PLAXIS

UPC

COMPUTATIONAL GEOTECHNICS COURSE

UNDRAINED BEHAVIOUR

(and modelling)

Antonio Gens

Technical University of Catalunya, Barcelona

➡ many of the slides were originally created by:

Helmut Schweiger (Technical University of Graz, Austria)
Ronald Brinkgreve (Plaxis bv / Delft University of Technology)
Cino Viggiani (Laboratoire 3S, Grenoble, France)

outline

- drained / undrained (conditions and analysis)
- drained / undrained soil behaviour
  - typical results from drained and undrained triaxial tests
  - strength parameters in effective stresses and total stresses
  - what is the critical case: drained or undrained?
- modelling undrained behaviour with Plaxis
  - general procedure
  - three methods
    - Method A: effective stresses
    - Method C: total stresses
    - Method B (hybrid method)
- undrained shear strength
  - undrained behaviour with Mohr-Coulomb Model
  - undrained behaviour with advanced models
  - influence of dilatancy
- summary

drained / undrained (conditions and analysis)

in undrained conditions, no water movement takes place and, therefore, excess pore pressures are built up

$$\Delta u \neq 0, \Delta\sigma \neq \Delta\sigma'$$

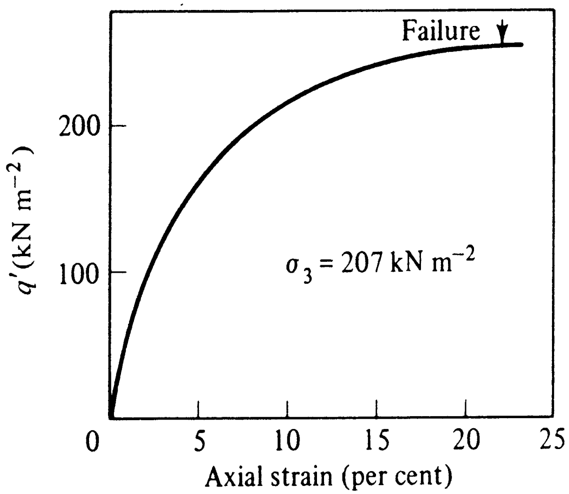
in drained conditions, no excess pore pressures are built up

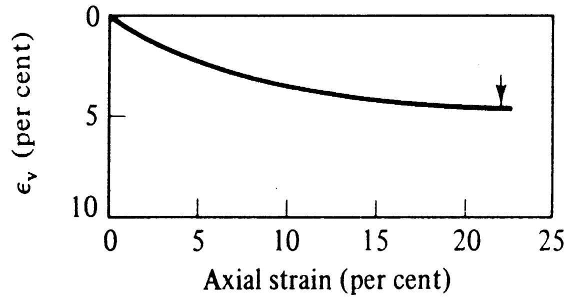
$$\Delta u = 0, \Delta\sigma = \Delta\sigma'$$

- drained analysis appropriate when
  - permeability is high
  - rate of loading is low
  - short term behavior is not of interest for problem considered
- undrained analysis appropriate when
  - permeability is low and rate of loading is high
  - short term behavior has to be assessed

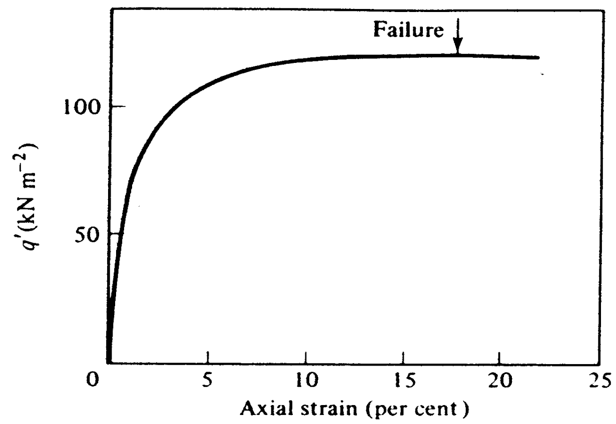
triaxial test (NC soils) – drained / undrained

