + \gamma_2 H_2)^2 \sin(2\theta)}$	-0.016	0.0003	$6.396 \cdot 10^{-5}$
γ_2	$\frac{-2cH_2}{(\gamma_1 H_1 + \gamma_2 H_2)^2 \sin(2\theta)}$	-0.0044	$1.974 \cdot 10^{-8}$	$1.974 \cdot 10^{-8}$
H_2	$\frac{-2c\gamma_2}{(\gamma_1 H_1 + \gamma_2 H_2)^2 \sin(2\theta)}$	-0.016	0.0003	$2.558 \cdot 10^{-4}$
$\sigma_{FS}^2 = \sum (\partial FS / \partial X_i)^2 \sigma_{X_i}^2$				0.0178

$$\sigma_{FS} \approx 0.1333, \quad \text{CoV} = 0.1333 / 1.767 = 7.54 \%$$

2. Repeat the calculations assuming the angle θ has a standard deviation of 2 degrees.

Variable X_i	$\partial FS / \partial X_i$	Value of $\partial FS / \partial X_i$ At mean value of parameters	$(\partial FS / \partial X_i)^2$	$(\partial FS / \partial X_i)^2 \sigma_{X_i}^2$
ϕ	$\frac{1}{\tan(\theta) \cos^2(\phi)}$	3.519	12.383	0.0151
c	$\frac{2}{(\gamma_1 H_1 + \gamma_2 H_2) \cdot \sin(2\theta)}$	0.016	0.0003	0.0023
θ	$\frac{-\tan(\phi)}{\sin^2(\theta)} \cdot \frac{4c \cos(2\theta)}{(\gamma_1 H_1 + \gamma_2 H_2) \sin^2(2\theta)}$	-4.959	24.592	0.030
γ_1	$\frac{-2cH_1}{(\gamma_1 H_1 + \gamma_2 H_2)^2 \sin(2\theta)}$	-0.0044	$1.974 \cdot 10^{-8}$	$1.974 \cdot 10^{-8}$
H_1	$\frac{-2c\gamma_1}{(\gamma_1 H_1 + \gamma_2 H_2)^2 \sin(2\theta)}$	-0.016	0.0003	$6.396 \cdot 10^{-5}$
γ_2	$\frac{-2cH_2}{(\gamma_1 H_1 + \gamma_2 H_2)^2 \sin(2\theta)}$	-0.0044	$1.974 \cdot 10^{-8}$	$1.974 \cdot 10^{-8}$
H_2	$\frac{-2c\gamma_2}{(\gamma_1 H_1 + \gamma_2 H_2)^2 \sin(2\theta)}$	-0.016	0.0003	$2.558 \cdot 10^{-4}$
$\sigma_{FS}^2 = \sum (\partial FS / \partial X_i)^2 \sigma_{X_i}^2$				0.0478

$$\sigma_{FS} \approx 0.2185, \quad \text{CoV} = 0.2185 / 1.767 = 12.37 \%$$

3. Estimate the failure probability of the slope assuming a normal distribution for FS.

$$\mu_{FS} = 1.767, \quad \sigma_{FS} = 0.2185, \quad P_f = P[\text{Failure}] = P[FS < 1]$$

Note P[...] mean the probability that

Distance between μ_{FS} and $FS = 1$ is $(1.767 - 1) / 0.2185 = 3.51$ standard deviations. In other words, the reliability index $\beta = 3.51$.

Assuming a normal distribution for FS, $P_f = \Phi(-\beta)$. From the Table of Standard Normal Probability (see table at the end of this note):

$$\Phi(-3.51) = 1 - \Phi(3.51) = 1 - 0.999776 = 2.24 \cdot 10^{-4}$$

$$\begin{aligned} M = T_{\max} - T &= \{(\gamma_1 \cdot H_1 + \gamma_2 \cdot H_2) \cdot \cos^2(\theta) \cdot \tan(\phi) + c\} - \{(\gamma_1 \cdot H_1 + \gamma_2 \cdot H_2) \cdot \cos(\theta) \cdot \sin(\theta)\} \\ &= c + (\gamma_1 \cdot H_1 + \gamma_2 \cdot H_2) \cdot \cos(\theta) \{\cos(\theta) \cdot \tan(\phi) - \sin(\theta)\} \\ &= c + (\gamma_1 \cdot H_1 + \gamma_2 \cdot H_2) \cdot \cos(\theta) \cdot \sin(\phi - \theta) / \cos(\phi) \end{aligned}$$

$$\mu_M \approx (18.5 + 18.5) \cdot \cos^2(22^\circ) \cdot \tan(33^\circ) + 10 - (18.5 + 18.5) \cdot \cos(22^\circ) \cdot \sin(22^\circ) = 47.97 \text{ kPa}$$

