LECTURE 2

ANALYSIS OF SETTLEMENT OF PILES & PILE GROUPS

OUTLINE

- Analysis of pile-soil interaction
- Settlement of single piles
- Estimation of parameters
- Settlement of pile groups
  - Interaction factor methods
  - Settlement ratio method
  - Equivalent raft method
  - Equivalent pier method
- Applications

ANALYSIS OF PILE-SOIL INTERACTION

- Load-transfer ("t-z") methods
- Simplified analytical solutions (Randolph)
- Boundary element methods
- Finite element methods

For given set of data, these methods give similar results.

Attention here is focused on solutions from boundary element method, using elastic continuum theory to characterize soil behaviour.

Allowances can be made readily for departures from elastic behaviour.

ADVANTAGES OF ELASTIC-BASED ANALYSES

- Continuous soil model; allows stress transmission
- Consistent model – parameters understood
- Can analyze group behaviour
- Can modify to allow for non-linear and cyclic loading effects.

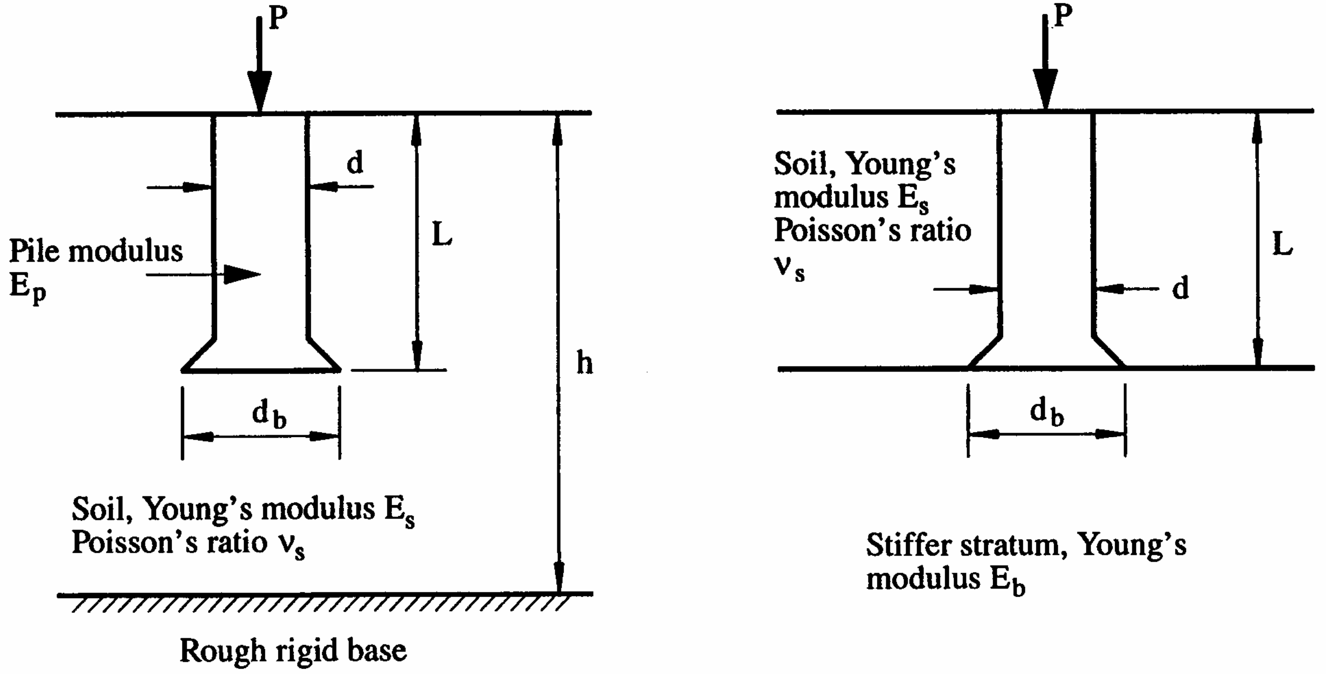
Can use:

- Parametric solutions (Poulos & Davis, Randolph & Wroth, Butterfield & Banerjee)
- Computer programs for problems involving layered soils or non-uniform pile

WHEN MAY A COMPUTER ANALYSIS BE NECESSARY?

- When problem falls outside range of available parametric solutions
- When detailed information on load transfer is desired
- When soil profile is layered
- When pile section is non-uniform
- When require load-settlement curve to failure
- For pile groups, when load and settlement distributions are required
- For examination of mechanisms of deformation

THE BASIC PROBLEMS

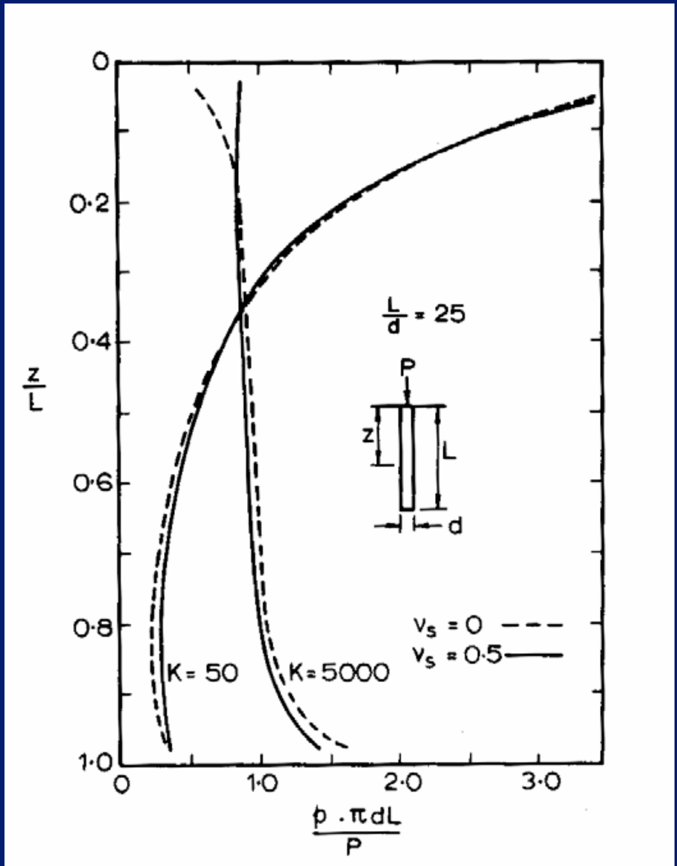


Pile stiffness factor $K = \frac{E_p}{E_s} R_A$

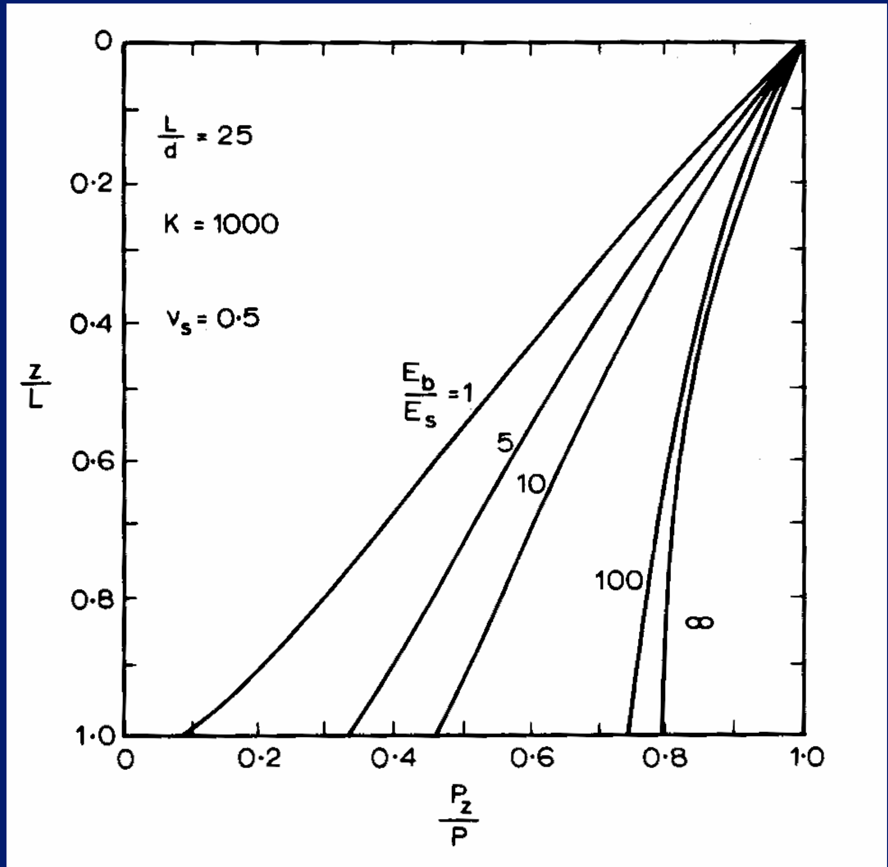
(a) Floating or Friction Pile

(b) End-Bearing Pile

LOAD TRANSFER CHARACTERISTICS



Stresses along friction piles effect of compressibility



Load distribution along pile- effect of modulus of bearing stratum

PROPORTION OF BASE LOAD ON PILE

$\beta = \beta_0 . C_K . C_\nu$

(friction pile)

$\beta = \beta_0 . C_K . C_\nu . C_b$

(end bearing pile)

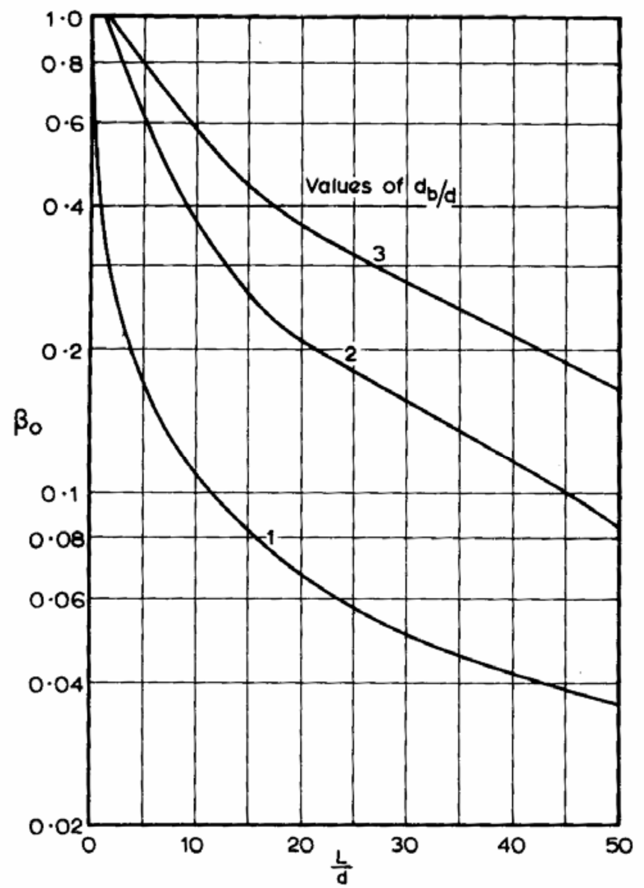


FIGURE 5.11 Proportion of base load, β_o.

PROPORTION OF BASE LOAD ON PILE – FACTORS C_K, C_ν

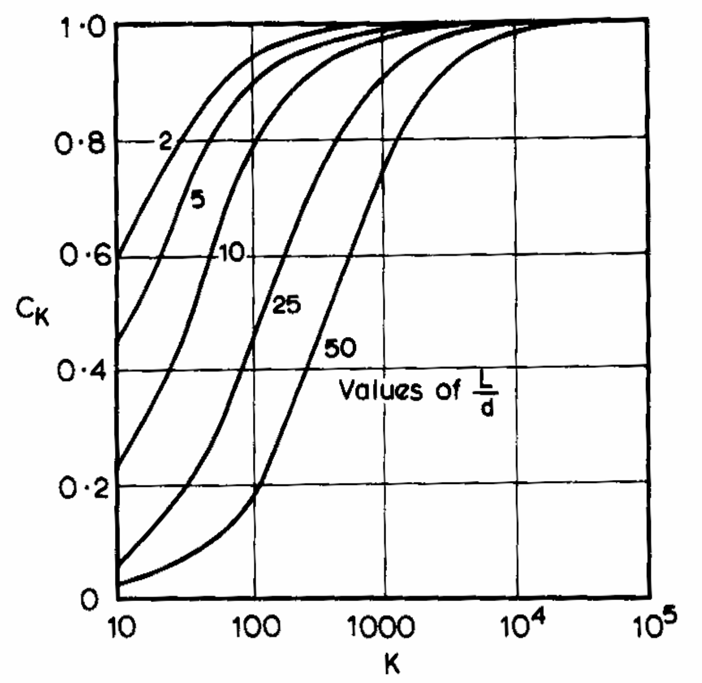


FIGURE 5.12 Compressibility correction factor for base load, C_K.

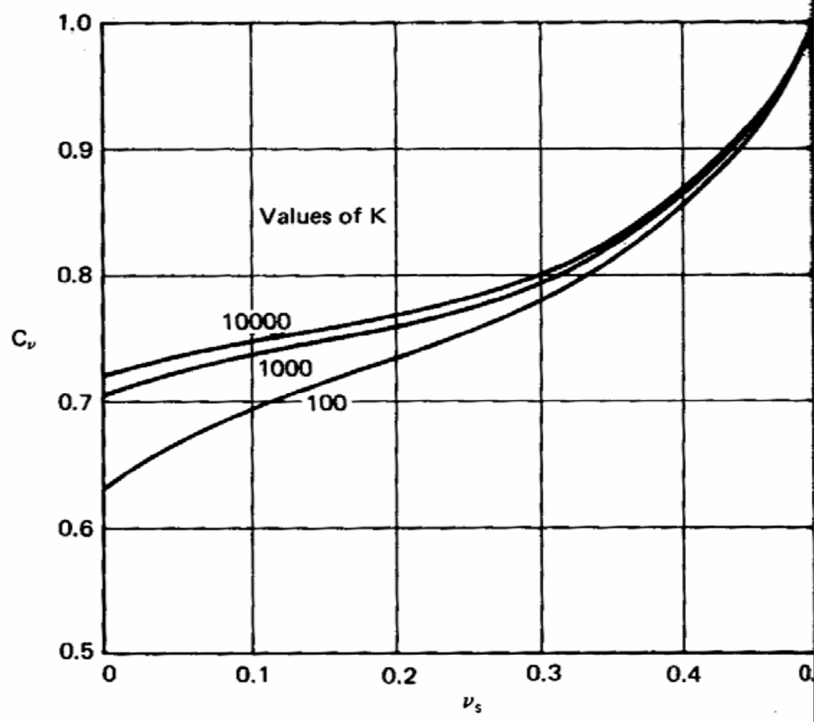
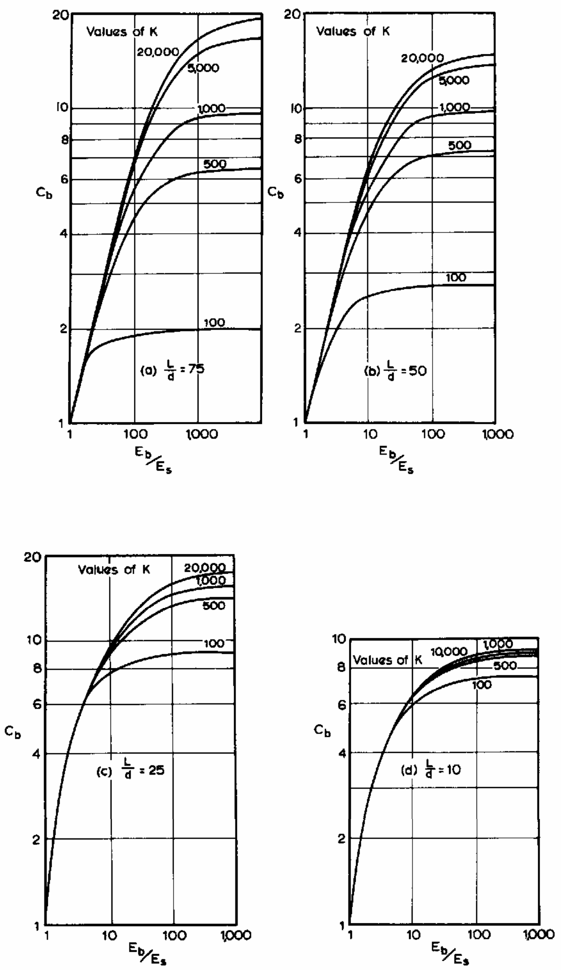


FIGURE 5.13 Poisson's ratio correction factor for base load, C_ν.

