Probabilistic Methods in Geotechnical Engineering: Risk and Reliability

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NGI

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Why do probabilistic analyses?

- Society, regulations and our clients demand to know the risks quantitatively
- Reliability-based design is becoming standard practice for structural engineers
- Probabilistic analyses complement the conventional deterministic analyses in achieving a safe design, and add great value to the results by modest additional effort

Aim:

Quantify the margin against “failure”

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Engineering failures result from:

- Extreme value of a single parameter
- Combination of small parameter variations
- Gross design or construction error (human factors)
- Unforeseen situations

There is no universal rule

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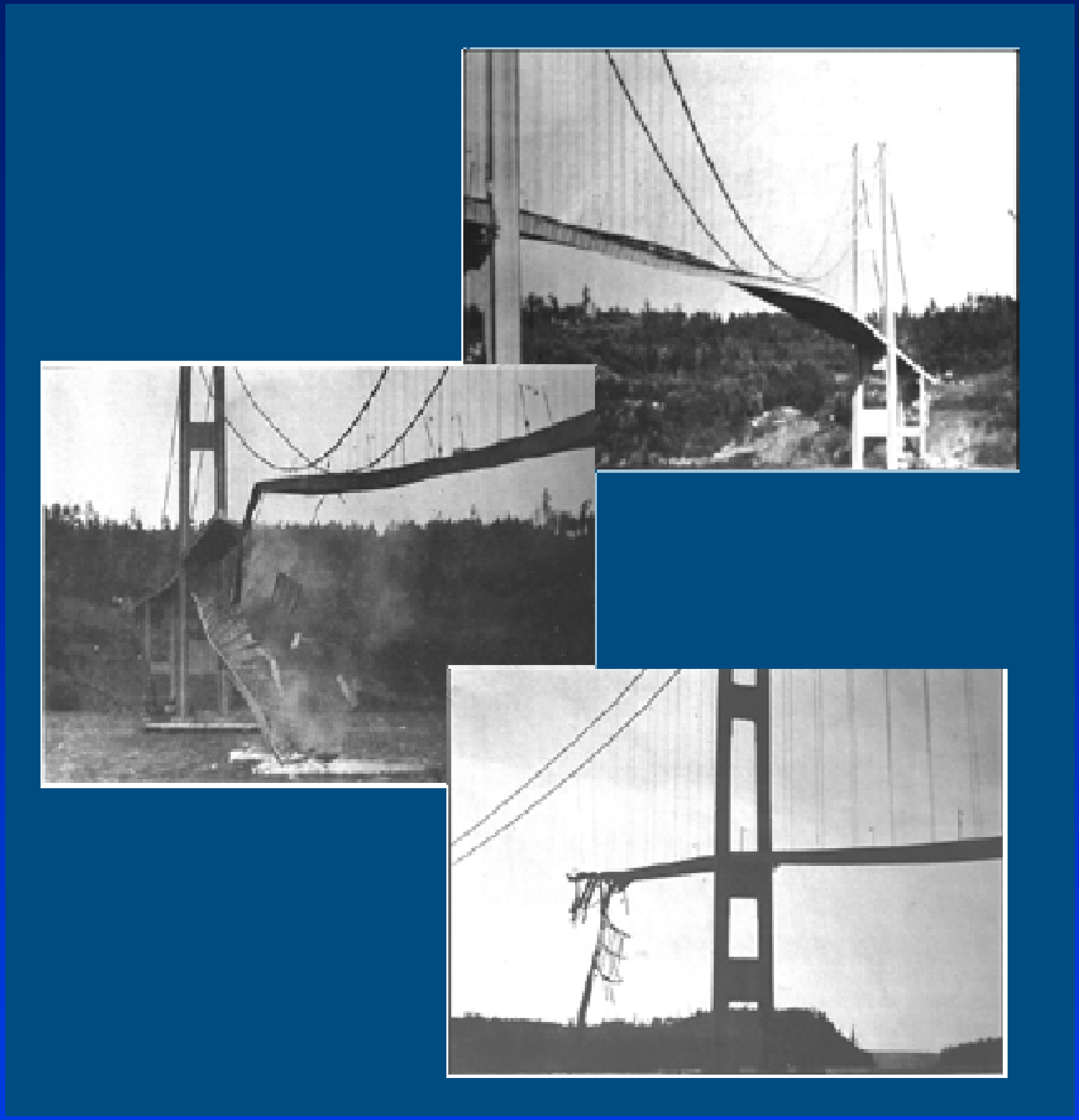

Bridge collapse due to unforeseen dynamic behaviour in certain wind conditions, Tacoma Narrows, USA.

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Example of failure in transporting construction materials due to falsely estimated load or falsely estimated weight of donkey (Ref: Michael Faber)

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Living with uncertainty

In any geotechnical and geological assessment, one must deal with uncertainties because geo-sciences are not exact.

It is better to be

probably *right*…

… than to be

exactly *wrong*

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Sources of Uncertainty

- Limited geo-exploration
- Measurement errors
- Spatial variability of soil and rock properties
- Limited parameter evaluation
- Limitations of calculation models

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Types of uncertainty

Uncertainties associated with an engineering problem can be divided into two groups:

- aleatory (inherent)
- epistemic (lack of knowledge)

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Aleatory Uncertainty

The natural randomness of a property.

The variation in a soil/rock property in the within a geological unit are aleatory uncertainties.

This type of uncertainty cannot be reduced.

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Epistemic Uncertainty

The uncertainty due to lack of knowledge.

Measurement uncertainty and model uncertainty are epistemic uncertainties.

This type of uncertainty can be reduced (by increasing number of tests, improving measurement method or evaluating calculation procedure with model tests,...)

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Sources of uncertainty in geomechanical parameters

Epistemic or Aleatory?

- Limited geo-exploration
- Measurement errors
- Spatial variability of soil and rock properties
- Limited parameter evaluation
- Limitations of calculation models

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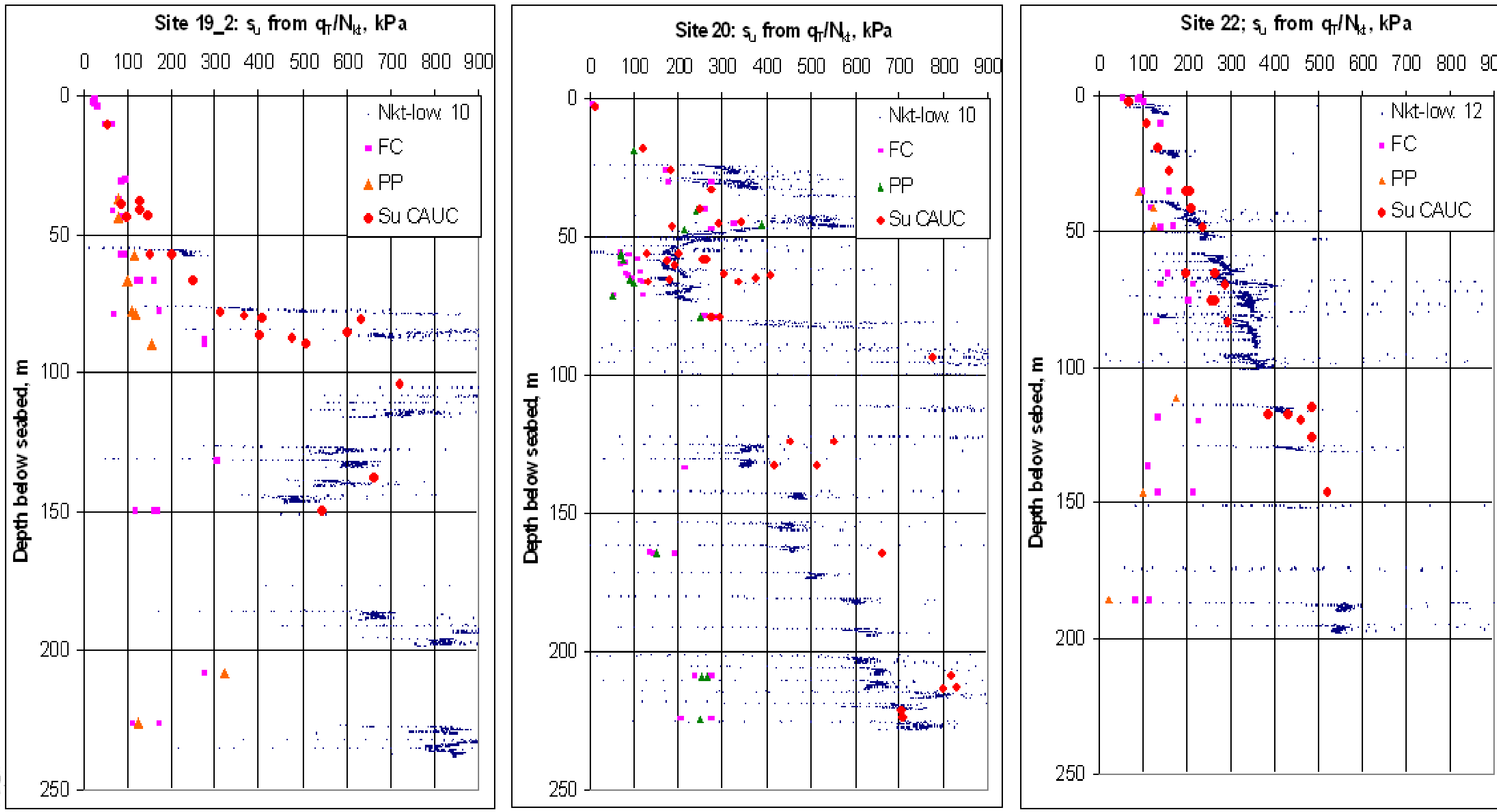
Soil parameters at Ormen Lange – Remoulded shear strength



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Basic Concepts of Probability

Random Variables

Quantities that can take on many values

Discrete random variables - finite number of values

- Number of borings encountering peat at a site
- Date of birth

Continuous random variables - infinite number of values

- Undrained strength of a clay layer
- Unit weight of soil

Basic Concepts of Probability

Continuous Random Variables

Distribution of values described by probability density function (pdf) that satisfies the following conditions:

$$f_X(x)dx \geq 0$$

$$\int_{-\infty}^{\infty} f_X(x)dx = 1$$

$$P[a \leq X \leq b] = \int_{a}^{b} f_X(x)dx$$

The probability that *X* is between *a* and *b* is equal to the area under the pdf between *a* and *b*

Basic Concepts of Probability

Continuous Random Variables

Distribution of values can also be described by a cumulative distribution function (CDF), which is related to the pdf according to

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$$

$$P[a \leq X \leq b] = F_X(b) - F_X(a)$$

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Basic Concepts of Probability

Statistical Characterization of Random Variables

Distribution of values can also be characterized by statistical descriptors

$$\bar{x} = \int_{-\infty}^{x} x f_X(x)\,dx$$

Mean

$$\sigma_X^2 = \int_{-\infty}^{x} (x-\bar{x})^2 f_X(x)\,dx$$

Variance

$$\sigma_X = \sqrt{\sigma_X^2}$$

Standard deviation

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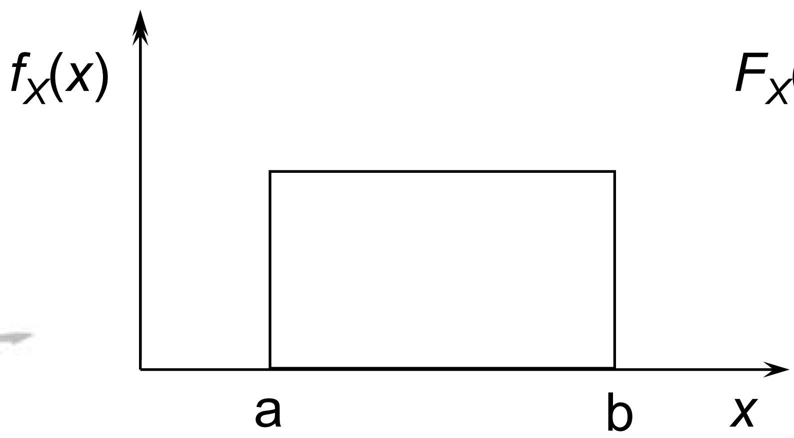
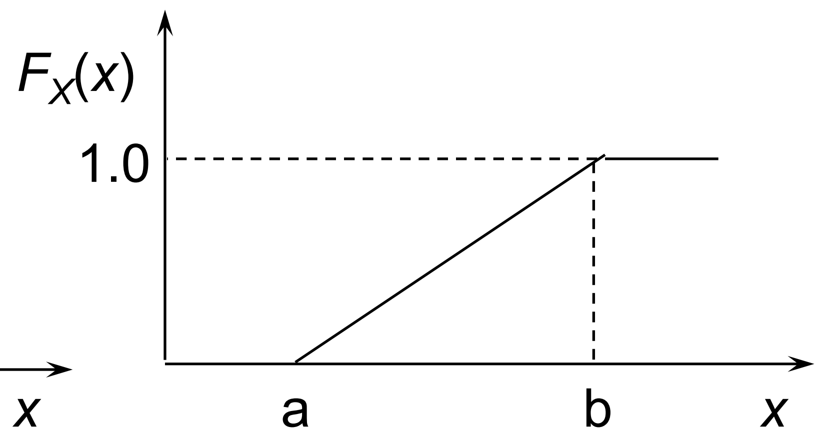
Basic Concepts of Probability

Common Probability Distributions

Uniform distribution

$$f_X(x) = \begin{cases} 0 & \text{for } x < a \\ 1/(b - a) & \text{for } a < x < b \\ 0 & \text{for } x > b \end{cases}$$