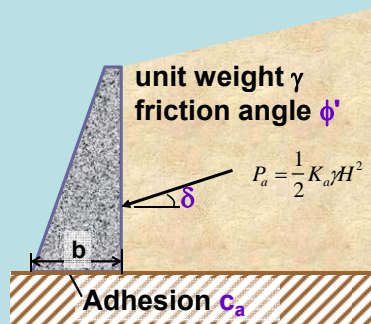
Variable X_i	$\partial M / \partial X_i$	Value of $\partial M / \partial X_i$ At mean value of parameters	$(\partial M / \partial X_i)^2$	$(\partial M / \partial X_i)^2 \cdot \sigma_{X_i}^2$
ϕ	$\frac{(\gamma_1 \cdot H_1 + \gamma_2 \cdot H_2) \cdot \cos^2(\theta)}{\cos^2(\phi)}$	220.00	48400	58.95 (kPa) ²
θ	$-(\gamma_1 \cdot H_1 + \gamma_2 \cdot H_2) \cdot \cos(2\theta - \phi) / \cos(\phi)$	-210.68	44387	54.06 (kPa) ²
c	1	1	1	9 (kPa) ²
γ_1	$H_1 \cdot \{\cos^2(\theta) \cdot \tan(\phi) - \cos(\theta) \cdot \sin(\theta)\}$	1.055	1.113	1.11 (kPa) ²
H_1	$\gamma_1 \cdot \{\cos^2(\theta) \cdot \tan(\phi) - \cos(\theta) \cdot \sin(\theta)\}$	3.797	14.42	3.60 (kPa) ²
γ_2	$H_2 \cdot \{\cos^2(\theta) \cdot \tan(\phi) - \cos(\theta) \cdot \sin(\theta)\}$	1.055	1.113	1.11 (kPa) ²
H_2	$\gamma_2 \cdot \{\cos^2(\theta) \cdot \tan(\phi) - \cos(\theta) \cdot \sin(\theta)\}$	3.797	14.42	14.42 (kPa) ²
$\sigma_M^2 = \sum (\partial M / \partial X_i)^2 \cdot \sigma_{X_i}^2$				142.25 (kPa) ²

$$\sigma_M \approx 11.93 \text{ kPa}, \quad \text{CoV} = 11.93 / 47.97 = 24.9 \%$$

$$\text{Reliability Index: } \beta = \frac{\mu_M}{\sigma_M} = 47.97 / 11.93 = 4.02$$

$$\text{Failure probability: } P_f = \Phi(-\beta) = 2.92 \cdot 10^{-5} = 0.003 \%$$

Retaining wall with random variables ϕ' , δ , and c_a Overturning mode:



$$F_s = \frac{\text{Resisting moment}}{\text{Overturning moment}}$$

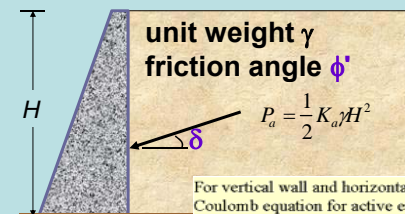
$$\text{PerFunc1} = M_R - M_O = g(\phi', \delta, \dots)$$

Sliding mode:

$$F_s = \frac{\text{Resisting force}}{\text{Pushing force}}$$

$$\text{PerFunc2} = b \times c_a - P_{ah} = g(c_a, \phi', \delta, \dots)$$

Active pressure on retaining wall



For vertical wall and horizontal surface of backfill (i.e., $\alpha = 90^\circ$ and $\lambda = 0^\circ$), the Coulomb equation for active earth pressure coefficient

$$K_a = \left(\frac{\sin(\alpha - \phi') / \sin \alpha}{\sqrt{\sin(\alpha + \delta) + \sqrt{\sin(\phi' + \delta) \sin(\phi' - \lambda) / \sin(\alpha - \lambda)}}} \right)^2$$

simplifies to:

$$K_a = \left(\frac{\cos(\phi')}{\sqrt{\cos(\delta) + \sqrt{\sin(\phi' + \delta) \sin(\phi')}}} \right)^2$$

Ditlevsen (1981), citing Veneziano (1974):

$$\beta = \min_{\underline{x} \in F} \sqrt{(\underline{x} - \underline{\mu})^T \underline{C}^{-1} (\underline{x} - \underline{\mu})}$$

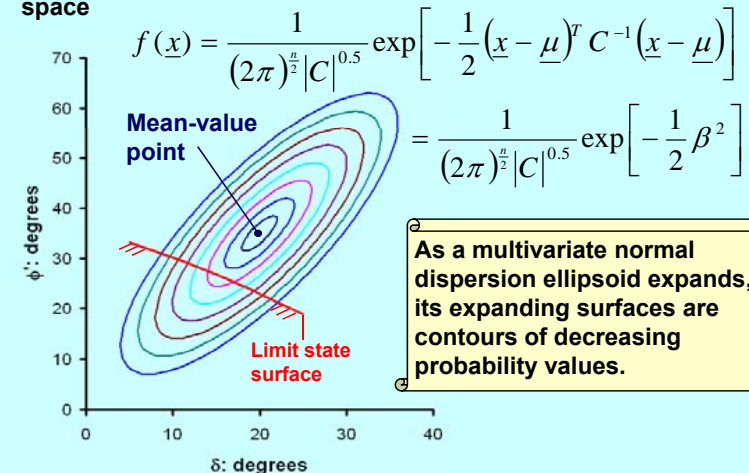
More convenient equivalent form:

$$\beta = \min_{\underline{x} \in F} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i} \right]^T [\underline{R}]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i} \right]}$$

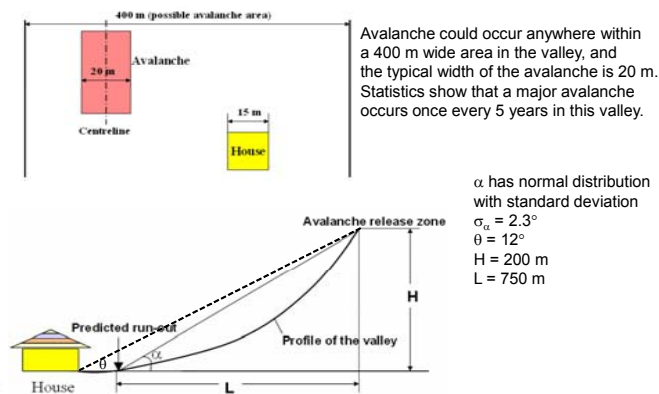
Low and Tang (1997, 2004): Constrained optimization in original space, using Excel array formulas:

Cell object: “= sqrt(mmult(transpose(nx), mmult(minvorse(pymatrix), nx)))”

Expanding ellipsoid perspective in original random variable space



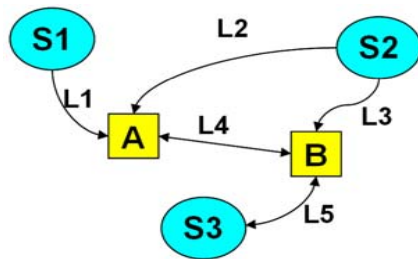
Example: Event tree construction



Example: Event tree construction (cont.)