C_K

C_ν

PROPORTION OF BASE LOAD ON PILE – FACTOR C_b



5.14 Base modulus correction factor for base load, C_b.

SETTLEMENT OF SINGLE PILES – ELASTIC ANALYSIS

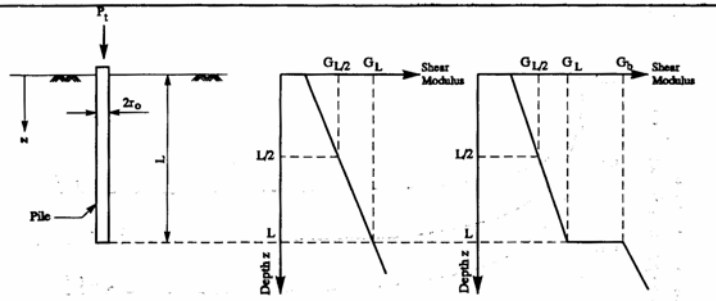
- **Closed Form Solutions**

  Randolph & Wroth (1978) – uniform & "Gibson" soil profiles, friction & end-bearing piles

- **Chart Solutions**

  Poulos & Davis (1980) – uniform soil profile
- Poulos (1979) – "Gibson" soil profile

Randolph & Wroth (1978) Equations



(a) Friction Pile (b) End-bearing Pile

Assumed Variation in Modulus with Depth

For an applied load P_t, the pile head settlement (ω_t) of a compressible pile is given by the following approximate closed form solution :

$$\frac{P_t}{\omega_t \, r_o \, G_L} = \frac{\dfrac{4\,\eta}{(1-\nu)\xi} + \dfrac{2\pi\rho}{\zeta}\,\dfrac{\tanh(\mu L)}{(\mu L)}\,\dfrac{L}{r_o}}{1 + \dfrac{1}{\pi\lambda}\,\dfrac{4}{(1-\nu)}\,\dfrac{\eta}{\xi}\,\dfrac{\tanh(\mu L)}{(\mu L)}\,\dfrac{L}{r_o}}$$

where

$\eta = r_b/r_o$ (r_o is the radius of pile shaft & r_b is radius of pile base)

$\xi = G_L/G_b$ (G_L & G_b is shear modulus of soil at depth L & pile base respectively)

$\rho = G_{L/2}/G_L$ (Rate of variation of shear modulus of soil with depth)

$\lambda = E_p/G_L$ (pile stiffness ratio)

$$\mu L = \left[\frac{2}{\zeta\lambda}\right]^{1/2}\frac{L}{r_o}$$

$\zeta = \ln\{[0.25 + (2.5\rho(1-\nu) - 0.25)\xi]\, L/r_o\}$

ν_s = Poisson's ratio of soil

The settlement profile with depth may be approximated as :

$$w = w_b \cosh(\mu(L-z)) \text{ where } w_b = \frac{P_b(1-\nu_s)}{4(r_b\,G_b)}$$

For a non-circular pile, E_p may be taken as : $E_p = (E\,A)_p/\pi\, r_o^2$

Where the slenderness ratio L/D is $\leq 0.25\,(E_p/G_L)^{½}$, the pile may be treated as effectively rigid and the pile head stiffness is given by :

$$\frac{P_t}{(w_t\, r_o\, G_L)} = \frac{4\eta}{(1-\nu)\xi} + \frac{2\pi\rho L}{r_o}$$

Where L/D is $\geq 1.5\,(E_p/G_L)^{1/2}$, the pile may be treated as infinitely long. In this case, the effective pile head stiffness is given by :

$$\frac{P_t}{(w_t\, r_o\, G_L)} = \pi\,\rho\,(2\lambda/\zeta)^{1/2}$$

or $P_t \approx 2\,\rho\, r_o\,(E_p\, G_{L_{ac}})^{1/2}$

For this case, G_L is the soil shear modulus at the bottom of the active pile length L_{ac}, where $L_{ac} = 3r_o(E_pG_L)^{1/2}$.

Note : Method based on Fleming et al (1992).

SETTLEMENT OF SINGLE PILES – CHART SOLUTIONS

Floating (Friction) Pile in Uniform Layer:

$S = P.\ I_0.\ R_K.\ R_h.\ R_\nu\ /\ d.\ E_s$

End Bearing Pile in Uniform Layer:

$S = P.\ I_0.\ R_K.\ R_b.\ R_\nu\ /\ d.\ E_s$

Pile in "Gibson" Soil:

$S = P.I_\rho\ /\ d.\ E_{sL}$

INFLUENCE FACTOR I_0

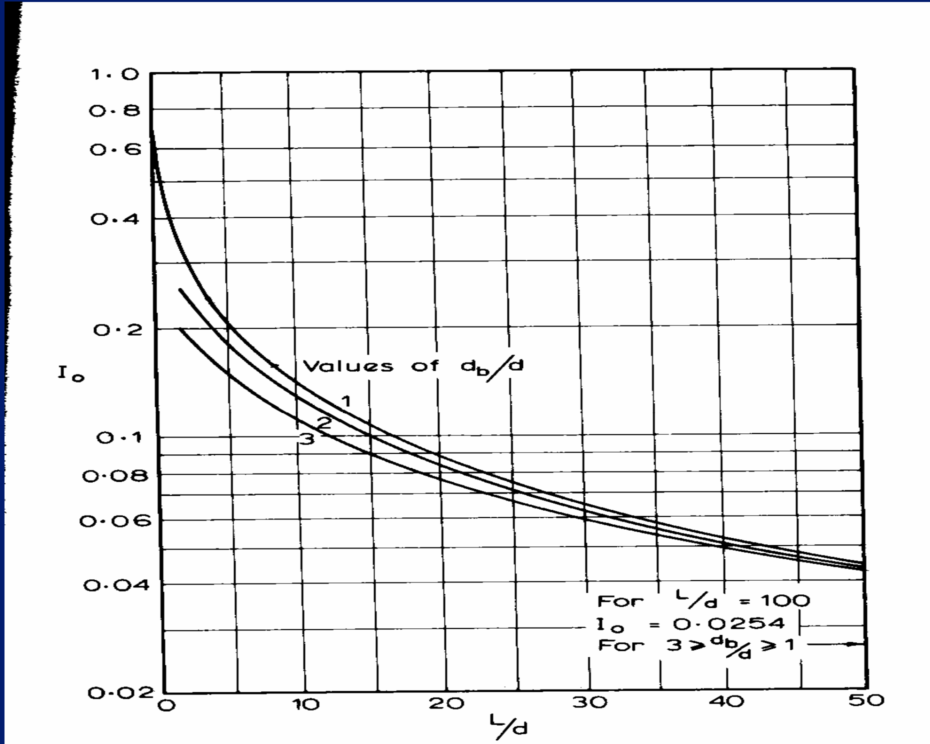


FIGURE 5.18 Settlement-influence factor, I_o.

- Increasing L/d reduces settlement (for same diameter)
- Effects of enlarged base are only significant for relatively short piles

CORRECTION FACTOR R_K

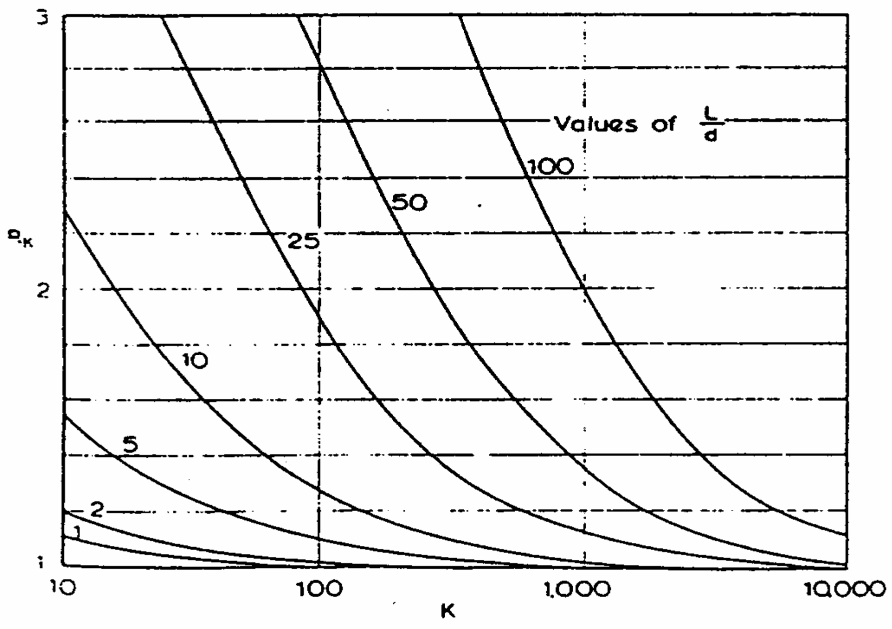
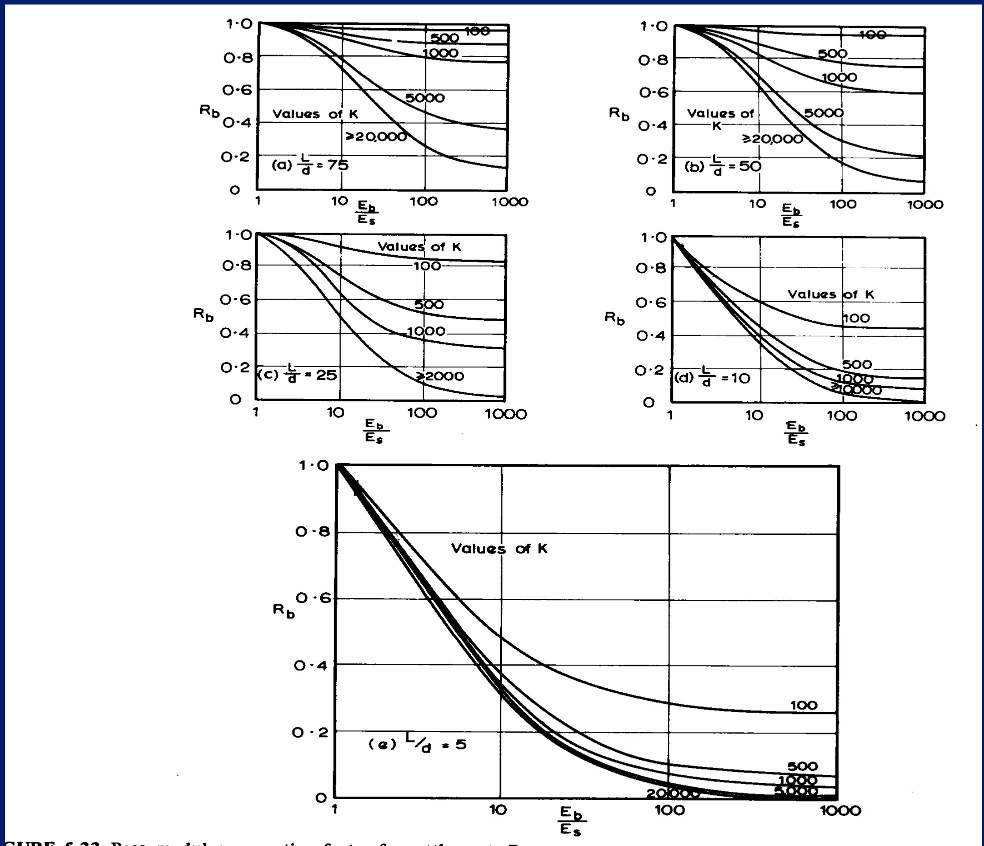


FIGURE 5.19 Compressibility correction factor for settlement, R_K.

- Pile compressibility is very important for longer piles
- For relatively long piles (L/d>50), a rigid pile requires that K > 5000 or so

CORRECTION FACTOR R_b



- Effects of bearing stratum are more pronounced for:
  - Shorter piles
  - Stiffer piles (larger K)

CORRECTION FACTOR R_h

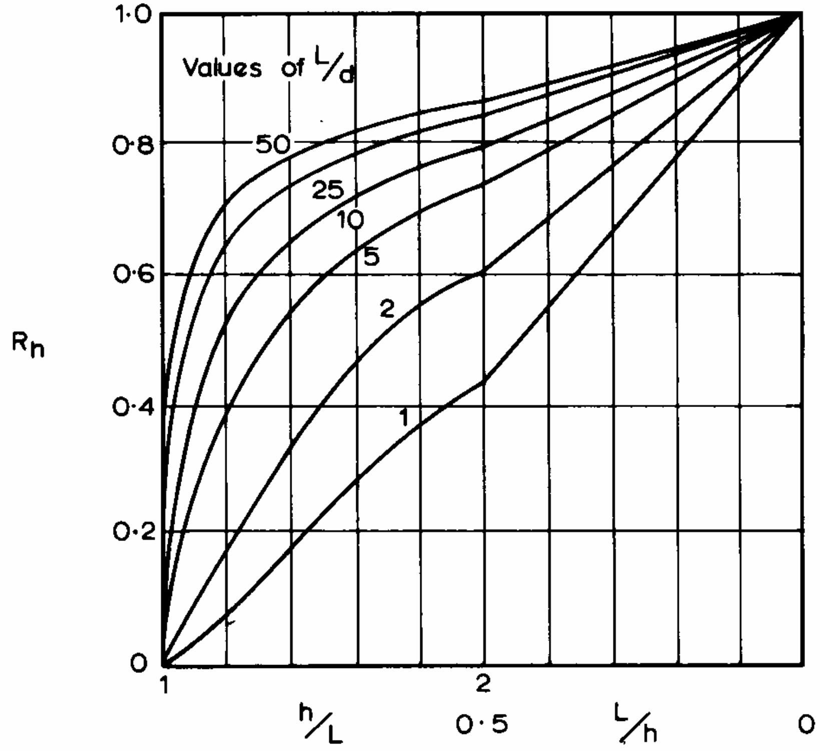


FIGURE 5.20 Depth correction factor for settlement, R_h.

- Effect of finite layer is most pronounced for shorter (and stiffer) piles
- Has relatively little effect for long compressible piles

CORRECTION FACTOR R_ν

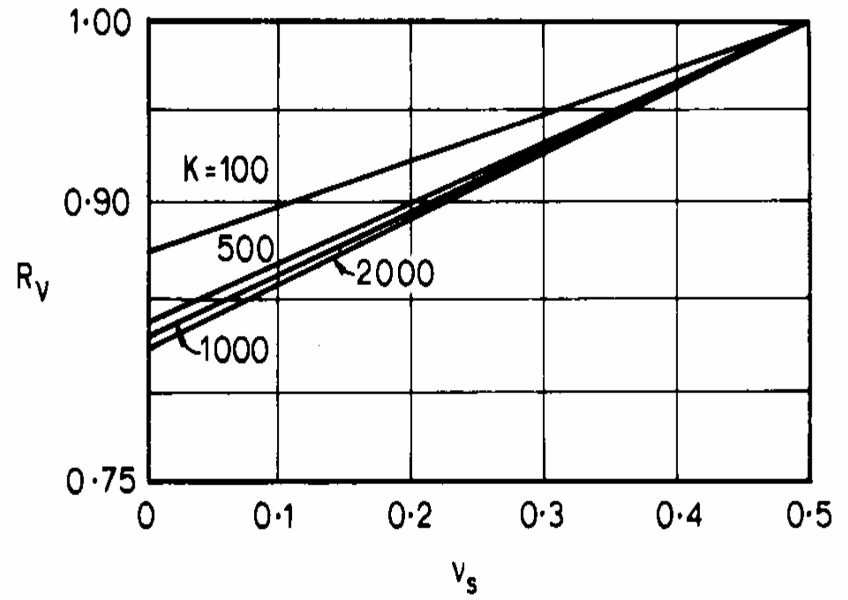
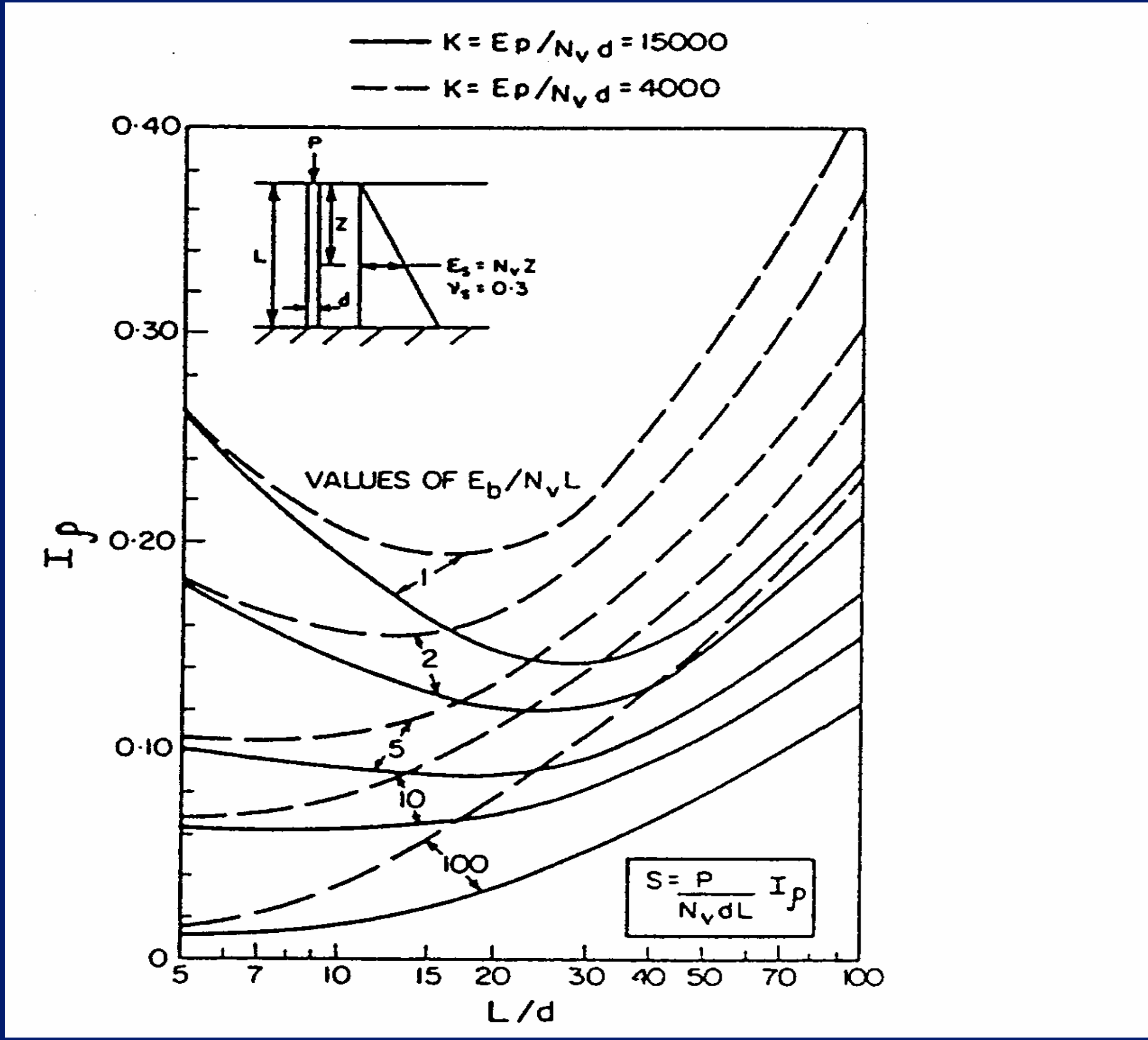


FIGURE 5.21 Poisson's ratio correction factor for settlement, R_ν.

