



# Tunnels Underground Excavations

Design Analyses  
Case Studies and Observed Performance

Giovanni Barla



*Department of Structural and Geotechnical Engineering*



## Lecture 1: Analytical and Numerical Methods

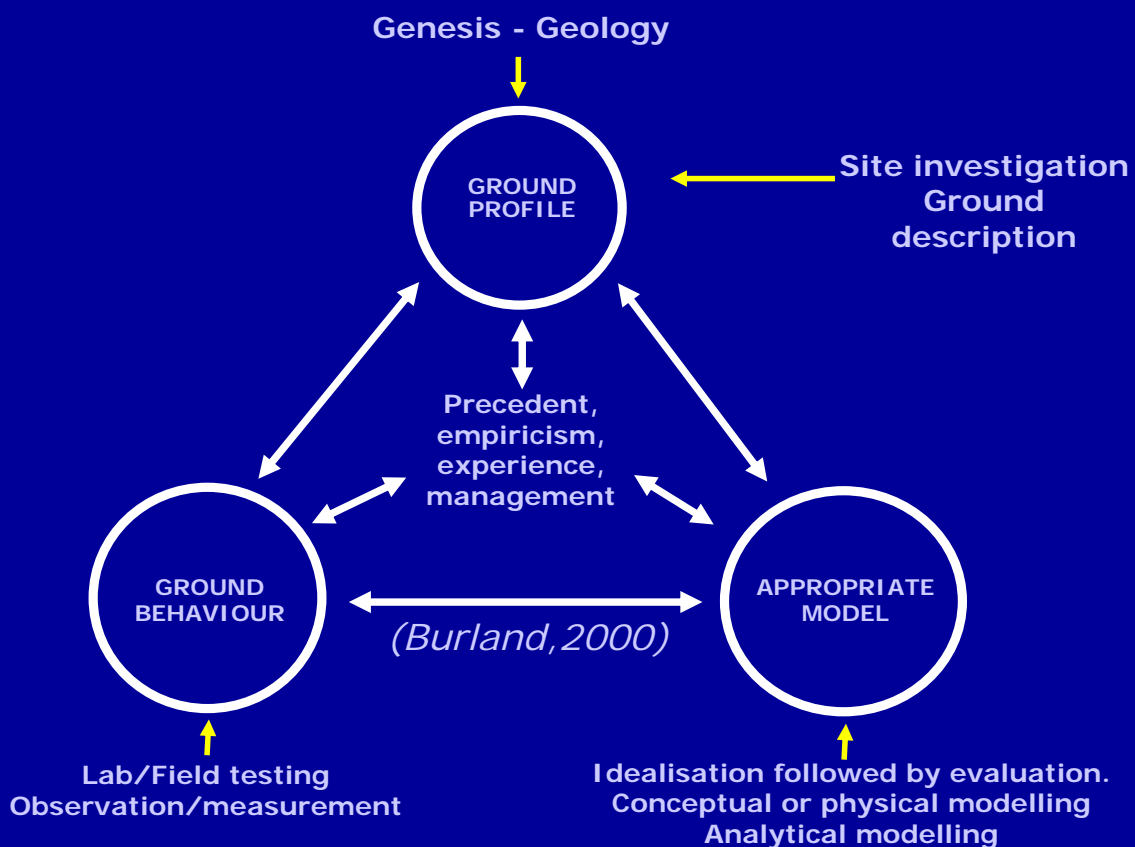




## Lecture Outline

- Geotechnical Triangle
- Interactive Design
- Continuum and Discontinuum Modelling
- Elastic and Elastoplastic Models
- Closed-form Solutions
- Numerical Methods

## THE GEOTECHNICAL TRIANGLE



# INTERACTIVE GEOTECHNICAL DESIGN



**Final Design** based on the most probable conditions (geology, hydrogeology, geotechnical parameters, construction methods,...)

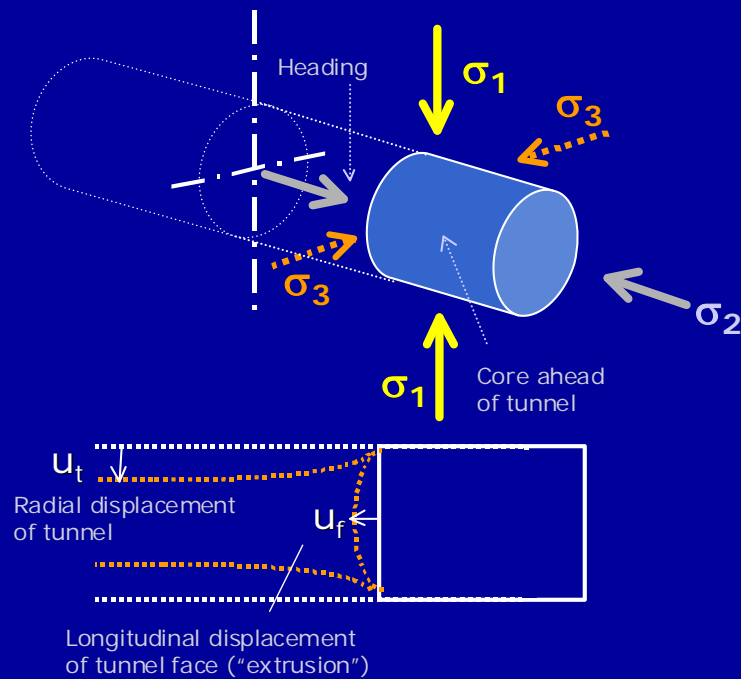
**Prediction of Rock Mass Behaviour** without (intrinsic conditions) or with reinforcement/support measures (identify excavation cross sections)

**Performance Monitoring** by a suitable well thought monitoring system to adopt during construction

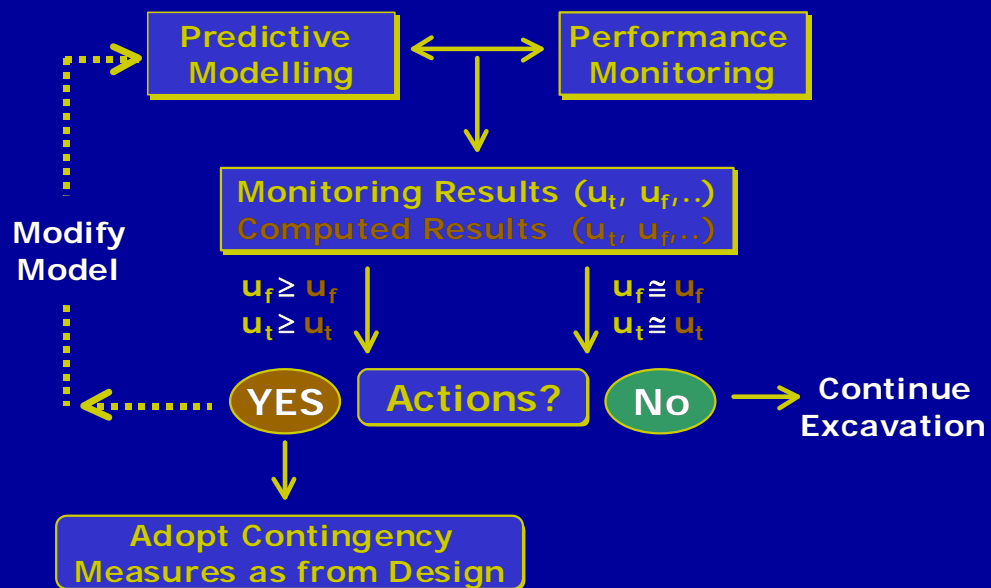
**Prediction of Rock Mass Behaviour** calls for the use of **Computational Models/Design Analyses**

There are no simple methods that can be adopted to predict the complex three-dimensional behaviour of underground structures, including the influence of reinforcement-support measures and rock mass-structure interaction

# Tunnel Response during face advance



## INTERACTIVE GEOTECHNICAL DESIGN



**The following requirements shall be met before construction is started:**

**(1) The limits of behaviour which are acceptable shall be established**

**The following requirements shall be met before construction is started:**

**(2) The range of possible behaviour shall be assessed and it shall be shown that there is an acceptable probability that the actual behaviour will be within the acceptable limits**

**The following requirements shall be met before construction is started:**

**(3) A plan of monitoring shall be devised which will reveal whether the actual behaviour lies within the acceptable limits. The monitoring shall make this clear at a sufficiently early stage and with sufficiently short intervals to allow contingency actions to be undertaken successfully**


**The following requirements shall be met before construction is started:**

**(4) The response time of the instruments and the procedures for analysing the results shall be sufficiently rapid in relation to the possible evolution of the system**

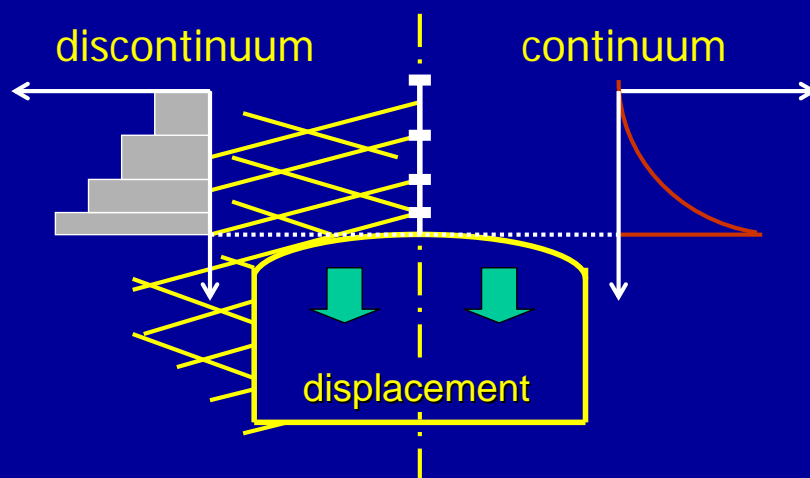


**The following requirements shall be met before construction is started:**

**(5) a plan of contingency actions shall be devised which may be adopted if the monitoring reveals behaviour outside acceptable limits**