- Draw an event tree for estimating the annual probability of an avalanche hitting the house.
- Evaluate the annual probability of an avalanche hitting the house using the event tree.
- If an avalanche hits the house, there is a 10% probability that it will be seriously damaged, 70% probability that it will suffer moderate damage, and otherwise it will suffer minor damage. The cost of serious damage is € 1 00 000, the cost of moderate damage is € 20 000, and the cost of minor damage is € 5 000. Extend your event tree to make it possible to evaluate the risk.

Example: Minimal cut set



You are the city engineer for City A and want to estimate the reliability of the system for water supply to this city under earthquake loading. Assuming that only the pipelines and water source S2 might fail, show the minimal cut set for evaluating system reliability (system failure is defined as City A losing drinking water).

Risk Assessment for Submarine Slides

Farrokh Nadim
International Centre for Geohazards,
Norwegian Geotechnical Institute

Griffith University Gold Coast Campus
16-17 February 2009




First Challenge: Terminology

- Probability
- Uncertainty
- **Threat (danger)**
- **Hazard**
- **Risk**
- Consequence
- Failure
- Vulnerability
-



Terminology: Danger (threat)

Danger (Threat): The natural phenomenon that could lead to damage, described in terms of its geometry, mechanical and other characteristics. The danger can be an existing one (such as a creeping slope) or a potential one (such as an earthquake).

The characterisation of a danger or threat does not include any forecasting.



Terminology: Hazard & Risk

Hazard: Probability that a particular danger (threat) occurs within a given period of time.

Risk: Measure of the probability and severity of an adverse effect to life, health, property, or the environment.



Risk and hazard

Hazard = Probability of occurrence of a dangerous event (/ Time unit)

(for example annual probability of slope failure)

Risk = Hazard x Potential worth of loss

(risk could be real or perceived)

Often we are not consistent, and mix up "risk" and "hazard"



More general definition of "Risk"

Risk = f (hazard, elements at risk, vulnerability)

- **Risk:** Expected losses (i.e. the probability of specified negative consequence to life, well-being, property, economic activity and other specified values) due to a particular threat for a given area and reference period
- **Elements at risk:** All objects with a damage potential located within a given area
- **Vulnerability:** Degree of loss resulting from the occurrence of a specific type and magnitude of event



Terminology: Vulnerability

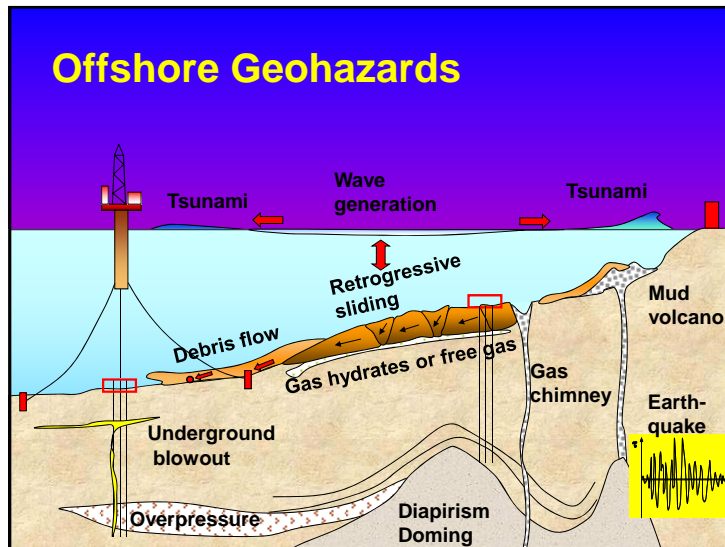
- **Vulnerability** relates to the consequences, or the results of an impact of a natural force, and *not* to the natural process or force itself.
- **Consequences** are generally measured in terms of damage and losses, either on a metric scale in terms of a given currency, or on a non-numerical scale based on social values or perceptions and evaluations.



Social sciences approach

- Any natural hazard, natural risk, and consequently any form of "natural" disaster is *caused by humans* (Geipel 1992).
- If the person – or society – that is threatened or endangered can make decisions and react to potential process occurrence, the hazard becomes a risk. Consequently, if an individual or a society has *no opportunity to make decisions*, the natural event is "just" a hazard, *not a risk* (Pohl & Geipel 2002).





