Mohr-Coulomb Soil Model

Things you should know about this model!

by

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Method of Analysis

Plaxis offers the following choices of analysis for short term performance of TERS in clay:

- **A. Mohr-Coulomb: effective stress, c'- ϕ', undrained**
- **B. Mohr-Coulomb: effective stress, c_u- ϕ_u, undrained**
- **C. Mohr-Coulomb: total stress, c_u- ϕ_u, non-porous, undrained**
- **D. Mohr-Coulomb: effective stress, c'- ϕ', consolidation**
- **E. Mohr-Coulomb: effective stress, c_u- ϕ_u, consolidation**
- **F. Soft Clay: effective stress, c'- ϕ', undrained**
- **G. Soft Clay: effective stress, c'- ϕ', consolidation**
- **H. Mod. Cam Clay: effective stress, c'- ϕ', undrained**
- **I. Mod. Cam Clay: effective stress, c'- ϕ', consolidation**
- **J. Advanced Hardening: effective stress, c'- ϕ', undrained**
- **K. Advanced Hardening: effective stress, c'- ϕ', consolidation**

Which one should we use?

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TERS Design using Finite Element Method

Soil Model

- Mohr-Coulomb
- Hyperoblic
- Soft Clay
- Mod. Cam Clay
- Advanced Hardening

Types of Analysis

- Total Stress
- Effective Stress
- Undrained
- Drained
- Consolidation

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What is undrained, drained and consolidation?

They refer to the different soil behaviour when subjected to external load which can be quantified in terms of **excess pore pressure** and **volume change**.

	Excess pore pressure	**Volume change**
Undrained	Generated but no dissipation	None
Drained	Fully dissipated	Yes
Consolidation	Dissipation is time dependent	Yes

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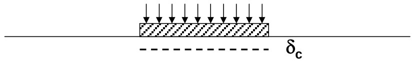
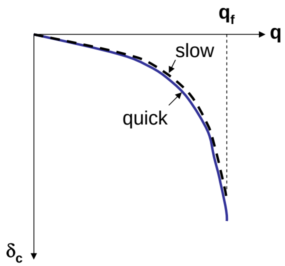
Behaviour of Sand





- Sand has high permeability.
- Response is almost rate independent
- Settlement is almost instantaneous
- Δu=0 except during earthquake
- Drained condition prevailed
- Use **effective stress** analysis

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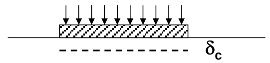
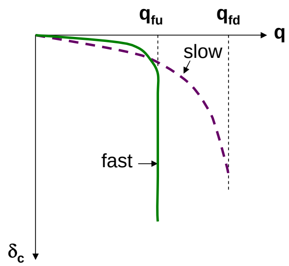
Behaviour of Clay





- **Clay has very low permeability.**
- **Response is rate dependent.**
- **Slow loading rate:**
  - **Δu = 0**
  - **Drained condition**
  - **Long term steady state**
  - **Effective stress**
- **Fast loading rate:**
  - **Δu > 0**
  - **undrained condition**
  - **Short term, end-of-construction**
  - **Total stress**
- **Undrained condition is usually more critical.**

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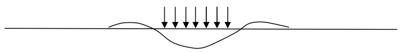
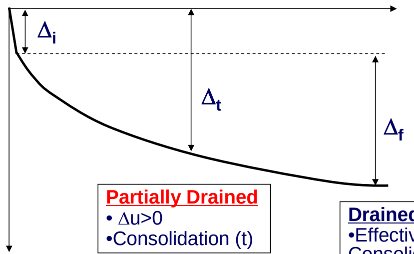
Behaviour of Clay under a Constant Load



Undrained (Δu>0)
•Total
•Effective
•Consolidation (t~0+)



Partially Drained
• Δu>0
•Consolidation (t)

Drained(Δu=0)
•Effective
Consolidation (t~∞)

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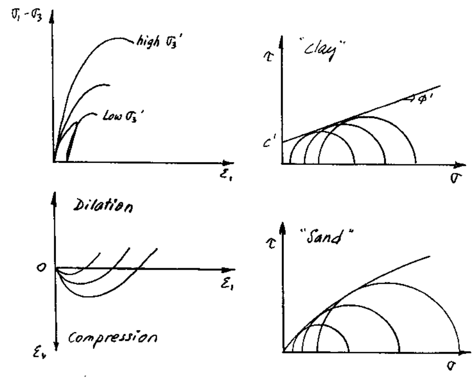
Drained Behaviour of Sand and Clay

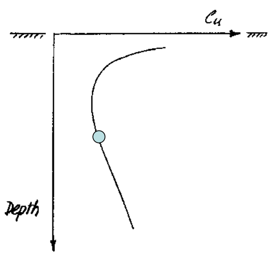


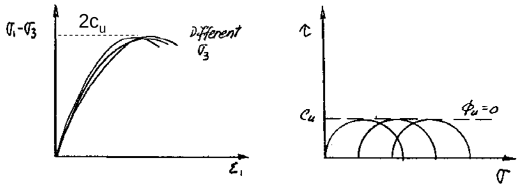
- Strength & modulus are stress dependent.
- Nonlinear.
- Inelastic.
- Stress path dependent.

$$(\sigma_1-\sigma_3)_f = \frac{2c'\cos\phi' + 2\sigma_3'\sin\phi'}{1-\sin\phi'}$$

Undrained Behaviour of Saturated Clay

- Strength & modulus are independent of σ_3
- Nonlinear
- Inelastic
- Failure
- Stress Path Dependent





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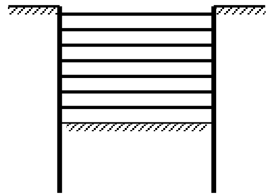
What type of analysis should we conduct?

- ➢ Undrained?
- ➢ Drained?
- ➢ Consolidation?



It depends on the permeability of soil and duration of construction.

