

LECTURE 3

LATERAL LOADING OF PILES

OUTLINE

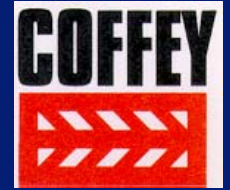
- Ultimate lateral capacity
- Lateral deflection
 - Methods of analysis
 - Solutions based on elastic continuum
 - Non-linear analyses
 - Assessment of parameters
- Interaction & group effects

ULTIMATE LATERAL CAPACITY OF PILES

SIGNIFICANCE

- Usually lateral deflections govern pile design for lateral loadings
- BUT, ultimate lateral resistance may be important for:
 - Short piers
 - Long slender piles
 - Non-linear analysis of deflections

FAILURE MODES & HEAD CONDITIONS



SHORT PILE MODE:

- Failure of the supporting soil

LONG PILE MODE:

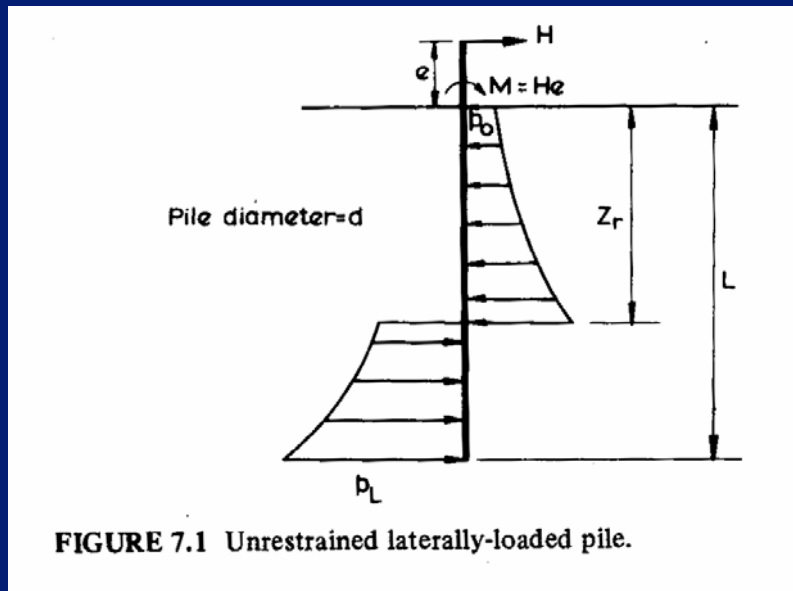
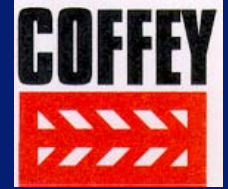
- Structural failure or yielding of the pile itself.

Both modes need to be analyzed, and the more critical mode established.

HEAD CONDITIONS

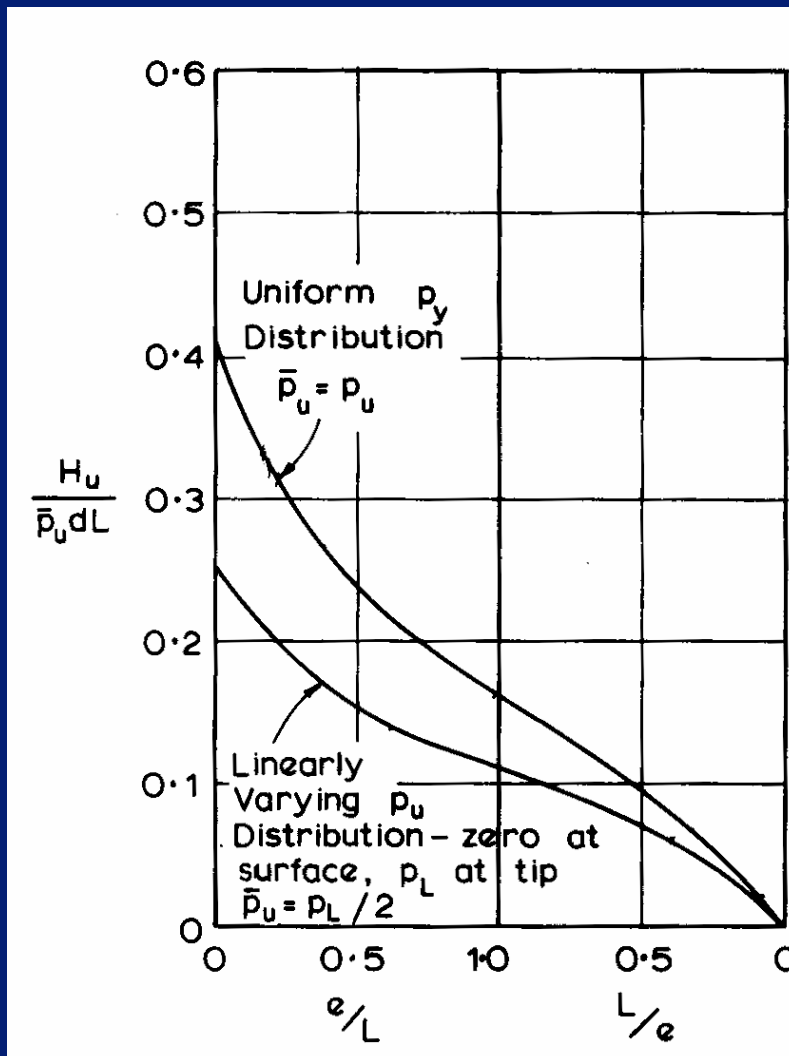
1. Free or unrestrained head – no head restraint
2. Fixed head or restrained head – No rotation of head.

GENERAL PRINCIPLES OF ANALYSIS



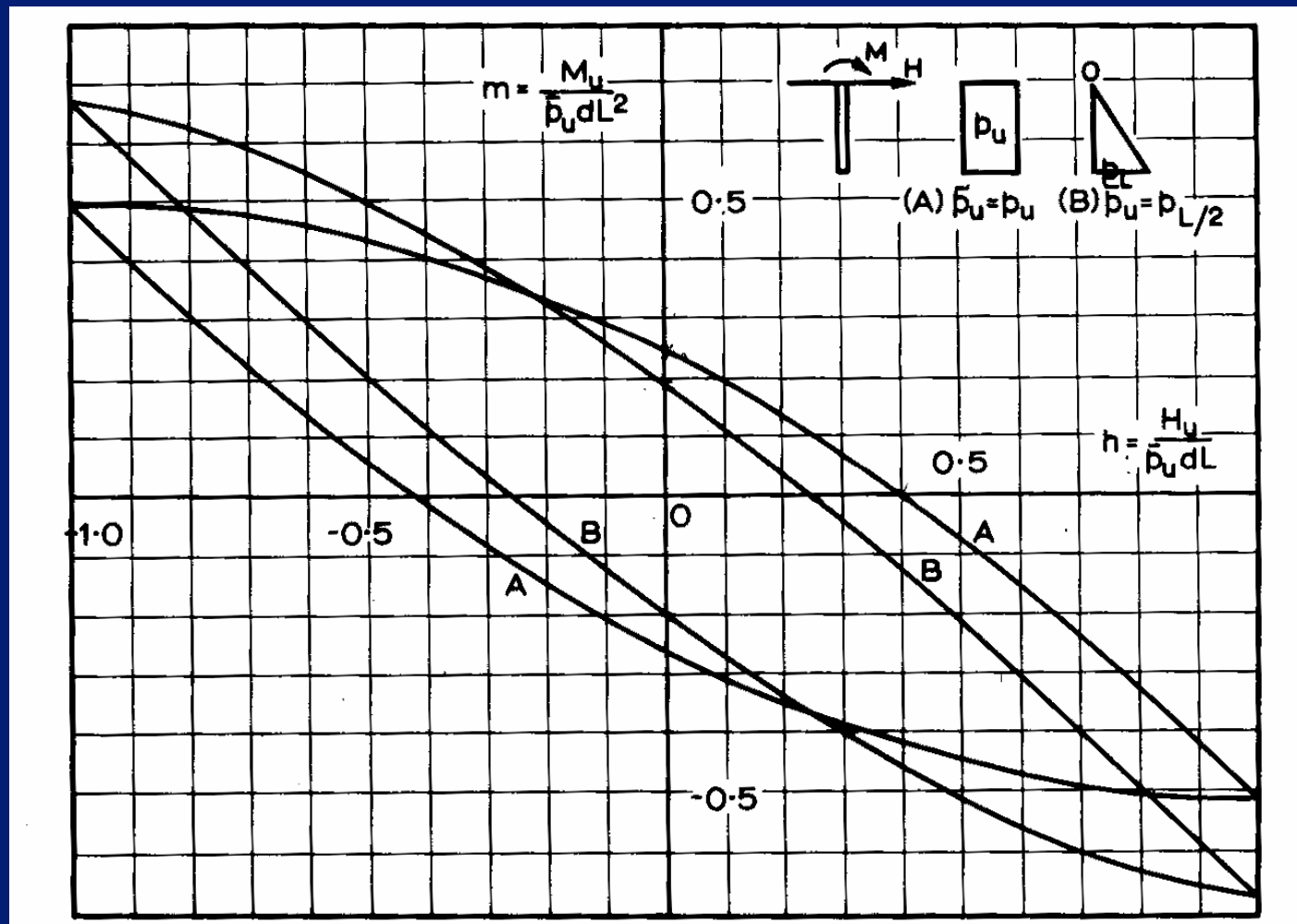
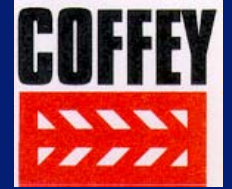
- Consider:
 - Horizontal load equilibrium
 - Moment equilibrium
- Need to specify:
 - Mode of failure
 - Distribution of ultimate lateral pile-soil pressure

SIMPLE CASES – FREE HEAD



- Constant p_y with depth (e.g. O/C clays)
- Linearly increasing p_y with depth (e.g. N/C clays, sands)

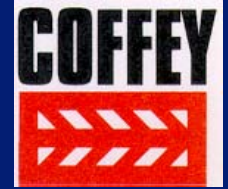
ALTERNATIVE PRESENTATION OF SIMPLE SOLUTIONS



H-M “yield”
Surface

Rigid free-head
pile

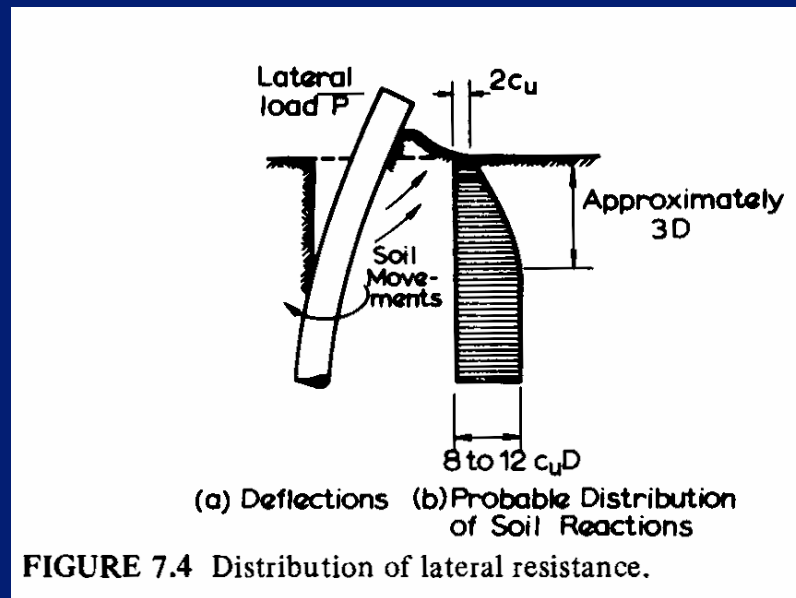
ULTIMATE LATERAL PILE-SOIL PRESSURE - CLAYS



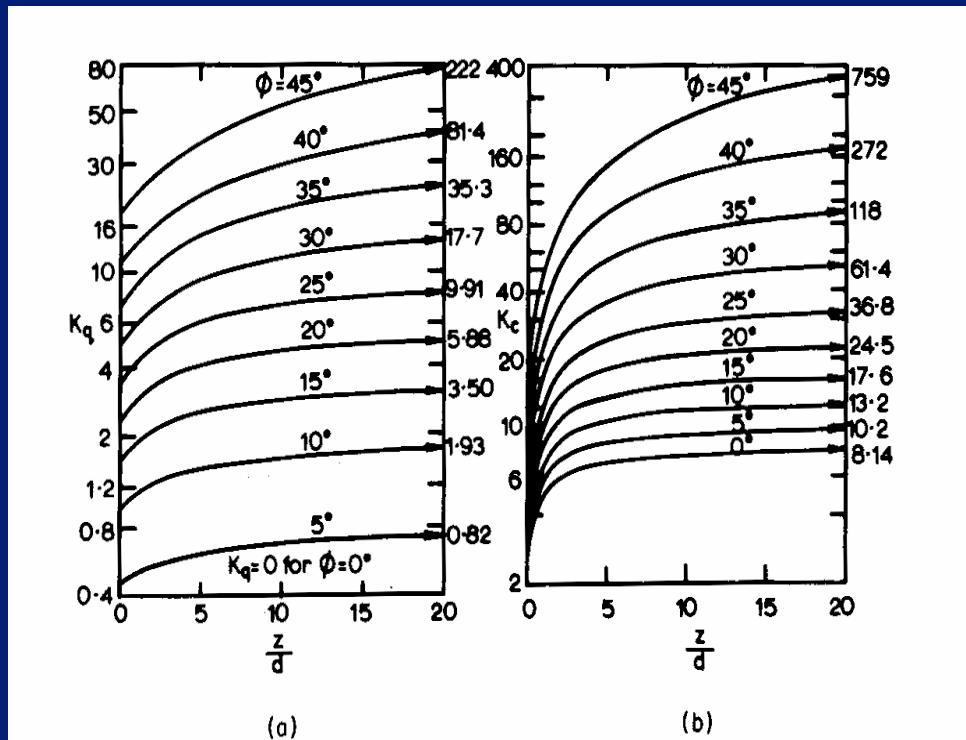
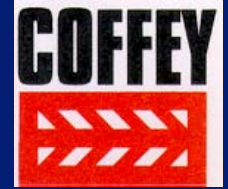
Usually adopt

$$p_y = N_c \cdot c$$

N_c varies from 2 at the surface to 8 – 12 at z/d greater than 3 (typically, 9 is used)



ULTIMATE LATERAL PILE-SOIL PRESSURE – SANDS & c-φ SOILS



Usually adopt

$$p_y = F \cdot p_p$$

where $F = 3 - 5$

Alternatively, use theory of Brinch Hansen

$$p_y = q \cdot N_q + c \cdot N_c$$

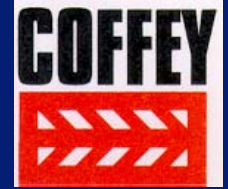
BROMS' THEORY

- Probably the most widely-used method.
- Relies on statics.
- Considers short & long-pile failure modes.