**Continuum and  
Discontinuum Modelling**

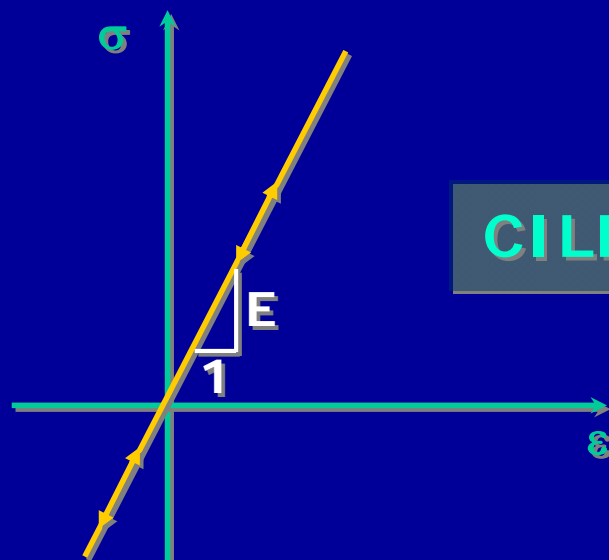




## Elastic and Elastoplastic Models

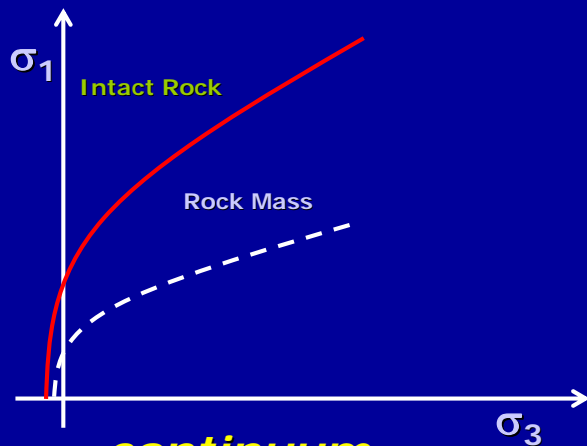


## Continuum Isotropic Linearly Elastic (CILE)

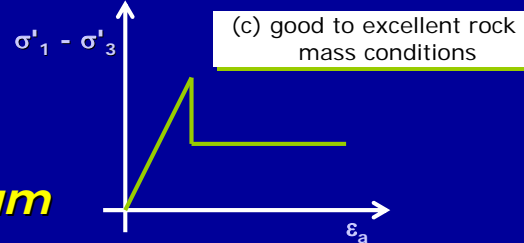
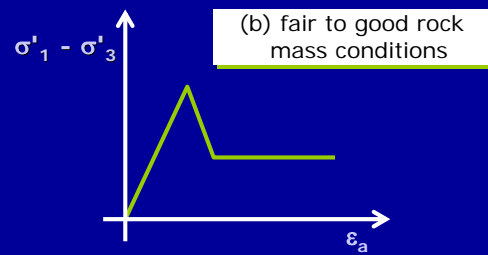
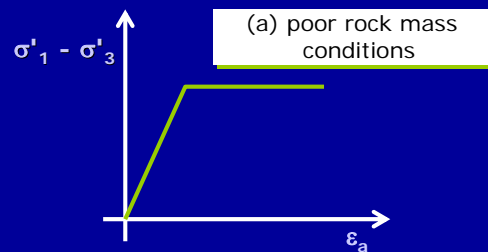


# ELasto-PLAstic (ELPLA) Isotropic Continuum

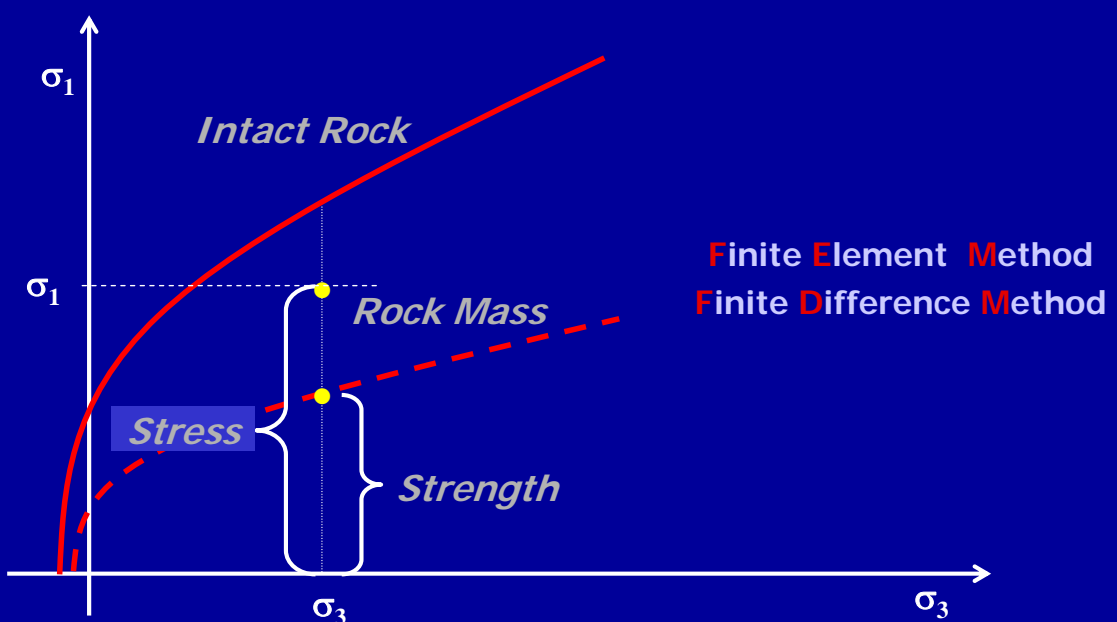
THE INTACT ROCK PROPERTIES NEED BE  
SCALED DOWN TO THE ROCK MASS  
PROPERTIES



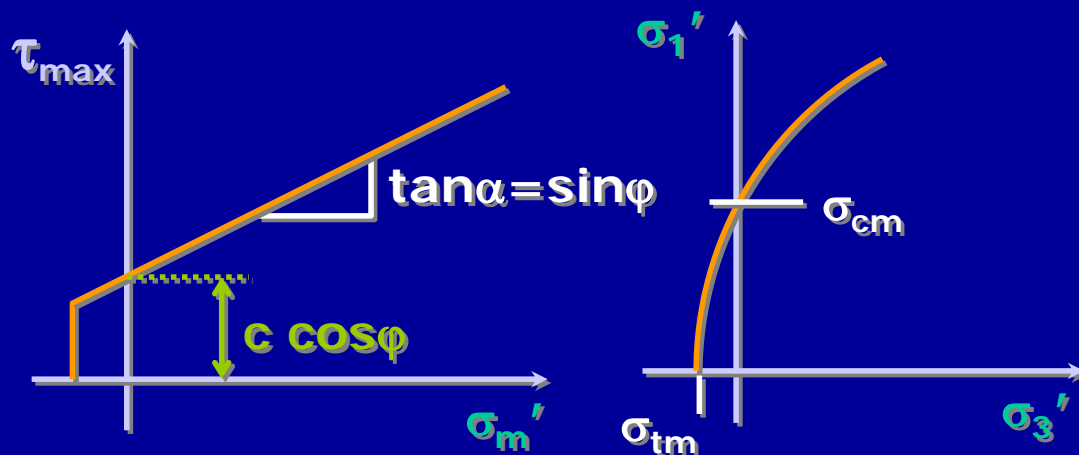
- **continuum**
- **equivalent continuum**



*Illustration of the scaling process and definition of Factor of Safety*



**CONTINUUM MODELLING**



**Mohr-Coulomb  
Criterion**

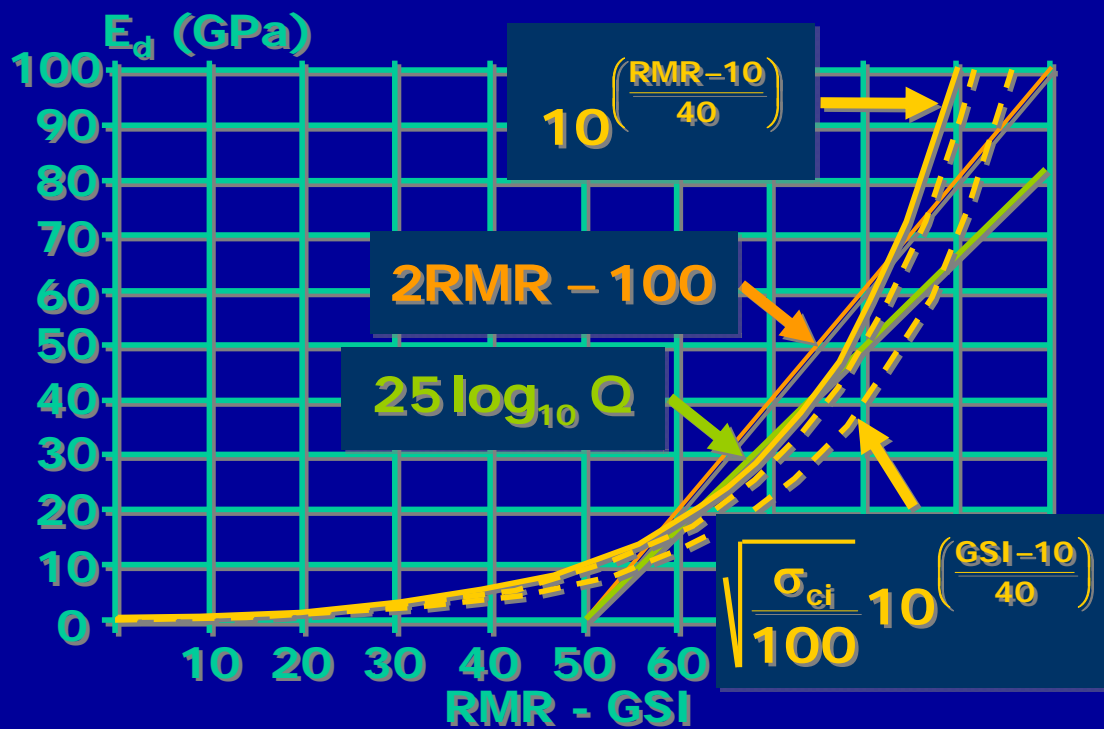
**Hoek-Brown  
Criterion**

### Continuous Isotropic Linearly Elastic Medium (CILE)

For a CILE medium we need the  
following parameters:  
the **elastic (Young's) modulus  $E$**   
and  
**the Poisson's ratio  $\nu$**



## Elastic Modulus $E_d$

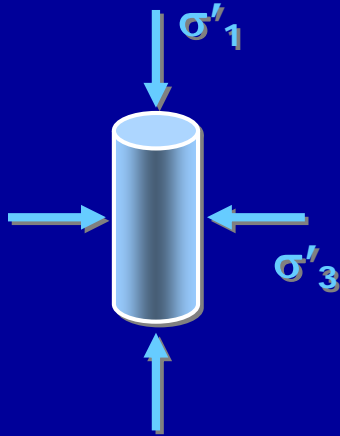


## ELasto-PLAstic (ELPLA) Isotropic Continuum

In this case, we also need the following parameters:

- for the M-C criterion:  $c, \phi$
- for the H-B criterion:  $\sigma_{ci}, m_b, s$

# Hoek and Brown criterion for the rock mass



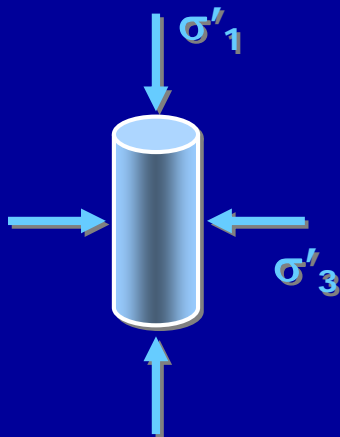
$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left( m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^\alpha$$

$\sigma_{ci}$

intact rock  
uniaxial  
compressive  
strength

$s, m_b$

Hoek–Brown  
constants  
for the rock  
mass



$\alpha$

exponent of the  
H-B criterion  
(usually  $\alpha = 0.5$ )

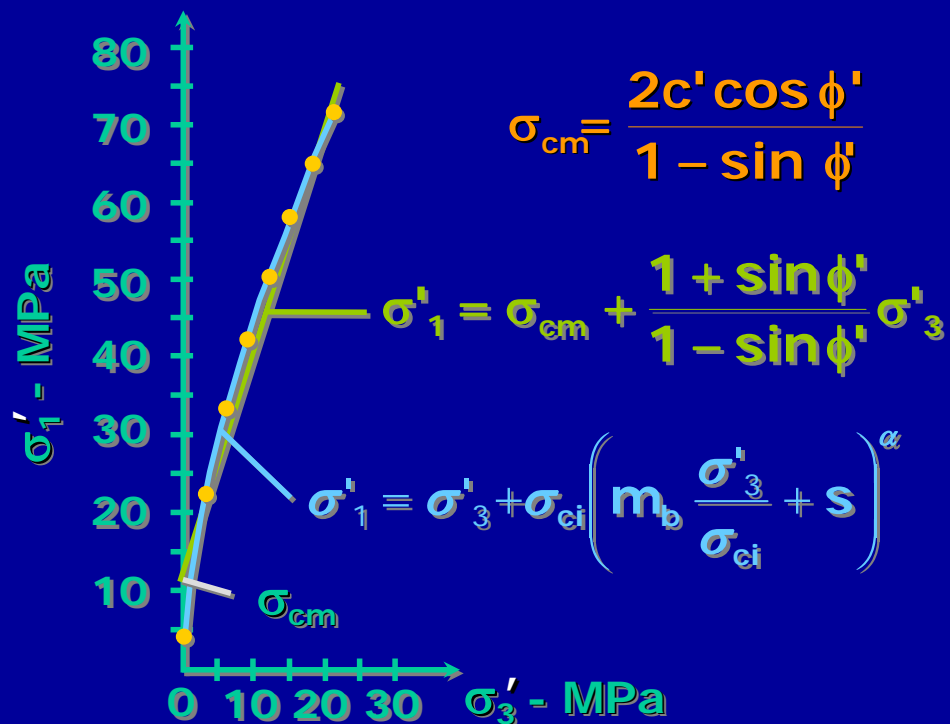
# Linearization Procedure

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left( m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^\alpha$$

$$m_b = m_i \exp\left(\frac{GSI - 100}{28}\right)$$

$$s = \exp\left(\frac{GSI - 100}{9}\right)$$

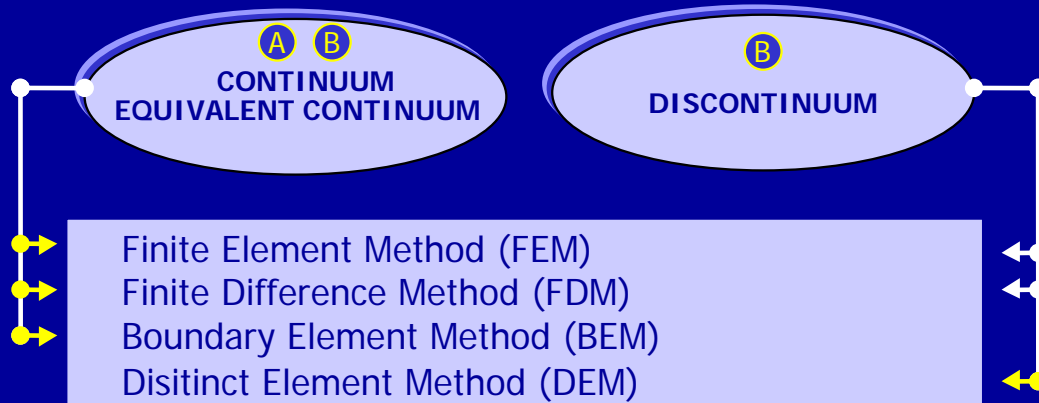
$\sigma_{ci}$  and  $m_i$   
from uniaxial  
and triaxial  
laboratory  
tests



