Article ID: 1671-3664(2007)02-0123-12 **DOI**:

Intergrain contact density indices for granular mixes — I: Framework

S. Thevanayagam[†]

Dept. of Civil, Struct. and Envir. Eng., State University of New York, Buffalo, NY 14260, USA

Abstract: Mechanical behavior such as stress-strain response, shear strength, resistance to liquefaction, modulus, and shear wave velocity of granular mixes containing coarse and fine grains is dependent on intergrain contact density of the soil. The global void ratio e is a poor index of contact density for such soils. The contact density depends on void ratio, fine grain content (C_F), size disparity between particles, and gradation among other factors. A simple analysis of a two-sized particle system with large size disparity is used to develop an understanding of the effects of C_F, e, and gradation of coarse and fine grained soils in the soil mix on intergrain contact density. An equivalent intergranular void ratio $(e_c)_{eq}$ is introduced as a useful intergrain contact density for soils at fines content of less than a threshold value C_{Fth}. Beyond this value, an equivalent interfine void ratio $(e_f)_{eq}$ is introduced as a primary intergrain contact density index. At higher values of C_F beyond a limiting value of fine grains content C_{FL}, an interfine void ratio e_f is introduced as the primary contact density index. Relevant equivalent relative density indices $(D_{rc})_{eq}$ and $(D_{rf})_{eq}$ are also presented. Experimental data show that these new indices correlate well with steady state strength, liquefaction resistance, and shear wave velocities of sands, silty sands, sandy silts, and gravelly sand mixes.

Keywords: Sand; silt; silty sand; sandy silt; cyclic strength; shear wave velocity; intergrain contact density

1 Introduction

Recent earthquake case histories indicate that natural soils and man-made sandy deposits containing a significant amount of fine grains (silty sands, clayey sands) and/or gravel liquefy and cause lateral spreading (Seed *et al*. 1983, Seed and Harder 1990, JGS 1996). Experience gained from past studies on clean sands does not always directly translate to such broadly graded granular soil mixes. Recognition of this has lead to several laboratory and field studies to evaluate the effects of increasing silt or gravel content on: (a) cyclic strength, (b) collapse potential, (c) steady state strength, (d) shear wave velocity, etc. Laboratory studies on clean sands mixed with non-plastic silts or plastic fines show that, at the same (global) void ratio, the steady state strength and cyclic strength of silty sand decrease with an increase in fines content (Chang, 1990; Chameau and Sutterer, 1994; Georgiannou *et al*., 1990, 1991a, 1991b; Vaid, 1994; Koester, 1994; Pitman *et al*., 1994; Singh, 1994; Yamamuro *et al*., 1999; Zlatovic and Ishihara, 1995, 1997). Beyond a certain transition range, this trend reverses and the strength increases with a further increase in fines content. The transition fines content range is about 20 to 30% for non-plastic fines (Vaid, 1994; Kuerbis *et al*., 1989; Singh, 1994; Koester, 1994). It is less than 20% for clayey fines (Georgiannou *et al*. 1990, 1991a, 1991b). The physical meaning of the transition fines content is not clear. The conclusions in the literature on whether the presence of fines is beneficial or not are contentious. No consensus exists on how to characterize liquefaction resistance, collapse resistance, and post-liquefaction strength of silty sands and sandy silts. Similar concerns prevail regarding gravely soils (Evans and Zhou, 1995).

Taking a different approach, field performance studies have sought to solve this problem by correlating SPT blow counts, CPT data, and shear wave velocity measurements with observations of liquefied sites and back-calculated post-liquefaction residual strength of failed embankments. Their use in practice relies on such intuitive reasoning as the impeding drainage effect of fines on SPT blow counts and/or their relationship with relative density (Seed *et al*., 1983; Seed, 1987; Seed and Harder, 1990; Robertson and Wride, 1997; Andrus and Stokoe, 1997). There are variations as well on the nature of such relationships (Stark and Mesri, 1992; Ishihara, 1993; Baziar and Dobry, 1995). Questions prevail among practicing engineers on broad applicability of the field correlations to all (new) sites.

Recently, new and/or additional thoughts concerning the relative roles of various particles in silty/gravely soils have been put forward to explain the observed behavior of such granular mixes (Thevanayagam, 1998;

Correspondence to: S. Thevanayagam, Dept. of Civil, Struct. and Envir. Eng., 212 Ketter Hall, SUNY at Buffalo, Buffalo, NY 14260, USA
Tel: 716-645-2114 extension 2430
E-mail: theva@eng.buffalo.edu
[†]Associate Professor
Supported by: National Science Foundation and USGS
Received May 8, 2007; **Accepted** May 22, 2007

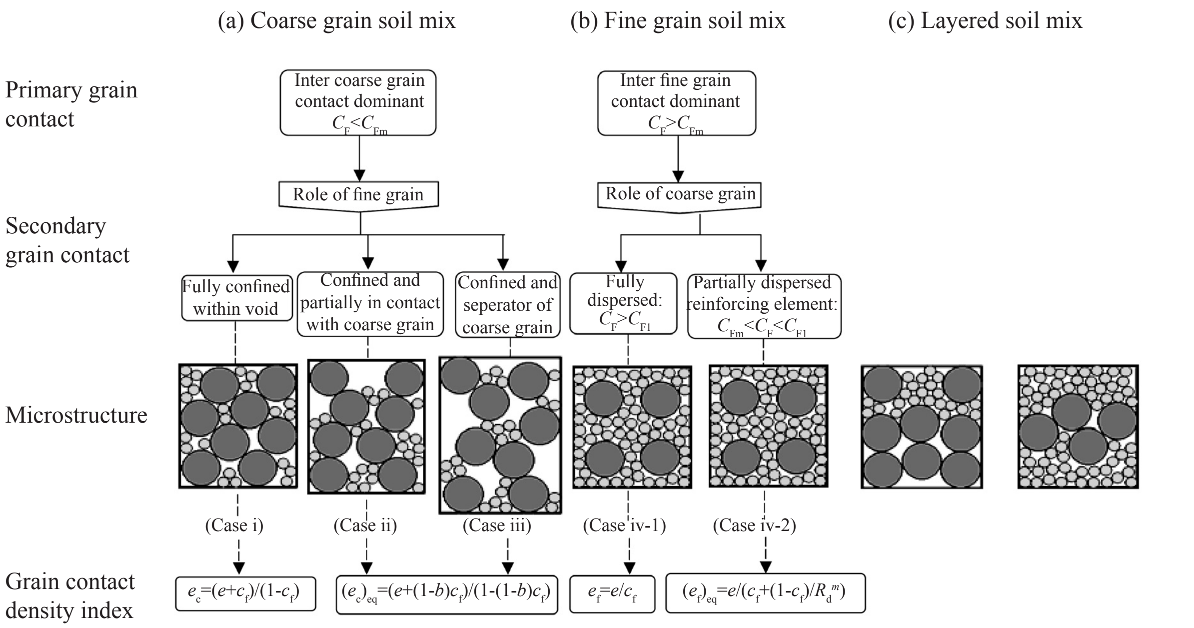
Thevanayagam and Mohan, 2000). The underlying idea is that the physical nature of silty sands and gravely sands is entirely different from clean sand. As the void ratio and proportion of the coarse and fine grains content of these soils change, the nature of their microstructure also changes. The relative participation of the particles of very different sizes in the internal *interparticle contact force chain* also changes. The global void ratio can be a reasonably good state parameter only if it correlates well with the internal force chain within the soil mass. Although it has been used as one of the primary state variables, starting from as early as Terzaghi, in the critical state soil mechanics theories (Roscoe *et al*., 1958, 1963), its broader application for all soils ranging from clean sands to sandy silts and gravely soils is not fully satisfactory. Due to the particle size disparity and availability of pores larger than some particles, some particles may remain inactive or move between pores without significantly affecting or contributing to the force chain, yet they contribute to the global void ratio. Alternately, when there are sufficient fine grains, the coarse grains are dispersed and contribute much less to the force chain than to the global void ratio. Global void ratio turns out to be a weak parameter to represent the internal force chain. In general, the stress-strain behavior is affected by a critical combination of *intergranular and interfine contacts* and the physical and physico-chemical *interactions* thereof. The combined effects of intergranular and interfine contacts must be delineated in dealing with silty sands and gravely soils in understanding their mechanical response and mechanisms leading to liquefaction and post-liquefaction deformation.