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Hydraulic Conductivity or Coefficient of Permeability *k*

Coefficient of Permeability (m/s) (BS 8004:1986)

1	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}
Clean gravels	Clean sands and sand-gravel mixtures			Very fine sands, silts and clay-silt laminate			Unfissured clays and clay-silts (> 20% clay)			
	Desiccated and fissured clays									

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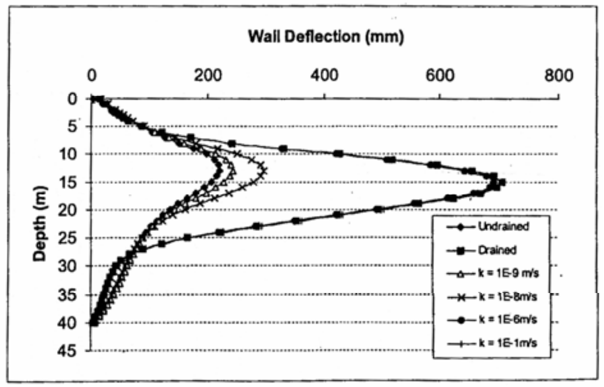
Effect of permeability on wall deflection



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Effect of permeability on ground settlement

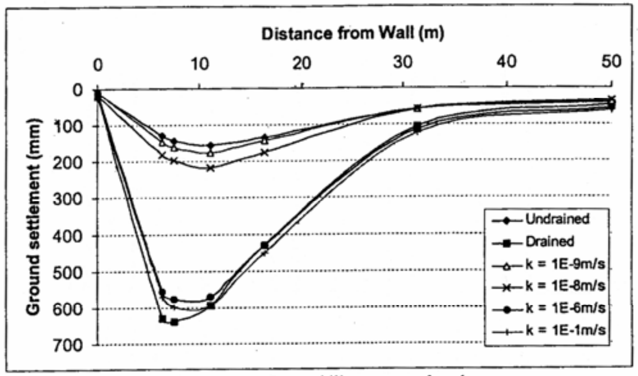

Figure 6.3a Effect of permeability on ground settlements

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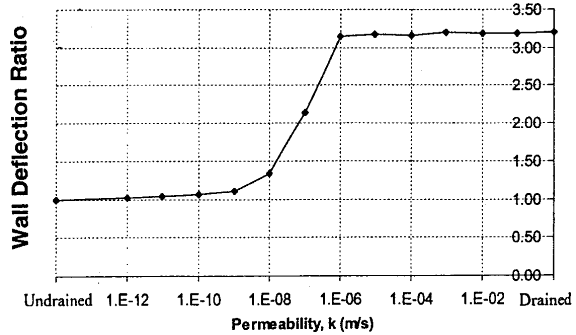
Is it important to conduct consolidation analysis for deep excavation in clay?

$$\text{Wall Deflection Ratio} = \frac{\text{Max. wall deflection computed for a given "k"}}{\text{Max. wall deflection under undrained condition}}$$



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Coefficient of Permeability "*k*"

k **(m/s)**

1 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10}

Clean gravels	Clean sands	Very fine sands	Silts & clayey sand	Clays
Drained			*Transition*	*Undrained*
Independent of construction period			*Short construction period*	

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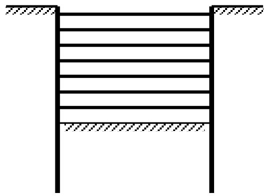
Excavation in Sand → Drained Analysis



Sand or any soil with

Permeability > 10^{-6} m/s

- **Drained analysis**
- **Undrained analysis is not appropriate.**
- **Consolidation analysis is "not appropriate".**

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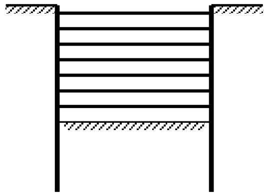
Excavation in Clay → Undrained Analysis



Clay or any soil with

Permeability < 10^{-8} m/s

- **Undrained analysis**
- **Drained analysis is not appropriate.**
- **Consolidation yields similar results as undrained *if proper soil parameters are used.***

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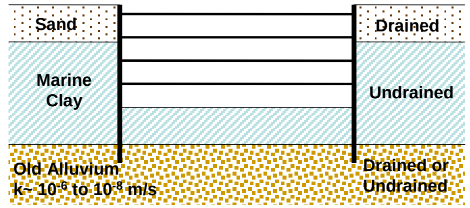
Excavation in Clay over Old Alluvium



Scenario	Sand	Clay	OA
1	Drained	Undrained	$k \leq 10^{-8}$ m/s → undrained
2	Drained	Undrained	$k \geq 10^{-6}$ m/s → drained
3	Drained	Consolidation	k between 10^{-6} & 10^{-8} m/s → consolidation

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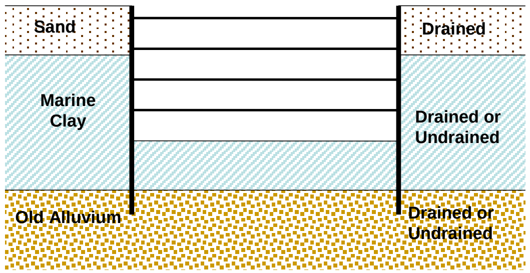
What are the implications of conducting an undrained or drained analysis?



For TERS, assuming Marine Clay to be drained may not yield meaningful results!

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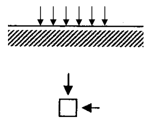
Total Stress Approach in Clay

$$\sigma = \sigma' + u$$



Initial:
σ_{1o}, σ_{3o} and u_o
σ'_{1o} and σ'_{3o}

Applied Load:
$\Delta\sigma_1$, $\Delta\sigma_3$ and Δu

- Involves only total stress.
- Involves total stress parameters e.g. c_u, ϕ_u=0, E_u, ν_u=0.495.
- Does not involve u and Δu.
- Commonly used in undrained analysis in saturated clay.

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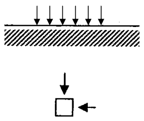
Effective Stress Approach in Clay and Sand

$$\sigma' = \sigma - u$$
$$u = u_o + \Delta u$$



Initial:
σ_{1o}, σ_{3o} and u_o
σ'_{1o} and σ'_{3o}

Applied Load:
$\Delta\sigma_1$, $\Delta\sigma_3$ and Δu

- Involves both total stress and pore pressure.
- Involves effective stress parameters e.g. c′, ϕ′, E′, ν′=0.1 to 0.4.
- Accuracy of Δu has major impact.
- For **clay,** ESA can be adopted in undrained, drained and consolidation analysis.
- For **sand ,** ESA should be used under drained condition.

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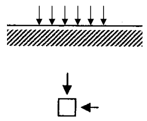
Undrained Analysis in Clay

$$\sigma = \sigma' + u$$



Initial:
σ_{1o}, σ_{3o} and u_o
σ'_{1o} and σ'_{3o}

Applied Load:
$\Delta\sigma_1$, $\Delta\sigma_3$ and Δu

- It can be a total stress analysis involving total stress parameters.
- It can be an effective stress analysis involving effective stress parameters as in Mohr-Coulomb, Soft Clay, Modified Cam Clay model and Hardening Soil model.
- It can be a consolidation analysis with t~0.
- No volume change only shear distortion.
- No dissipation of pore pressure.

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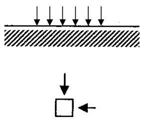
Drained Analysis in Clay and Sand

$$\sigma' = \sigma - u$$
$$u = u_o \text{ or steady state seepage}$$



Initial:
σ_{1o}, σ_{3o} and u_o
σ'_{1o} and σ'_{3o}

Applied Load:
$\Delta\sigma_1$, $\Delta\sigma_3$ and Δu

- It can be an effective stress analysis involving effective stress parameters.
- It can be a consolidation analysis with t~∞.
- Excess pore pressure fully dissipated $\Delta u=0$.
- $u = u_o$ = hydrostatic pore pressure or steady state seepage.