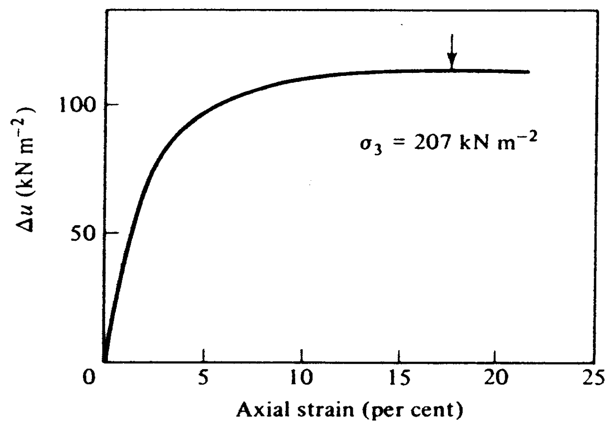
drained





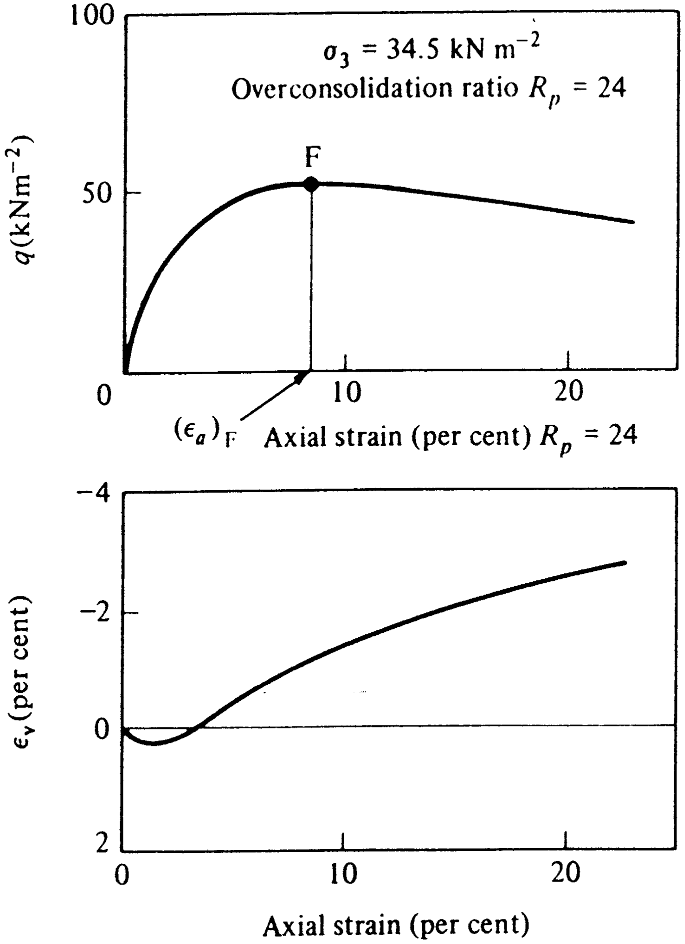
undrained



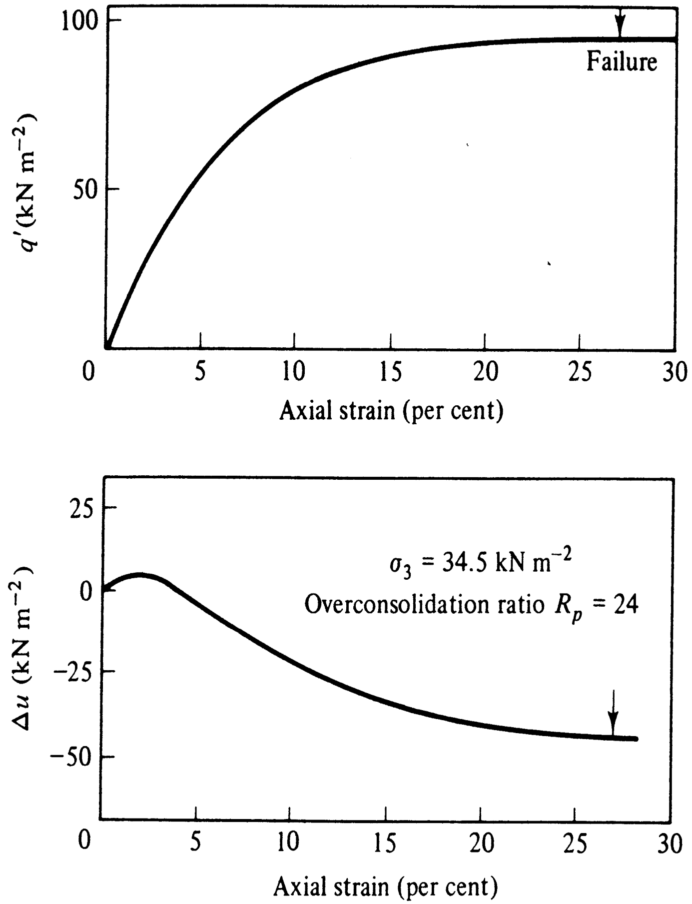


triaxial test (OC soils) – drained / undrained

drained



undrained

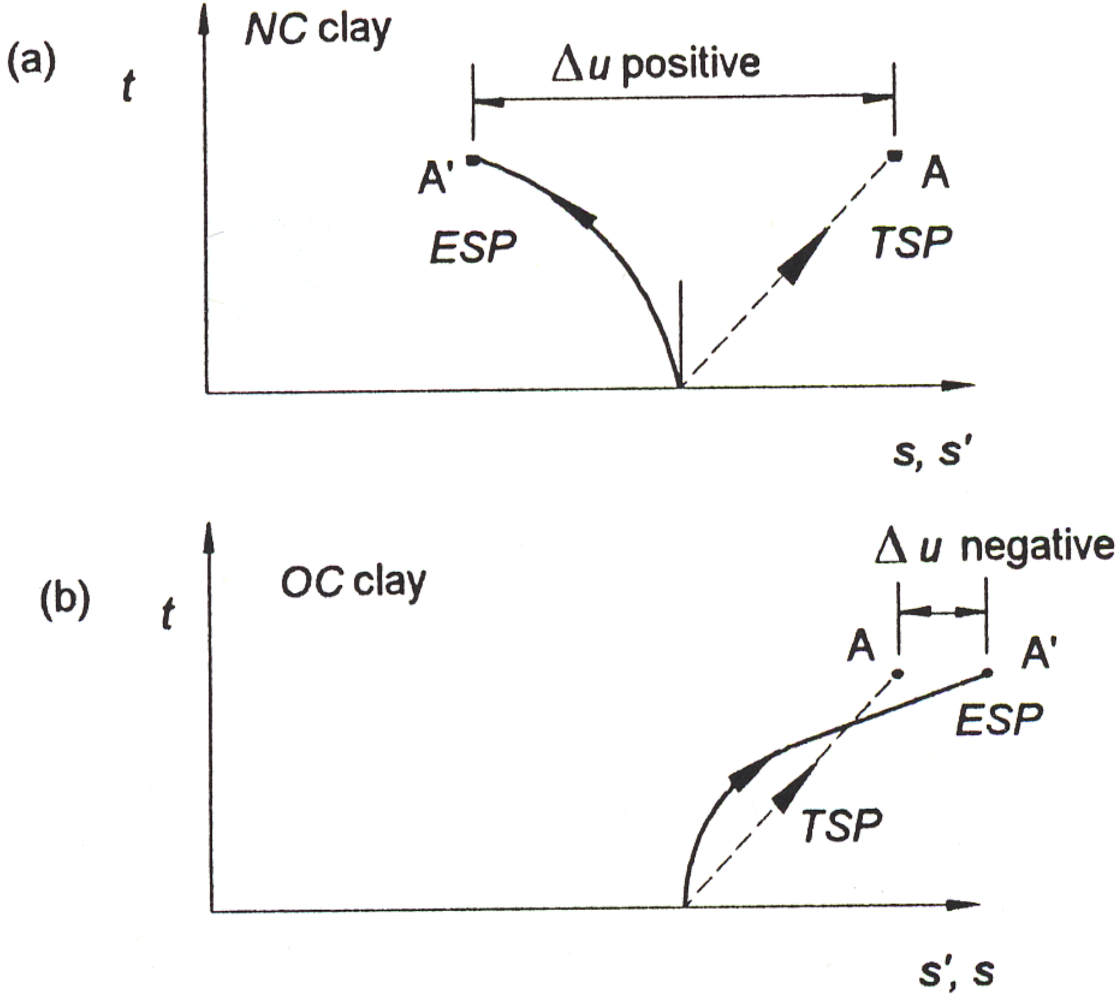


stress paths in undrained triaxial test – NC / OC

$$t = \frac{\sigma_1 - \sigma_3}{2}$$

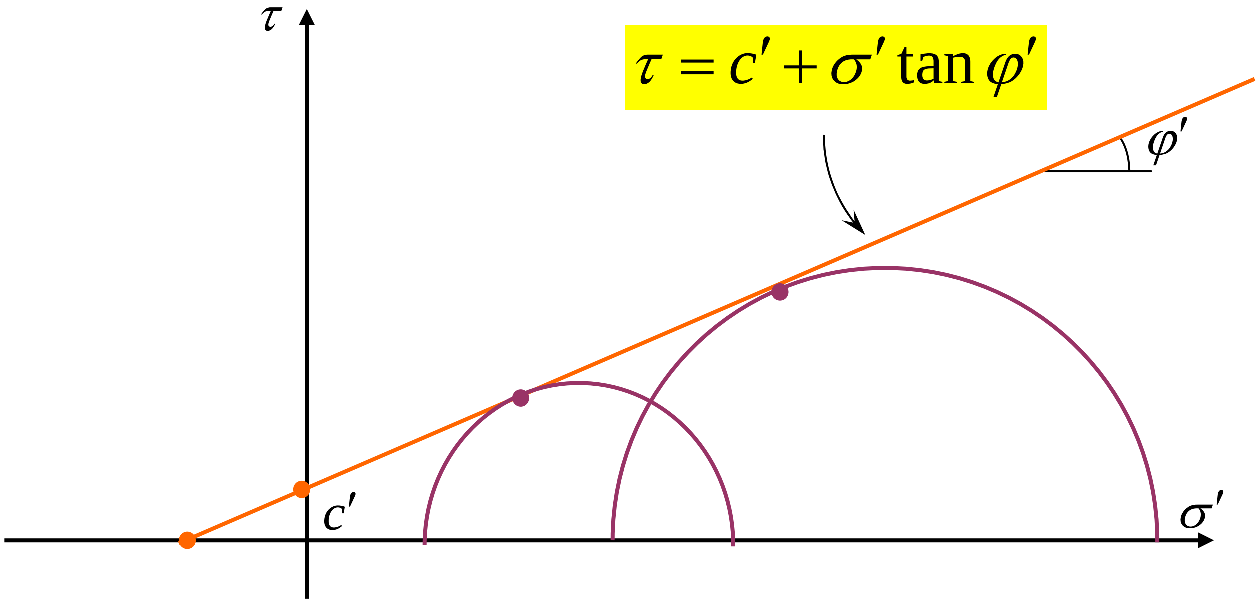
$$s' = \frac{\sigma_1' + \sigma_3'}{2}$$

$$s = \frac{\sigma_1' + \sigma_3'}{2}$$



Strength parameters

- Mohr-Coulomb parameters in terms of effective stress



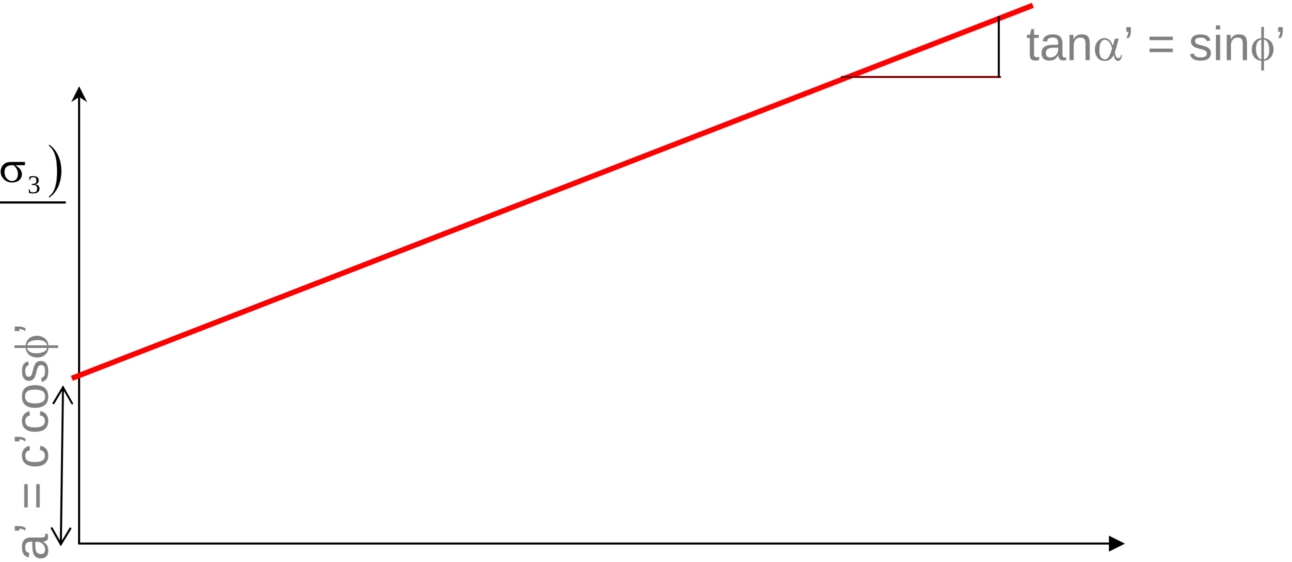
in terms of effective stresses

$$\frac{\sigma_1 - \sigma_3}{2} = \left[\frac{\sigma_1' + \sigma_3'}{2} + \frac{c'}{\tan\varphi'} \right] \sin\varphi' \quad ; \quad t = \left[s' + \frac{c'}{\tan\varphi'} \right] \sin\varphi'$$

Strength parameters

- Mohr-Coulomb parameters in terms of effective stress

$$t = \frac{(\sigma_1 - \sigma_3)}{2}$$

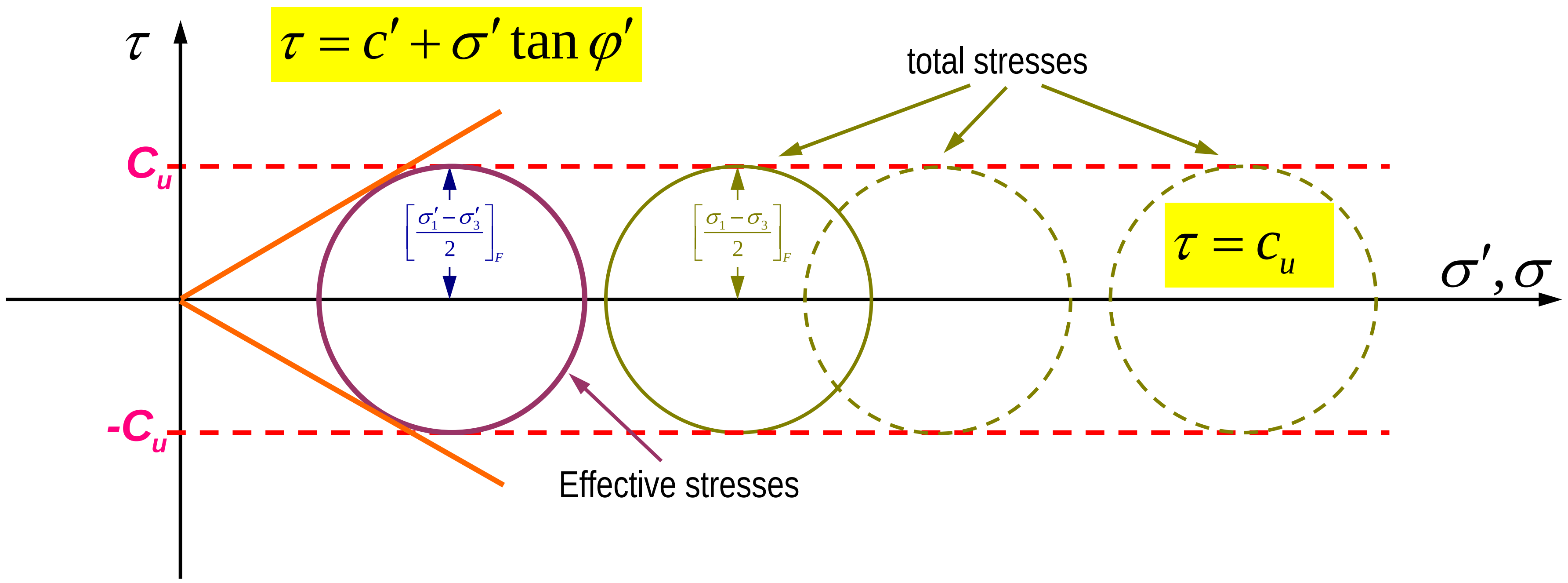


$$s' = \frac{(\sigma'_1 + \sigma'_3)}{2}, \; s = \frac{(\sigma_1 + \sigma_3)}{2}$$

$$\frac{\sigma_1' - \sigma_3'}{2} = \left[\frac{\sigma_1' + \sigma_3'}{2} + \frac{c'}{\tan\varphi'}\right]\sin\varphi' \quad ; \quad t = \left[s' + \frac{c'}{\tan\varphi'}\right]\sin\varphi' \quad ; \quad t = s'\sin\varphi' + c'\cos\varphi'$$

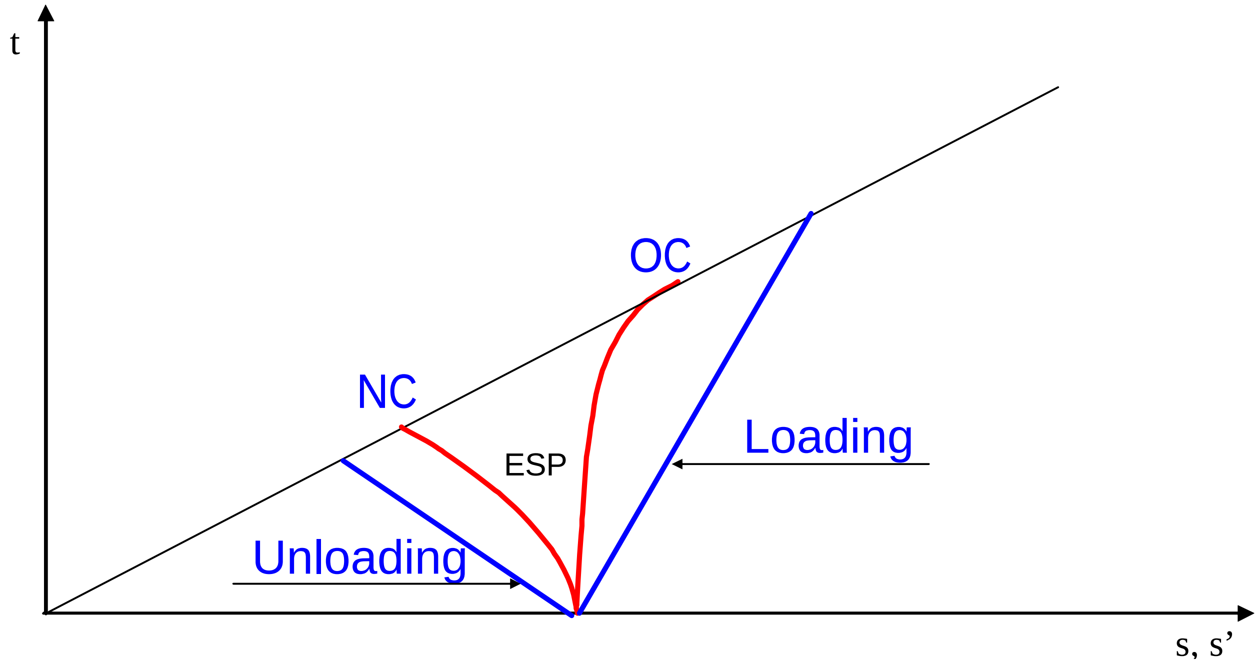
Strength parameters

- Mohr-Coulomb parameters in terms of total stresses
  - Only undrained conditions!



- Soil behaves as if it was cohesive
- $c_u \ (= s_u)$: undrained shear strength
- c_u only changes if drainage occurs (no change if undrained conditions prevail)

What is the critical case: drained or undrained?