This paper focuses on the nature of the microstructure of granular mixes containing coarse and fine grains. A simple approach for classification of such soils, with due consideration of the contributions of coarse and fine grains to the mechanical response of these soils is presented. A set of contact density indices, namely equivalent intergranular void ratio $(e_c)_{eq}$, equivalent interfine void ratio $(e_f)_{eq}$, and interfine void ratio (e_f) is proposed as the primary indices of contact density for mixes containing low, intermediate, and high fine grain contents, respectively. These indices are expressed in terms of global void ratio, fines content, and gradation characteristics of the soil. These indices can be used for interpretation of the behavior of gap-graded granular mixes in a consistent framework. A companion paper (Thevanayagam, 2007) presents a detailed experimental evaluation of this framework to characterize liquefaction resistance of sands and non-plastic silty soils.

2 Framework

2.1 Microstructure

Consider a granular mix containing two distinct sizes of spherical particles of diameters d and D, respectively (Fig.1). Microstructure of this granular mix can be constituted by many different ways. Each microstructure leads to a different particle contact



b= Portion of the fine grains that cotribute to the active intergrain contacts: e=global void ratio: C_F=fine grains content
C_{Fth}=Threshhold fine grains content, $C_{Fth}<(100e/e_{max,HF})\%$; C_{F1}=limit fines content, $C_{F1}>100(1-\pi(1+e)/(6s^3))\%>C_{Fth}$
m: Reinforcement factor; $R_d=D/d$=particle size disparity ratio; $s=1+a/R_d$, $a=10$; $e_{max,HF}$: the maximum void ratio of host fine

Fig. 1 Intergranular soil mix classification

network and corresponding internal force chain network along contacts between particles. Hence, each microstructure exhibits a different stress-strain response and mechanical behavior. Among many variations, a few extreme limiting categories of possible microstructure are as follows.

2.1.1 Case i

The first category in Fig.1(a) occurs when the fine grains are fully confined within the void spaces between the coarse-grains with no contribution whatsoever in supporting the coarser grain skeleton. This requires that the fine grain particle size (*d*) is much smaller than the pore size between the coarse grains. The particle size disparity R_d (=*D/d*) must be greater than about 6.5 (Appendix I). It is also essential that the fine grain content (C_F) is less than a certain threshold value (C_{Fth}) such that the inter-coarse grain voids are not completely filled with the fine grains making it impossible to make contact force chain among the fine grains themselves. The threshold state occurs when C_F is sufficiently high so that direct fine-grain-to-fine-grain contacts begin to constitute a strong force chain. Soil microstructure that satisfies the two constraints ($C_F \ll C_{Fth}$ and $R_d \gg 6.5$) is categorized as case i in this paper. C_{Fth} is given by (Appendix II):

$$C_{Fth} \le \frac{100\,e}{e_{max,HF}}\% \qquad (1)$$

where $e_{max,HF}$ is the maximum void ratio of the host fine grain soil.

Consider changing the microstructure shown for case i in two ways: (1) alter the position of some of the fine grains, or (2) add more fine grains. The consequences are significant.

2.1.2 Cases ii and iii

If one alters the position of some of the fine grains while maintaining the fine grain content, the microstructure corresponding to cases ii and iii in Fig.1(a) are obtained with a concurrent increase in global void ratio. Essentially, the microstructure for these cases is made up of a coarse grain skeleton where some of the coarse grain contacts are separated by the fine grains while some fine grains are confined within the voids between the coarse grains. Hence, a portion of the fine grains becomes active participants in the internal contact force chain. These fine grains are termed the 'separating fines'.

2.1.3 Case iv

If FC exceeds sufficiently beyond C_{Fth}, the fourth category (Fig.1(b)) occurs naturally. The fine grains make active contacts among themselves while the coarse grains become gradually dispersed and act as embedded reinforcement elements within the fine grain matrix until they are separated sufficiently apart. This occurs until limiting fines content C_{FL}, beyond which the effects of coarse grains is negligible and the behavior of the soil mix is entirely governed by the fine-grain-to-fine-grain contact. The transition zone between C_{Fth} and C_{FL} is called case iv 2 and the zone corresponding to $C_F > C_{FL}$ is called case iv-1. The C_{FL} is given by (Appendix III):

$$C_{FL} \ge 100\left[1-\frac{\pi(1+e)}{6\,s^3}\right]\% > C_{Fth} \qquad (2)$$

where $s=1+a(d/D)=1+a/R_d$, and a is a constant, approximately equal to 10.

2.1.4 Case v

The fifth category (Fig.1(c)) occurs when the coarse and fine grains constitute a fully layered system. This is called case v. It is also possible to create a composite system that contains some of the cases i through v.

2.2 First order indices of active contacts

Contact is the mechanism by which particles in granular media compose the internal force chain network, in the absence of inter-particle gravitational and physio-chemical force fields. The latter is relevant for soils containing plastic fines. Stress-strain response, strength, compressibility, and modulus are dictated by the nature of this network. A primary index parameter that represents the density of active contacts (per grain) is essential to deduce the mechanical response of granular mixes. Physics imposes certain space constraints to the motion of particles. Two objects cannot occupy the same space at the same time. This imposes a kinematic constraint on how the contacts and therefore the internal force chain would evolve with subsequent deformation involving motion of particles. Although the particles that are actively in contact constitute the force chain, the inactive grains also have a secondary influence on the evolution of the force chain by imposing kinematic space constraints to the motion of the active grains. Physically, void ratio is an index of mass (solid volume) density. It is a good index for space constraints. However, for certain arrays of spherical particles of equal size, it also becomes an index of active contact density. For other particle shapes and packing, it is only a crude index of active contacts. It is not expected to hold as a unifying index from one sand to another. It is only a secondary index of active contacts.

2.2.1 Intergranular (e_c) and interfine (e_f) void ratios

A first order solution is to consider a granular mix as a composite matrix consisting of two skeletons, the coarse grain skeleton and fine grain skeleton, identify the particles that do not actively participate in the force chain (in Fig.1), and define an index void ratio of active contacts excluding the aforementioned particles. For the cases i through iii ($C_F < C_{Fth}$), if all fine grains are considered to be inactive and voids, the resulting intergranular void ratio e_c (Mitchell, 1993; Vaid 1994) is given by:

$$e_c = \frac{e + c_f}{1 - c_f} \qquad (3)$$

where c_f=C_F/100. The e_c is also known as the *sand skeleton void ratio* in the literature (Kuerbis *et al*., 1989). For case iv (C_F>C_{Fth}), if the effect of coarse grain is completely neglected, the relevant interfine contact void ratio (Thevanayagam, 1998) is given by:

$$e_f = \frac{e}{c_f} \tag{4}$$

In both cases, the specific gravities of the coarse and fine grains are assumed to be nearly the same. The underlying tenet is that the soil behavior is governed by the coarse grain skeleton alone in cases i through iii and it is governed by the fine grain skeleton alone in case iv. But this tenet is fundamentally flawed because the neglected fine and coarse grains in cases i through iii and iv, respectively, influence the global mechanical response. For cases i through iii, although it may be possible to use e_c as an index of active contacts, it is expected that the mechanical behavior of such mixes would be stronger than the host coarse grain soil *at the same* e_c. Similarly, for case iv, the mechanical response of a granular mix is expected to be stronger than the host fine grain soil at the same e_f. The magnitude of e_f in cases i through iii, and e_c in case iv may be used as the secondary indices to assess the degree of such secondary influences. A set of equivalent contact indices $(e_c)_{eq}$ and $(e_f)_{eq}$ that reflect the primary and secondary influences together is necessary for characterization of granular mixes.

2.3 Second order indices of active contacts

2.3.1 Equivalent intergranular void ratio, $(e_c)_{eq}$

At C_F<C_{Fth}, the influence of fine grains supporting the coarse grain skeleton and the kinematic constraints that the fine grains confined within the voids impose on the deformation of the coarse grain skeleton must be accounted for in devising an equivalent index of active contacts. The amount of fine grains that contributes to the force chain would depend on pore size distribution within the coarse grain skeleton, particle shape characteristics, and the size disparity ratio R_d. Denoting the portion of the fine grains contributing to the force chain by '*b*', the relevant equivalent intergranular contact index void ratio $(e_c)_{eq}$ is given by

$$(e_c)_{eq} = \frac{e + (1-b)c_f}{1-(1-b)c_f} \tag{5}$$

where 0<*b*<1. From a physical point of view, *b*=0 means that none of the fine grains contribute to the force chain and *b*=1 means that all fine grains contribute to the force chain. Also, when R_d=1, *b* would be 1 and when R_d is very large, b would tend to be zero. For a broadly graded mix, *b* depends on R_d and gradation of the fine and coarse grain soils contained in the mix (Thevanayagam *et al*., 2003; Kanagalingam and Thevanayagm, 2006).