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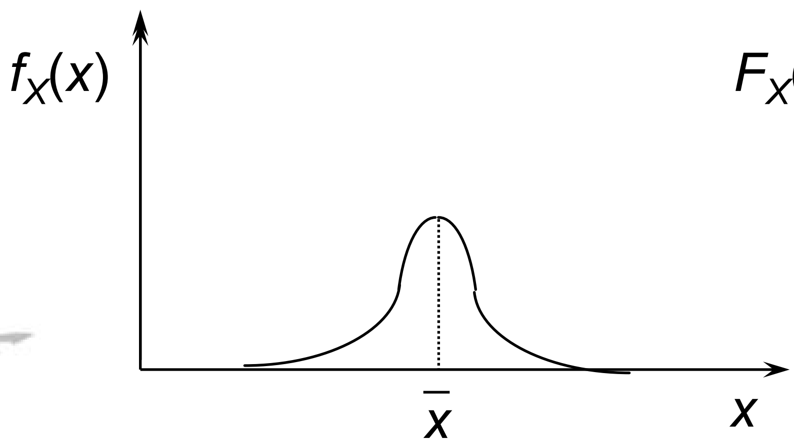
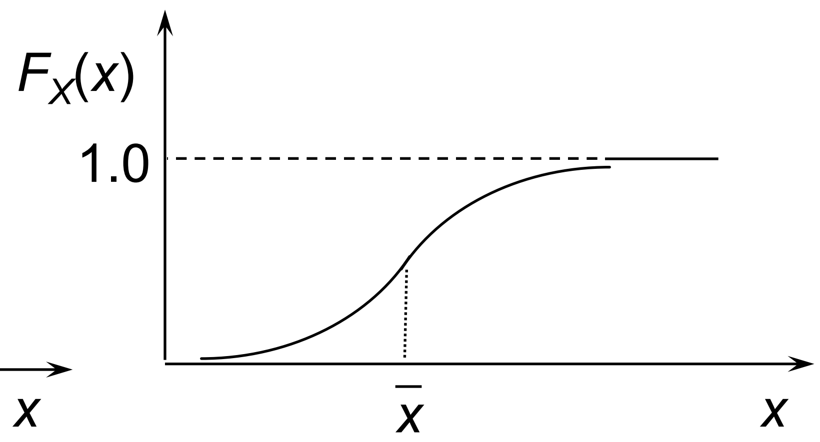
Basic Concepts of Probability

Common Probability Distributions

Normal distribution

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma_X} \exp\left[-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma_X}\right)^2\right]$$





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Basic Concepts of Probability

Common Probability Distributions

Standard normal distribution

Mean = 0

Standard deviation = 1

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2} z^2\right]$$

Values of standard normal CDF commonly tabulated

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Basic Concepts of Probability

Common Probability Distributions

Standard normal distribution

Mapping from random variable to standard normal random variable

$$Z = \frac{X - \bar{x}}{\sigma_X}$$

Compute Z, then use tabulated values of CDF

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Basic Concepts of Probability

Common Probability Distributions

Example: Given a normally distributed random variable, *X*, with $\bar{x} = 270$ and $\sigma_x = 40$, compute the probability that $X < 300$

$$Z = \frac{X - \bar{x}}{\sigma_X} = \frac{300 - 270}{40} = 0.75$$

Looking up Z = 0.75 in CDF table,

$$F_Z(0.75) = 1 - F_Z(-0.75) = 0.7734$$

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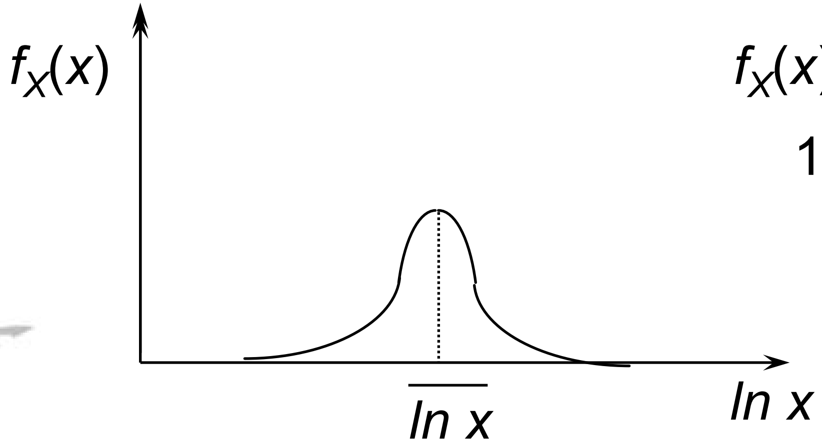
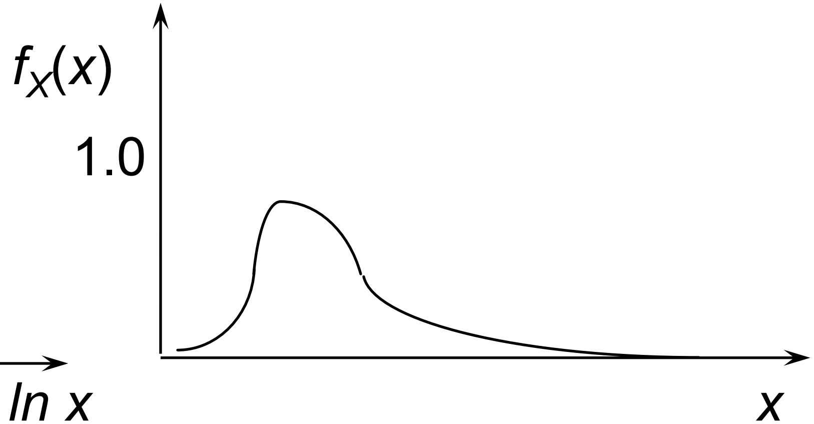
Basic Concepts of Probability

Common Probability Distributions

Lognormal distribution

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma_{\ln x}} \exp\left[-\frac{1}{2} \left(\frac{\ln x - \overline{\ln x}}{\sigma_{\ln x}} \right)^2 \right]$$





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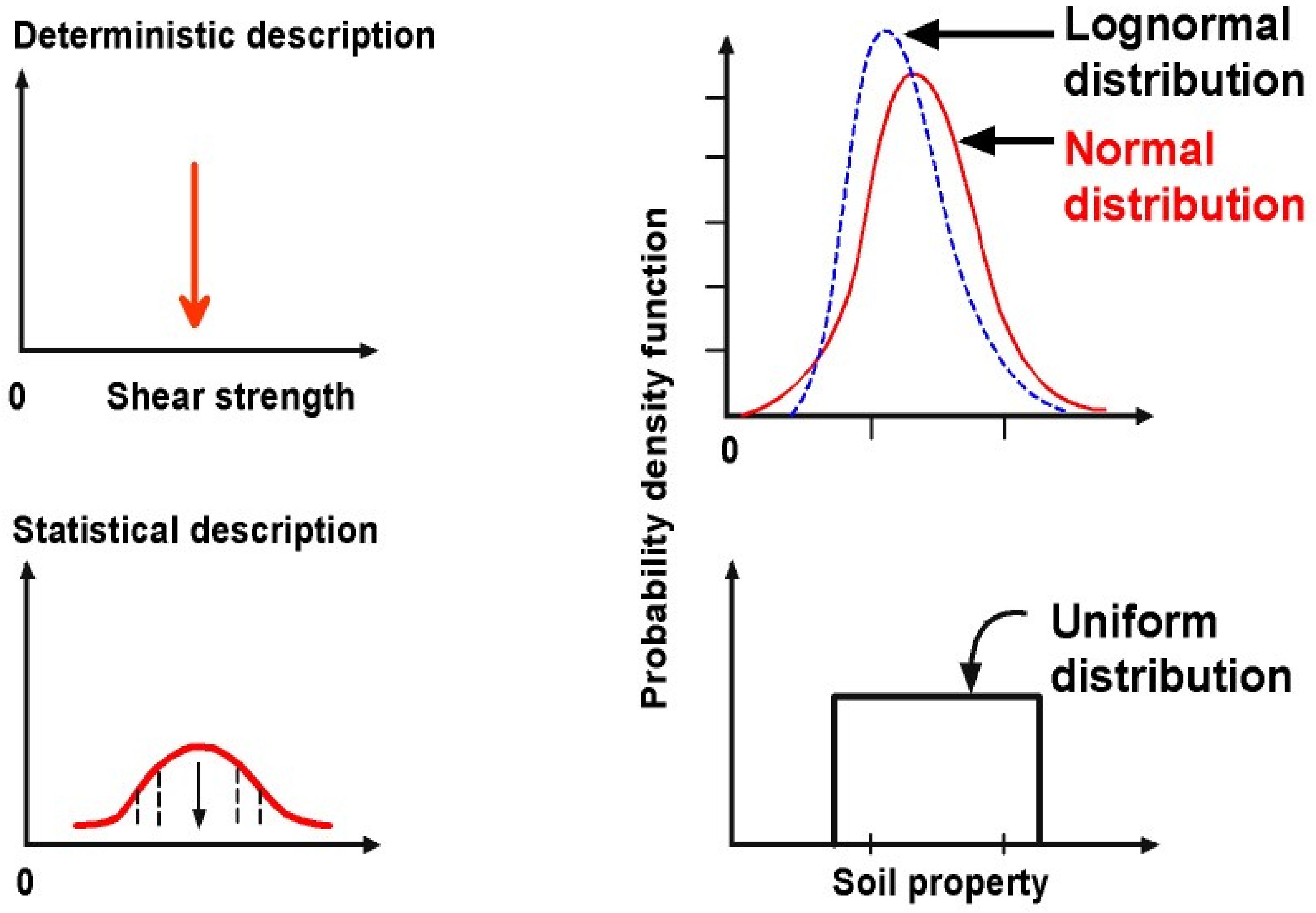

(a) (b)

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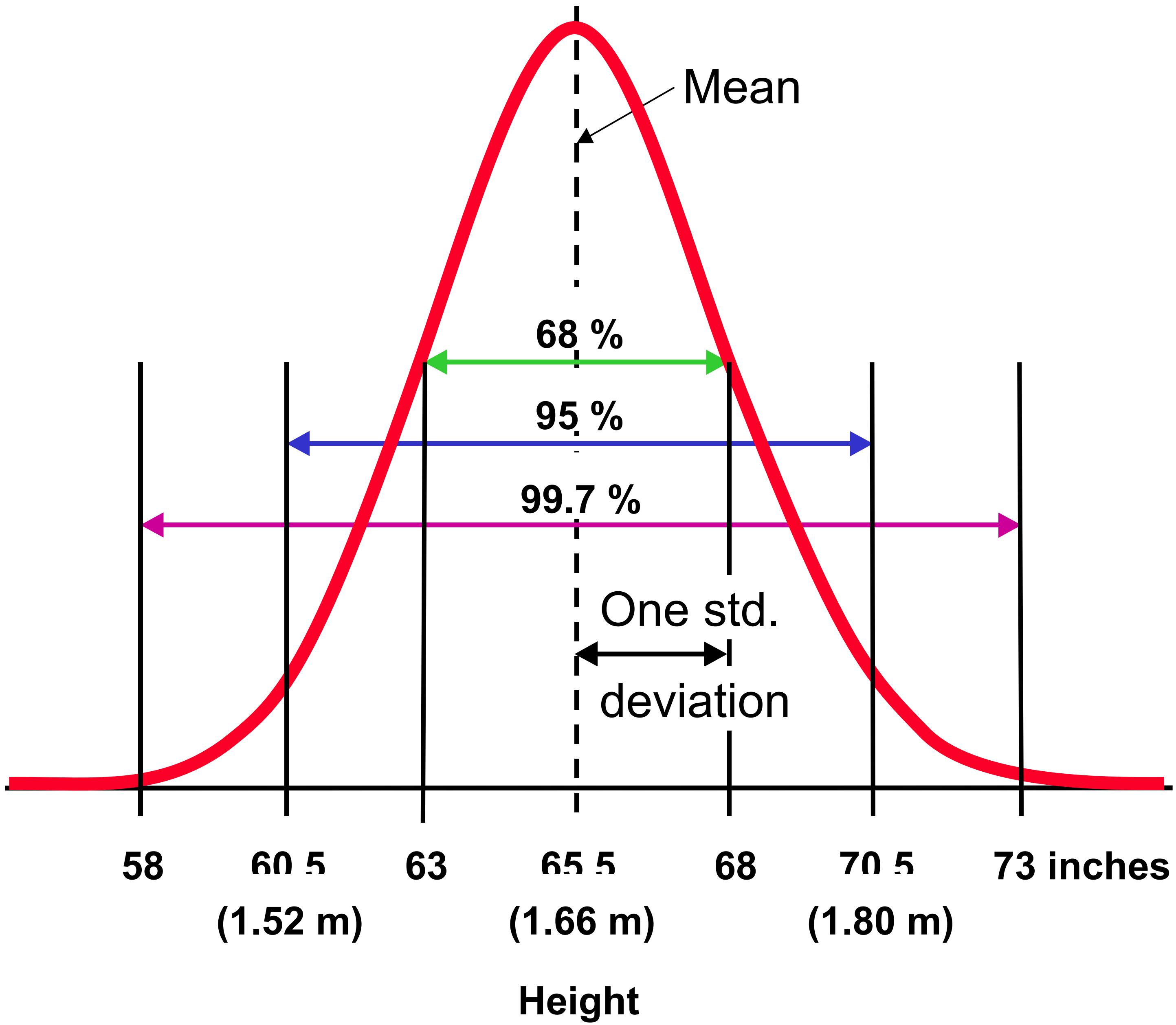

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Necessary contributors to parameter evaluation

- Experience
- Expert judgement

You, as the "expert", are expected to evaluate how large the uncertainties are.

