ANALYSIS OF PILE GROUPS & PILED RAFT FOUNDATIONS

Coffey Geosciences

PILE GROUP ANALYSIS - OUTLINE

- Simple statical methods
- Equivalent bent analyses
- Hybrid analyses
- Elastic-based methods
  - Lateral loading
  - General loadings
- Finite element analyses

SIMPLE STATICAL METHOD ("Rivet Group")

INHERENT ASSUMPTIONS

- Piles pinned to cap
- No 2nd order effects i.e. no loads due to pile deflections
- Vertical load shared equally among piles
- Each pile carries extra vertical load due to applied moment, proportional to distance from centroid of group

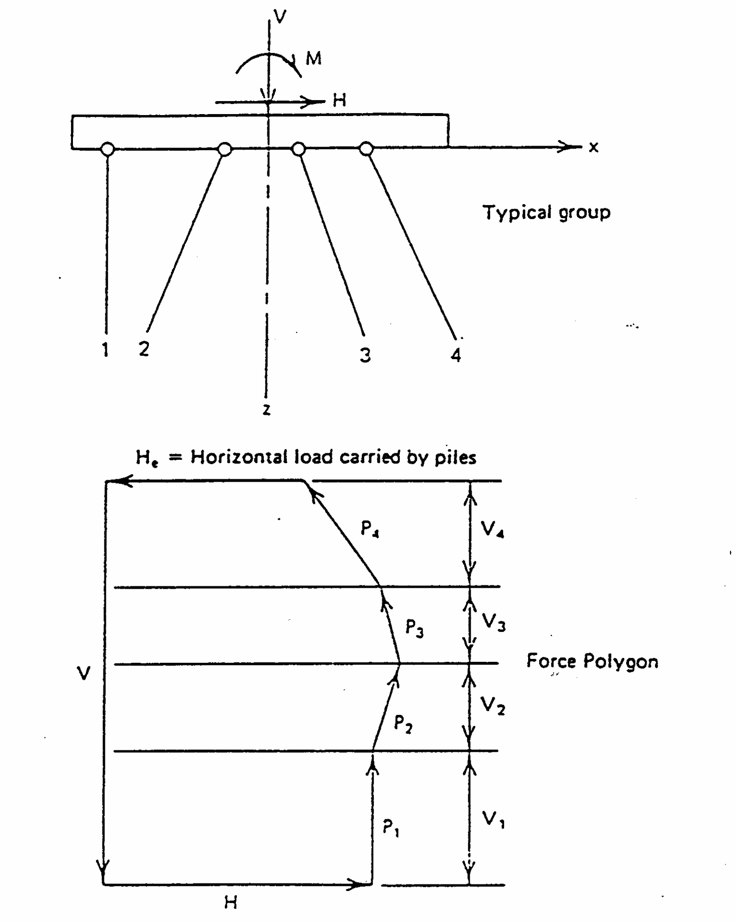
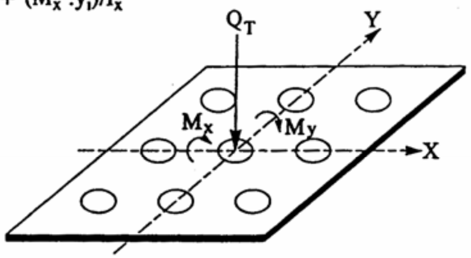


FIGURE 9.1 Simple statical method for determination of group load distribution.

SIMPLE STATICAL METHOD
(“Rivet Group”)

$$P_i = Q_T/n_p + (M_y^*.x_i)/I_y + (M_x^*.y_i)/I_x$$



and

$$M_x^* = \frac{M_x - M_y.I_{xy}/I_y}{1 - I_{xy}^2/(I_x.I_y)}$$

$$M_y^* = \frac{M_y - M_x.I_{xy}/I_x}{1 - I_{xy}^2/(I_x.I_y)}$$

where

- P_i = axial load on an individual pile i
- Q_T = total vertical load acting at the centroid of the pile group
- n_p = number of piles in the group
- M_x, M_y = moment about centroid of pile group with respect to the x and y axes respectively
- I_x, I_y = moment of inertia of pile group with respect to x and y axes respectively
- I_{xy} = product of inertia of pile group about the centroid
- x_i, y_i = distance of pile i from y and x axis respectively
- M_x^*, M_y^* = effective moment with respect to x and y axes respectively, taking into account the symmetry of the pile layout.

For a symmetrical pile group layout, $I_{xy} = 0$ and M_x^*, $M_y^* = M_x$, M_y

In this case, the above expression becomes :

$$P_i = \frac{Q_T}{n_p} \pm \left(\frac{M_y\, x_i}{\sum_{i=1}^{n_p} x_i^2} \right) \pm \left(\frac{M_x\, y_i}{\sum_{i=1}^{n_p} y_i^2} \right)$$

Notes : The assumptions made in this method are :

(a) pile cap is perfectly rigid,
(b) pile heads are hinged to the pile cap and no bending moment is transmitted from the pile cap to the piles, and
(c) piles are vertical.

EQUIVALENT BENT METHOD

- Principle is to reduce group to equivalent structural frame or “bent”
- Ideally, allowance should be made for effects of pile-soil-pile interaction
- Most earlier methods did not do this

PROCEDURE

1. Estimate equivalent length of cantilever to obtain same lateral deflection as pile
2. Estimate equivalent X-sectional area of cantilever to obtain same axial settlement as pile

PROBLEMS

1. Lateral deflection characteristics of pile and cantilever are NOT same
2. Group effect need to be taken into account

EQUIVALENT BENT METHOD

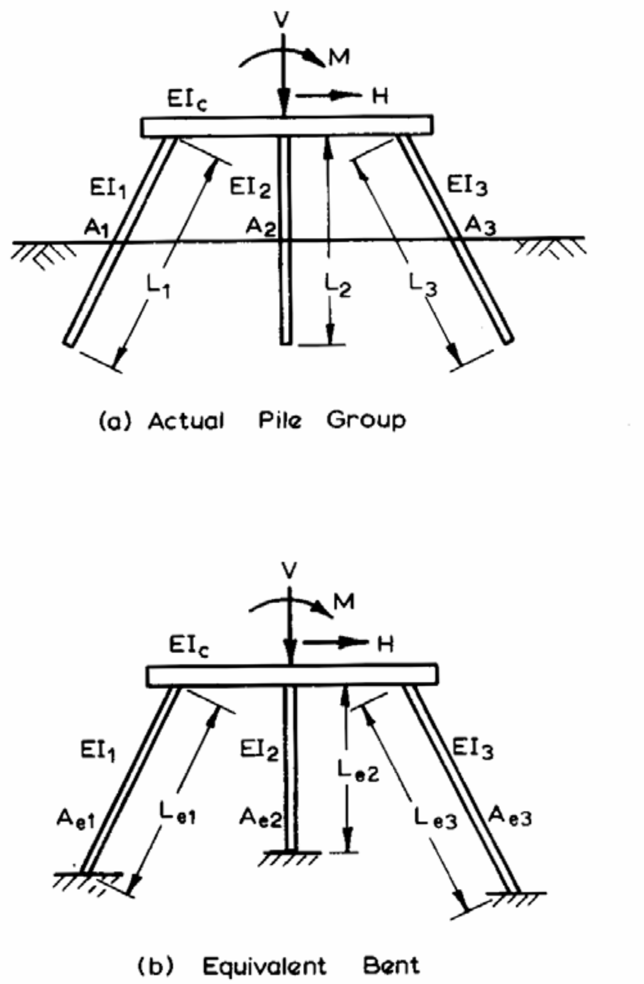


FIGURE 9.2 Principle of equivalent-bent approach.