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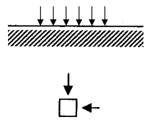
Consolidation Analysis in Clay

$$\sigma' = \sigma - u$$
$$u = u_o + \Delta u$$



Initial:
σ_{1o}, σ_{3o} and u_o
σ'_{1o} and σ'_{3o}

Applied Load:
$\Delta\sigma_1$, $\Delta\sigma_3$ and Δu

- It is an effective stress analysis involving effective stress parameters.
- It can generate results for any period of time.
- $t \sim 0^+$ → undrained analysis
- t~∞ → drained analysis
- Accuracy depends of Δu.

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Total or Effective Stress Approach for Saturated Clay

Total Stress

Applicable only to undrained condition

Pore pressure has no effect on the results.

Parameters required for Mohr-Coulomb model is c_u and E_u

Effective Stress

Applicable only to undrained, partially drained and drained conditions i.e. from end-of –construction to long term steady state.

Pore pressure has great effect on the results.

Parameters required are:

- Mohr-Coulomb model: c', ϕ', φ, E' and ν' (Method A)
- Modified Cam Clay: c', ϕ', φ, κ, λ and v_{ur}

Effective Stress with Strength in Total Stress (Method B)

Applicable only to undrained condition (Mohr-Coulomb model)
Parameters required are: c_u, $\phi_u=0$, $\varphi=0$, E' and $\nu'=0.35$

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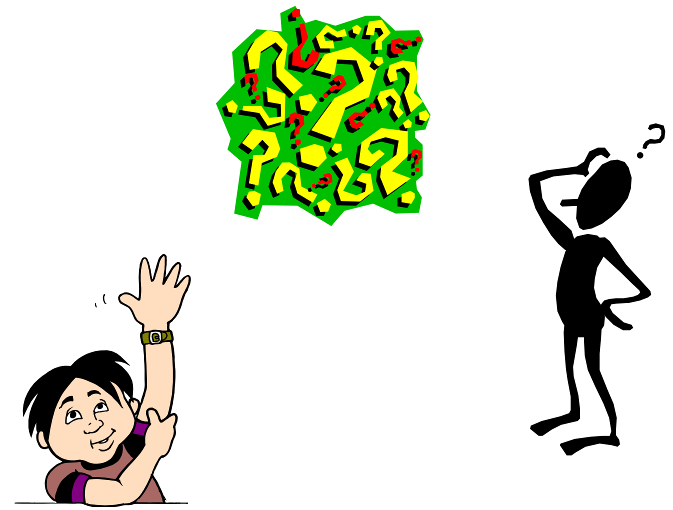

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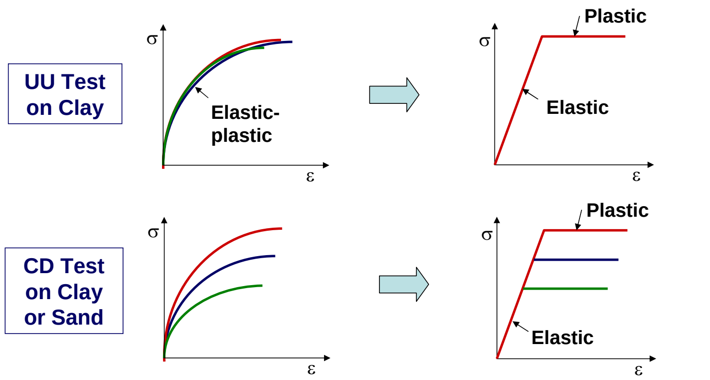
Can Mohr-Coulomb Model simulate Real Soil Behaviour?



Real Soil

Mohr-Coulomb Soil

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Can a Elastic Model simulate Real Soil Behaviour?

Elastic Model

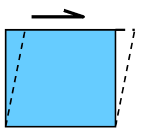
Shear stress produces shear strain:

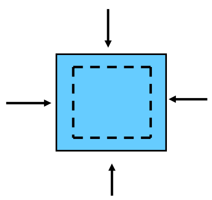
$\tau \rightarrow \gamma$

$\tau \rightarrow$ **no** ε_v

Normal stress produces volumetric strain:

$\sigma \rightarrow \varepsilon_v$





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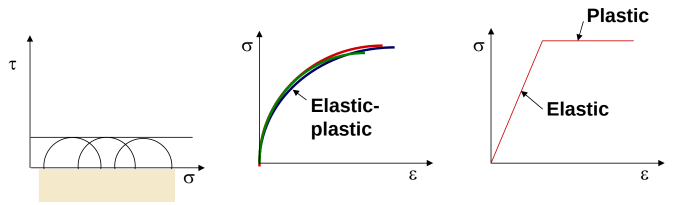
Can a elastic soil simulate undrained behaviour of clay?



Real Soil Behaviour	Elastic Model (ν=0.5)
$\tau \rightarrow \gamma$	$\tau \rightarrow \gamma$
$\tau \rightarrow$ no ε_v	$\tau \rightarrow$ no ε_v
$\sigma \rightarrow$ no ε_v	$\sigma \rightarrow$ no ε_v
Stress independent	Stress independent

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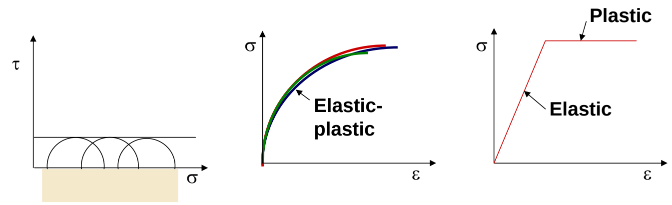
Can a elastic soil simulate undrained behaviour of clay?



Yes! If we use c_u and E_u.

Can we use c' - ϕ' and E' ?

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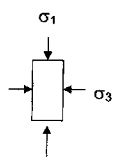
Pore Pressure Response of an Elastic Soil

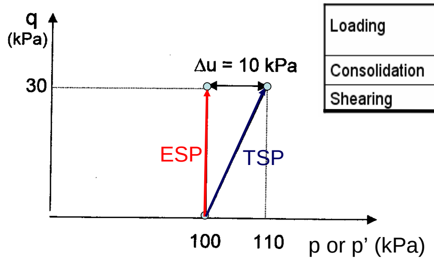
CU Test



Characteristics of an elastic soil:

- Shear stress [Δq or $\Delta(\sigma_1-\sigma_3)$] does not generate pore pressure
- Normal stress generates pore pressure

$\Delta u = \Delta p = (\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) / 3$



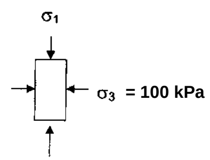
Loading	σ_1 (kPa)	σ_2 (kPa)	σ_3 (kPa)	p (kPa)	p' (kPa)	q (kPa)
Consolidation	100	100	100	100	100	0
Shearing	130	100	100	110	100	30

$A = \Delta u / \Delta(\sigma_1-\sigma_3)$

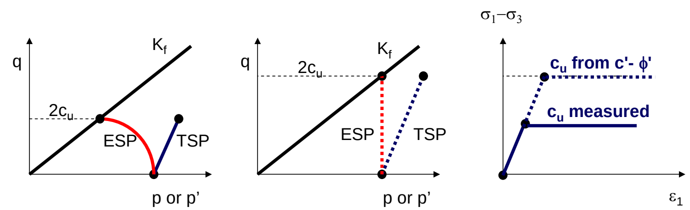
$A = 10 / 30 = 1/3$

CU Test

Consolidated Undrained Triaxial Compression Test



Real Soil Mohr-Coulomb σ - ε curve



c'- ϕ' over-predicted c_u !!!