- Effect of Poisson's ratio of soil is generally small, especially for more compressible piles (smaller K values)

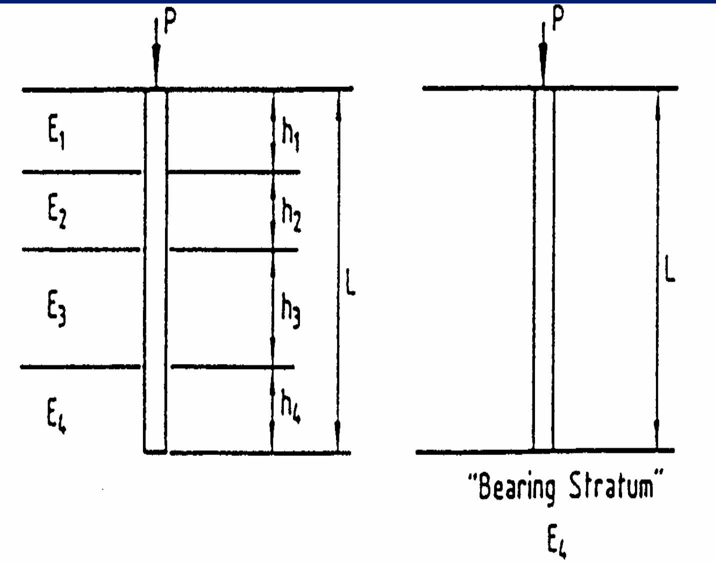
TYPICAL SOLUTIONS FOR SOIL WITH LINEARLY INCREASING MODULUS



APPROXIMATE APPROACH FOR LAYERED SOILS

- Use average soil modulus along shaft

  $E_{sav} = \Sigma (E_i h_i) / L$
- For base, use average modulus within region affected by base
- Then, treat as end-bearing pile



(a) Actual Pile in Layered Soil

(b) Equivalent End-Bearing Pile Through Homogeneous Soil

AVERAGE SOIL MODULUS FOR PILE BASE IN LAYERED SOILS

- Estimate weighted modulus as:

$$E_{sb} = \frac{\Sigma W_i h_i}{\Sigma W_i h_i / E_{si}}$$

- where
- h_i, E_{si} = thickness, modulus of layer I
- W_i = weighting factor

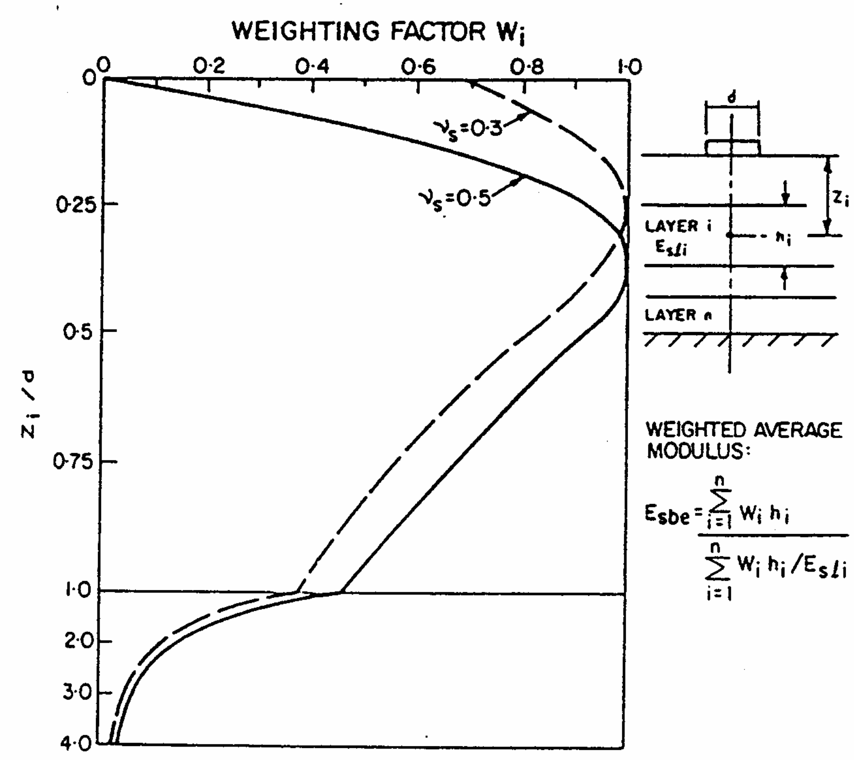
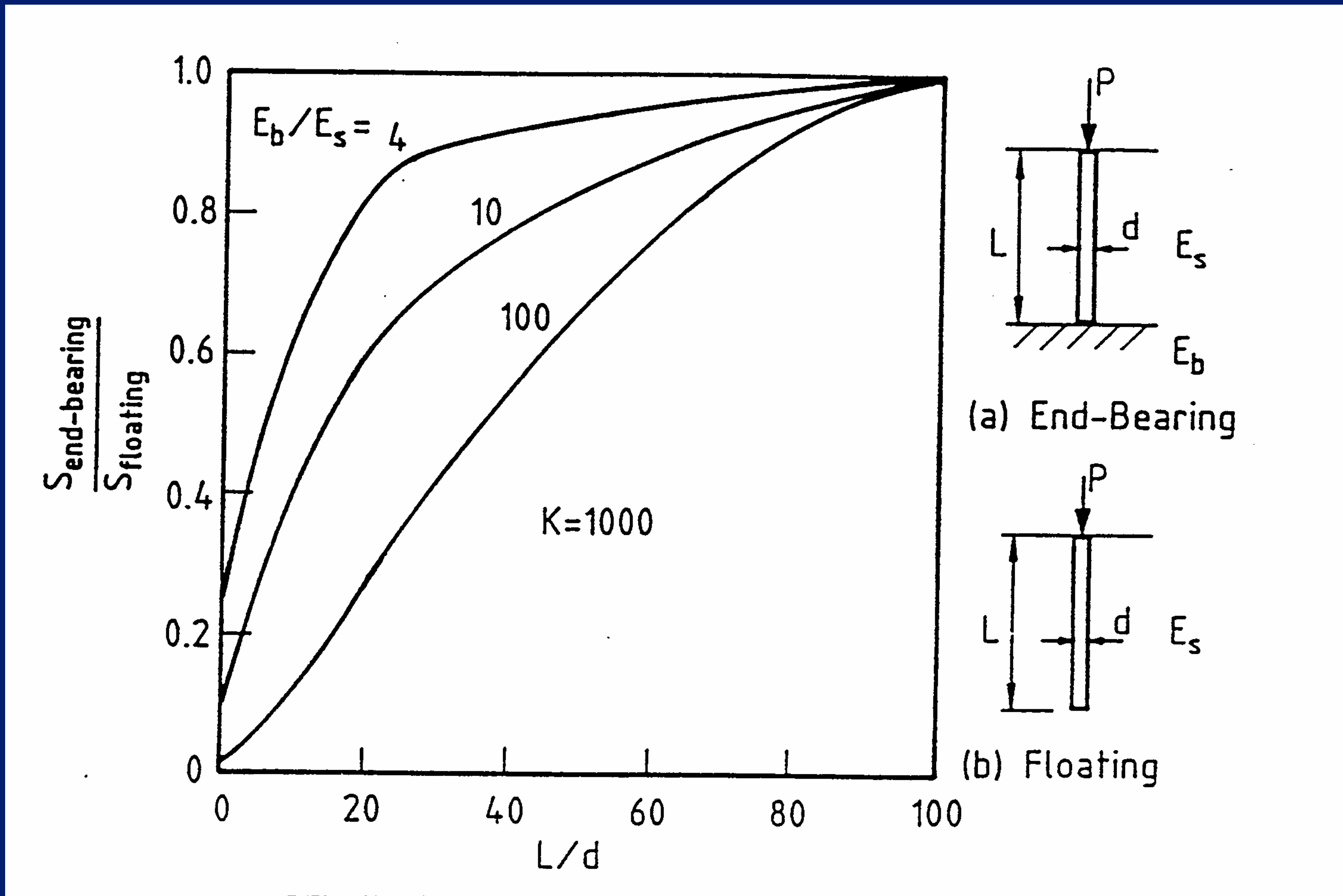


FIG. 6. Weighting Factor for Es.. ıation of Equivalent Modulus below Circular Area

MAIN CHARACTERISTICS OF BEHAVIOUR

- Major part of settlement is IMMEDIATE SETTLEMENT (typically >80%)
- Effect of compressibility is important for long slender piles
- For long compressible piles, settlement is little influence by soil stiffness at pile tip
- For piles of normal proportions in clay, the load-settlement behaviour is largely liner at normal working loads. Thus, elastic theory can be used directly
- Nonlinear effects are important when piles derive much of their capacity from base resistance, e..g.
  - Piles in sand
  - Piles with enlarged base
  - Large diameter bored piles

END BEARING vs FRICTION PILE SETTLEMENTS



SIMPLIFIED METHOD FOR LOAD-SETTLEMENT CURVE TO FAILURE

Superpose relationships between:

- shaft load vs settlement
- base load vs settlement

SHAFT LOAD vs SETTLEMENT (until shaft ultimate capacity is developed)

$P_s = P\,(1 - \beta)$

$S = I_\rho .\ P_s / \{E_s .\ d.\ (1-\beta)\}$

BASE LOAD vs SETTLEMENT

1. Until shaft ultimate capacity is developed

   $Pb = \beta .\ P$

   $S = I_\rho .\ P_b / \{E_s .\ d.\ \beta\}$

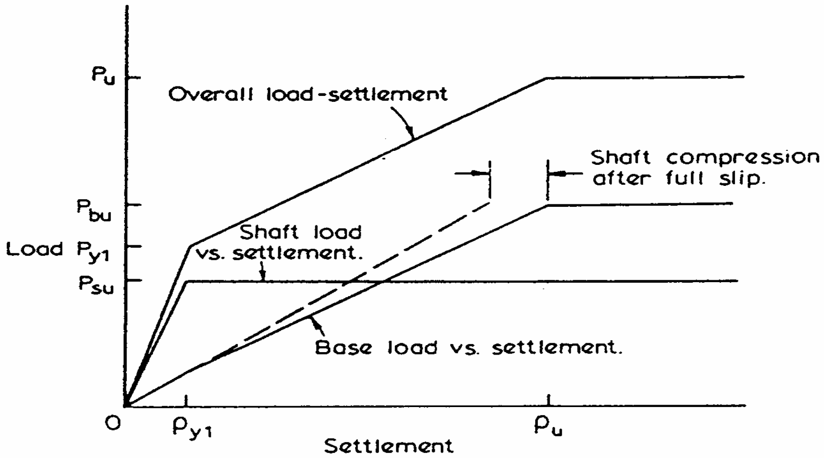
2. After shaft has slipped: add additional settlement $\Delta\rho$, where

   $\Delta\rho = [P_b - P_{su} .\ \beta/(1-\beta)].\ L/\ (E_p .\ A_p)$

SIMPLIFIED METHOD FOR LOAD-SETTLEMENT CURVE TO FAILURE

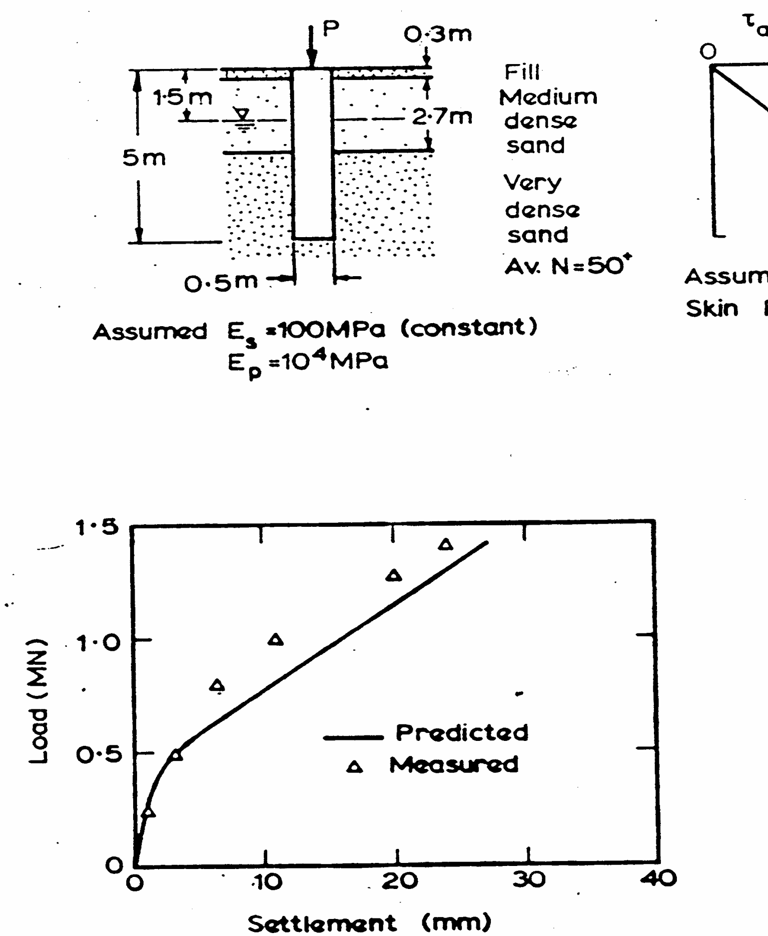
$$\rho_{y1} = \left(\frac{I}{E_s d}\right)\left(P_{y1}\right)$$

$$\rho_u = \left(\frac{I}{E_s d}\right)\left(\frac{P_{bu}}{\beta}\right) + \left[P_{bu} - \frac{P_{su}\beta}{(1-\beta)}\right]\frac{L}{A_p E_p}$$



Construction of load-settlement curve.

EXAMPLE OF LOAD-SETTLEMENT CURVE



PREDICTED AND MEASURED SETTLEMENTS OF GROUTED PILE - SURFER'S PARADISE

ESTIMATION OF SOIL PARAMETERS

Soil modulus (Es) is the key parameter.