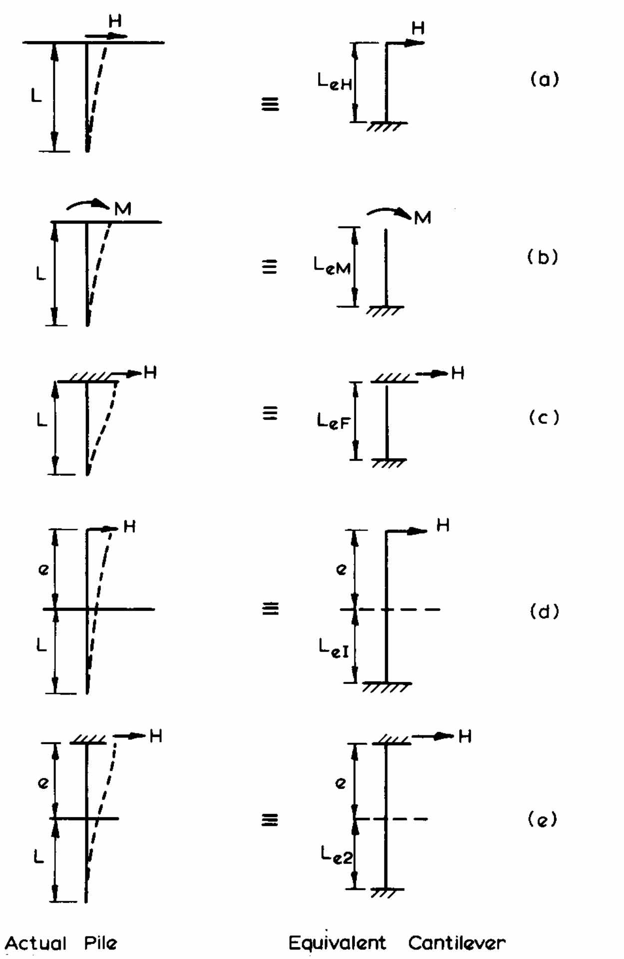


FIGURE 9.3 Equivalent cantilevers for laterally loaded piles.

HYBRID ANALYSES

Combination of:

- Load transfer analyses for single pile axial & lateral response
- Use of elastic theory to estimate pile-soil-pile interaction

Examples:

- Focht & Koch (1973 OTC)
- O'Neill et al (1977 OTC)

ADVANTAGES:

- Ability to model non-linear behaviour
- Interaction is confined to elastic components of deflection & rotation

BUT:

- Difficult to obtain compatible parameters for single pile & group effects

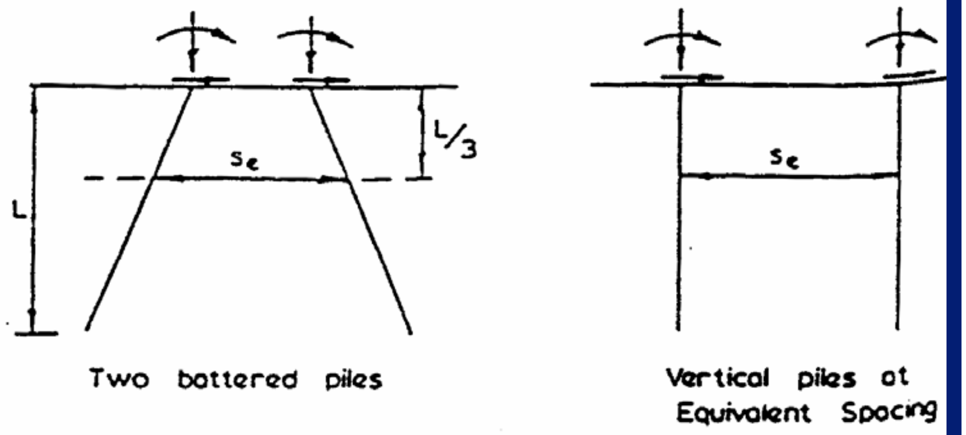
ELASTIC-BASED ANALYSIS

- Uses elastic theory for BOTH single pile response & pile-soil-pile interaction
- Examples are:
  - PIGLET (Randolph)
  - DEFPIG (Poulos)

ASSUMPTIONS IN DEFPIG

- Piles are battered in same plane
- Horizontal loads act in this plane
- For raked piles:
  - Axial displacement due to axial load = vertical displacement due to vertical load
  - Normal deflection due to normal load = horizontal deflection due to horizontal load
  - Interaction between battered piles same as between 2 vertical piles at an equivalent spacing

ANALYSIS OF GROUPS UNDER GENERAL LOADING



Equivalent spacing of battered piles.

$$\begin{bmatrix} A_v & B_v & C_v \\ A_h & B_h & C_h \\ A_\theta & B_\theta & C_\theta \end{bmatrix} \cdot \begin{Bmatrix} V \\ H \\ M \end{Bmatrix} = \begin{Bmatrix} \rho_v \\ \rho_h \\ \theta \end{Bmatrix}$$

where the coefficients of the sub-matrices are as follows:

$$\begin{aligned}
A_{vij} &= \rho_{a1}\alpha_{ij}\cos\psi_j\cos\psi_i + \rho_{NQ1}\alpha_{\rho Hij}\sin\psi_j\sin\psi_i \\
B_{vij} &= \rho_{a1}\alpha_{ij}\cos\psi_i\sin\psi_j - \rho_{NQ1}\alpha_{\rho Hij}\sin\psi_i\cos\psi_j \\
C_{vij} &= \rho_{NM1}\alpha_{\rho Mij}\sin\psi_i \\
A_{hij} &= \rho_{a1}\alpha_{ij}\sin\psi_i\cos\psi_j - \rho_{NQ1}\alpha_{\rho Hij}\cos\psi_i\sin\psi_j \\
B_{hij} &= \rho_{a1}\alpha_{ij}\sin\psi_j\sin\psi_i + \rho_{NQ1}\alpha_{\rho Hij}\cos\psi_j\cos\psi_i \\
C_{hij} &= \rho_{NM1}\alpha_{\rho Mij}\cos\psi_i \\
A_{\theta ij} &= -\theta_{N1}\alpha_{\theta Hij}\sin\psi_j \\
B_{\theta ij} &= \theta_{N1}\alpha_{\theta Hij}\cos\psi_j \\
C_{\theta ij} &= \theta_{M1}\alpha_{\theta Mij}
\end{aligned}$$

ρ_{a1} = axial deflection of single pile caused by unit axial load

ρ_{NQ1} = normal deflection of single pile caused by unit normal load

ρ_{NM1} = normal deflection of single pile caused by unit moment

θ_{N1} = rotation of single pile caused by unit normal load

θ_{M1} = rotation of single pile caused by unit moment

Have 3 equations for each pile, + 3 equilibrium equations

SOME PILE GROUP PROGRAMS USING ELASTIC-BASED ANALYSIS

Program	Analytical Approach	Capabilities
PGROUP	Elastic boundary element analysis	Linear variation of soil modulus with depth Horizontal loading in one direction Piles raked in plane of vert. & horiz. loading Pile cap contact modelled
	Rigorous analysis, but accuracy limited by level of discretisation of piles, particularly when considering lateral response. Requires considerable computer resources and relatively slow and labour intensive to run.	
DEFPIG	Boundary element analysis of single piles with inter–action factors for pile groups	Any variation of soil modulus Yielding and slip allowed for by limiting stress Horizontal loading in one direction Piles raked in plane of vert. and horiz. loading Pile cap contact modelled
	Versatile program, with main limitation being the approximate treatment of variations in modulus with depth. Microcomputer version available, but relatively slow and non–interactive.	
PIGLET	Elastic closed form solutions (single piles), interaction factors for groups	Linear variation of soil modulus with depth Separate modulus for axial and lateral loading Horiz. loading in any direction, torsional loading Piles raked in any direction
	Fast and simple program but limited to linear response, extensively checked against more rigorous analysis. Microcomputer based, with fully interactive data input/editing. Standard version can analyse groups of up to 100 piles (version also available for 300 piles).	