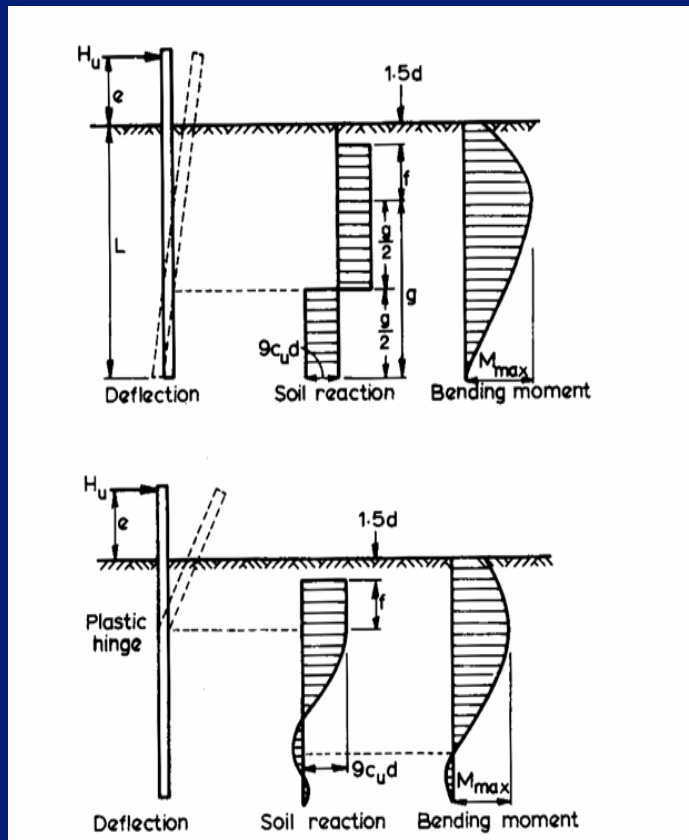
Analyses Available:

- **Cohesive soils**
 - Short & long pile failure – unrestrained head
 - Short & long-pile failure – restrained head
- **Cohesionless Soils**
 - Short & long pile failure – unrestrained head
 - Short & long-pile failure – restrained head

BROMS' THEORY FOR PILES IN CLAY



Short Pile

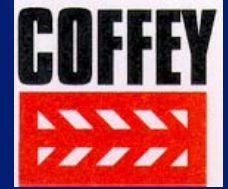


Long Pile

- Assume “dead” zone to $1.5d$ below surface
- Assume $p_y = 9c$ below this depth
- Employ statics to obtain failure load for short-pile and long-pile cases

Failure modes for free-head pile

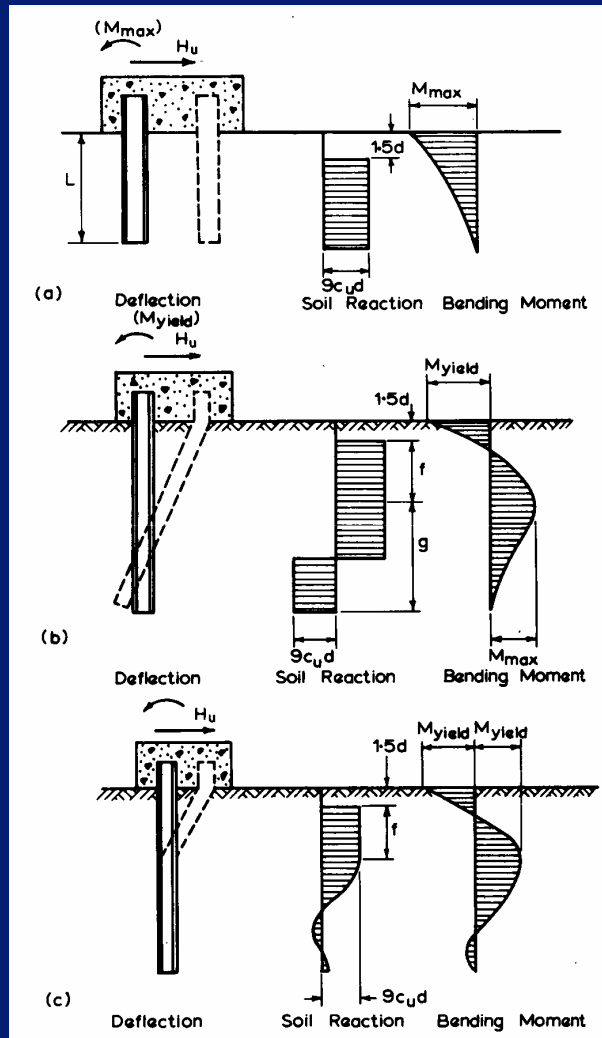
BROMS' THEORY FOR PILES IN CLAY



Short Pile

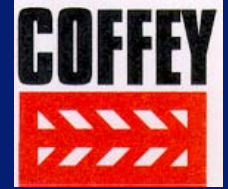
Intermediate Pile

Long Pile



Failure modes for fixed-head pile

BROMS' THEORY FOR PILES IN CLAY – ULTIMATE LATERAL LOAD



Need:

For soil -

c_u

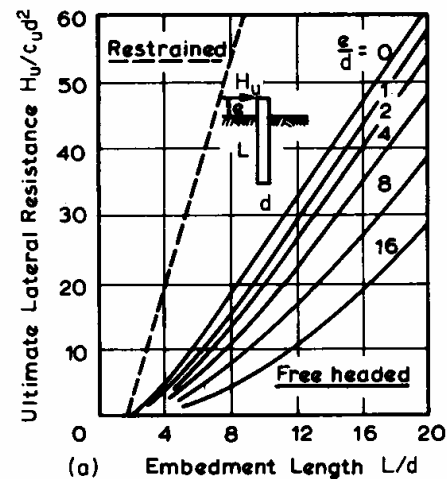
For pile -

M_{yield}

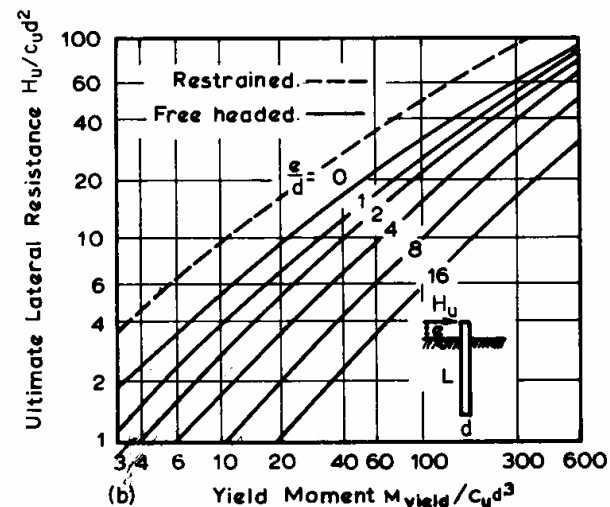
L

D

e

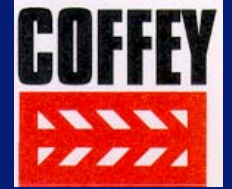


SHORT PILE
FAILURE MODE



LONG-PILE
FAILURE MODE

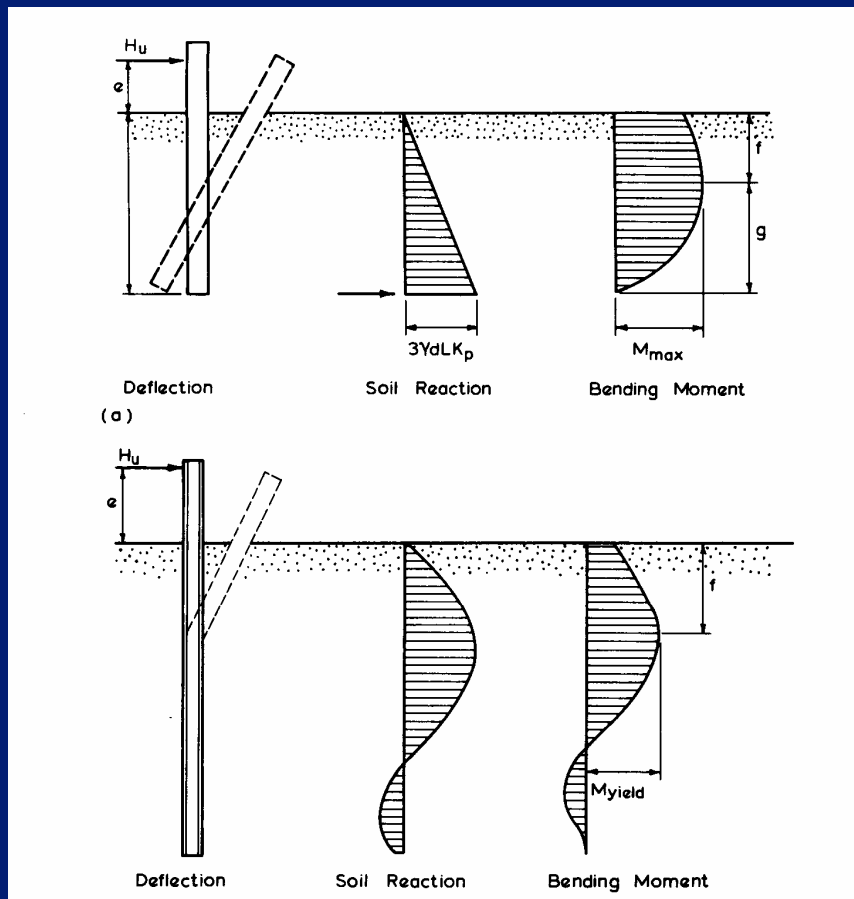
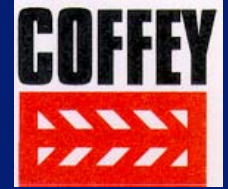
BROMS' THEORY FOR PILES IN COHESIONLESS SOILS



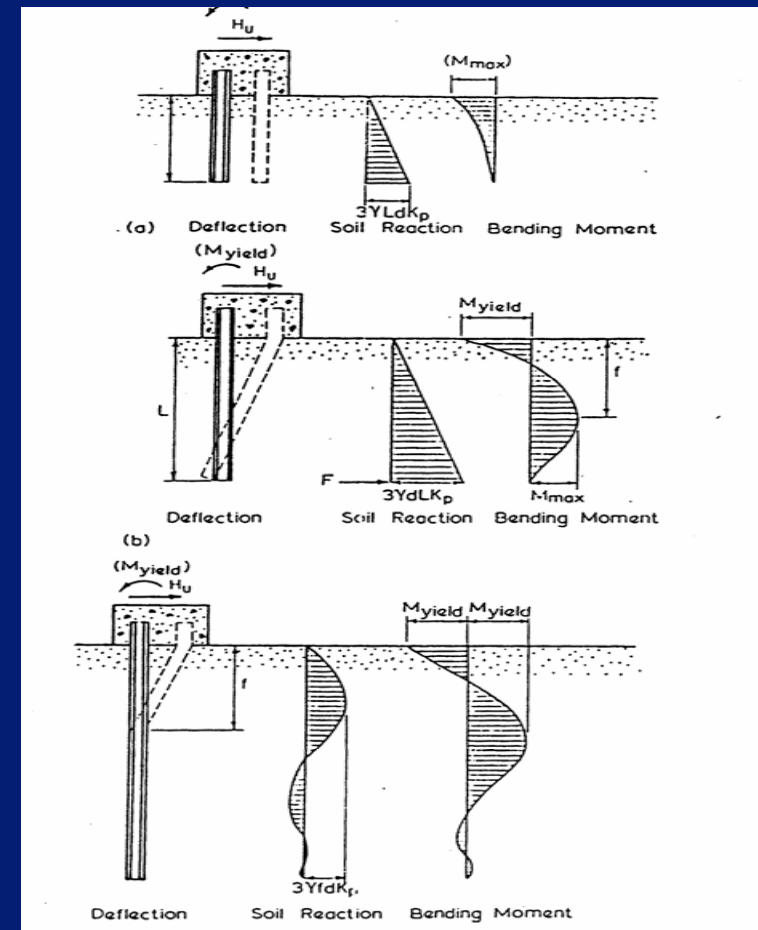
ASSUMPTIONS:

- Ignore active earth pressure behind wall
- Assume shape of pile section does not affect p_y
- Soil reaction below point of rotation is represented by a point load (to simplify algebra)
- $p_y = 3p_p$

BROMS' THEORY FOR PILES IN COHESIONLESS SOILS – FAILURE MODES

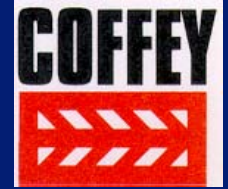


Free Head Pile

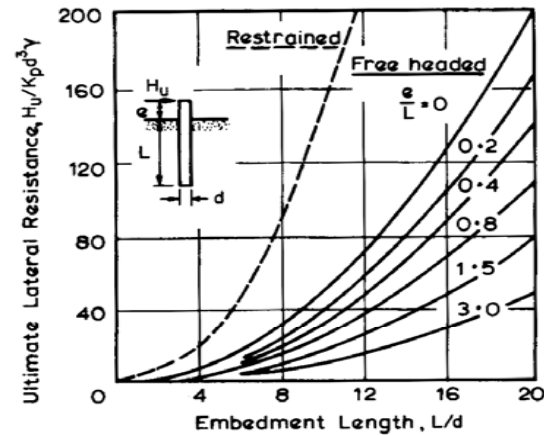


Fixed Head Pile

BROMS' THEORY FOR PILES IN COHESIONLESS SOILS – FAILURE LOAD

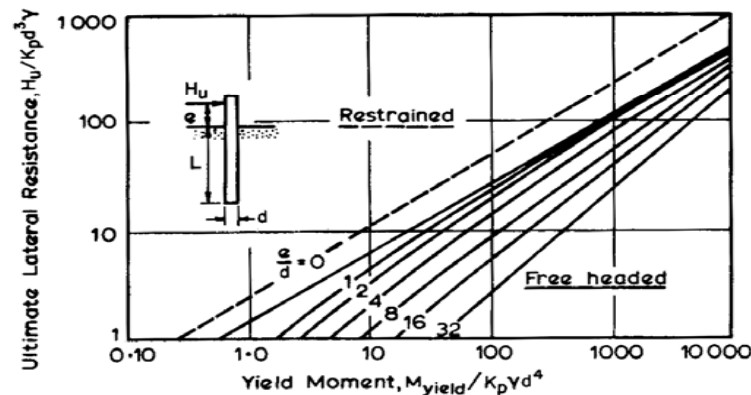


Short Pile Mode



(a)

Long Pile Mode



(b)

Need:

For soil -

K_p

γ

For pile -

M_{yield}

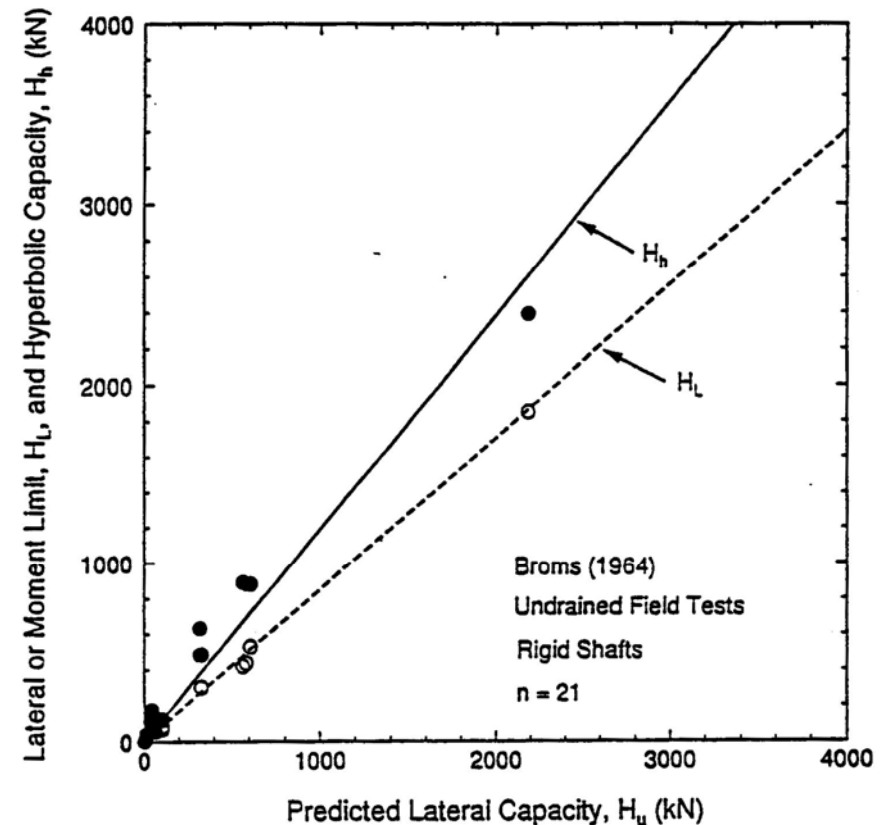
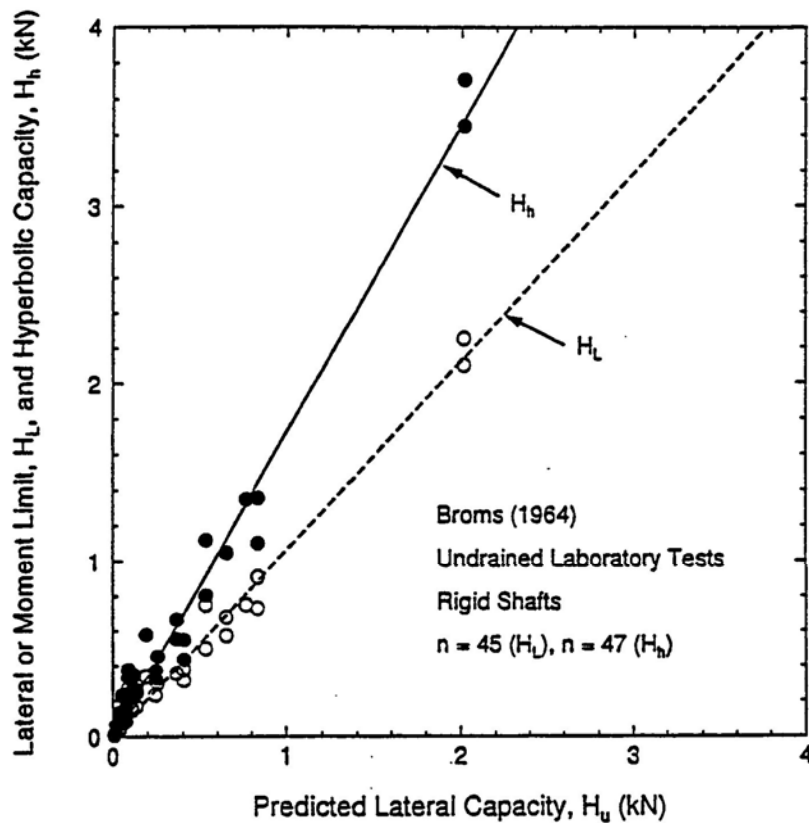
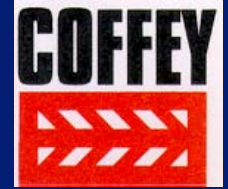
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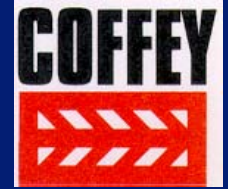
BROMS' THEORY – EVALUATION