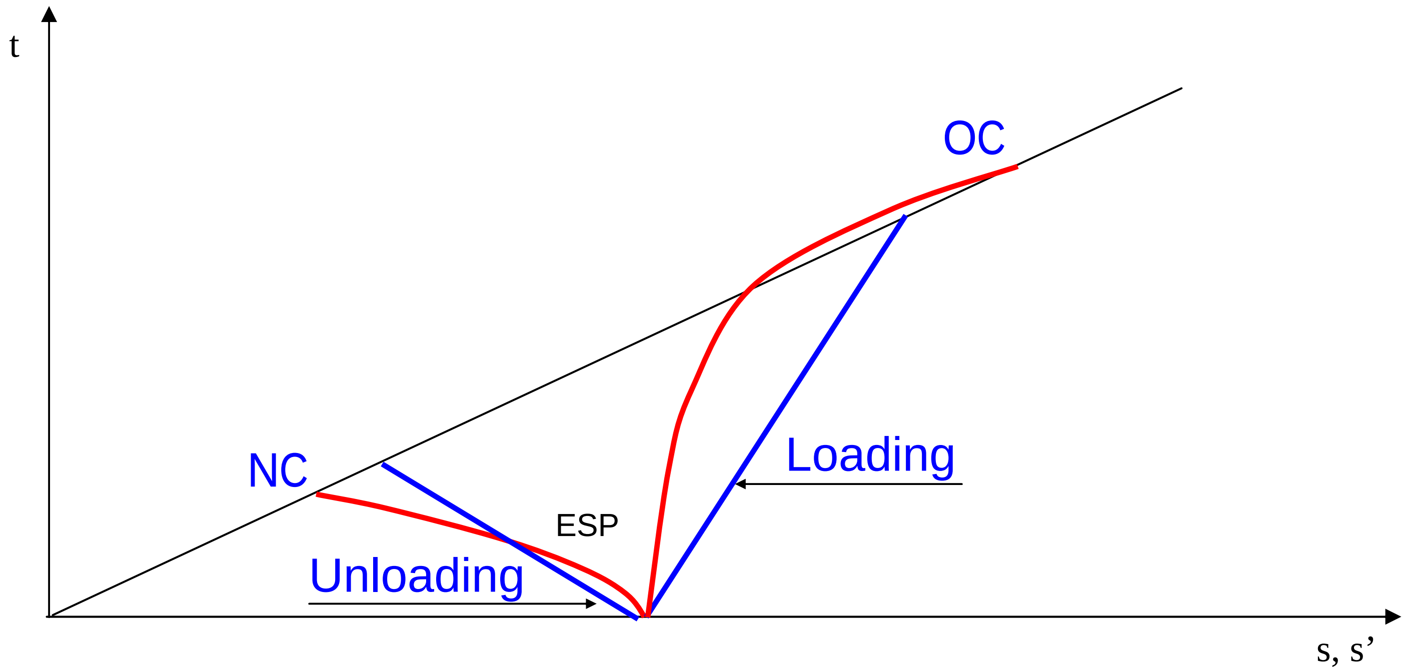


note that for soils in general:

- factor of safety against failure is *lower for short term* (undrained) conditions for loading problems (e.g. embankment)
- factor of safety against failure is *lower for long term* (drained) conditions for unloading problems (e.g. excavations)

however ...

What is the critical case: drained or undrained?



- For very soft NC soil, factor of safety against failure may be *lower for short term* (undrained) conditions for unloading problems (e.g. excavations)
- For very stif OC soil, factor of safety against failure may be *lower for short term* (undrained) conditions for loading problems (e.g. embankment)

FE modeling of undrained behavior

- what Plaxis does when specifying type of material behaviour: ***undrained***
- both changes in σ' and u are considered
- constitutive equations are formulated in terms of σ

$$\Delta\sigma' = D'\Delta\varepsilon$$

- we need to compute $\boldsymbol{D}$

$$\Delta\sigma = D\Delta\varepsilon$$

FE modeling of undrained behavior

principle of effective stress → $\Delta\sigma = \Delta\sigma' + \Delta\sigma_f$

with $\Delta\sigma_f = [\Delta p_w \quad \Delta p_w \quad \Delta p_w \quad 0 \quad 0 \quad 0]^T$

since the strains are the same in each phase,

$$\Delta\sigma' = D' \Delta\varepsilon$$

$$\Delta\sigma_f = D_f \Delta\varepsilon \quad D_f = K_e \begin{vmatrix} 1_3 & 0_3 \\ 0_3 & 0_3 \end{vmatrix} \quad K_e \cong \frac{K_f}{n}$$

pore fluid stiffness, related to the bulk modulus of pore fluid (water) K_f

- We need ***D*** $\quad \Delta\sigma = D\Delta\varepsilon$

$$\Delta\sigma' + \Delta\sigma_f = \Delta\sigma = D\,\Delta\varepsilon = D'\Delta\varepsilon + D_f\Delta\varepsilon = (D' + D_f)\Delta\varepsilon$$

$$\boxed{D = D' + D_f}$$

FE modeling of undrained behavior

- Example: linear elastic model + plane strain

$$D = D' + D_f$$

$$K = \frac{E}{3(1-2\nu)} \qquad G = \frac{E}{2(1+\nu)}$$

$$G = G'$$

$$\begin{bmatrix} \dot{\sigma}'_{xx} \\ \dot{\sigma}'_{yy} \\ \dot{\sigma}'_{zz} \\ \dot{\sigma}'_{xy} \end{bmatrix} = \begin{bmatrix} K'+\frac{4}{3}G & K'-\frac{2}{3}G & K'-\frac{2}{3}G & 0 \\ K'-\frac{2}{3}G & K'+\frac{4}{3}G & K'-\frac{2}{3}G & 0 \\ K'-\frac{2}{3}G & K'-\frac{2}{3}G & K'+\frac{4}{3}G & 0 \\ 0 & 0 & 0 & G \end{bmatrix} \begin{bmatrix} \dot{\varepsilon}^e_{xx} \\ \dot{\varepsilon}^e_{yy} \\ \dot{\varepsilon}^e_{zz} \\ \dot{\gamma}^e_{xy} \end{bmatrix}$$

$$\mathbf{\Delta\sigma' = D' \, \Delta\varepsilon}$$

$$\begin{bmatrix} \dot{\sigma}_{xx} \\ \dot{\sigma}_{yy} \\ \dot{\sigma}_{zz} \\ \dot{\sigma}_{xy} \end{bmatrix} = \begin{bmatrix} K+\frac{4}{3}G & K-\frac{2}{3}G & K-\frac{2}{3}G & 0 \\ K-\frac{2}{3}G & K+\frac{4}{3}G & K-\frac{2}{3}G & 0 \\ K-\frac{2}{3}G & K-\frac{2}{3}G & K+\frac{4}{3}G & 0 \\ 0 & 0 & 0 & G \end{bmatrix} \begin{bmatrix} \dot{\varepsilon}^e_{xx} \\ \dot{\varepsilon}^e_{yy} \\ \dot{\varepsilon}^e_{zz} \\ \dot{\gamma}^e_{xy} \end{bmatrix}$$

$$\mathbf{\Delta\sigma = D \, \Delta\varepsilon}$$

FE modeling of undrained behavior

- Example: linear elastic model + plane strain

$$D = D' + D_f$$

$$D = \begin{bmatrix} K' + \frac{4}{3}G & K' - \frac{2}{3}G & K' - \frac{2}{3}G & 0 \\ K' - \frac{2}{3}G & K' + \frac{4}{3}G & K' - \frac{2}{3}G & 0 \\ K' - \frac{2}{3}G & K' - \frac{2}{3}G & K' + \frac{4}{3}G & 0 \\ 0 & 0 & 0 & G \end{bmatrix} + \begin{bmatrix} K_e & K_e & K_e & 0 \\ K_e & K_e & K_e & 0 \\ K_e & K_e & K_e & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} K' + \frac{4}{3}G + K_e & K' - \frac{2}{3}G + K_e & K' - \frac{2}{3}G + K_e & 0 \\ K' - \frac{2}{3}G + K_e & K' + \frac{4}{3}G + K_e & K' - \frac{2}{3}G + K_e & 0 \\ K' - \frac{2}{3}G + K_e & K' - \frac{2}{3}G + K_e & K' + \frac{4}{3}G + K_e & 0 \\ 0 & 0 & 0 & G \end{bmatrix}$$

FE modeling of undrained behavior

- Example: linear elastic model + plane strain

$$D = D' + D_f$$

$$D = D' + D_e = \begin{bmatrix} K' + \frac{4}{3}G + K_e & K' - \frac{2}{3}G + K_e & K' - \frac{2}{3}G + K_e & 0 \\ K' - \frac{2}{3}G + K_e & K' + \frac{4}{3}G + K_e & K' - \frac{2}{3}G + K_e & 0 \\ K' - \frac{2}{3}G + K_e & K' - \frac{2}{3}G + K_e & K' + \frac{4}{3}G + K_e & 0 \\ 0 & 0 & 0 & G \end{bmatrix}$$

$$D = \begin{bmatrix} K + \frac{4}{3}G & K - \frac{2}{3}G & K - \frac{2}{3}G & 0 \\ K - \frac{2}{3}G & K + \frac{4}{3}G & K - \frac{2}{3}G & 0 \\ K - \frac{2}{3}G & K - \frac{2}{3}G & K + \frac{4}{3}G & 0 \\ 0 & 0 & 0 & G \end{bmatrix}$$

$$K' + \frac{4}{3}G + K_e = K + \frac{4}{3}G$$

$$K = K' + K_e$$

FE modeling of undrained behavior

all the above (which is valid for any soil (or model) for which the principle of effective stress applies) can be easily combined with the FEM

- instead of specifying the components of D, specify D', and K_e

$$D = D' + D_f$$

then same as in the drained case

- when calculating stresses,

$$\Delta\sigma_f = K_e \Delta\varepsilon_v$$
$$\Delta\sigma' = D' \Delta\varepsilon$$

$$\Rightarrow \quad \Delta\sigma = \Delta\sigma' + \Delta\sigma_f$$

a value must be set for K_e

the pore-fluid is assigned a bulk modulus that is substantially larger than that of the soil skeleton (which ensures that during undrained loading the volumetric strains are very small)

FE modeling of undrained behavior

PLAXIS automatically adds stiffness of water when undrained material type is chosen using the following approximation:

$$\mathbf{K}_{total} = \mathbf{K}' + \frac{\mathbf{K}_w}{\mathbf{n}} = \frac{\mathbf{E}_u}{3(1-2\nu_u)} = \frac{2\mathbf{G}(1+\nu_u)}{3(1-2\nu_u)}$$

$$\mathbf{K}_{total} = \frac{\mathbf{E}'(1+\nu_u)}{3(1-2\nu_u)(1+\nu')}$$

assuming $\nu_u = 0.495$

Notes:

- this procedure gives reasonable results only for $\nu' < 0.35$!
- in Version 8 B-value can be entered explicitely for undrained materials
- real value of $K_w/n \sim 1 \cdot 10^6$ kPa (for n = 0.5)

modeling undrained behavior with PLAXIS

<u>method A</u> (analysis in terms of *effective* stresses):

type of material behaviour: *undrained*

effective strength parameters (MC: c', φ', ψ')

effective stiffness parameters (MC: E_{50}', ν')

<u>method B</u> (analysis in terms of *effective* stresses):

type of material behaviour: *undrained*

total strength parameters $c = c_u$, $\varphi = 0$, $\psi = 0$

effective stiffness parameters E_{50}', ν'

<u>method C</u> (analysis in terms of *total* stresses):

type of material behaviour: *drained*

total strength parameters $c = c_u$, $\varphi = 0$, $\psi = 0$

total stiffness parameters E_u, $\nu_u = 0.495$

FE modeling of undrained behavior (method A)

- analysis in terms of effective stress
- type of material behaviour: *undrained*
- u changes (excess pore water pressures generated)
- constitutive equations are formulated in terms of σ'

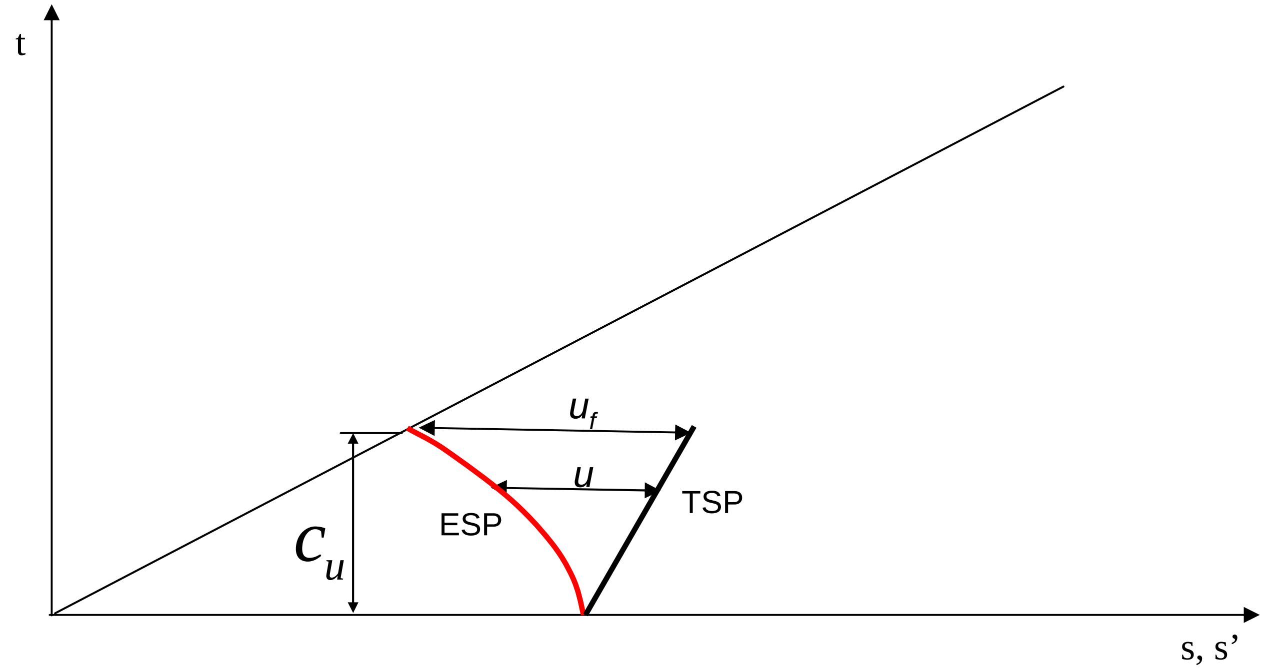
$$\Delta\sigma' = D'\Delta\varepsilon$$

In the case of Mohr Coulomb model:

effective strength parameters c', φ', ψ
effective stiffness parameters E_{50}', ν'