Tsunami damage

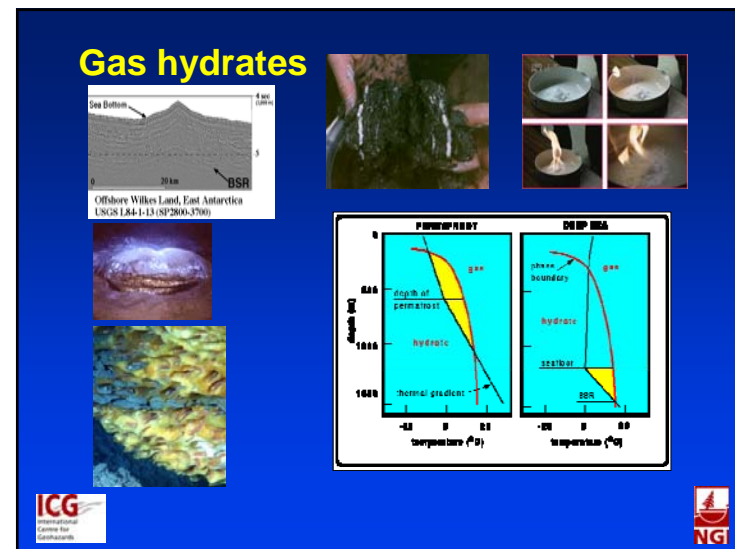
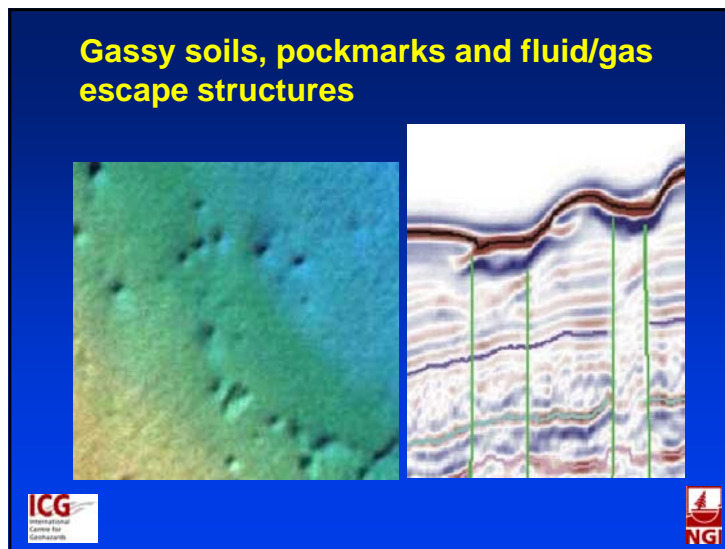
Lithography, Lisbon (Portugal), All Saint's Day 1755

Papua New Guinea Tsunami Vicinity Map*
July 19, 1998

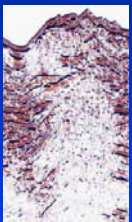
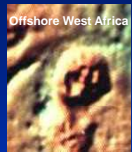
On 17th July 1998, at 08:49 (UTC), a magnitude 7.1 earthquake off the north coast of Papua New Guinea generated a locally very destructive tsunami. The tsunami damage was anomalously large for a quake of the magnitude:

- A fast-moving wall of sand-laden water left detritus in trees up to 17.5 metres above sea level,
- No structures were left standing along the 19 kilometres of coast fronting Sissano Lagoon,
- Concrete was stripped to the reinforcing,
- Some ripped-out trees were carried more than a kilometre,
- More than 2189 people died.

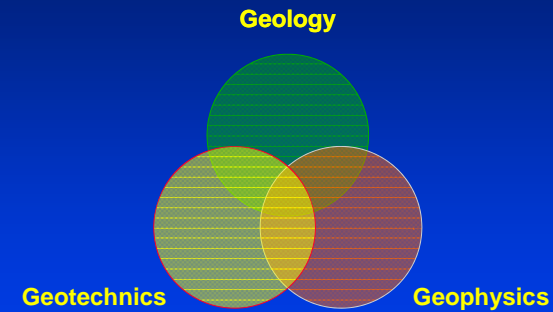
1998 Hawaii tsunami caused by Aleutian earthquake
<http://www.usc.edu/depts/tsunami/saleia/>



Mud volcanoes



Disciplines supplement and complement each other

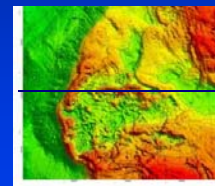
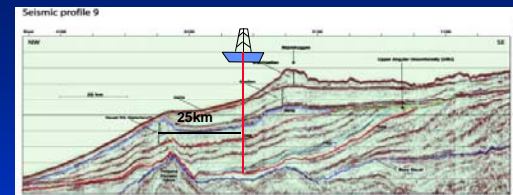


Geophysics

- Often available before offshore sampling
- Regional overview
 - Stratigraphy
 - Structural patterns
 - Geohazards: shallow gas, hydrates, diapirs, old slides
- No ground truth
 - Geo-fantasies can occur
 - Ages often unknown without correlation



Seismic data and a few cores



The seismic profile is nearly 200km long.

The diameter of a core is 10cm!

It takes at least a geologist (and a fair amount of geo-fantasy) to interpret accurately



Geotechnical concerns

- Ability to define relevant and critical failure modes
- Assessment of probability of occurrence
- Calculate/predict consequences
- Uncertainties to addressed:
 - Limited site investigations and extrapolation over large areas and depths
 - Assessment of in situ effective stress and pore pressure conditions
 - Gas hydrates existence and quantification
 - Modelling of triggering mechanisms



Submarine slope stability on gentle slopes

- Field development on the continental slopes
- Enormous historic and paleo slides observed
- Gravity forces increasingly important even at very low slope angles of 0.5 to 3°
- Triggering mechanism not well understood
- Large runout distances, retrogressive sliding upslope/laterally and tsunami generation may threaten 3rd parties in large areas



Submarine Slope Stability on Gentle Slopes (2)