(Kulhawy & Chen, 1995)



BROMS' THEORY – EVALUATION

(Kulhawy & Chen, 1995)



<i>Data Type</i>	<i>Calc./Msd Limit Load H_L</i>	<i>Calc. / Msd. Ult. Load H_h</i>
Undrained lab + field data (68 tests)	0.78	0.50
Drained lab + field data (65 tests)	1.38	0.89

H_L = load for initial failure
or yield

H_h = hyperbolic extrapolation
for ultimate load

In general, Broms' method appears to be moderately conservative

PILE IN LAYERED COHESIVE SOIL

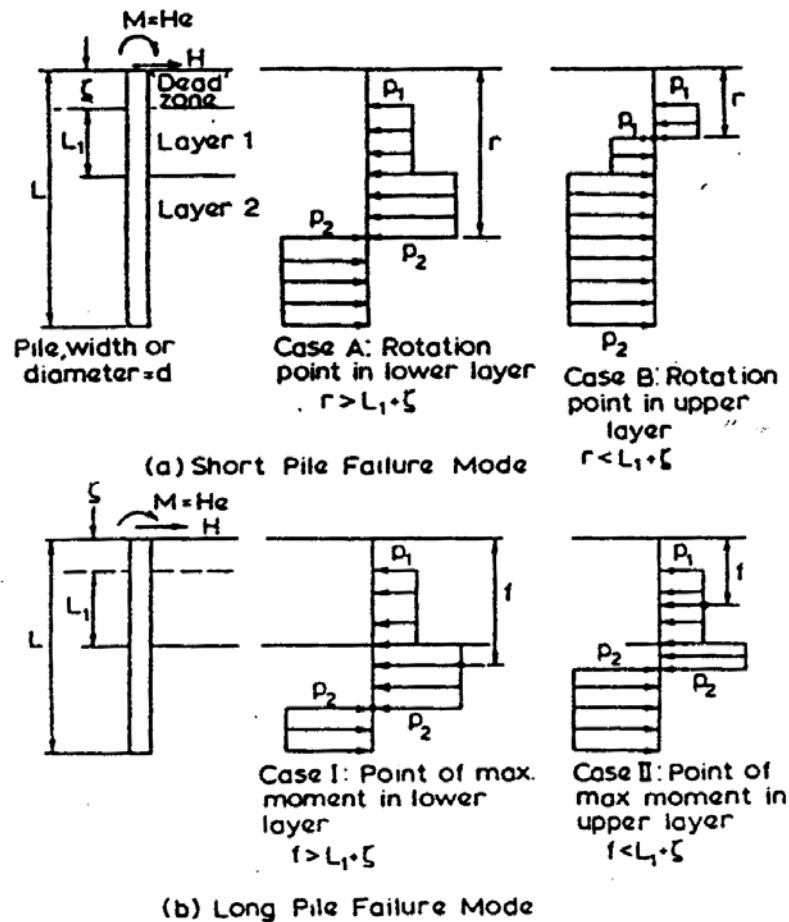
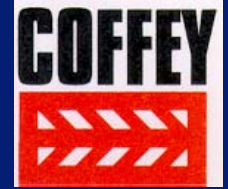


Fig. 1. Failure of Free Head Pile.

Again, consider short- and long-pile failure modes. Obtain failure load via statics.

PILE IN LAYERED COHESIVE SOIL

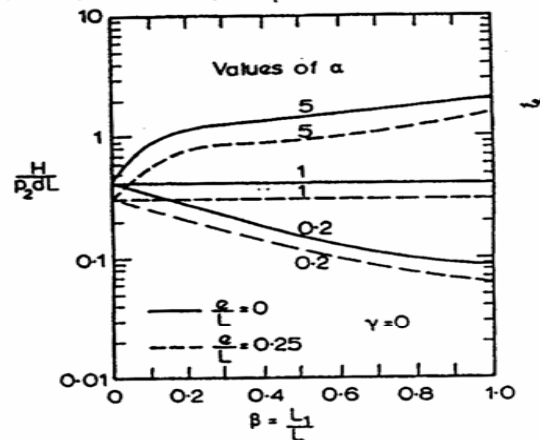
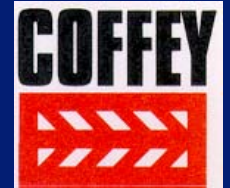


Fig. 5. Short-Pile Failure : Free-Head Pile.

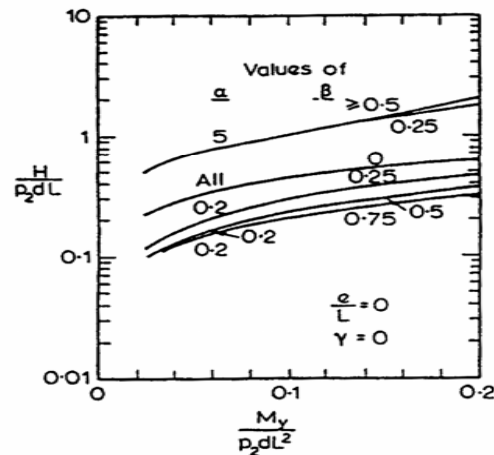


Fig. 6. Long-Pile Failure : Free-Head Pile.

Quadratic equation for ultimate lateral load capacity:

$$aH^{*2} + bH^* + C = 0$$

where $H^* = H / (p_2 d L)$
 $M^* = M_y / (p_2 d L^2)$
 $\alpha = p_1 / p_2$
 $\beta = L_1 / L$
 $\gamma = \zeta / L$

Coefficients a, b, c given by Poulos (1985)

PILE GROUPS

- Take group capacity as lesser of:
 - Sum of individual pile capacities;
 - Capacity of block containing piles + soil.
- Need to consider both short-pile and long-pile modes for single piles.
- Consider only short-pile mode for block.
- DO NOT USE Broms' solutions for blocks in clay! "Dead zone" of $1.5B$ is not realistic.
- Suggest use of solutions for uniform p_y , with a smaller "dead" zone (via an eccentricity of load), or solutions of Fleming et al (1992)..

LATERAL DEFLECTION OF PILES

METHODS OF ANALYSIS

1. SUBGRADE REACTION METHOD

Soil modelled as series of independent springs (linear or nonlinear).

A point in the soil only deflects if it has stress acting on it. Therefore, this model does not consider stress transmission.

$$\rho(i, j) = 0 \quad \text{for } i \neq j$$

$$\rho(i, j) = p(j) / k(j) \quad \text{for } i = j$$

2. ELASTIC - BASED METHOD

Soil modelled as an elastic mass.

Allowance is made for soil yielding by specifying limiting values of pile - soil pressure at various points along the pile.

$$\rho(i, j) = p(j) \cdot d \cdot I(i, j) / E_s(i)$$

where $I(i, j)$ = displacement influence factor, evaluated from Mindlin's equations.

3. FINITE ELEMENT METHOD

Requires 3 - Dimensional analysis for proper nonlinear analysis.

METHODS OF ANALYSIS

ADVANTAGES AND DISADVANTAGES OF ALTERNATIVE METHODS OF ANALYSIS

METHOD	ADVANTAGES	DISADVANTAGES
Subgrade Reaction	<p>Simpler analysis</p> <p>Any type of "p - y" pile - soil response can be analysed</p> <p>Considerable experience in use & evaluation of parameters</p>	<p>Soil "spring" stiffness is dimension - dependent</p> <p>Cannot consider group action</p>
Elastic - based analysis	<p>Can analyse interaction between piles and consider group behaviour</p> <p>can allow for effect of cut or slope near pile</p> <p>Can extend to batter piles</p> <p>Can use to analyse effect of external soil movements</p>	<p>Consideration of non-homogeneous soil profile is approximate</p> <p>Only limited experience in practical use</p> <p>Uncertainties remain in evaluation of relevant soil parameters</p>

SUBGRADE REACTION THEORY

Linear Analysis

- Soil “spring” behaviour:
 - $p = k \cdot \rho$
- Usual cases:
 - $k = \text{constant}$
 - $k = n_h \cdot z / d$

Dimensionless

Parameters:

1. $k = \text{constant}$

$$\beta = (k_h \cdot d / E_p I_p)^{1/4}$$

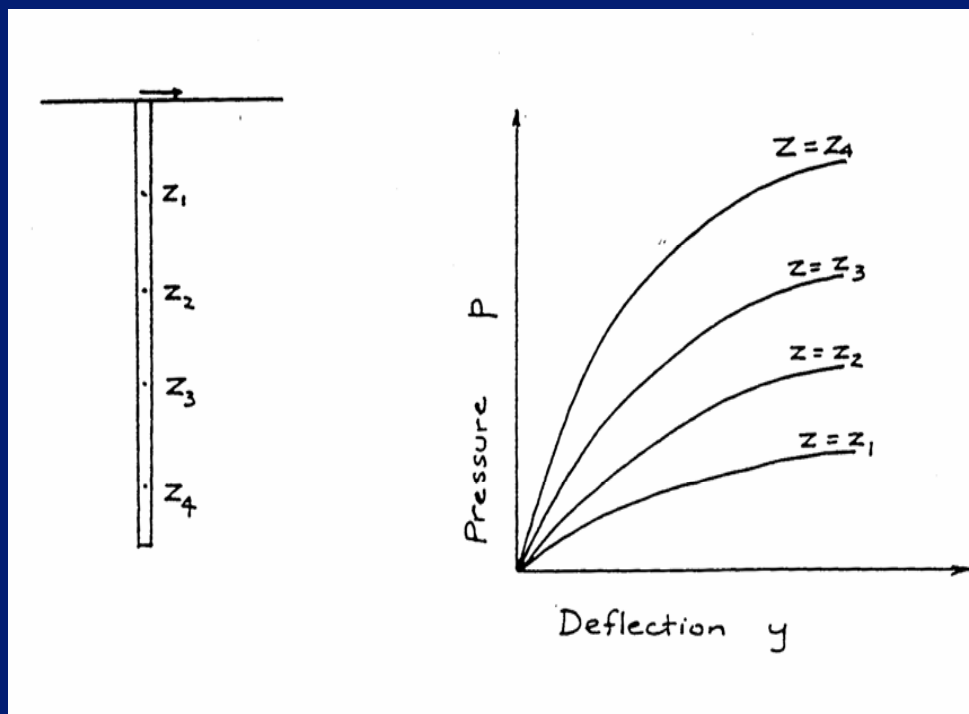
$$L_{cr} = 2.5 / \beta$$

2. Linearly Varying k

$$T = (E_p I_p / n_h)^{1/5}$$

$$L_{cr} = 4T$$

NONLINEAR “p-y” ANALYSES



- Widely used, both onshore and offshore
- Uses empirical relationships between pressure p and deflection r at a point on the pile
- These are used in a beam analysis to obtain load-deflection curves for the pile

TYPICAL “p-y” CURVES - CLAY

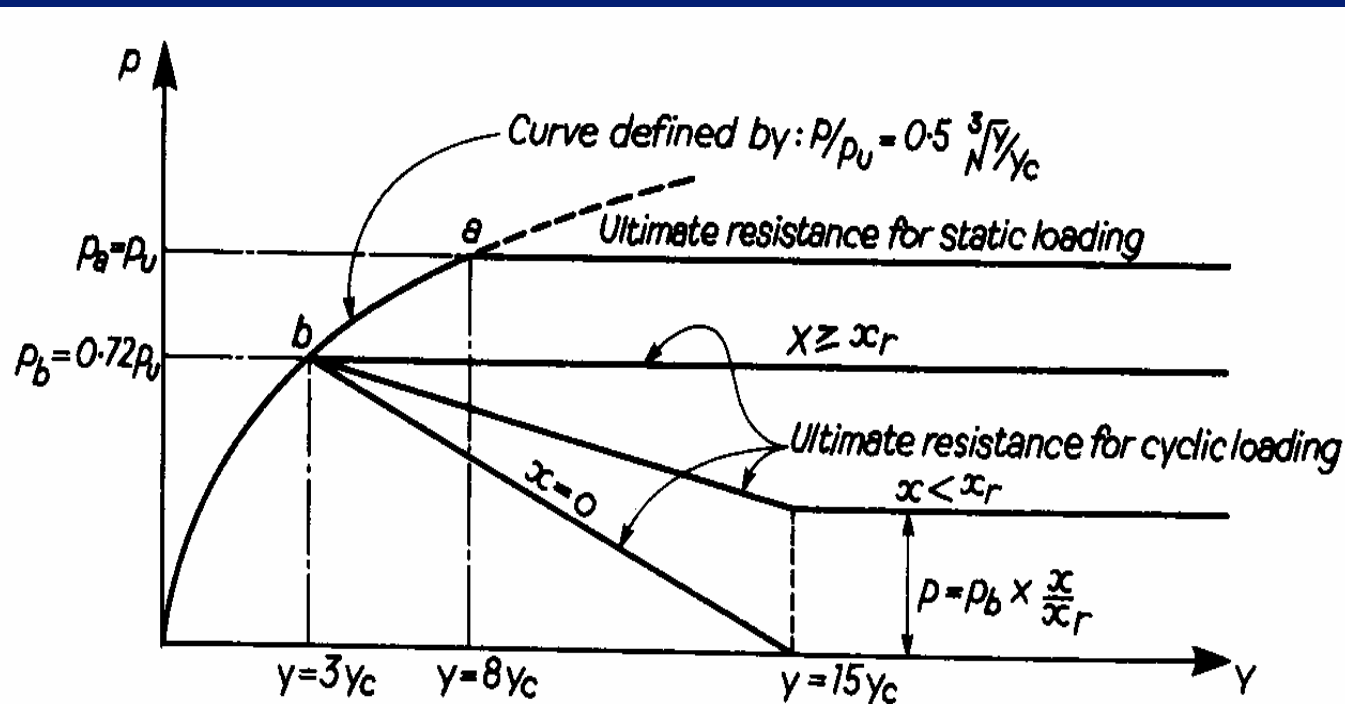


Fig. 6.42 Determining shape of p-y curve in soft to firm clay (after Matlock^(6.16))

$$x_r = 6d / (\gamma d / c_u + J)$$

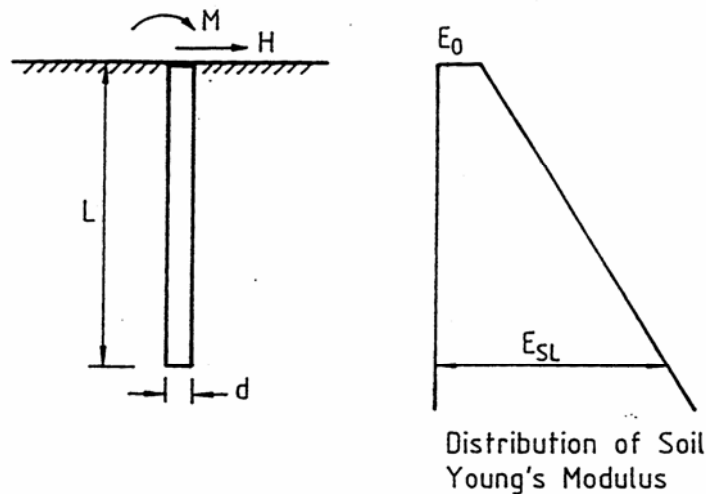
J=0.5 soft clay
=0.25 stiff clay

$$p_u = N_c \cdot c_u \cdot d$$

$$y_c = 2.5 \varepsilon_c \cdot d$$

$\varepsilon_c = 0.02$ soft clay;
0.0005 for
brittle & stiff clay

CONTINUUM SOIL MODEL

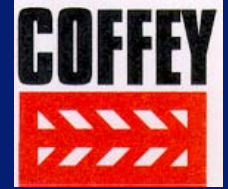


E_p = Pile modulus
 I_p = Pile moment of inertia
 E_s = Soil Young's modulus (uniform soil)
 ν_s = Soil Poisson's ratio
 G = Soil shear modulus
 k = Modulus of subgrade reaction
 N_h = Rate of increase of Young's modulus with depth
 n_h = Rate of increase of subgrade reaction modulus with depth