2.3.2 Equivalent interfine void ratio $(e_f)_{eq}$

At C_{Fth}<C_F<C_{FL}, the global void ratio *e overestimates* the actual density of active contacts in the granular mix. This is because the dispersed coarse grains in the mix do not contribute as many active contacts as if the soil was prepared at the same void ratio by substituting each coarse grain by an equal (solid) volume of fine grains. The reason is that solid volume of a dispersed coarse grain, which directly influences the global void ratio, grows in proportion to the power of three of particle size. Surface area, which influences the nature of contacts with the surrounding fine grains, grows in proportion to the power of only two. For equal solid volume, the substituted fine grains have a larger surface for contact than a dispersed coarse grain of equal (solid) volume embedded in the fine grain medium. The density of contacts in the mix is smaller than in the fine grain soil at the same *e*. The use of e_c as an index of active contacts is also not valid since it completely ignores the existence of interfine grain contacts. It grossly underestimates the active contacts. Similarly, the interfine void ratio e_f also *underestimates* the active contacts, since it completely ignores the contribution by the dispersed coarse grains to the contact force chain.

One way to theoretically quantify the effect of each dispersed coarse grain on the macro response of the mix is to estimate the *effective* number of active contacts it makes with the fine grains surrounding it and simply replace each dispersed coarse grain in the mix by an equivalent number of fine grains that will make the same number of active contacts made by the coarse grains. The equivalent void ratio $(e_f)_{eq}$ of the resulting equivalent fine grain medium is taken as the index of active contacts, leading to (Appendix IV):

$$(e_f)_{eq} = \left[\frac{e}{c_f + \dfrac{(1-c_f)}{R_d^m}}\right] < e_f \le e_{max,HF} \qquad e_c > e_{max,HC} \tag{6}$$

where *m*= a coefficient satisfying 0<*m*<1 and it depends on grain size and shape characteristics, gradation, and packing (Kanagalingam and Thevanayagam, 2006). An expression for m for a few special cases of spherical or disc shaped particles is presented in Appendix IV. An experimental determination of m is most preferred.

2.4 Minimum void ratios of granular mixes

Conceptually, Fig.2 shows the regions belonging to the four cases i through iv and the transition boundaries between these cases. The absolute lower bound lines for minimum void ratio of the granular mix e_{min} is obtained by substituting e_c= $e_{min,HC}$ and e_f= $e_{min,HF}$ in Eqs. 3 and 4, respectively. These limits are given by:

$$e_{\min} = e_{\min,\mathrm{HC}}[1 - c_\mathrm{f}] - c_\mathrm{f} \qquad \text{for } C_\mathrm{F} < C_\mathrm{Fth} \tag{7a}$$

$$e_{\min} = e_{\min,\mathrm{HF}}[c_\mathrm{f}] \qquad \text{for } C_\mathrm{F} > C_\mathrm{Fth} \tag{7b}$$

These are shown in Fig.2 However, due to the constraints presented by the presence of fine grains, in the case of $C_\mathrm{F}<C_\mathrm{Fth}$, and due to constraints presented by the coarse grains, in the case of $C_\mathrm{Fth}<C_\mathrm{F}<C_\mathrm{FL}$, the $e_{\min}$ is expected to be higher than the values given by Eq.(7). The $e_{\min}$ could be obtained by substituting $(e_\mathrm{c})_\mathrm{eq} = e_{\min,\mathrm{HC}}$ and $(e_\mathrm{f})_\mathrm{eq} = e_{\min,\mathrm{HF}}$ in Eqs.(5) and (6), respectively. This leads to:

$$e_{\min} = e_{\min,\mathrm{HC}}[1 - (1-b)c_\mathrm{f}] - (1-b)c_\mathrm{f} \qquad \text{for } C_\mathrm{F} < C_\mathrm{Fth} \tag{8a}$$

$$e_{\min} = e_{\min,\mathrm{HF}}\left[c_\mathrm{f}\,\frac{(1-c_\mathrm{f})}{R_\mathrm{d}^m}\right] \qquad \text{for } C_\mathrm{Fth} < C_\mathrm{F} < C_\mathrm{FL} \tag{8b}$$

$$e_{\min} = e_{\min,\mathrm{HF}}[c_\mathrm{f}] \qquad \text{for } C_\mathrm{F} > C_\mathrm{FL} \tag{8c}$$

2.5 Maximum void ratios of granular mixes

If the fine grains in the soil mix are fully confined within the intergranular voids or if the coarse grains are fully dispersed, with no influence on the maximum void ratio $e_{\max}$ of the soil mix, the $e_{\max}$ could be obtained by substituting $e_\mathrm{c}=e_{\max,\mathrm{HC}}$ and $e_\mathrm{f}= e_{\max,\mathrm{HF}}$ in Eqs. (3) and (4), respectively. On the other hand, if the soil mix is a fully layered system of coarse grained soil and fine grained soil, the $e_{\max}$ profile is given by the line connecting the coordinates ($e_{\max,\mathrm{HC}}$,0) and ($e_{\max,\mathrm{HF}}$,100) in Fig.2. These are the two limiting cases for $e_{\max}$ of a granular mix. However, due to interactions between coarse and fine grains and its influence on $e_{\max}$, it is obtained by substituting $(e_\mathrm{c})_\mathrm{eq} = e_{\max,\mathrm{HC}}$ and $(e_\mathrm{f})_\mathrm{eq} =e_{\max,\mathrm{HF}}$, in Eqs. (5) and (6), respectively. It leads to:

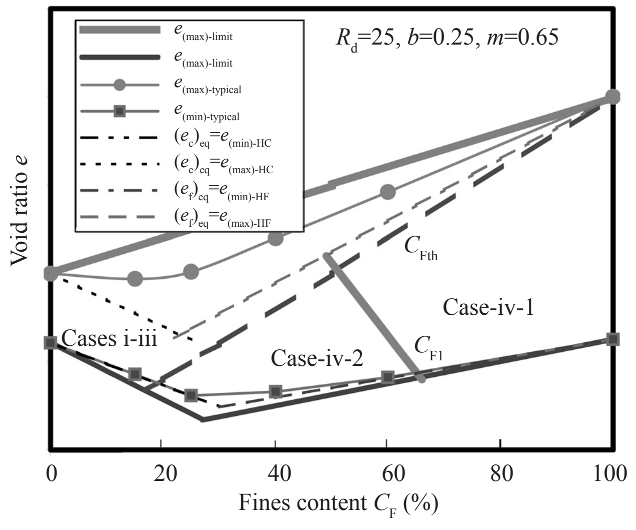


Fig. 2 Conceptual diagram for cases i through iv, $e_{\max}$, $e_{\min}$, C_Fth and C_FL

$$e_{\max} = e_{\max,\mathrm{HC}}[1 - (1-b)c_\mathrm{f}] - (1-b)c_\mathrm{f} \tag{9a}$$

$$e_{\max} = e_{\max,\mathrm{HF}}\left[c_\mathrm{f} + \frac{(1-c_\mathrm{f})}{R_\mathrm{d}^m}\right] \tag{9b}$$

2.6 Equivalent relative densities

Considering that intergrain contact density and contact force chain network are primary factors that affect the mechanical stress-strain response of a soil, and based on the equivalent void ratio concept which accounts for contact density of grains in a soil mix, a soil mix at a fines content of less than C_Fth would be expected to behave similar to a coarse grained soil prepared using the host coarse grains at a void ratio equal to $(e_\mathrm{c})_\mathrm{eq}$. Similarly, a soil mix at fines content greater than C_Fth would be expected to behave similar to a fine grained soil prepared using the host fine grains at a void ratio equal to $(e_\mathrm{f})_\mathrm{eq}$. This concept would allow the study of granular mixes in a consistent framework. If one were to study more than one granular mix, each made of a different pair of coarse grain sizes and/or fine grain sizes, then the equivalent void ratio concept would be applicable for each soil mix at different fine grains content, individually. If one were to study the entire group, an appropriate equivalent relative density index is more appropriate. When comparing the behavior of a mix with other soil mixes prepared by mixing other host grains, the equivalent intergranular and interfine contact relative densities, $(D_\mathrm{rc})_\mathrm{eq}$ and $(D_\mathrm{rf})_\mathrm{eq}$, as defined below may be used as the contact indices:

$$(D_\mathrm{rc})_\mathrm{eq} = \left[\frac{(e_{\max,\mathrm{HC}} - (e_\mathrm{c})_\mathrm{eq})}{(e_{\max,.\mathrm{HC}} - e_{\min,\mathrm{HC}})}\right] \qquad \text{for } C_\mathrm{F} < C_\mathrm{Fth} \tag{10a}$$

$$(D_\mathrm{rf})_\mathrm{eq} = \left[\frac{(e_{\max,\mathrm{HF}} - (e_\mathrm{f})_\mathrm{eq})}{(e_{\max,.\mathrm{HF}} - e_{\min,\mathrm{HF}})}\right] \qquad \text{for } C_\mathrm{F} > C_\mathrm{Fth} \tag{10b}$$

This framework presents a unified way to study the behavior of granular mixes using a consistent set of contact density indices. A brief analysis of this framework using experimental data is presented below.