Infinite slope analysis

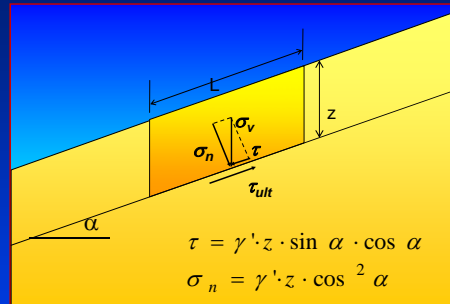
Strength:

$$\tau_{ultd} = (\sigma_n - \Delta u) \cdot \tan \phi'$$

$$\tau_{ultu} = s_u = k(\sigma_n - \Delta u)$$

Pore pressure ratio:

$$r = \frac{\Delta u}{\gamma' \cdot z}$$



$$\tau = \gamma' \cdot z \cdot \sin \alpha \cdot \cos \alpha$$

$$\sigma_n = \gamma' \cdot z \cdot \cos^2 \alpha$$

Drained factor of safety:

$$FS_d = \frac{(\cos^2 \alpha - r) \tan \phi'}{\sin \alpha \cdot \cos \alpha}$$

Undrained factor of safety:

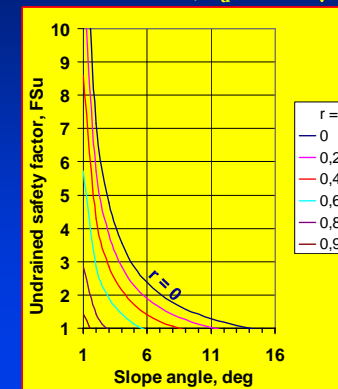
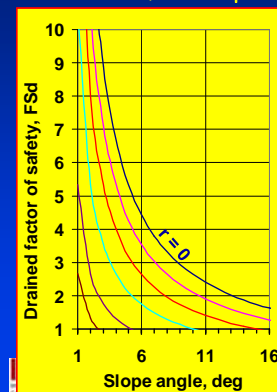
$$FS_u = \frac{k \cdot (\cos^2 \alpha - r)}{\sin \alpha \cdot \cos \alpha}$$



Submarine Slope Stability on Gentle Slopes (3)

Safety factors vs. α and $r = \Delta u / \gamma' z$

Drained; $c' = 0$ $\phi' = 25^\circ$ Undrained NC; $s_u = 0.25 \gamma' z$



Submarine Slope Stability on Gentle Slopes (4)

Pore pressure generating mechanisms

- Rapid sedimentation → Underconsolidation
- Earthquake and shear strain induced pore pressure generation in collapsible and sensitive soils
- Pressure decrease and temperature increase in gassy soils (Climate and human induced)
 - Gas exsolution and free gas expansion
 - Melting of gas hydrates and gas expansion
- Underground blow-outs → pressurizing shallow layers
- Smectite -Illite conversion → Water release $T > 60^{\circ}\text{C}$

**The Storegga Slide (8000 yrsBP)**

Headwall ~ 300 km
Run-out ~ 800 km
Volume ~ 5.600 km³
Area ~ 34.000 km²

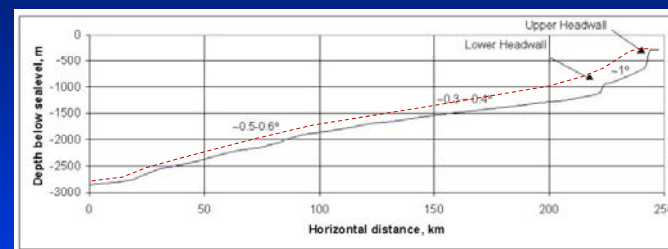
**Answer: The Storegga Slide (~8200 yr. BP)**

Headwall ~ 300 km
Run-out ~ 800 km
Volume ~ 3.000 km³

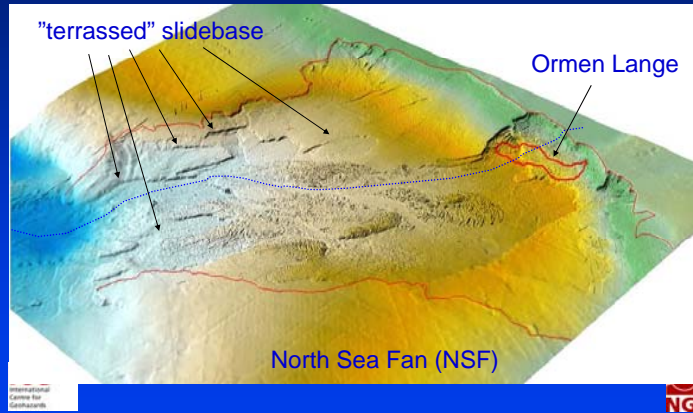
Tsunami:
The slide generated a tsunami that hit the coastlines of Norway, Scotland, Shetland and the Faeroes



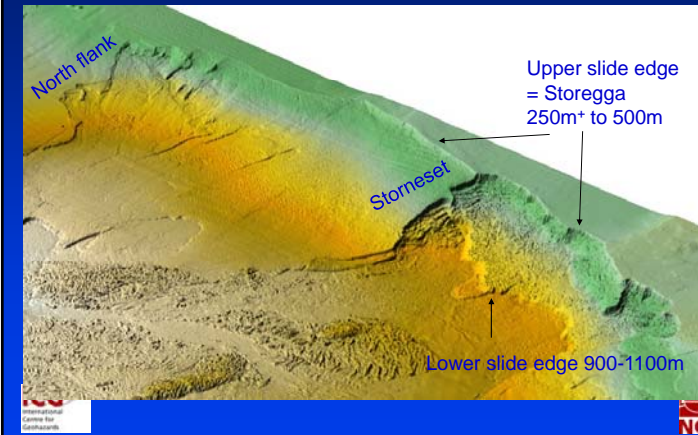
In situ conditions:

Profile from shelf edge to deep basin**How could the Storegga slide develop?**

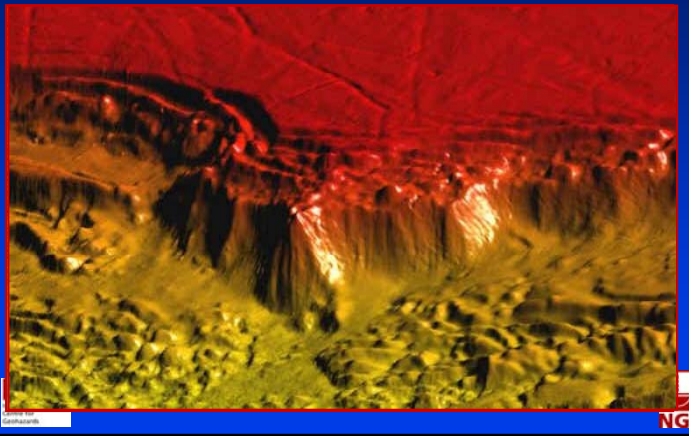
**Answer:
Located in the Storegga slide area**



The shelf edge and the central part



**Technical challenge:
Bathymetry in the Ormen Lange area**



Simulation of the Storegga slide tsunami

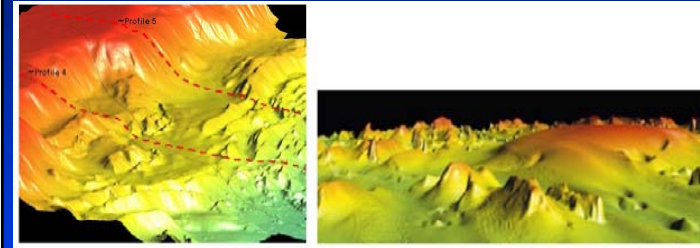


The main questions for the oil and gas industry were:

- Do we have access to this area?
- Is the natural risk related to new slides too high?
- Can field development influence slope stability? New Storegga slide? New tsunami?
- Is it safe to develop the Ormen Lange gas field close to the steep headwalls (30 - 40 deg.) of the Storegga Slide?
- How can we explain the Storegga slope failure when the slope angle was close to 1° prior to the sliding?



Technical challenge: Local bathymetry - routing

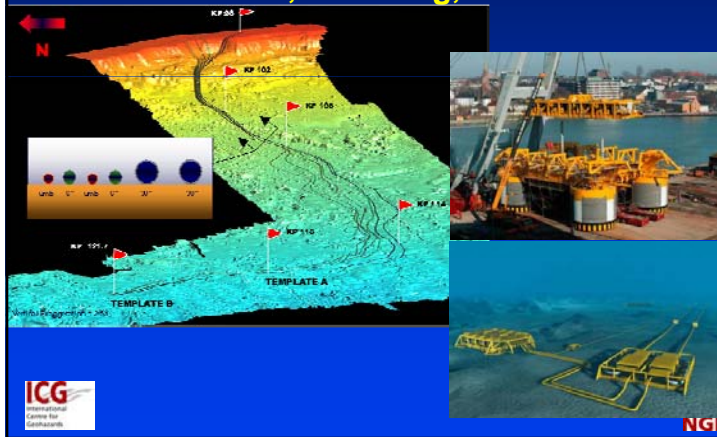


Upper headwall pipeline crossing area

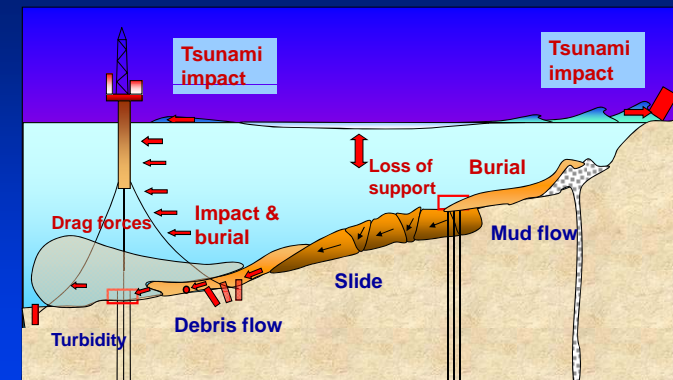
Field development area



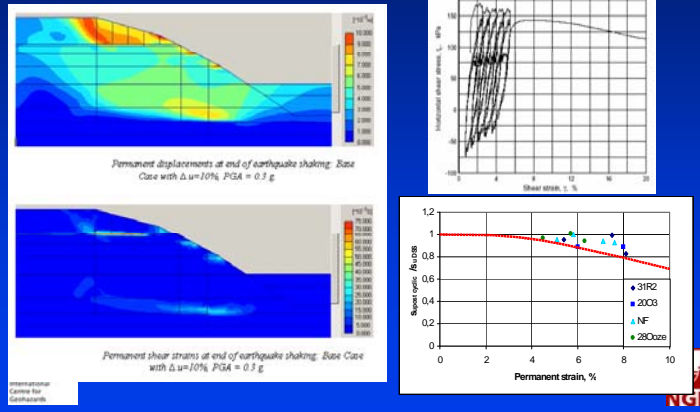
Investments in deepwater field development area: Wells, subsea equipment, pipelines, MEG and umbilicals, trenching, rockfill



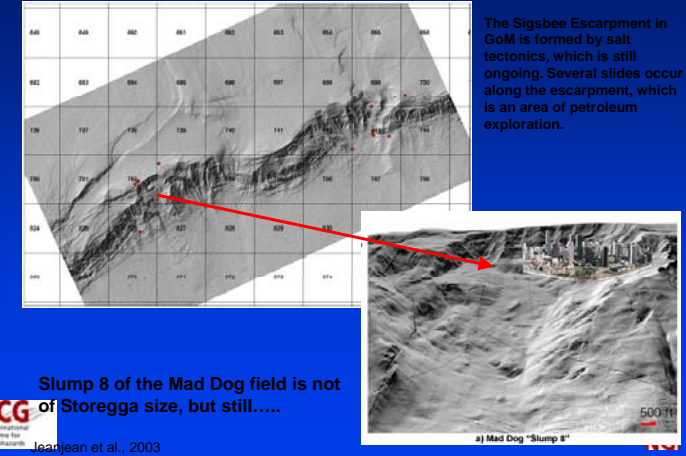
Slide consequences



2-D Earthquake analyses: Post earthquake accumulated displacements and strains



Not only in high latitudes!