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Data interpretation

Human interpretation and engineering judgment are still the most important issue in automated data processing and analysis

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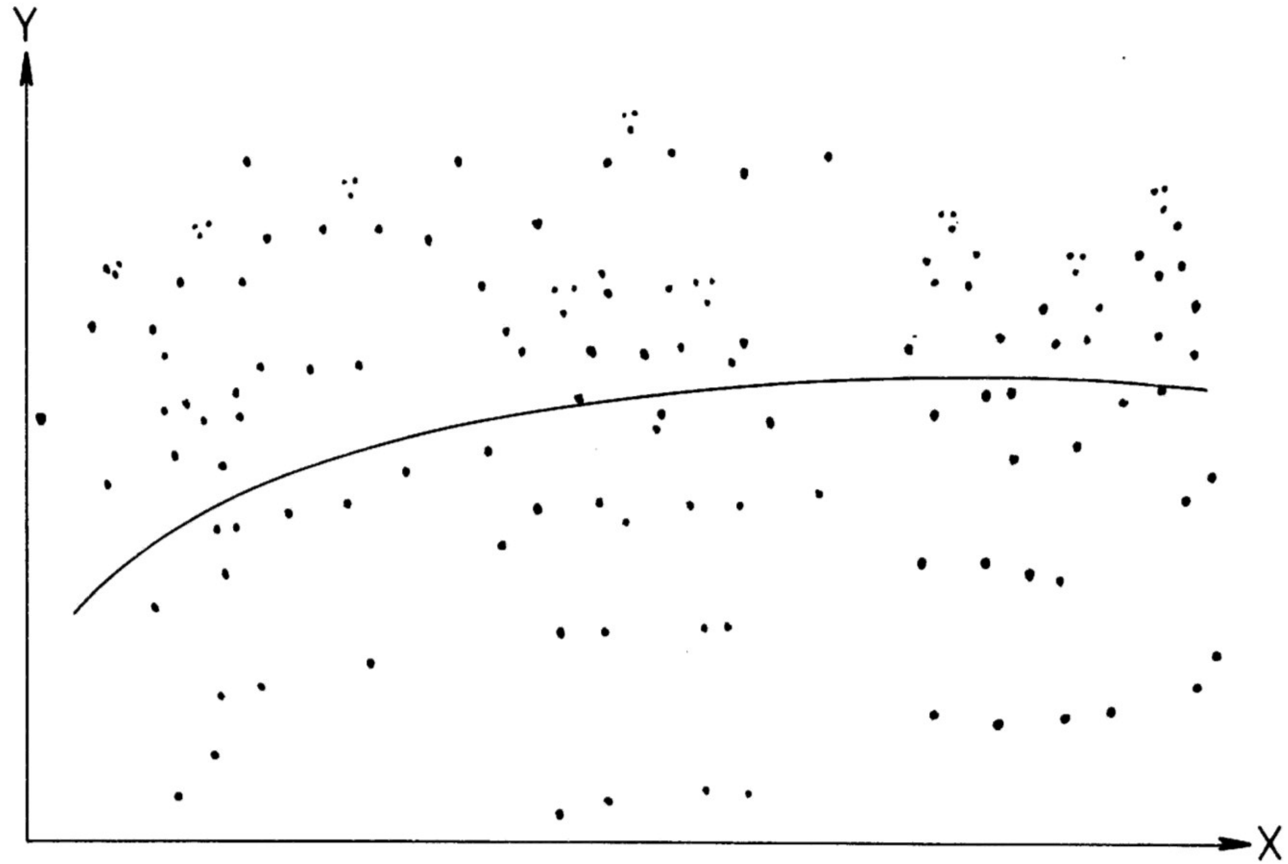
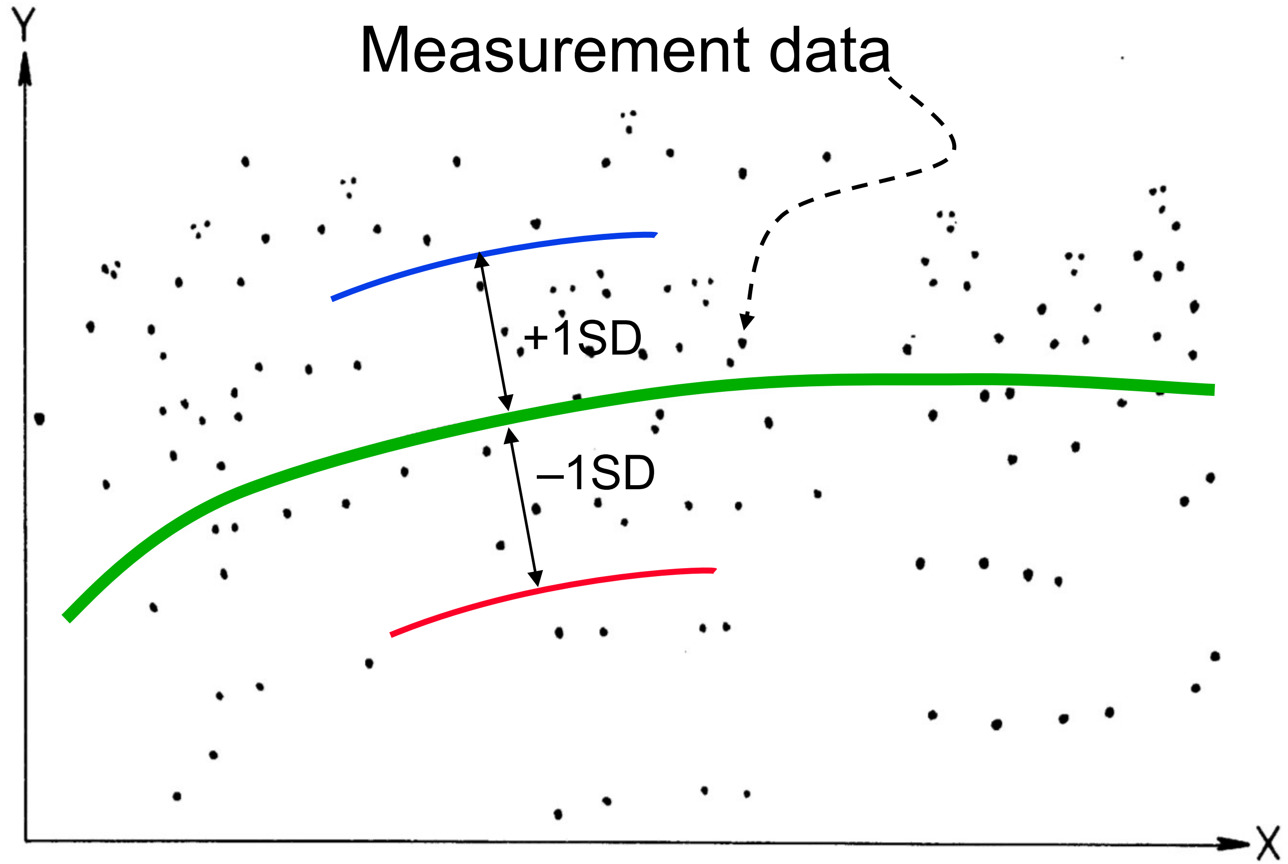
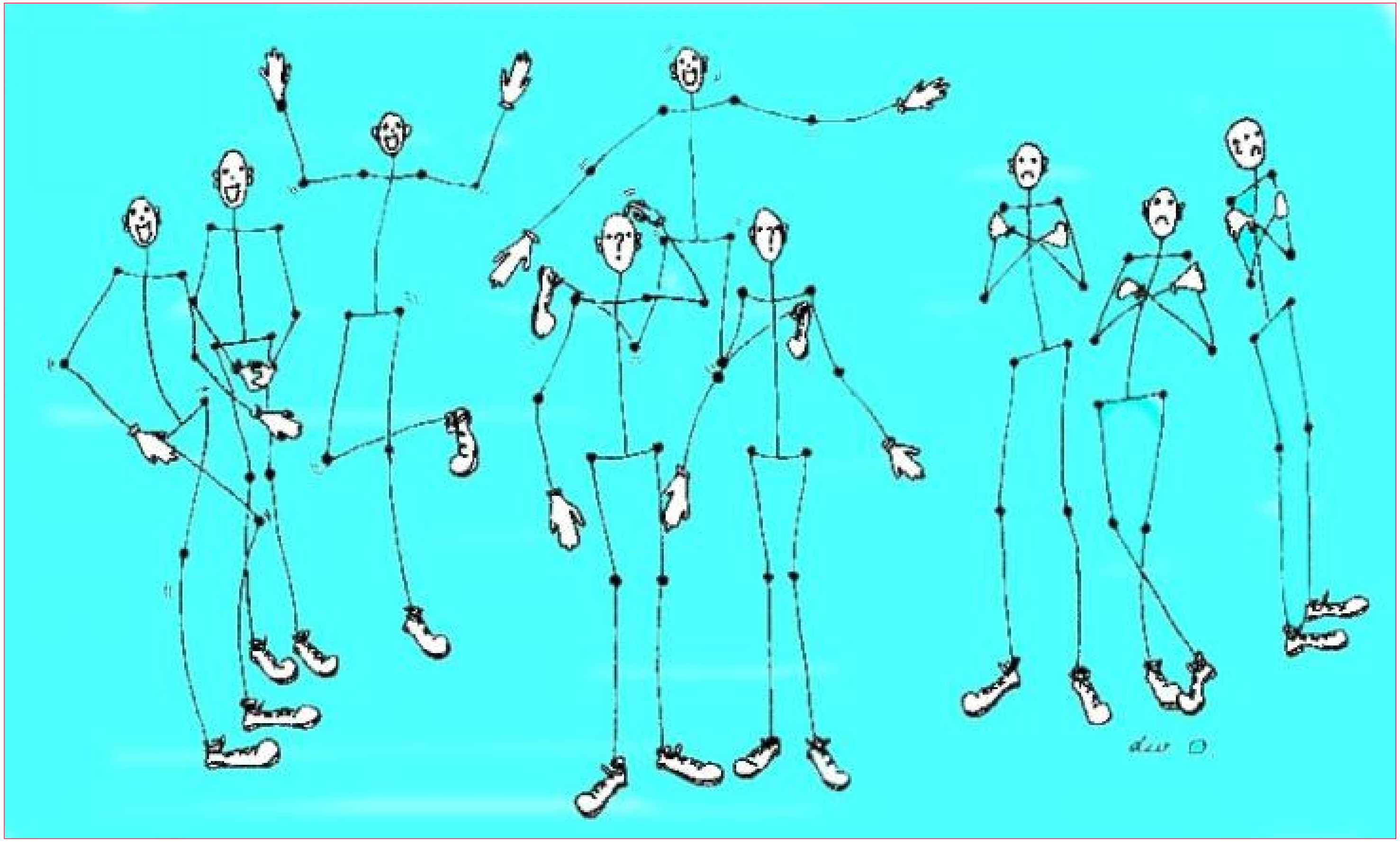

Y
X

Data interpretation



it's mathematically correct



Engineering judgement gives the best interpretation

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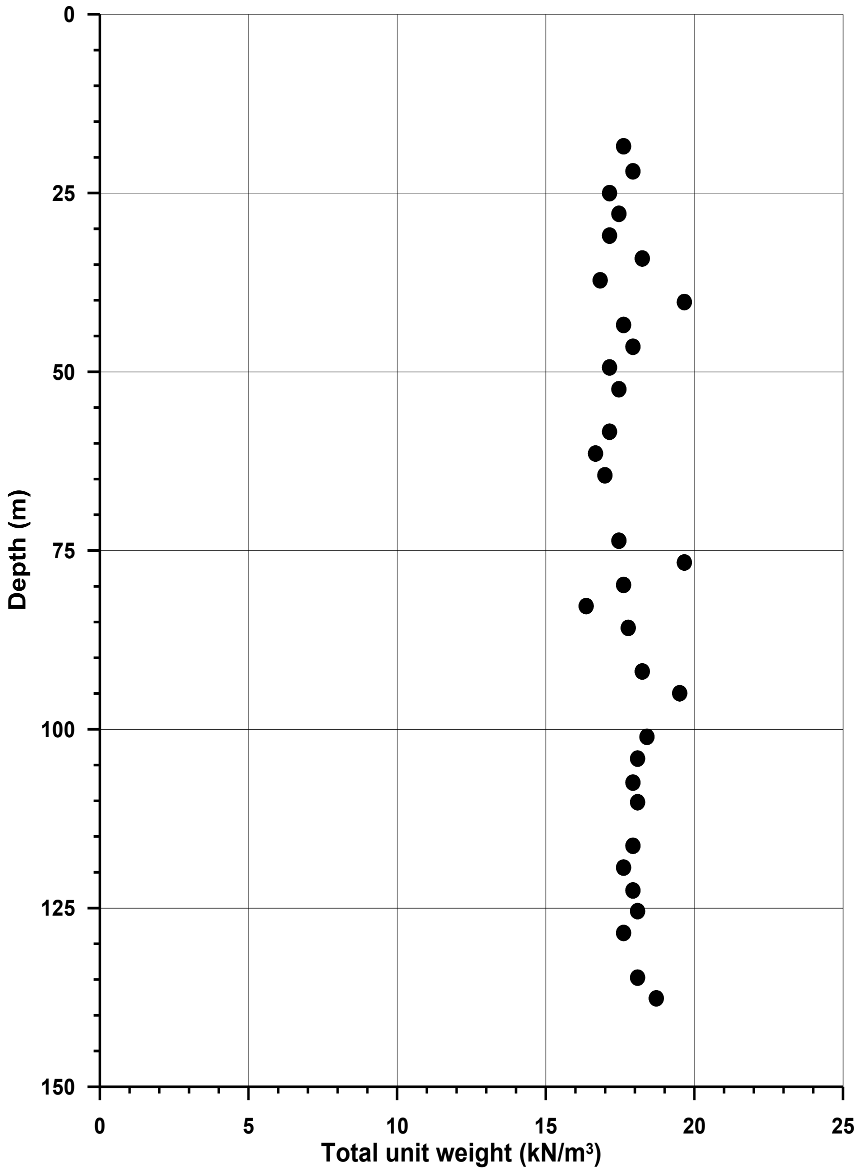
Example from an offshore site Investigation