Table 1 Soils used for evaluation

Coarse grain	Fine grain	Test type	C_F (%)	D_{50} (mm)	R_d	b	m	Raw data
OS-F55 sand	Crushed silica fines	CIU-TC CIU-CT	0 to 100	0.25	25.0	0.25 0.35	0.65	Thevanayagam *et al*., 2003
FJ#80 sand	Crushed silica fines	CIU-TC and CIU-CT	0 to 100	0.18	18.0	0.30 0.65	-	Thevanayagam *et al*., 2003
Monterey 0/30 sand	Yatesville silt	CIU-CT	0 to 100	0.43	14.3	0.25	0.45	Polito and Martin, 2001
Yatesville sand	Yatesville silt	CIU-CT	0 to 100	0.18	6.0	0.60	0.60	Polito and Martin, 2001
Gravel	Tone river sand	Shear wave velocity	25 to 100	-	25	-	0.55	Kokusho *et al*., 1995

Note: CIU-TC = isotropic consolidated undrained triaxial compression, CIU-CT = isotropic consolidated undrained cyclic triaxial. Different b values are reported for CIU-TC and CIT-CT

3 Experimental evaluation

In order to evaluate the effectiveness of this framework, and the utility of the contact density indices to characterize the mechanical properties of granular mixes, several experiments were conducted on a number of different sand-silt mixes constituted at different silt contents ranging from 0 to 100%. These experiments included undrained triaxial tests, cyclic triaxial tests, and tests to obtain e_{min} and e_{max} of these soils. Details of these tests and the data are presented in Thevanayagam *et al*. (2002, 2003), Kanagalingam (2006) and Shenthan (2006). Further experimental data available from the literature were also collected and analyzed. Table 1 summarizes the relevant sources of these data and the soils involved in this evaluation.

3.1 Minimum and maximum void ratios

As a first step, Fig.3 shows the minimum and maximum void ratios measured for Ottawa sand – silt mixes at silt contents ranging from 0 to 100%. Details of the tests and the data are presented in Thevanayagam *et al*. (2003) and Kanagalingam (2006). Also shown in this figure are the theoretical limiting lines for e_{min} and e_{max} given by Eqs. (7) through (10). The lines corresponding to C_{Fth} and C_{FL} given by Eqs.1 and 2, respectively, are also shown in this figure. In general, the maximum and minimum void ratio lines closely follow the theoretical limits. Such observations were found for several other soils as well (Thevanayagam *et al*., 2003). These comparisons indicate a general validity of the framework presented in this paper.

3.2 Steady state line

Figures 4(a)-(b) present the steady state data ($p'_{ss} = (\sigma'_1+2\sigma'_3)/3$) obtained from triaxial compression tests on initially isotropic consolidated specimens for OS-F55 sand-silt mixes. Figure 4(a) shows the data for $C_F<C_{Fth}$ and Fig.4(b) shows the data for $C_F>C_{Fth}$ and plotted against $(e_c)_{eq}$ and $(e_f)_{eq}$, respectively. The data for all silty sands for C_F up to C_{Fth} fall in the vicinity of the data for the respective host clean sands. The data for all sandy silts for C_F beyond C_{Fh} fall in the vicinity of the data for the respective host silt. Although not shown in this paper, no unique relationship was found between p'_{ss} and the global void ratio e (Thevanayagam *et al*., 2003), indicating that the global void ratio is not an appropriate index for characterizing steady state response of granular mixes.

Figures 5(a)-(b) show the steady state data for two different granular mixes (OS-F55 sand-silt mix and FJ#80 sand-silt mix) plotted against equivalent relative density $(D_{rc})_{eq}$ and $(D_{rf})_{eq}$ for $C_F<C_{Fth}$ and $C_F>C_{Fth}$, respectively. The data for both mixes fall in a narrow band surrounding the data for the host coarse and fine grain soils, respectively.

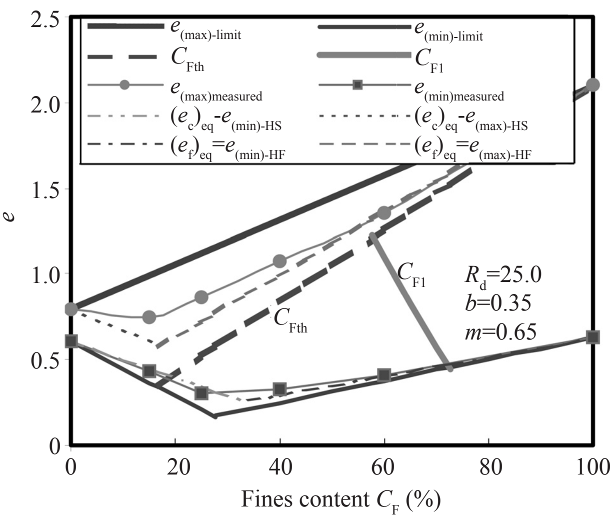


Fig. 3 e_{max}, e_{min}, C_{Fth} and C_{FL} for OS-F55 sand-silt mix

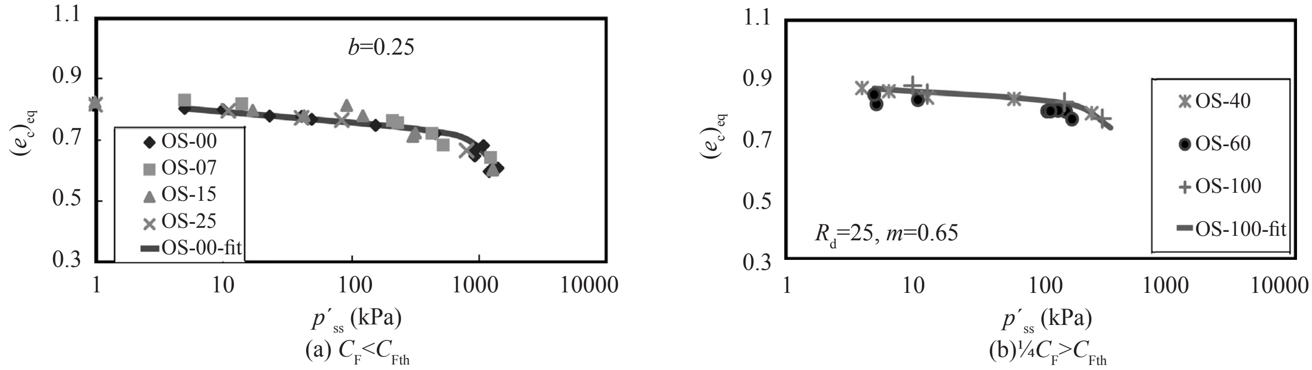


Fig. 4 p'_{ss} **vs** $(e_c)_{eq}$ **and** $(e_f)_{eq}$ **(Note: OS-15=OS-F55 sand with 15% silt)**

