Wednesday 6th December--2

PVD in Ground Improvement

Equivalent Drain Diameter for Band Shaped Drain

Hansbo (1981) introduced the equivalent diameter for a prefabricated band-shaped drain, as given in Equation

$$d_w = r_w = 2\frac{a+b}{\pi}$$

Rixner et al., (1986) suggested that the more appropriate dw is given by the less complex relationship as in Equation 2.2

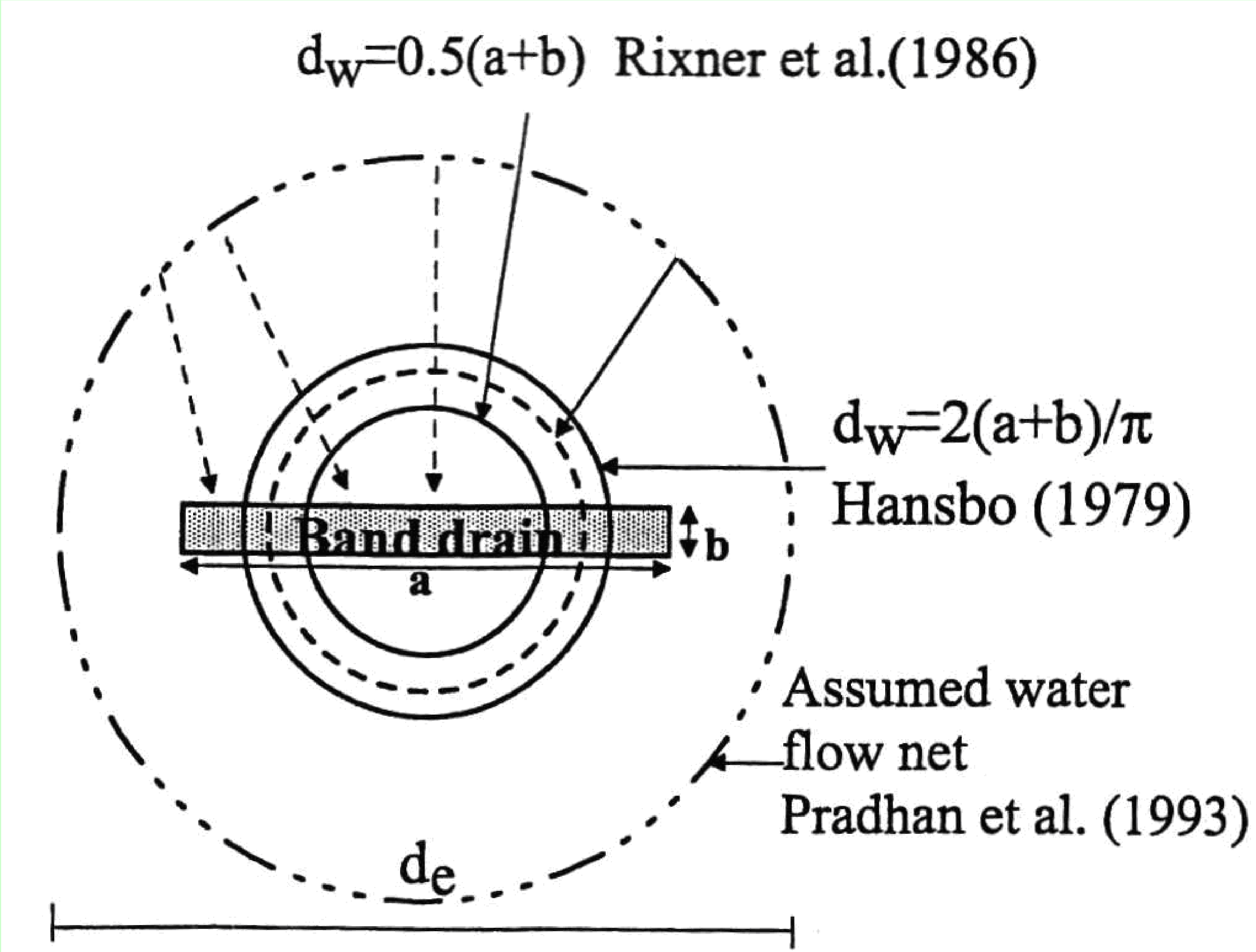
$$d = \frac{a + b}{2}$$

where, a = the width of the PVD and b = the thickness of the PVD

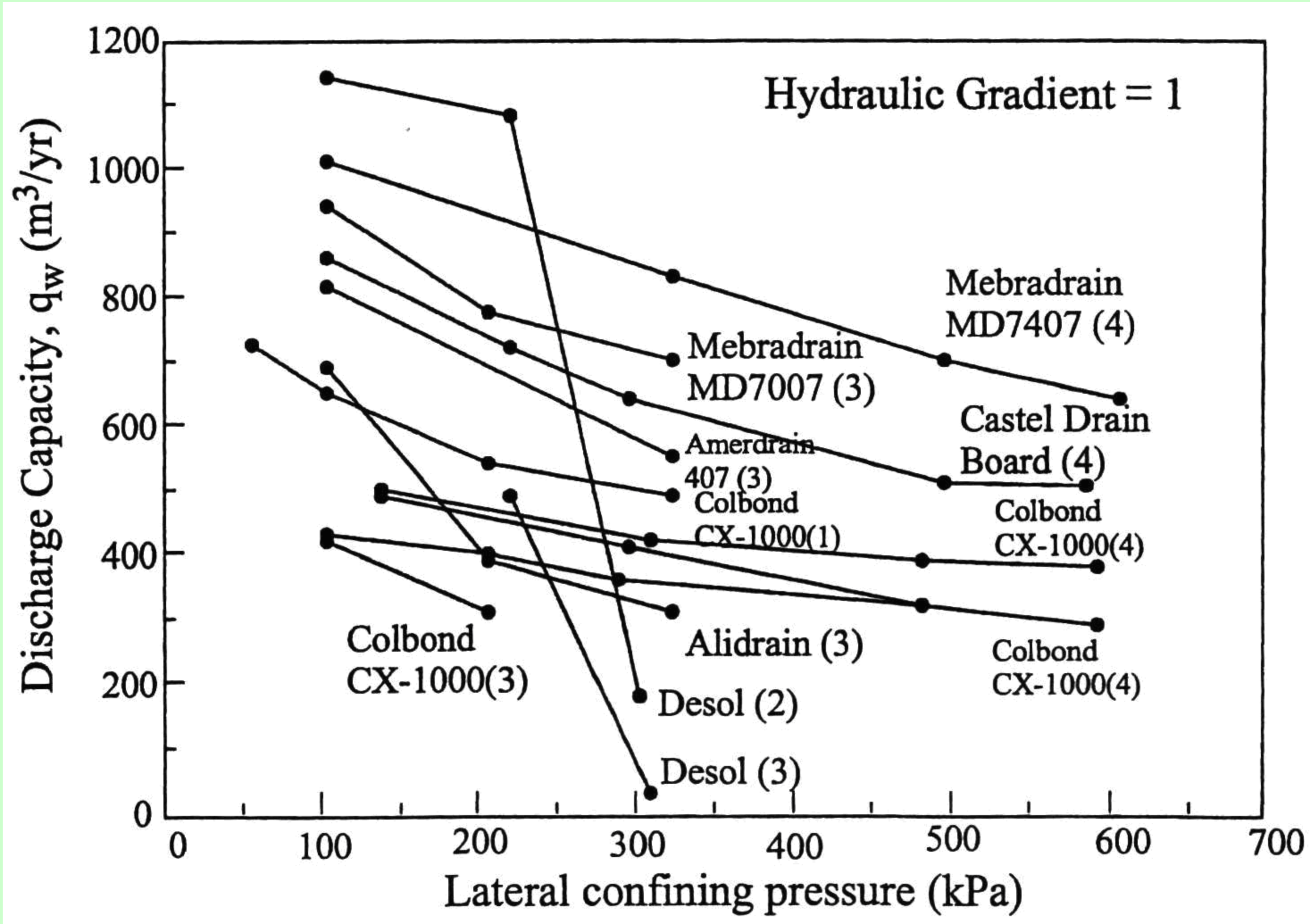
Pradhan et al. (1993) suggested that the equivalent diameter of band-shaped drains should be estimated by considering the flow net around the soil cylinder of diameter (de). The mean square distance of their flow net is calculated as:

$$\bar{s}^2 = \frac{1}{4} d_e^{\,2} + \frac{1}{12} a^2 - \frac{2a}{\pi^2} d_e$$

$$d_w = d_e - 2\sqrt{\bar{s}^2} + b$$



Equivalent diameters of band-shaped vertical drains



Typical values of vertical discharge capacity (Rixner et al., 1986)

The drain material (sand drain) and the filter jacket of PVD have to perform two basic but contrasting requirements, which are retaining the soil particles and at the same time allowing the pore water to pass through. The general guideline of the drain permeability is given by:

$$k_{filter} > 2\, k_{soil}$$

An effective filtration can minimise soil particles from moving through the filter. A commonly employed filtration requirement is given by:

$$\frac{O_{95}}{D_{85}} \leq 3$$

O95 indicates the approximate largest particle that would effectively pass through the filter.
D85 indicates the diameter of clay particles corresponding to 85% passing.

The discharge capacity of the prefabricated vertical drain is required to analyse the drain (well) resistance factor

the actual discharge capacity, is then given by

$$q_w = F_t \cdot F_c \cdot F_{fc} \cdot q_{req}$$

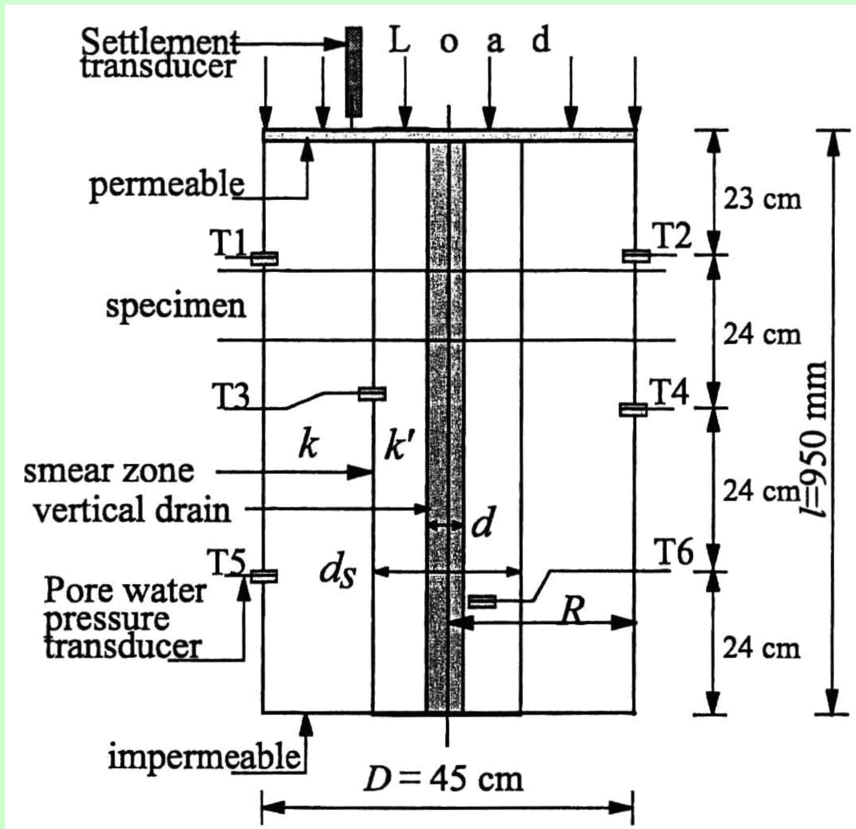
where F_t, F_c, and F_{fc} are the influence factors due to time, drain deformation and clogging, respectively