- the total stiffness matrix is computed as: $D = D' + D_f$

FE modeling of undrained behavior (method A)



- single set of parameters in terms of effective stress (undrained, drained, consolidation analysis consistent)
- realistic prediction of pore pressures (if model is appropriate)
- the undrained analysis can be followed by a consolidation analysis (correct pore pressures, correct drained parameters)
- C_u is a consequence of the model, not an input parameter!!

modeling undrained behavior with PLAXIS

method A (analysis in terms of *effective* stresses):
type of material behaviour: *undrained*
effective strength parameters c', φ', ψ'
effective stiffness parameters E_{50}', ν'

method B (analysis in terms of *effective* stresses):
type of material behaviour: *undrained*
total strength parameters $c = c_u$, $\varphi = 0$, $\psi = 0$
effective stiffness parameters E_{50}', ν'

method C (analysis in terms of *total* stresses):
type of material behaviour: *drained*
total strength parameters $c = c_u$, $\varphi = 0$, $\psi = 0$
total stiffness parameters E_u, $\nu_u = 0.495$

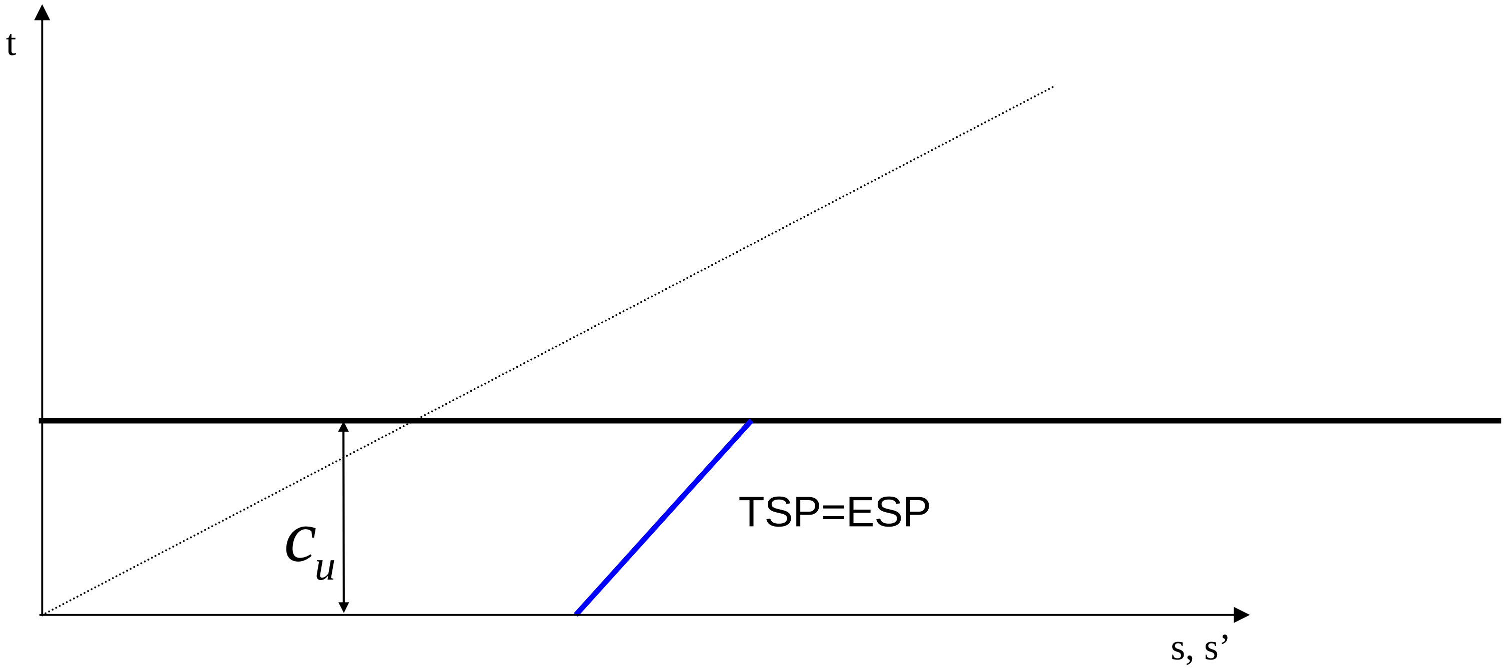
FE modeling of undrained behavior (method C)

- analysis in terms of total stress
- type of material behaviour: *drained* (in spite of modelling an undrained case)
- u does not change
- constitutive equations are formulated in terms of σ

total strength parameters $c = c_u$, $\varphi = 0$, $\psi = 0$
total stiffness parameters E_u, $\nu_u = 0.495$

$$\Delta\sigma = D\,\Delta\varepsilon$$

FE modeling of undrained behavior (method C)



- parameters in terms of total stress
- no prediction of pore pressures (only total stresses are obtained)
- the undrained analysis can not be followed by a consolidation analysis
- C_u is an input parameter!!

modeling undrained behavior with PLAXIS

method A (analysis in terms of *effective* stresses):
type of material behaviour: *undrained*
effective strength parameters c', φ', ψ'
effective stiffness parameters E_{50}', ν'

method B (analysis in terms of *effective* stresses):
type of material behaviour: *undrained*
total strength parameters $c = c_u$, $\varphi = 0$, $\psi = 0$
effective stiffness parameters E_{50}', ν'

method C (analysis in terms of *total* stresses):
type of material behaviour: *drained*
total strength parameters $c = c_u$, $\varphi = 0$, $\psi = 0$
total stiffness parameters E_u, $\nu_u = 0.495$

FE modeling of undrained behavior (method B)

- analysis in terms of effective stress
- type of material behaviour: *undrained*
- u changes
- constitutive equations are formulated in terms of σ' (but strength in total stresses!)

total strength parameters $c = c_u$, $\varphi = 0$, $\psi = 0$
effective stiffness parameters E_{50}', ν'

$$\Delta\sigma' = D'\Delta\varepsilon$$

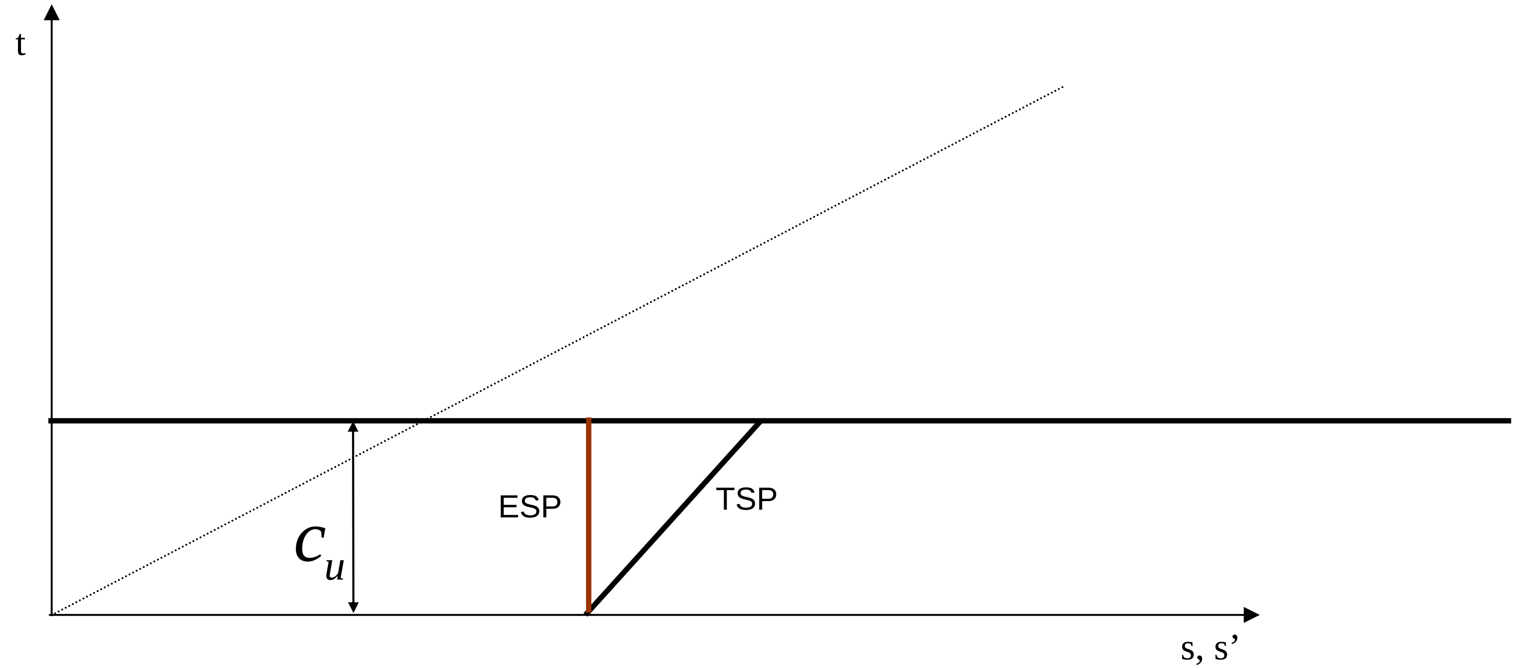
$$\Delta\sigma = D\,\Delta\varepsilon$$

$$D = D' + D_f$$

Resulting undrained stiffness parameters

$$E_u = \frac{3}{2}\frac{E'}{1+\nu'} \quad ; \quad \nu_u = 0.495$$

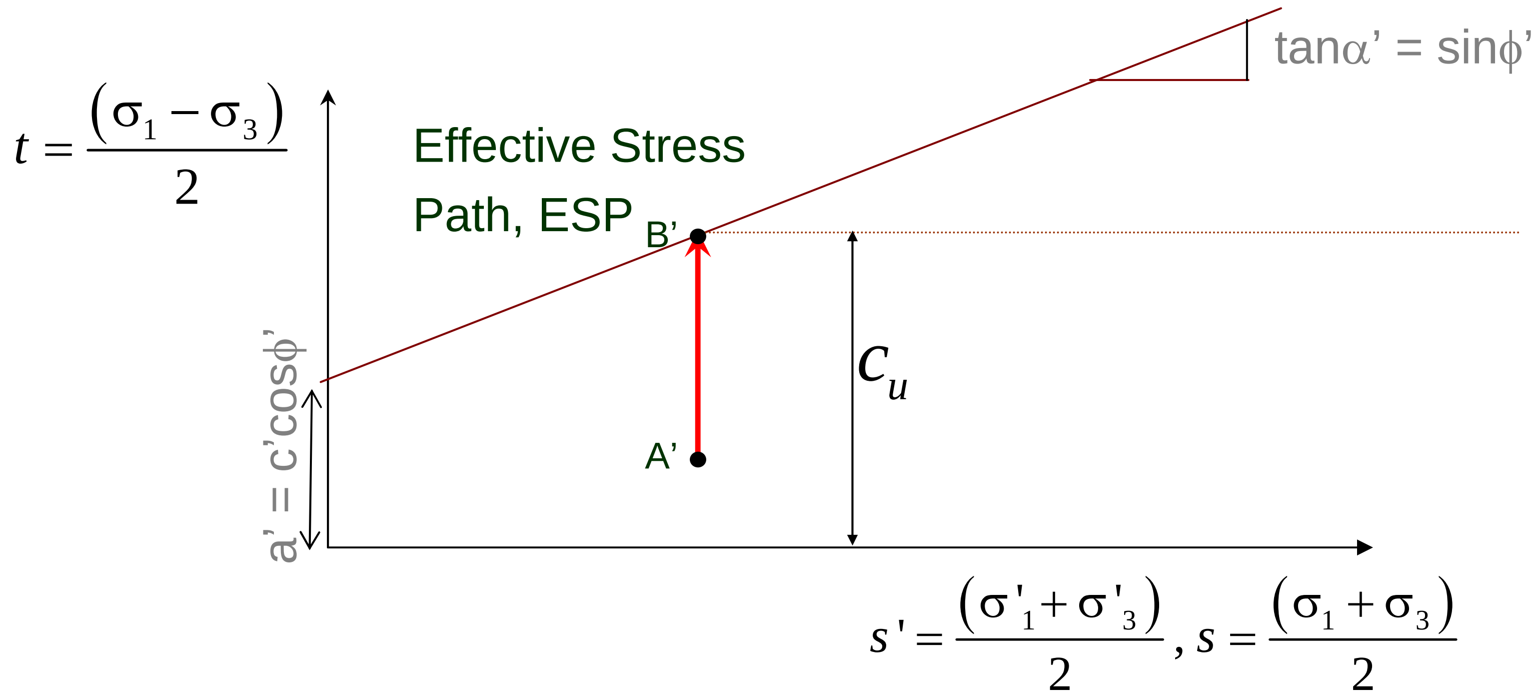
FE modeling of undrained behavior (method B)



- parameters in terms of total stress and effective stress
- prediction of pore pressures (generally unrealistic)
- the undrained analysis should not be followed by a consolidation analysis (pore pressures unrealistic)
- C_u is an input parameter!!