Total unit weight vs. Depth below seabed



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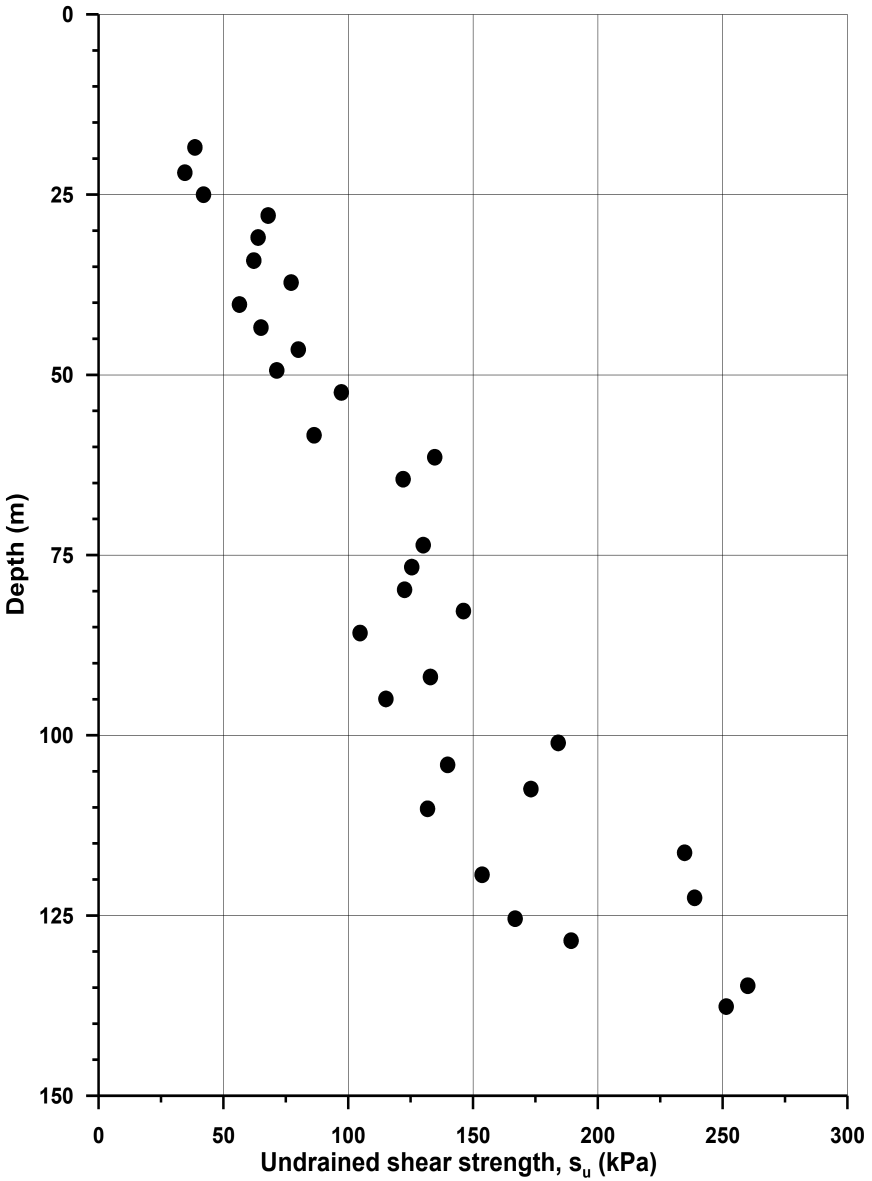
Example from an offshore site Investigation

Undrained shear strength vs. Depth below seabed



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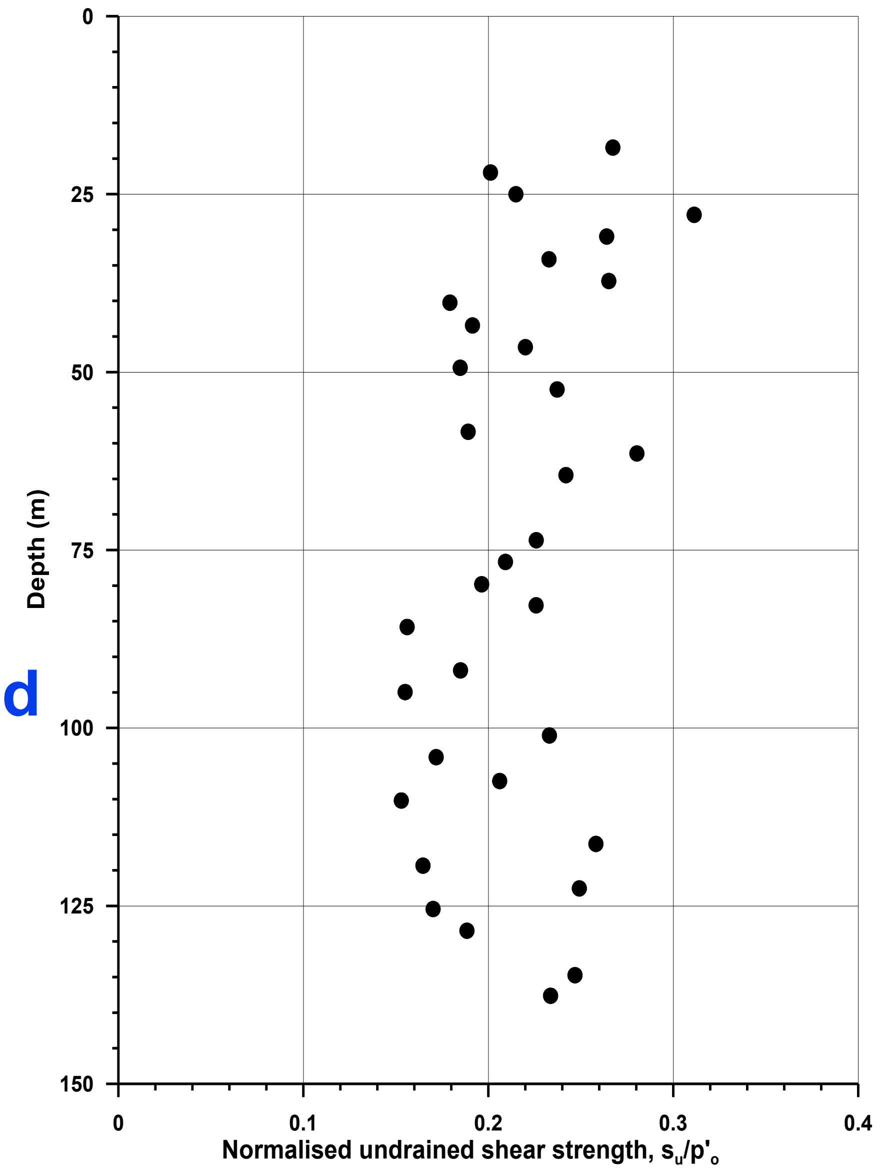
Example from an offshore site Investigation

Normalised undrained shear strength (s_u/p'_o) vs. Depth below seabed



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Conventional deterministic measures of safety

Factor of Safety:

FS = Resistance / Load

$FS \geq 1 \Rightarrow$ Acceptable, safe situation

$FS < 1 \Rightarrow$ Unacceptable, unsafe situation

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Conventional deterministic measures of safety

Margin of Safety:

M = Resistance – Load

$M \geq 0 \Rightarrow$ Acceptable, safe situation

$M < 0 \Rightarrow$ Unacceptable, unsafe situation

Conventional deterministic measures of safety

Factor of safety and margin of safety are not sufficient indicators of safety because the uncertainties in the analysis parameters affect the results.

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Probabilistic measures of safety

- Reliability index, β
- Probability of failure, P_f

P_f and β include information about the uncertainty in load and resistance

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Results of reliability/uncertainty-based analysis

- Probability of failure
- Reliability index and most probable combination of parameters causing failure
- Sensitivity of results to any change in the uncertain parameters

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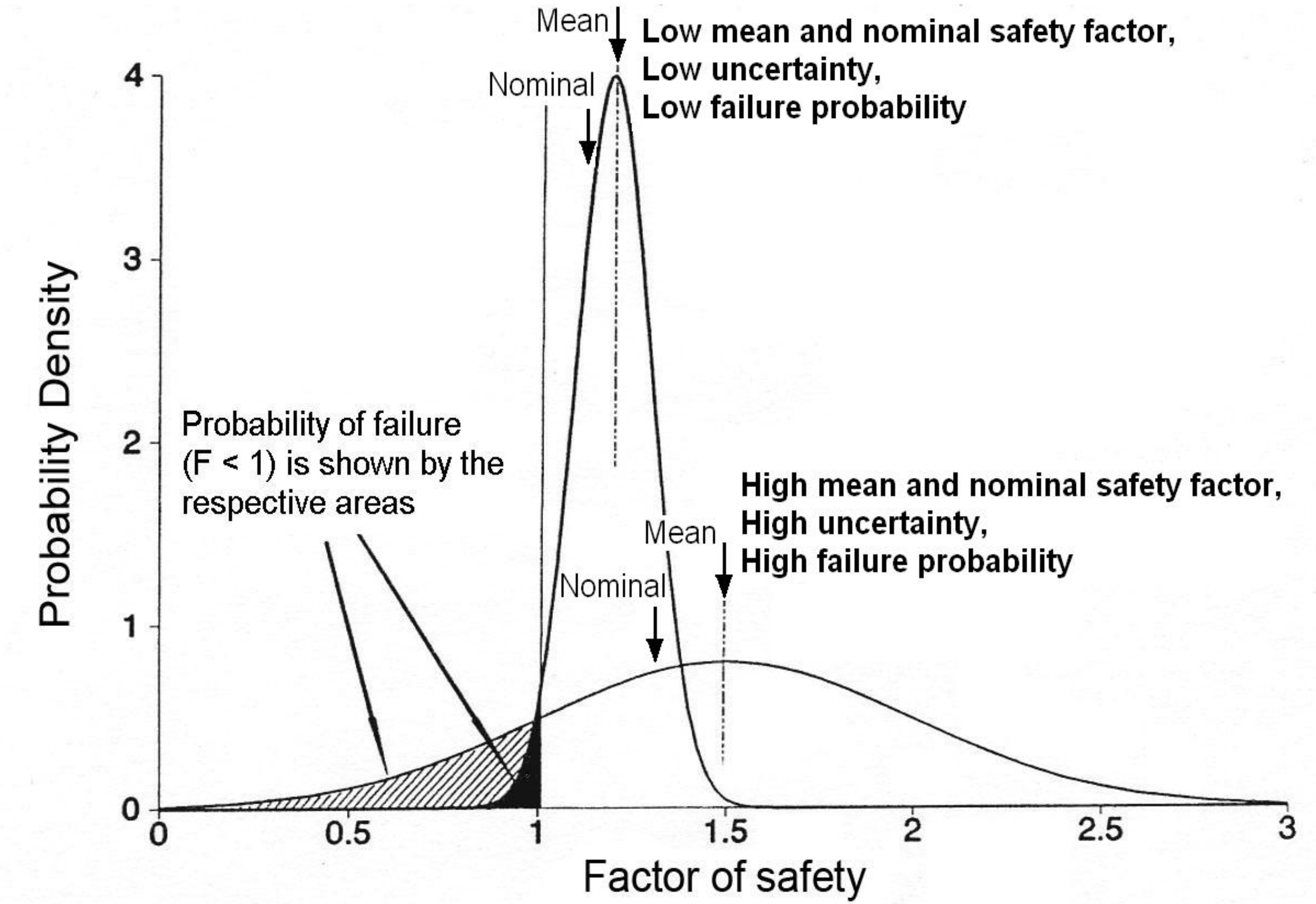
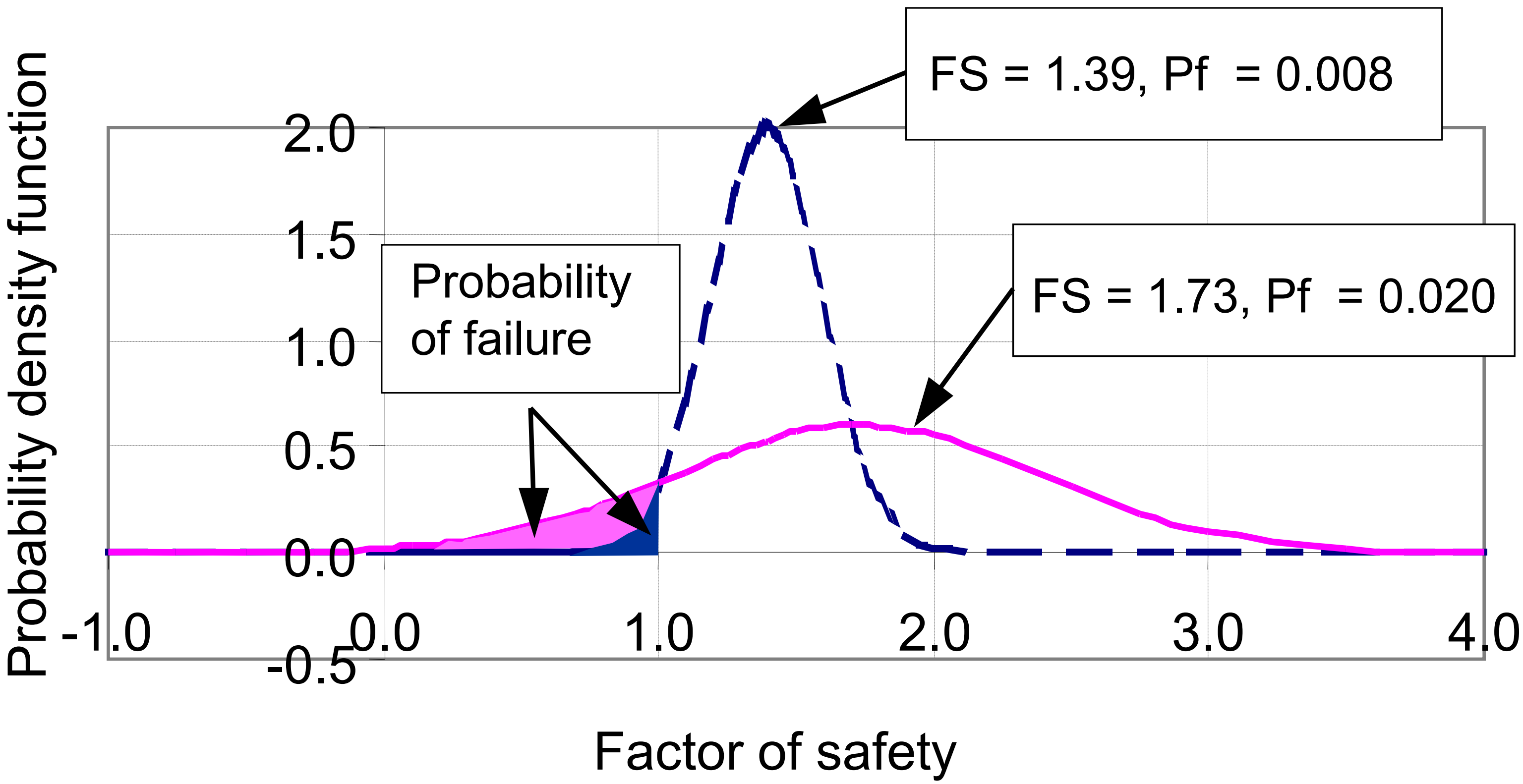