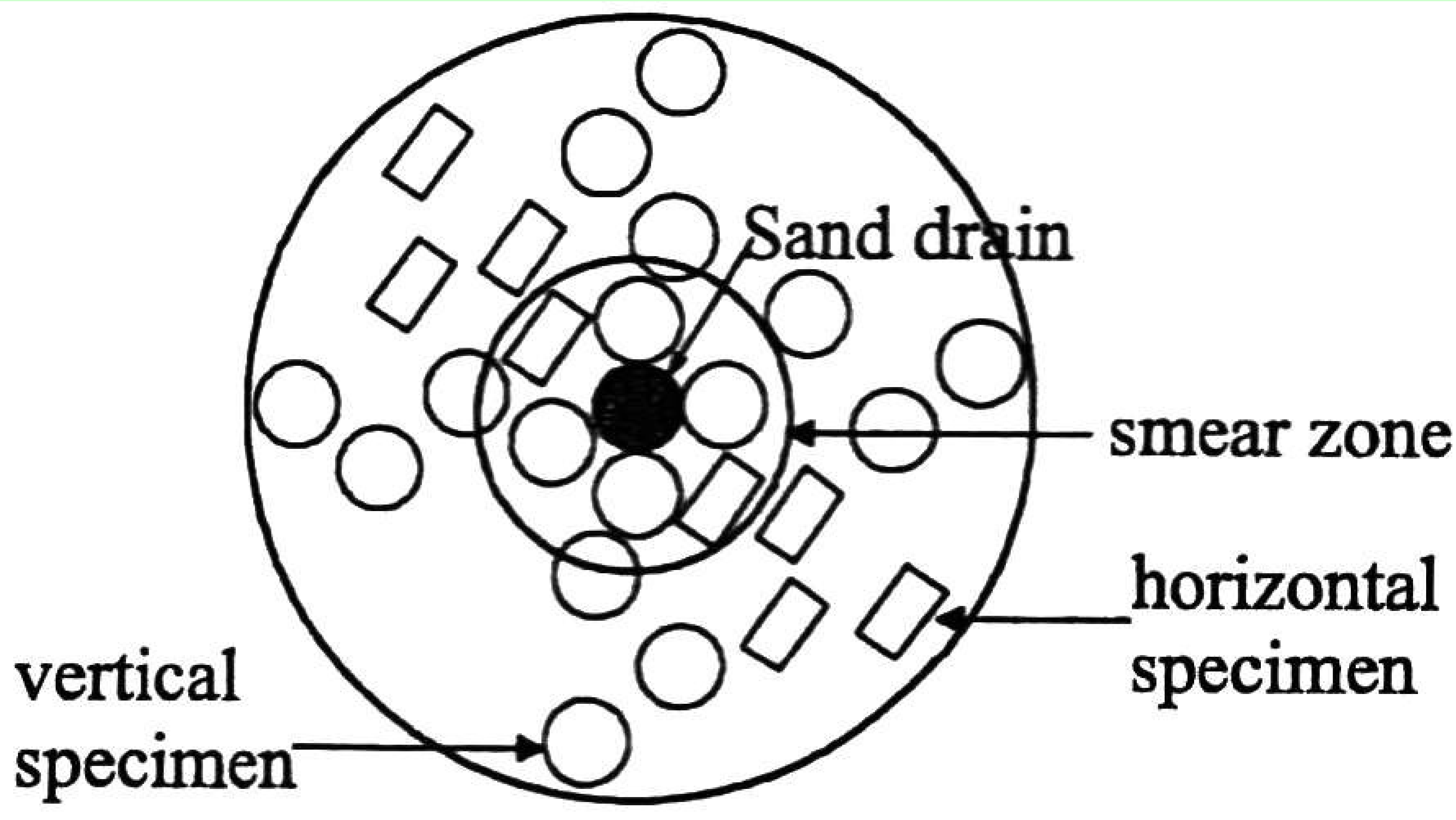
The term q_{req} is the theoretical discharge capacity calculated from Barron's theory of consolidation, which is given by

$$q_{req} = \frac{\varepsilon_f U_{10} l \pi c_h}{4T_h}$$

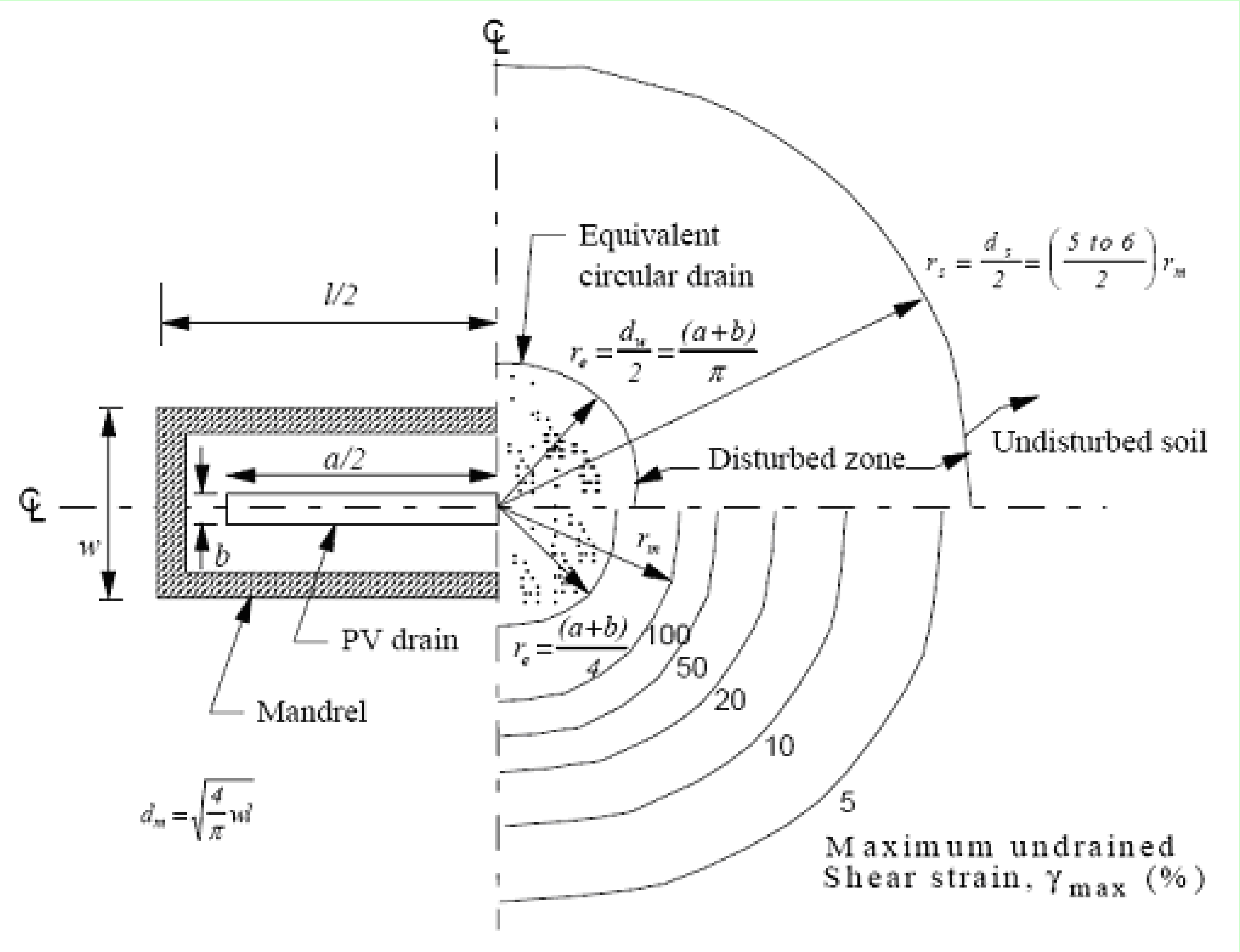
where, ε_f is the final settlement of the soft soil equivalent to 25% of the length of the drain installed to the soft ground, U_{10} is the 10 percent degree of consolidation, l is the depth of the vertical drain, c_h is horizontal coefficient of consolidation and T_h is the time factor for horizontal (lateral) consolidation.