Undrained shear strength from the Mohr Coulomb model

Plane strain: effective stress path rises vertically

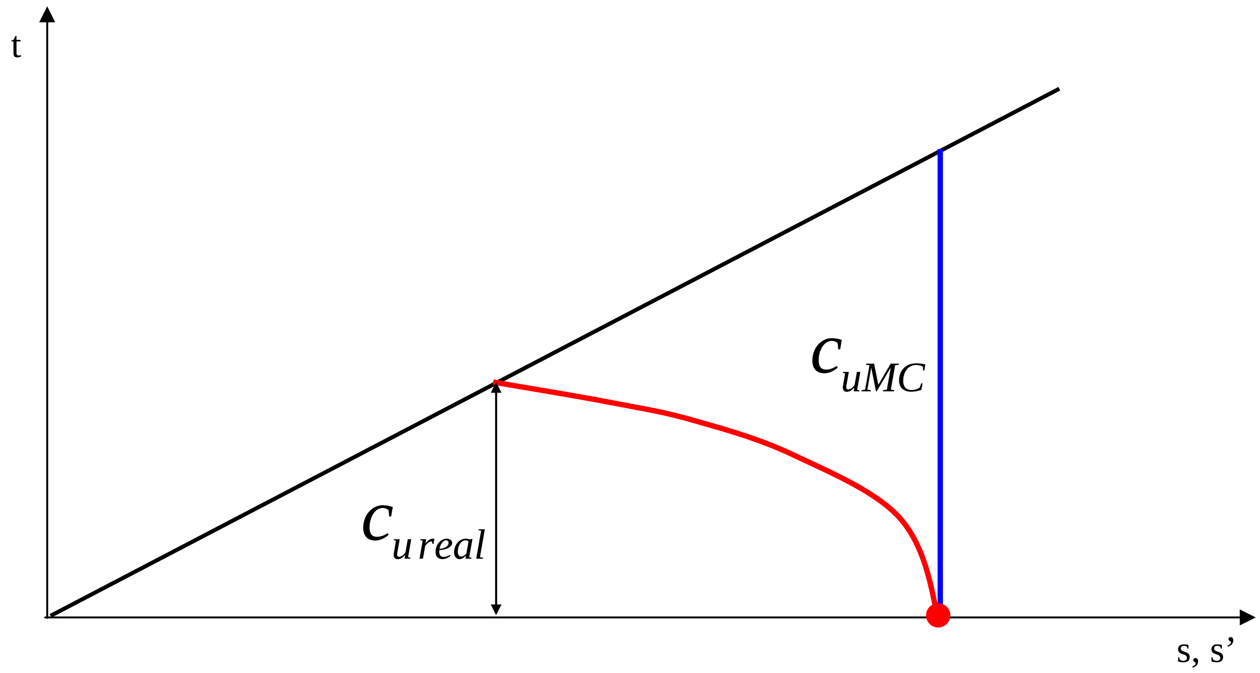


$$c_u = c'\cos\phi' + s_o'\sin\phi' = c'\cos\phi' + \frac{1}{2}\left\{\sigma'_{vo} + \sigma'_{ho}\right\}\sin\phi'$$

$$\frac{c_u}{\sigma'_{v0}} = \frac{c'\cos\phi'}{\sigma'_{v0}} + \frac{1}{2}\left\{1 + K_0\right\}\sin\phi'$$

Undrained shear strength from the Mohr Coulomb model

- The Mohr Coulomb model in terms of effective stresses OVERESTIMATES the undrained shear strength of soft clays!



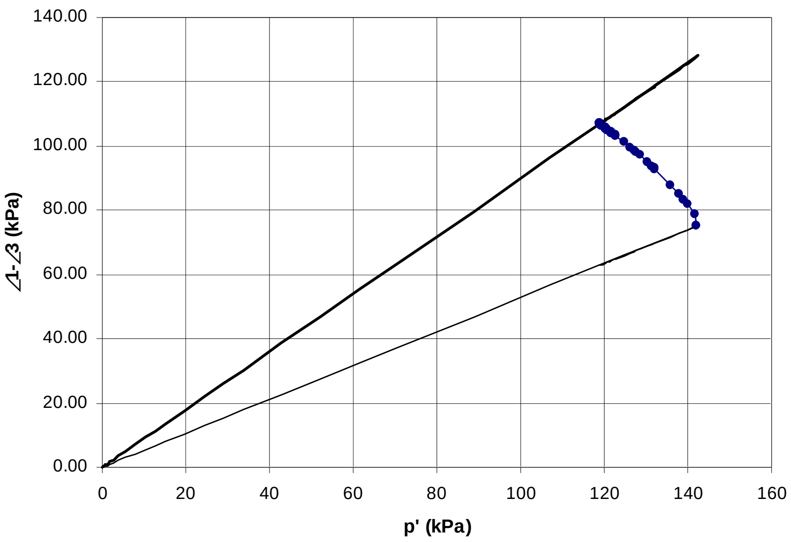
Undrained shear strength from advanced models

- Although it is possible, in a few simple cases, to obtain an analytical expression for $\mathbf{c_u}$, it is advisable to perform a numerical “laboratory” test to check the value of undrained shear strength actually supplied by the model
- It is important to perform the numerical “laboratory” test under the same condition as in the analysis
  - Plane strain, triaxial, simple shear
  - Correct initial stresses
  - Compression, extension, simple shear
- Not all $\mathbf{c_u}$ values are achievable with a particular model

Soft soil model

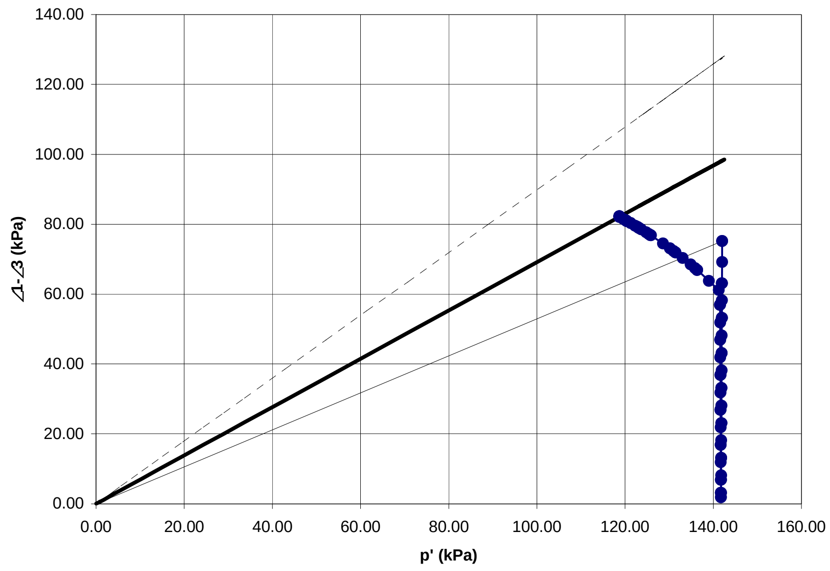
Parameters

$$c' = 0.1\ kPa \quad \phi' = 23^{o} \quad K_0^{NC} = 1 - \sin\phi' = 0.609 \quad \nu_{ur} = 0.15 \quad \lambda^* = 0.11 \quad \kappa^* = 0.0275$$



Triaxial (compression)

$c_u/\sigma_v' = 0.279$



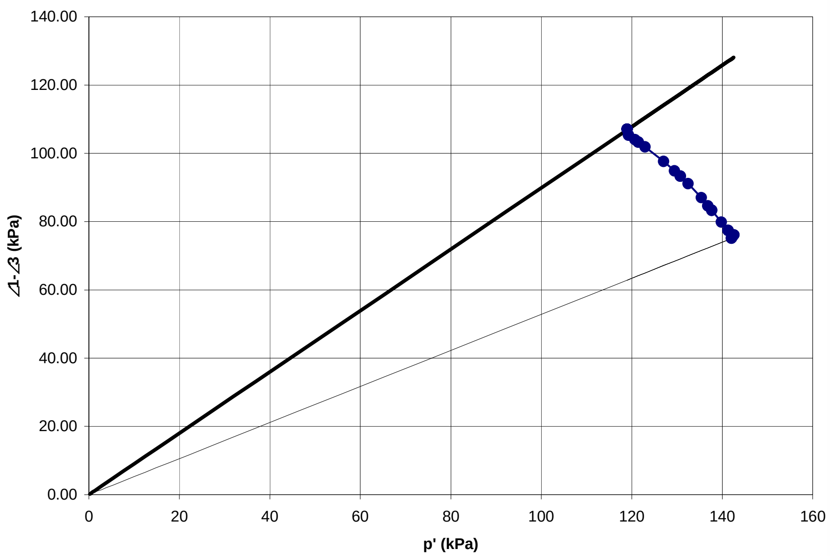
Triaxial (extension)

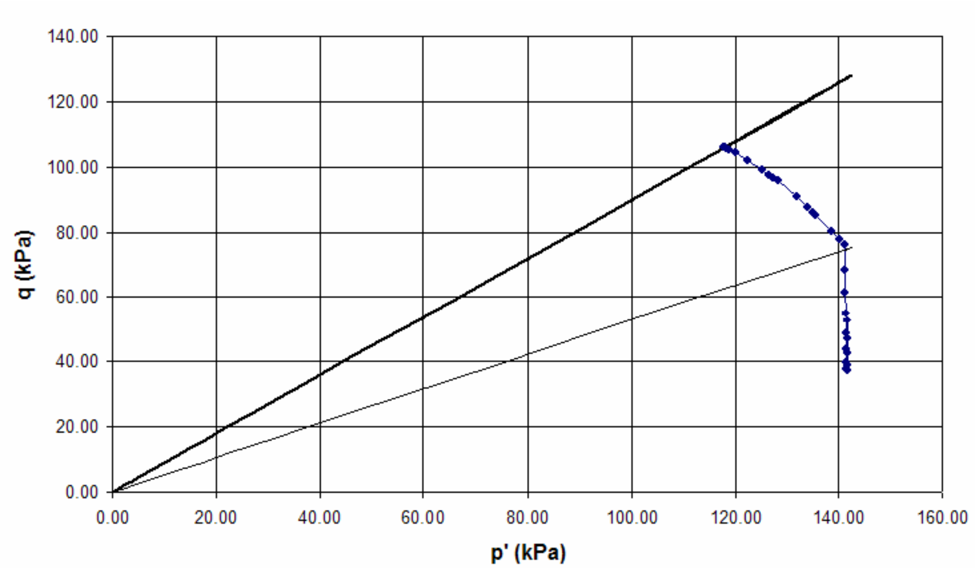
$c_u/\sigma_v' = 0.214$

Soft soil model

Parameters

$c' = 0.1\ kPa \quad \phi' = 23^{o} \quad K_0^{NC} = 1 - \sin\phi' = 0.609 \quad \nu_{ur} = 0.15 \quad \lambda^* = 0.11 \quad \kappa^* = 0.0275$





Plane strain (compression)

Plane strain (extension)