COMPARISON OF METHODS OF GROUP ANALYSIS

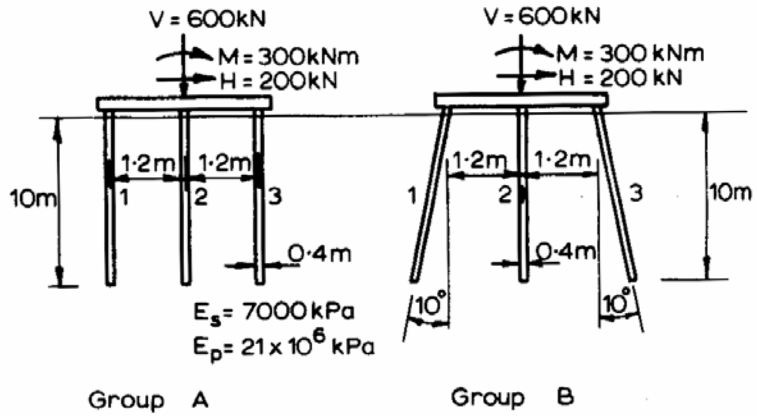


FIGURE 9.16 Pile groups considered in comparison of methods.

TABLE 9.8 COMPARISON OF METHODS OF GROUP ANALYSIS

	Quantity	Simple Statical Analysis	Equivalent-Bent Analysis	Elastic Continuum Analysis
Group *A*	V_1 (kN)	75	67.2	50.5
	V_2 (kN)	200	200.0	163.4
	V_3 (kN)	325	332.8	386.1
	H_1 (kN)	66.7	66.6	75.9
	H_2 (kN)	66.7	66.7	48.2
	H_3 (kN)	66.7	66.6	75.9
	M_1 (kN m)	0	−6.2	−39.6
	M_2 (kN m)	0	−6.2	−23.5
	M_3 (kN m)	0	−6.2	−39.6
	ρ_{v_3} (mm)		17.5	14.8
	ρ_h (mm)		8.9	11.8
	θ		0.00581	0.00248
Group *B*	V_1 (kN)	75	59.3	65.4
	V_2 (kN)	200	200.3	174.8
	V_3 (kN)	325	329.6	359.8
	H_1 (kN)	52.0	76.7	20.3
	H_2 (kN)	52.0	75.5	26.3
	H_3 (kN)	52.0	47.8	153.3
	M_1 (kN m)	0	−43.3	−6.4
	M_2 (kN m)	0	−26.9	−5.1
	M_3 (kN m)	0	66.9	−41.8
	ρ_{v_3} (mm)		16.4	12.9
	ρ_h (mm)		8.2	10.4
	θ		0.00490	0.00233

COMPARISON OF METHODS OF GROUP ANALYSIS

Conclusions:

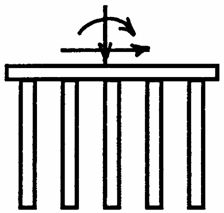
- Vertical pile loads are not very sensitive to analysis method
- Considerable difference between head moments, and group deflections & rotations from equivalent bent method & elastic (DEFPIG) analysis

THE SIGNIFICANCE OF CONSIDERING INTERACTION

EFFECT OF INCLUDING INTERACTION

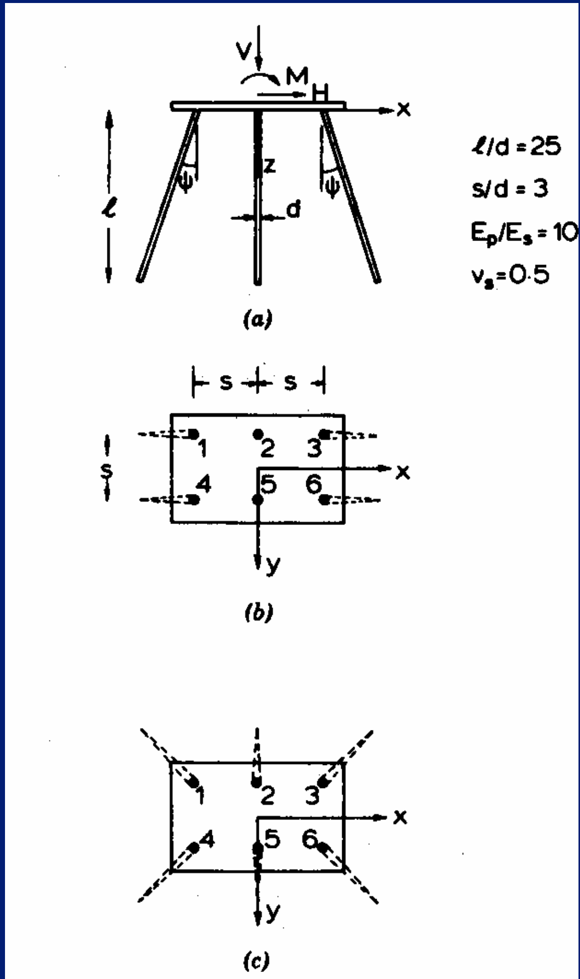
Value	Without Interaction	With Interaction
Vertical Load (MN) - pile 4	10.4	10.6
Horizl. Load (MN) - pile 4	0.55	0.54
Head Moment (MN m) - pile 4	-1.65	-2.16
Vertical Defln. (mm) - pile 4	14	26
Horizl. Defln. (mm)	40	82