Indraratna and Redana (1998) proposed that the estimated smear zone is about 3-4 times the cross-sectional area of the mandrel. The proposed relationship was verified using the specially designed large-scale consolidometer (Indraratna and Redana, 1995). The schematic section of the consolidometer and the location of the recovered specimen are shown in Figures





Locations of small specimens obtained to determine the consolidation and permeability characteristics (Indraratna and Redana, 1998)



Approximation of the smear zone around the mandrel.

Factors Influencing the Vertical Drain Efficiency

Smear Zone

Jamiolkowski et al.(1981) proposed that the diameter of the smear zone (d_s) and the cross sectional area of mandrel can be related as

$$d_s = \frac{(5\ to\ 6)d_m}{2}$$

where d_m is the diameter of the circle with a area is equal to the cross sectional area of the mandrel.

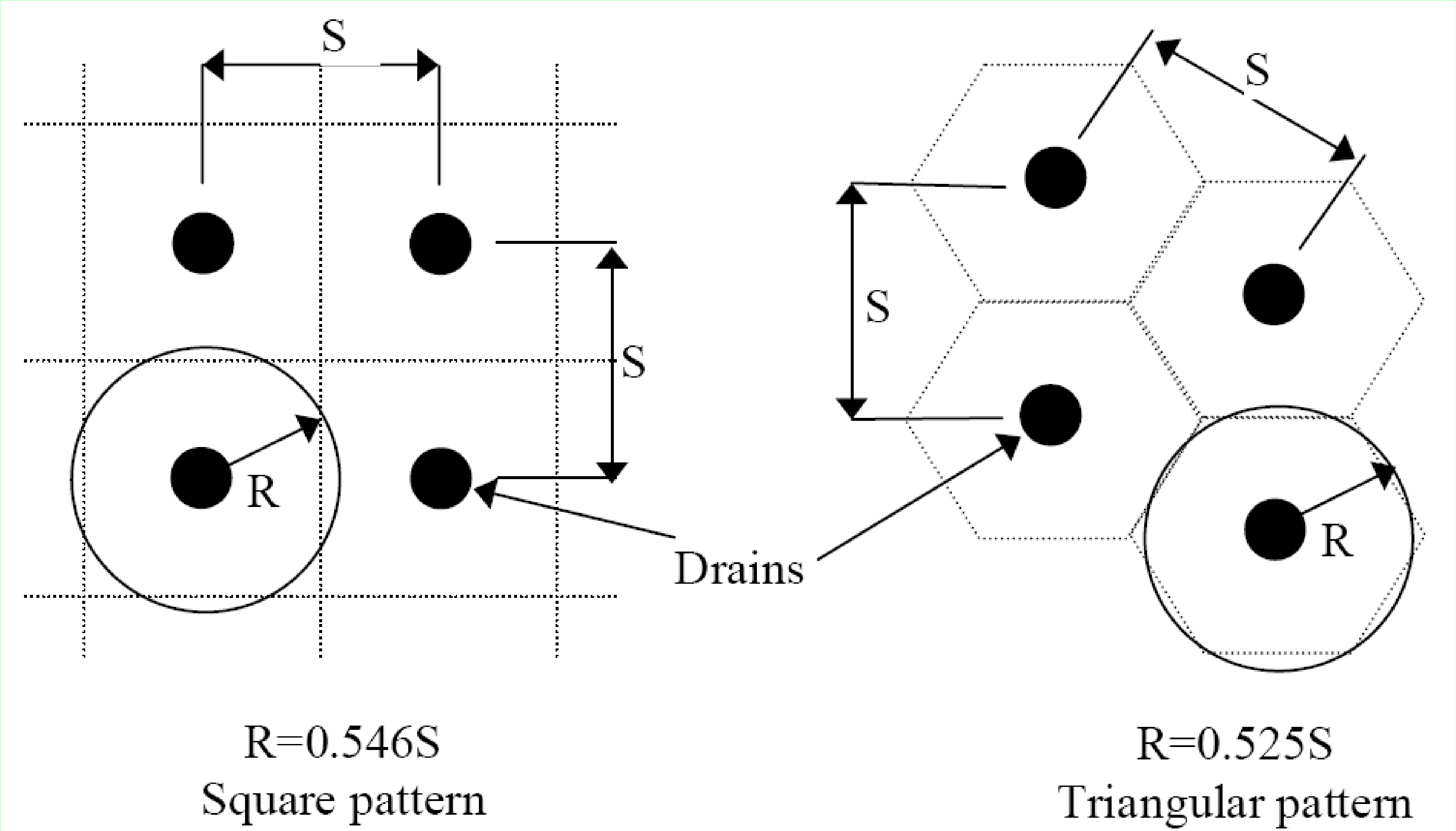
Table 2.1 Short-term discharge capacity (m^3/year) of eight band drains measured in laboratory (Hansbo, 1981)

Drain Type	Lateral pressure (kPa)			
	40	80	250	500
Geo-drain	26	20	20	16
Other drain types	21	20	18	10
	24	22	14	12
	15	14	14	12
	10	5	1	Clogged
	21	19	17	15
	--	17	13	12
	19	17	9	4

Vertical drains are commonly installed in square or triangular patterns. The influence zone (R) is a controlled variable, since it is a function of the drain spacing (S) as given by:

$R = 0.546\ S$ for drains installed in a square pattern

$R = 0.525\ S$ for drains installed in a triangular pattern



Plan of drain well pattern and zone of influence of each well

Jamiolkowski and Lancellota (1981) suggested that the smear zone is given by

$$d_s = (5 \; to \; 6) r_m$$

r_m is the radius of a circle with an area equal to the mandrel's greatest cross- sectional area, or the cross-sectional area of the anchor or tip, which ever is greater.

Development of Vertical Drain Theory

- The basic theory of radial consolidation around a vertical sand drain system is an extension of Terzaghi's (1925) one-dimensional consolidation theory.
- The theory of vertical drain was probably first solved by Kjellman (1948). His solution based on equal vertical strain hypothesis, was developed on the assumption that horizontal sections remain horizontal throughout the consolidation process.
- Barron (1948) presented the most comprehensive solution to the problem of radial consolidation by drain wells. He studied the two extreme cases of free strain and equal strain and showed that the average consolidation obtained in these cases is nearly the same. The 'free strain hypothesis' assumes that the load is uniform over a circular zone of influence for each vertical drain, and that the differential settlements occurring over this zone have no effect on the redistribution of stresses by arching of the fill load.

Development of Vertical Drain Theory

- Barron (1948) considered the influence of well resistance and smear on the consolidation process due to vertical well drains.
- Takagi (1957) extended Barron‘s solution to incorporate a variable rate of loading
- Richart (1959) presented a convenient design chart for the effect of smear, where the influence of variable void ratio was also considered.
- Hansbo (1960) presented a solution by pointing out that the Darcy‘s law might not be valid when the hydraulic gradient is in the range of magnitudes prevailing during most consolidation processes in practice. However, in this equal strain solution, the effect of smear and well resistance were not considered.
- A simplified solution to the problem of smear and well resistance was proposed by Hansbo (1979, 1981), giving results almost identical with those given by Barron (1948) and Yoshikuni and Nakanodo (1974).