For linear theory:

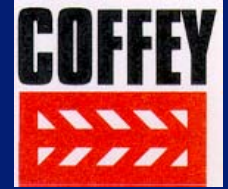
- Components due to shear & moment can be superposed
- Critical length exists (as for subgrade reaction theory)
- Need accurate assessment of soil parameters near surface

SOLUTIONS FROM CONTINUUM SOIL MODEL



- Soil characterized by:
 - Young's modulus E_s
 - Ultimate lateral pressure p_y
- Cases:
 - Uniform E_s & p_y
 - Linearly varying E_s & p_y - ($E_s = N_h.z$)
- Dimensionless Parameters:
 - Flexibility Factor
 - $K_R = E_p I_p / E_s L^4$
 - $K_N = E_p I_p / N_h L^4$
 - Critical Length
 - $L_c / L = 4.44 K_R^{1/4}$
 - $L_c / L = 3.30 K_N^{1/5}$

SOLUTIONS FROM CONTINUUM SOIL MODEL



1. Constant Modulus With Depth

$$\rho = \frac{H}{E_s L} I_{\rho H} + \frac{M}{E_s L^2} I_{\rho M}$$

$$\theta = \frac{H}{E_s L^2} I_{\theta H} + \frac{M}{E_s L^3} I_{\theta M}$$

$$\rho_F = \frac{H}{E_s L} I_{\rho F} \quad (\text{Fixed-head})$$

2. Linearly Increasing Modulus With Depth

$$\rho = \frac{H}{N_h L^2} I'_{\rho H} + \frac{M}{N_h L^3} I'_{\rho M}$$

$$\theta = \frac{H}{N_h L^3} I'_{\theta H} + \frac{M}{N_h L^4} I'_{\theta M}$$

$$\rho_F = \frac{H}{N_h L^2} I'_{\rho F} \quad (\text{Fixed-head})$$

SOLUTIONS FROM CONTINUUM SOIL MODEL

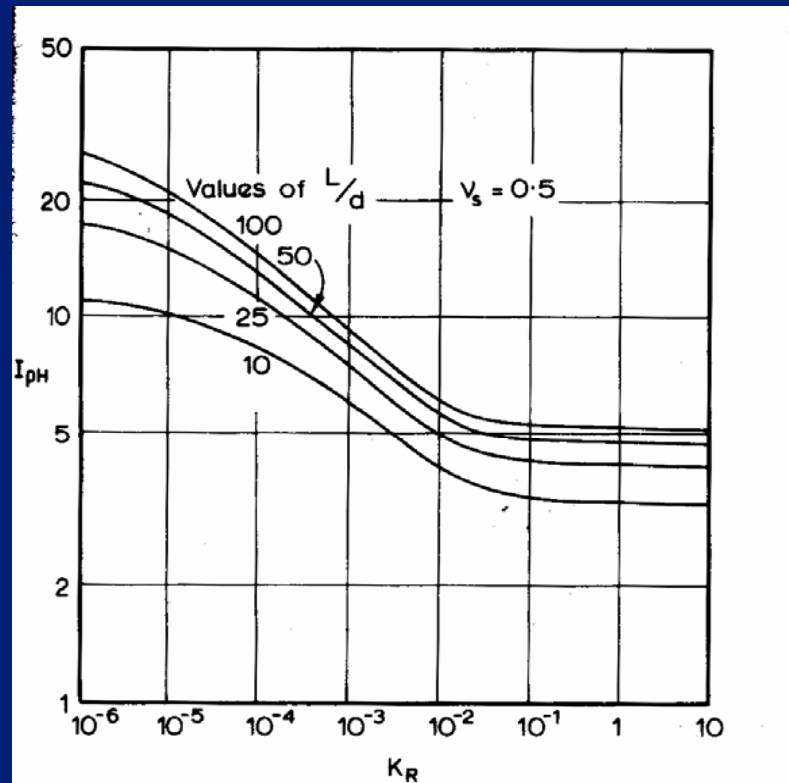
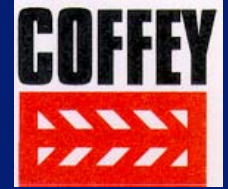


FIGURE 8.13 Values of $I_{\rho H}$ —free-head floating pile, constant soil modulus.

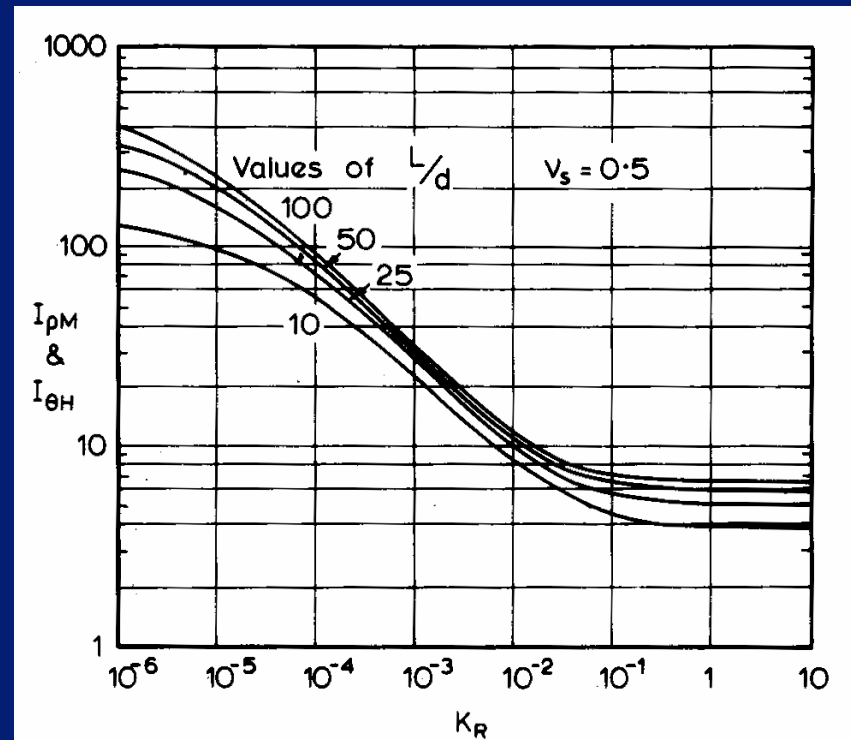


FIGURE 8.14 Values of $I_{\rho M}$ and $I_{\theta H}$ —free-head floating pile, constant soil modulus.

SOLUTIONS FROM CONTINUUM SOIL MODEL

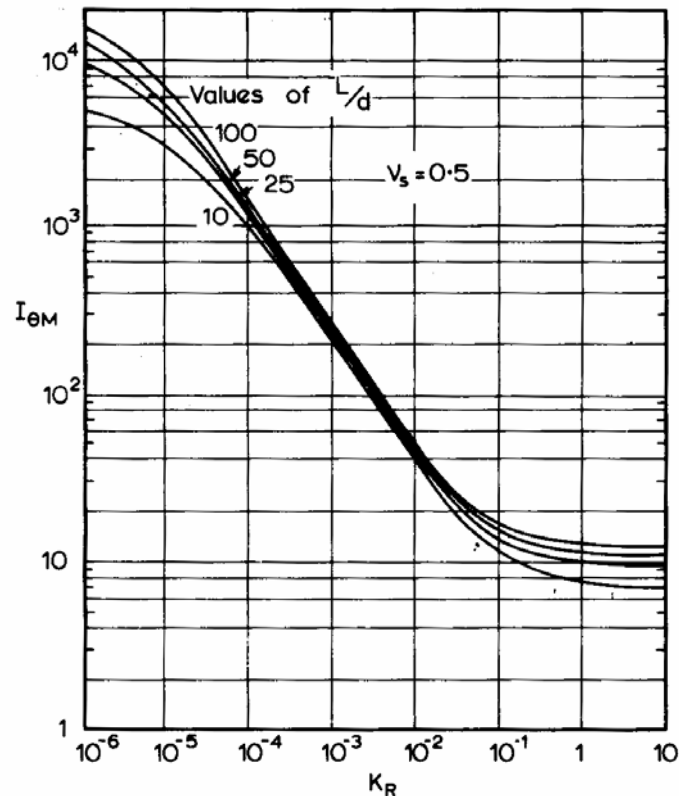
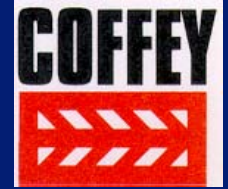


FIGURE 8.15 Values of $I_{\theta M}$ —free-head floating pile, constant soil modulus.

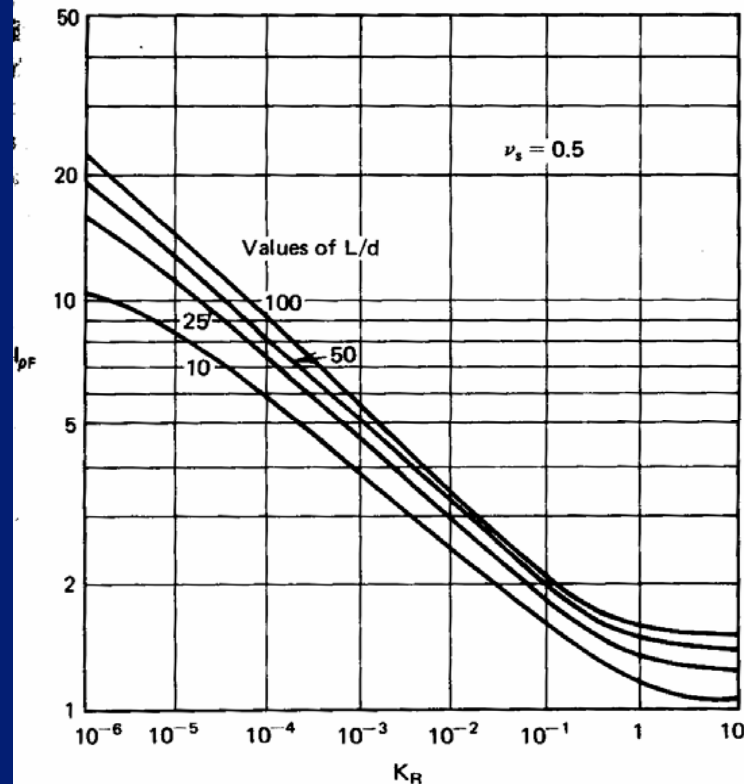


FIGURE 8.19 Influence factor $I_{\rho F}$ —fixed-head floating pile, constant soil modulus.

SOLUTIONS FROM CONTINUUM SOIL MODEL (Poulos & Hull, 1989)

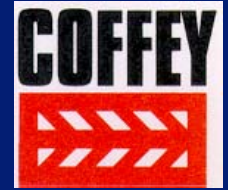


Table 3. Solutions For Lateral Pile Response (Linear Elastic Soil)

Pile Head Condition	Groundline Deflection	Groundline Rotation	Pile Head Fixing Moment
	u_e	θ_e	M_{Fe}
Free	$\frac{H}{E_e L_e} I_1$ $+ \frac{M}{E_e L_e^2} I_2$	$\frac{H}{E_e L_e^2} I_2$ $+ \frac{M}{E_e L_e^3} I_3$	-
Fixed	$\frac{H}{E_e L_e} I_4$	0	$- H L_e I_5$

H = applied horizontal load at groundline

M = applied moment at groundline

E_e = soil Young's modulus at depth equal to effective length L_e of pile

L_e = effective length of pile

= critical length L_c if $L > L_c$ (flexible pile)

= or actual length L if $L < L_c/3$ (rigid pile)