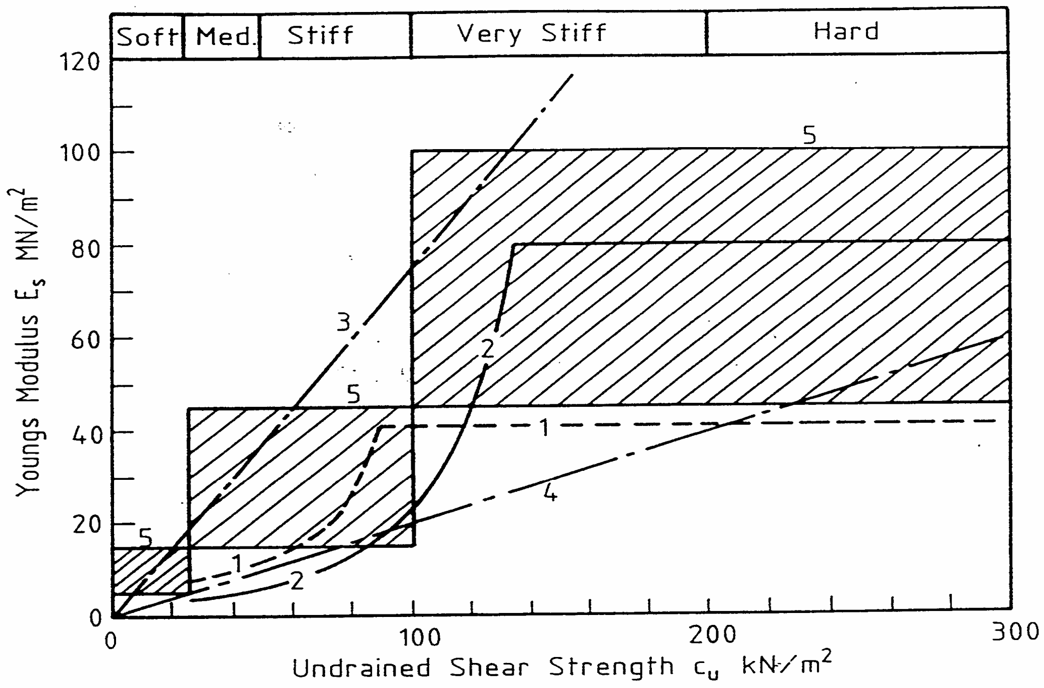
- **Laboratory Testing**
  - Not usually useful because of
    - Differences between stress paths in lab and field
    - Difficulty of accounting for installation effects
- **Interpretation of Load Tests**
  - Fit theory to observed behaviour
  - Usually the most satisfactory method

ESTIMATION OF SOIL PARAMETERS

Empirical Correlations

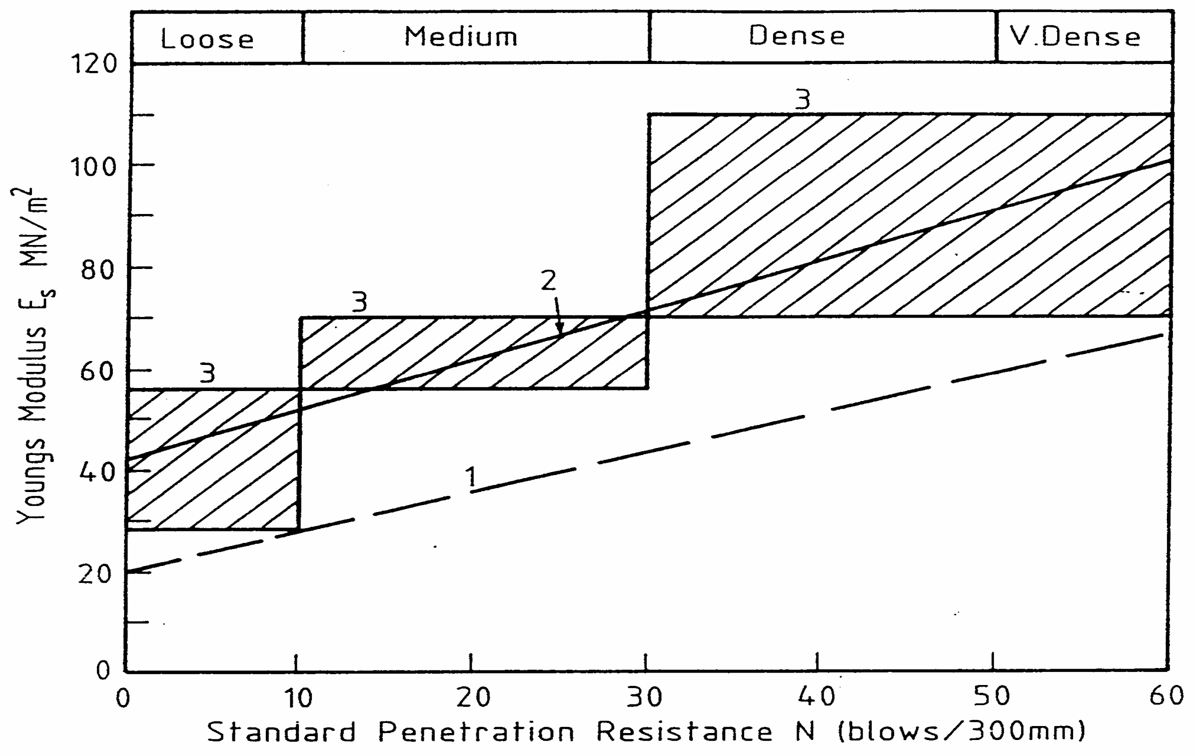
- With laboratory data
- With field & in-situ test data
  - SPT
  - CPT
  - PMT
- Most correlations are for SECANT MODULUS at a typical design load level
- Can also correlate initial tangent modulus and degrade with stress level e.g. hyperbolic curve

ESTIMATION OF SOIL PARAMETERS – PILES IN CLAY



Legend	Remarks	Reference
1	Driven piles	Poulos (1972)
2	Bored piles	Poulos (1972)
3	Driven piles (E_s=750c_u)	Aschenbrenner and Olsen (1984)
4	Bored piles, lower bound (E_s=200c_u)	Callinan and Kulhawy (1985)
5	Suggested ranges	Terzaghi and Peck (1967)

ESTIMATION OF SOIL PARAMETERS – PILES IN SAND



Legend	Remarks	Reference
1	Normally-consolidated sand	D'Appolonia et al (1970)
2	Over-consolidated sand	D'Appolonia et al (1970)
3	Driven piles	Poulos (1972)

ESTIMATION OF SOIL PARAMETERS – CRUDE CORRELATIONS WITH CPT

Soil Type	*Es / qc*
Normally consolidated sand	5
Over-consolidated sand	7.5
Clay	15

USE OF INITIAL (SMALL STRAIN) SHEAR MODULUS – Mayne 2002

- Makes use of initial shear modulus derived from shear wave velocity measurements
- Seismic cone, cross-hole, down-hole measurements
- Initial shear modulus $G_i = \rho v_s^2$
- Young's modulus $E_i = 2(1+\nu) G_i$

APPLICATION TO PILE SETTLEMENT CALCULATION

- Allow for non-linearity via Fahey-Carter degradation function:

$$E = E_i \, [1 - f.(P/P_u)^g]$$

- f = 1, g=0.3 recommended by Mayne
- Settlement is:

$$S = P.I_\rho \, / \, [d.E_i \, (1-(P/P_u)^{0.3})]$$

- In this way, obtain non-linear load-settlement curve

EXAMPLE OF APPLICATION

I85 Bridge site, Georgia, USA

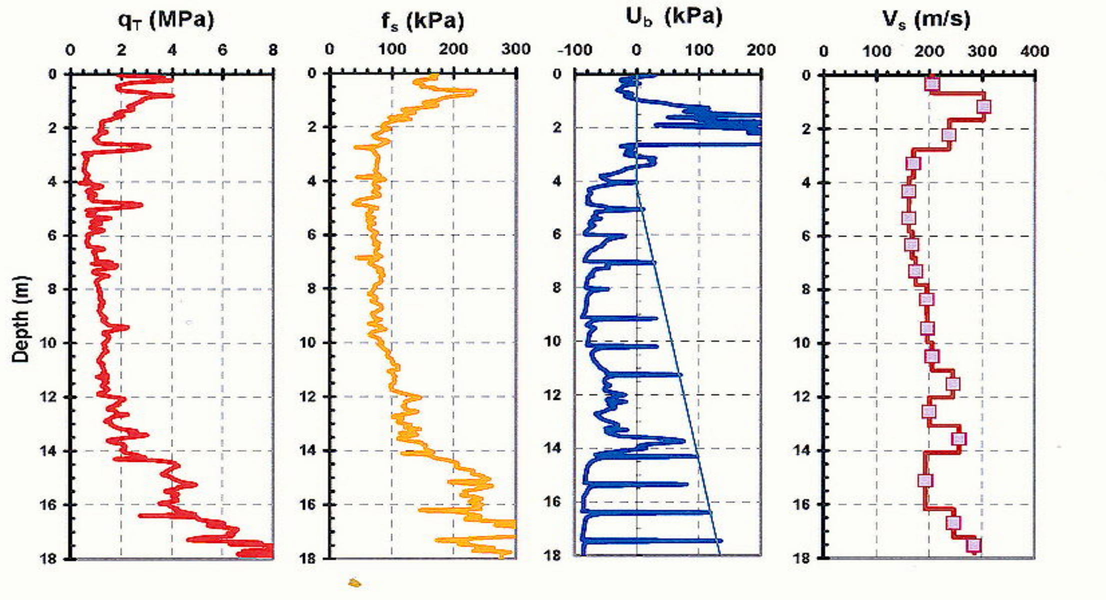


Figure 6. Seismic Cone Sounding at I-85 Drilled Shaft Bridge Site near Newnan, Georgia.

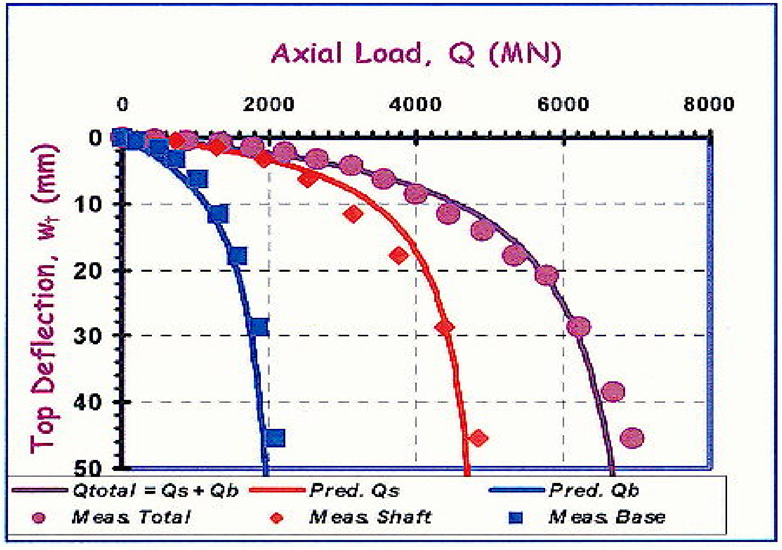


Figure 7. Measured and Predicted Load-Transfer-Deflections for the Drilled Shaft at Newnan, Georgia.

EXAMPLE OF APPLICATION

Jackson County Power Facility, Georgia

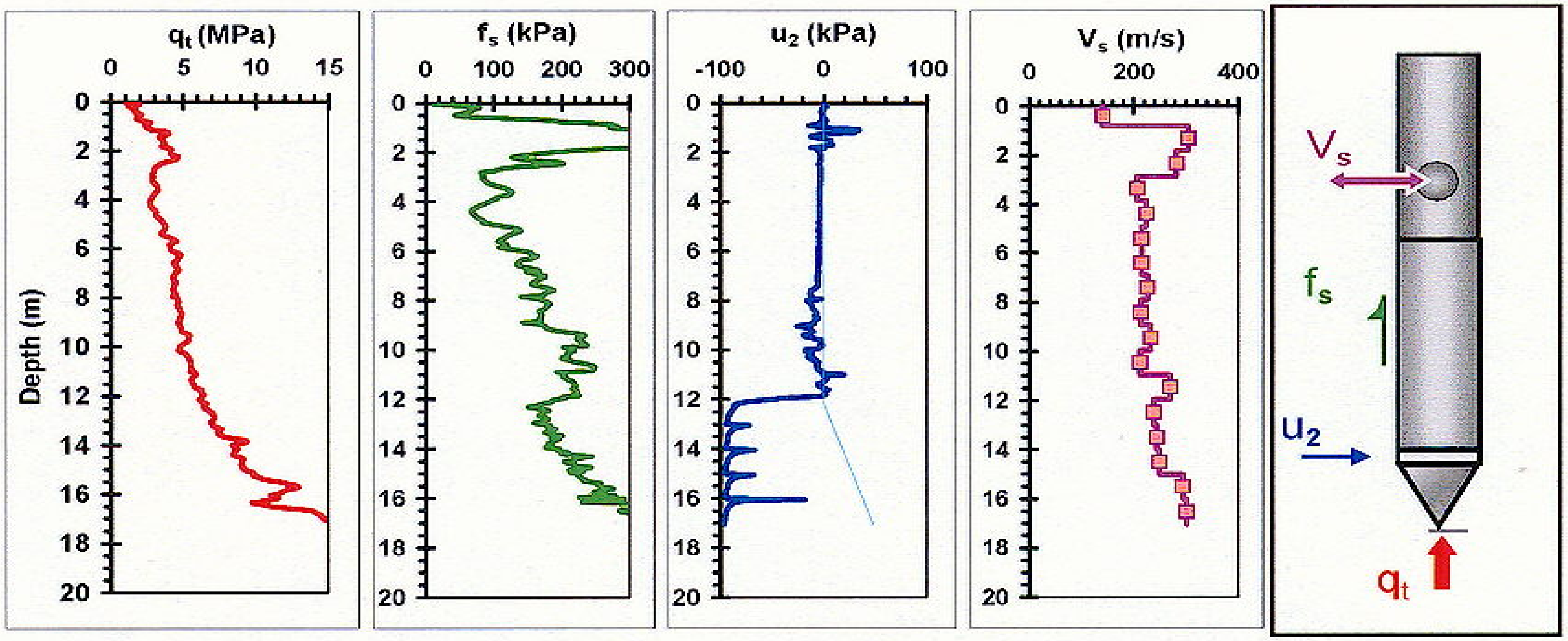


Figure 8. Results of SCPTu at Jackson County Electrical Power Facility, Center, Georgia.

EXAMPLE OF APPLICATION

Jackson County Power Facility, Georgia

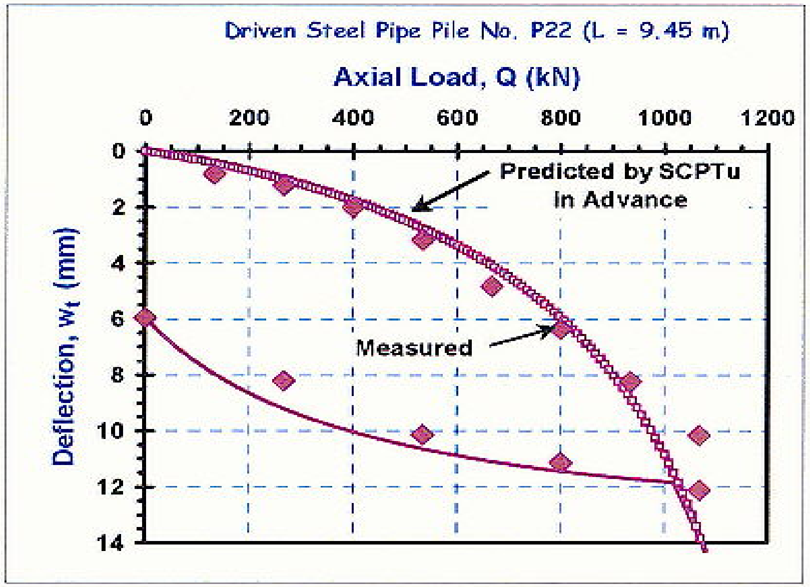


Figure 9. Class "A" Prediction and Measured Load Test Results for Short Pipe Pile at Center, Georgia.

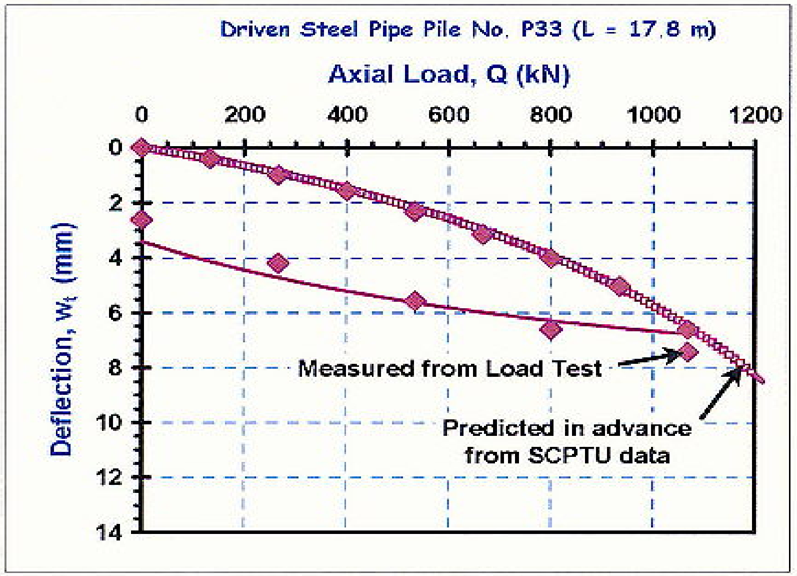


Figure 10. Class "A" Prediction and Measured Load Test Results for Long Pipe Pile at Center, Georgia.