Barron‘s solution is based on the following assumptions:

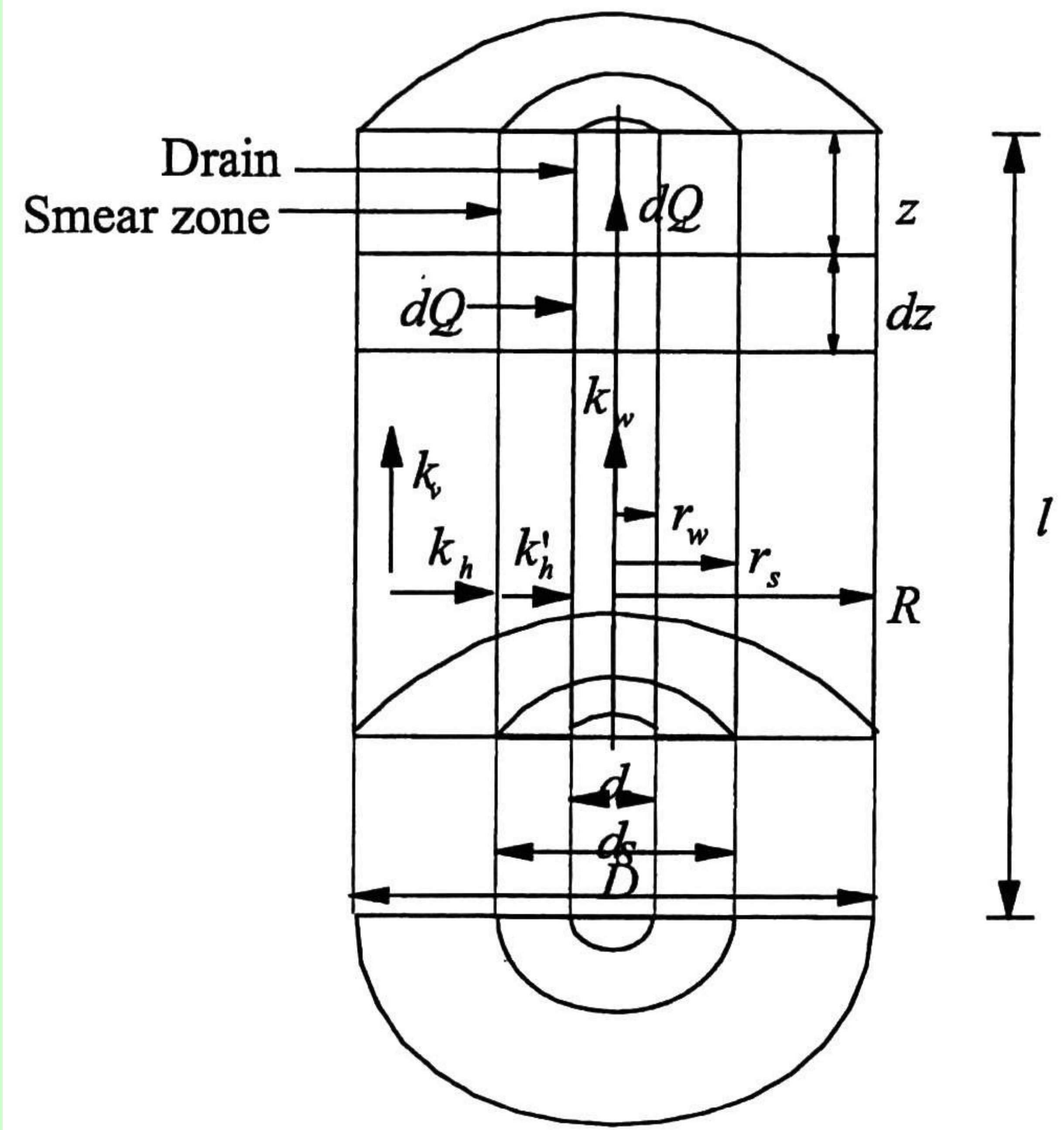
(a) all vertical loads are initially carried by excess pore water pressure, u, which means that the soil is saturated,

(b) the applied load is assumed to be uniformly distributed and all compressive strain within the soil occurs in the vertical direction,

(c) the zone of influence of the drain is assumed to be circular and axisymmetric,

(d) permeability of the drain is infinite in comparison with that of the soil, and

(e) Darcy‘s law is valid.



Schematic of soil cylinder with vertical drain (after Hansbo,1979)

The three dimensional consolidation of radial drainage (Barron, 1948) is given by

$$\frac{\partial \overline{u}}{\partial t} = c_v \left(\frac{\partial^2 u}{\partial z^2} \right) + c_h \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right)$$

For radial flow only, the above equation becomes

$$\frac{\partial \overline{u}}{\partial t} = c_h \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right)$$

where t is the time elapsed after the load is applied, u is the excess pore water pressure at radius r and at depth z.

The solution of the excess pore pressure for radial flow only, u_r of the above equation based on 'equal strain' assumption is given by:

$$u_r = \frac{4\bar{u}}{D^2 F(n)} \left[R^2 \, ln\left(\frac{r}{r_w}\right) - \frac{r^2 - r_w^2}{2} \right] \quad (2.19)$$

where, D is the diameter of soil cylinder, the drain spacing factor, $F(n)$ is given by:

$$F(n) = \frac{n^2}{n^2 - 1} ln(n) - \frac{3n^2 - 1}{4n^2} \quad (2.20)$$

where, $n = R / r_w$ is drain spacing ratio

The average excess pore water pressure is given by:

$$\bar{u} = u_{av} = u_o \, exp\left(\frac{-8T_h}{F(n)}\right) \tag{2.21}$$

The average degree of consolidation, $\bar{U}_h$, in the soil body is given by:

$$\bar{U}_h = 1 - exp\left(\frac{-8T_h}{F(n)}\right) \tag{2.22}$$

where the time factor T_h is defined as:

$$T_h = \frac{c_h \, t}{D^2} \tag{2.23}$$

The coefficient of radial drainage consolidation, c_h, is represented by:

$$c_h = \frac{k_h\left(1+e\right)}{a_v \gamma_w} \tag{2.24}$$

where γ_w is unit weight of water, and a_v is the coefficient of compressibility of the soil, e is the void ratio, and k_h is the horizontal permeability of the soil.