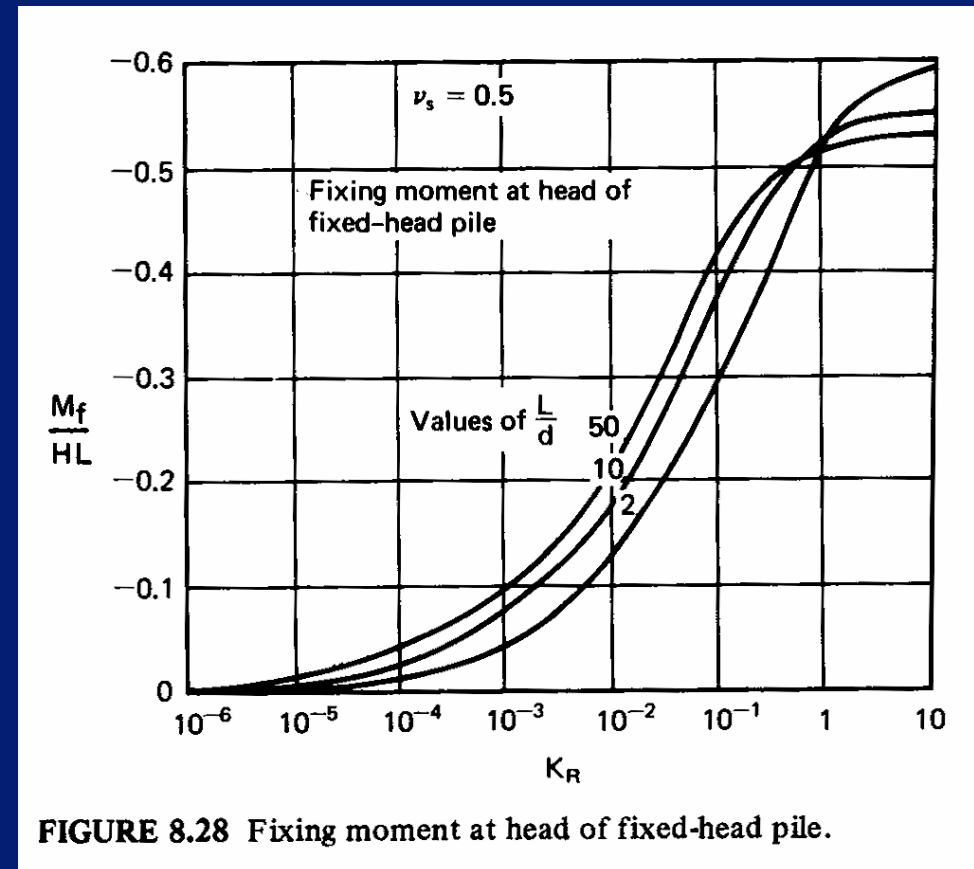
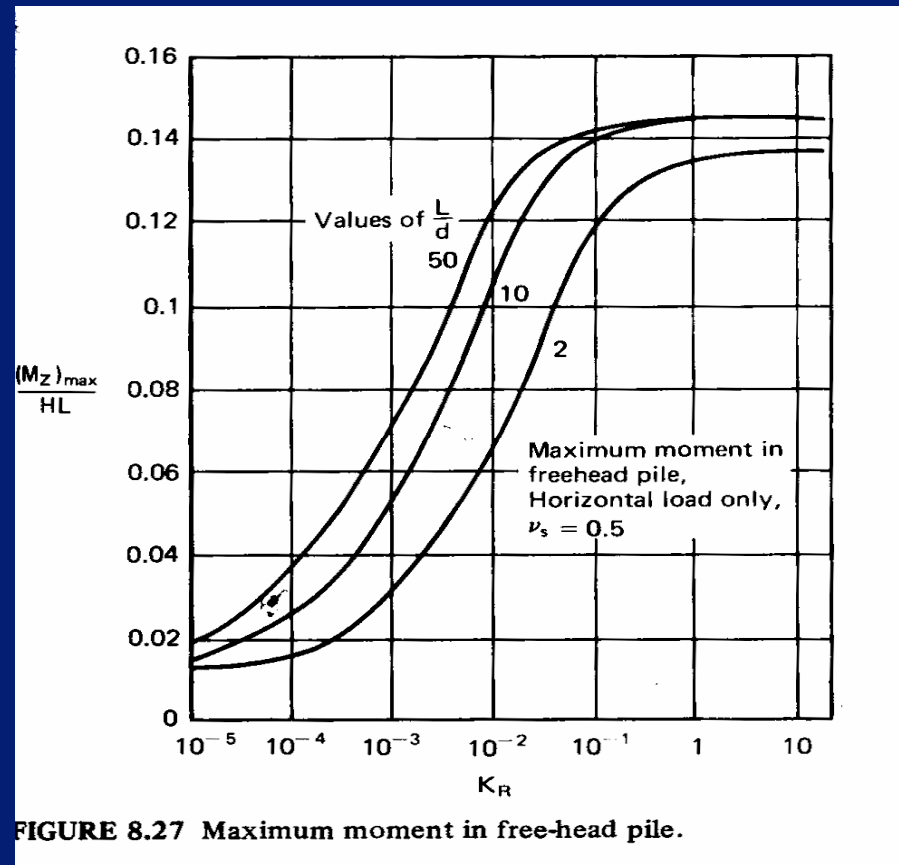
I_1 - I_5 = influence factors depending on L_e/d (where d = pile diameter or breadth) (see Table 4)

Table 4. Solutions For Pile Head Displacement and Rotation – Linear Elastic Response

Case	Factor	Uniform Soil		Gibson Soil	
		A	B	A	B
Flexible Piles ($L > L_c$)	I_1	1.646	3.395	13.10	11.09
	I_2	5.520	9.082	34.63	18.03
	I_3	64.98	37.95	156.1	37.14
	I_4	1.326	1.641	5.659	4.139
	I_5	0.09764	0.04245	0.2278	0.04402
Rigid Piles ($L < L_c/3$)	I_1	0.976	2.196	3.181	9.701
	I_2	0.701	3.225	2.409	12.71
	I_3	1.086	6.292	1.844	18.65
	I_4	0.539	0.545	0.773	1.081
	I_5	0.547	-0.0140	0.764	-0.347

For each factor, $I = A + B \log_{10}(L_e/d)$
Solutions are for soil Poisson's ratio $\nu_s = 0.5$.

MOMENTS INDUCED IN PILE



COMPARISONS BETWEEN THEORIES (Linear)

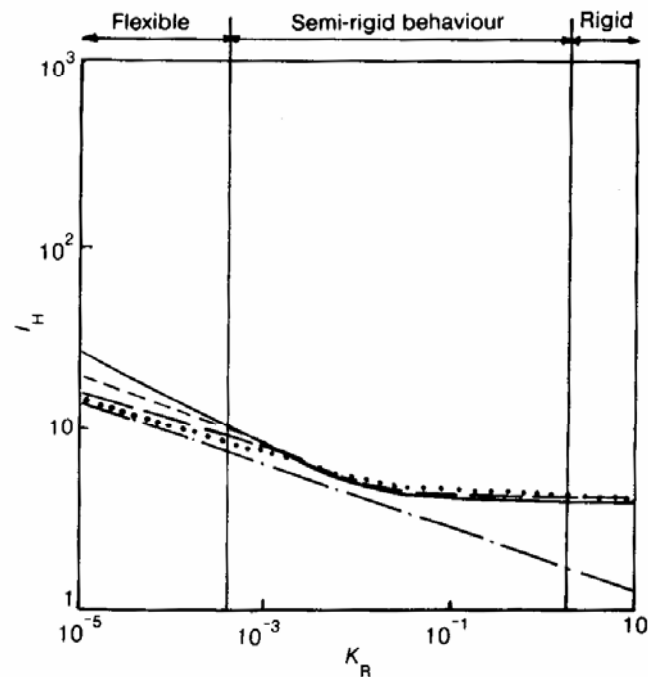
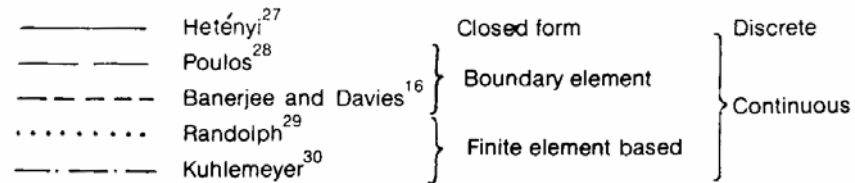
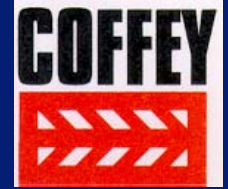


Fig. 17. Constant modulus: influence coefficients for horizontal load, I_H

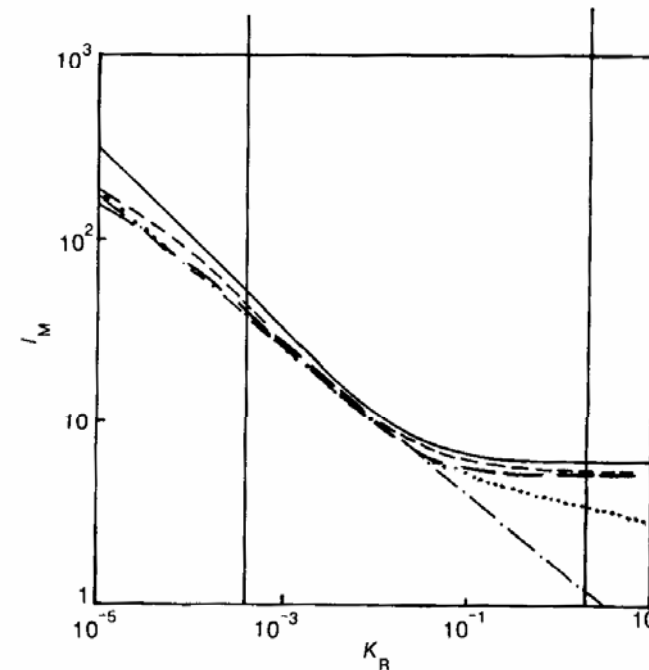
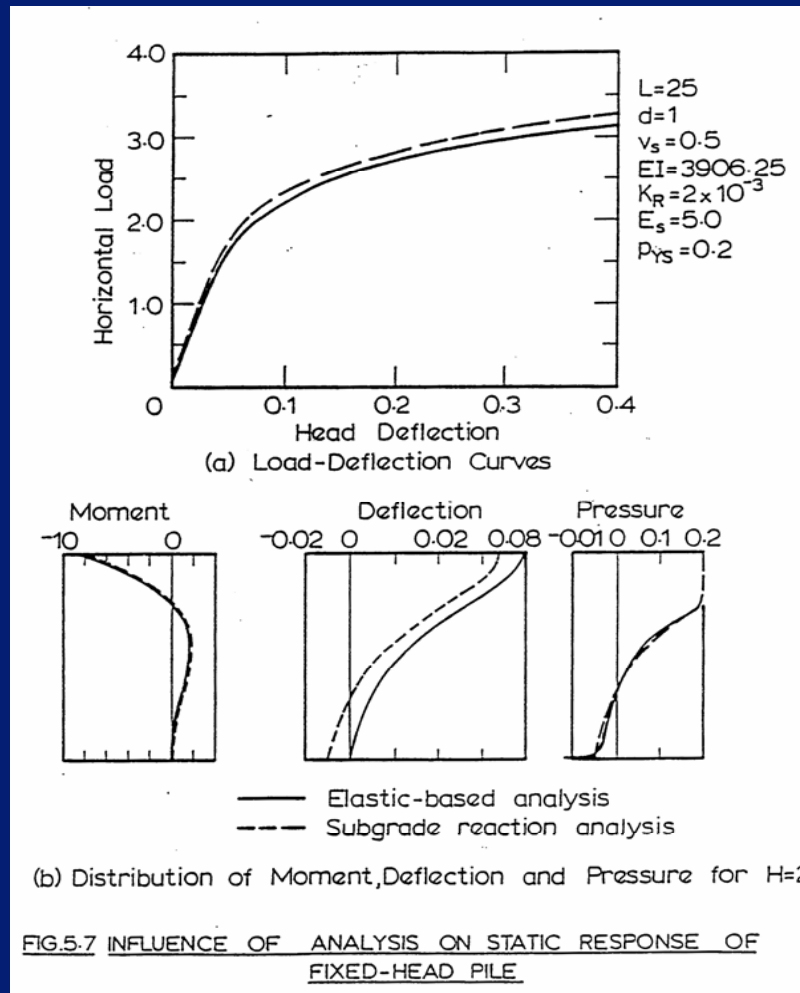
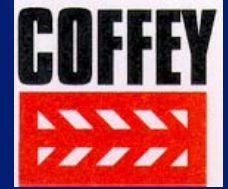


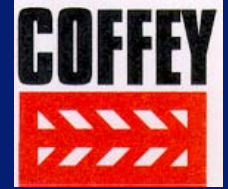
Fig. 18. Constant modulus: influence coefficients for moment, I_M

COMPARISONS BETWEEN THEORIES (Non-Linear)



- Subgrade reaction & elastic continuum analyses are quite similar
- Elastic solution shows general additional movement due to pile-soil interaction through the soil
- Bending moments are very close, as are pressures

MODIFICATIONS FOR NON-LINEAR SOIL-PILE BEHAVIOUR



Can allow for non-linear lateral response by modifying elastic solutions via yield factors obtained from non-linear analysis.

Lateral Displacement:

$$\rho = \rho_e / F_u$$

Fixing Moment at Pile Head:

$$M_f = M_{fe} / F_m$$

ρ_e , M_{fe} are calculated from elastic theory.

The yield correction factors F_u , F_m depend on:

- Load level
- Relative flexibility of pile.

Non-linearity can be significant for:

- Piles in stiff clay
- Relatively flexible piles.

Solutions available for:

- Uniform E_s & p_y (except near surface) - relevant to stiff clay
- Linearly varying E_s & p_y ("Gibson soil") - relevant to soft clay

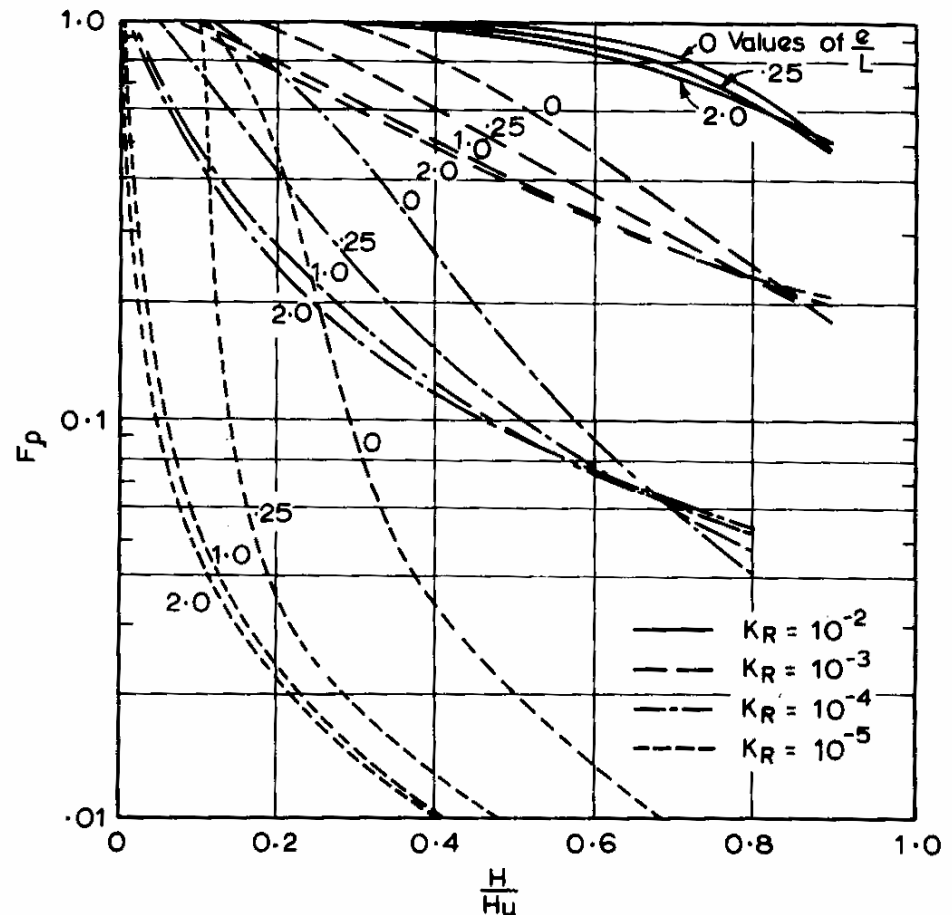
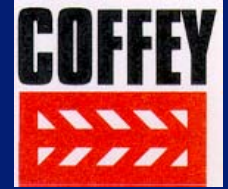


FIGURE 8.16 Yield-displacement factor F_p —free-head floating pile, uniform E_s and p_y .