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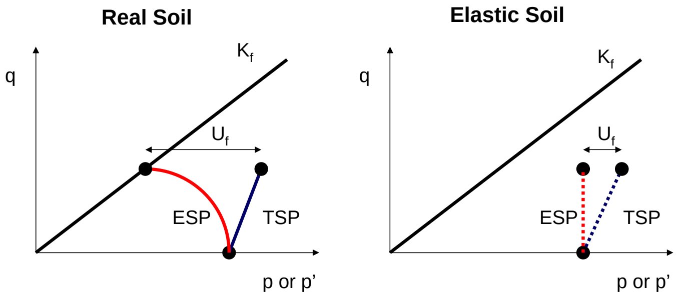
Is the pore pressure response correct?

Let's look at CU test on a normally consolidated clay.



The predicted pore pressure is much smaller than the measured!

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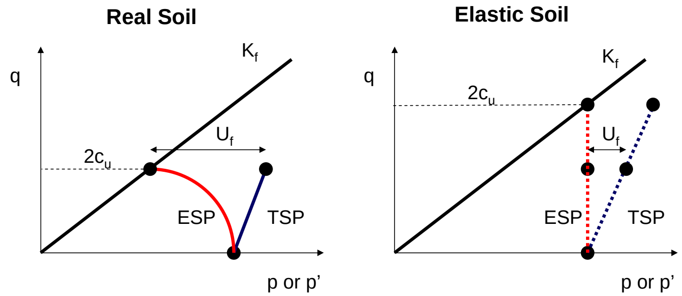
Method "A"

Effective stress Mohr-Coulomb Method using c' and ϕ'

It over-estimates the undrained shear strength and under-estimates the excess pore pressure of a normally consolidated clay.



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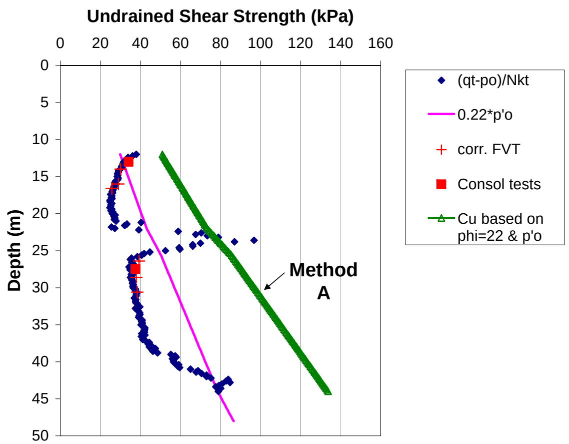
Over-estimation of c_u at a Reclaimed Site



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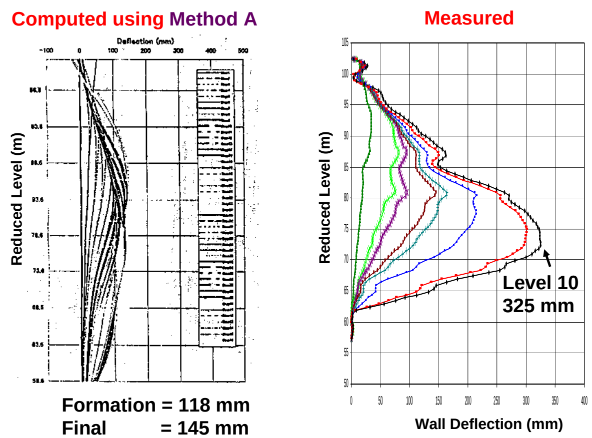
Nicoll Highway – Results of Undrained Analysis using Method A



Why is **Method A** giving the wrong deflection?

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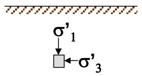
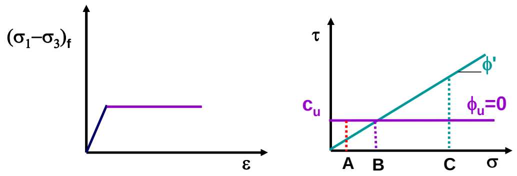
Does Method A always over-estimate c_u for NC clay?



This site has a constant c_u.



$$(\sigma_1-\sigma_3)_f = \frac{2c'\cos\phi' - 2\sigma_3'\sin\phi'}{1-\sin\phi'}$$

For NC Clay, it under-estimates c_u at low stress and over-estimates it at high stress.

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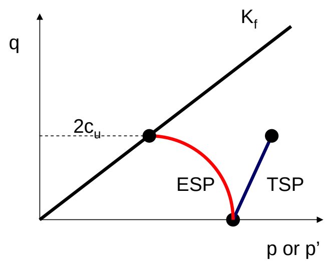
Method "B"

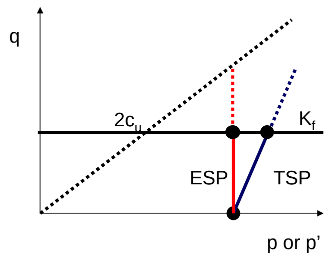
Effective stress Mohr-Coulomb Method using c_u and ϕ_u=0

It forces the soil to fail at a specified undrained shear strength.