EXAMPLE OF APPLICATION

Atlanta, Georgia

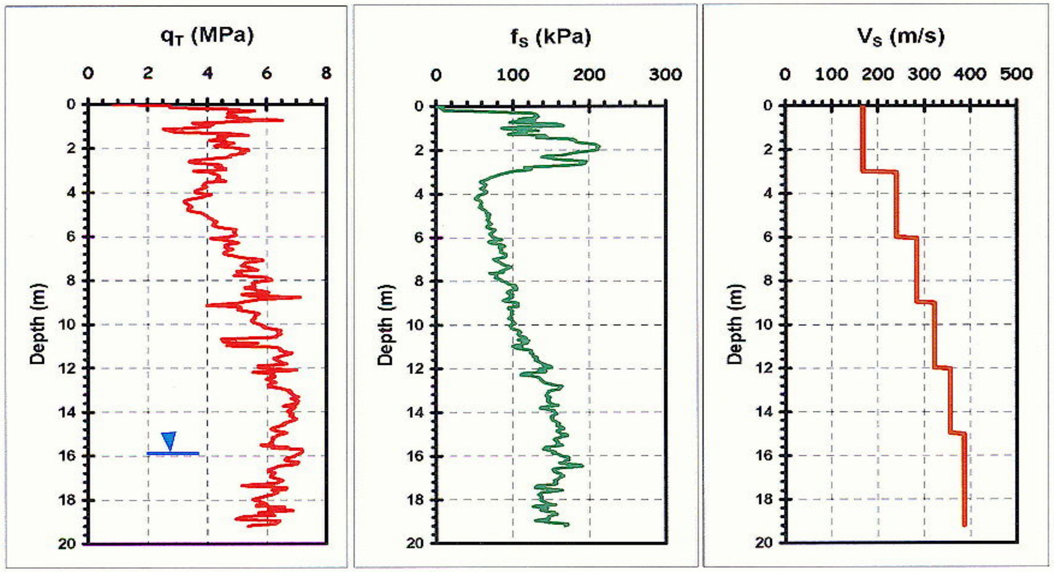


Figure 12. Composite Record for Equivalent Seismic Piezocone at GT Campus, Atlanta, Georgia.

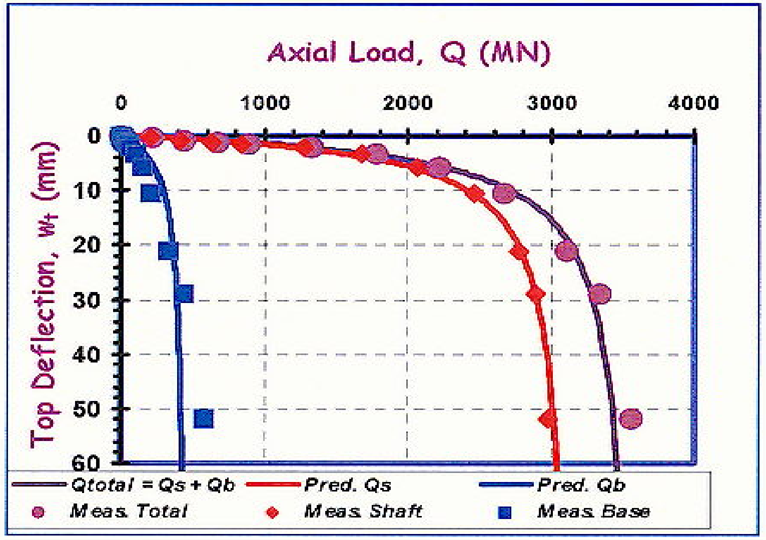


Figure 11. Results of Axial Load-Deflection for Floating Shaft Foundation in Atlanta, Georgia.

SETTLEMENT OF PILE GROUPS METHODS OF ANALYSIS - Hand

- Interaction factor method
- Settlement ratio method
- Equivalent raft method
- Equivalent pier method

INTERACTION FACTOR METHOD

Interaction factor

$$\alpha = \frac{\textbf{Extra settlement caused by pile 2}}{\textbf{Settlement of pile 1 under own load}}$$

α depends on:

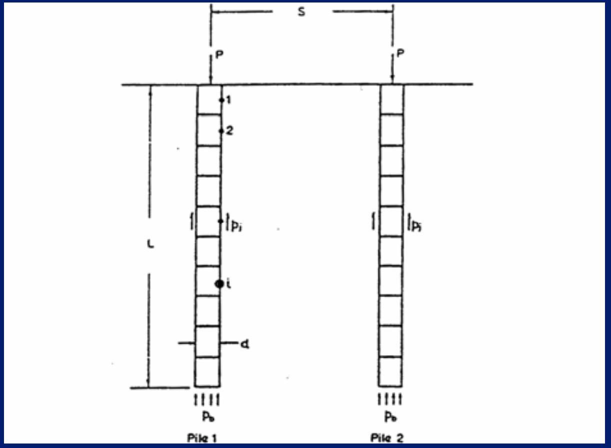
L/d

K = Ep/Es

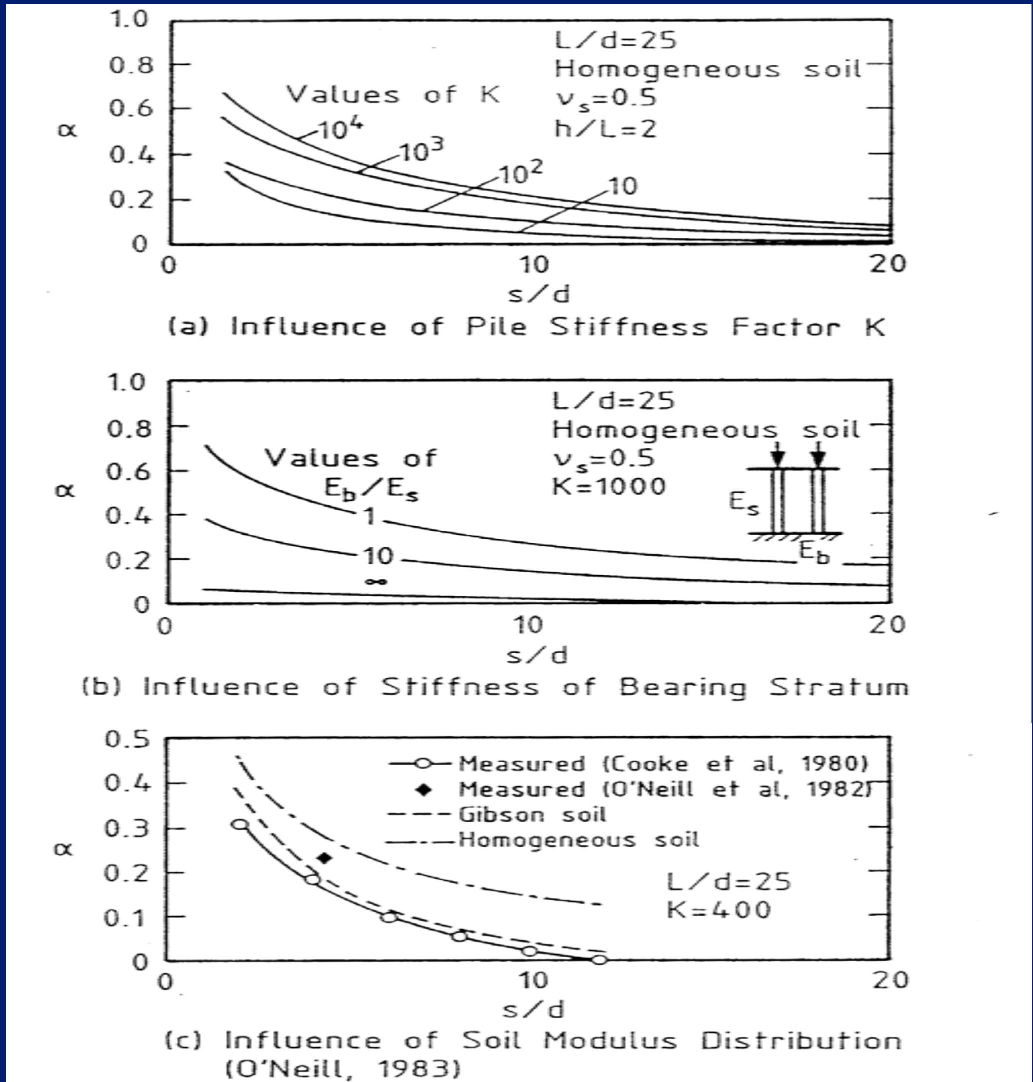
s/d

Distribution of Es

Pile tip conditions



INTERACTION FACTORS - EFFECT OF VARIOUS PARAMETERS



INTERACTION FACTORS - CHARACTERISTICS

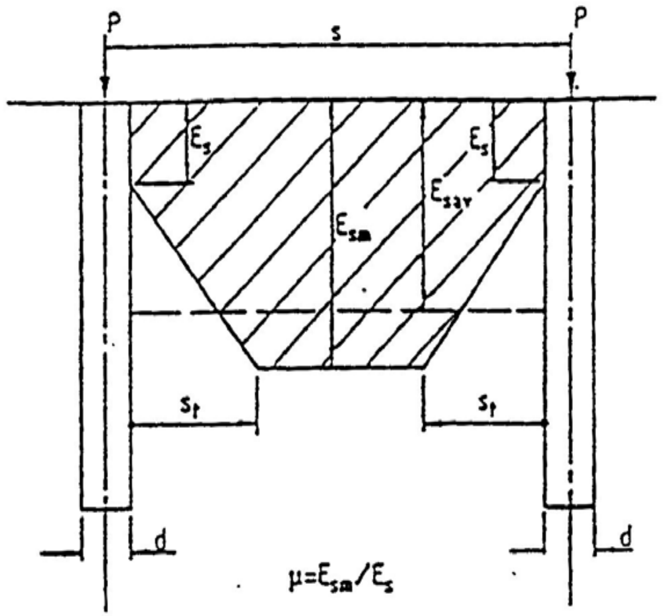
α

- Decreases as s/d increases
- Decreases as K decreases
- Decreases as L/D decreases
- Less for end bearing than friction piles
- Less for non-homogeneous than homogeneous profiles
- Increases as ν_s increases
- Increases if base enlarged
- Decreases if soil between piles is stiffer

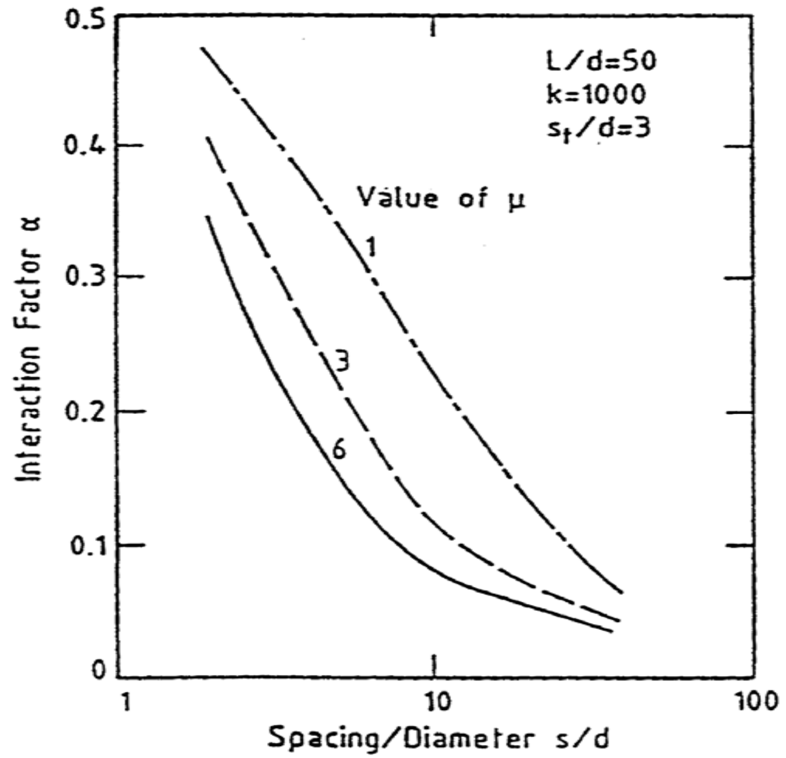
INTERACTION FACTORS - Warnings!

- Generally, will tend to over-estimate interaction within a relatively large group, due to effects of:
  - Greater stiffness of soil between piles
  - α decreases more rapidly with spacing than theory suggests
  - The intervening piles within the group tend to reduce interaction.
- Should check group settlement with simpler approach
- Should make allowances for stiffer soil, & more rapid decay of α

INTERACTION FACTORS -
EFFECT OF STIFFER SOIL BETWEEN PILES



(a). Assumed Distribution of Soil Modulus between Two Piles: E_s = Soil Modulus Adjacent to Pile; E_{sm} = soil-mass modulus (for Very Small Strain Levels = μE_s; E_{sav} Average Soil Modulus for Computing Interaction; and s_t = Transition Distance



(b). Influence of Mass Modulus Ratio μ on Axial Interaction Factor

Note the significant reduction in α for μ >1

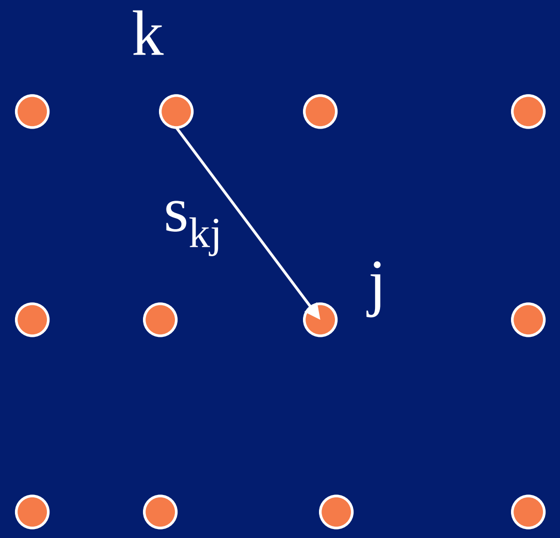
GROUP ANALYSIS VIA INTERACTION FACTORS

Superposition of 2-pile interaction factors can be used to analyze settlement & load distribution in groups – approximate but convenient.

For pile k,

$S_k = S_1 \Sigma P_j a_{kj}$

$(a_{kj} = 1.0$ for $k = j)$



For all piles:

$\{S\} = S_1 [A] \{P\}$

For equilibrium:

$P_G = \Sigma P_j$

SOLUTION OF GROUP SETTLEMENT FOR TYPICAL PILE HEAD CONDITIONS

- **Known loads (flexible pile cap)**
  - Settlement of each pile is calculated directly.
  - Will have differential settlements within the group

- **Rigid Pile Cap**
  - Settlement of all piles equal
  - Pile loads unknown
  - Form equations for each pile in group
  - Solve equations for unknown loads & group settlement