Slump 8 of the Mad Dog field is not of Storegga size, but still.....

Jeanjean et al., 2003

Uncertainty in soil shear strength

- The uncertainty in the undrained soil shear strength is derived from the **probabilistic description** of the parameters entering the **SHANSEP** equation in each layer:

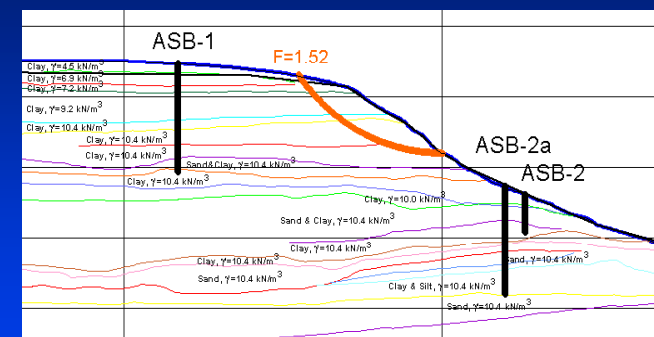
$$\alpha, \gamma', m, h, \Delta\sigma, \kappa$$

NOTE:

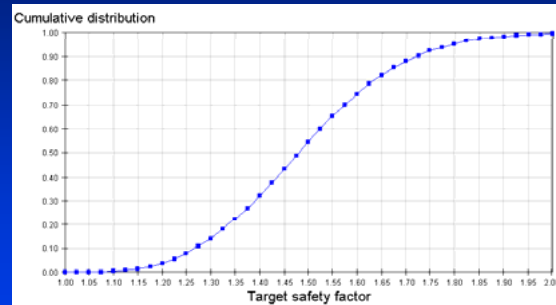
Strength Anisotropy: $s_{u, \text{ at inclination } \theta} = s_u (1 + (\kappa - 1) \sin 2\theta)$
 κ = shear strength anisotropy factor (1.0 - 1.5)



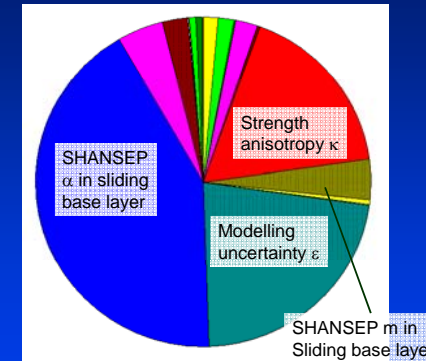
Atlantis Field, Slump E – Undrained stability



Distribution of safety factor - Slump E



Sensitivity factors for random variables - Slump E



Parameters contributing most to total uncertainty:

1. Soil shear strength parameters α and m (increasing importance with depth)
2. Modelling uncertainty
3. Anisotropy parameter
4. Elevation of seabed prior to previous slide
5. Maximum past pressure in deep layers

Mad Dog Prospect Slumps 8_1, 8_2, 8_3 and 8_4

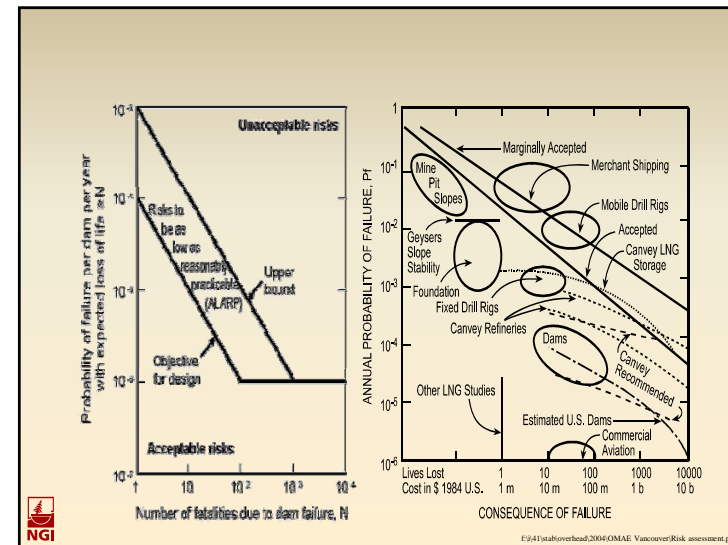
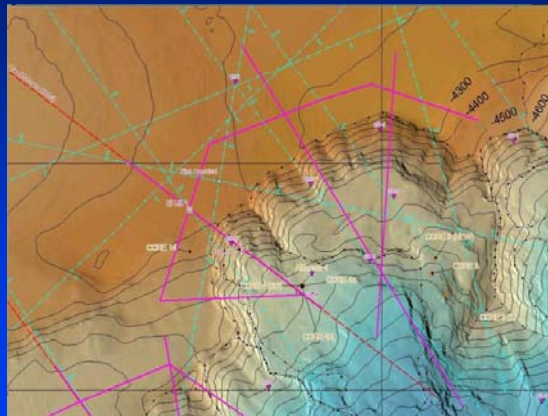
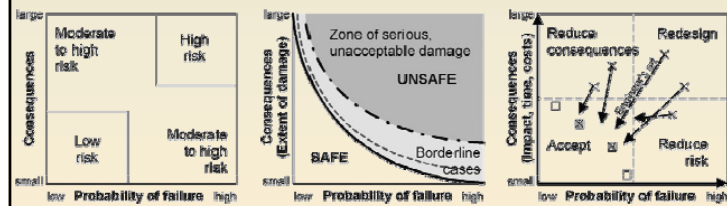
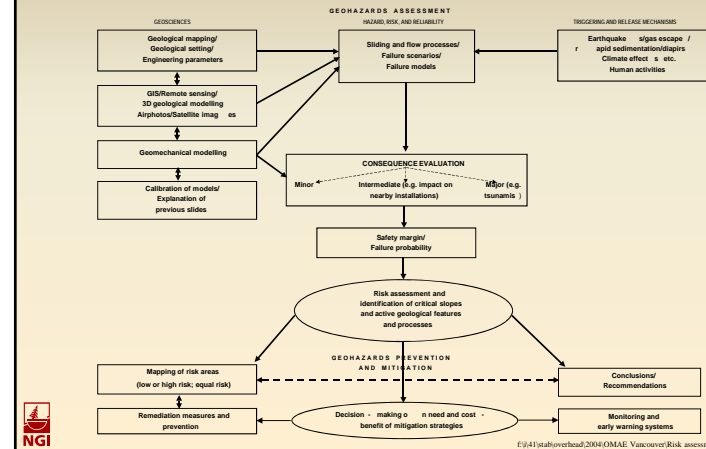


