Experience of consolidation process from test areas with and without vertical drains

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Abstract: Two different methods of how to analyse the consolidation process arising due to loading of soft clay deposits are presented, one of which is the classical approach based on validity and the other on non-validity of Darcy's law at small hydraulic gradients. The two methods of analysis are compared with results of full-scale loading tests on areas without vertical drains and areas provided with vertical drains.

Two test areas in Sweden without vertical drains are presented, which have been followed-up by pore pressure and settlement observations for half a century. The results obtained in these test areas is of particular interest since the observed delay in excess pore pressure dissipation in relation to what would be expected according to Terzaghi's one-dimensional consolidation theory has been explained to be due to the influence of creep *(*secondary consolidation*)*. However, the consolidation process based on non-Darcian flow yields better agreement with the pore pressure observations than the theory based on the assumed effect of creep.

Well-instrumented test areas provided with vertical sand drains or prefabricated band-shaped drains, established in Sweden, Thailand and Italy, are also presented. In one of the cases described, surface loading was replaced by the vacuum method. Although the consolidation characteristics of the soil in the test areas and the contract project are quite different, a better correspondence between practice and theory based on non-Darcian flow is obtained in all the studied cases than with theory based on Darcian flow.

Finally, a study made in connection with the extension of the Stockholm-Arlanda project with band drain installation is presented. In this case, due to heavy loading causing high hydraulic gradients, an acceptable correlation between theory and practice is obtained also by the assumption of Darcian flow.

INTRODUCTION

Land reclamation works and the need of improving infrastructures in areas with bad soil conditions have created an increasing interest in soil improvement. Among the methods utilised for this purpose, we find vertical drainage in combination with preloading, deep soil mixing with e.g. lime/cement, electro-osmosis and heavy tamping (dynamic consolidation). Case records, including all these soil improvement methods, have been published (e.g. Moseley & Kirsch, 2004; Hansbo, 1994, 2004). In this paper, various aspects of how to analyse the consolidation process arising due to loading of soft clay deposits are discussed (a complete review of the development of the theoretical approach to the problem of vertical drainage is presented by the author in Moseley & Kirsch, 2004). The results obtained by the analytical solutions are compared with observations made in a number of test areas with vertical drain installations and test areas without vertical drains. The analysis of the consolidation process is carried out on one hand on the classical assumption that Darcy's law is valid and on the other on the assumption of non-Darcian flow, as suggested by the author (Hansbo, 1960) in connection with the observations made at the Skå-Edeby test field. The influence on the consolidation process of disturbance effects due to vertical drain installation on one hand and of limited discharge capacity of the drains on the other is discussed and accounted for in the analysis.

THEORETICAL APPROACH

A deviation from Darcy's law has been discussed by several authors (e.g. Brenner, 1946; Kézdi, 1958; Florin, 1959, Miller & Low, 1963; Olsen, 1985; Dubin & Moulin, 1986). Considering the electric double-layer and the texture of the clay, a deviation from Darcy's law is to be expected. Thus, the forces, which bind the mobile phase of the water, gradually become stronger the smaller the distance to the rigid phase. Moreover, the pore space may be filled with mobile particles of colloidal or greater size, which may be bound by sorption forces. As a logical working hypothesis, it can be assumed that, within certain limits, the clay becomes increasingly porous because of the hydrodynamic forces created by increasing hydraulic gradient.

According to laboratory investigations on Skå-Edeby clay carried out by the author (Hansbo, 1960) a deviation from Darcy's law was observed at small hydraulic gradients. It was concluded that Darcy's law $v = ki$, where v = seepage velocity, k = coefficient of permeability and i = hydraulic gradient, should be replaced by non-Darcian flow, defined by exponential flow correlation $v = \kappa i^n$ when $i \le i_l$ and by linear correlation $v = \kappa n i_l^{n-1}(i - i_0)$ when $i > i_l$, where $i_0 = i_l(n-1)/n$, Fig. 1.