- **Analysis requires:**
  - Interaction factors
  - Settlement of single pile

EXPRESSION OF ANALYSIS RESULTS

- **SETTLEMENT RATIO R_s**

  R_s = Average group settlement / Settlement of single pile at same average load

  $$n > Rs > 1$$

- **SETTLEMENT EFFICIENCY FACTOR R_G**

  R_G = Stiffness of pile group / Sum of individual stiffnesses of all piles in group

  $$1/n < R_G < 1.0 \quad (R_s = n.\ R_G)$$

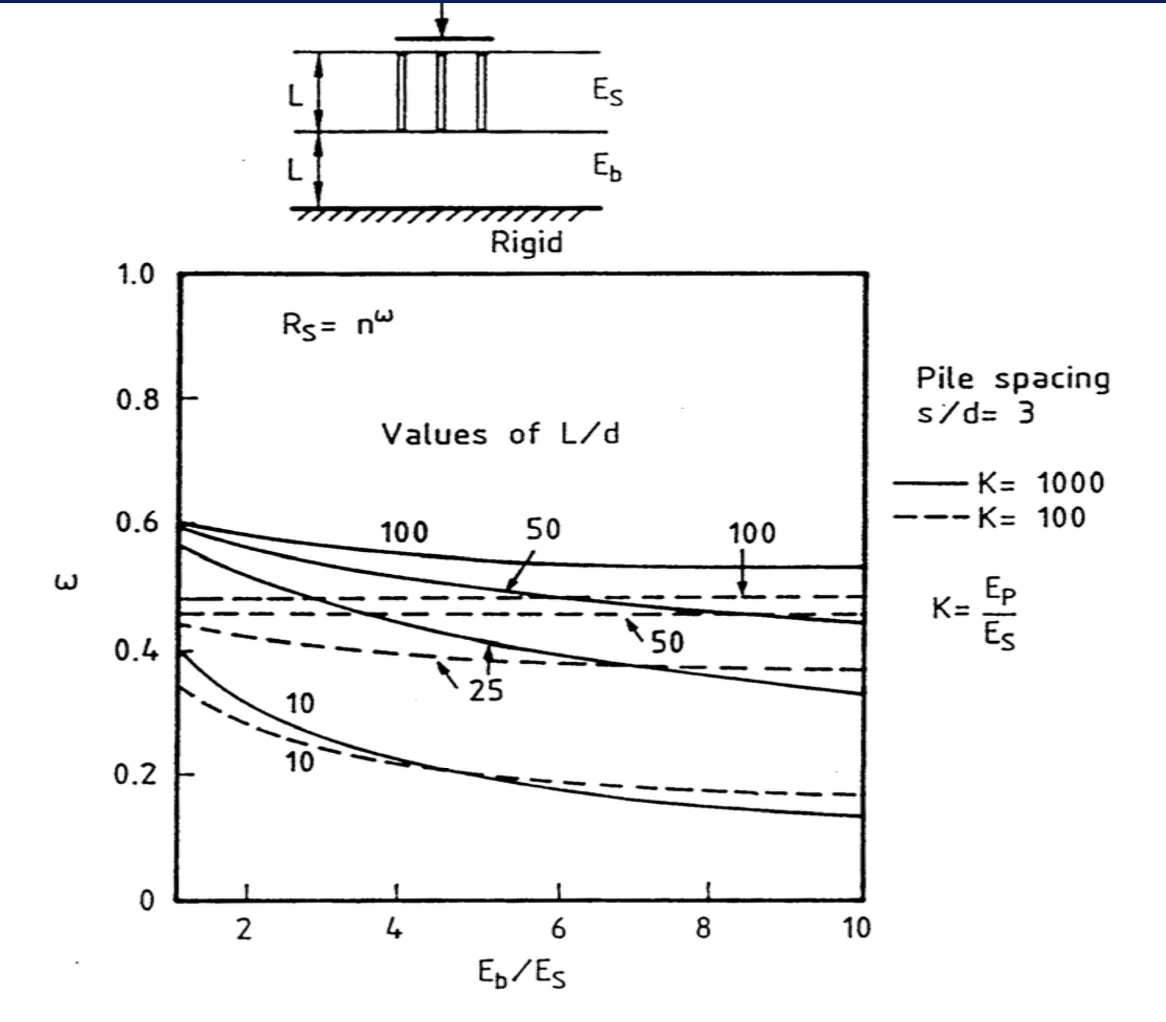
SETTLEMENT RATIO METHOD FOR GROUP SETTLEMENT

$$S_G = R_s . P_{av} . S_1$$

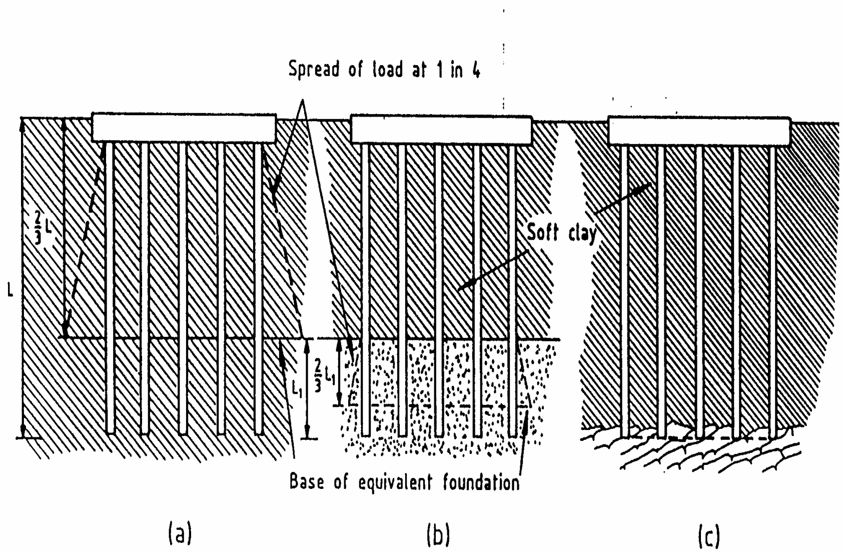
R_s from:

- Tabulated values (Poulos, 1979)
- Randolph's approximation
  - $R_s \sim n^w$
  - Values of w from theoretical analysis
  - As first approximations:
    - w ~ 0.5 for floating groups in clay
    - w ~ 0.33 for floating groups in sand

THEORETICAL SOLUTIONS FOR EXPONENT w



SETTLEMENT OF PILE GROUPS - EQUIVALENT RAFT METHOD



EQUIVALENT RAFT APPROACH

(a) Group of piles supported mainly by skin friction.
(b) Group of piles driven through weak clay to combined skin friction and end-bearing in stratum of dense granular soil
(c) Group of piles supported in end-bearing on hard incompressible stratum. (After Tomlinson, 1986)

EQUIVALENT RAFT APPROACH

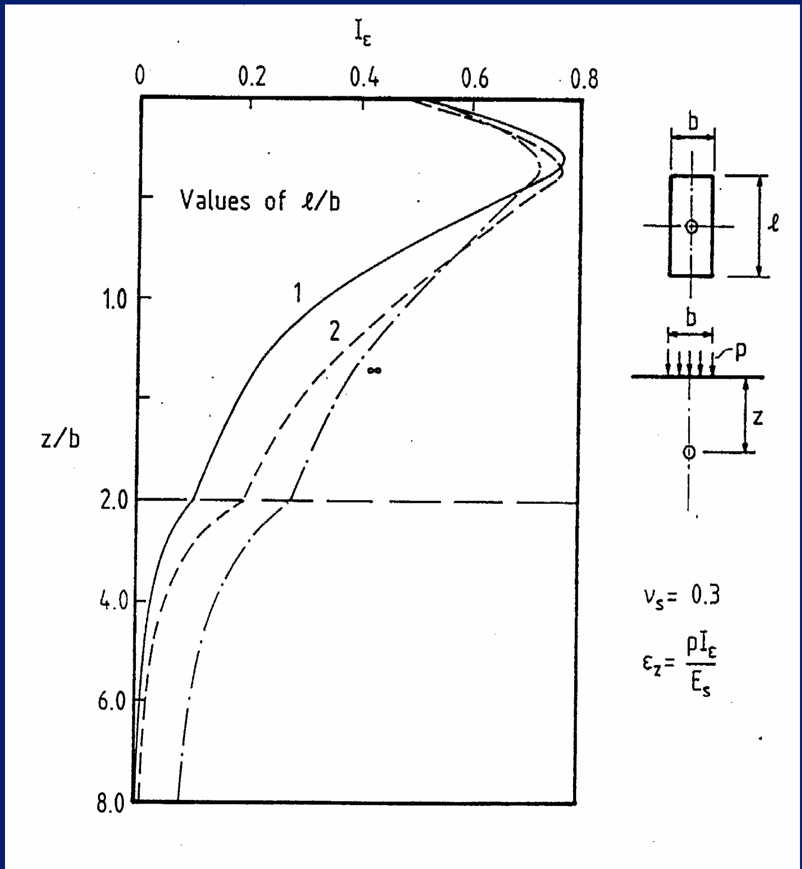
Compute settlement of raft at an equivalent depth along the pile shaft.

Allow for:

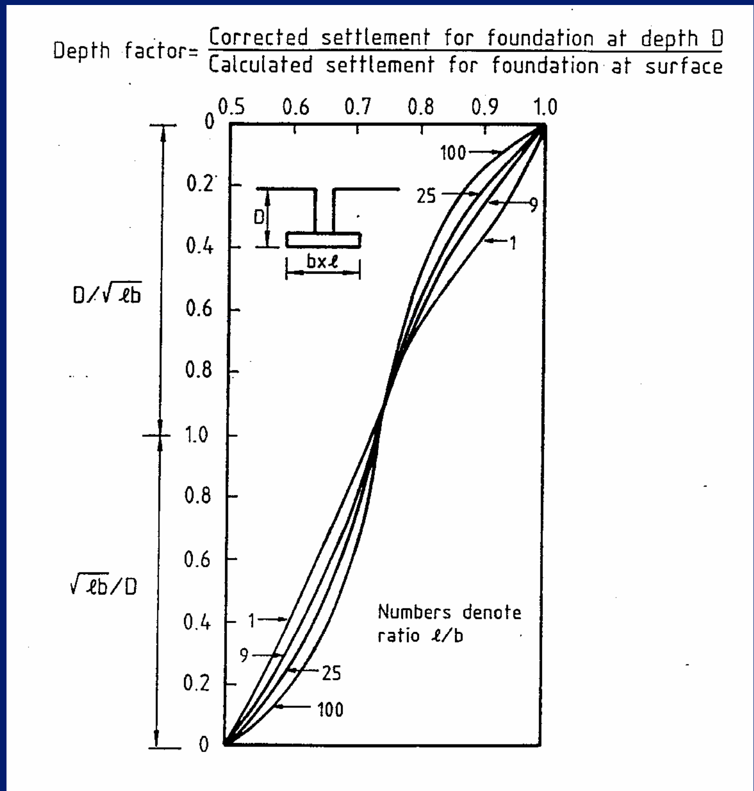
- Pile compression above the equivalent depth
- Embedment of equivalent raft.

Can use elastic theory or conventional settlement analysis.

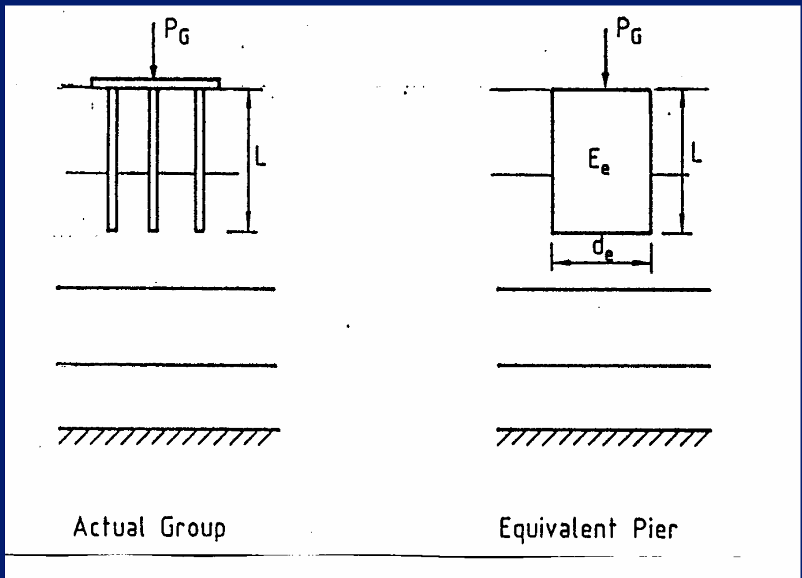
SETTLEMENT OF PILE GROUPS - EQUIVALENT RAFT METHOD



$$S = \sum \varepsilon_z \, . \, dh. \, F_D$$



SETTLEMENT OF PILE GROUPS - EQUIVALENT PIER METHOD



Pile group is replaced by an equivalent pier (piles + soil). Use equivalent diameter & stiffness of pier to compute settlement.

a) for predominantly friction piles:

$$d_e = 1.27 \sqrt{A_G}$$

b) for predominantly end-bearing piles:

$$d_e = 1.13 \sqrt{A_G}$$

where A_G = plan area of pile group.

The equivalent pier modulus, E_e, is approximated as:

$$E_e = E_p \frac{A_p}{A_G} + E_s \left(1 - \frac{A_p}{A_G}\right)$$

where E_p = Young's modulus of piles
E_s = average Young's modulus of soil within the group
A_p = total cross-sectional area of the piles in the group.

SETTLEMENT OF PILE GROUPS - EQUIVALENT PIER METHOD

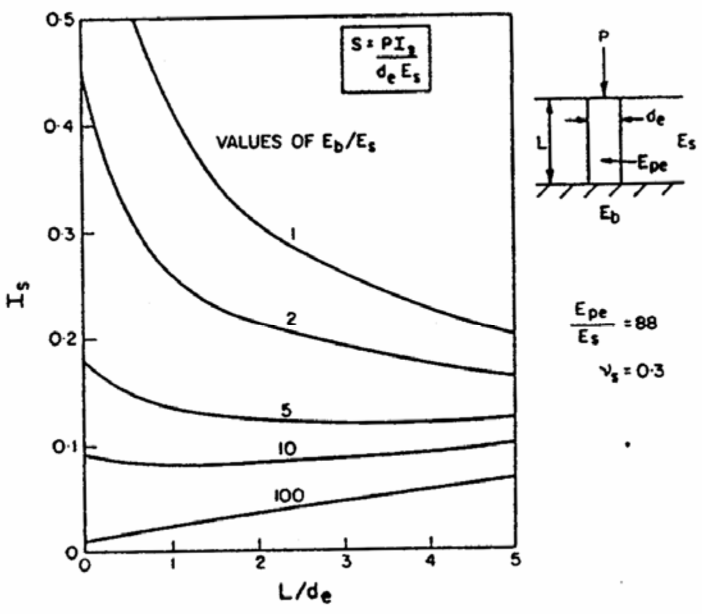


FIG. 4. Settlement of Equivalent Pier in Soil Layer

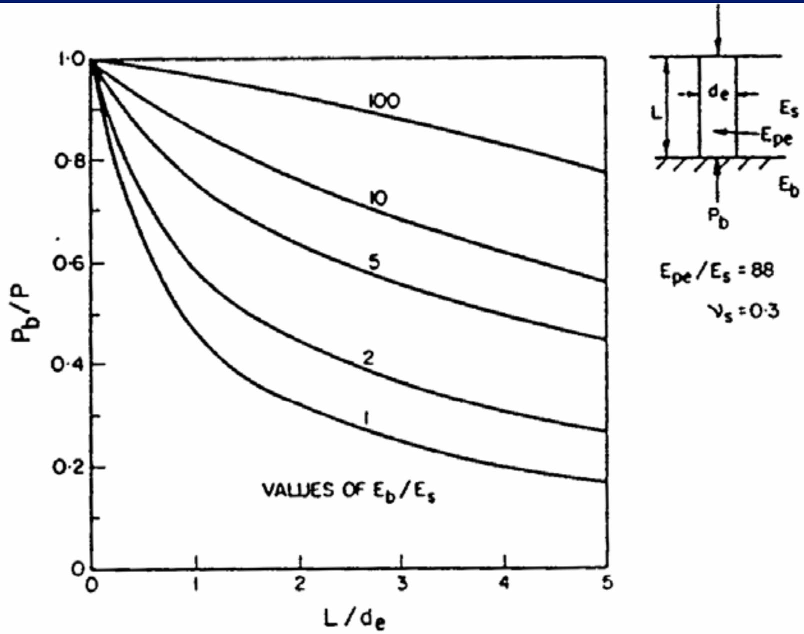


FIG. 5. Proportion of Base Load for Equivalent Pier

SETTLEMENTS DUE TO UINDERLYING COMPRESSIBLE LAYERS

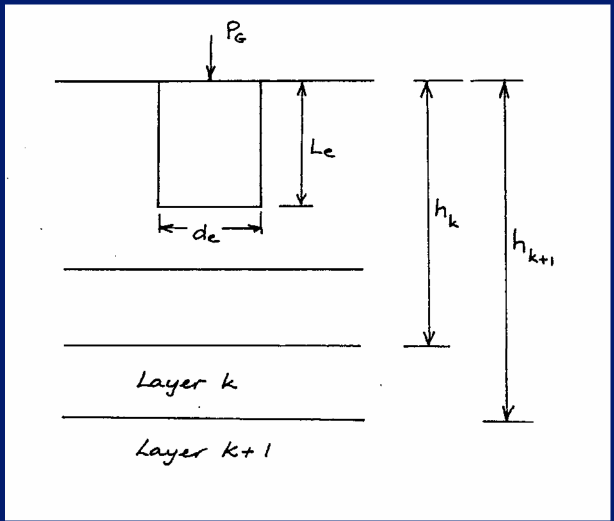
- Interaction factor approach
  - Require interaction factors for the appropriate soil profile
- Approximate Approach
  - S = settlement of group in founding layer + additional settlement due to underlying layers (ΔS)
  - Compute ΔS from:
    - 1-D analysis
    - Equivalent pier analysis, via calculation of settlement of underlying layers from elastic theory