Real Soil



Elastic Soil



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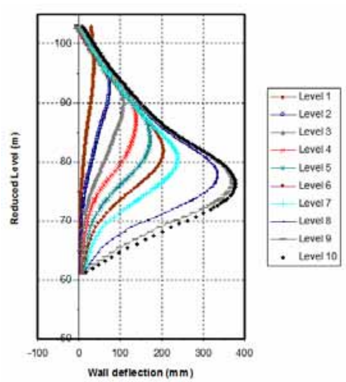
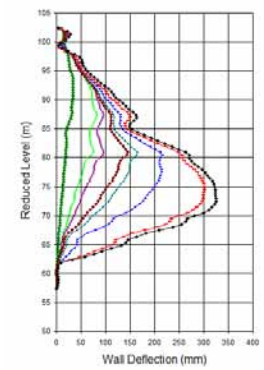
Nicoll Highway – Results of Undrained Analysis using Method B

Computed using Method B



Measured



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Using Mohr-Coulomb model for Undrained Analysis

Method A → c' and φ' produces wrong c_u

Method B or C → Forces Plaxis to use specified c_u

	Method **A**	Method **B**	Method **C**
Stress Type	Effective	Effective	Total
Strength	c ′ and φ ′	c_u and ϕ_u	c_u and ϕ_u
Modulus	E ′	E ′	E_u
Poisson's Ratio	ν ′	ν ′ = 0.35	ν_u = 0.495
K_o or K_{ot}	K_o	K_o	K_{ot}

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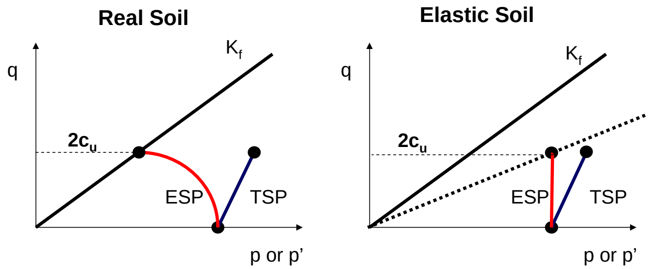
What can we do to salvage Method A?



Use a smaller ϕ' so that c' -ϕ' can yield the correct c_u!

$$(\sigma_1-\sigma_3)_f = \frac{2c' \cos\phi' + 2\sigma'_3 \sin\phi'}{1 - \sin\phi'} \Rightarrow \phi' = \sin^{-1}(c_u/\sigma'_v)$$

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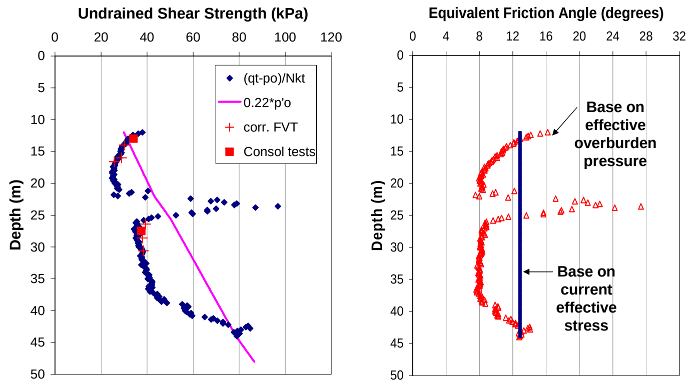
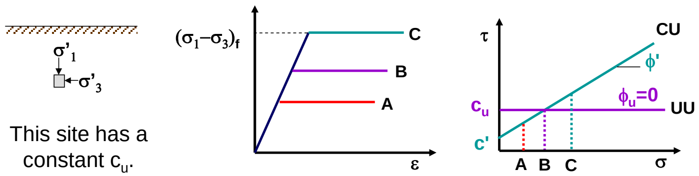
Equivalent Friction Angle of Soft Clay for Method A



We can use Method A with $\phi'=8^\circ$ for undrained analysis but not drained or consolidation analysis!

Can Method A be used for Overconsolidated Clay?



$$(\sigma_1-\sigma_3)_f = \frac{2c' \cos\phi' + 2\sigma'_3 \sin\phi'}{1 - \sin\phi'}$$

For OC Clay, it under-estimates c_u at low stress and over-estimates it at high stress.

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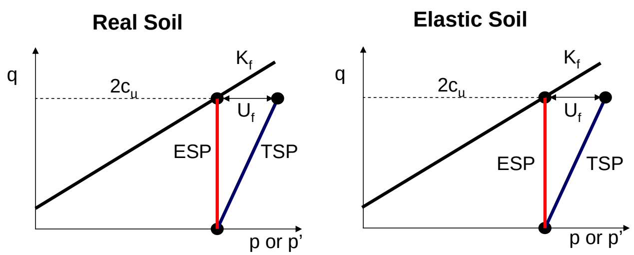
Potential application of Method A to over-consolidated clay



1. Make sure the measured stress path is similar to that of Elastic Soil.
2. Divide the stratum into sub-layers with different c_u or c' and ϕ' for each layer.

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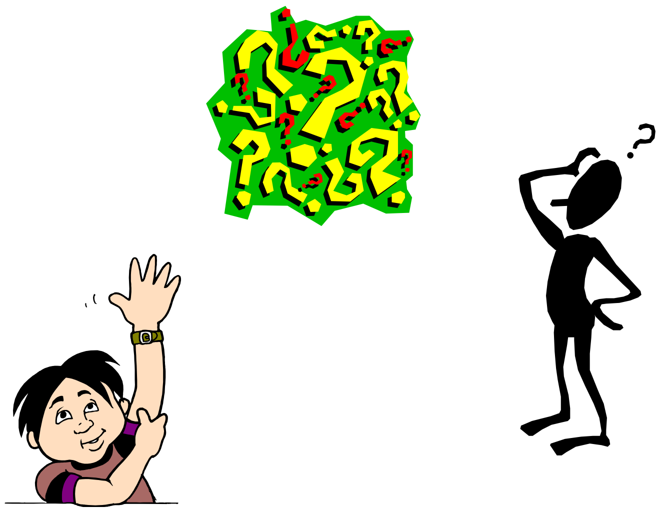

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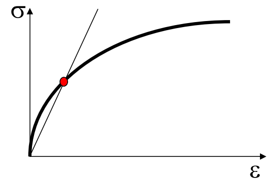
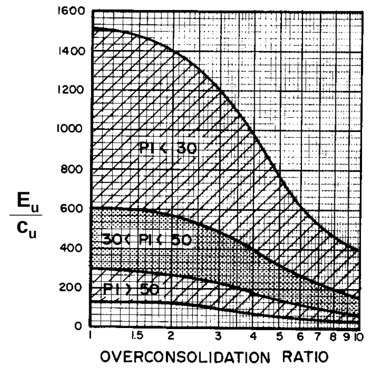
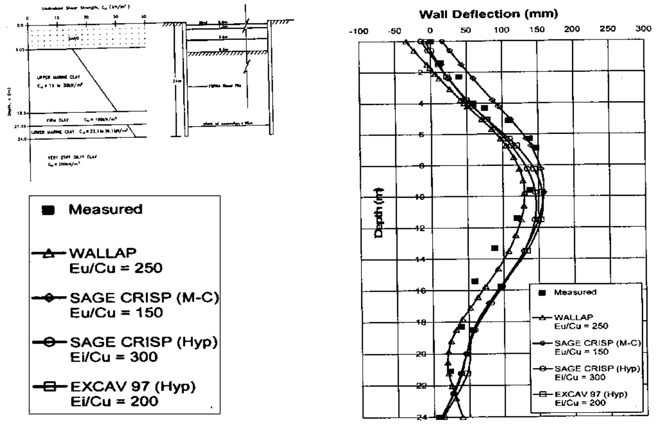
How do we determine E_u?