Permeability tests carried out by the author (Hansbo, 1960) indicated i_0 values of 1 to 4 and i_l values of 3 to 12, corresponding to n = 1.5. Several authors have reported deviations from Darcy's law. Florin (1959) reported relations between flow and hydraulic gradient similar to those proposed by the author but suggested they could be replaced by straight lines, in his case intersecting the gradient axis at i_0 values as high as 17 to 31. Permeability tests carried out by Dubin and Moulin (1986) on high-plasticity, moderately organic clay showed similar correlations to those of the author. According to their tests, the i_l value was in the range of 14 to 20. A threshold gradient, in reality corresponding to i_0, which has to be exceeded before pore water flow takes place, as suggested by among others Brenner (1946), Buisson (1953) and Kézdi (1958), was not observed in either of these investigations.

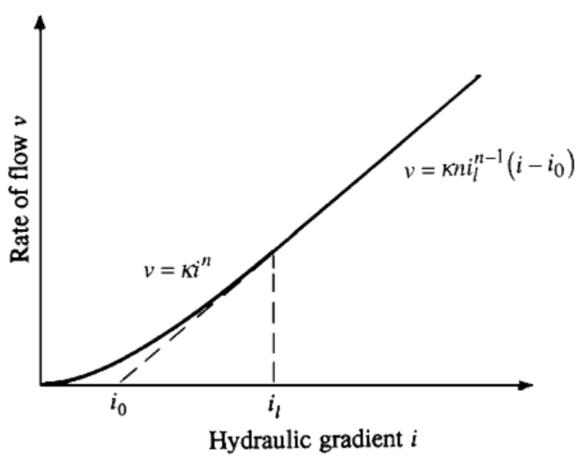

Fig. 1. Non-Darcian flow. Assumed correlation between rate of flow v and hydraulic gradient i.

1. ONE-DIMENSIONAL CONSOLIDATION

In the following, the analysis of one-dimensional consolidation will be carried out in the two ways described: analysis based on Darcian flow and analysis based on non-Darcian flow.

In the case of one-dimensional consolidation, based on Darcian flow, the theoretical correlation between excess pore water pressure *u* and consolidation time *t* at various depths *z* is given by the relation:

$$\frac{\partial u}{\partial t} = M \frac{\partial}{\partial z}\left(\frac{k}{\gamma_w}\frac{\partial u}{\partial z}\right) \tag{1}$$

where k is the permeability, M is the oedometer modulus and γ_w is the unit weight of water.

The permeability k is generally decreasing linearly with increasing relative compression ε in the $\log k/\varepsilon$-diagram. Consequently, k is a function of the effective stress σ' (and thus of the excess pore water pressure u). Another fact to be considered is that in the derivation of equation (1) it was assumed that the oedometer modulus is independent of a change in effective stress σ'. In reality, the decrease in k with increasing relative compression is generally counteracted by an increase in M. Thus, according to the results of oedometer tests, the coefficient of consolidation $c_v = kM/\gamma_w$ of normally consolidated clay is generally found to be fairly constant under a moderate increase of relative compression and with increasing instead of decreasing tendency.

This justifies the classical Terzaghi solution (Terzaghi, 1925), where the product of permeability and oedometer modulus is assumed constant, which yields:

$$\frac{\partial u}{\partial t} = \frac{kM}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = c_v \frac{\partial^2 u}{\partial z^2} \tag{2}$$

In the case of exponential flow ($i \le i_l$), we have $v = \kappa i^n = \kappa\left(\frac{1}{\gamma_w}\frac{\partial u}{\partial z}\right)^n$ and $\frac{\partial v}{\partial z} = \frac{1}{M}\frac{\partial u}{\partial t}$.

Assuming, as in the Terzaghi solution, that $\kappa M/\gamma_w$ is independent of u, we have:

$$\frac{\partial u}{\partial t} = \frac{\kappa M n}{(\gamma_w)^n} \left(\frac{\partial u}{\partial z} \right)^{n-1} \frac{\partial^2 u}{\partial z^2} \tag{3}$$

When $i > i_l$, we have $v = \kappa n i_l^{n-1} \left(\frac{1}{\gamma_w} \frac{\partial u}{\partial z} - i_0 \right)$ and $\frac{\partial v}{\partial z} = \frac{1}{M} \frac{\partial u}{\partial t}$, which yields:

$$\frac{\partial u}{\partial t} = \frac{\kappa M n}{\gamma_w} i_l^{n-1} \frac{\partial^2 u}{\partial z^2} \tag{4}$$

These equations can be solved numerically, e.g. by finite element or finite difference methods.