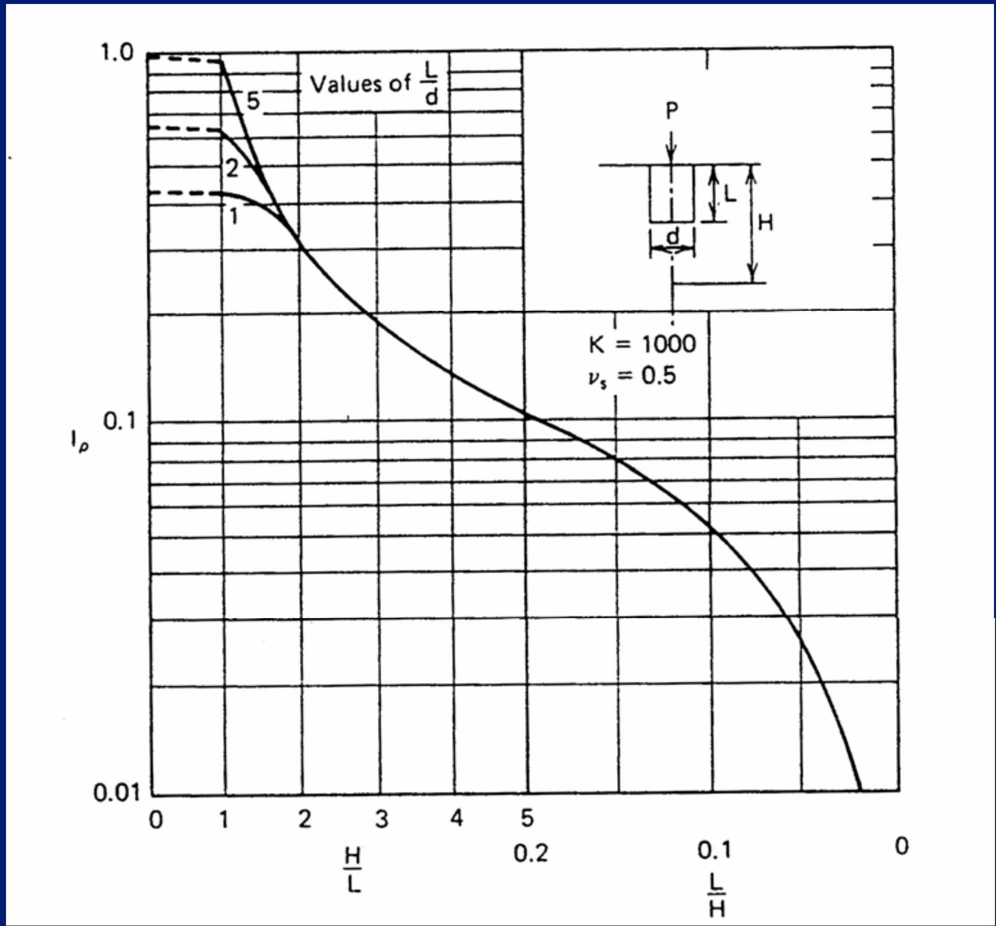
SETTLEMENT OF PIER DUE TO UINDERLYING COMPRESSIBLE LAYERS

$$\Delta S = P_G\{\Sigma(I_k - I_{k+1})\}/L_e$$

where I_k = displacement factor for depth=top of layer k

I_{k+1} = displacement factor for depth=top of layer k+1

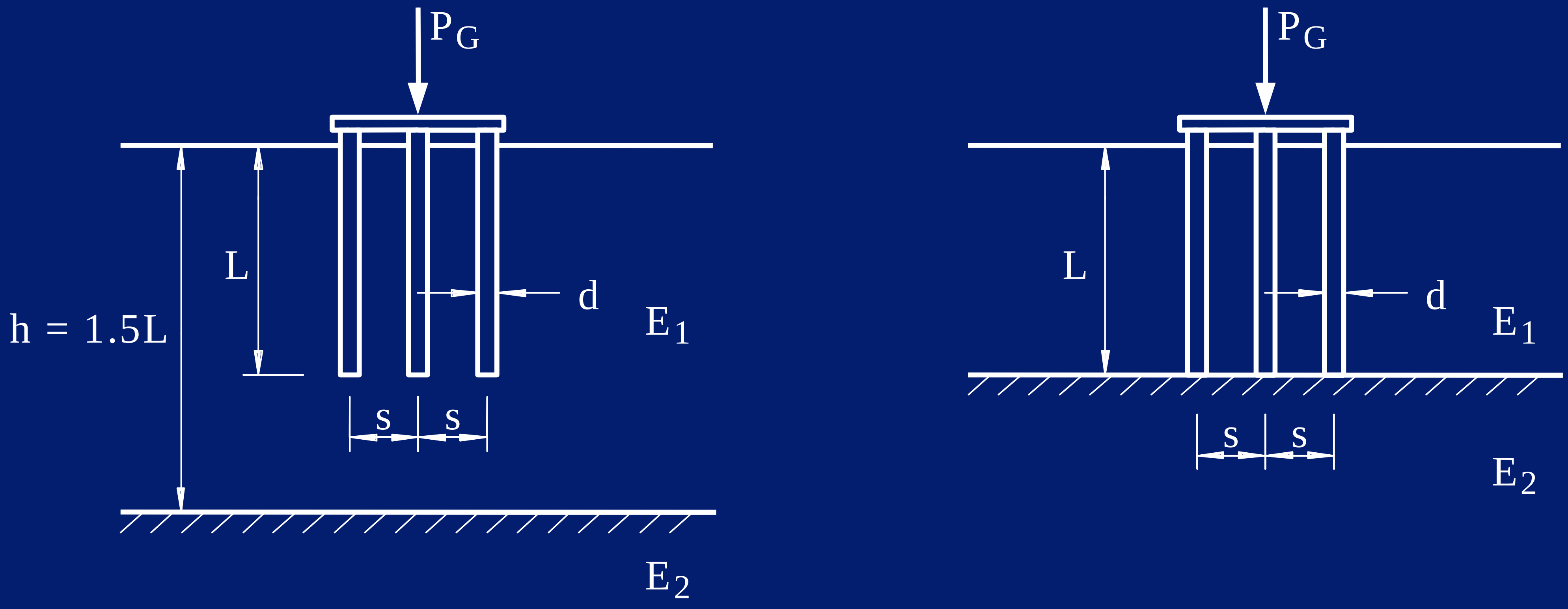




COMPUTER PROGRAMS FOR GROUP SETTLEMENT

Program	*Originator*	*Features*
PIGLET	Randolph	Simplified linear analysis with interaction factors
PGROUP	Banerjee	Linear boundary element analysis
DEFPIG	Poulos	Simplified boundary element analysis with interaction factors; non-linear capability
PGROUP	O'Neill	Hybrid: t-z (&p-y) analyses for single pile, elastic theory for group interaction