$c_u/\sigma_v' = 0.279$

$c_u/\sigma_v' = 0.277$

influence of dilatancy on undrained shear strength

if we set $\psi \neq 0$ then, negative volumetric plastic deformations occur at failure:

$$\Delta\varepsilon_v = \Delta\varepsilon_v^e + \Delta\varepsilon_v^p \quad \text{(elastic-plastic behavior)}$$

$$\Delta\varepsilon_v = 0 \quad \text{(undrained conditions)}$$

$$\Delta\varepsilon_v^p < 0 \Rightarrow \Delta\varepsilon_v^e > 0 \Rightarrow \Delta p' = K'\Delta\varepsilon_v^e > 0$$

$$\text{At failure: } \Delta q = M\,\Delta p' \Rightarrow \Delta q > 0$$

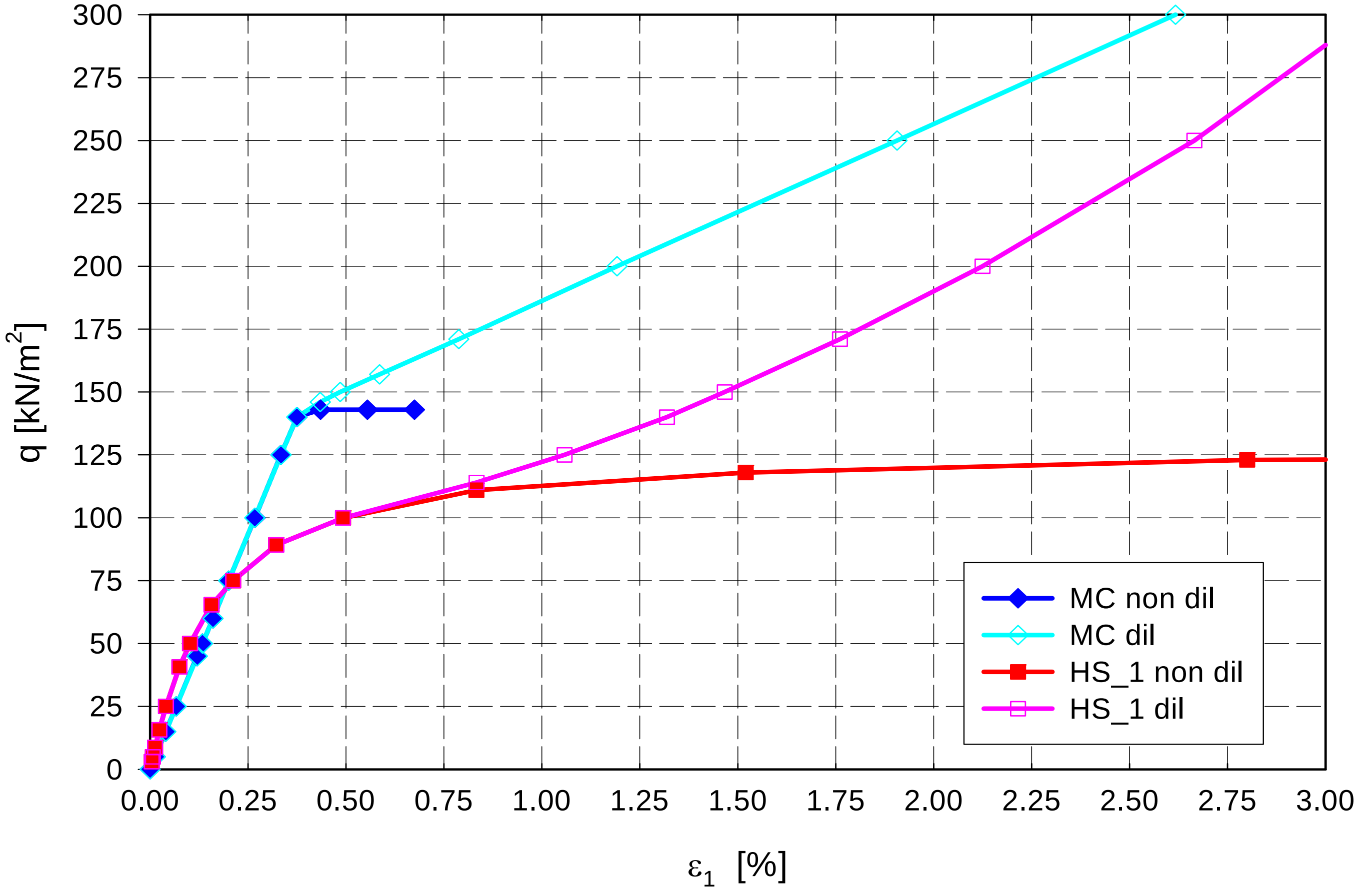
$$\Delta t = \Delta s' \sin\varphi' \Rightarrow \Delta t > 0$$

result: unlimited increase of q (or t), i.e. infinite strength!!

Therefore, in undrained analysis, dilatancy, Ψ, must be set to zero!

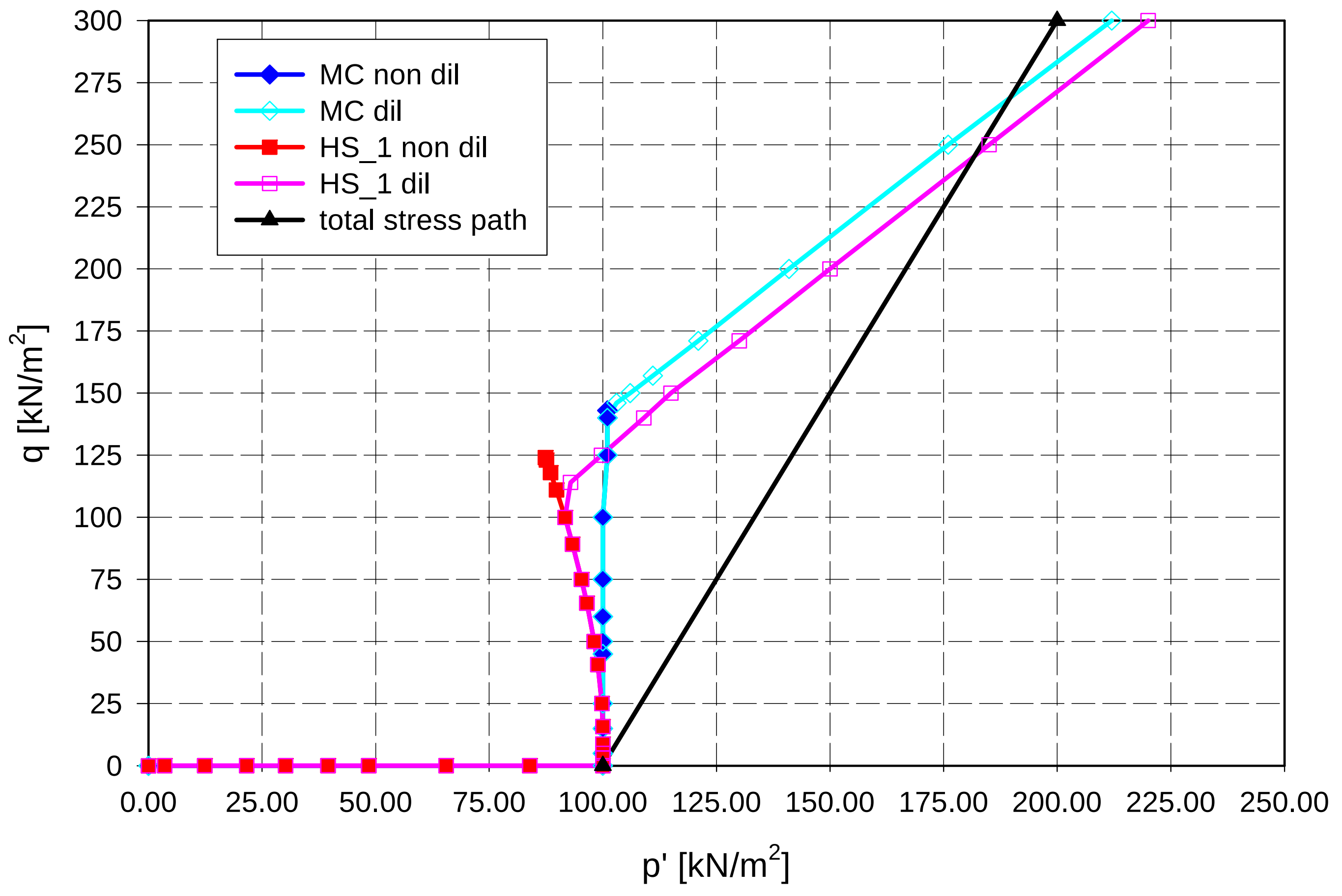
influence of constitutive model and dilatancy

simulation of undrained triaxial compression test – MC / HS model - q vs ε_1



influence of constitutive model and dilatancy

simulation of undrained triaxial compression test – MC / HS model - q vs p'