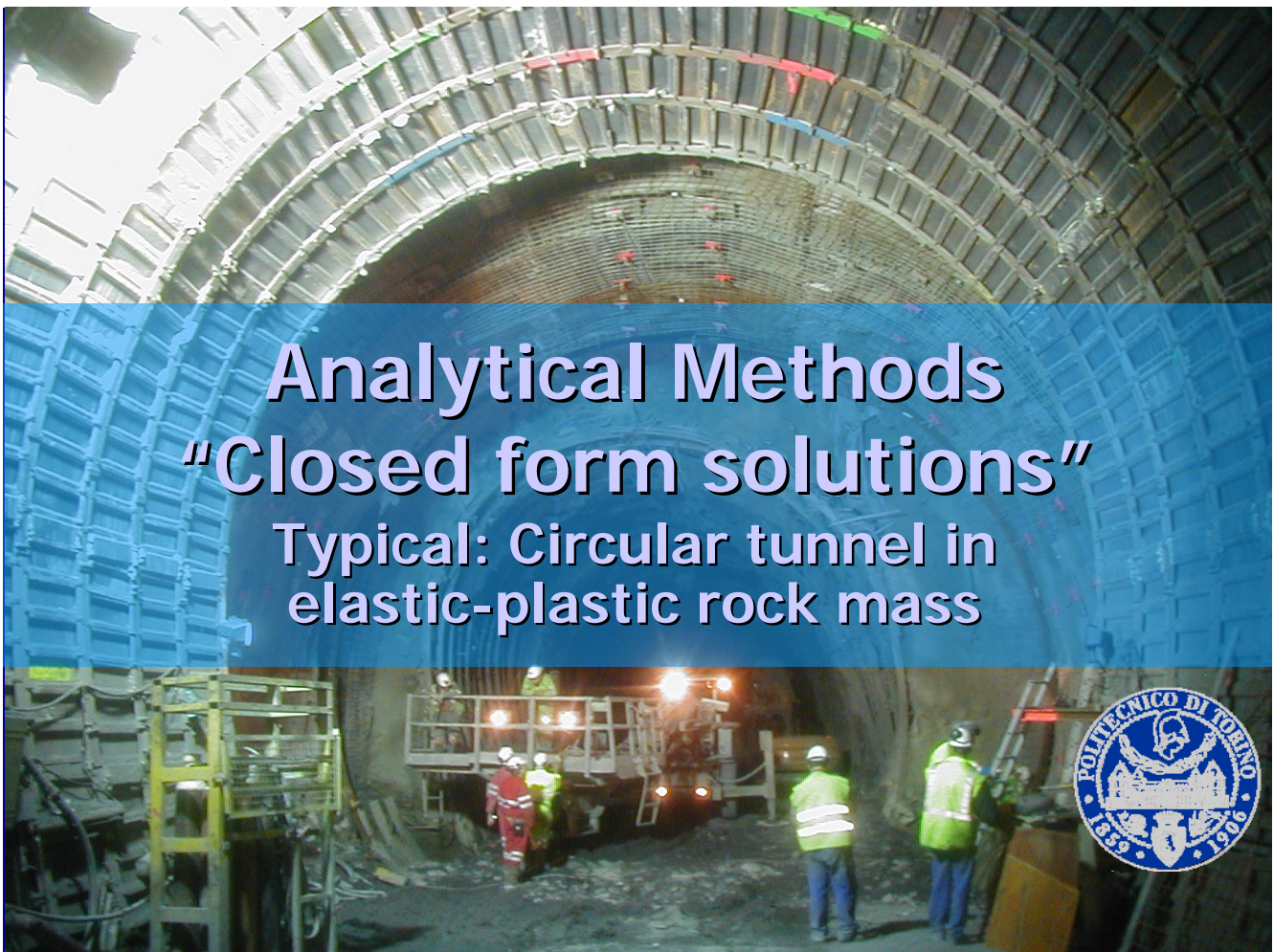


# Design Analysis Methods

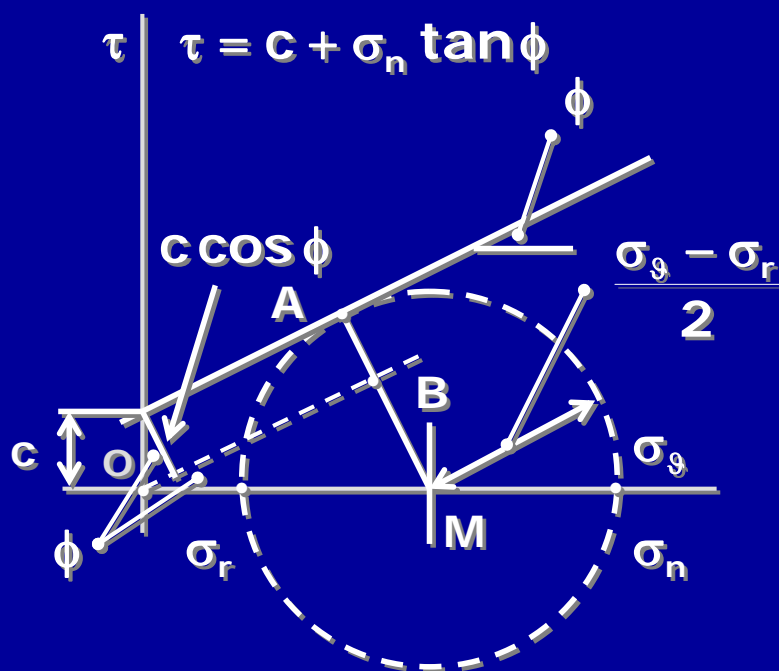
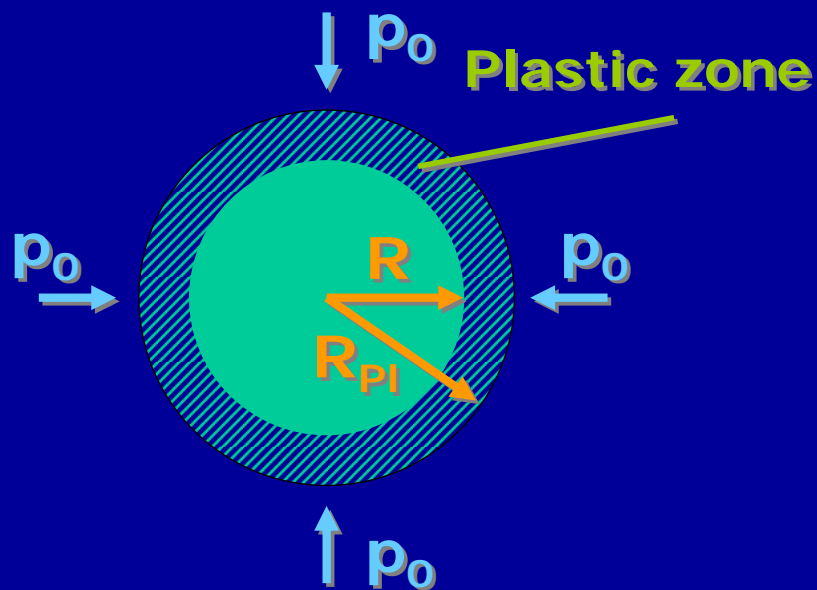
- (1) Closed form solutions  
(2) Numerical methods



**Analytical Methods**  
**"Closed form solutions"**  
Typical: Circular tunnel in elastic-plastic rock mass



## (a) Simplified solution



$$\sigma_\theta = ?$$

The result finally  
obtained is



with  $p_i$

$$R_{pl} = R \left( \frac{(p_o \sin \phi + c \cos \phi) (1 - \sin \phi)}{p_i \sin \phi + c \cos \phi} \right)^{\frac{1 - \sin \phi}{2 \sin \phi}}$$

The result finally  
obtained is



with  $R_{pl}$

$$p_i = \frac{1}{\sin \phi} \left( (p_o \sin \phi + c \cos \phi) (1 - \sin \phi) \left( \frac{R}{R_{pl}} \right)^\alpha - c \cos \phi \right)$$

$$\text{where } \alpha = \frac{2 \sin \phi}{1 - \sin \phi}$$



## **(b) Complete Solution**

### **Assumptions**

**Initial State of stress: isotropic  $k_0=1$**

**R.M. homogeneous and isotropic**

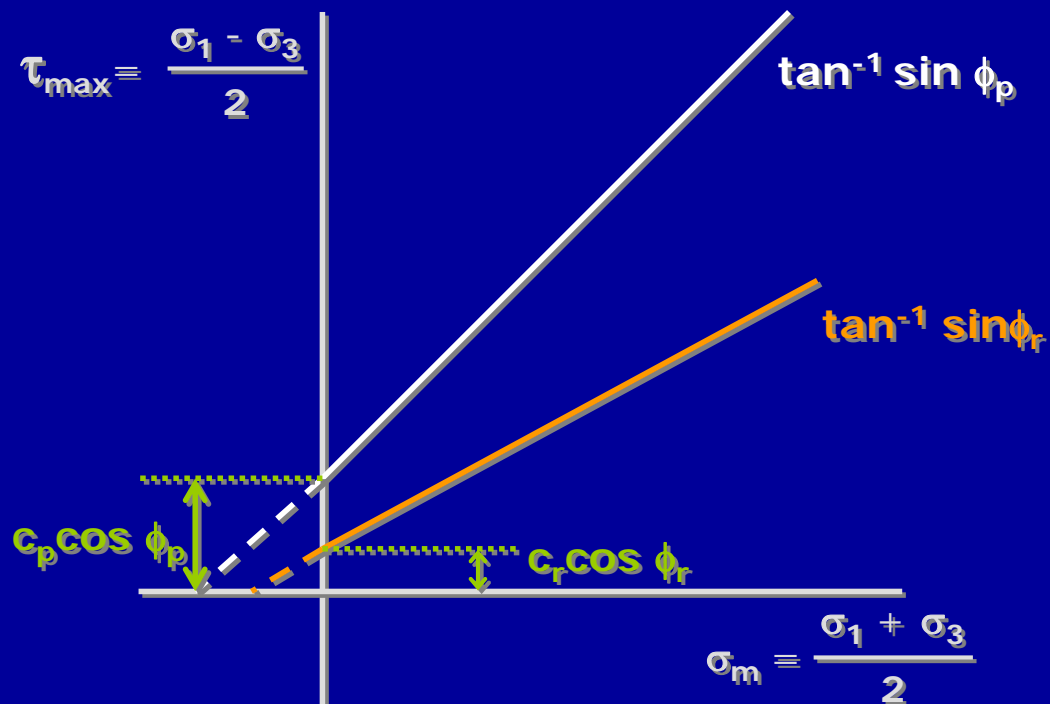
**Deep circular tunnel**

**Plane strain conditions**

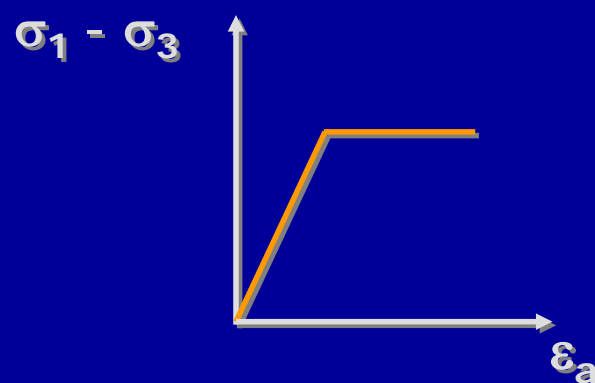
**Constitutive Models  
Closed Form Solutions**



**Mohr – Coulomb  
yield criterion**

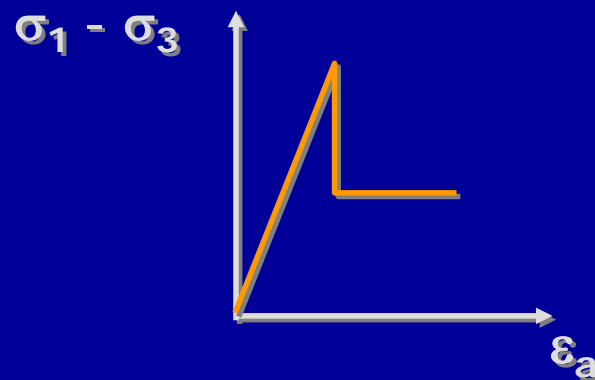


## Constitutive Model 1



**Elastic-plastic Ideally Plastic,  
for poor to fair rock mass  
quality**

## Constitutive Model 2



Elastic-plastic Ideally  
Brittle, for good to  
excellent rock mass quality

## Equilibrium equation for axis-symmetric problem

1

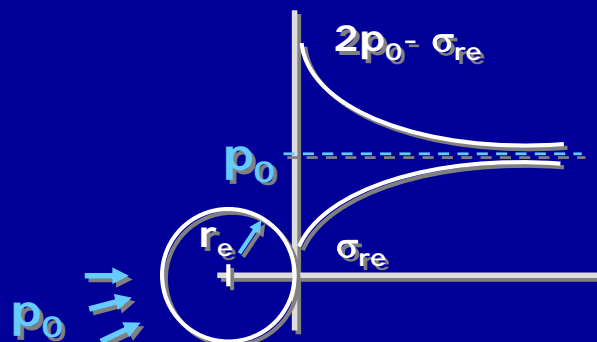
$$\frac{d\sigma_r}{dr} + \frac{\sigma_\theta - \sigma_r}{r} = 0$$

## Lamé's Solution

State of stress in a CILE ground  
around a circular hole  
with radius  $r_e$ , inside pressure  
 $\sigma_{re}$  and at infinity  $p_o$

2

$$\begin{cases} \sigma_\theta = p_o + (p_o - \sigma_{re}) \left( \frac{r_e}{r} \right)^2 \\ \sigma_r = p_o - (p_o + \sigma_{re}) \left( \frac{r_e}{r} \right)^2 \end{cases}$$



## Strains $\varepsilon_{re}$ ed $\varepsilon_{\theta e}$ in CILE

3

$$\begin{cases} \varepsilon_{re} = -\frac{1+\nu}{E}(p_0 - \sigma_{re}) \\ \varepsilon_{\theta e} = \frac{1+\nu}{E}(p_0 - \sigma_{re}) \end{cases}$$

## Yield Criterion (Mohr - Coulomb)

4

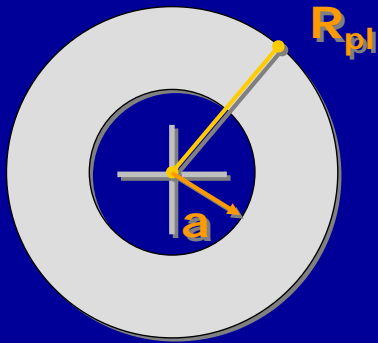
$$\begin{cases} \tau_{\max} = c \cos \phi + \sigma_m \sin \phi \\ \sigma_1 = 2c \sqrt{N_\phi} + N_\phi \sigma_3 \\ N_\phi = \frac{1 + \sin \phi}{1 - \sin \phi} \end{cases}$$

peak:  $c_p, \phi_p$   
residual:  $c_r, \phi_r$



1

## Plastic Radius



$$p_{cr} = p_0 (1 - \sin \phi_p) - c_p \cos \phi_p$$

## Critical Pressure

$$R_{pl} = a \left\{ \frac{(p_0 + c_r \cot g \phi_r)}{p_i + c_r \cot g \phi_r} - \frac{(p_0 + c_p \cot g \phi_p) \sin \phi_p}{p_i + c_r \cot g \phi_r} \right\}^{\frac{1}{N_r - 1}}$$

## Plastic Radius

$$R_{pl} = a \left\{ \frac{(p_0 + c_r \cot g\phi_r)}{p_i + c_r \cot g\phi_r} - \frac{(p_0 + c_p \cot g\phi_p) \sin \phi_p}{p_i + c_r \cot g\phi_r} \right\}^{\frac{1}{N_r - 1}}$$