Industry standard → $E_u = 300\ c_u$

Calibration of E_u – Rochor Complex



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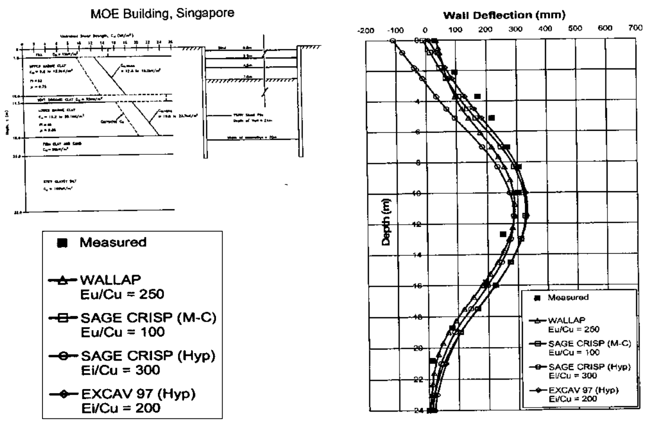
Calibration of E_u – MOE Building



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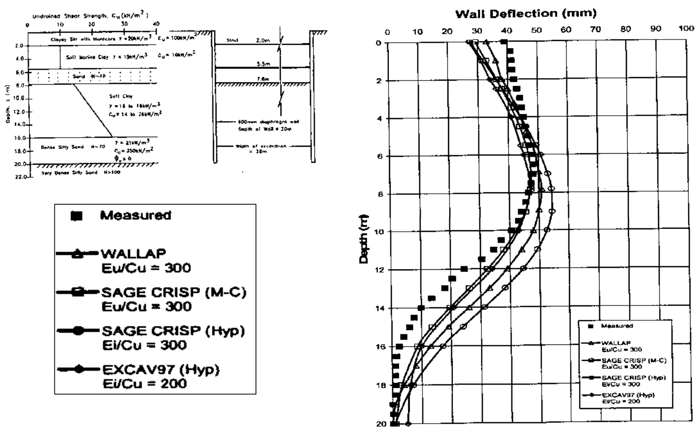
Calibration of E_u – Syed Alwi Condo



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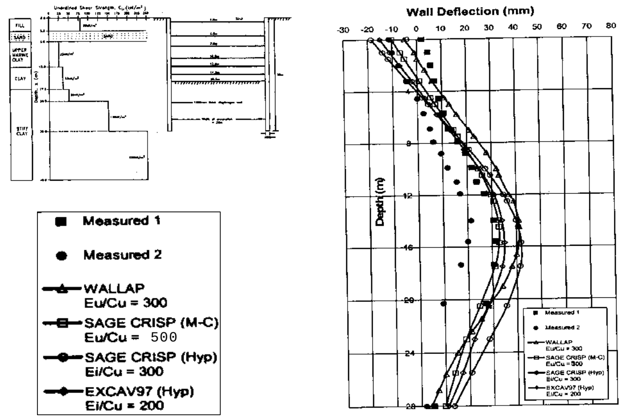
Calibration of E_u – Lavender MRT Station



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E_i/c_u or E_u/c_u Values obtained from Back-Analysis

	MOE Building	Rochor Complex	Syed Alwi Project	Lavender Station
WALLAP, Mohr Coulomb, E_u/c_u	250	250	300	300
SAGE CRISP, Mohr-Coulomb, E_u/c_u	100	150	300	500
SAGE CRISP, Hyperbolic, E_i/c_u	300	300	300	300
EXCAV97, Hyperbolic, E_i/c_u	200	200	200	200

Comparison of maximum wall deflection (mm)

	MOE Building	Rochor Complex	Syed Alwi Project	Lavender Station
Measured	305	146	48	31
WALLAP, Mohr Coulomb, E_u/c_u	290	130	51	41
SAGE CRISP, Mohr Coulomb, E_u/c_u	326	157	51	33
SAGE CRISP, Hyperbolic, E_i/c_u	287	145	57	42
EXCAV97, Hyperbolic, E_i/c_u	330	155	47	35

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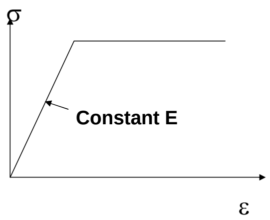
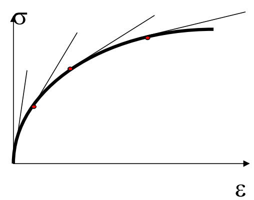
Mohr-Coulomb

E_u/c_u ~ 100 to 500



Hyperbolic Model

E_i/c_u ~ 300



Why?

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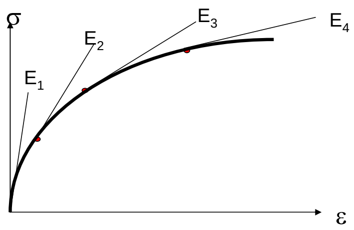
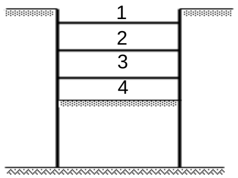
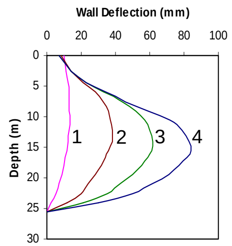
Soil Modulus changes at various stages of excavation







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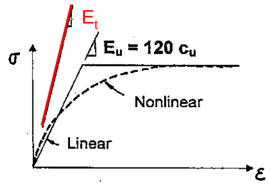
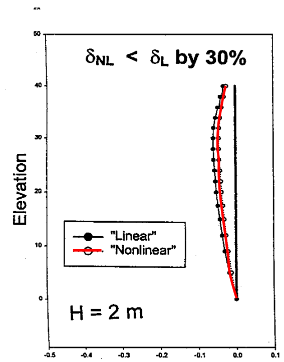
At early stage of excavation,

Mohr-Coulomb, Linear E → larger δ
Hyperbolic, Non-linear E → smaller δ





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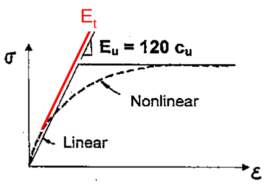
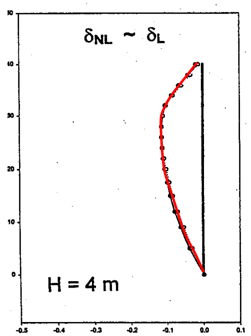
At intermediate stage of excavation,