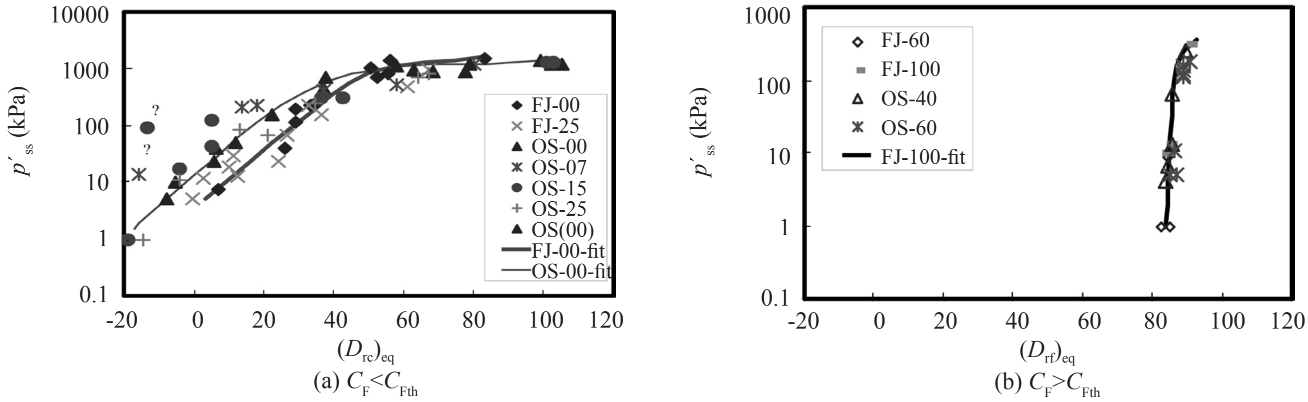


Fig. 5 p'_{ss} **vs** $(D_{rc})_{eq}$ **and** $(D_{rf})_{eq}$ **(Note: FJ-60=FJ#80 sand with 60% silt)**

3.3 Liquefaction resistance

Figures 6(a)-(b) show the energy per unit volume of soil (E_L at 5% double amplitude axial strain) required to cause liquefaction versus $(e_c)_{eq}$ and $(e_f)_{eq}$ for $C_F<C_{Fth}$ and $C_F>C_{Fth}$, respectively, for the OS-F55 sand-silt mix. The specimens were initially isotropically consolidated to 100 kPa and subjected to stress-controlled cyclic triaxial tests at a cyclic stress ratio (CSR) R_{CS} of 0.2. Figures 7(a)-(b) show R_{CS} required to trigger liquefaction in 15 cycles, for two different sand-silt mixes (Monterey 0/30 sand-silt and Yatesville sand-silt), plotted against $(D_{rc})_{eq}$ for $C_F<C_{Fth}$ and $(D_{rf})_{eq}$ for $C_F>C_{Fth}$, respectively. The specimens in this case were also initially isotropically consolidated. The raw data for these soils were obtained from Polito and Martin (2002). The data for both sandy silts ($C_F>C_{Fth}$) fall in the vicinity of the host silt (Yatesville silt) irrespective of the sand grain characteristics. In general, the data for silty sands for C_F less than C_{Fth} fall in a narrow band near the data trend for the respective host sands.

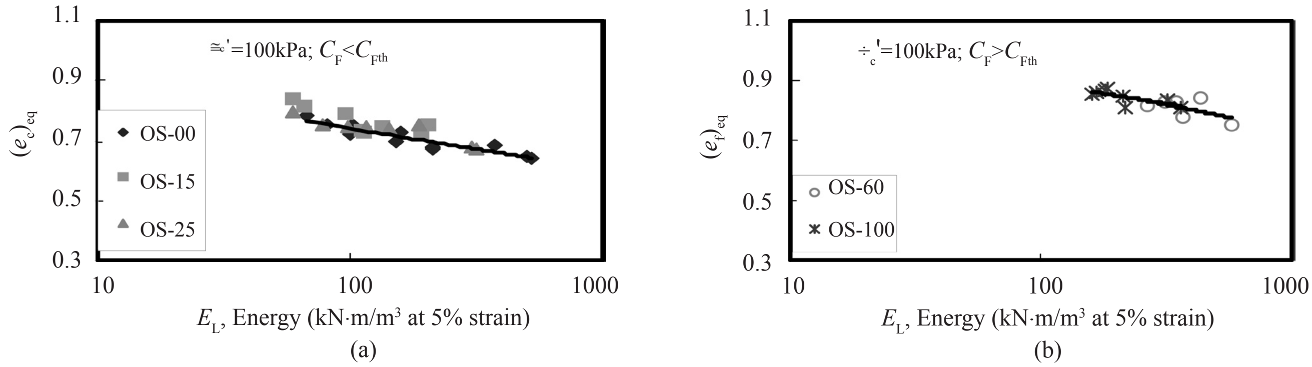
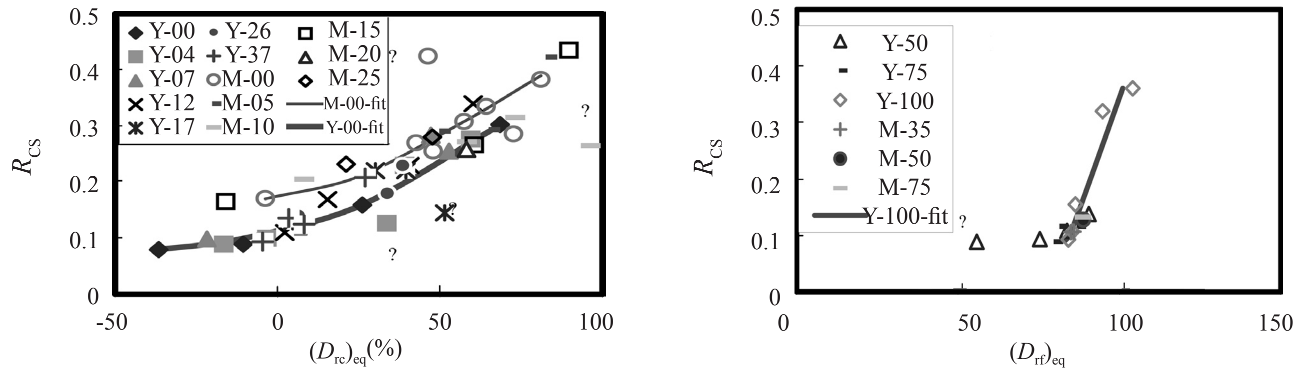


Fig. 6 E_L **versus** $(e_c)_{eq}$ **and** $(e_f)_{eq}$ **at** R_{CS}**=0.2**



(a) $C_F<C_{Fth}$ (b) $C_F>C_{Fth}$

Fig. 7 $\boldsymbol{R_{CS}}$ **vs** $\boldsymbol{(D_{rc})_{eq}}$ **and** $\boldsymbol{(D_{rf})_{eq}}$

3.4 Shear wave velocity

Figure 8 shows the normalized shear wave velocity corresponding to 100 kPa confining stress for gravelly sands containing up to 75% gravel plotted against $(e_f)_{eq}$. The raw data for these soils were obtained from Kokusho *et al*. (1994). In this case, sand was considered as the fine grain and gravel as the coarse grain to obtain $(e_f)_{eq}$ using Eq.(6). The data for a sand-gravel mix fall in a narrow band surrounding the data for sand.

3.5 Effect of gradation on parameters b and m

Table 1 also shows the values for parameters *b* and *m* for each of the sand mixes. It was considered that *b* and *m* are influenced by the gradation of the coarse and fine grain soils involved in the soil mixes (Thevanayagam *et al*., 2003; Kanagalingam and Thevanayagam, 2006; Ni *et al*., 2004). To evaluate this hypothesis, the values of *m* and *b* were plotted against an index representing the gradations of the soils. Fig.9 shows this relationship for *b* and *m* with C_{uc}, C_{uf}, and R_{d50} $(=D_{50}/d_{50})$, where C_{uc} and C_{uf} are the coefficients of uniformity of the respective coarse and fine grain soils contained in each soil mix.

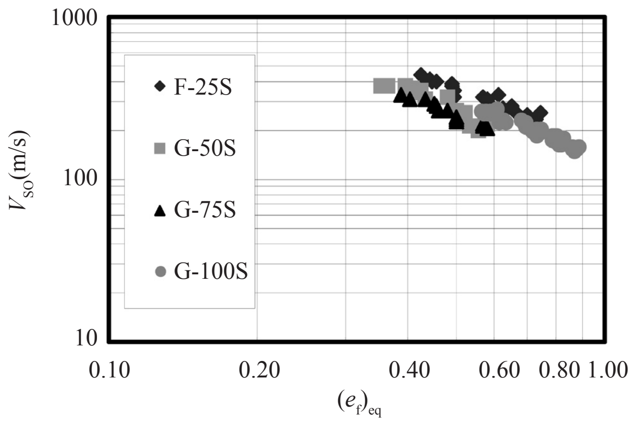


Fig. 8 Shear wave velocity - $\boldsymbol{v_{so}}$ **versus** $\boldsymbol{(e_f)_{eq}}$

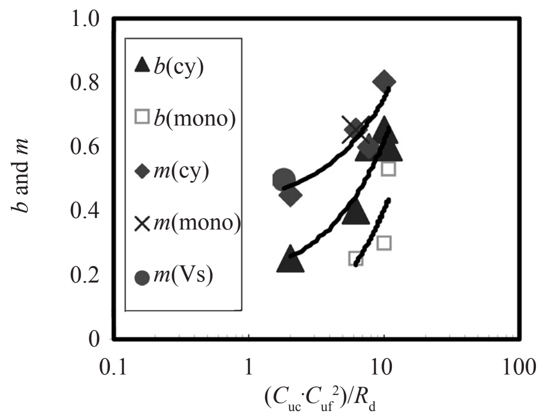


Fig. 9 ***b*** **and** ***m*** **versus** $\boldsymbol{C_{uc}\cdot C_{uf}^2/R_{d50}}$

4 Conclusions

Based on a study of a granular mix made of two distinct size of particles with large size disparity between them, a framework for development of contact density indices for a granular mix is presented. This framework takes into account qualitatively the contributions of the coarse grains and fine grains to the global intergrain contact density and contact force network. The contributions by the coarse and fine grains to the mechanical response depend on fines content, size disparity between fines and coarse grains, global void ratio, and gradation of the coarse and fine grain soils present in the granular mix.