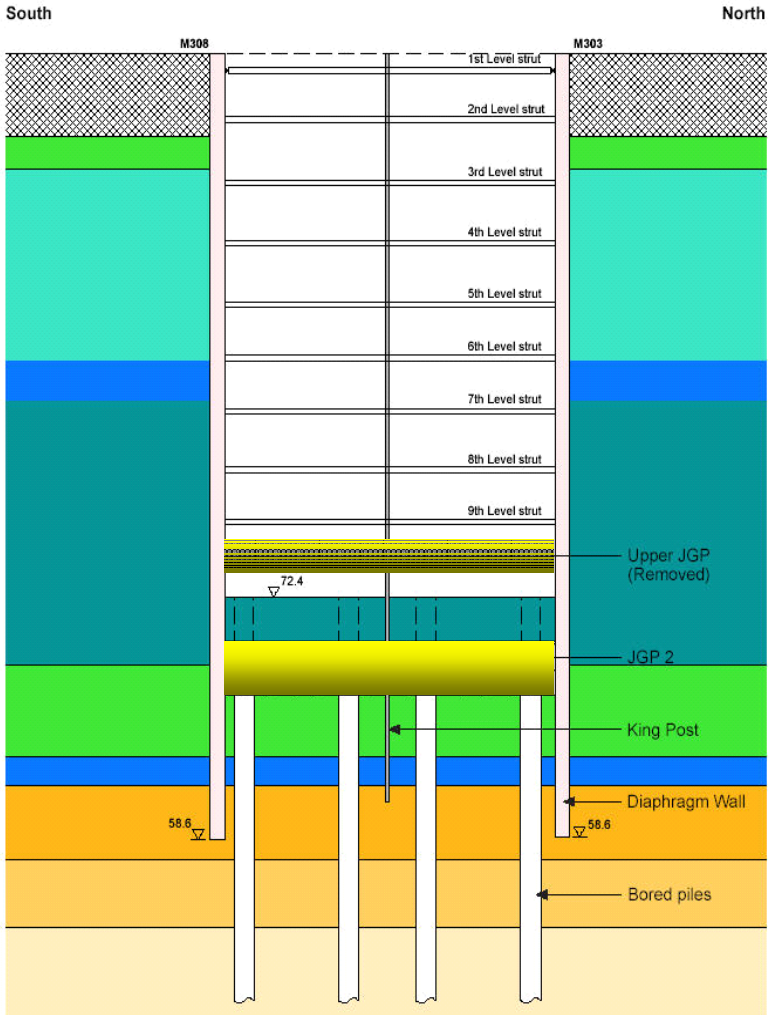


summary

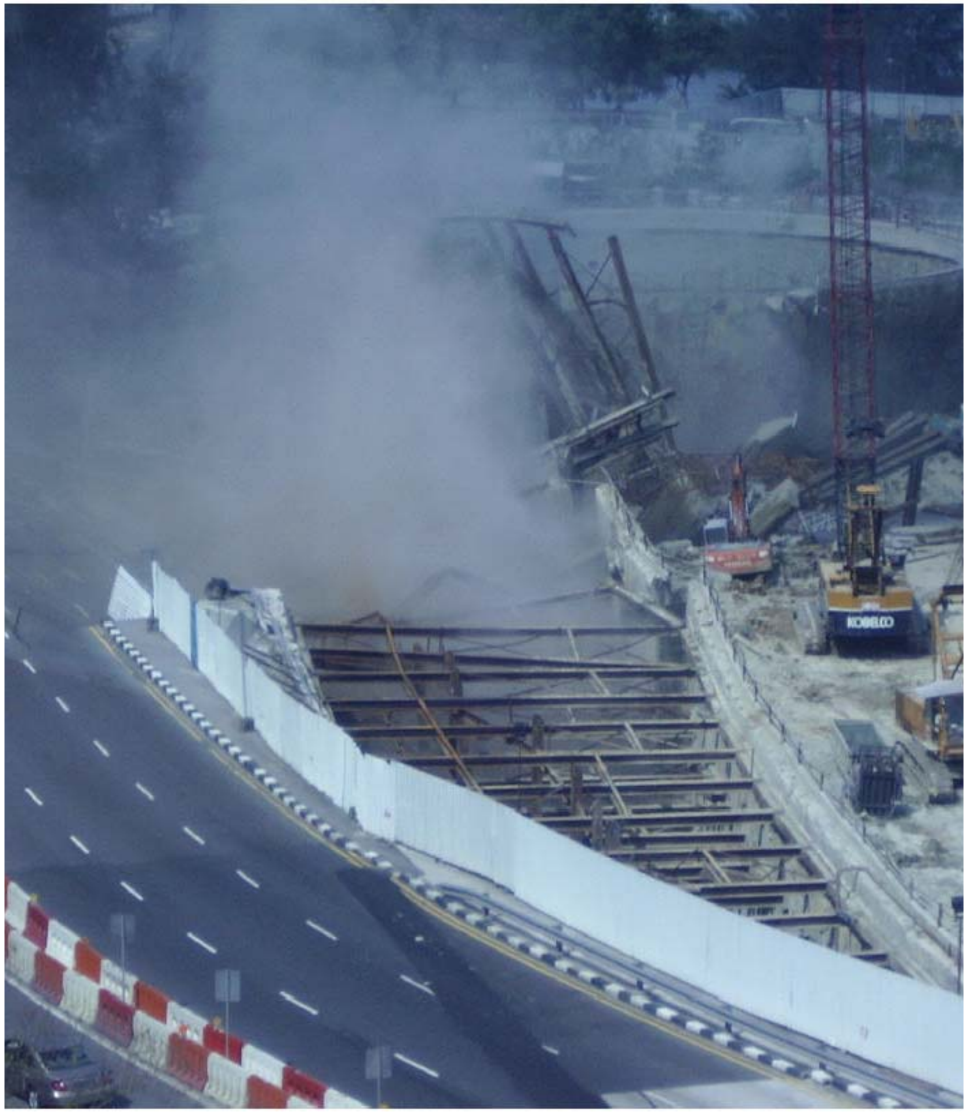
- FEM analysis of undrained conditions can be performed in effective stresses and with effective stiffness and strength parameters (Method A)
- **Method A** must be used:
  - if consolidation/long term analysis are required
  - advanced soil models are adopted
- undrained shear strength is a result of the constitutive model
- care must be taken with the choice of the value for dilatancy angle
- **Methods B and C** provide alternative ways to analyze undrained problems but:
  - the constituive model dos not generally represent the true soil behaviour (before failure)
  - potentially useful for stability problems in undrained conditions (specification of undrained shear strength is straightforward)

Nicoll Highway Collapse, Singapore

Nicoll Highway Collapse, Singapore



Nicoll Highway Collapse

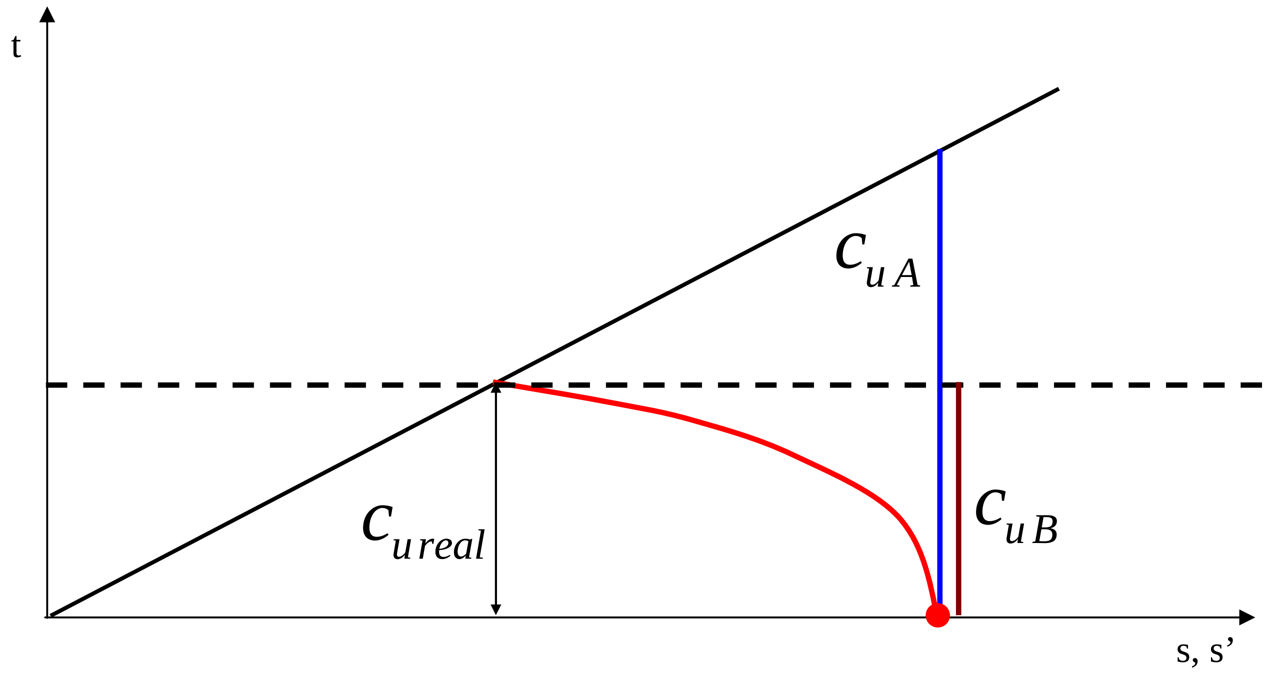


15:33, 20 April 2004

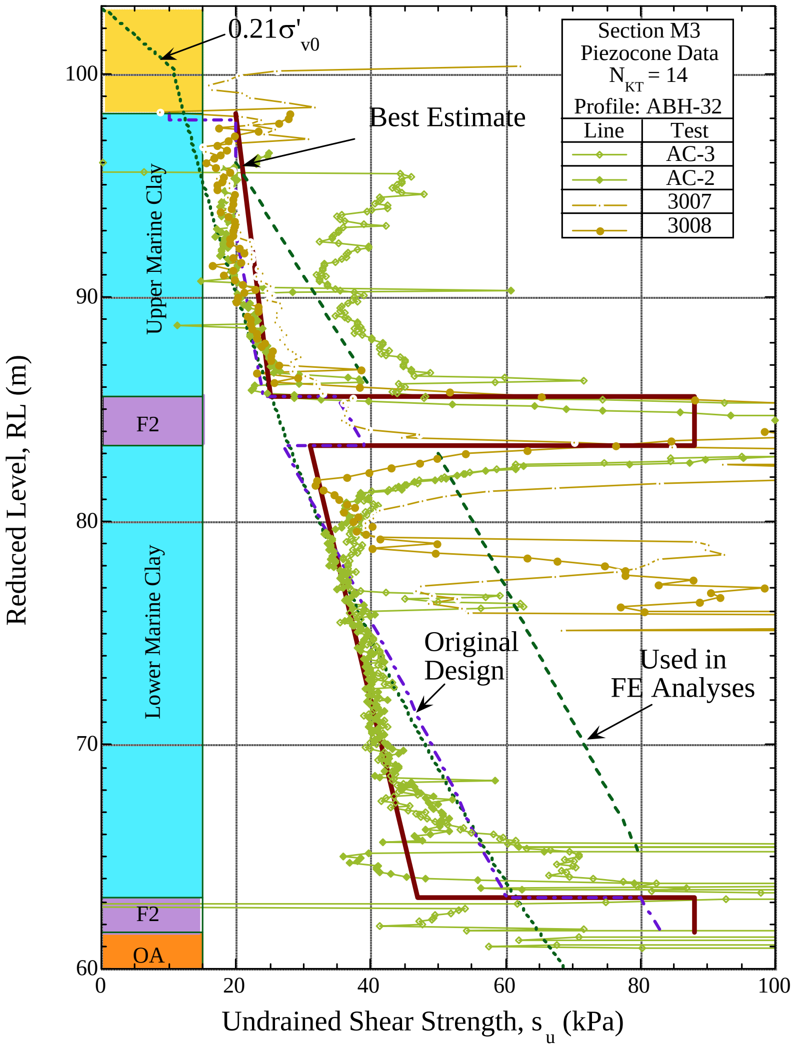
Nicoll Highway Collapse

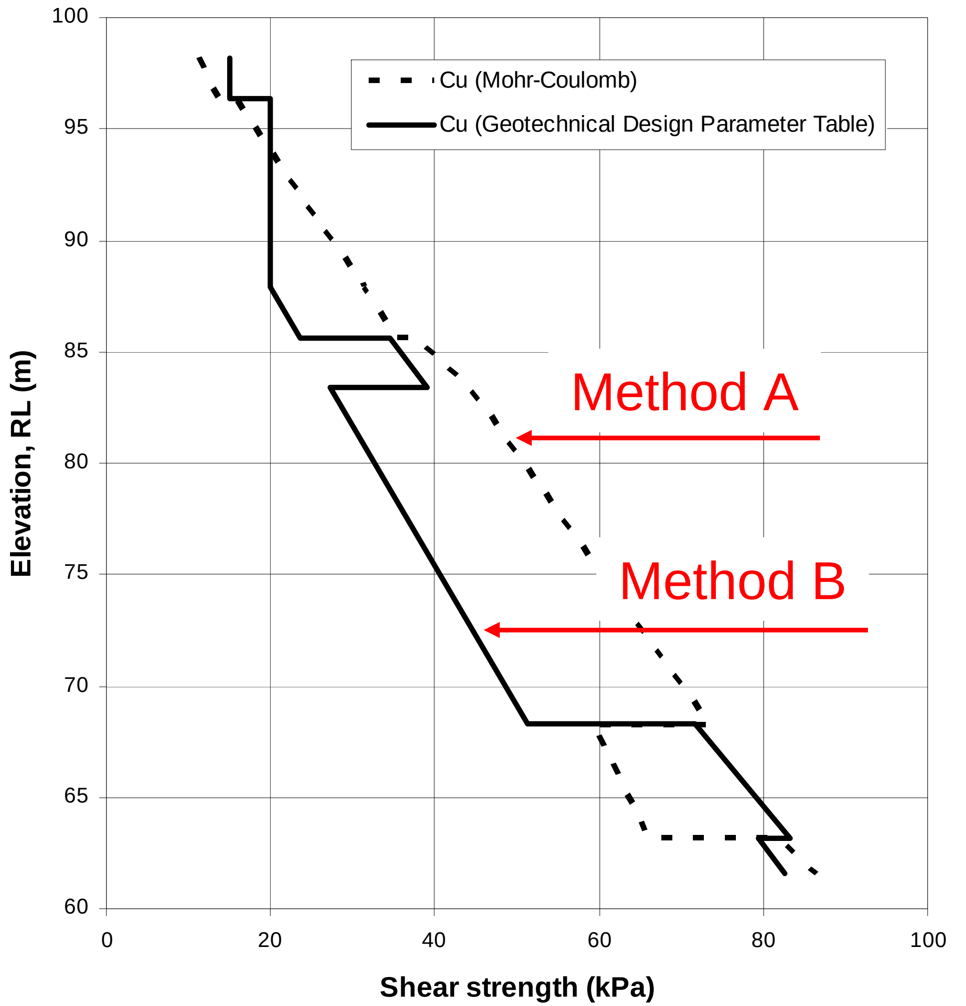
Nicoll Highway Collapse

- Undrained stabilty problem. Method A and Mohr Coulomb constituive model used for design analysis