The solution taking account of smear effect is given by:

$$u_r = \bar{u}_r \frac{1}{V}\left[ln\frac{r}{r_s} - \frac{r^2 - r_s^2}{2R^2} + \frac{k_h}{k'_h}\left(\frac{n^2 - s^2}{n^2}\right) ln\ s \right]$$

The smear factor v is given by

$$v = F(n, s, k_h, k'_h) = \left[\frac{n^2 - s^2}{n^2} ln\frac{n}{s} - \frac{3}{4} + \frac{s^2}{4n^2} + \frac{k_h}{k'_h}\left(\frac{n^2 - s^2}{n^2}\right) ln\ s \right]$$

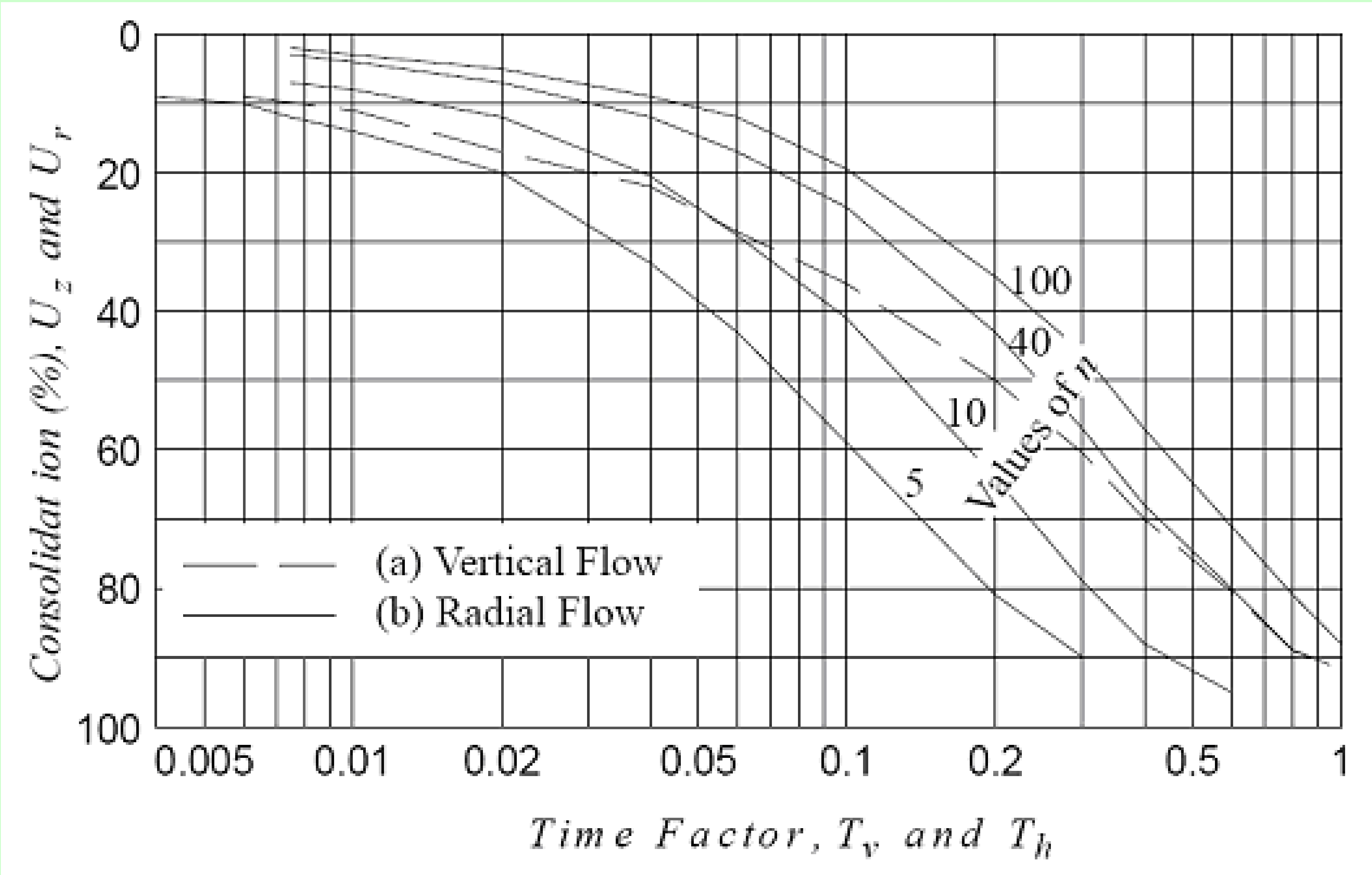
$$\bar{u} = u_{av} = u_o\, exp\left(\frac{-8T_h}{v}\right)$$

In the above expression, s is the extent factor of the smear zone with respect to the size of the drain and is given by $s = r_s / r_w$

The average degree of consolidation including smear is now given by

$$\overline{U}_h = 1 - exp\left(\frac{-8T_h}{v}\right)$$

Curves of average radial excess pore water pressure, $\bar{u}_r$, and average degree of consolidation, $\overline{U}_h$ (purely radial flow) versus time factor T_h for various values of n are shown in Figure



Approximate Equal Strain Solution

Hansbo (1981) derived an approximate solution for vertical drain based on the ‘equal strain hypotheses’ by taking both smear and well resistance into consideration. By applying Darcy‘s law, the rate of flow of internal pore water in the radial direction can be estimated. The total flow of water from the slice, dz, to the drain, dQ_1, is equal to the change of flow of water from the surrounding soil, dQ_2, which is proportional to the change of volume of the soil mass. The average degree of consolidation, $\overline{U}$, of the soil cylinder with vertical drain is given by:

$$\overline{U}_h = 1 - exp\left(\frac{-8T_h}{\mu}\right)$$

$$\mu = ln\left(\frac{n}{s}\right) + \left(\frac{k_h}{k'_h}\right) ln(s) - 0.75 + \pi\, z(2\,l - z)\frac{k_h}{q_w}\left[1 - \frac{\frac{k_h}{k'_h} - 1}{\frac{k_h}{k'_h}\left(\frac{n}{s}\right)^2}\right]$$

In a simplified form:

$$\mu = ln\left(\frac{n}{s}\right) + \left(\frac{k_h}{k'_h}\right) ln(s) - 0.75 + \pi\ z(2\,l - z)\frac{k_h}{q_w}$$

The effect of smear only is given by

$$\mu = ln\left(\frac{n}{s}\right) + \left(\frac{k_h}{k'_h}\right) ln(s) - 0.75$$

The effect of well resistance only is given by

$$\mu \approx ln(n) - 0.75 + \pi\ z(2\,l - z)\frac{k_h}{q_w}$$

If both smear and well resistance are ignored, this parameter becomes

$$\mu = ln(n) - 0.75$$

Plane Strain Consolidation Model

Most finite element analyses on embankments are conducted based on the plane strain assumption. However, this kind of analysis poses a problem, because the consolidation around vertical drains is axisymmetric. Therefore, to employ a realistic 2-D finite element analysis for vertical drains, the equivalence between the plane strain and axisymmetric analysis needs to be established. The matching of axisymmetric and plane strain conditions can be done in three ways:

1. geometric matching approach œ the spacing of the drains is matched while keeping the permeability the same

2. permeability matching approach œ permeability coefficient is matched while keeping the spacing of drains to be the same.