Mean
Low mean and nominal safety factor,
Low uncertainty,
Low failure probability
Nominal
Probability Density
Probability of failure
(F < 1) is shown by the
respective areas
High mean and nominal safety factor,
High uncertainty,
High failure probability
Mean
Nominal
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3
Factor of safety



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Deterministic vs. Probabilistic Analyses

CONVENTIONAL ANALYSIS

Input parameters

Load(s)
Resistance
strength, unit weight, ……

Shear strength

$\Sigma F_x = 0 \rightarrow$ **Load effects**
Factor of safety =
(Resistance/Load effs)

Output of analysis

Factor of safety

Factor of safety

ANALYSIS ACCOUNTING FOR UNCERTAINTIES

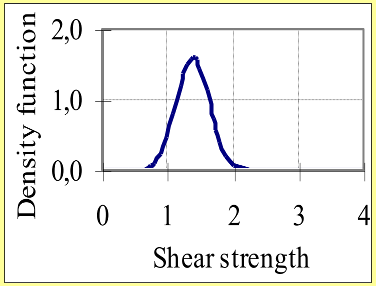
Input parameters

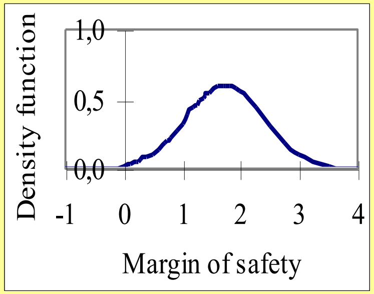
Load(s)
Resistance
strength, unit weight, ……
Model uncertainty

$\Sigma F_x = 0 \rightarrow$ **Load effects**
G_x **= Resistance – Load effs**
$P_f = P[G_x \leq 0]$
$(P_f = P$ **[Load > Resistance])**

Output of analysis

Probability of failure
Reliability index
Parameter(s) which cause failure





F_X = equilibrium equation
G_X = limit state function
P_f = probability of failure

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Reliability and risk in geological and geotechnical evaluations

- **WHY** do risk analysis?
- **HOW** to do risk analysis?

Terminology

- Probability
- Uncertainty
- Hazard
- Risk
- Consequence
- Failure
- Vulnerability
- ……..

Terminology: Danger (threat)

Danger (Threat): The natural phenomenon that could lead to damage, described in terms of its geometry, mechanical and other characteristics. The danger can be an existing one (such as a creeping slope) or a potential one (such as a rockfall). The characterisation of a danger or threat does not include any forecasting.

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Terminology: Hazard & Risk

Hazard: Probability that a particular danger (threat) occurs within a given period of time.

Risk: Measure of the probability and severity of an adverse effect to life, health, property, or the environment. Quantitatively, **Risk = Hazard x Potential Worth of Loss**. This can be also expressed as "**Probability of an adverse event times the consequences if the event occurs**".

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Terminology: Hazard & Risk

Quantitatively:

Risk = Hazard x Consequence, or

Risk = Hazard x Potential Worth of Loss

Loss could be:

- ***Loss of human life***
- ***Economic loss***
- ***Loss of reputation***

Often we are not consistent, and mix up "risk" and "hazard"

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Conventional Factor of Safety

Criterion: Load < Strength / FS

Factor of safety (FS) accounts for

- *Variations in loads & materials*
- *Inaccuracies in design equations and modelling approximations*
- *Construction effects etc.*

UNCERTAINTIES IMPLICITLY RECOGNIZED

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Reliability-Based Design

- Reliability analysis is the consistent evaluation of **probability of failure** using probability theory
- **Reliability-based design** (RBD) is any methodology that uses reliability analysis, explicitly or otherwise
- RBD requires access to **tools for doing reliability analysis** and a conscious choice of **acceptable probability of failure**

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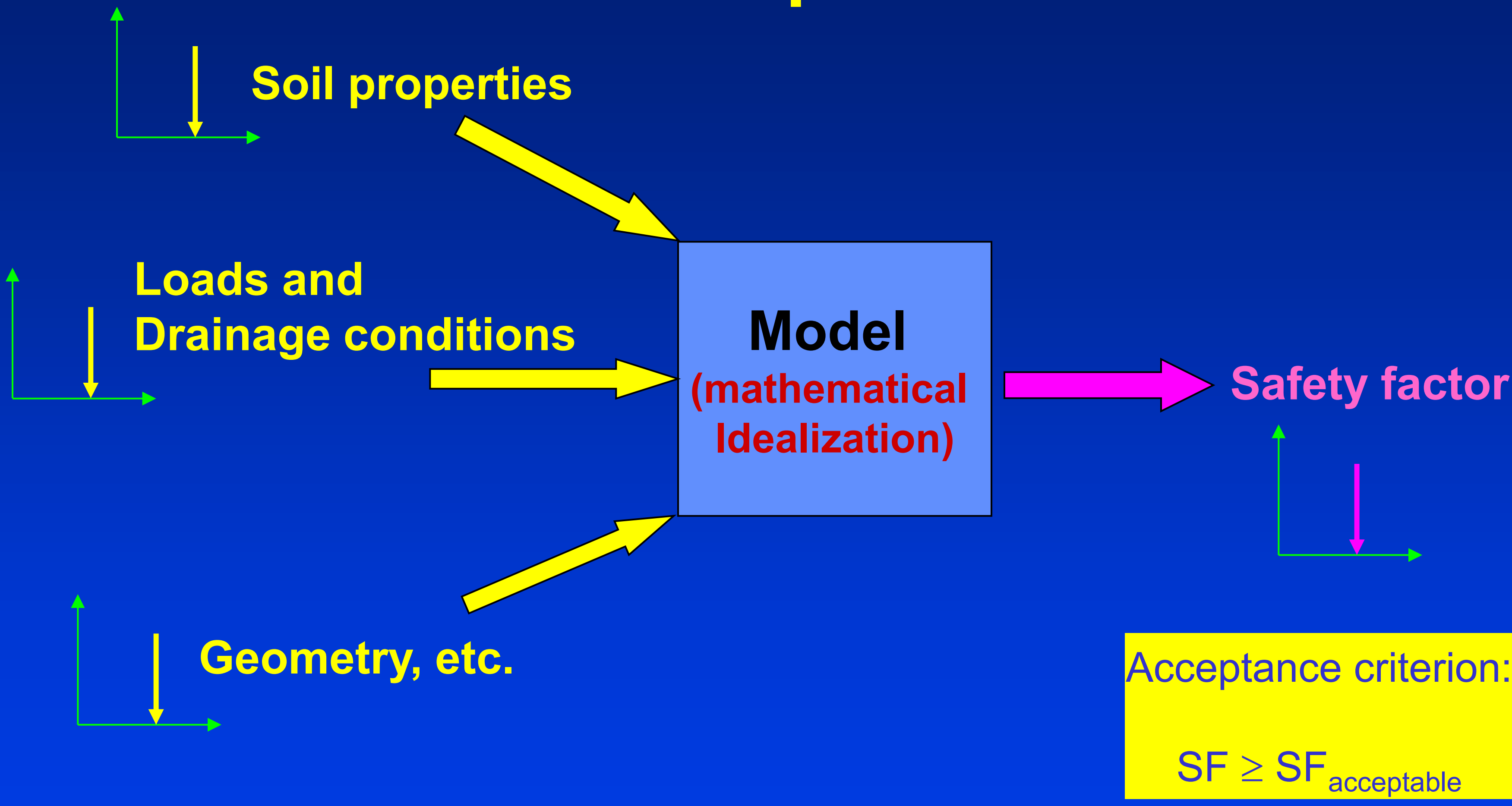
Deterministic stability evaluation of soil slopes



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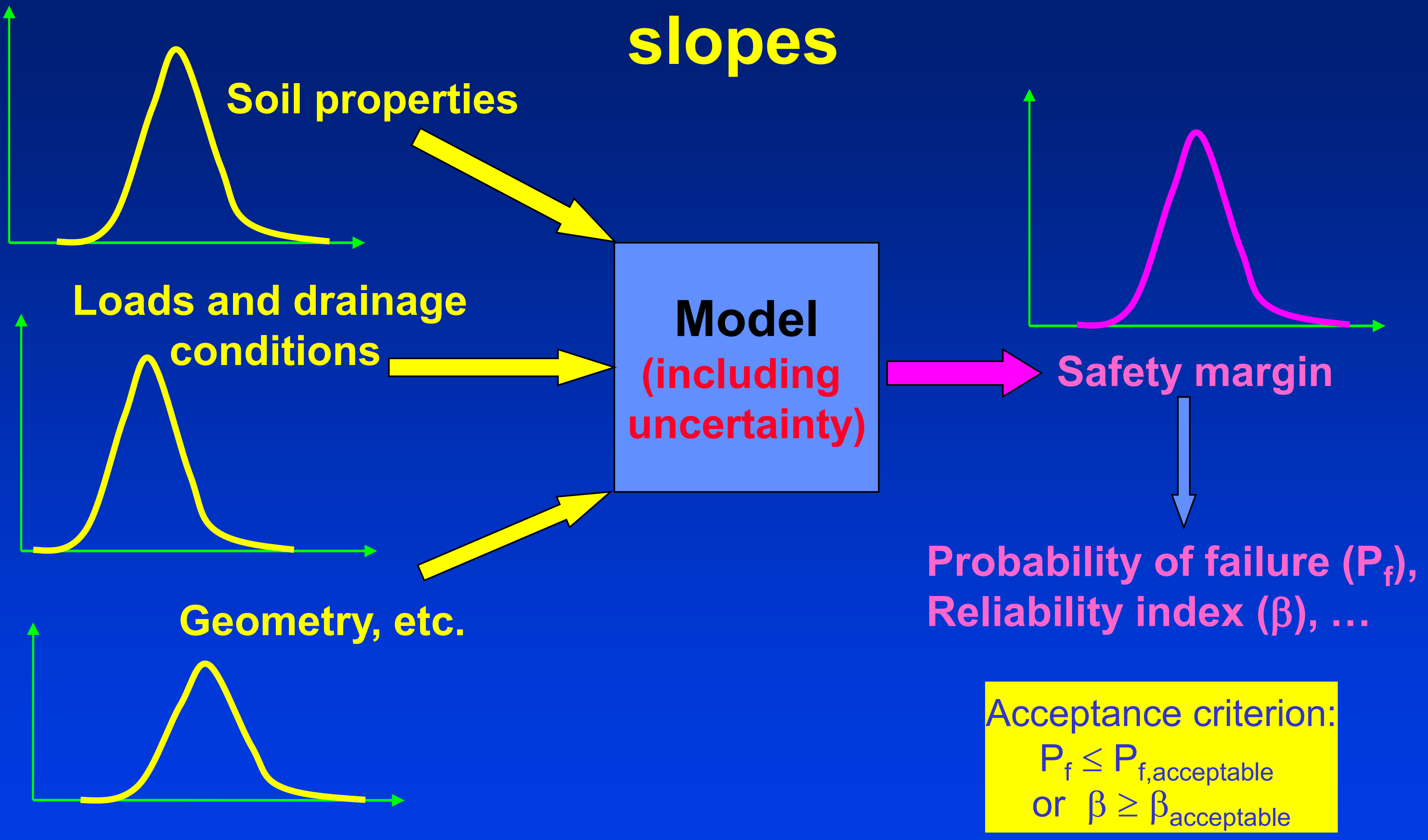
Probabilistic stability evaluation soil slopes



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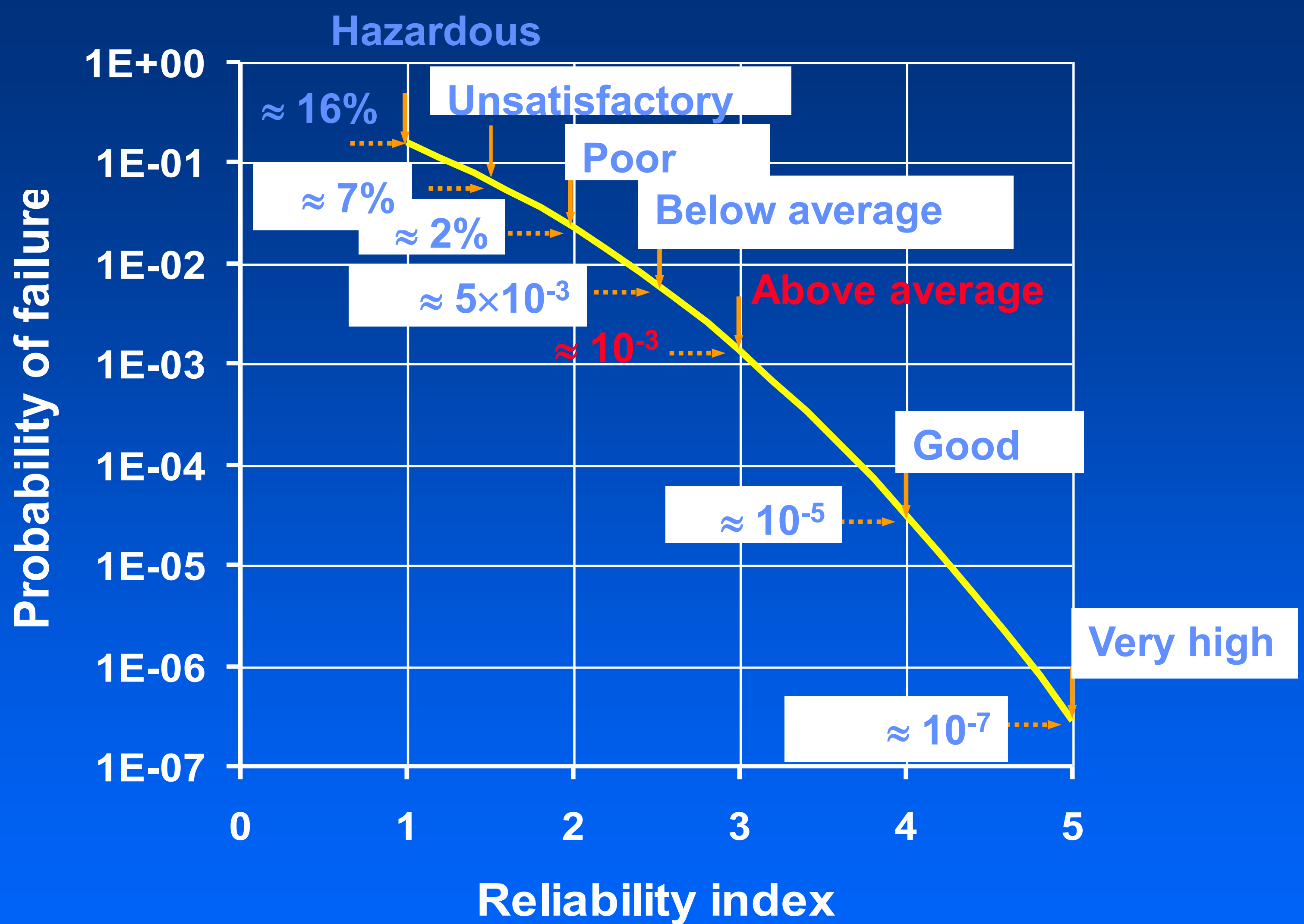