Mohr-Coulomb, Linear E → simliar δ
Hyperbolic, Non-linear E → smililar δ





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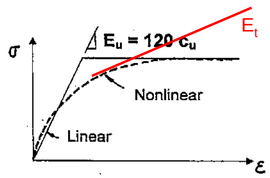
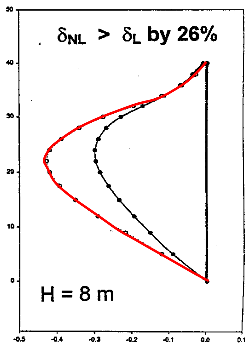
At final stage of excavation,

Mohr-Coulomb, Linear E → smaller δ
Hyperbolic, Non-linear E → larger δ





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RECOMMENDATIONS

Program & Soil Model	E_u/c_u; E_i/c_u; K_s/c_u
RIDO: spring constant	15 - 200
WALLAP: spring constant	250
EXCAV97: Hyperbolic	200
SAGE CRISP: Hyperbolic	300
SAGE CRISP: Mohr-Coulomb	100 - 500
PLAXIS: Mohr-Coulomb	100 - 300

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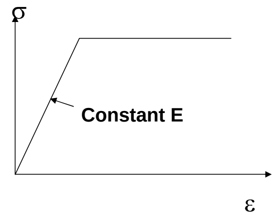
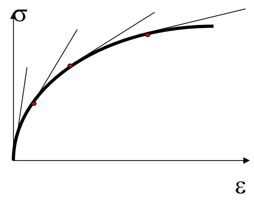
Mohr-Coulomb

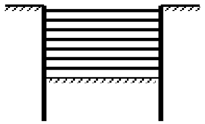
E_u/c_u ~ 100 to 500



Hyperbolic Model

E_i/c_u ~ 300





At the final stage of excavation, the relevant modulus may vary from project to project.

So… what E_u/c_u should we use?

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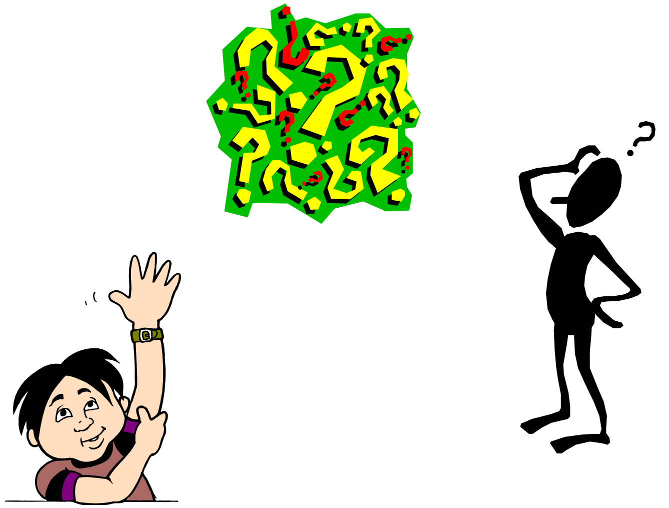

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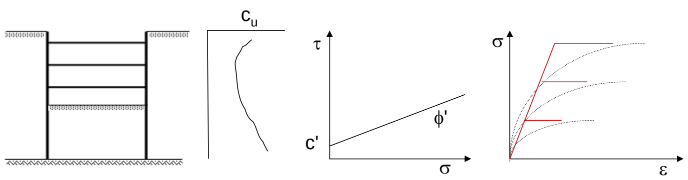
Drained Analysis using M-C Model



$$G = \frac{E'}{2(1+\nu')} = \frac{E_u}{2(1+\nu_u)}$$

$E_u \sim 300\ c_u$ $\nu_u = 0.5$ $\nu' \sim 0.1$ to 0.4

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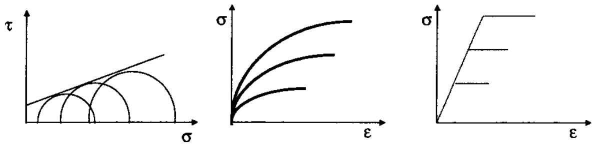
How reliable is drained analysis using M-C model?

Can M-C model simulate drained behaviour of soil?



Real Soil Behaviour	**Elastic Model**
$\tau \rightarrow \gamma$	$\tau \rightarrow \gamma$
$\tau \rightarrow \varepsilon_v$	$\tau \rightarrow$ no ε_v
$\sigma \rightarrow \varepsilon_v$	$\sigma \rightarrow \varepsilon_v$
E' is stress dependent	E' is stress independent

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What are the implications?

1. It may not produce the correct deformation.
2. Results may be sensitive to Poisson's ratio.
3. Difficult to determine Poisson's ratio (ν=0 to 0.5).
4. It may not produce the correct pore pressure response.
5. When using c'-ϕ' in consolidation analysis, it may generate the wrong "undrained" shear strength.

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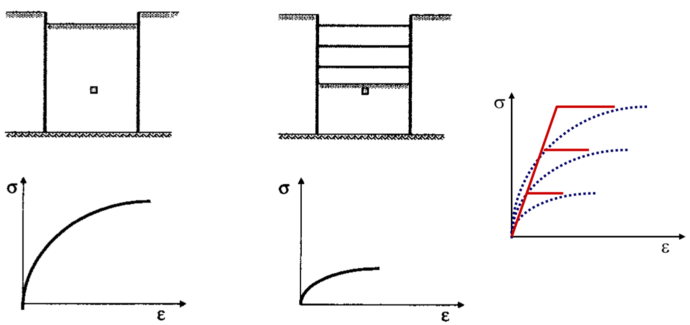
Stress Dependent Behaviour of Soil under Drained Condition



E' = 20 MPa **E' = 3 MPa**

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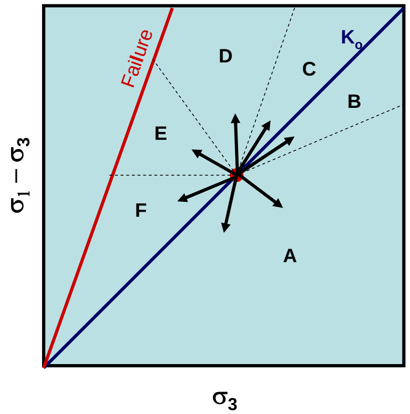
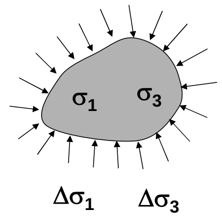
Stress Paths in an Elastic Medium





E – Questionable Zone

F – Danger Zone

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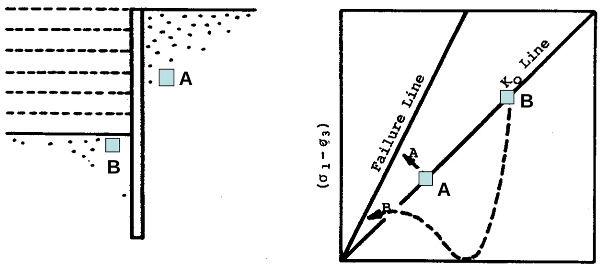
Typical Stress Paths in Excavation