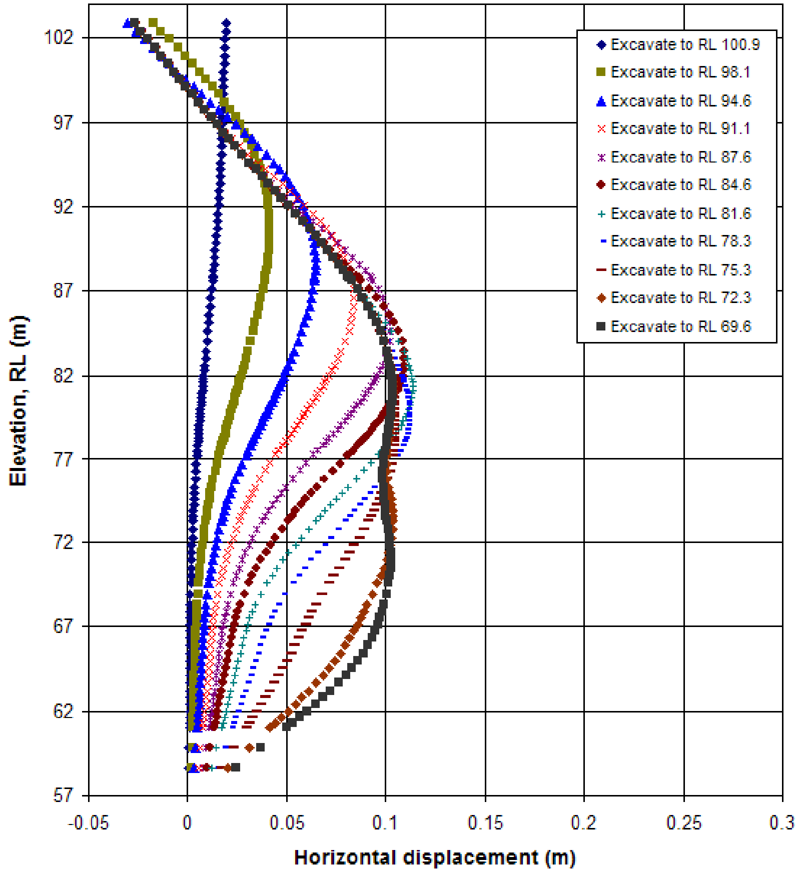
Nicoll Highway Collapse



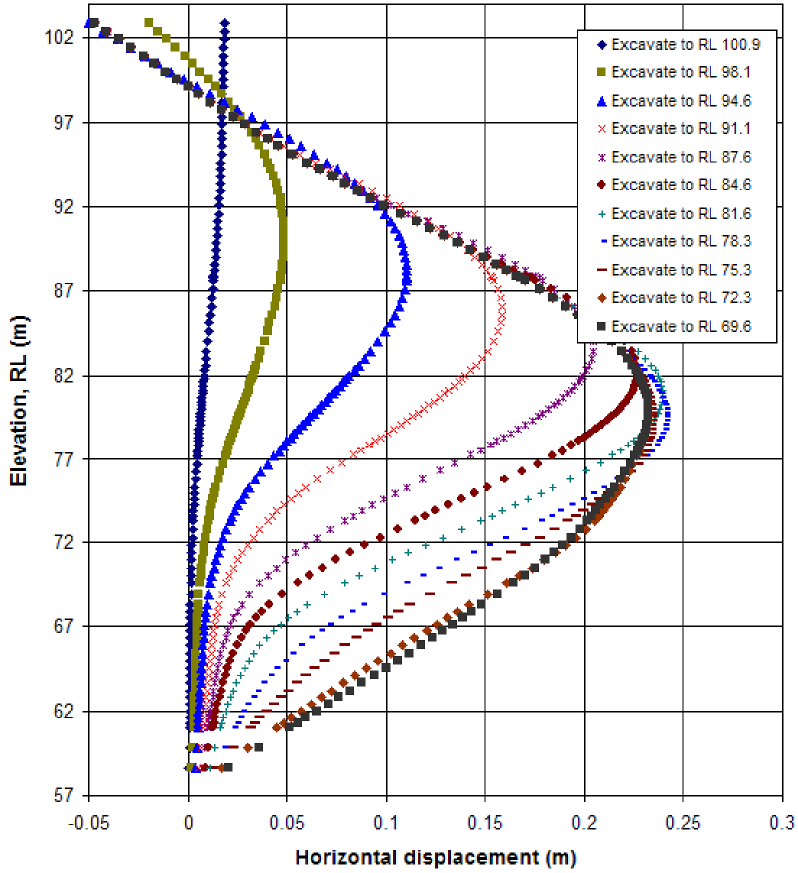


Overestimation of shear strength: 43%-62.1%

Nicoll Highway Collapse



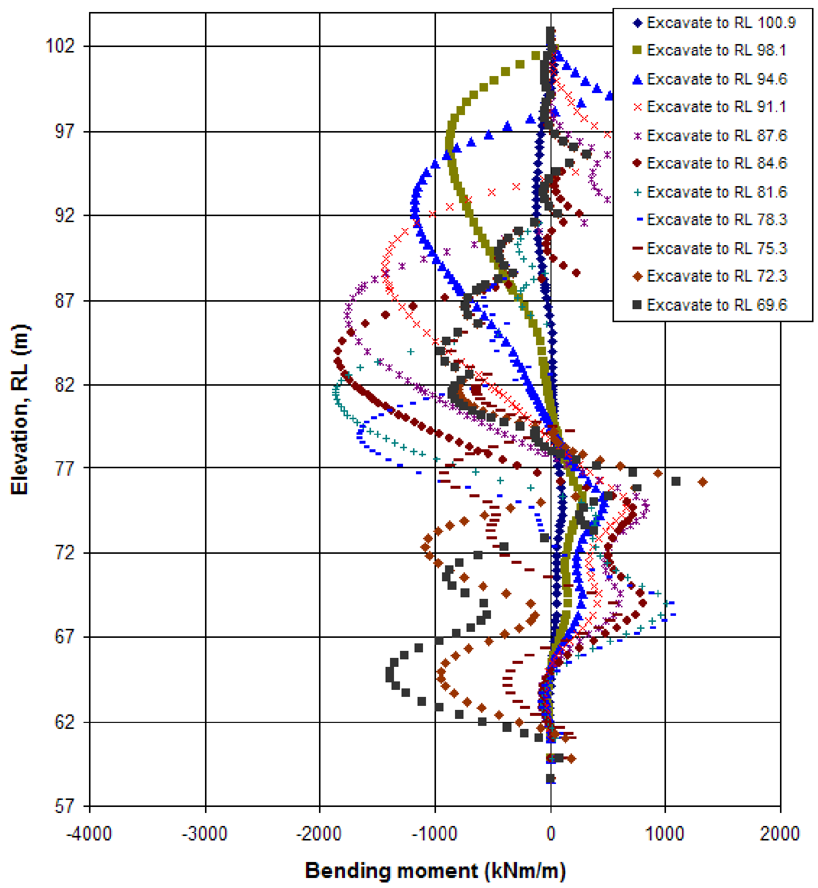
Method A



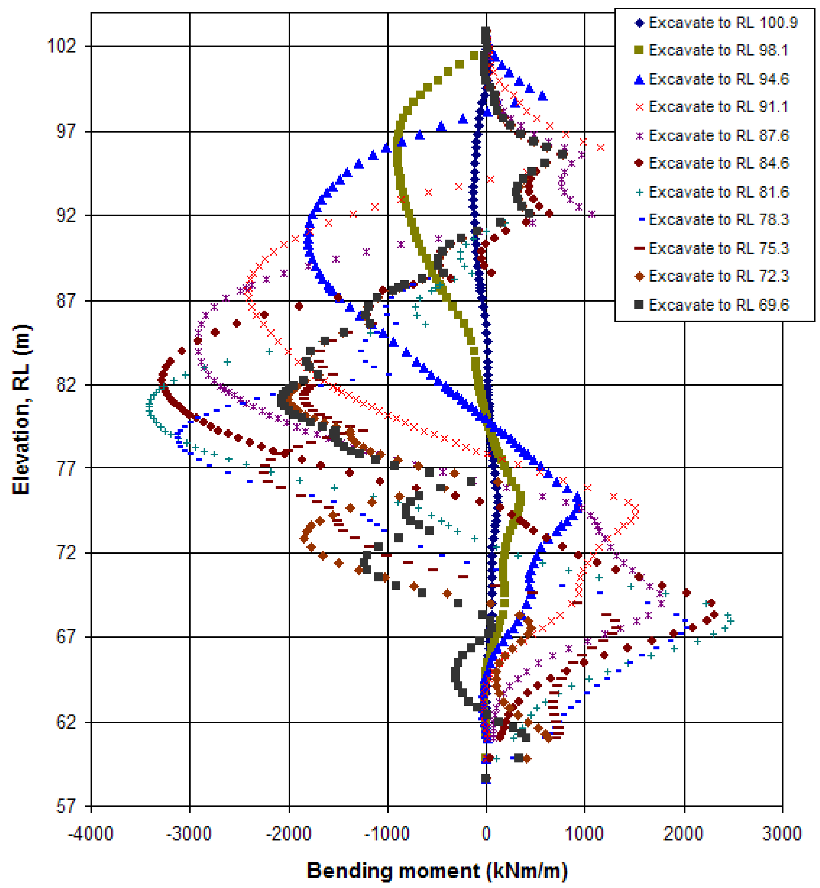
Method B

Underestimation of wall displacements (about a factor of 2)

Nicoll Highway Collapse



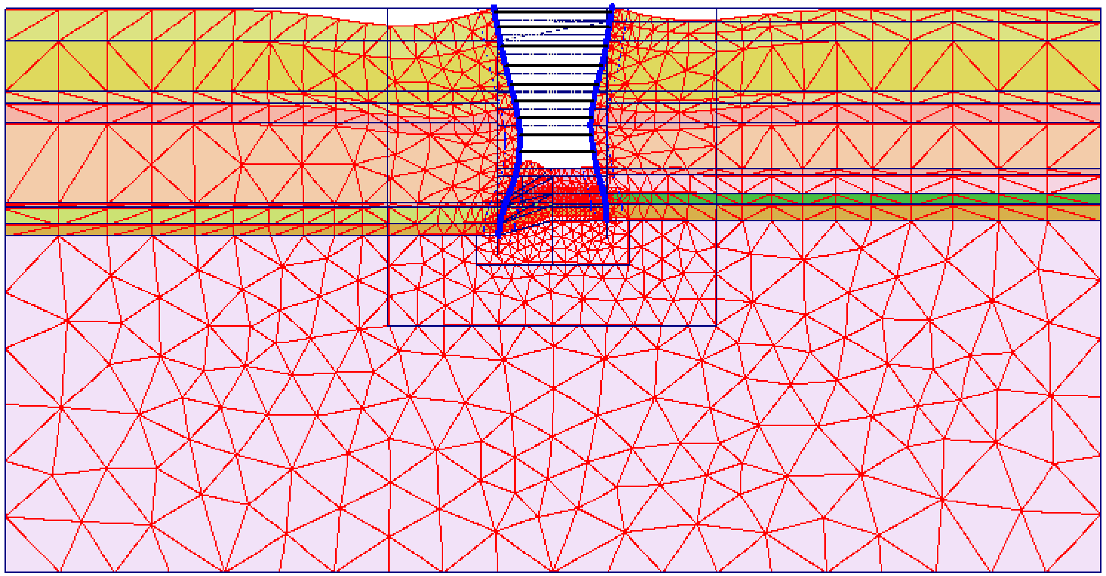
Method A



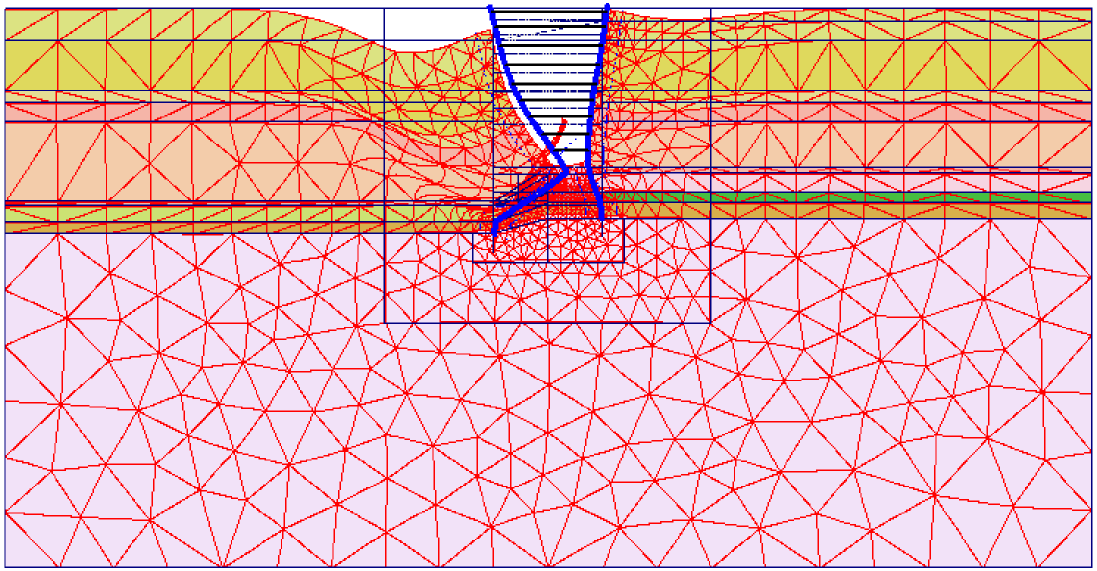
Method B

Underestimation of bending moment (about a factor of 2)

Nicoll Highway Collapse



Method A
(no collapse)



Method B
(collapse)

Predictions (backanalysis)