Broadly, there exist three zones of soil behavior:

(a) one that is governed primarily by the coarse grain skeleton with a little contribution by the fine grains at FC less than a threshold value C_{Fth},

(b) one that is governed by the fine grain skeleton at fines content above a limiting value C_{FL}, and

(c) one that is governed primarily by the fine grains with a little contribution by the coarse grain at an intermediate range of C_F between C_{Fth} and C_{FL}.

The contact index void ratios for these three cases are $(e_c)_{eq}$, e_f, and $(e_f)_{eq}$, respectively. Theoretical expressions for these indices and the transition fines contents C_{Fth} and C_{FL} are also presented. They are expressed in terms of global void ratio, fines content, and soil gradation characteristics. Theoretical expressions for the maximum and minimum void ratios of granular mixes have also been presented. A set of equivalent intergranular and intefine relative densities $(D_{rc})_{eq}$ and $(D_{rf})_{eq}$, have been presented as useful index to study a broad range of granular mixes.

Experimental evaluation of this framework using data for minimum and maximum void ratios of granular mixes, steady state strength, liquefaction resistance, shear wave velocity for a variety of sands, silty sands, sandy silts, and gravelly sands shows that in general the framework is valid. The mechanical response of a granular mix at fines content of less than C_{Fth} is similar to the host coarse grain soils prepared at a void ratio equal to $(e_c)_{eq}$ or relative density $(D_{rc})_{eq}$. The mechanical response of a granular mix at fines content greater than C_{Fth} is similar to the host fine grain soils prepared at a void ratio equal to $(e_f)_{eq}$ or relative density $(D_{rf})_{eq}$.

Note that the framework presented herein pertains to gap-graded soil with large particle size disparity. Therefore, any extrapolations to well graded soils must be done carefully. Furthermore, this framework does not explicitly include effects such as cementation that may be found in field soils. It may be modified accordingly using appropriate values for the parameters b and m.

Acknowledgements

Financial support for this research by the National Science Foundation and USGS is gratefully appreciated. This research also benefited from other research conducted at University at Buffalo on liquefaction mitigation sponsored by MCEER and FHWA. T. Kanagalingam helped to prepare the figures.

References

Andrus R and Stokoe K (1997), "Liquefaction Resistance Based on Shear Wave Velocity," *NCEER Report, 97-0022*, National Center for Earthq. Eng. Res., State Univ. of New York at Buffalo, NY, pp. 89-128.

Baziar MH and Dobry R (1995), "Residual Strength and Large-deformation Potential of Loose Silty Sands," *J. Geotech. Eng. Div*., ASCE, **121**(12): 896-906.

Chameau JL and Sutterer K (1994), "Influence of Fines in Liquefaction Potential and Steady State Considerations," *13th Intl. Conf. Soil Mech. and Found Eng*., India, pp. 183-84.

Chang NY (1990), "Influence of Fines Content and Plasticity on Earthquake-induced Soil Liquefaction," *Contract Rep. DACW3988-C-0078*, US Army WES, Vicksburg,MS.

Evans MD and Zhou S (1995), "Liquefaction Behavior of Sand-gravel Composites," *ASCE, J. Geotech. Eng*., **121**(3): 287-298.

Georgiannou VN, Hight DW and Burland JB (1990), "The Undrained Behaviour of Clayey Sands in Triaxial Compression and Extension," *Geotechnique*, **40**(3): 431-49

Georgiannou VN, Hight DW and Burland JB (1991a), "Undrained Behaviour of Natural and Model Clayey Sands," *Soils and Foundations*, **31**(3): 17-29

Georgiannou VN, Hight DW and Burland JB (1991b), "Behaviour of Clayey Sands Under Undrained Cyclic Triaxial Loading," *Geotechnique*, **41**(3): 383-93.

Ishihara K (1993), "Liquefaction and Flow Failure During Earthquakes," *Geotechnique*, **43**(3): 351-41.

JGS (1996), "Special Issue on Geotechnical Aspects of the January 17 1995 Hyogoken-Nambu Earthquake," *Soils and Foundations*, Japanese Soc. Geotech. Eng.

Kanagalingam T (2006), "Liquefaction Resistance of Granular Mixes Based on Contact Density and Energy Considerations," *PhD Dissertation*, University at Buffalo, SUNY, Buffalo, NY.

Kanagalingam T and Thevanayagam S (2006), "Contribution of Fines to the Compressive Strength of Mixed Soils," Discussion, *Geotechnique*.

Koester JP (1994), "The Influence of Fines Type and Content on Cyclic Strength," *Proc. ASCE Conv*., Atlanta, Geotech. Spec. Pub. **44**: 17-32.

Kokusho T, Yoshida Y and Tanaka Y (1995), "Shear Wave Velocity in Gravelly Soils with Different Particle Gradings," *ASCE Geotech*, STP **56**: 92-106.

Kuerbis R, Nagussey D and Vaid YP (1989), "Effect of Gradation and Fines Content on the Undrained Response of Sand," *Proc. Conf. on Hyd. Fill Struc*., ASCE Geotech. Spec. Publ. **21**: 330-45.

Mitchell JK (1993), *Fundamentals of Soil Behavior*, Second ed. Wiley Interscience Publ.

Ni Q, Tan TS, Dasari GR and Hight DW (2004), "Contribution of Fines to the Compressive Strength of Mixed Soils," *Geotechnique*, **54**(9): 561-569

Pitman TD, Robertson PK and Sego DC (1994), "Influence of Fines on the Collapse of Loose Sands," *Can. Geotech. J.*, **31**: 728-39

Polito CP and Martin II, JR (2001), "Effects of Nonplastic Fines on the Liquefaction Resistance of Sands," *J. Geotech. Engrg. Div*., ASCE, **127**(5): 408-415.

Robertson PK and Wride CE (1997), "Evaluation of Cyclic Liquefaction Potential Based on the CPT," *Seismic Behavior of Ground and Geotech Structures*, ed. P.S. Pinto, Balkema Publs., pp. 269-278.

Roscoe KH (1970), "The Influence of Strains in Soil Mechanics," *Geotechnique*, **20**(2), 129-170.

Roscoe KH, Schofield AN and Thurairajah A (1963), "Yielding of Clays in States Wetter Than Critical," *Geotechnique*, **13**: 211-240.

Roscoe KH, Schofield AN and Wroth CP (1958), "On the Yielding of Soils," *Geotechnique*, **8**: 22-52.

Seed HB (1987), "Design Problems in Soil Liquefaction," *J. Geot.. Eng. Div.* , ASCE, **113**(8): 827-45.

Seed RB and Harder LF Jr. (1990), "SPT-based Analysis of Cyclic Pore Pressure Generation and Undrained Residual Strength," *Proc. Seed Mem. Symp*., Berkeley, **2**: 351-76.

Seed HB, Idriss IM and Arango I (1983), "Evaluation of Liquefaction Potential Using Field Performance Data," *J. Geotech. Eng.*, ASCE, **109**(3): 458-482.

Shenthan T (2006), "Liquefaction Mitigation in Silty Soils Using Stone Columns Supplemented with Wick Drains," *PhD Dissertation*, University at Buffalo, SUNY, Buffalo, NY.

Singh S (1994), "Liquefaction Characteristics of Silts," *ASCE Geotech*. Sp. Pub. **44**: 105-16.