MODIFICATIONS FOR NON-LINEAR SOIL-PILE BEHAVIOUR



Fixed-head
Piles (Poulos &
Hull, 1989)

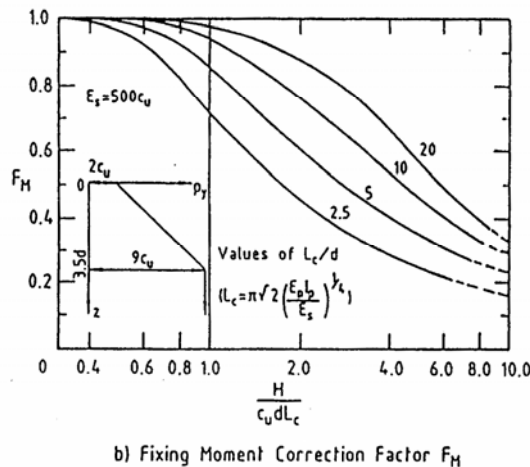
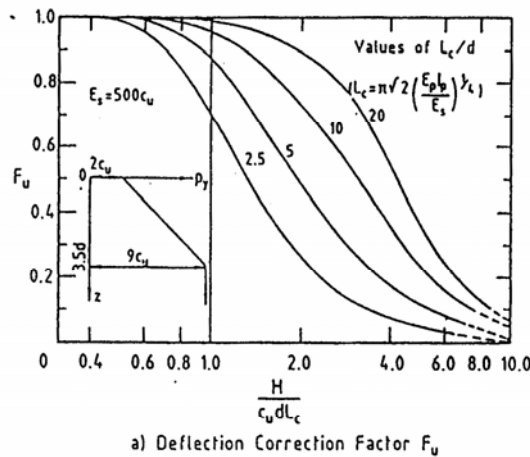


Figure 2. Nonlinear Correction Factors for a Flexible Fixed-Head Pile in Stiff Clay

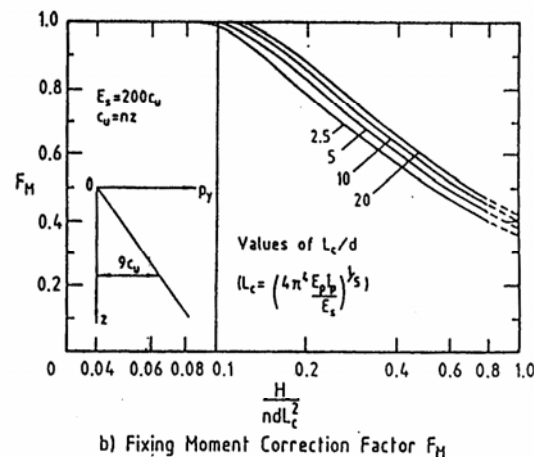
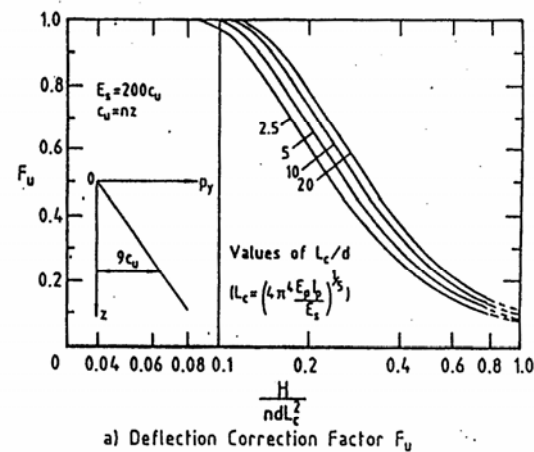


Figure 3. Nonlinear Correction Factors for a Flexible Fixed-Head Pile in Soft Clay

COMPARISONS BETWEEN MEASURED AND CALCULATED BEHAVIOUR

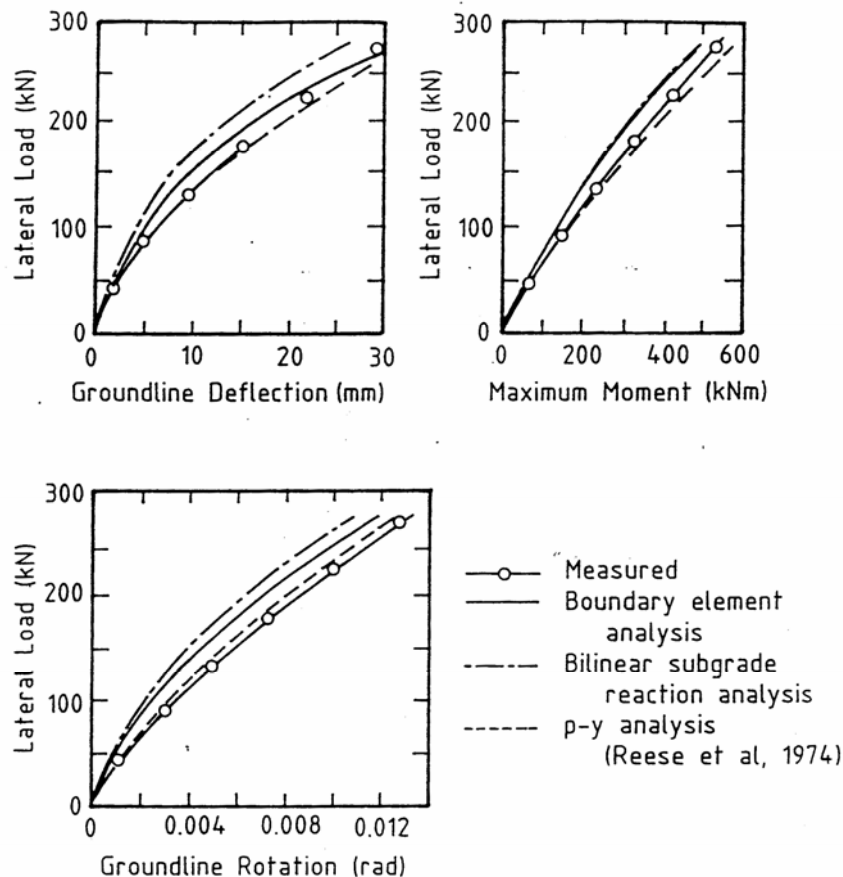
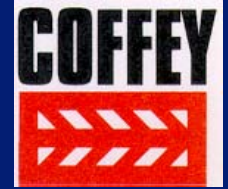
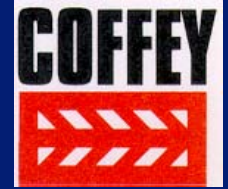


FIG.5 MUSTANG ISLAND TESTS. COMPARISONS BETWEEN MEASURED AND CALCULATED BEHAVIOUR

- All methods show acceptable agreement
- p-y analysis agrees closely because it was used in calibration of method
- Fair results with elastic –plastic subgrade reaction model

ESTIMATION OF SOIL PARAMETERS (MODULUS)



1) INTERPRETATION OF PILE LOAD TESTS

Fit observed deflections to theory

2) INSITU TESTS

eg pressuremeter
plate load tests (lateral)

3) EMPIRICAL CORRELATIONS

a) Clays

$$E_s / c_u = 300 \pm 100$$

$$p_y = 9 c_u$$

b) Sands

$$E_s = 1.6 N \text{ MPa (Kishida & Nakai)}$$

$$N_h = 0.19 D_r^{1.16} \text{ MPa / m}$$

(tangent value , Reese et al, 1974)

CORRELATIONS FOR MODULUS IN CLAYS

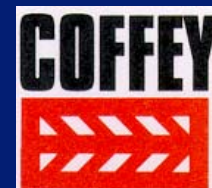


Table 7.12 Empirical correlations for Young's modulus in clays, for laterally loaded piles.

Relationship	Theory	Reference	Remarks
$E_{si}/c_u = 300-600$	non-linear subgrade reaction	Jamiolkowski & Garassino (1977)	initial tangent modulus for driven piles in soft clays
$E_{si}/c_u = 180-450$	non-linear boundary element	Poulos (1973)	tangent modulus from model tests on jacked piles
$E_{si}/c_u = 280-400$	non-linear subgrade reaction	Kishida & Nakai (1977)	tangent modulus
$E_s/c_u = 100-180$	linear boundary element	Banerjee (1978)	secant value
c_u (kPa) N_{hi} (MPa m ⁻¹)	non-linear subgrade reaction	Sullivan <i>et al.</i> (1979)	tangent values of rate of modulus increase:
12-25 8			
25-50 27			
50-100 80			
100-200 270			
200-400 800			

$$E_{si} = N_{hi} z$$

CORRELATIONS FOR MODULUS IN SANDS

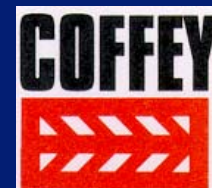


Table 7.13 Empirical correlations for Young's modulus in sands, for laterally loaded piles.

Relationship	Theory	Reference	Remarks	
$N_{hi} = 0.19D_R^{1.16}$ MPa m ⁻¹ where D_R = relative density (%)	non-linear subgrade reaction	Jamiolkowski & Garassino (1977)	tangent value for driven piles in saturated sands	
Condition	N_{hi} MP a/m			
Loose	5.4	non-linear subgrade reaction	Reese <i>et al.</i> (1974)	tangent value for driven piles in submerged sands
Medium	16.3			
Dense	34.0			
$N_h = 8\text{--}19$ MPa m ⁻¹ (av. 10.9)	linear boundary element	Banerjee (1978)	secant value	
$E_{si} = 1.6N$ MPa where N = SPT value	non-linear subgrade reaction	Kishida & Nakai (1977)	tangent value	

CYCLIC LOADING

MAIN EFFECTS

- Cyclic deflection increases
- Sustainable cyclic load decreases
- Degradation effects more severe for stiffer soils
- Failure can be quite abrupt for small increases in cyclic load level

ANALYTICAL PROCEDURES

1. p-y analyses with modified curves
2. Matlock et al degradation model
 - Degradation occurs when have reversal of plastic strain
3. Modified boundary element analysis: soil modulus & p_y degrade, depending on cyclic deflection & no. of cycles

CYCLIC LOADING APPROACHES

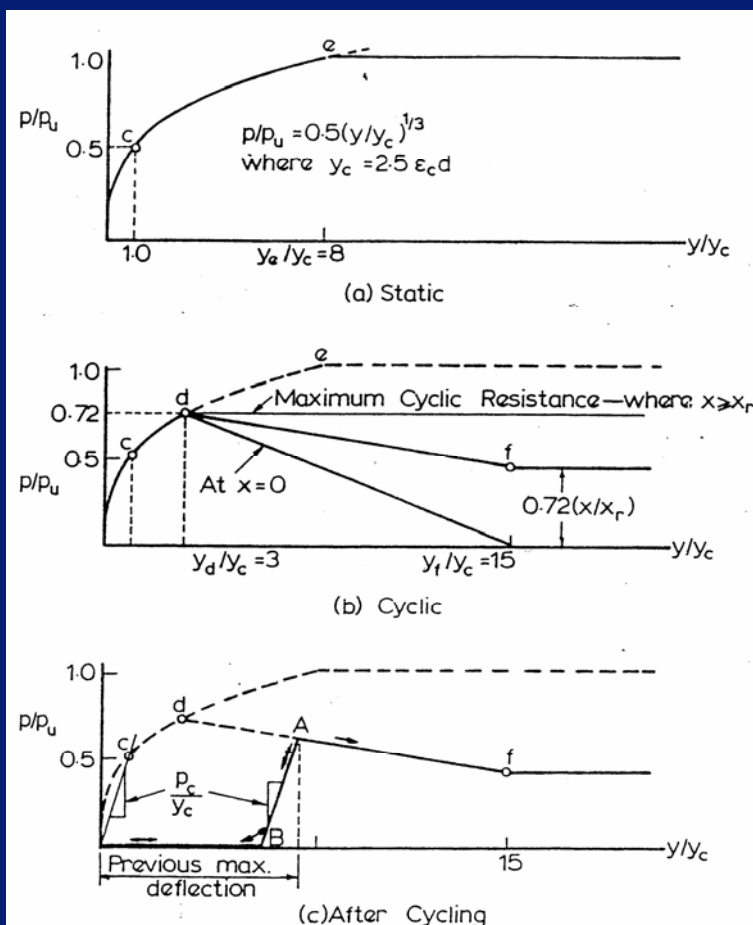


FIG.12 CRITERIA FOR PREDICTING p - y CURVES FOR (a) SHORT-TIME STATIC LOADING, (b) EQUILIBRIUM UNDER INITIAL CYCLIC LOADING AND (c) RELOADING AFTER CYCLING
(Matlock, 1970)

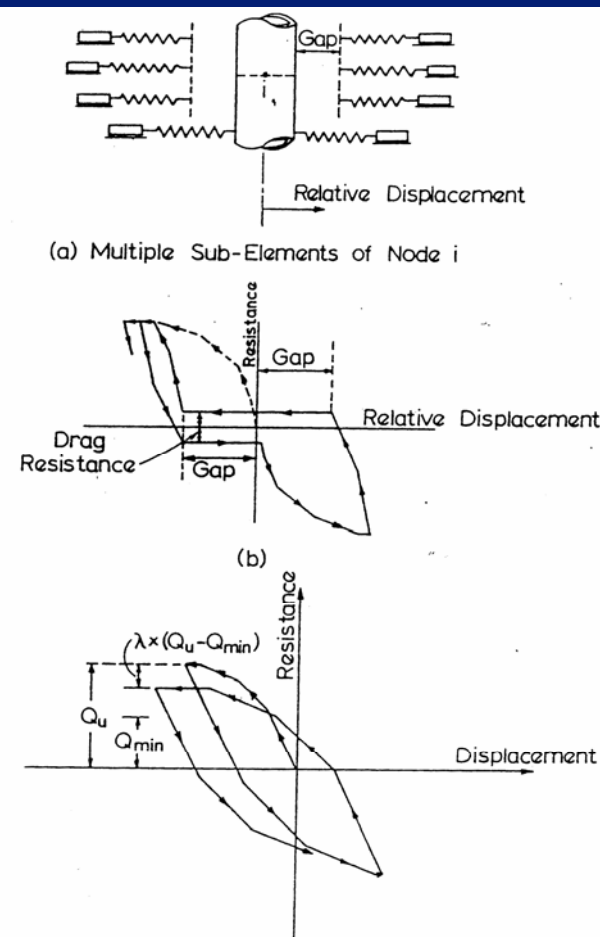
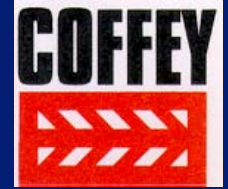


FIG.13 MODEL DEVELOPED BY MATLOCK ET AL (1978)

CYCLIC LOADING APPROACH VIA CONTINUUM ANALYSIS



1. Compute static lateral deflection for maximum lateral loading, ρ_{cl}
2. Estimate:
 - Critical strain ϵ_{cr}
 - Ultimate lateral load capacity under static loading (H_{UR})
 - Number of cycles N
3. Look up curves to obtain ratio of ρ_c / ρ_{cl} , where ρ_c = maximum deflection under cyclic loading, and ρ_{cl} = static lateral deflection
4. Compute $\rho_c = \rho_{cl} \cdot \rho_c / \rho_{cl}$