Figure 2 Qualitative risk prioritisation matrix



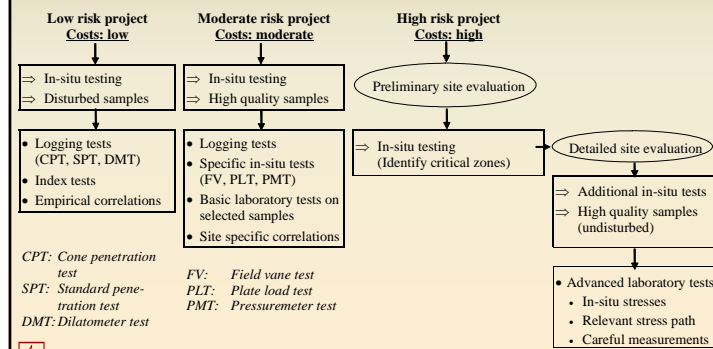
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Figure 3 Typical components of a geohazard study and multi-disciplinary interaction



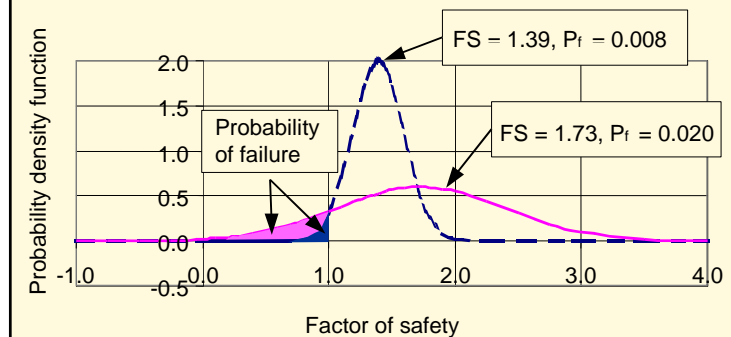
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Figure 4 Risk-based soil investigations



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Figure 5 Safety factor and probability of failure for most heavily loaded pile

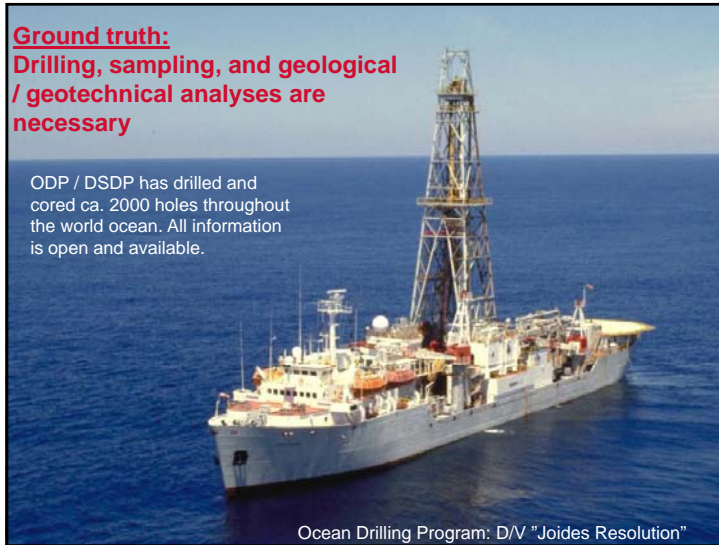


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Ground truth:

Drilling, sampling, and geological / geotechnical analyses are necessary

ODP / DSDP has drilled and cored ca. 2000 holes throughout the world ocean. All information is open and available.



Ocean Drilling Program: D/V "Joides Resolution"

Conclusions

- Geohazard assessment require multi-discipline geoscience cooperation and understanding
- Thorough understanding of natural and human induced effects in order to identify the relevant failure scenarios for field development
- Areal extent and volumes of potential slides on continental slopes can be very large:
 - *Project risk (total damage, local damage - repair)*
 - *3rd party risk*



Conclusions

Challenges for the geotechnical discipline:

- In situ conditions; pore pressure, gas hydrates
- Gassy soils and gas hydrate material models
- Brittle/sensitive soils; sampling disturbance, testing
- Analysis methods for retrogressive sliding that explain observed megaslides and slide initiation processes
- Slide dynamics and consequence assessment; run-out, impact, tsunami
- Assessment of uncertainties in risk analysis