Stark TD and Mesri G (1992), "Undrained Shear Strength of Liquefied Sands for Stability Analysis," *J. of Geotech. Eng*., ASCE, **118**(11): 1727-47.

Thevanayagam S (1998), "Effect of Fines and Confining Stress on Undrained Shear Strength of Silty Sands," *J. Geotech. & Geoenv. Engrg. Div*., ASCE, **124**(6): 479-491; (also see *closure* in vol. 125, issue 11).

Thevanayagam S (1999), "Liquefaction and Shear Wave Velocity Characteristics of Silty/Gravely Soils," *Proc.15th US-Japan Workshop on Bridge Eng.,* Public Works Res. Institute, Tsukuba City, Tokyo, Japan, pp.133-147.

Thevanayagam S (2007), "Intergrain Contact Density Indices for Granular Mixes – II: Liquefaction Resistance," *J. Earthquake Engineering and Engineering Vibrations*, June.

Thevanayagam S and Mohan S (2000),"Intergranular State Variables and Stress-strain Behavior of Silty Sands" *Geotechnique*, vol.1, pp.1-24.

Thevanayagam S, Shenthan T, Mohan S and Liang J (2002), "Undrained Fragility of Sands, Silty Sands and Silt," *ASCE, J. Geotech. & Geoenv. Eng.*, **128**(10): 849-859.

Thevanayagam S, Shenthan T and Kanagalingam T (2003), "Role of Intergranular Contacts on Mechanisms Causing Liquefaction & Slope Failures in Silty Sands," *Research Report*,http://erp-web.er.usgs.gov/reports/abstract/2001/pt/01hqgr0032-report.pdf,U.S.Geological Survey, Dept. of Interior, USA, 396p.

Vaid YP (1994), "Liquefaction of Silty Soils," *ASCE Geotech. Spec*. Publ. **44**: 1-16.

Yamamuro JA, Covert KM and Lade PV (1999), "Static and Cyclic Liquefaction of Silty Sands," *Proc. Workshop, Physics and Mechanics of Soil Liquefaction*, Balkema press.

Yamamuro J A and Lade P V (1998), "Steady-state Concepts and Static Liquefaction of Silty Sands," *J. Geotech. and Geoenv. Engrg. Div.*, ASCE, **124**(9): 868-877.

Zlatovic S and Ishihara K (1995), "On the Influence of Nonplastic Fines on Residual Strength," *Proc. IS-TOKYO'95, First Intl. Conf. Earthquake Geotechnical Engrg.*, Tokyo, eds. K. Ishihara, publs. A. A. Balkema, pp. 239-244.

Zlatovic S and Ishihara K (1997), "Normalized Behavior of Very Loose Non-plastic Soils: Effects of Fabric" *Soils and Foundations*, **37**(4): 47-56.

Notations

The following symbols are used in this paper.

CPT= cone penetration test

C_{uc} = uniformity coefficient of coarse-grained soil

C_{uf} = uniformity coefficient of fine-grained soil

d= diameter of fine grains

D= diameter of coarse grains

D_r= relative density

$(D_{rc})_{eq}$= equivalent intergranular relative density

$(D_{rf})_{eq}$= equivalent interfine relative density

E_L = energy required to cause liquefaction per unit vol of soil.

e = global void ratio

e_f= interfine void ratio

e_c = inter-coarse granular void ratio

$(e_c)_{eq}$= equivalent intergranular void ratio

$(e_f)_{eq}$= equivalent interfine void ratio

$(e_c)_{th}$= threshold inter-coarse granular void ratio

e_{max}= maximum void ratio

$e_{max,HC}$= maximum void ratio of the host coarse grains

$e_{max,HF}$= maximum void ratio of the host fine grains

e_{min}= minimum void ratio

$e_{mix,HC}$= minimum void ratio of the host coarse grains

$e_{mix,HF}$= minimum void ratio of the host fine grains

C_F = fine grains content

C_{Fth}= threshold fine grains content

C_{FL}= limiting fine grains content

N_L= number of cycles to cause initial liquefaction

p= mean effective stress

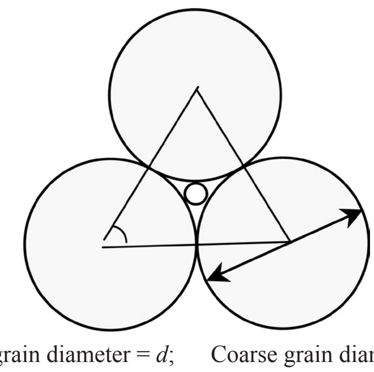
q= deviatoric stress

R_d= particle size disparity ratio

SPT = standard penetration test

Appendix I: Particle size disparity constraint (R_d)

Consider a two-size spherical particulate system with size disparity R_d. If the fine grains were to remain confined within the coarse grain skeleton (case i), d must be smaller than the minimum possible pore throat opening between the coarse grains (Fig.I-1) for all



Fine grain diameter = d; Coarse grain diameter = D
$d < D/6.46$

Fig. I-1 Fully confined fines

packing configurations:

$$R_d = \frac{D}{d} > \frac{1}{(4/3)\sin(\pi/3)-1} \approx 6.46 \qquad \text{(I-1)}$$

For real soils, however, for a number of reasons (shape, angularity, interlocking, packing, gradation, etc.) R_d must be much larger than 6.46.

Appendix II: Threshold fines content, C_{Fth}

If the voids within the coarse grain skeleton in Fig.1(a) are not completely filled with the fine grains and R_d is large enough, then during deformation, the fine grains can move from one void space to another. For this to happen rather freely, the apparent interfine void ratio e_f (=e/c_f) in each pore space must be higher than a possible the $e_{max,HF}$ value corresponding to the loosest packing possible where the fine grains cannot sustain any shear stress; viz. $e/c_{fth} > e_{max,HF}$:

$$C_{Fth} \le \frac{100\, e_c}{1+e_c+e_{max,HF}}\% = \frac{100\, e}{e_{max,HF}}\% \qquad \text{(II-1)}$$

Once the interfine grain void ratio drops below $e_{max,HF}$ the fine grains begin to make active contacts among themselves and begin to compose a strong internal force chain.

An accurate determination of C_{Fth} requires a detailed knowledge of pore geometry and the number of fine grain particles that could be placed loosely within each inter-coarse granular pore space. An approximate way to estimate C_{Fth} is to start with a unit solid volume of fine grain matrix packed at its loosest state ($e_{max,HF}$) and introduce coarse grains into it at the desired coarse grain packing (and the corresponding e_c) and remove the fine grains that each coarse grain displaces, without altering the fine grain packing and the total volume of the mix (1+$e_{max,HF}$). The e is altered as a result. The number of (full) fine grains N_d that each coarse grain displaces is an integer number between the number of (full) fine grains that can be contained at the loosest packing within a sphere of its size (D) and a sphere of size (D+2d).

If the C_F at the end of the above process is C_{Fth}, global void ratio is e, and the total volume of the mix is (1+$e_{max,HF}$), then the solid volumes of the fine and coarse grains are [C_{Fth}(1+$e_{max,HF}$)/(1+e)] and [(1-C_{Fth})(1+$e_{max,HF}$)/(1+e)], respectively. The total loss of fine grain solid volume from the initial state is [1- C_{Fth}(1+$e_{max,HF}$)/(1+e)]. On the other hand, each coarse grain (solid volume=$\pi D^3/6$) replaces a solid volume of [$N_d \pi d^3/6$] of the fine grains. Hence, the amount of solid volume of fine grains replaced by coarse grains is {[(1-C_{Fth})(1+$e_{max,HF}$)/(1+e)][$N_d \pi d^3/6$]/[$\pi D^3/6$]}[=$N_d/(R_d)^3$][(1-C_{Fth})(1+$e_{max,HF}$)/(1+e)]. This must be equal to the total loss of fine grains {=[1- C_{Fth}(1+$e_{max,HF}$)/(1+e)]}, leading to:

$$C_{Fth} = 100\left[\frac{(1+e)-(1+e_{max,HF})\dfrac{N_d}{R_d^3}}{(1+e_{max,HF})-(1+e_{max,HF})\dfrac{N_d}{R_d^3}}\right]\% \qquad \text{(II-2)}$$

Theoretically, for a simple cubic array of uniform sized spherical particles, the maximum void ratio $e_{max,HF}$ is 0.91 (=6/π–1). The magnitude of N_d/R_d^3, estimated laboriously, is about 0.689 and 0.582 for R_d=16 and 32, respectively, and approaches π/6 for very large R_d. For other particle shapes N_d/R_d^3 asymptotically approaches 1/(1+$e_{max,HF}$). C_{Fth} increases with an increase in e and R_d and decreases with an increase in $e_{max,HF}$. Hence, C_{Fth} can be smaller for silty sand prepared by mixing silt with a fine sand than a coarse sand. For plastic fines, the $e_{max,HF}$ can be high compared to a non plastic silt. C_{Fth} can be smaller for clayey sands compared to silty sands.