V=56MN, H=15 MN, M=1450 MNm



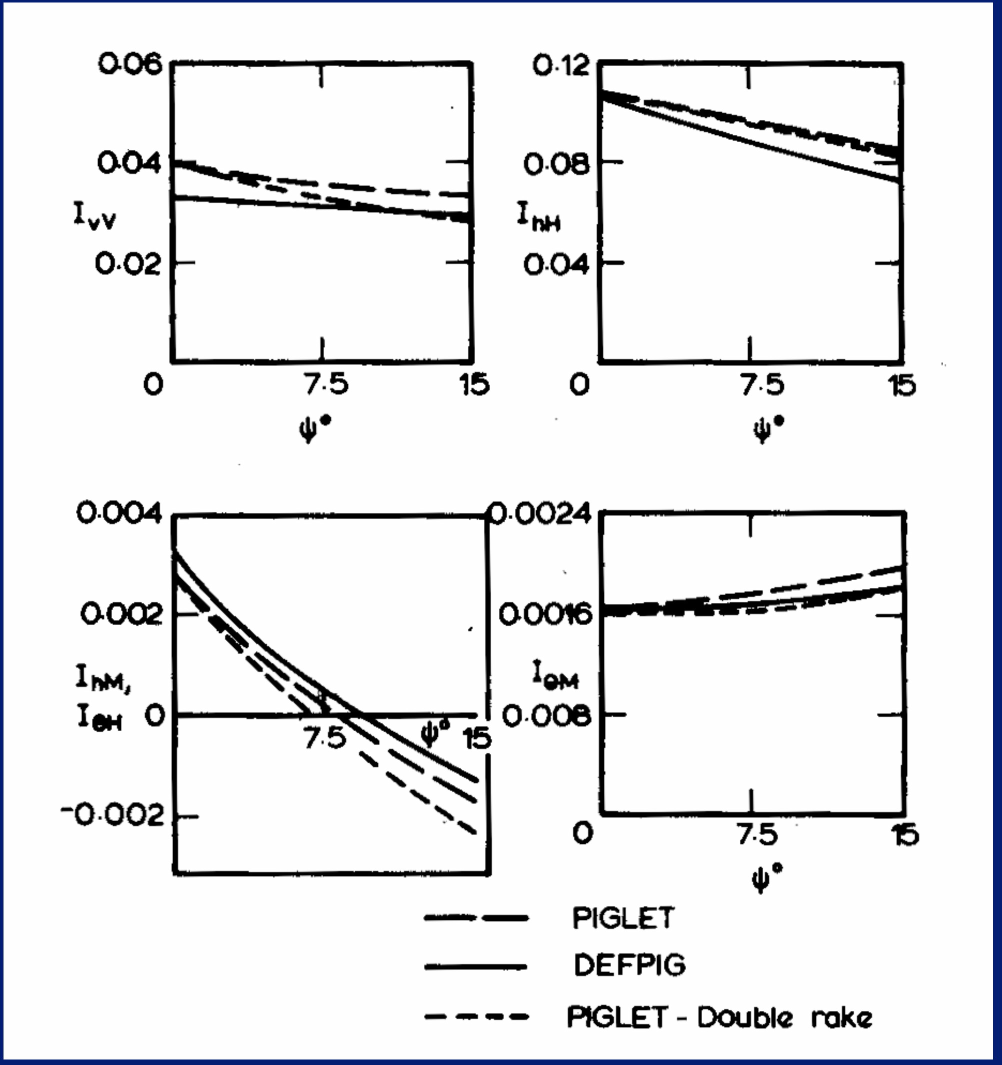
- The loads are not greatly affected
- The pile bending moments are affected somewhat
- The group deflection & settlement are affected significantly

COMPARISON OF PILE GROUP ANALYSIS PROGRAMS



- 6-pile group analyzed via 3 approaches:
  - DEFPIG
  - PIGLET (single rake)
  - PIGLET (double rake).
- Elastic analyses in all cases

COMPARISON OF PILE GROUP ANALYSIS PROGRAMS



COMPARISON OF PILE GROUP ANALYSIS PROGRAMS

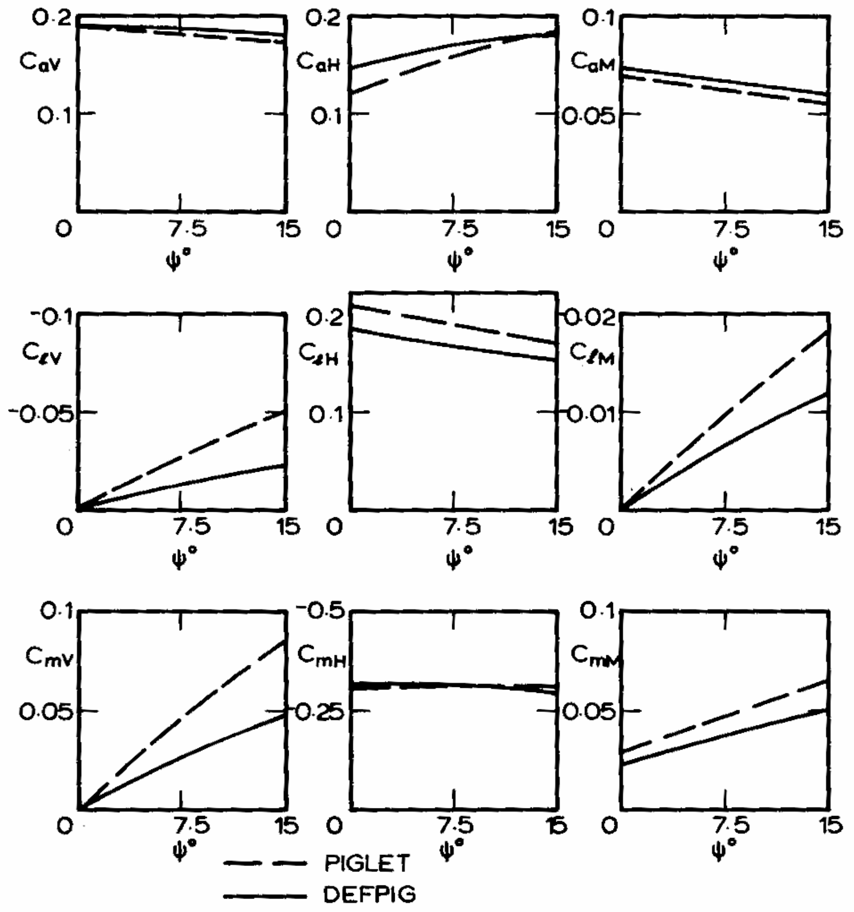


FIG. 3.—Variation of Load Influence Factors with ψ

COMPARISONS WITH FIELD DATA (Matlock et al, 1980)