**NOTE**

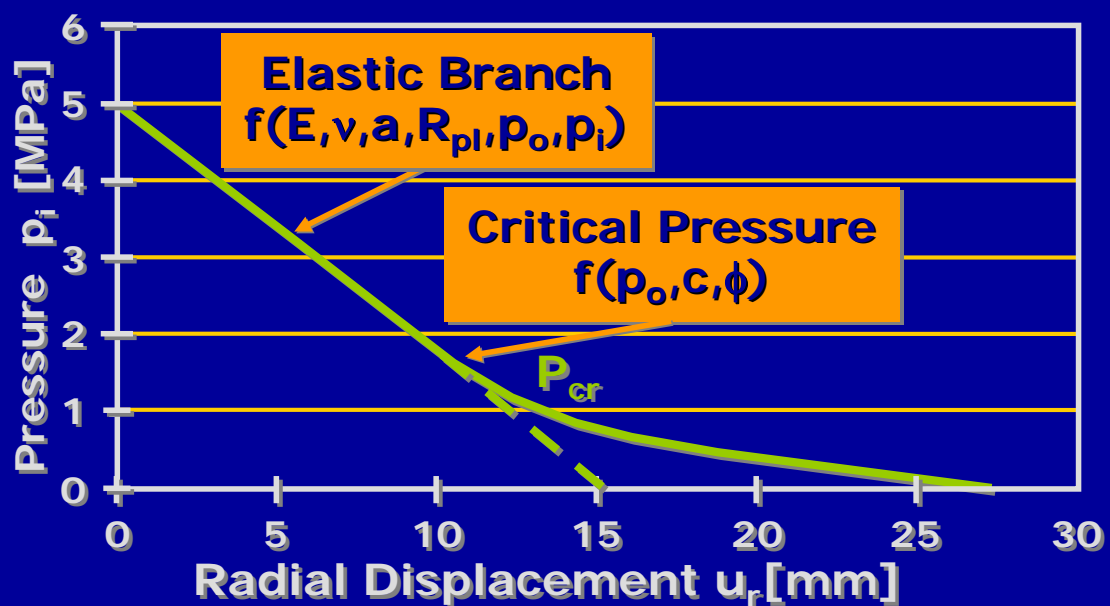
For  $c_p = c_r$  and  $\phi_p = \phi_r$   
We obtain the simplified  
solution (a)

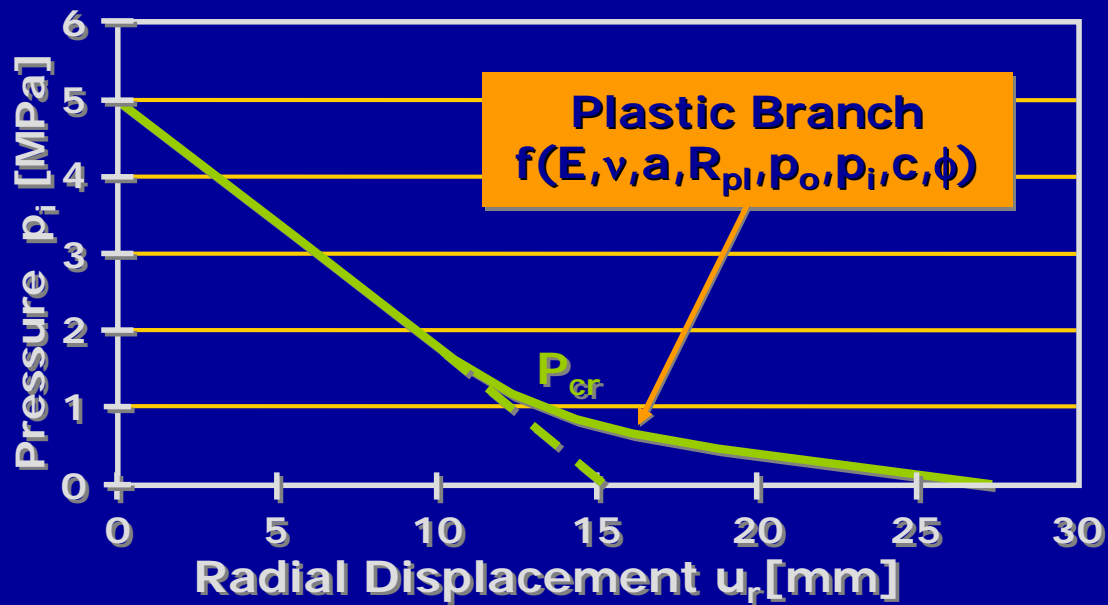
Compute the radial  
displacement in the  
elastic zone,  
i.e. for  $r \geq R_{pl}$

$$u_r = \frac{1 + \nu}{E} \cdot (p_0 - p_{cr}) \cdot \frac{R_{pl}^2}{r}$$

Compute the radial displacement  
in the plastic zone,  
i.e. for  $a \leq r \leq R_{pl}$

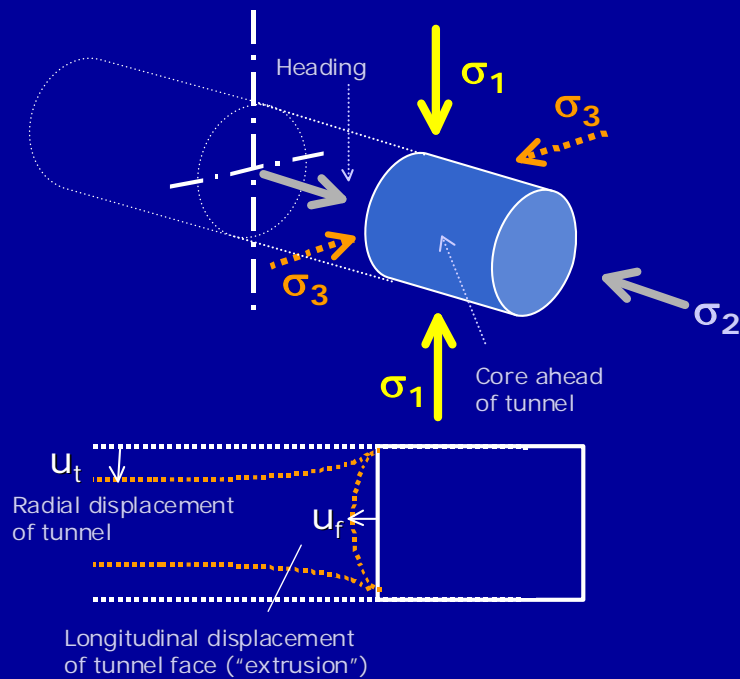
$$u_r = \frac{1+\nu}{E} \left\{ \frac{R_{pl}^{k+1}}{r^k} (p_o + c_p \cot g\phi_p) \sin\phi_p + \right. \\ \left. + (1-2\nu) (p_o + c_r \cot g\phi_r) \left( \frac{R_{pl}^{k+1}}{r^k} - r \right) + \right. \\ \left. - \frac{(p_i + c_r \cot g\phi_r) [1 + N_r k - \nu(k+1)(1+N_r)]}{(N_r + k) a^{N_r-1}} \cdot \right. \\ \left. \cdot \left[ \frac{R_{pl}^{(N_r+k)}}{r^k} - r^{N_r} \right] \right\} \quad N_r = \frac{1+\sin\phi_r}{1-\sin\phi_r}; \quad k = \frac{1+\sin\psi}{1-\sin\psi}$$





The **characteristic curve (ground reaction curve)** gives a relationship between the radial displacement  $u_r$  at the tunnel contour and the applied pressure  $p_i$

# Tunnel Response during face advance



$$u_r = \lambda(x) u_r(\infty) = \lambda(x) [(p_o a) / (2G)]$$

$$\lambda(x) = 0,28 + 0,72 \left[ 1 - \left( \frac{0,84a}{0,84a + x} \right) \right]$$

Panet & Guenot (1982)