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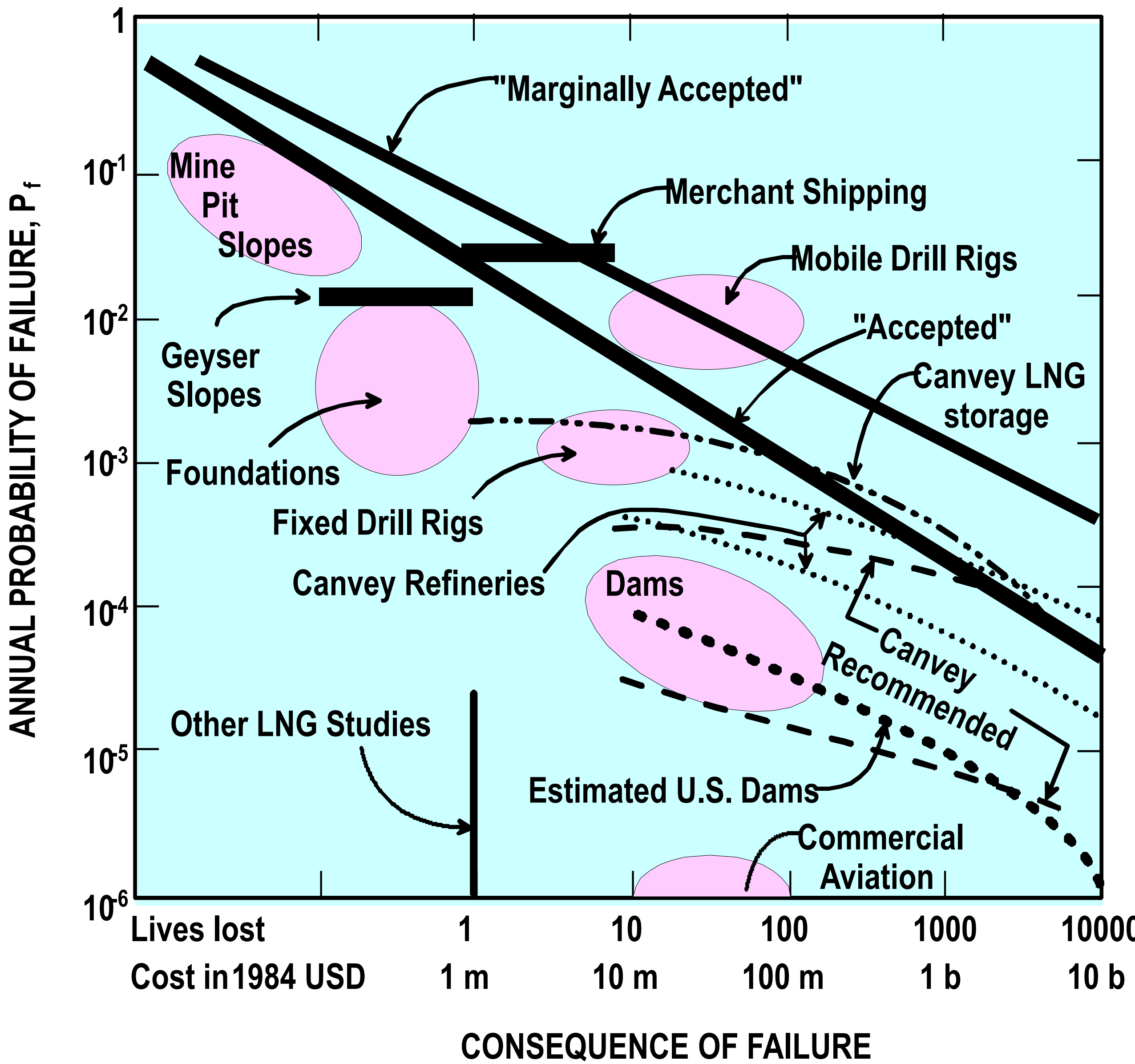
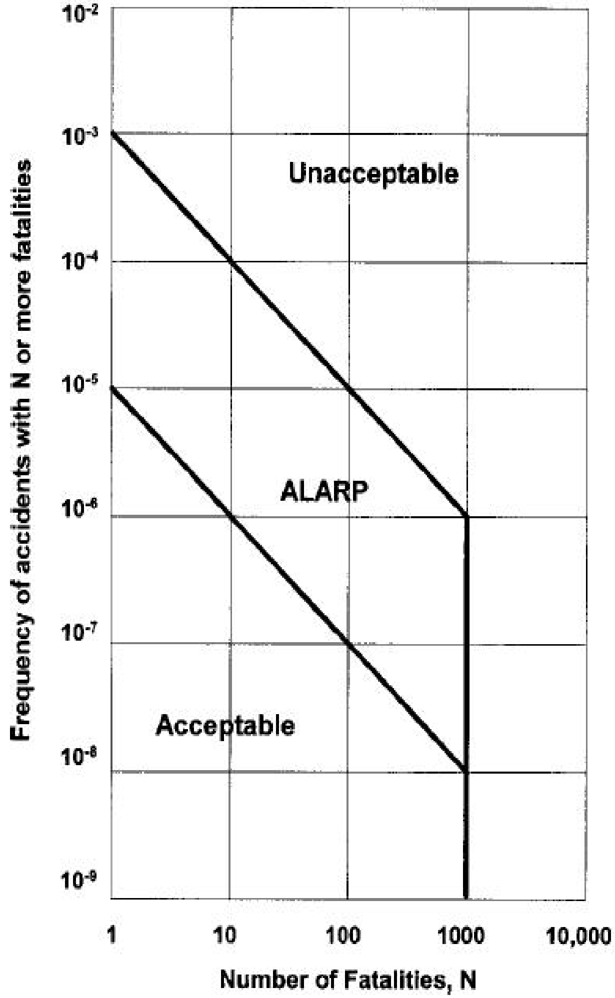
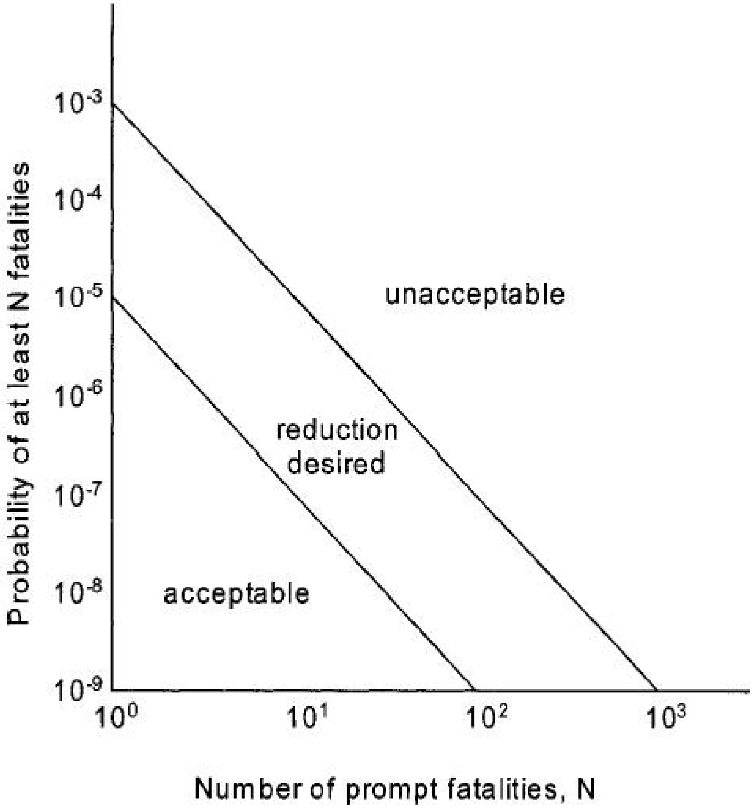


ANNUAL PROBABILITY OF FAILURE, P_f
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
"Marginally Accepted"
Mine Pit Slopes
Merchant Shipping
Mobile Drill Rigs
Geyser Slopes
"Accepted"
Canvey LNG storage
Foundations
Fixed Drill Rigs
Canvey Refineries
Dams
Canvey Recommended
Other LNG Studies
Estimated U.S. Dams
Commercial Aviation
Lives lost
1
10
100
1000
10000
Cost in 1984 USD
1 m
10 m
100 m
1 b
10 b
CONSEQUENCE OF FAILURE

ACCEPTABLE RISK LEVELS

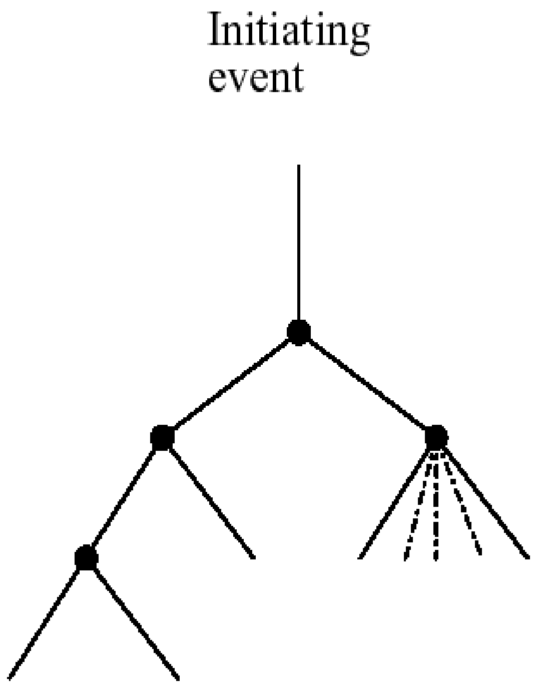


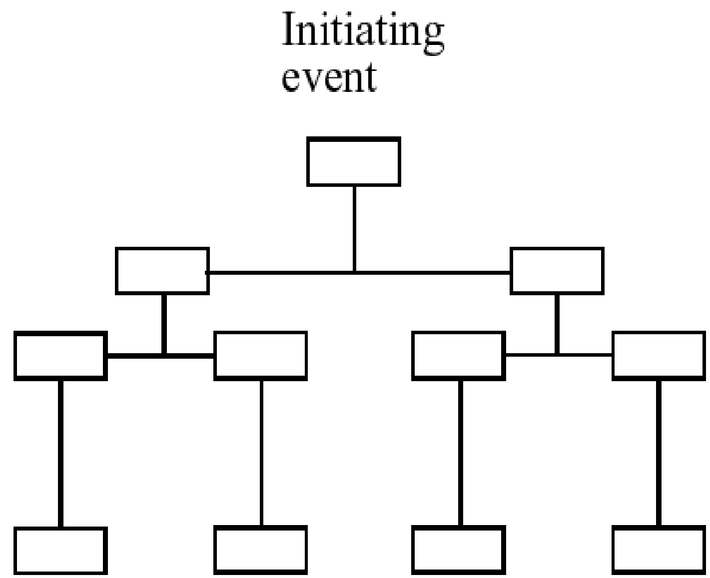
F–N diagram adopted by Hong Kong Planning Department for planning purposes (Hong Kong Government Planning Department 1994). "ALARP" stands for "as low as reasonably practicable."



F–N diagram proposed for Netherlands for planning and design (Versteeg 1987). "Prompt fatalities" is term used in original reference and refers to failures that occur in short term rather than because of lingering effects.

Event tree method





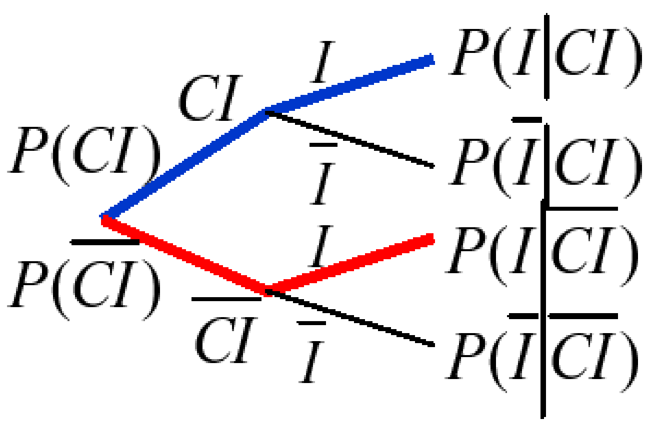


Illustration of the principal appearance of an event tree.