3. combination of permeability and geometric matching approach œ plane strain permeability is calculated for a convenient drain spacing.

Indraratna and Redana (1997) represented the average degree of consolidation in plane strain condition as

$$\overline{U}_{hp} = 1 - \frac{\bar{u}}{\bar{u}_0} = 1 - exp\left(\frac{-8T_{hp}}{\mu_p}\right)$$

where $\bar{u}_0$ = initial pore pressure; $\bar{u}$ = pore pressure at time t (average values); T_{hp}

= time factor in plane strain

$$\mu_p = \left[\alpha + \beta \frac{k_{hp}}{k'_{hp}} + \theta\left(2\, l\, z - z^2\right)\right]$$

where, k_{hp} and k'_{hp} are the undisturbed horizontal and corresponding smear zone

permeability, respectively

The geometric parameters α, β and the flow term θ are given by:

$$\alpha = \frac{2}{3} - \frac{2b_s}{B}\left(1 - \frac{b_s}{B} + \frac{b_s^2}{3B^2}\right)$$

$$\beta = \frac{1}{B^2}\left(b_s - b_w\right)^2 + \frac{b_s}{3B^3}\left(3b_w^2 - b_s^2\right)$$

$$\theta = \frac{2k_{hp}^2}{k'_{hp}\, q_z B}\left(1 - \frac{b_w}{B}\right)$$

where, q_z = the equivalent plane strain discharge capacity.

At a given stress level and at each time step, the average degree of consolidation for both axisymmetric (U_h) and equivalent plane strain (U_{hp}) conditions are made equal, hence:

$$\overline{U}_h = \overline{U}_{hp}$$

Combining

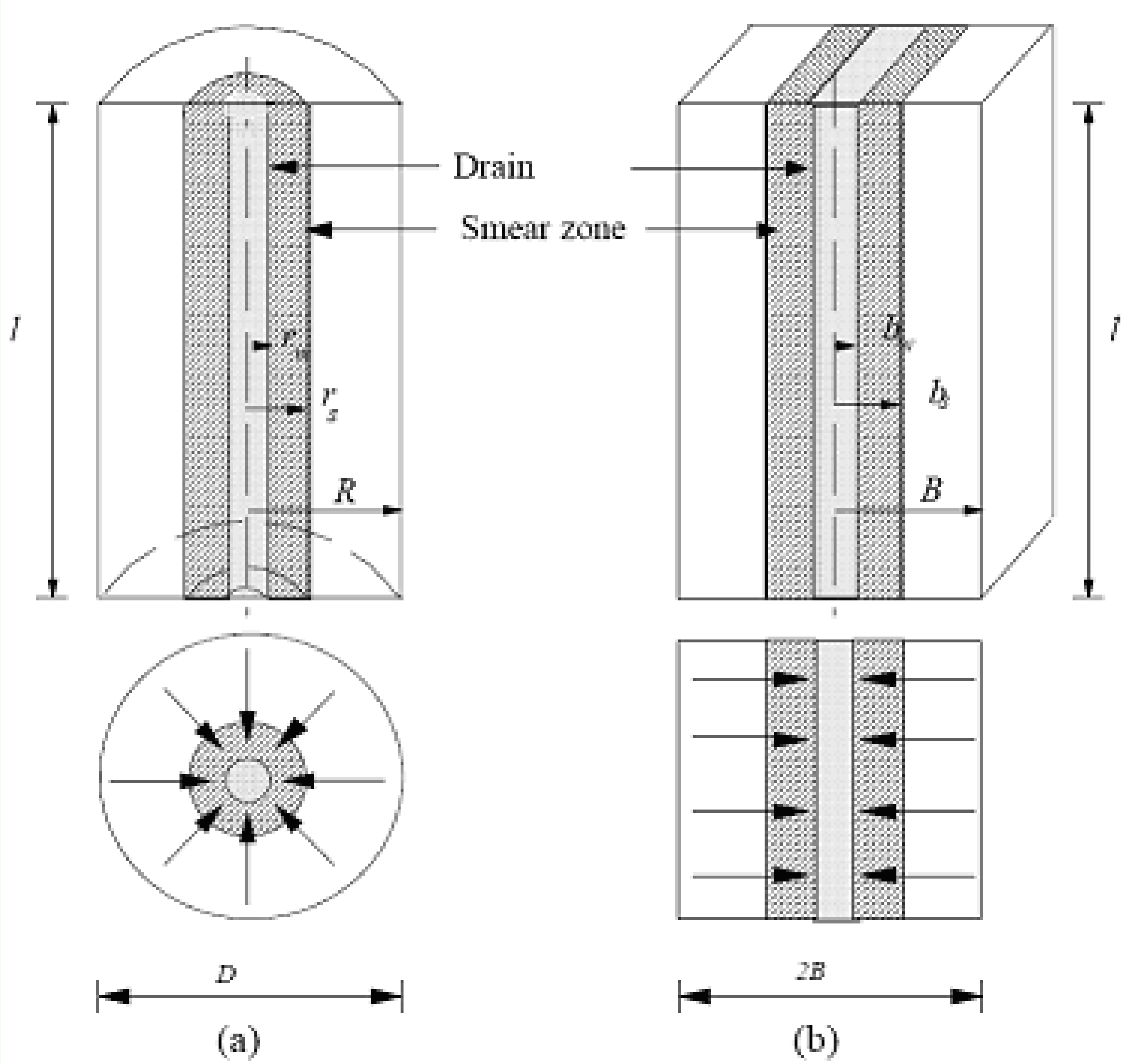
$$\overline{U}_{hp} = 1 - \frac{\overline{u}}{\overline{u}_0} = 1 - exp\left(\frac{-8T_{hp}}{\mu_p}\right) \qquad \text{and} \qquad \overline{U}_h = \overline{U}_{hp}$$

with the original Hansbo (1981) theory, the time factor ratio can be given by following equation:

$$\frac{T_{hp}}{T_h} = \frac{k_{hp}}{k_h}\frac{R^2}{B^2} = \frac{\mu_p}{\mu}$$

By assuming the magnitudes of R and B to be the same, Indraratna and Redana (1997) presented the relationship between k_{hp} and k'_{hp} as follow

$$k_{hp} = \frac{k_h \left[\alpha + \beta \frac{k_{hp}}{k'_{hp}} + \theta \left(2\, l\, z - z^2 \right) \right]}{\left[ln\left(\frac{n}{s} \right) + \frac{k_h}{k'_h} ln(s) - 0.75 + \pi \left(2\, l\, z - z^2 \right) \frac{k_h}{q_w} \right]}$$