ILE ground



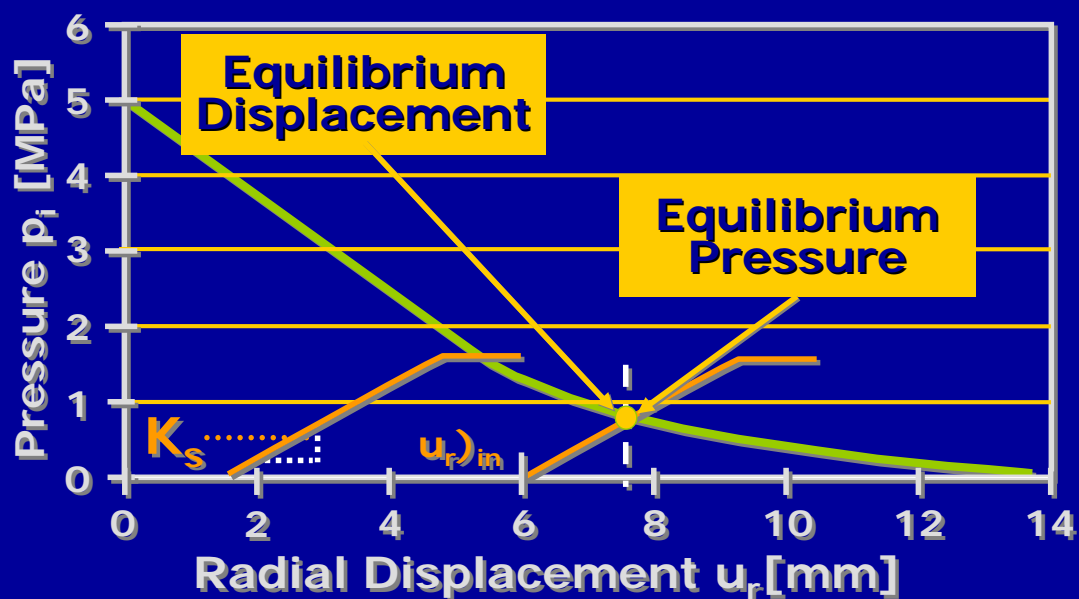
$$u_r = \lambda(x) u_r(\infty)$$

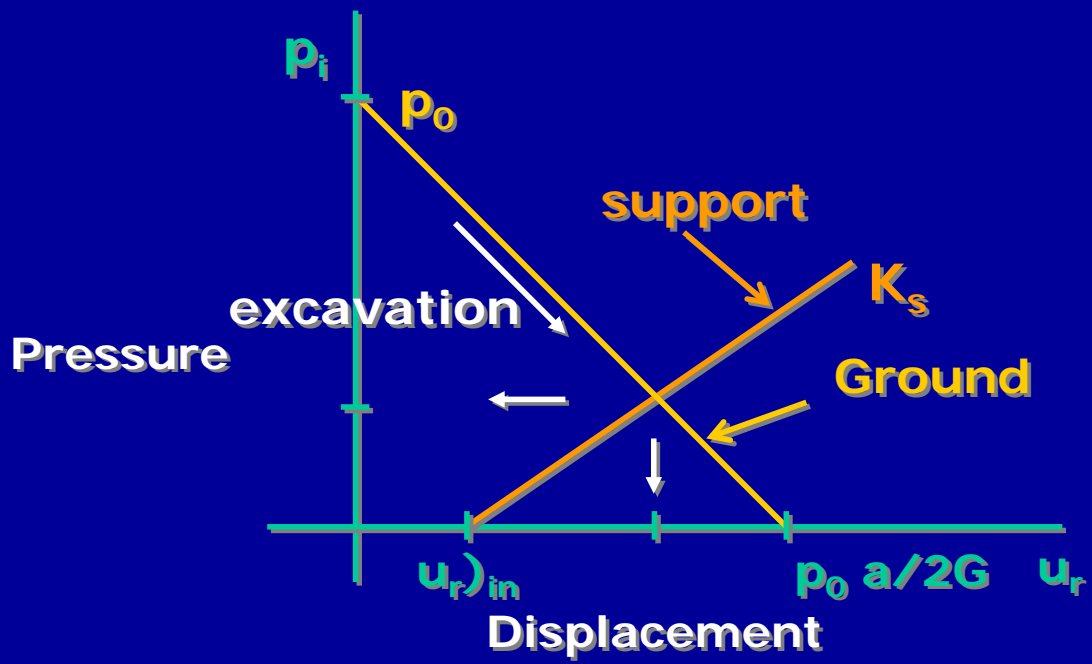
$$\lambda(x) = 1 - \left[ \frac{1}{1 + \frac{x}{0,84R_{pl}}} \right]^2$$

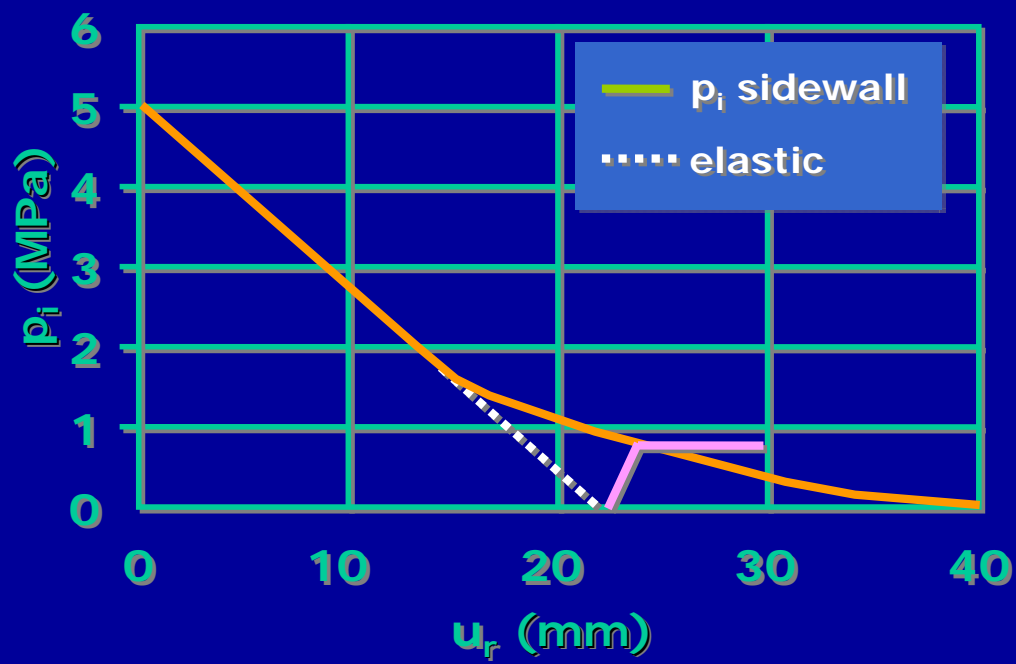
Panet & Guenot (1982)

ELPLA ground

## Ground reaction

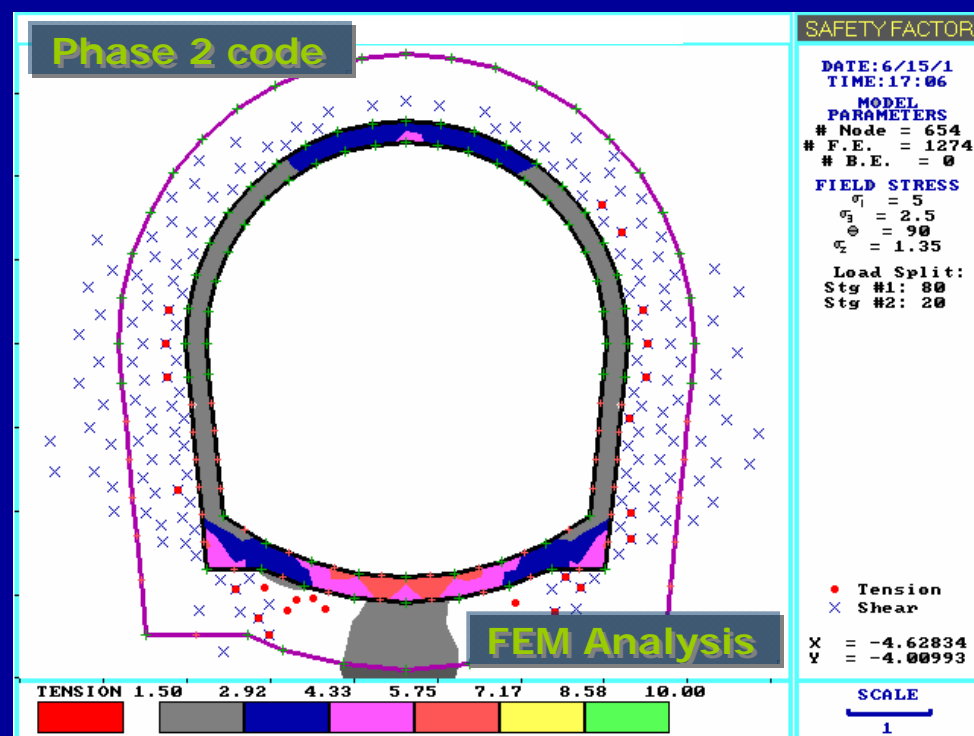
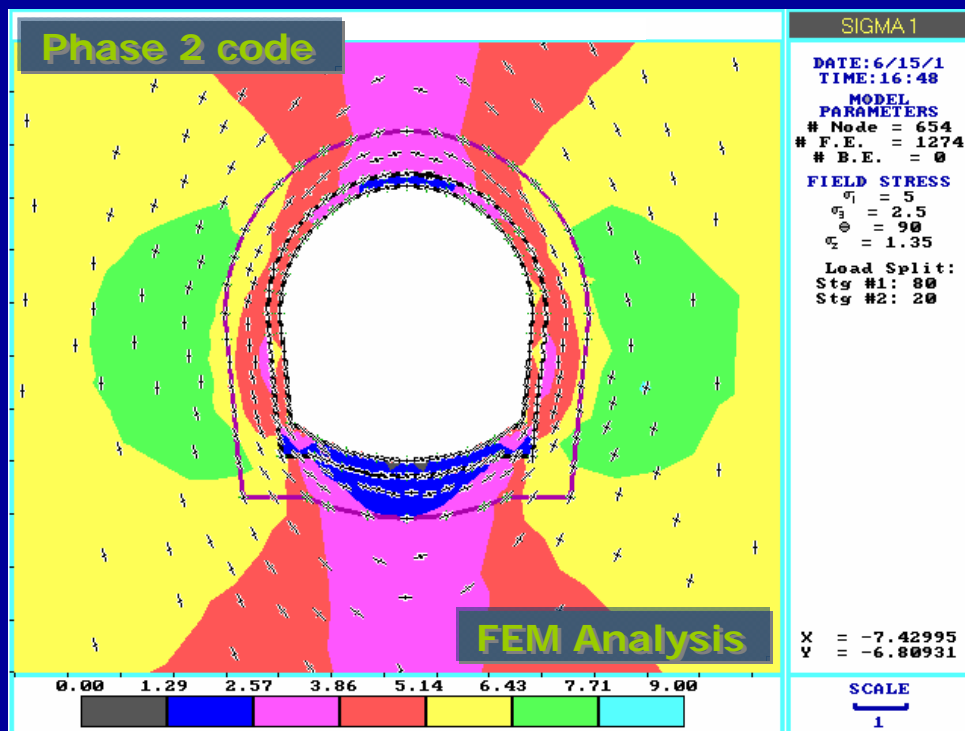




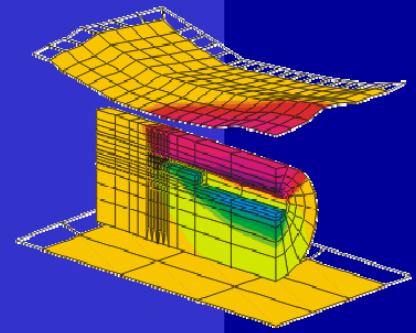
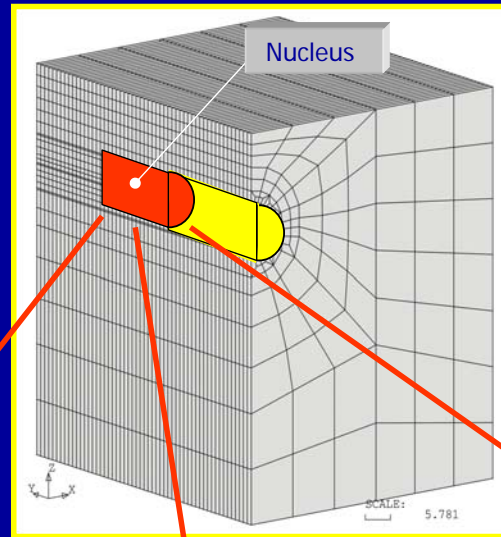
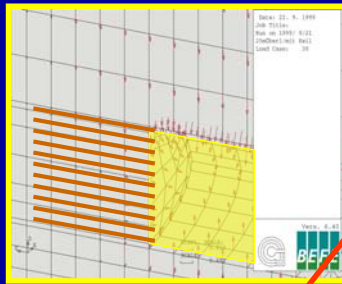