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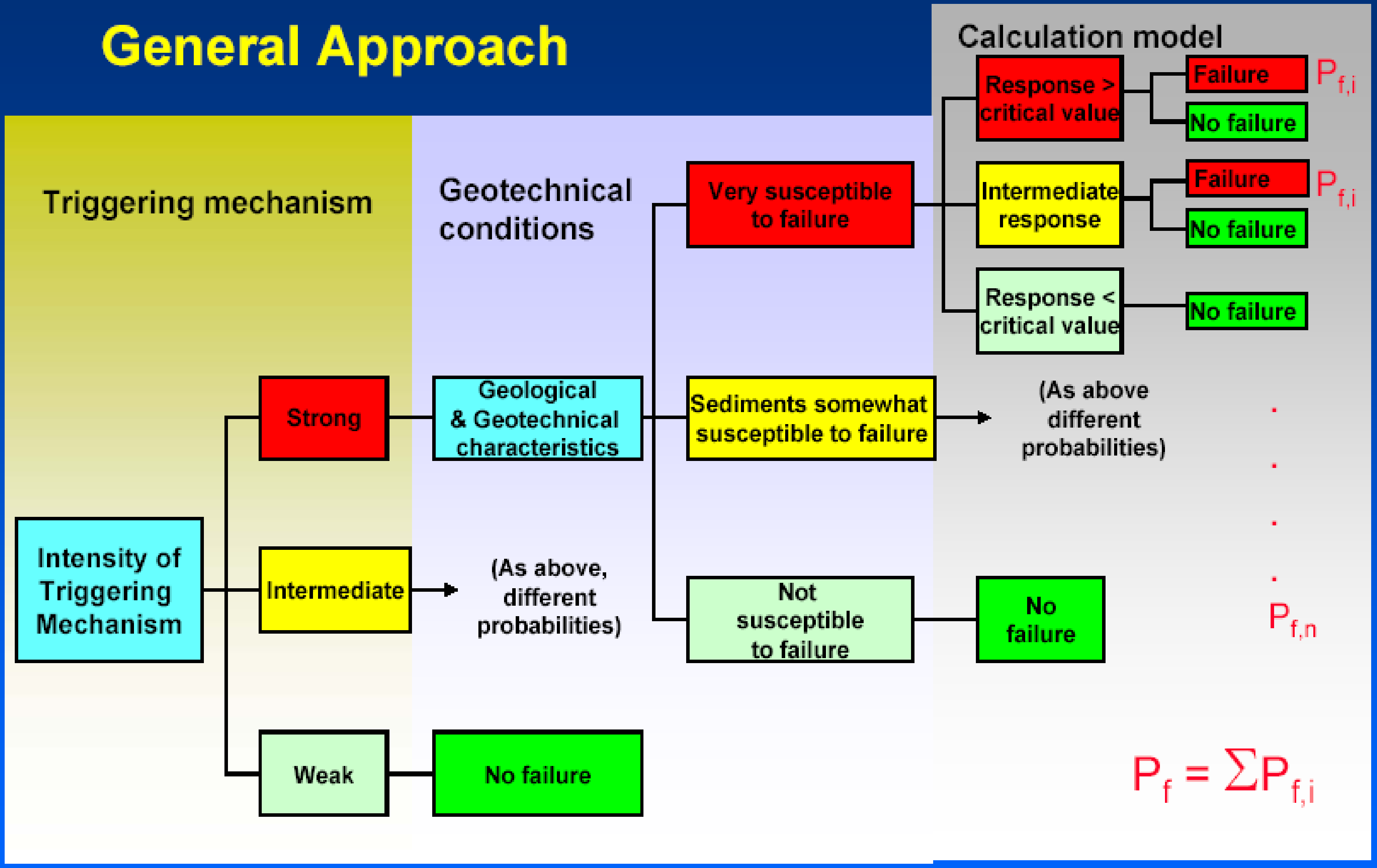
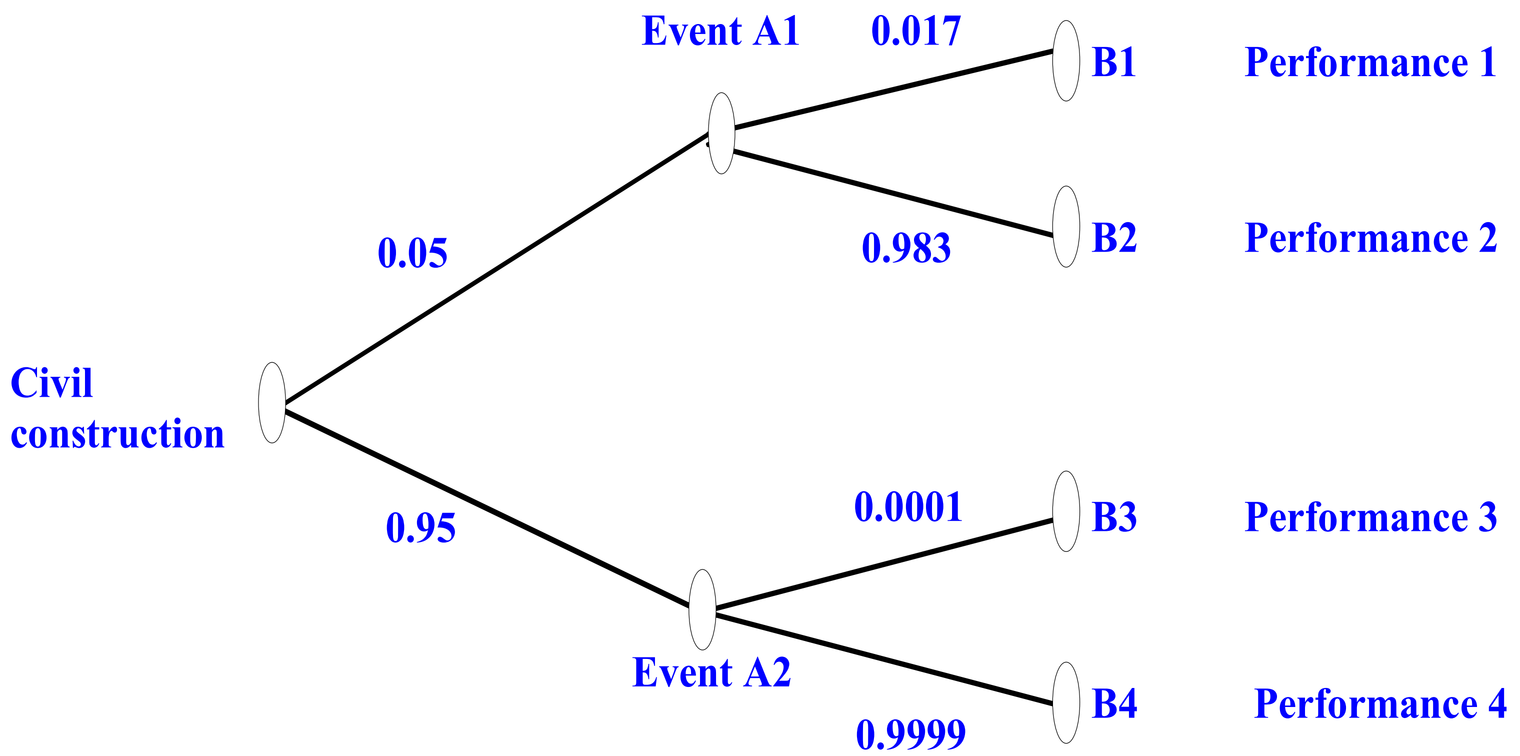
General Approach




Event A1
0.017
B1
Performance 1
0.05
0.983
B2
Performance 2
Civil construction
0.95
0.0001
B3
Performance 3
Event A2
B4
0.9999
Performance 4

Risk Analysis of Dams

- focus on safety and reliability of existing dams
- establish a diagnosis or set priorities among possible failure modes, to act as support in decision-making on issues related to dam safety modifications

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Probabilistic analysis is systematic application of engineering judgement

1) Dam site inspection and document review
2) Failure mode screening (defining all failure modes)
3) Construction of event tree, listing failure (events and their interrelationship)
4) Probability assessment of reach event (often subjective)
5) Failure probability from product of probability of each event along any one branch of the event tree
6) Iteration

Descriptors of uncertainty

0.001	**Virtually impossible**, due to known physical conditions or process that can be described and specified with almost complete confidence
0.01	**Very unlikely**, although the possibility cannot be ruled out on the basis of physical or other reasons
0.10	**Unlikely**, but it could happen
0.50	**Completely uncertain**, with no reason to believe that one possibility is more or less likely than the other
0.90	**Likely**, but it may not happen
0.99	**Very likely**, but not completely certain
0.999	**Virtually certain** due to know physical conditions or process that can be described and specified with almost complete confidence

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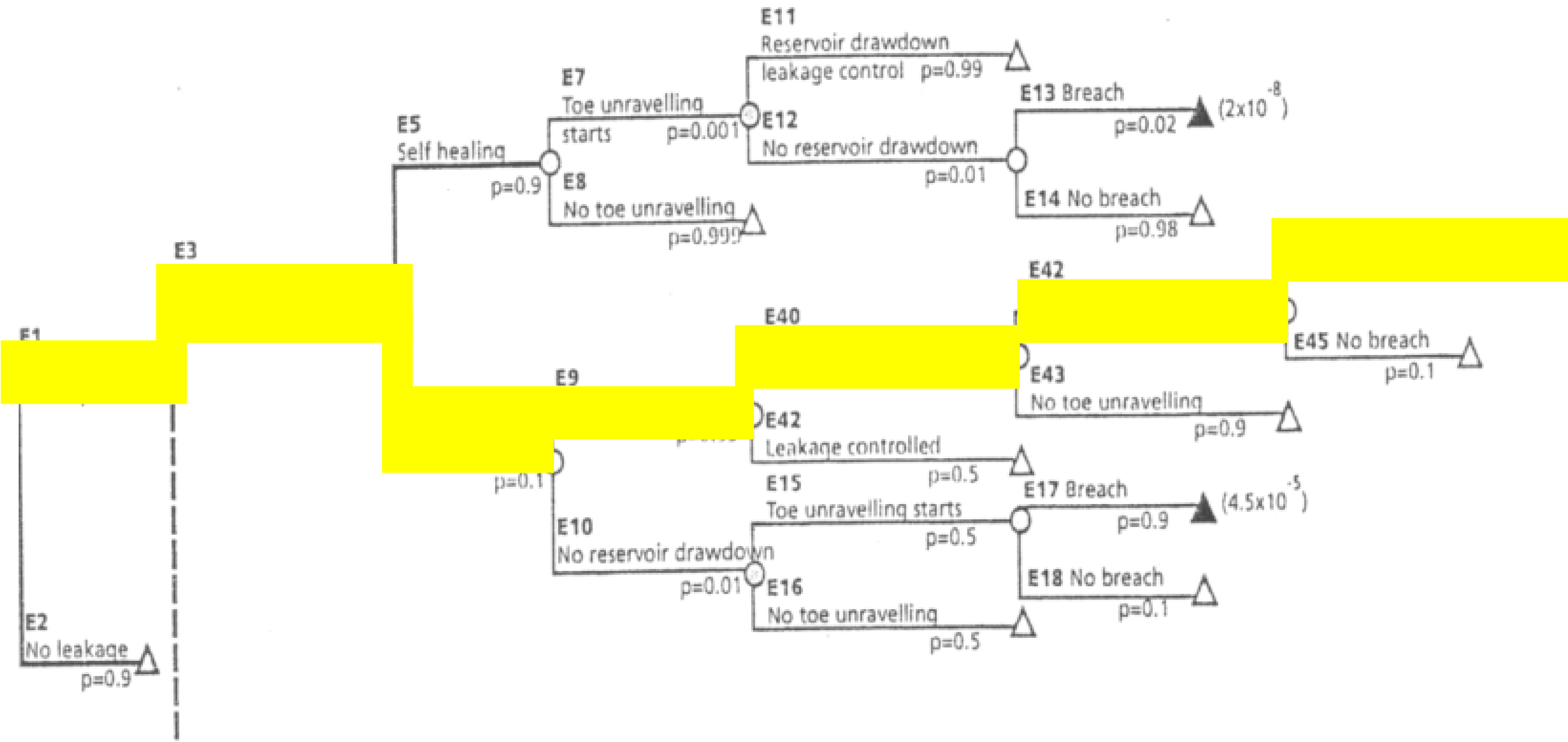

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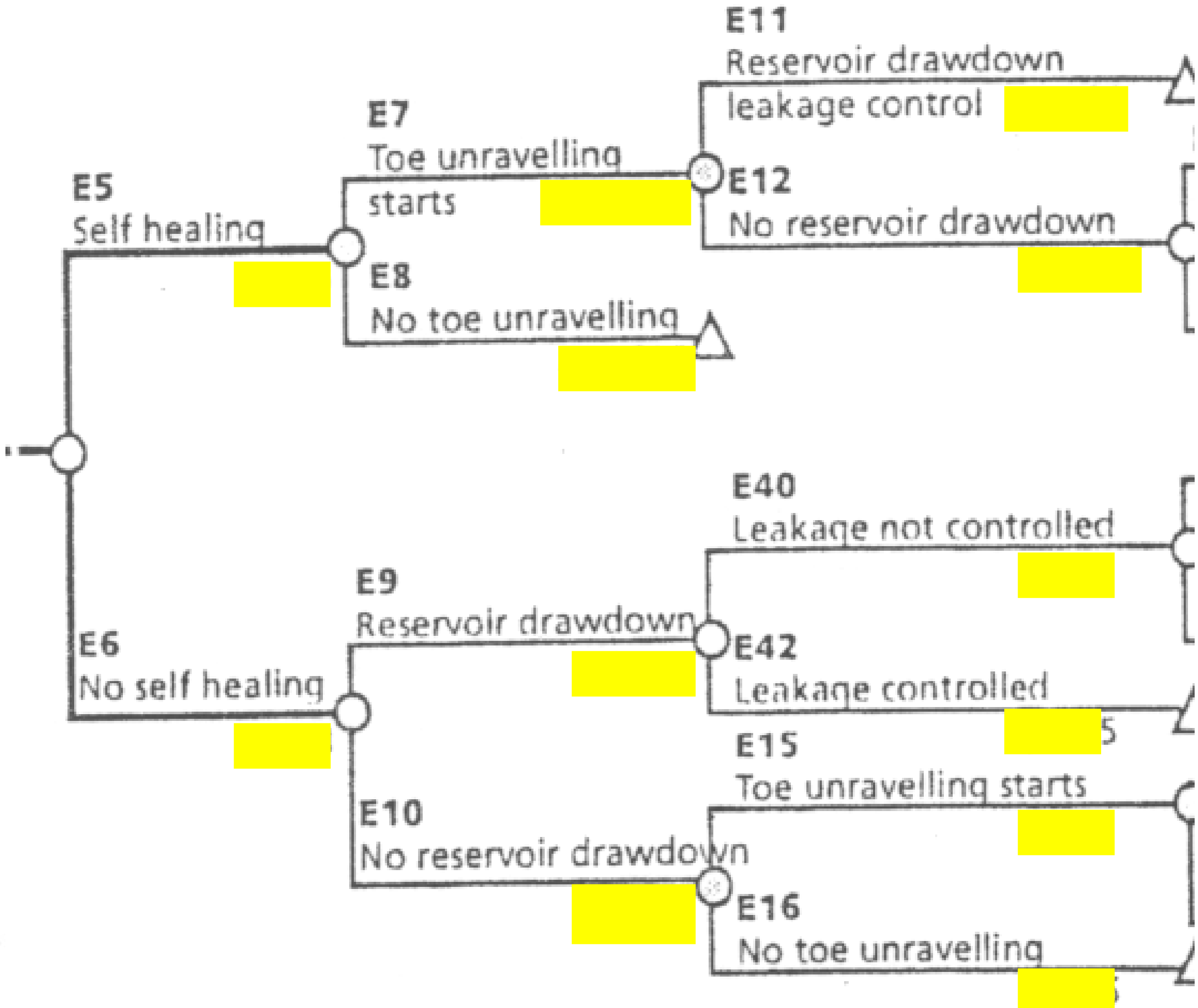
E11
Reservoir drawdown
leakage control
E7
Toe unravelling
starts
E12
No reservoir drawdown
E5
Self healing
E8
No toe unravelling
E40
Leakage not controlled
E9
Reservoir drawdown
E42
Leakage controlled
E6
No self healing
E15
Toe unravelling starts
E10
No reservoir drawdown
E16
No toe unravelling