Conversion of an axisymmetric unit cell into plane strain condition: a) Axisymmetric Radial Flow b) Plane Strain

Consolidation around Vertical Drains

Rate of Consolidation

The main reason for using pre fabricated vertical drain is to reach the desired degree of consolidation within a specified time period. But in a vertical drain system, both radial and vertical consolidation should be considered in calculating the specified time period. Carillo (1942) gave the combined effect as

$$1 - U = (1 - U_r)(1 - U_v)$$

where, U is the overall degree of consolidation; U_r is the average degree of consolidation due to radial drainage; U_v is the average degree of consolidation due to vertical drainage

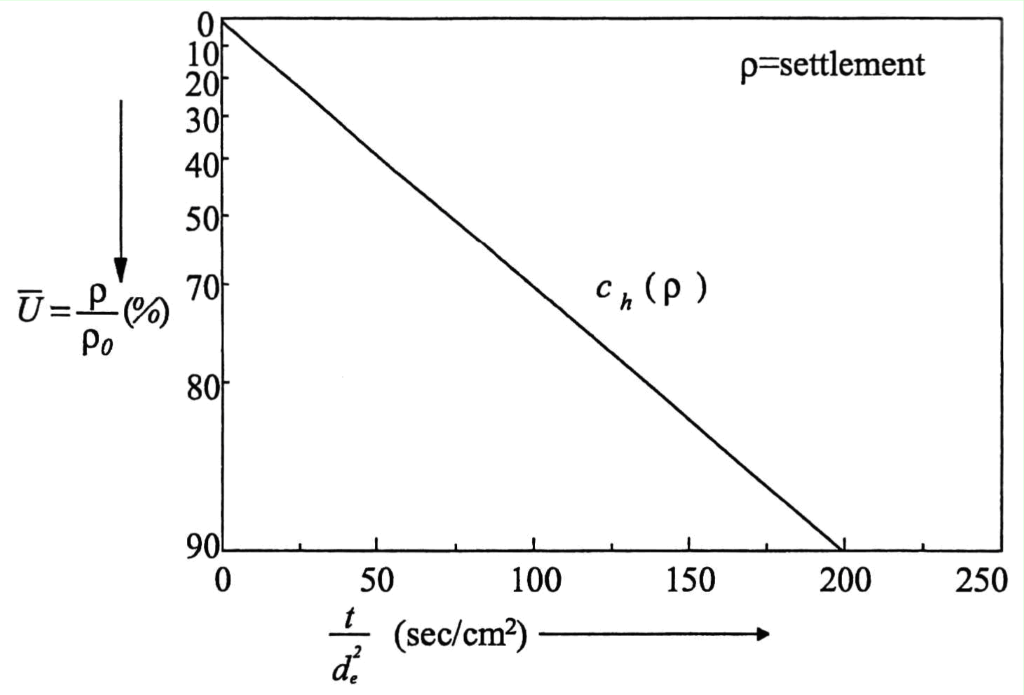
Coefficient of Consolidation with Radial Drainage

Log U vs. t Approach

Aboshi and Monden (1963) presented a curve fitting method using log *U* and linear *t*. This method is developed by taking 'log' on both sides of Barron's solution, which results in the following expression:

$$T_h = -\frac{F(n)}{8} ln\left(1 - \overline{U}\right)$$

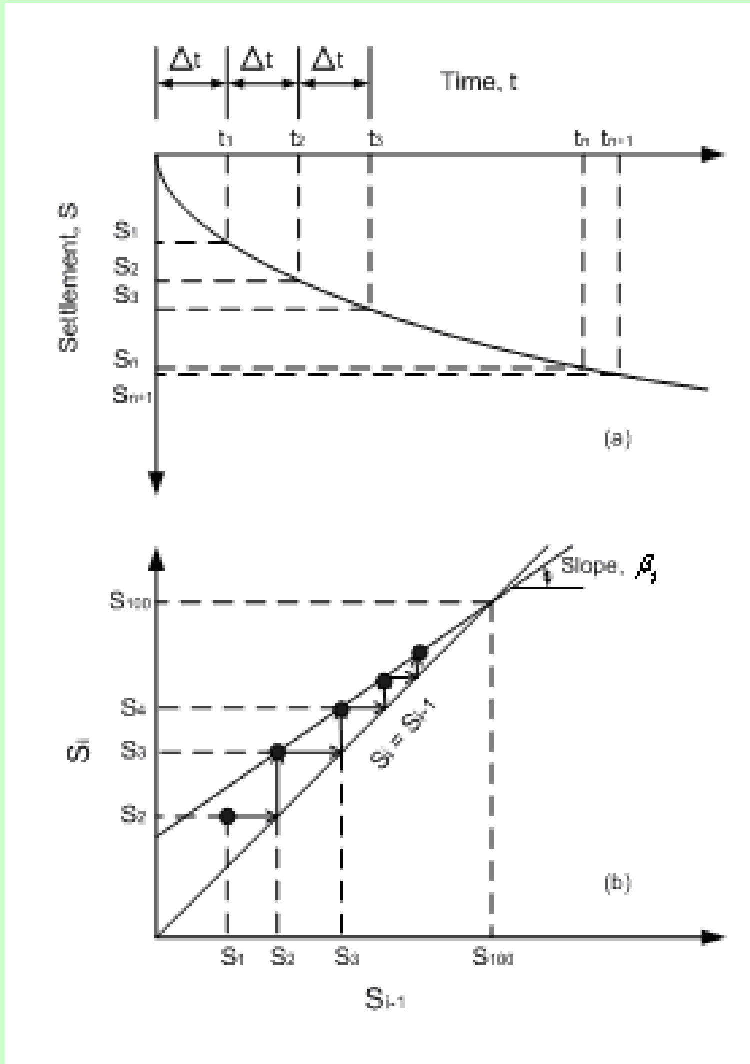
This equation represents the theoretical time factor for radial consolidation of perfect drains without considering the effect of smear. The coefficient of radial consolidation (c_h) is determined by plotting the logarithm of the average degree of consolidation against the linear consolidation time (log U vs. T_h), where a linear slope provides the c_h value (as shown in Figure).



Aboshi and Monden (1963) method for determining c_h

Plotting Settlement Data

Asaoka (1978) developed a method where a series of settlements (ρ_1 , …, ρ_{I-1}, ρ_i , ρ_{i+1} etc.), which are observed at constant time intervals are plotted as shown in Figure .



The coefficient of radial drainage consolidation in this method is derived using Barron's (1948) solution, which is given by:

$$c_h = -\frac{D^2 v}{8}\frac{\ln \beta}{\Delta t}$$