## Numerical Methods "Continuum Modelling"





# 3-D ANALYSES



Fictitious Pressure



(a)

Equivalent Nucleus

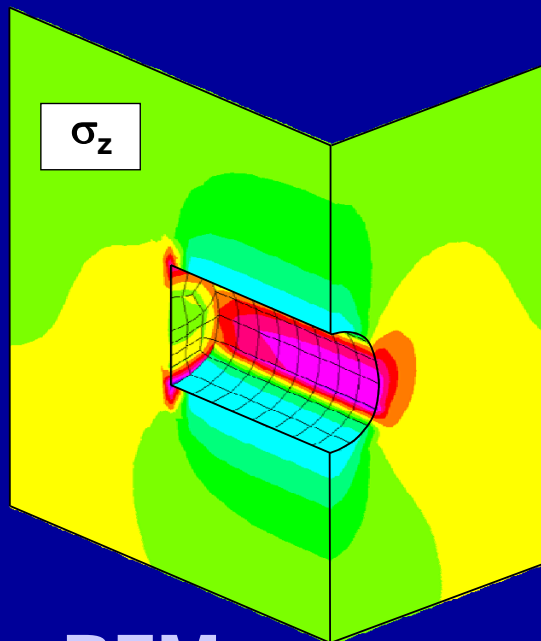


(b)

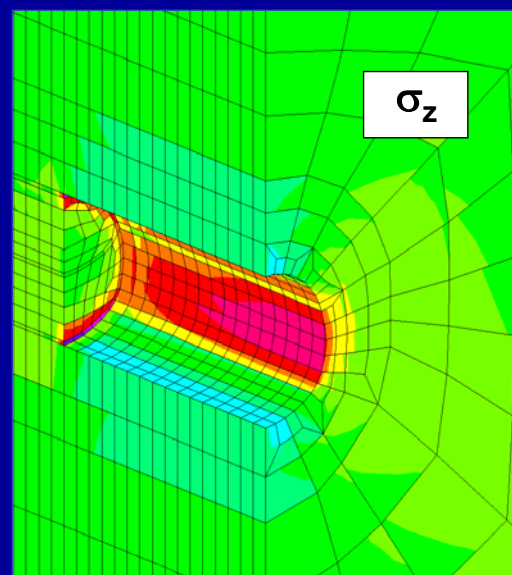
Reinforcing Elements



(c)

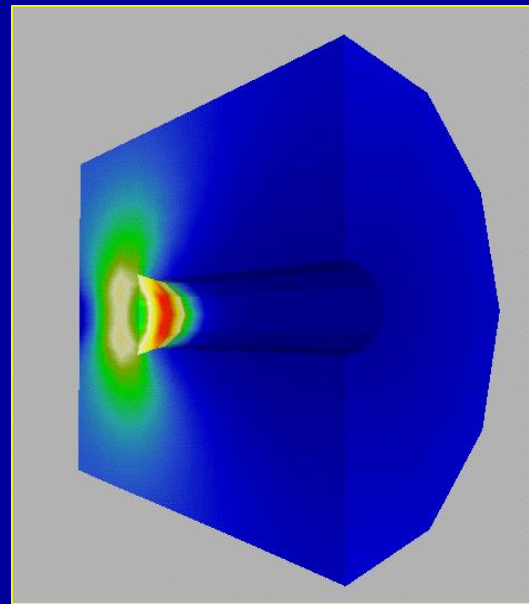
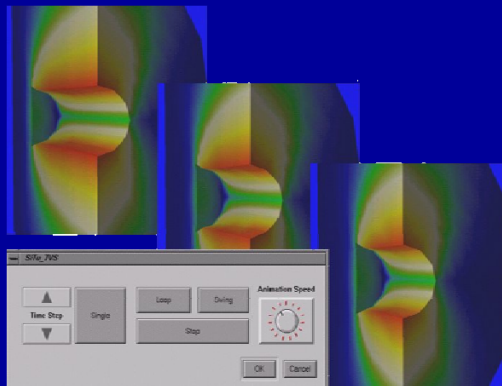


BEM



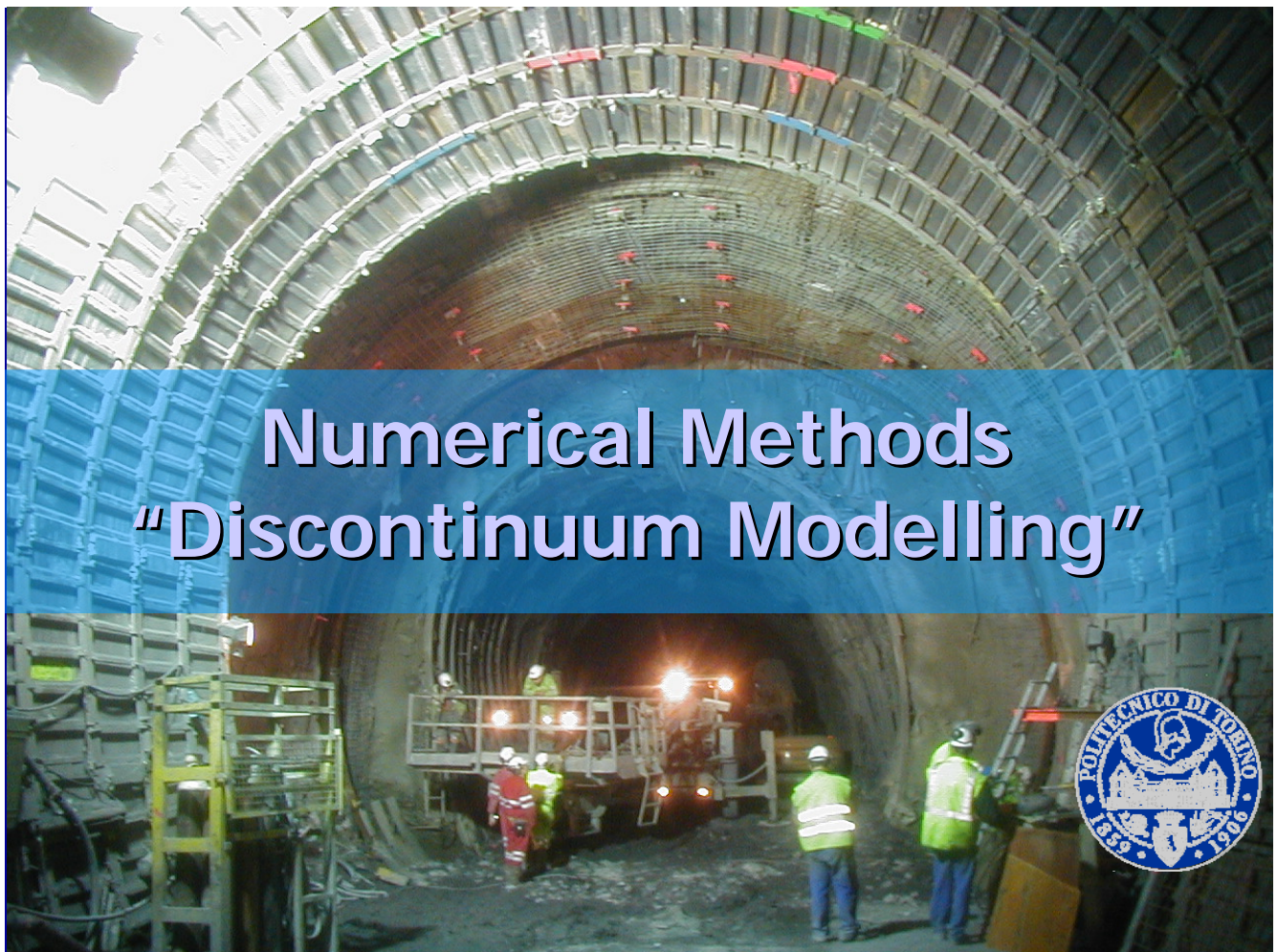
FEM





*Need for visualization of results of modelling*

from Beer, 2002



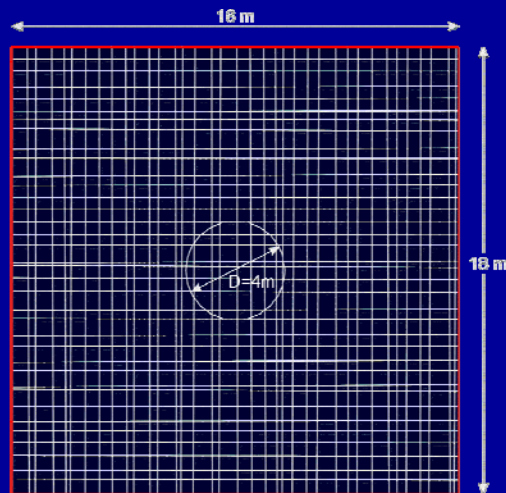


# Discontinuum modelling Distinct Element Method –UDEEC code

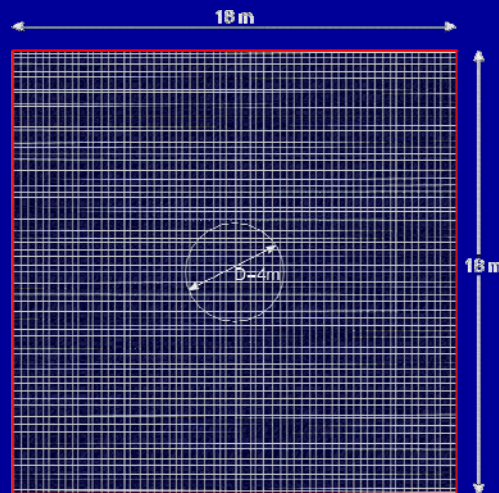
- Circular excavation in a flysch rock mass
- Two models with different joint spacing (respectively 0.5 and 0.3 m)
- Tunnel diameter = 4.00 m
- Two sets of persistent joints (horizontal bedding and vertical jointing)
- Average depth = 350 m

*The mechanical properties of blocks and discontinuities have been chosen in order to attain a Q value of:*

- 2.0 for first model
- 1.0 for second model



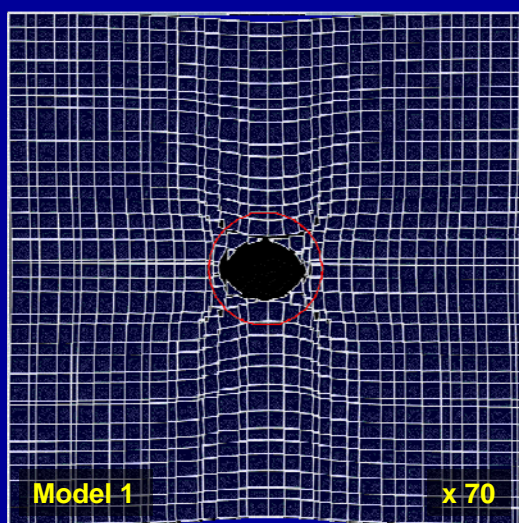
Q = 2.0 (poor)