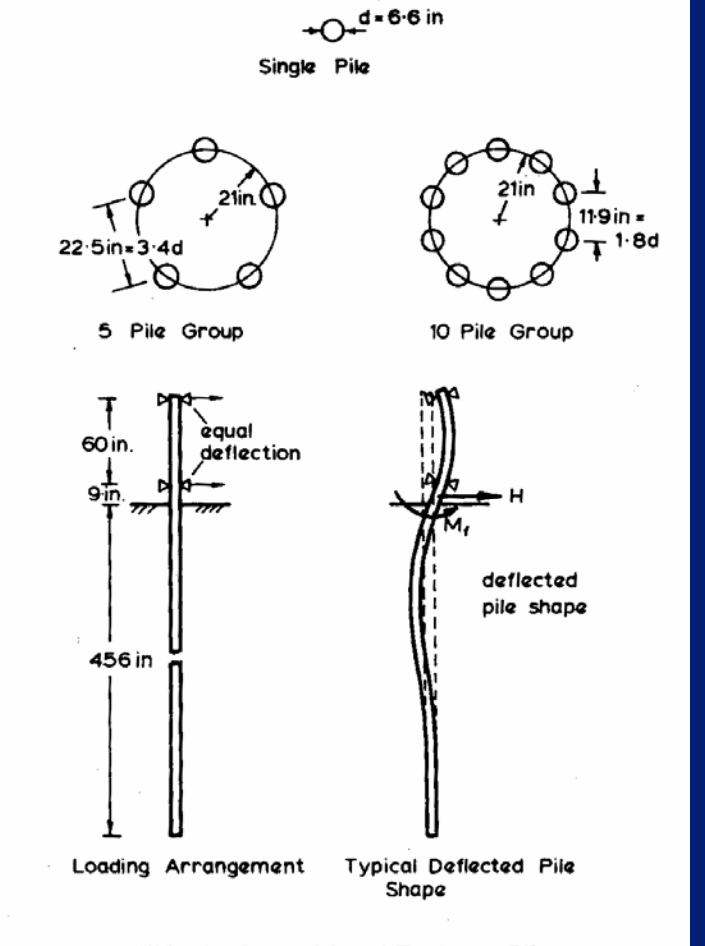


FIG. 4.—Lateral Load Tests on Piles

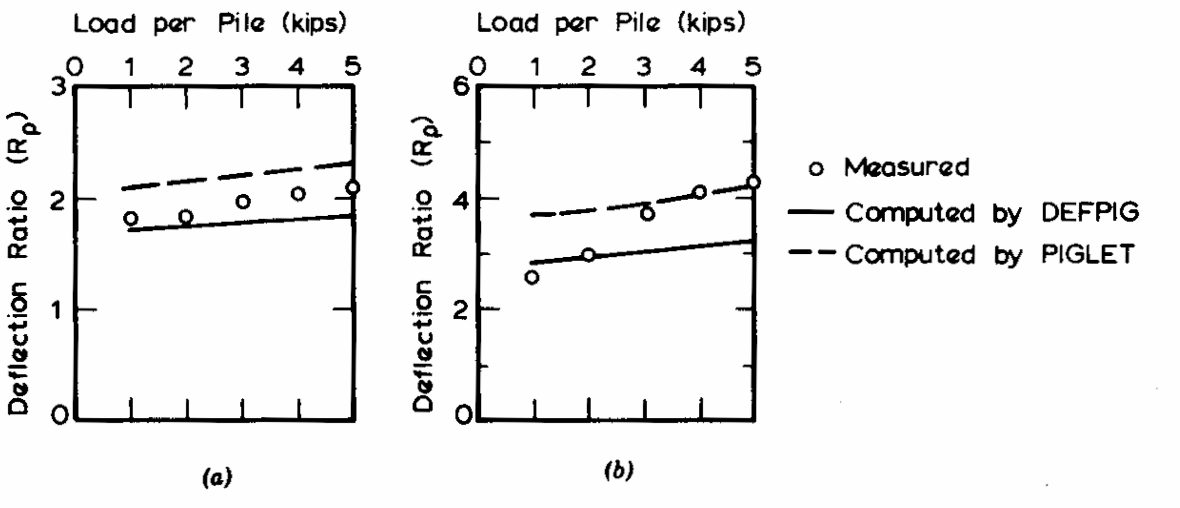


FIG. 5.—Comparisons between Measured and Computer Group Deflection Ratios: (*a*) Five-Pile Group; (*b*) Ten-Pile Group

COMPARISONS WITH FIELD DATA (Matlock et al, 1980)

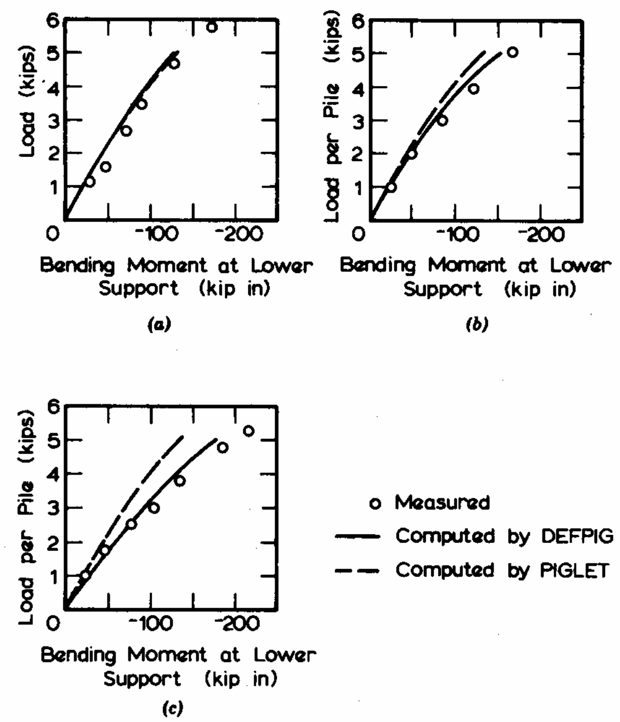


FIG. 6.—Comparisons between Computed and Measured Moments: (a) Single Pile; (b) Five Pile; and (c) Ten Pile

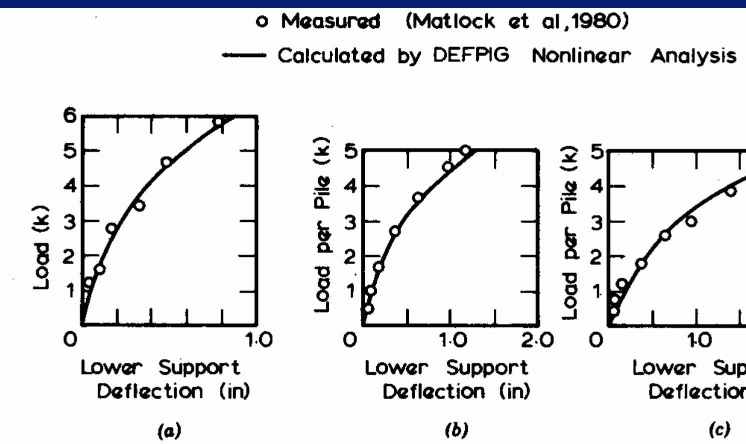


FIG. 7.—Comparisons between Measured and Calculated Load-Deflection Response: (a) Single Pile; (b) Five Pile; and (c) Ten Pile

FINITE ELEMENT ANALYSES

Two-dimensional plane strain analysis

- Model piles as 2-D “walls” with equivalent stiffness in axial & lateral directions

- **PROBLEMS:**
  - Walls & piles behave differently
  - 3-D loading is difficult to simulate in 2-D
  - Pile bending moments not easy to assess