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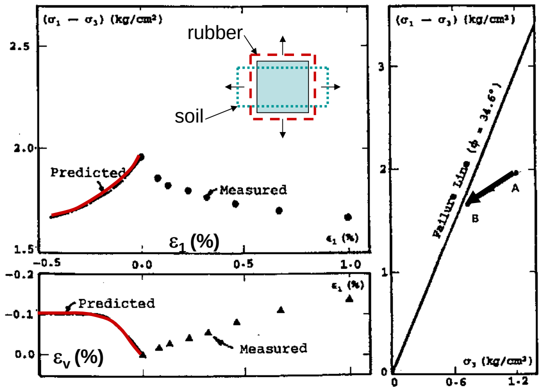
Stress Path in Zone F under Drained Condition



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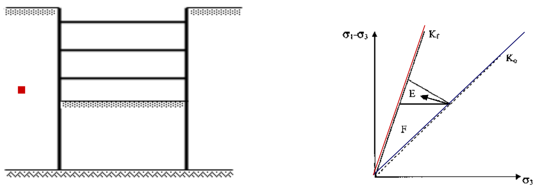
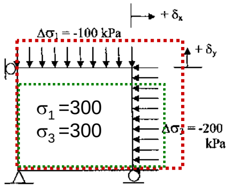
Stress Path in Zone E under Drained Condition





Poisson's Ratio	Horizontal Displacement (mm)	Vertical Displacement (mm)
0.2	+26	+15
0.32	+22	+2
0.4	+18	-9
0.495	+9	-15

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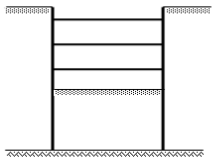
Effect of Poisson's Ratio



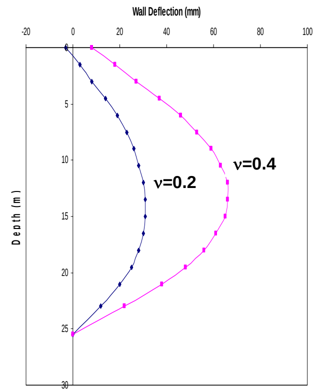
C'=5 kPa

φ'=35°

E'=8000 kPa

H=9 m

	ν=0.2	ν=0.4
M_{max} ,kNm/m	298	477
Strut 1, kN/m	77	114
Strut 2, kN/m	226	335
Strut 3, kN/m	163	178



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Can a Elastic Model simulate Real Soil Behaviour?

Undrained condition with total stress analysis:

❖ The Elastic Model can produce reasonable results.

Drained condition with effective stress analysis:

❖ The Elastic Model can produce reasonable results for certain stress paths (Zones A to D).

❖ Results involving stress paths in Zone E may be questionable.

❖ Results involving stress paths in Zone F is unreliable.

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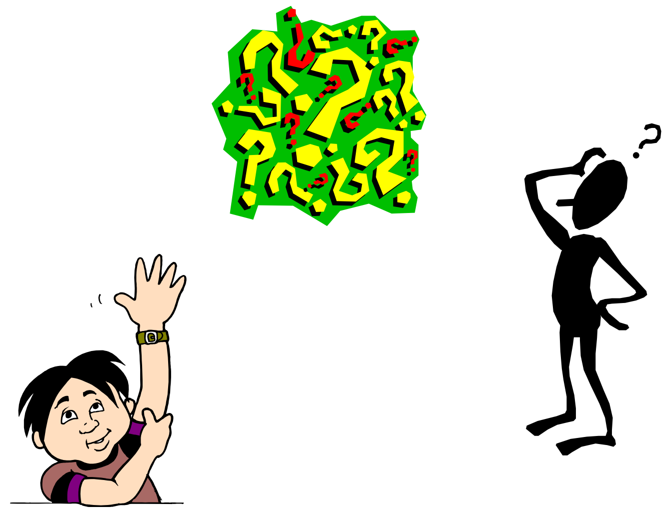

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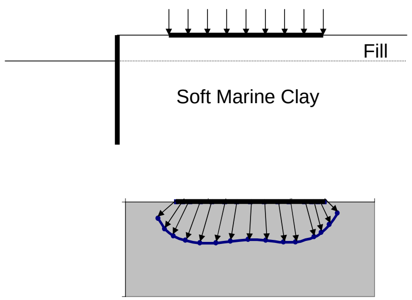
Can we trust the results?



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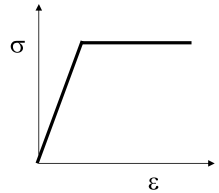
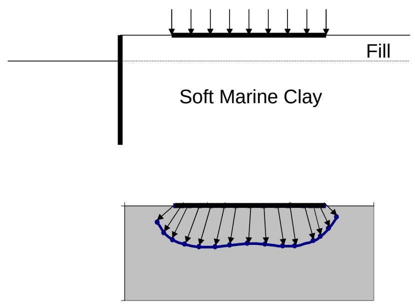
Results using Mohr-Coulomb Model





Linear Analysis

$\delta_{V,MAX}$ = **33 mm**

$\delta_{H,MAX}$ = **28 mm**

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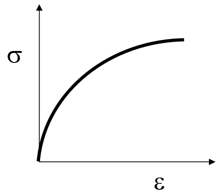
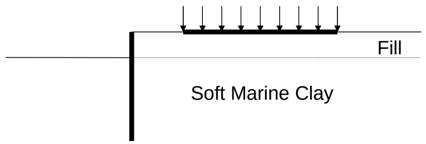
Results using Hyperoblic Model

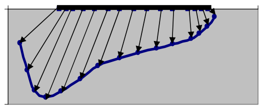




Non-Linear Analysis

$\delta_{V,MAX}$ = **72 mm**

$\delta_{H,MAX}$ = **59 mm**



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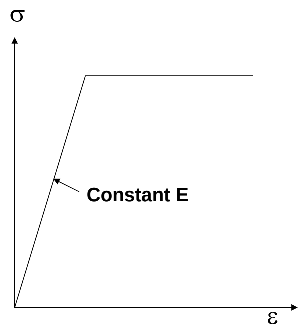
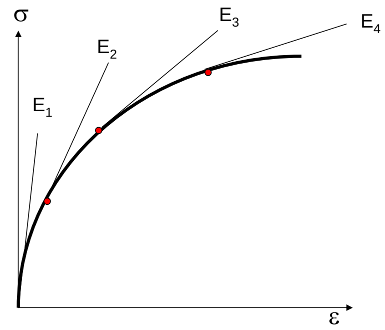
Linear vs Non-Linear Model

Mohr- Coulomb Model



Real Soil Behaviour



You must understand the shortcomings of the soil model used!

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