Q = 1.0 (very poor)

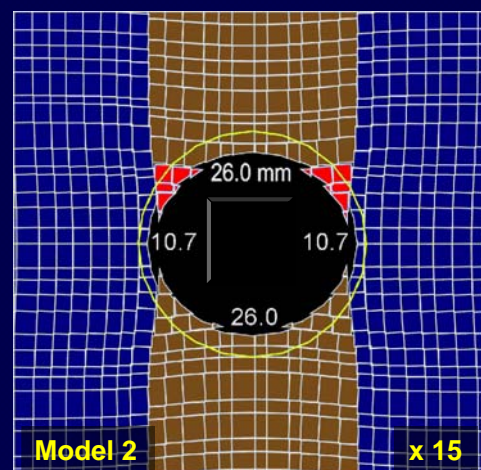
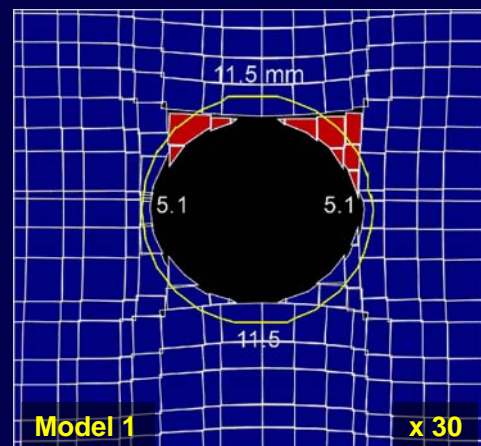
$\sigma_v = 9.45 \text{ MPa}$   
 $\sigma_h = 4.73 \text{ MPa}$   
 (Stress ratio  $K_0 = 0.5$ )  
 $\sigma_c = 45 \text{ MPa}$

## Stability analysis based on numerical approach



Displacements magnified by 70 times

- Instability of roof blocks
- Displacement ranging from 5 to 26 mm
- Displacement increasing as the joint spacing decreases
- Different evolution of displacement field







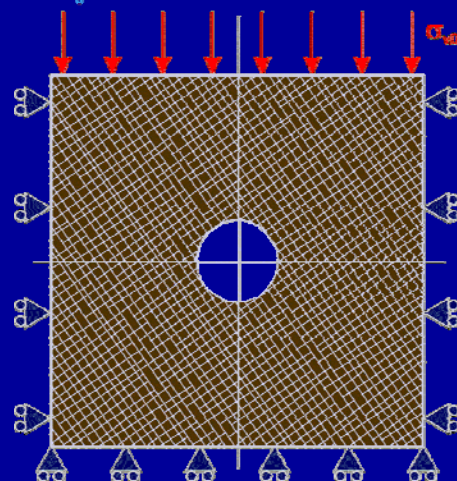
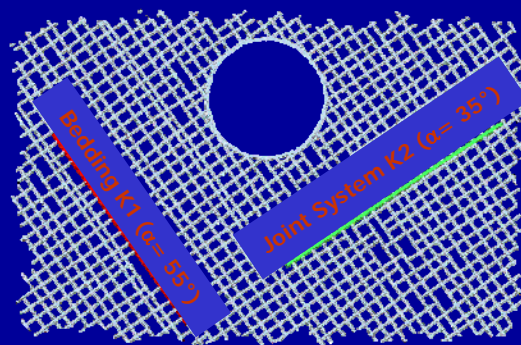
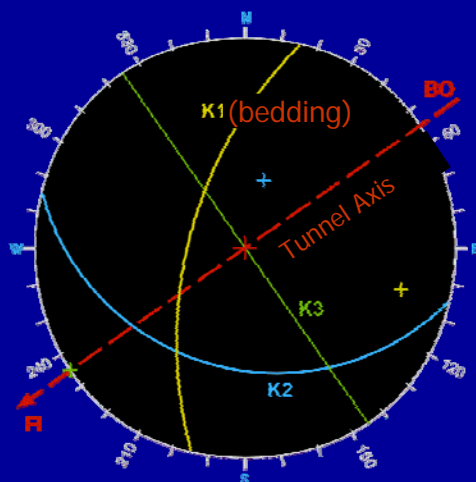
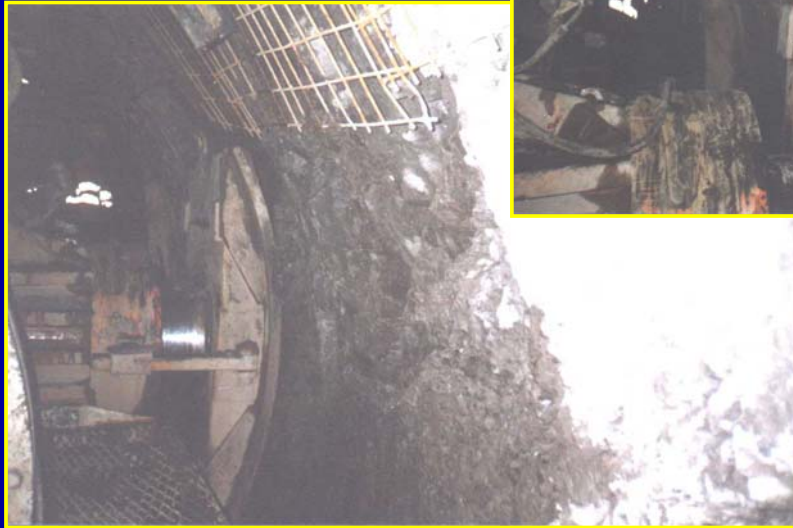
**Typical support system behind the cutter head (liner plates, steel mesh, bolting)**



**Instability at the roof occurs immediately behind the cutter head with the need to install support (liner plates, steel mesh, etc.), with a significant slow-down in progress rate**

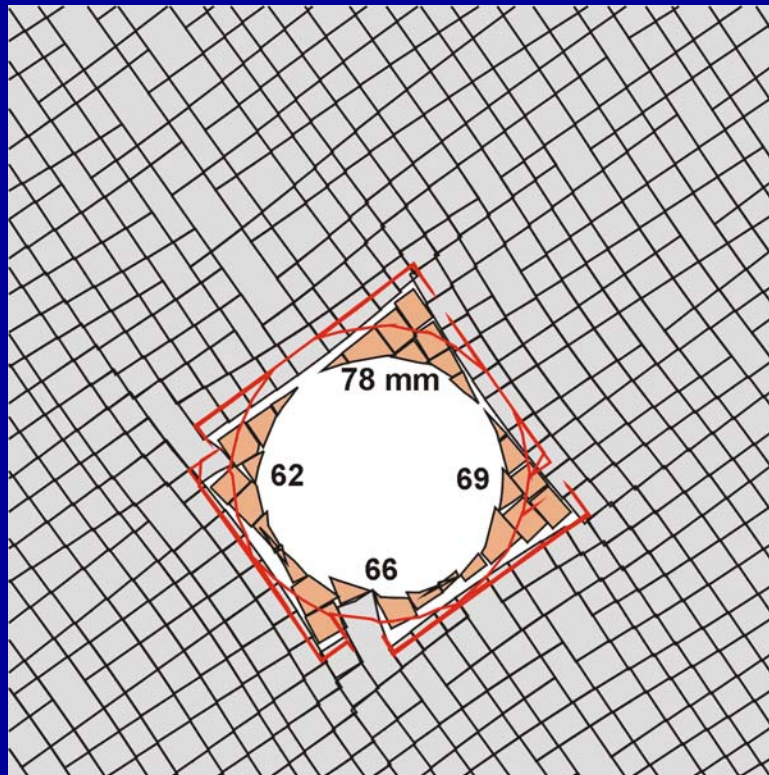


Instability at the tunnel walls with serious difficulties in positioning the grippers and applying the thrust required



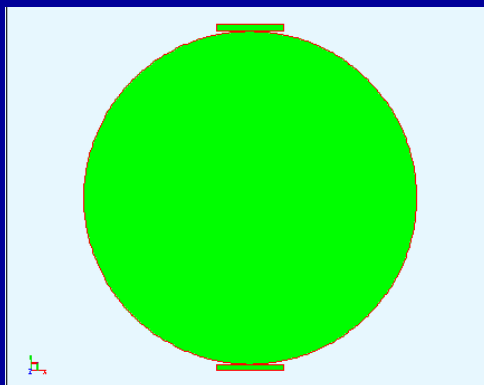
$K_0 = 0.5$   
 $K_0 = 1.0$   
 $K_0 = 1.5$

STRESS RATIO



## ***Transition from Continuum to Discontinuum (ELFEN code)***

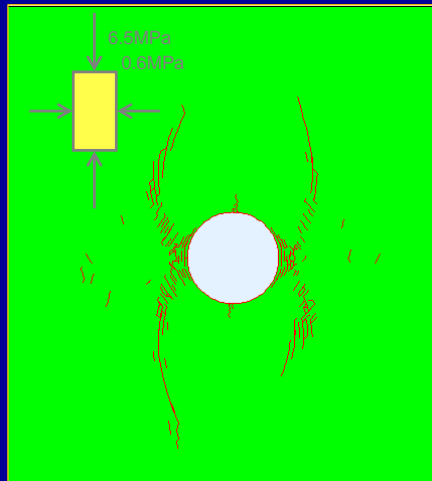
ELFEN is a dynamic solver based on the Finite Element Method (FEM) and specialized in problems of transition from continuum to discontinuum. It includes specific Non Linear Fracture Mechanics (NLFM) algorithms to simulate the fracturing and produce discrete fractures





## ***Transition from Continuum to Discontinuum (ELFEN code)***

ELFEN is a dynamic solver based on the Finite Element Method (FEM) and specialized in problems of transition from continuum to discontinuum. It includes specific Non Linear Fracture Mechanics (NLFM) algorithms to simulate the fracturing and produce discrete fractures



Modelling of problem by ELFEN



Fracturing in the roof of the raise bore pilot hole. Photo courtesy of B. Niederburger, 2001

# **Tunnels Underground Excavations**

**Design Analyses  
Case Studies and Observed Performance**

**Giovanni Barla**



*Department of Structural and Geotechnical Engineering*