Case study of Viddalsvatn dam in Norway

Loading	Annual probability of failure
Flood	1.2×10^{-6}
Earthquake	1.1×10^{-5}
Internal erosion	5.5×10^{-4}

- The total annual probability of failure for all modes is the sum of the three components, or 5.6×10^{-4}
- The results represent a relative order of magnitude for the different scenarios

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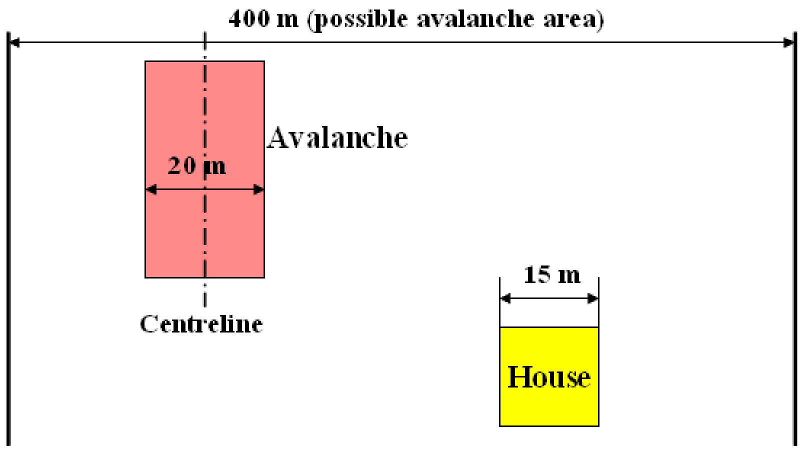
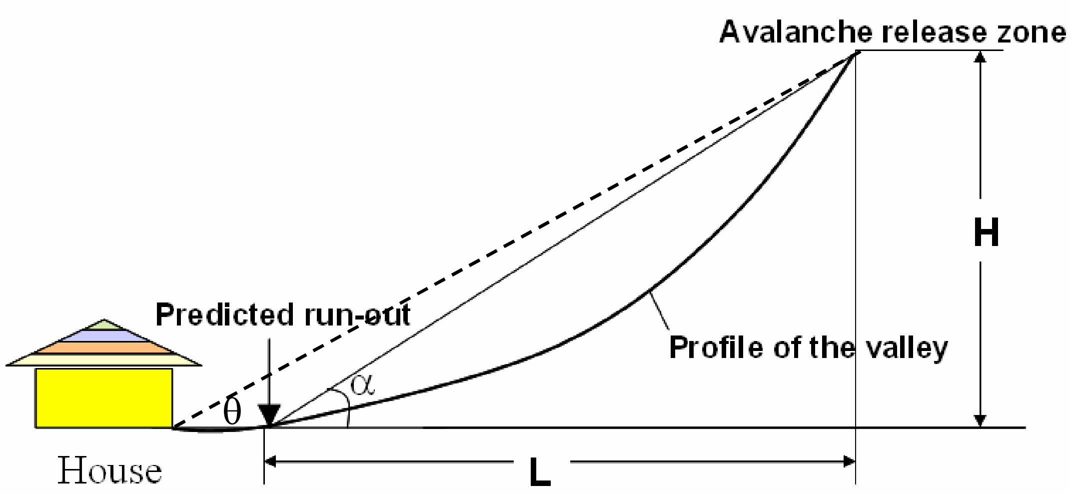
Example: Event tree construction



Avalanche could occur anywhere within a 400 m wide area in the valley, and the typical width of the avalanche is 20 m. Statistics show that a major avalanche occurs once every 5 years in this valley.



α has normal distribution with standard deviation
$\sigma_\alpha = 2.3°$
$\theta = 12°$
$H = 200$ m
$L = 750$ m

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Risk/uncertainty-based analysis

The approach is effectively a systematic application of engineering judgement

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Risk analysis

Pros (for)

- Encourages to scrutinize problem as a whole
- Helps communication
- Encourages gathering, compilation and organisation of data for systematic examination of problem
- Identifies the optimum among alternative solutions
- Emphasizes where decisions have to be made
- Provides a framework for contingency planning and continued evaluation

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Risk analysis

Cons (against)

- More complex calculation (?)
- Need to include judgement
- Uncertainties can be too large to enable a good basis for decision-taking
- Not always possible to have explicit formulation of a thought process
- Danger of leaving consideration that cannot be quantified out of the process
- Does not account for human error

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Risk/uncertainty-based analysis

It is possible to use whatever **data** are available, to supplement them with **judgement** and to do a few simple **calculations** to get an idea of the uncertainty and the combined effects of possible variation in parameters.

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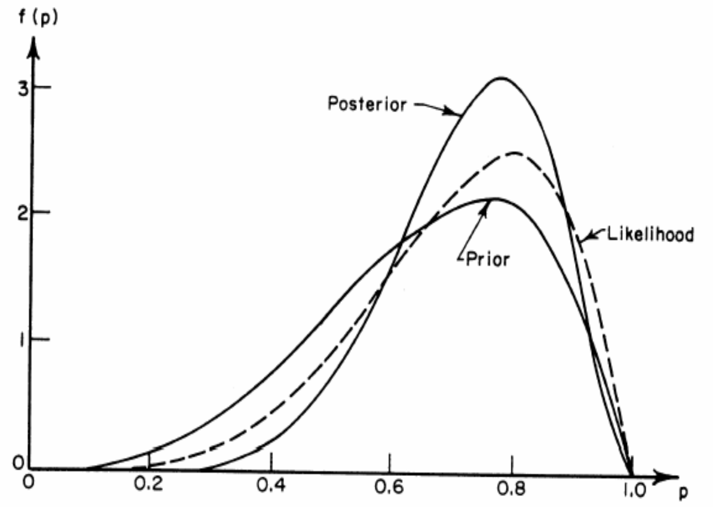
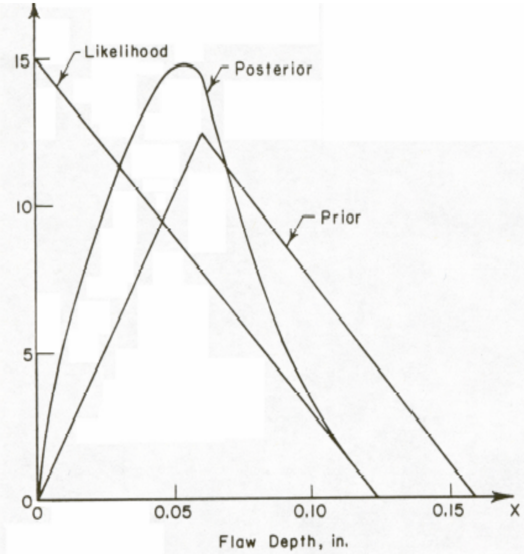
Bayesian Updating

Bayesian updating is a powerful technique for combining subjective judgement and data from different sources.



Prior, likelihood, and posterior functions



Distribution of flaw depth

Posterior distribution = Prior x Likelihood x nomalising factor

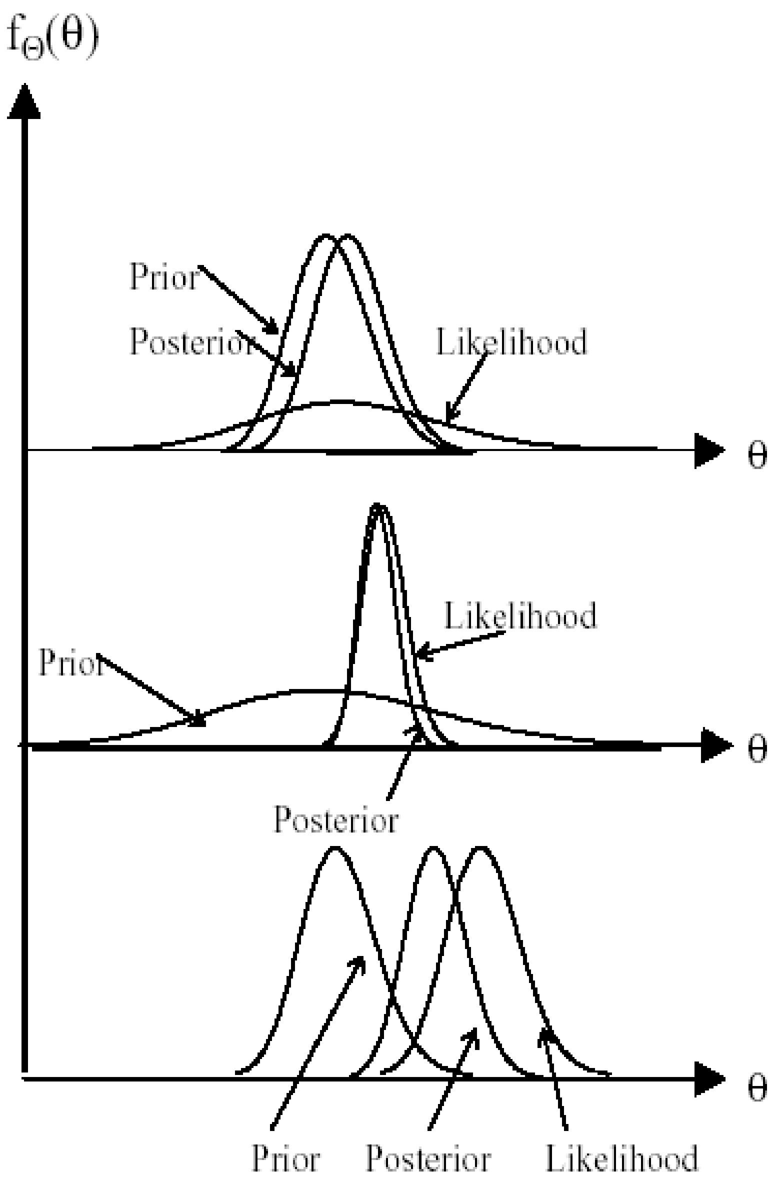
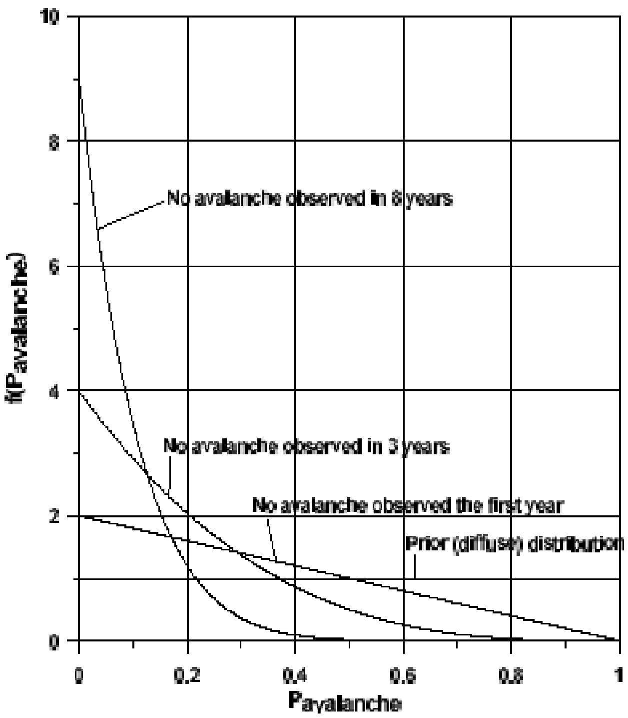


Illustration of updating of uncertainty models.

Bayesian updating – Example application to annual probability of avalanche



No avalanche in n years:

$$f(P_{annual}) = (n+1)(1 - P_{annual})^n$$

Probability distribution for annual avalanche occurrence after 0, 1, 3, and 8 years of observation of no avalanche

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Bayesian updating – Some useful equations (assuming normal distribution)

- **Prior estimates:**

  Mean = μ_1 , Stand. Dev. = σ_1

- **Likelihood estimates:**

  Mean = μ_2 , Stand. Dev. = σ_2

- **Posterior estimates (updated estimates):**

$$\mu_{updated} = (\mu_1 / \sigma_1^2 + \mu_2 / \sigma_2^2) / (1 / \sigma_1^2 + 1 / \sigma_2^2)$$

$$\sigma^2_{updated} = (\sigma_1^2 \cdot \sigma_2^2) / (\sigma_1^2 + \sigma_2^2)$$

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