FINITE ELEMENT ANALYSES
2-D Piled Raft

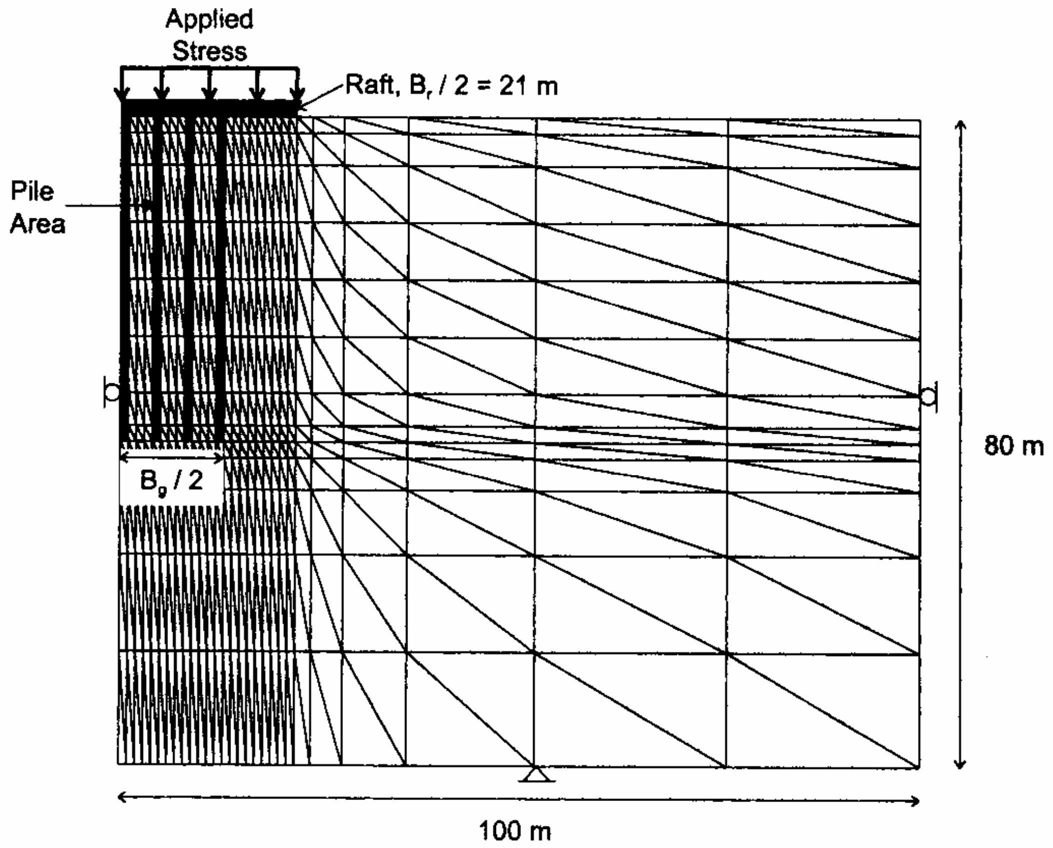
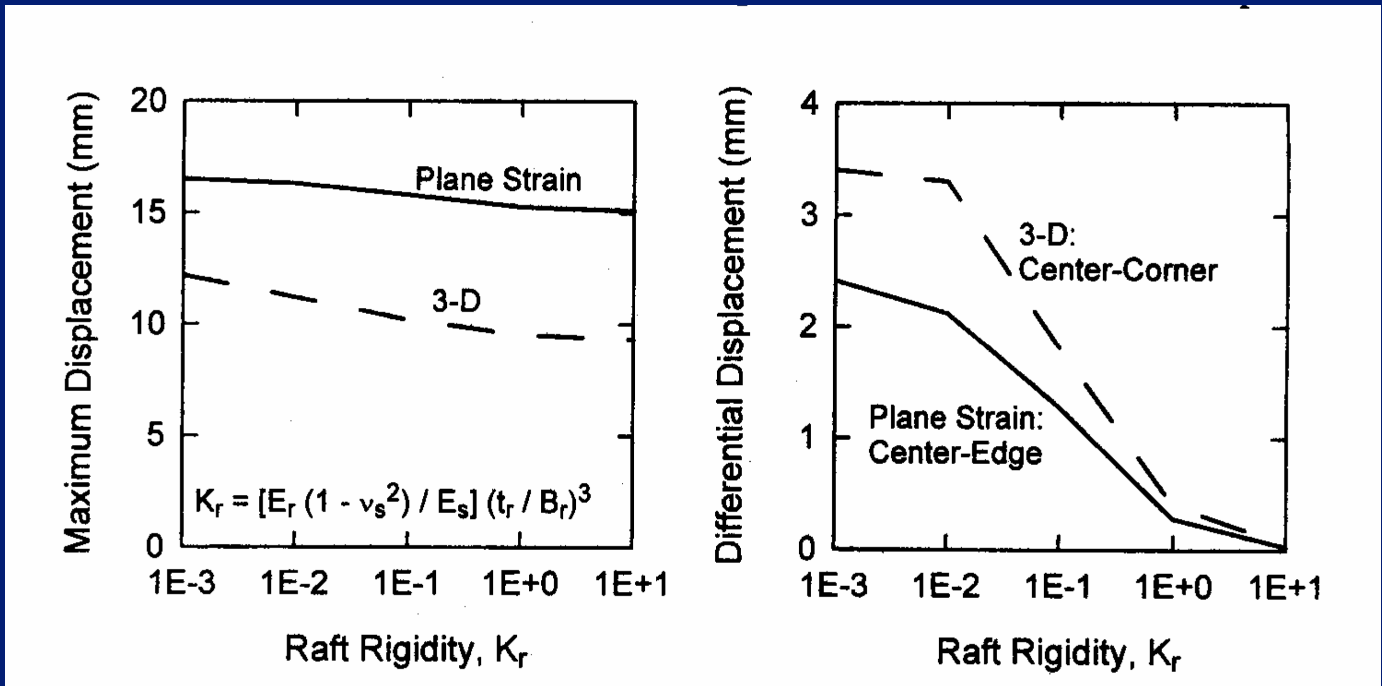


FIG. 2. Typical Finite-Element Mesh (798 Six-Node Triangular Linear Elastic Elements)

Prakoso & Kulhawy, 2001

FINITE ELEMENT ANALYSES
2-D Piled Raft



Prakoso & Kulhawy, 2001

2-D analysis over-predicts maximum settlements & under-predicts Differential settlements

FINITE ELEMENT ANALYSES

Two-dimensional axisymmetric analysis

- Model piles as 2-D "rings" with equivalent stiffness in axial direction

- **PROBLEMS:**
  - Rings & piles behave differently
  - Can only consider vertical loading with normal analyses

FINITE ELEMENT ANALYSES (Pressley & Poulos, 1986)