Appendix III: Limiting fines content, C_{FL}

If the coarse grains were to play an insignificant role on the behavior of the granular mix, the spacing between the coarse-grains must be greater than a minimum value so that they do not influence the shearing along the fine grains. At spacing less than this, the coarse grains can have a reinforcing effect on the behavior of the granular mix somewhat similar to inclusion of short fibers in (clean) sand. The minimum spacing is governed by many constraints including: (1) microgeometry of the locus of motion of a fine grain, (2) thickness of the zone that a fine grain in motion influences, and (3) stress concentrations within the fine grain matrix in the vicinity of the coarse grains. Based on Roscoe's observations (1970), the limiting center-to-center spacing between the coarse grains is about (D+10d). Let the C_F at this state be C_{FL}.

Assume that coarse grains are dispersed uniformly, in a simple cubic array within the fine-grain matrix, at a center-to-center spacing of sD. Consider a cube of size sD. The solid volume of the coarse grains in this cube =$\pi D^3/6$. The total volume of the mix is $(sD)^3$ and the global void ratio is e. Using a simple phase diagram relationship, [$\pi D^3/6$]/$(sD)^3$=(1-c_{fL})/(1+e)} where c_{fL}= C_{FL}/100, leading to:

$$C_{Fl} \ge 100\left[1-\frac{\pi(1+e)}{6\,s^3}\right]\% = 100\left[\frac{\dfrac{6s^3}{\pi}-1}{\dfrac{6\,s^3}{\pi}+e_f}\right]\% \ge C_{Fth}\,; \qquad e_f \le e_{max,HF} \qquad \text{(III-1)}$$

where s=1+a(d/D)=1+a/R_d where a=10 (approximately). Slightly different expressions may be found for different arrays. Simple cubic array, however, yields the largest spacing and therefore yields the largest C_{FL}. Also, the corresponding limiting inter-coarse granular void ratio $(e_c)_l$ is given by:

$$(e_c)_l = \frac{6s^3}{\pi} - 1 \qquad \text{(III-2)}$$

The $(e_c)_l$ is the limiting coarse grain intergranular void ratio beyond which the coarse grains have little effect on the behavior of the fine grain matrix. Also note that case iii was defined as a state where the coarse grains are essentially separated by the fine grains, but the fine grains are still not dense enough. By setting s=1 in Eq.(III-2), the corresponding threshold inter-coarse granular void ratio $(e_c)_{th}$ is given by:

$$(e_c)_{th} = \frac{6}{\pi} - 1 = 0.91 \qquad \text{(III-3)}$$

Theoretically this is also the maximum theoretical void ratio $e_{max,HC}$ for spherical particles arranged in a simple cubic array (p139, Mitchell 1993). For real soils, the $e_{max,HC}$ varies in the vicinity of 0.90±0.1 for other reasons and hence the magnitude of $(e_c)_{th}$ would be equal to the corresponding $e_{max,HC}$.

Appendix IV: Interfine contact index $(e_f)_{eq}$

The $(e_f)_{eq}$ is defined as the void ratio of the equivalent fine grain soil obtained by replacing each coarse grain in the granular mix by an equivalent number of fine grains (N_e) such that the total number of contacts in the medium remains the same. This void ratio is taken as the index of active contact density in the medium.

For spherical particles, the equivalent fine grain solid volume added to the mix per unit solid volume of coarse grain removed from the matrix=$(N_e \pi d^3/6)/(\pi D^3/6)$= $[N_e/(R_d)^3]$. Neglecting the differences in specific gravities of solids, the *net* equivalent fine grain solid volume= $[c_f+(1\text{-}c_f)N_e/(R_d)^3]$ per unit total solid volume of coarse and fine grains in the mix. It leads to $(e_f)_{eq}$:

$$(e_f)_{eq} = \left[\frac{e}{c_f + \dfrac{(1-c_f)N_e}{R_d^3}} \right] < \ e_f \le e_{max,HF} \qquad \text{(IV-1)}$$

N_e depends on void ratio e, R_d, packing of the fine grain skeleton, and proximity between coarse grains.

Disc-like particles: Consider a disc-like two-size particle arrangement shown in Fig.IV-1. The size disparity ratio is R_d [=diameter ratio (D/d)] and thickness ratio is (T/t)= R_t. Let N_f be the number of contacts that a fine grain (far from the coarse grain) makes with other fine grains surrounding it. Let N_c be the number of contacts that a dispersed coarse grain makes with the fine grains surrounding it. Neglect the influence of the larger disc on the packing of the smaller discs located away from the larger disc. Then N_e=N_c/N_f. For the case shown in Fig.IV-1, the angle, made by two adjacenet fine grains in contact with a coarser grain, at the center of the coarse grain is arcsin[2/(R_d+1)]. N_c approaches $2\pi R_t$/arcsin[2/(R_d+1)]. N_c=[$\pi R_t(R_d+1)$] for large R_d. N_f=6. Hence, N_e approaches [$\pi R_t(R_d+1)/6 = \beta R_t R_d$, $\beta=\pi/6(1+1/R_d)<1.0$ approaching $\pi/6$ for large R_d]. Assuming R_d = R_t, the equivalent fine grain solid volume per unit actual solid volume of the dispersed coarse grain is given by $N_e(d/D)^2(t/T)=\pi(R_d+1)/(6R_d^2)$ [$=\beta/R_d$], leading to $(e_f)_{eq}$:

$$(e_f)_{eq} = \left[\frac{e}{c_f + \dfrac{(1-c_f)\beta}{R_d}} \right] < \ e_f \le e_{max,HF} \qquad \text{(IV-2)}$$

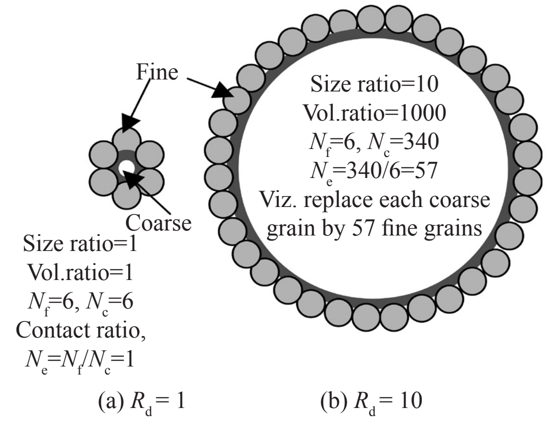


Fig. IV-1 Contact ratio-N_e vs R_d: disc-like particle mix

Spherical particles: Consider spherical particles arranged in a simple cubic array. The corresponding N_f=6. The locus of the centers of the fine grains in contact with a dispersed coarse grain is given by a sphere of diameter (R_d+1)d with its center coinciding with the center of the dispersed coarse grain. Accordingly N_c is approximately given by [$\pi(R_d+1)^2$], for large R_d. N_e is approximately given by [$\pi(R_d+1)^2/6 = \beta R_d^2$, $\beta<1$]. The equivalent fine grain solid volume per unit solid volume of the dispersed coarse grain is given by $N_e(d/D)^3=\pi(R_d+1)^2/(6R_d^3)$ [$=\beta/R_d$, $\beta<1$, β approaches $\pi/6$ for large R_d], leading to the same expression as Eq.(VI-2) for $(e_f)_{eq}$.

In reality, however, the reinforcement effect of a coarse grain embedded in a fine grain matrix is due to more reasons than mere contacts in its immediate vicinity. Mechanical response depends on the active contacts as well as the kinematic constraints for deformation. A coarse grain also influences contacts beyond its immediate vicinity and introduces kinematic constraints for deformation of other fine particles beyond its immediate vicinity. The *effective* N_e must be larger than βR_d^2. But it must be smaller than $(R_d)^3$ for the reasons discussed before. Physically this means that N_e grows faster than the growth rate of surface area growth ratio $(R_d)^2$ but slower than solid volume growth ratio $(R_d)^3$. To account for this deficiency in Eq.(IV-2), N_e is approximated to be $(R_d)^{3-m}$, where $0<m<1.0$, leading to Eq.(6). The parameter m could be obtained experimentally.