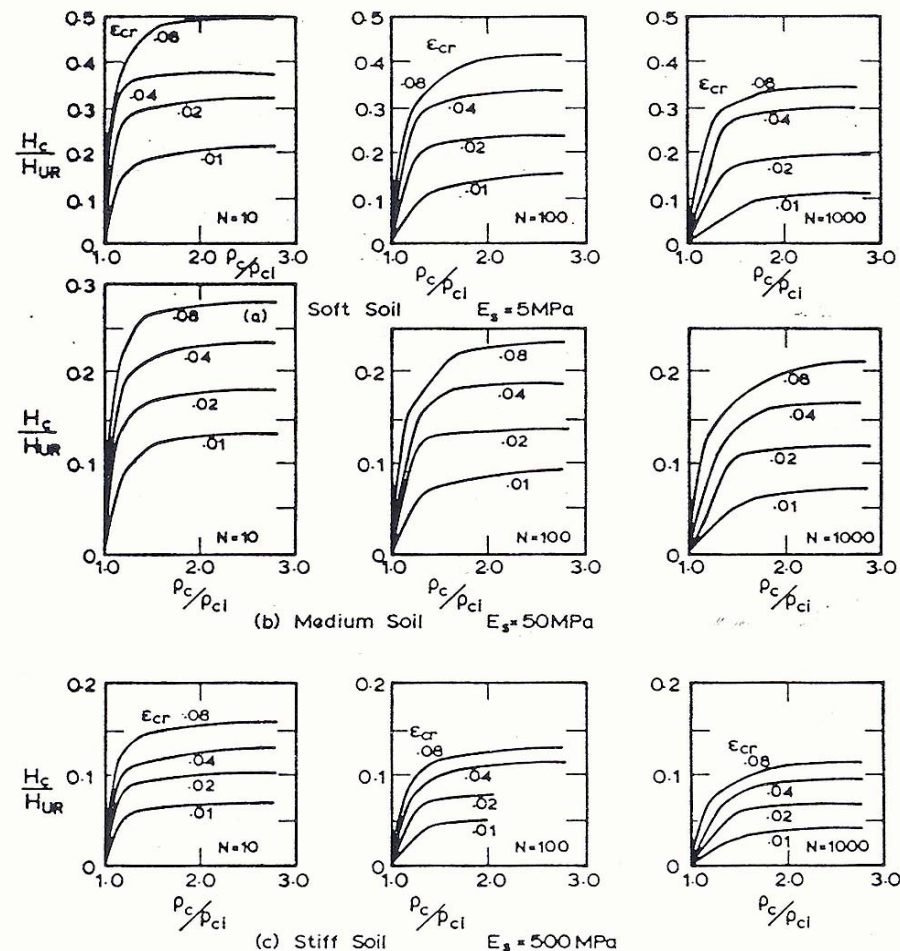
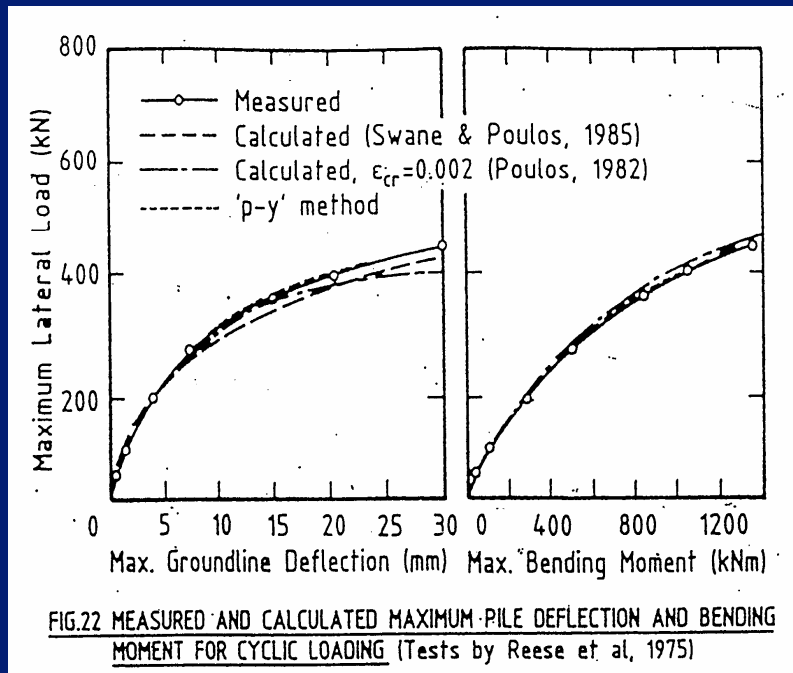
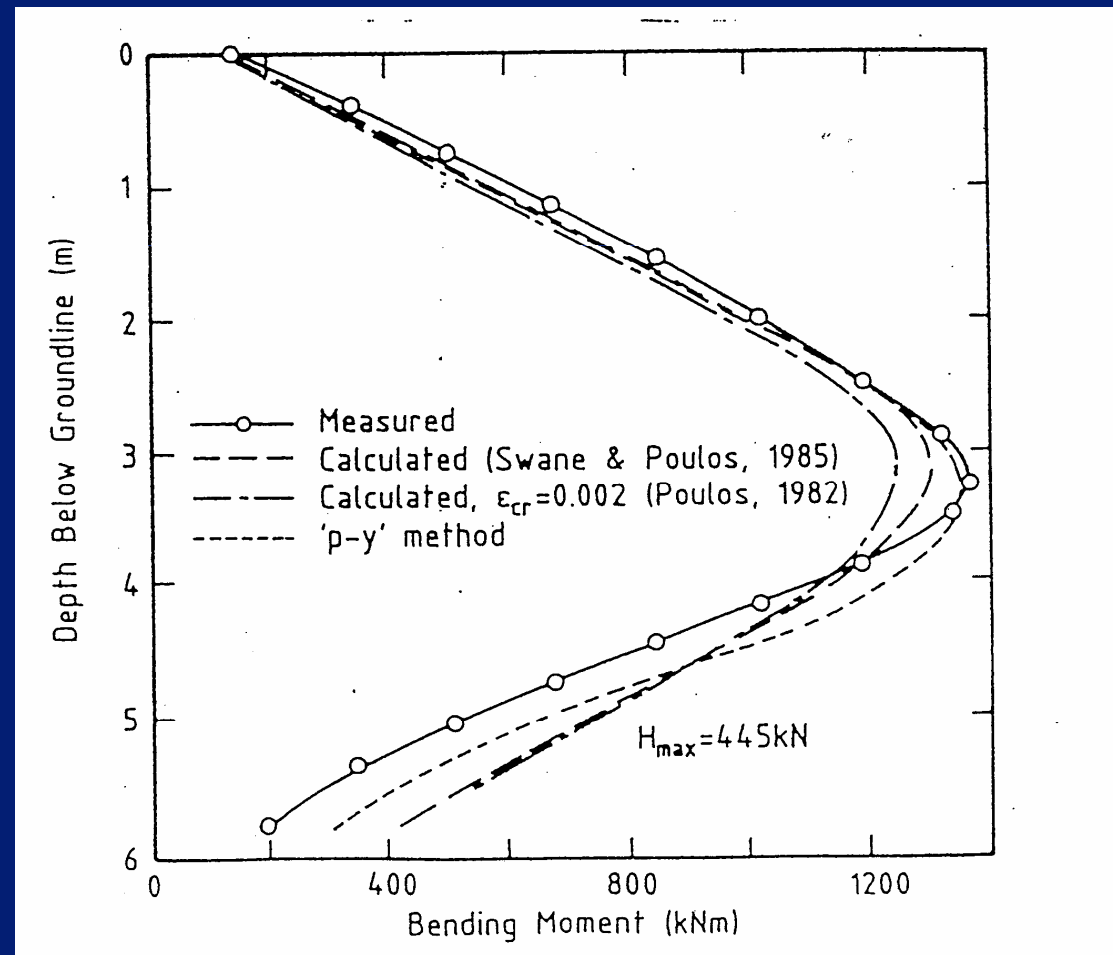


FIG.9 NORMALISED CYCLIC LOAD-DEFLECTION CURVES.
HOMOGENEOUS SOIL.

MEASURED vs CALCULATED BEHAVIOUR

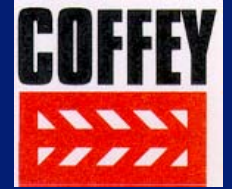


Load-deflection &
Load-maximum moment



Bending moment distribution along pile

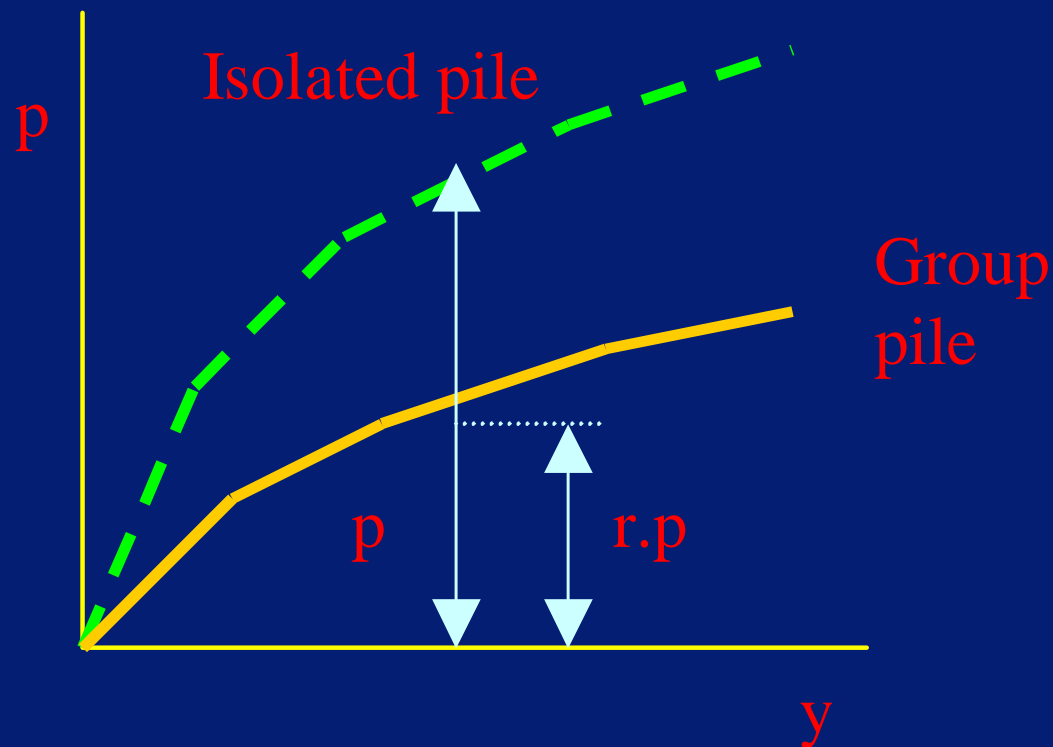
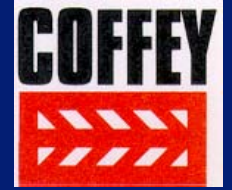
ALLOWANCE FOR GROUP EFFECTS



- US practice employs the concept of “p-multipliers”.
- These scale down the p-y curves for a single pile to allow for pile-soil-pile interaction in piles within a group.
- Typical example shown in table, from program FLPIER.

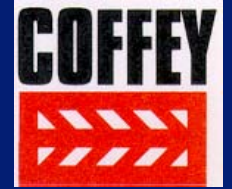
<i>Row</i>	<i>p-multiplier</i>
Lead	0.8
1 st trail	0.4
2 nd trail	0.2
3 rd trail	0.3

ALLOWANCE FOR GROUP EFFECTS – “p-Multiplier”



Apply multiplier (≤ 1) to p-y curve for each pile in group. Multiplier derived empirically in most cases.

LATERAL INTERACTION BETWEEN PILES



- Can adopt the interaction factor approach, as for settlement of groups.
- BUT, have some additional problems:
 - Have 5 interaction factors:
 - $\alpha_{\rho H}, \alpha_{\rho M}, \alpha_{\theta H}, \alpha_{\theta M}, \alpha_{\rho F}$ (but $\alpha_{\rho M} = \alpha_{\theta H}$ from reciprocal theorem)
 - Interaction factors depend both on spacing (s/d) and “departure angle” between direction of loading & piles
- Values plotted in Poulos & Davis (1980).
- Useful approximations by Randolph (1981).

TYPICAL LATERAL INTERACTION FACTOR DIAGRAM

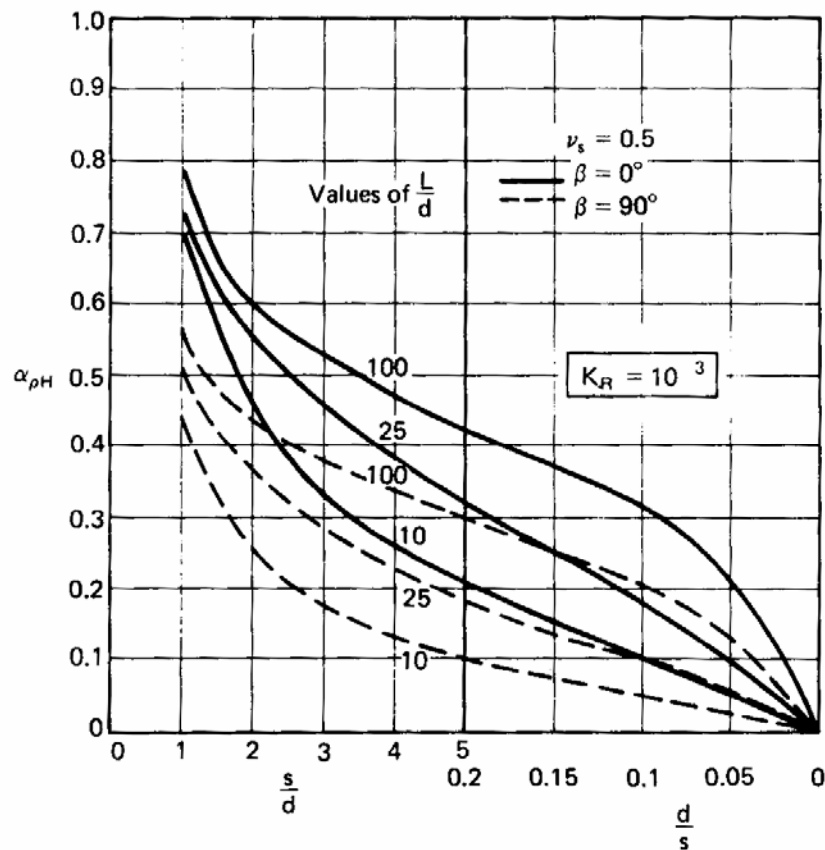
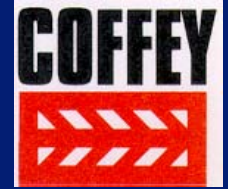
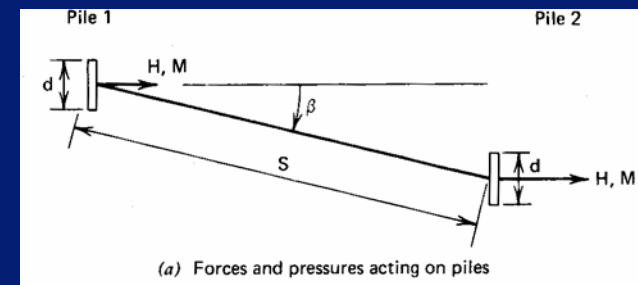
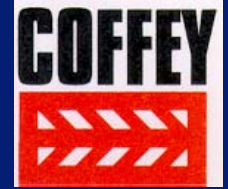


FIGURE 8.63 $\alpha_{\rho H}$ for $K_R = 10^{-3}$.



APPROXIMATIONS FOR LATERAL INTERACTION FACTORS



APPROXIMATIONS FOR LATERAL INTERACTION FACTORS

(Randolph, Geotechnique, June 1981)

1 Non-Homogeneity Factor

$$\rho_c = \frac{G_{lc/4}^*}{G_{lc/2}^*}$$

2 Critical length

$$l_c = 2r_o \left(\frac{E_p}{m^* r_o} \right)^{2/9}$$

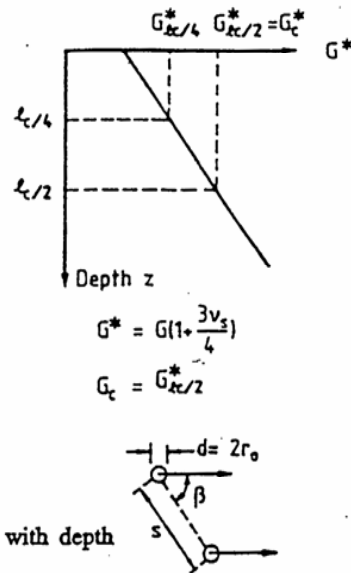
$$\text{where } m^* = m \left(1 + \frac{3\nu_s}{4} \right)$$

$$m = dG/dz$$

= rate of increase of shear modulus with depth

r_o = pile radius

E_p = pile modulus



3 Interaction Factors

$$\alpha_{\rho F} = 0.6 \rho_c \left(\frac{E_p}{G_c} \right)^{1/7} \frac{r_o}{s} (1 + \cos^2 \beta)$$

i) If $\alpha_{\rho F} > 0.5$ from this expression

$$\text{use } \alpha_{\rho F} = 1 - (4\alpha_{\rho F})^{-1}$$

$$\alpha_{\rho H} = 0.5 \rho_c \left(\frac{E_p}{G_c} \right)^{1/7} \frac{r_o}{s} (1 + \cos^2 \beta)$$

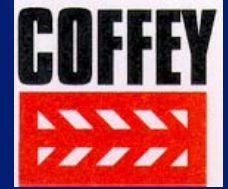
ii) If $\alpha_{\rho H} > 0.5$ from this expression

$$\text{use } \alpha_{\rho H} = 1 - (4\alpha_{\rho H})^{-1}$$

$$\text{iii) } \alpha_{\rho M} = \alpha_{\theta H} = \alpha_{\rho H}^2$$

$$\text{iv) } \alpha_{\theta m} = \alpha_{\rho H}^3$$

EQUATIONS FOR Laterally LOADED GROUP



1. Lateral Deflections

$$\{ \rho \} = [A_{\rho H}] \{ H \} + [A_{\rho M}] \{ M \}$$

2. Rotations

$$\{ \theta \} = [A_{\theta H}] \{ H \} + [A_{\theta M}] \{ M \}$$

3. Equilibrium

$$H_G = \sum H_i$$
$$M_G = \sum M_i$$

4. Boundary Conditions

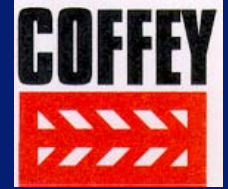
For rigid cap:

$$\{ \rho \} = \rho_G \{ 1 \}$$

$$\{ \theta \} = \theta_G \{ 1 \}$$

Note: These equations hold only for pinned - head piles or fixed - head piles, where lateral response can be de-coupled from the axial response.