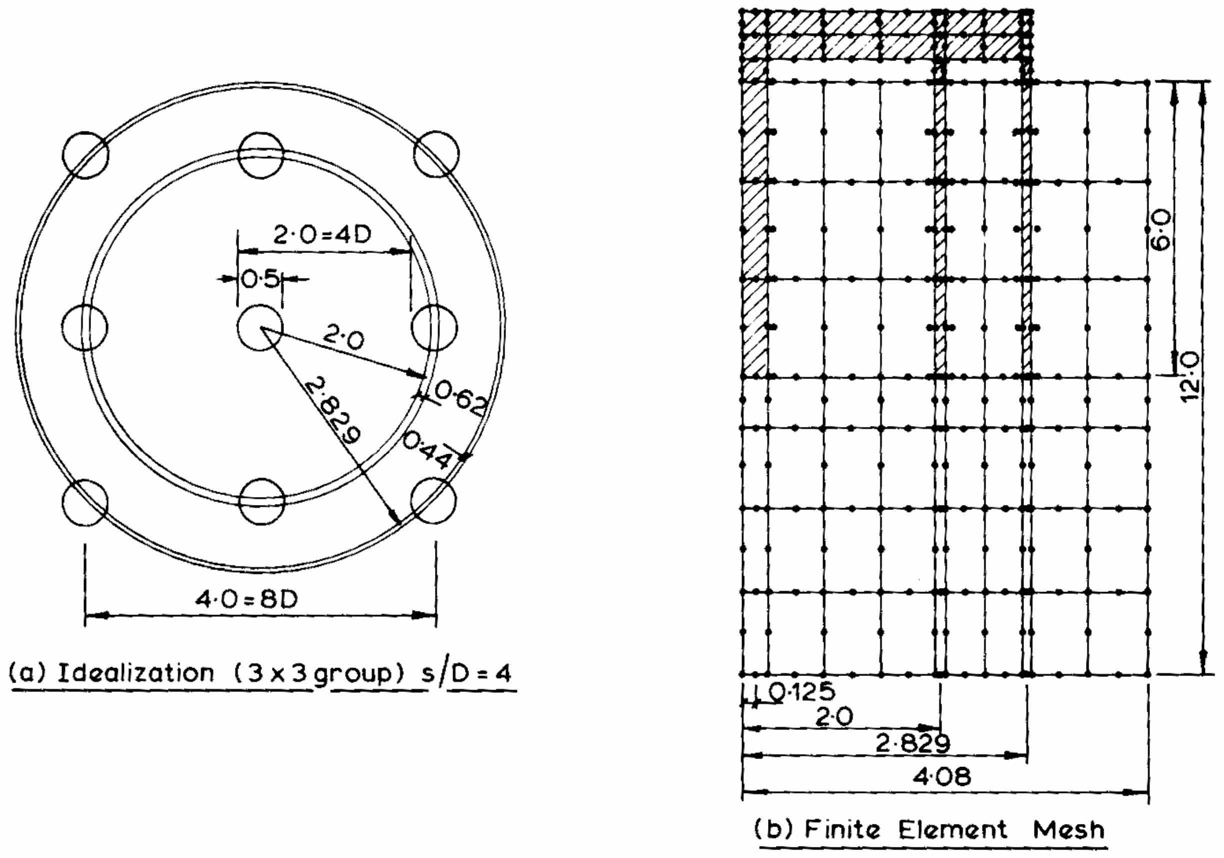


Figure 1. Modelling of pile group

FINITE ELEMENT ANALYSES (Pressley & Poulos, 1986)

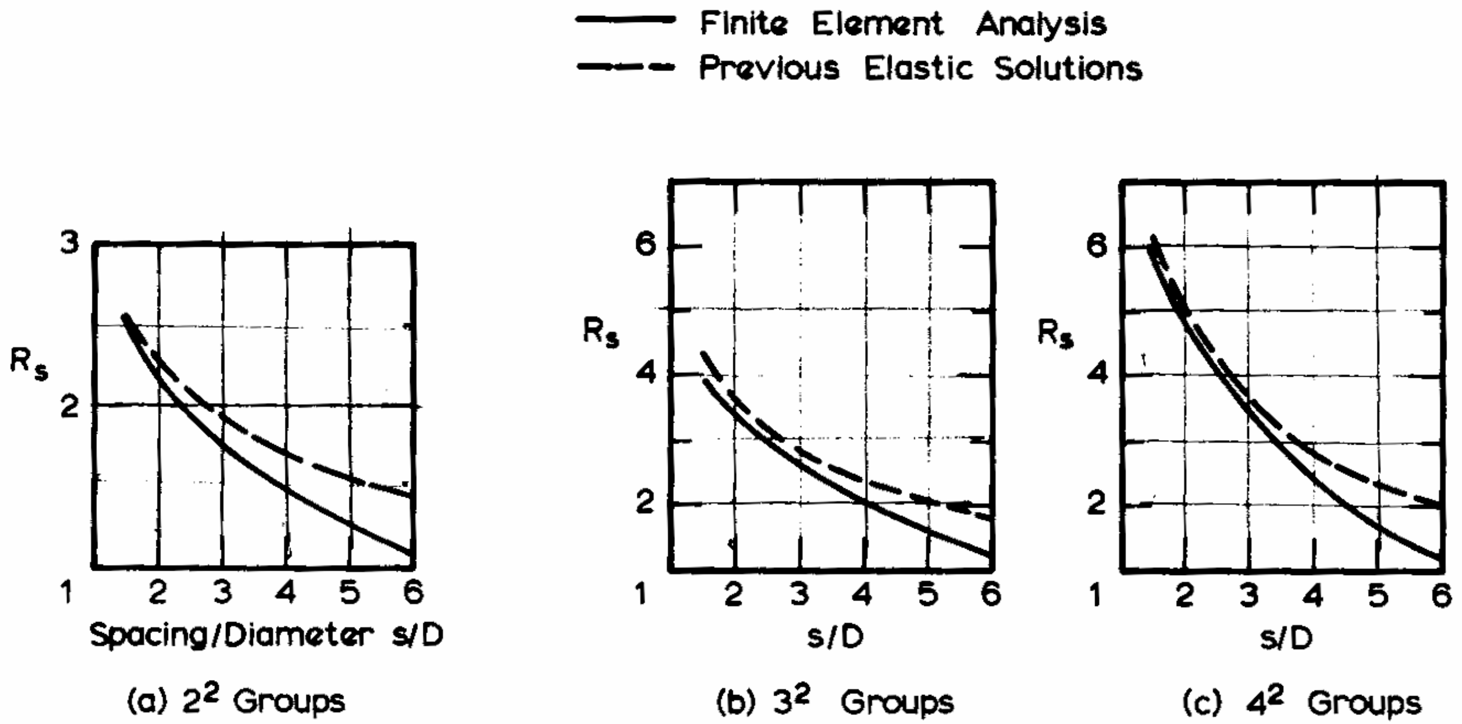


Figure 2. Comparisons between solutions for group settlement ratio R_s: ——— Finite element analysis; – – – – previous elastic solutions

FINITE ELEMENT ANALYSES (Pressley & Poulos, 1986)