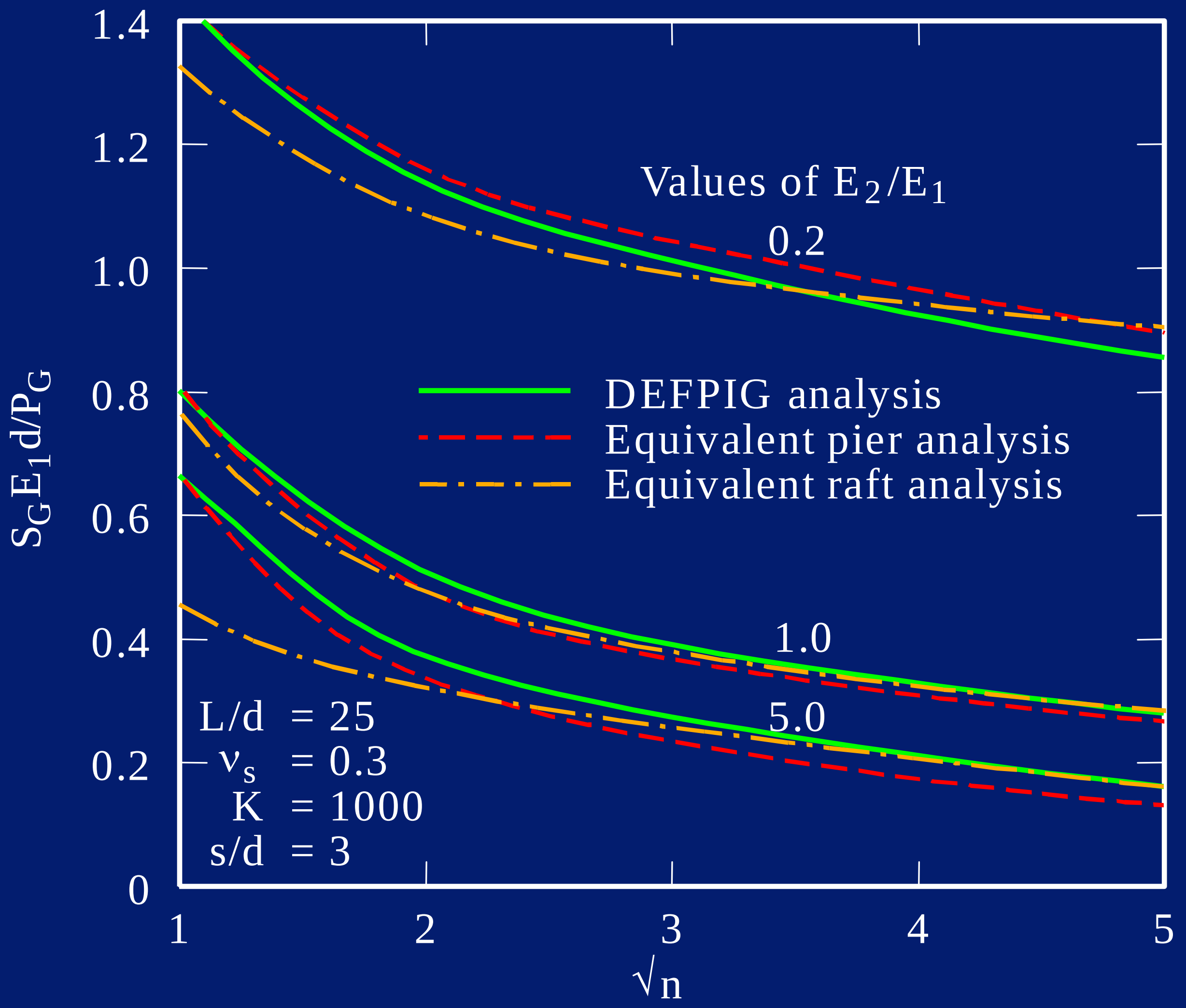
PILE GROUP CASES ANALYSED



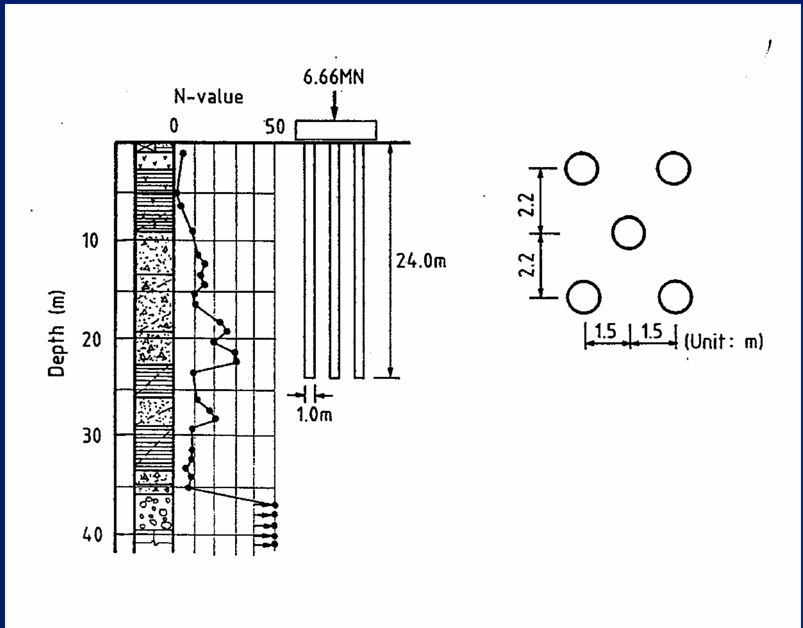
(a) Floating pile group

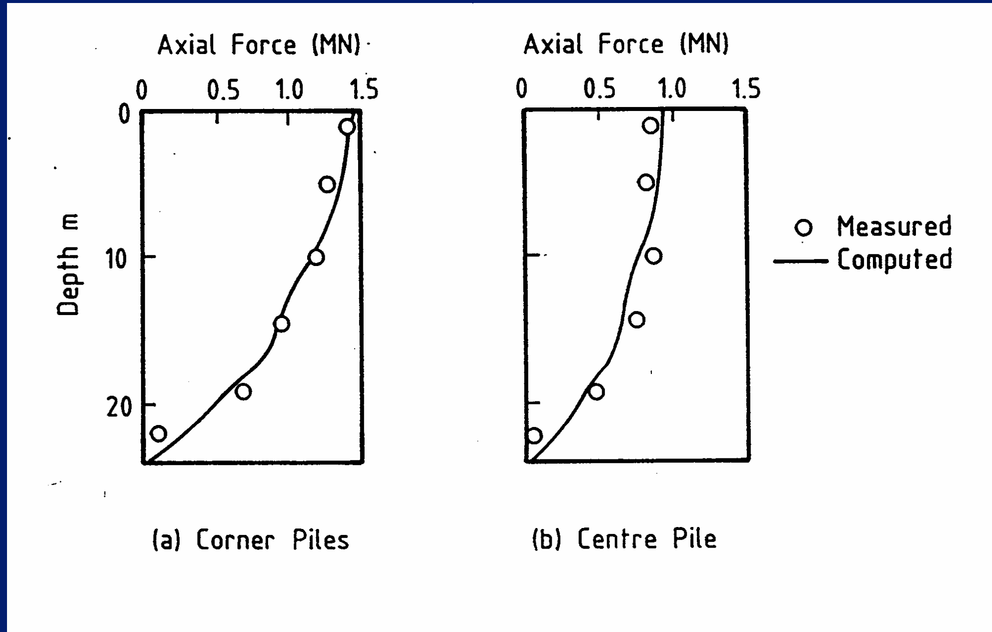
(b) End bearing pile group

COMPARISON BETWEEN SOLUTIONS FOR GROUP SETTLEMENT



APPLICATION TO CASE IN JAPAN

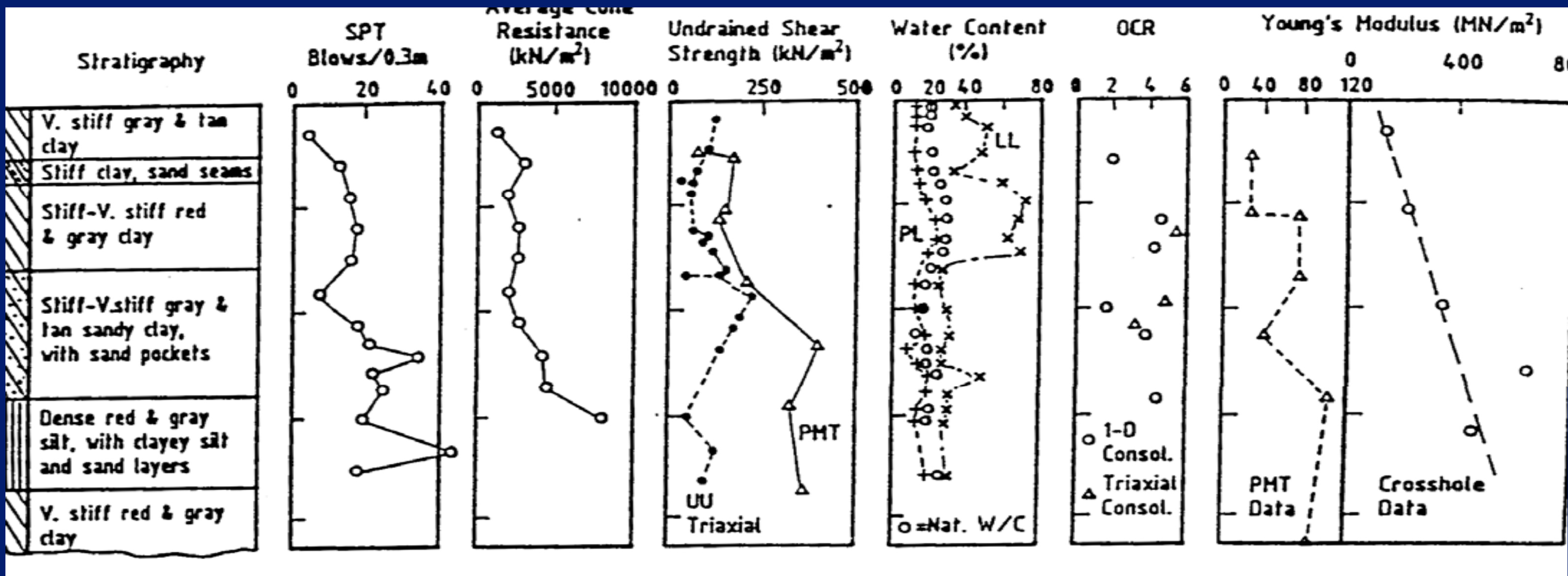




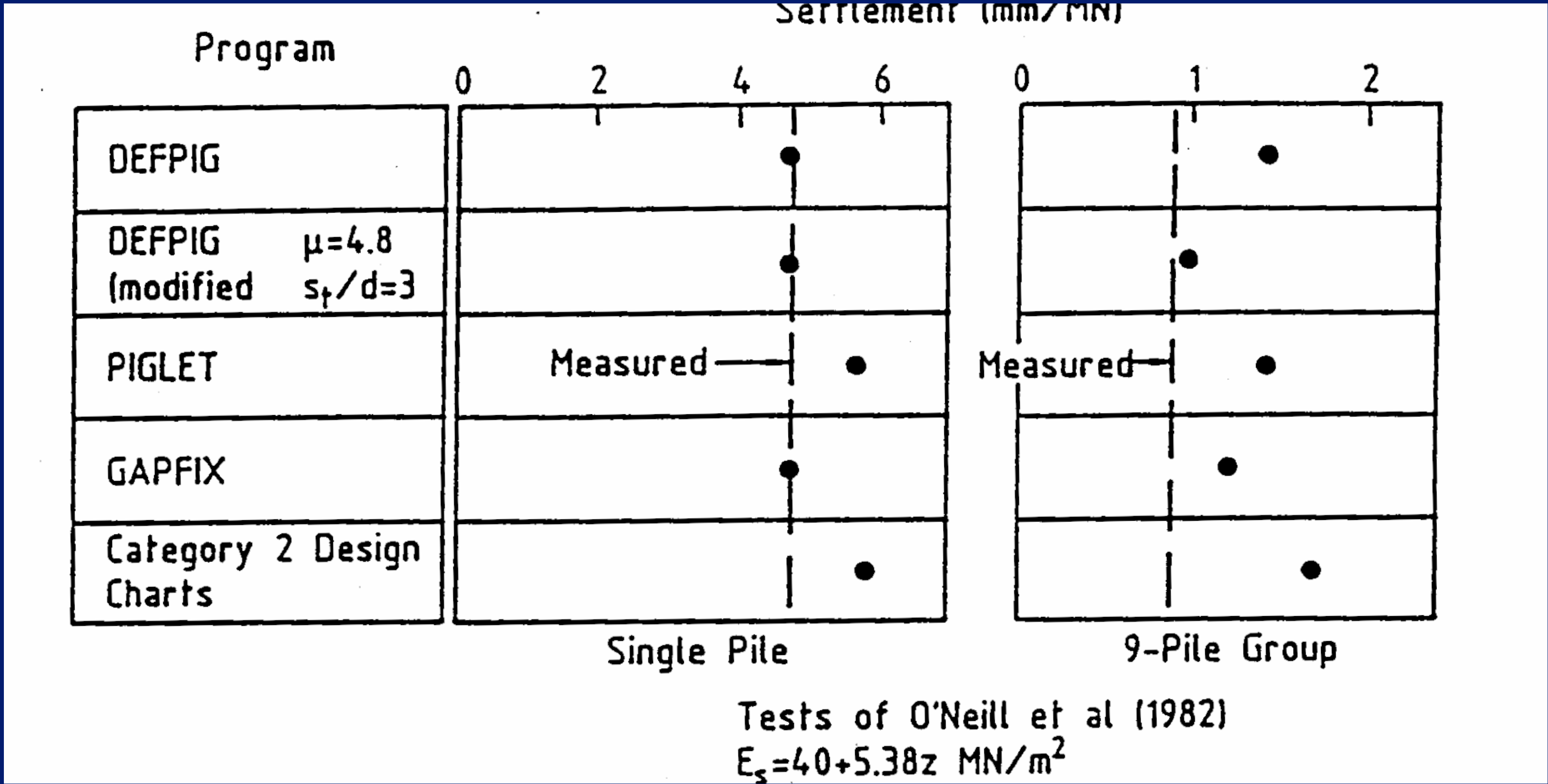
SENSITIVITY STUDY- CASE OF O'NEILL (1982)

Factors for decision:

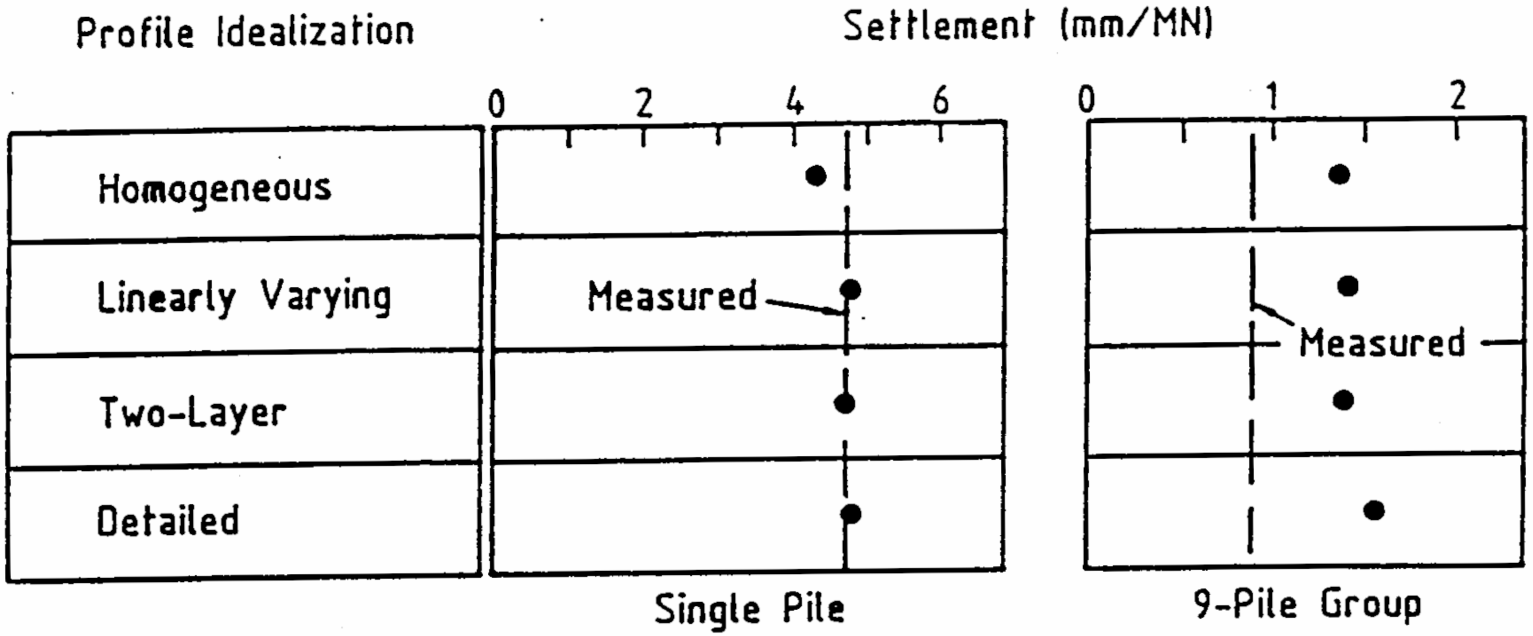
- Method of analysis & associated soil model
- Idealization of soil profile
- Geotechnical parameters



SENSITIVITY STUDY- EFFECT OF ANALYSIS METHOD

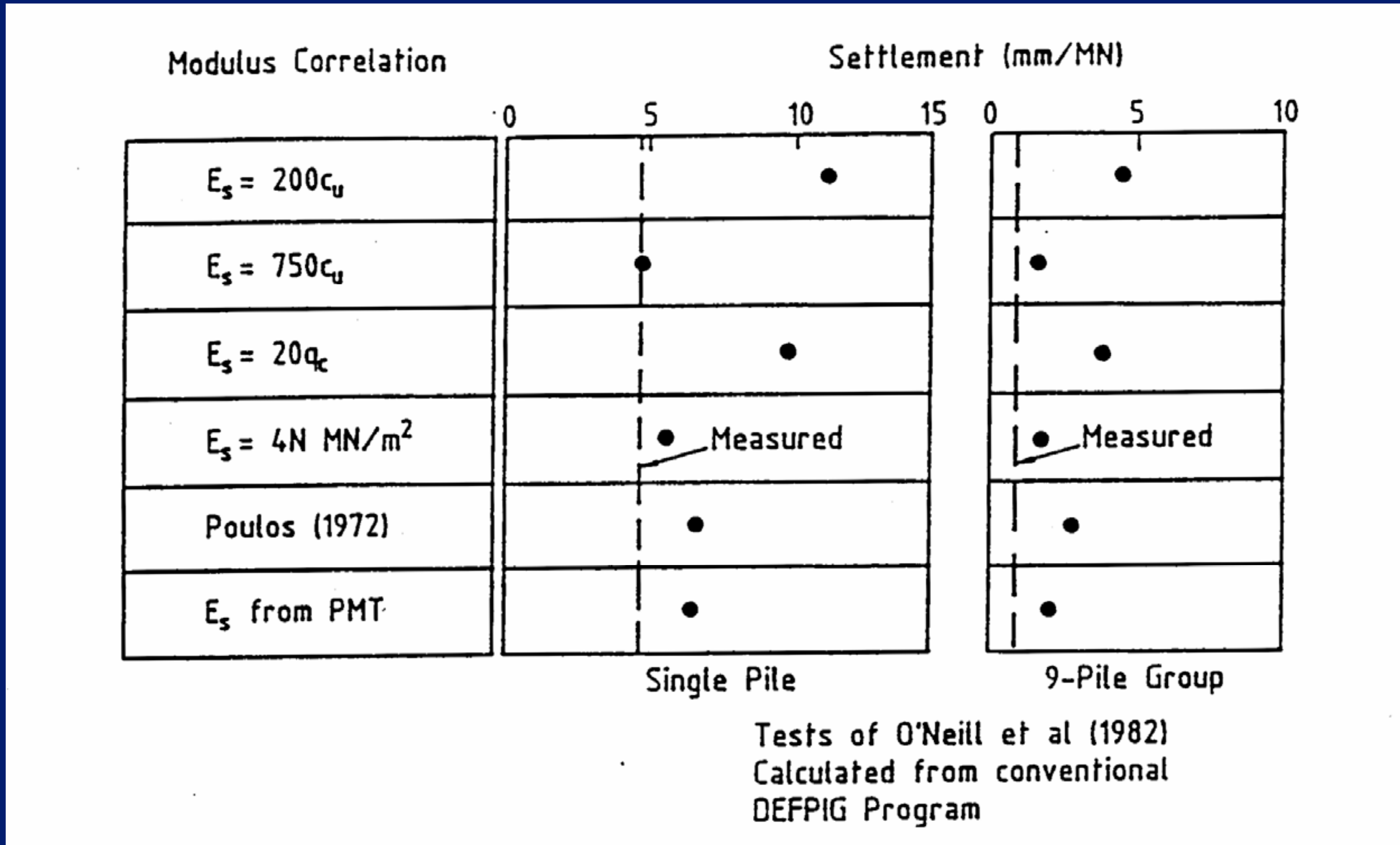


SENSITIVITY STUDY- EFFECT OF SOIL PROFILE IDEALIZATION

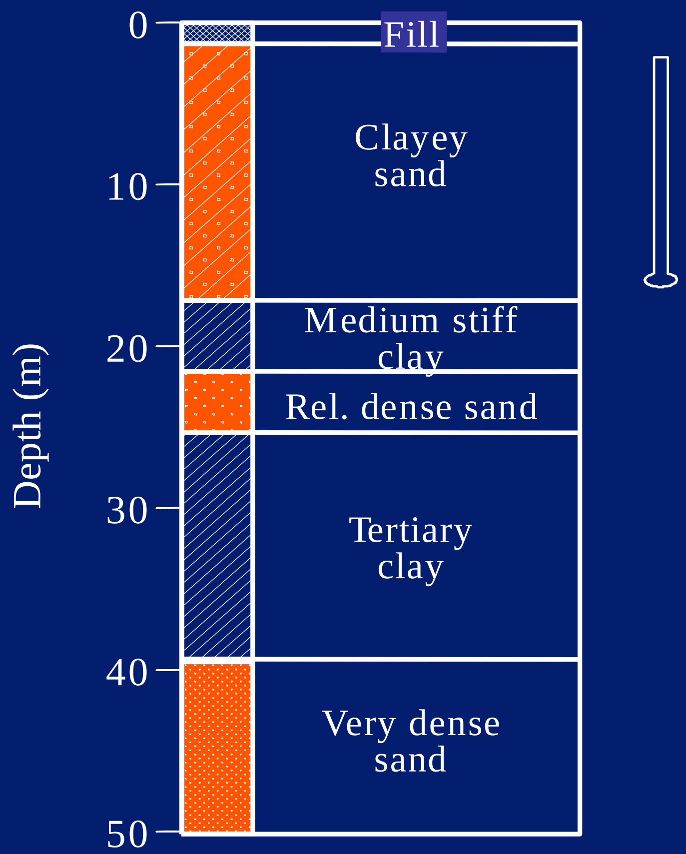


Tests of O'Neill et al (1982)
Calculated from conventional
DEFPIG Program $E_s = 750c_u$

SENSITIVITY STUDY- EFFECT OF SOIL MODULUS CORRELATION

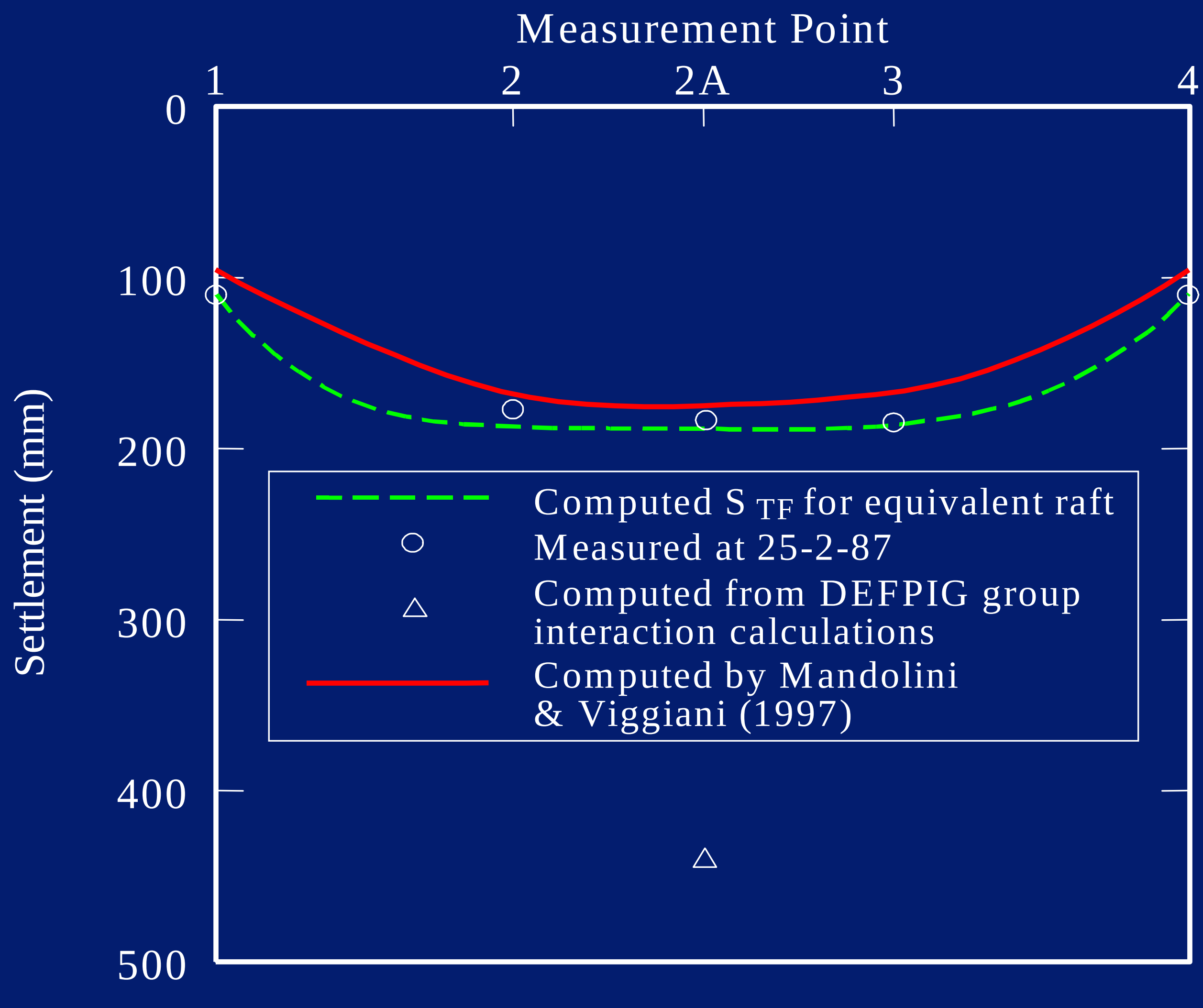


CASE OF GOOSENS & VAN IMPE (1991) - SOIL PROFILE & PARAMETERS



Average cone resistance. MPa	Young»s Modulus E_s MPa
<1	3
10	100
6	60
8	80
1.5	11
12	42
3.5	26
?	200 (assumed)

COMPARISON BETWEEN SETTLEMENT



LESSONS FROM COMPARISONS

- Assessment of soil modulus values is critical
- The method of analysis is less critical (provided it is sound)
- Beware of analyzing very large groups of piles with the interaction factor method. There is a potential for significant over-estimation of settlements
- Equivalent raft and pier analyses are ***useful checks*** on the order of group settlement and should **always** be carried out in addition to computer analyses