SIMPLIFIED APPROACH FOR ESTIMATING LATERAL GROUP DEFLECTION



FIXED HEAD GROUP

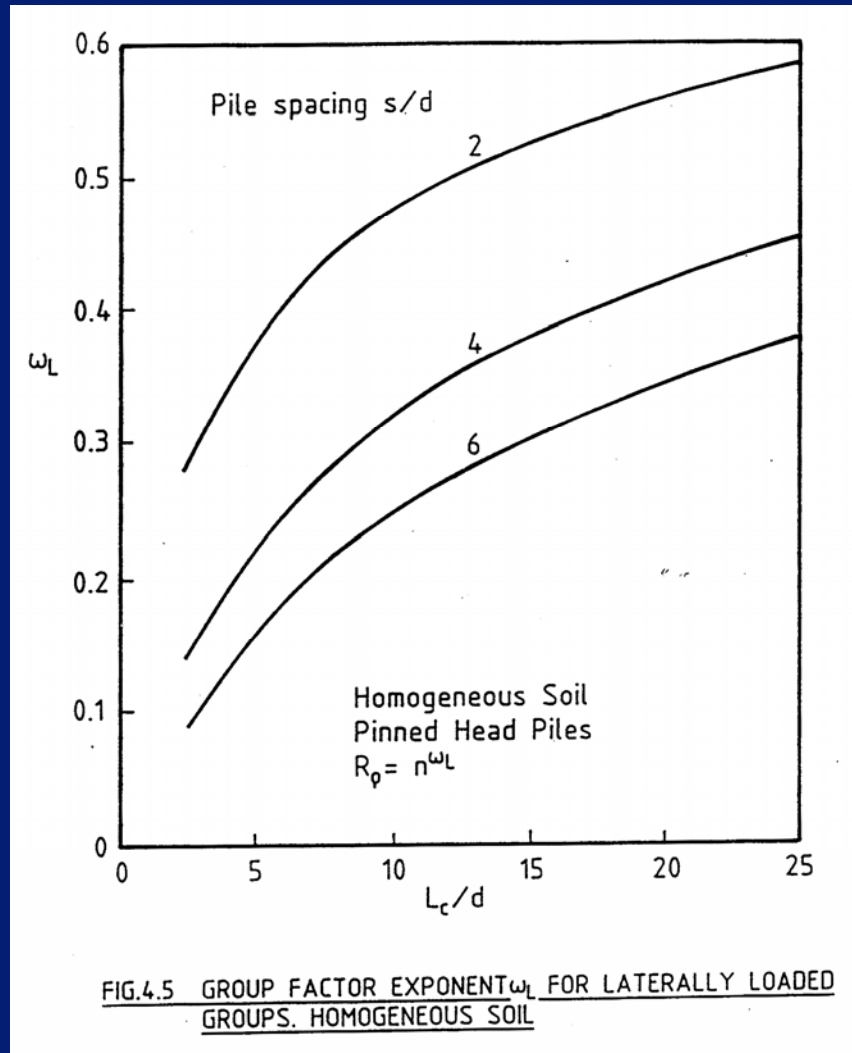
$$\rho_{hF} = \rho_{F1} \cdot H_{av} \cdot R_{\rho F}$$

where

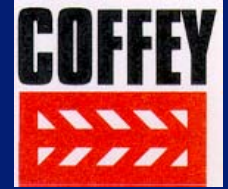
ρ_{hF} = single pile deflection / unit load

H_{av} = average lateral load

$R_{\rho F}$ = group deflection factor
 $\approx n^{\omega_L}$



SOME PRACTICAL DIFFICULTIES WITH THEORETICAL LATERAL INTERACTION FACTORS



- Elastic theory suggests that interaction between 2 piles is the same, for the same spacing & orientation
- Experiments indicate this is NOT SO.
- Interaction is smaller for $\zeta = 0^\circ$ than for $\zeta = 180^\circ$.

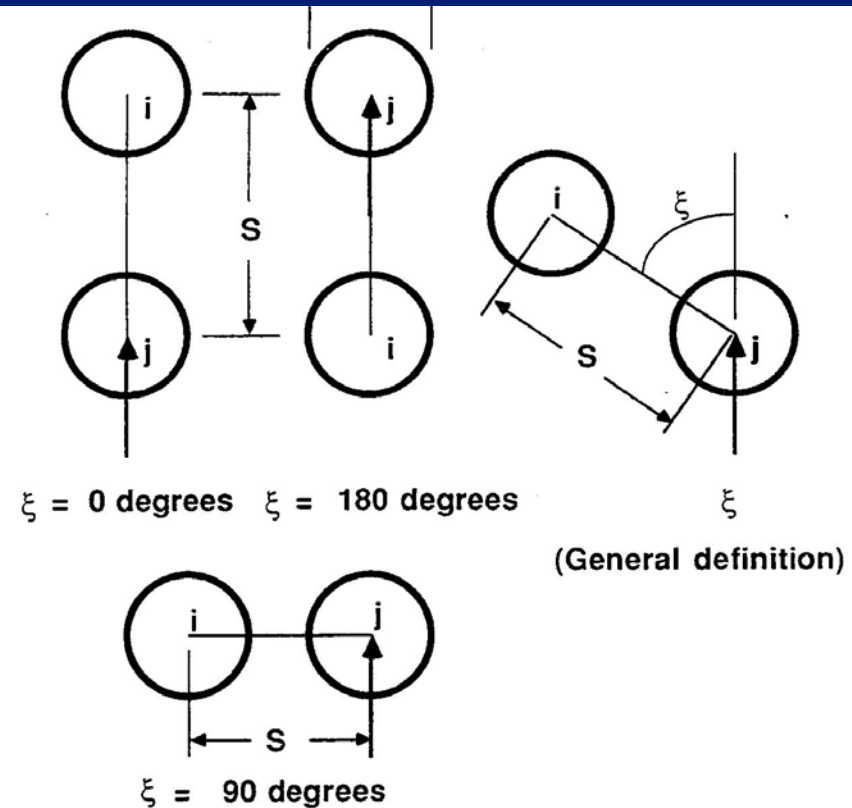
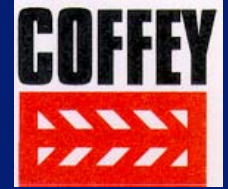


FIG. 7. Definitions of ξ , S , and D

SOME PRACTICAL DIFFICULTIES WITH THEORETICAL LATERAL INTERACTION FACTORS



- Also, field tests indicate that interaction factors increase with increasing load level, but decrease with increasing number of cycles

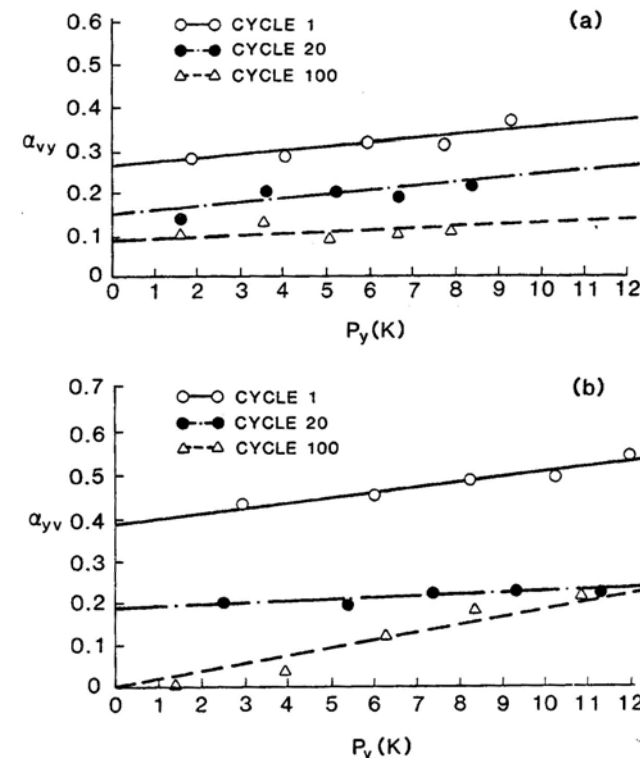
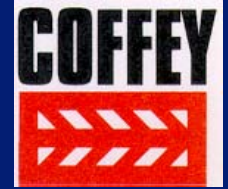
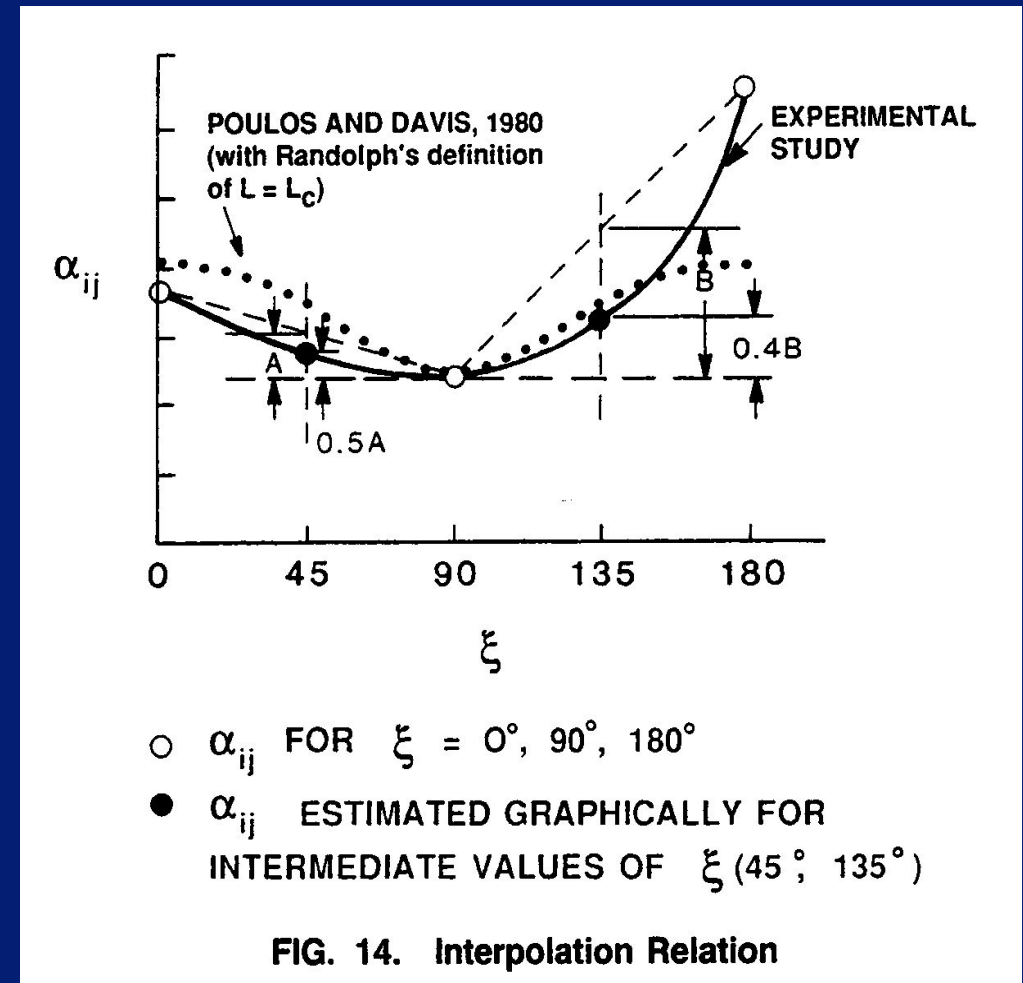


FIG. 8. Interaction Factors for Two-Pile Test on Piles V and Y: (a) $\xi = 0^\circ$, $S/D = 3$, Loading North; (b) $\xi = 0^\circ$, $S/D = 3$, Loading South (1 K = 4.45 kN)

SOME PRACTICAL DIFFICULTIES WITH THEORETICAL LATERAL INTERACTION FACTORS

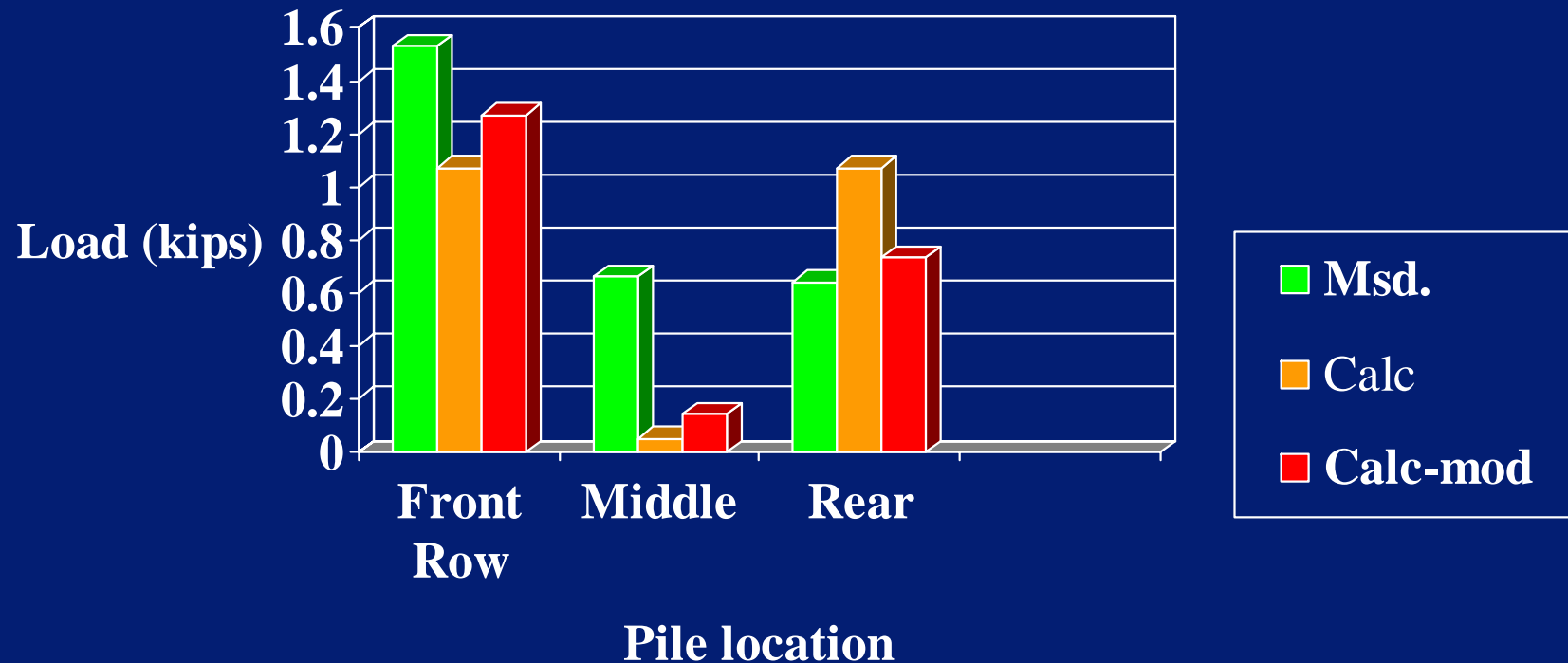


- The conventional interaction factors can be adjusted to produce more realistic group behaviour.
- Scaling process described by Ochoa and O'Neill (1989).
- Factor α_{ij} for $\zeta = 180^\circ$ should only be applied when pile j is in leading row of group.
- Otherwise, use factor for $\zeta = 0^\circ$ when $\zeta = 180^\circ$.



MEASURED & COMPUTED PILE LOADS IN 9-PILE GROUP (Ochoa & O'Neill, 1989)

Measured & Calculated Pile Loads



SUMMARY

- In reality, the leading piles of a laterally loaded group tend to be stiffer and take a larger proportion of the lateral load than the inner or rear piles.
- This effect can be reproduced by the theoretical analysis if the adjustments are made to the interaction factors, as proposed by Ochoa & O'Neill (1989).