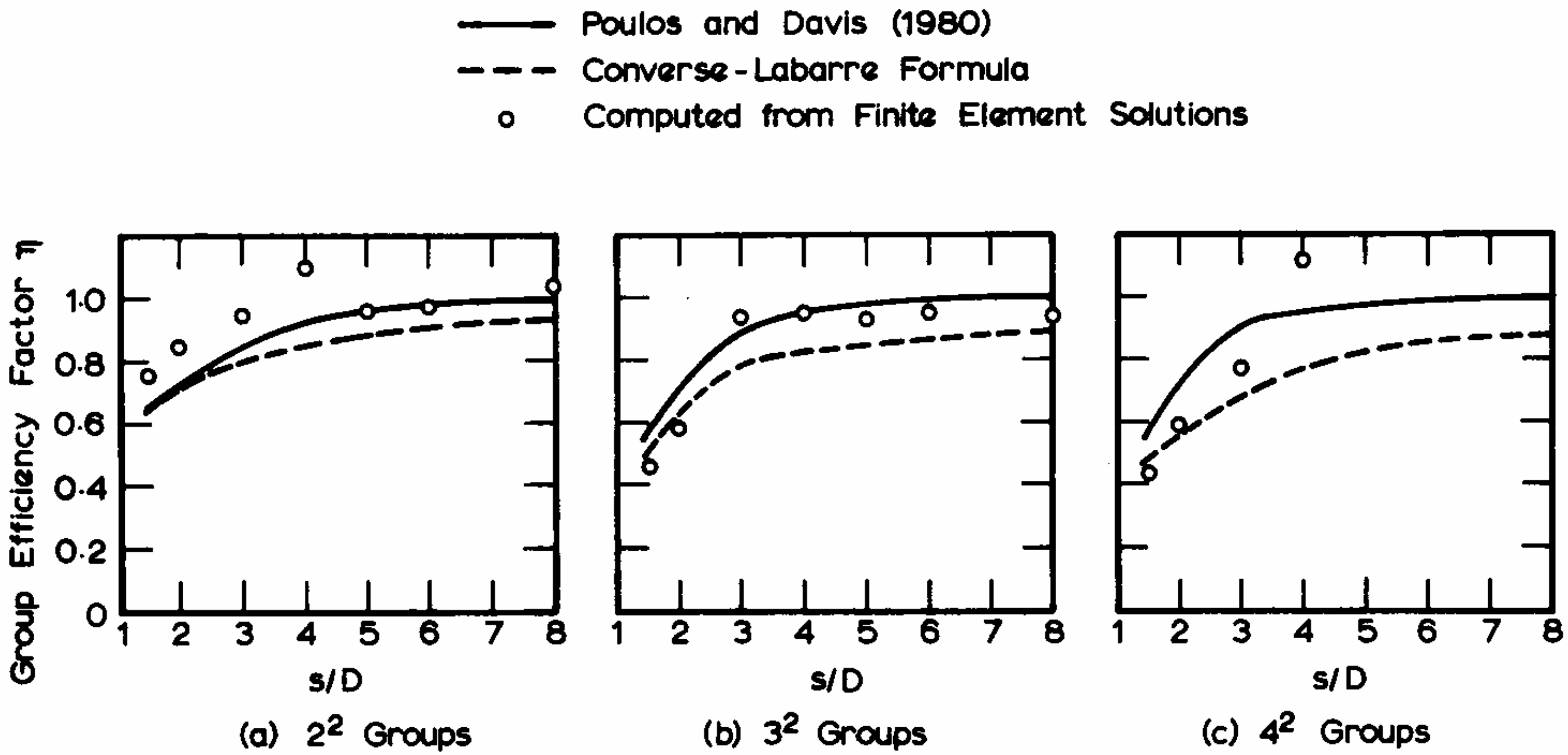


Figure 8. Group efficiency factors:——— Poulos and Davis;[2] ----converse-Labarre formula; ° computed from finite element solutions

FINITE ELEMENT ANALYSES

Three-dimensional analysis

- Use solid brick elements to model soil, piles & pile cap

- **PROBLEMS:**
  - Time & effort involved
  - BUT, newer programs are facilitating this type of analysis

3-D FINITE ELEMENT ANALYSES (Ottaviani, 1975)

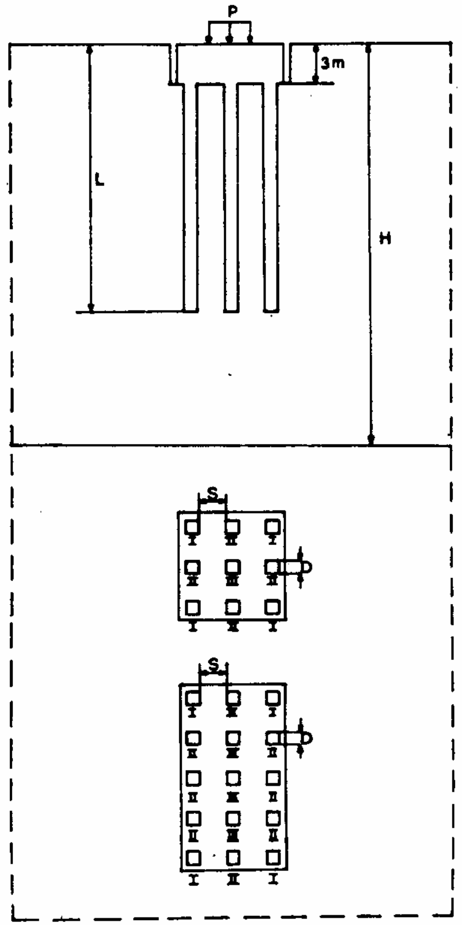


Fig. 1. Geometry of pile groups

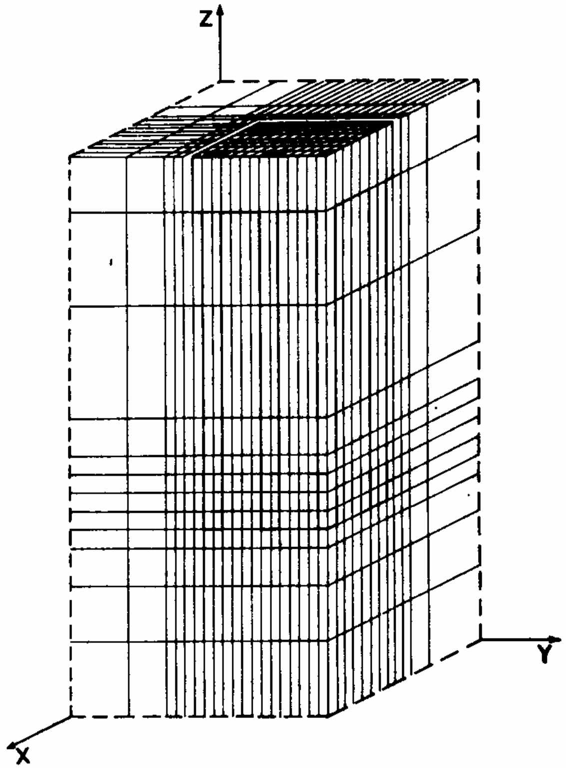


Fig. 2. Finite element scheme of pile groups

3-D FINITE ELEMENT ANALYSES (Ottaviani, 1975)

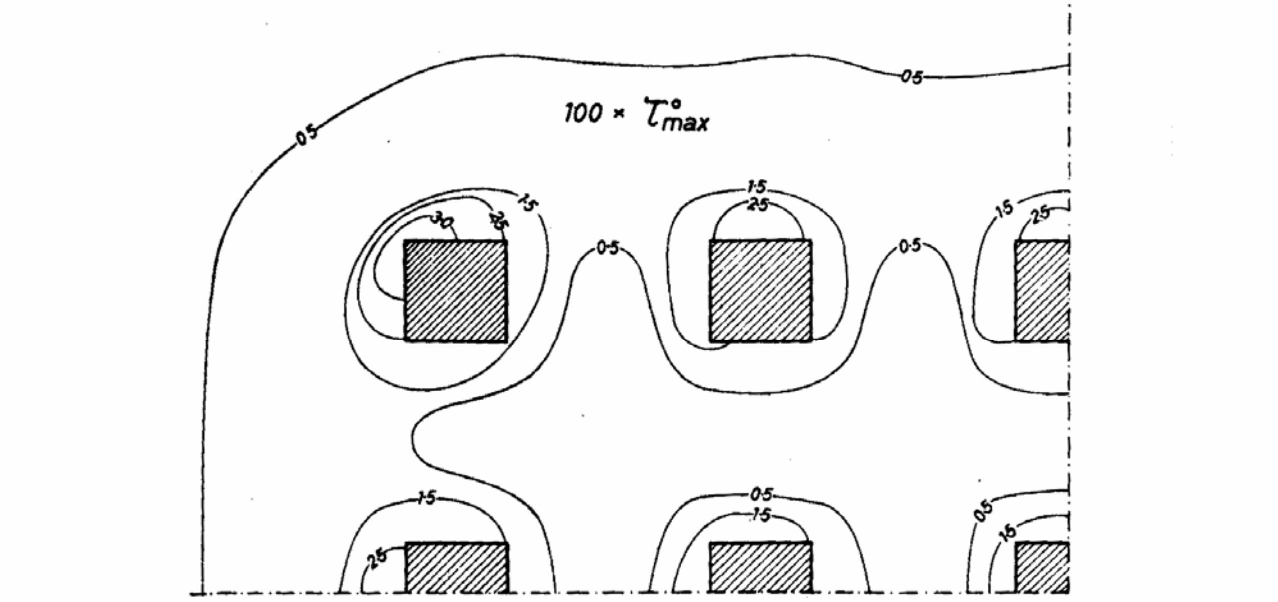


Fig. 21. Maximum shear stress distribution on horizontal section just above piles' base of 5 × 3 group with cap ($L = 20$ m, $\lambda = 800$)

Note that stress distribution around piles is NOT symmetrical, as assumed in many simplified analyses, especially for corner piles.