2. VERTICAL DRAINAGE

The aim of vertical drain installation is to shorten the drainage paths and thereby reduce the time required for the excess pore water pressure, induced by the loading operation, to dissipate. The effectiveness of the vertical drain installation depends on the drain spacing and the discharge capacity of the drains (denoted q_w at a hydraulic gradient equal to one, i.e. $q_w = k_w \pi d_w^2 / 4$, see Fig. 2). The solution to the consolidation problem is greatly simplified if one assumes that horizontal sections due to arching remain horizontal throughout the consolidation process — the so-called *equal strain theory* (Barron, 1944, 1947). As shown by Barron (1947), this can be justified since the difference in average degree of consolidation between the solutions obtained by assuming the strains to develop freely (no arching) — the so-called *free strain theory* — and the solution obtained by assuming *equal strain theory* is negligible. From practical experience, we also know that with the drain types used nowadays there is no evidence of differential settlement between the immediate vicinity of the drains and the centre between the drains. Therefore, in the following, only results obtained according to the *equal strain theory* will be treated.

The solution based on Darcian flow can be expressed by the relation (Hansbo, 1981):

$$\overline{U}_h = 1 - \exp\left(-\frac{8 c_h t}{\mu D^2} \right) \tag{5}$$

where

$$\mu = \frac{D^2}{D^2 - d_w^2}\left[\ln\left(\frac{D}{d_s}\right) + \frac{k_h}{k_s}\ln\left(\frac{d_s}{d_w}\right) - \frac{3}{4}\right] + \frac{d_s^2}{D^2 - d_w^2}\left(1 - \frac{d_s^2}{4D^2}\right) +$$

$$+ \frac{k_h d_w^2}{k_s\left(D^2 - d_w^2\right)}\left(\frac{d_s^4 - d_w^4}{4D^2 d_w^2} - \frac{d_s^2}{d_w^2} + 1\right) + \frac{k_h}{q_w}\pi z(2l - z)\left(1 - \frac{d_w^2}{D^2}\right)$$

$$c_h = k_h M / \gamma_w$$

Equation (5) gives results that are in good agreement with those obtained by advanced analytical and numerical solutions (e.g. Yoshikuni & Nakanado, 1974; Onoue, 1988; Zeng & Xie, 1989; Lo, 1991).

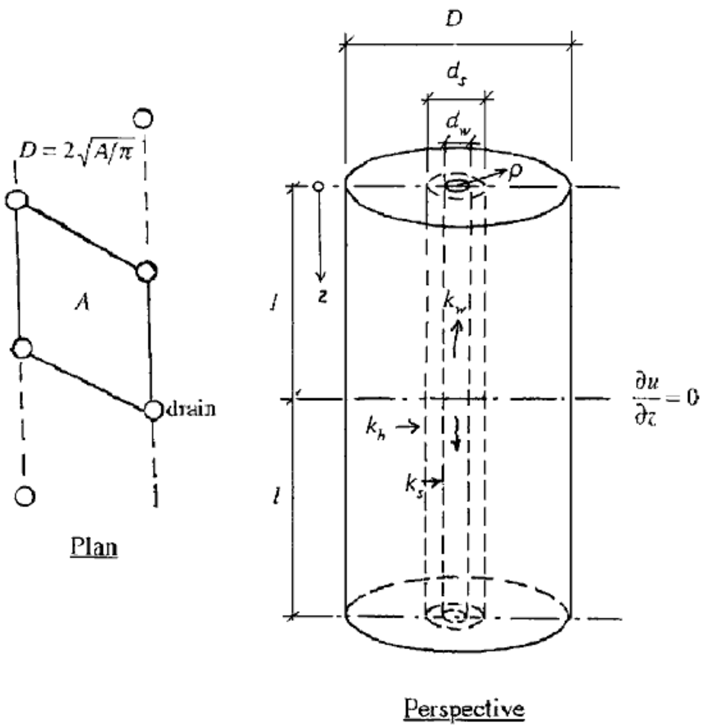

Fig. 2. Terms used in the analysis of vertical drains: D = diameter of soil cylinder dewatered by a drain; d_w = drain diameter; d_s = diameter of zone of smear; l = length of drain when closed at bottom (2l = length of drain when open at bottom); z = depth coordinate; k_w = permeability in the longitudinal direction of the drain; k_h = permeability (in the horizontal direction) of soil; k_s = permeability (in the horizontal direction) of zone of smear; ρ = radius vector.

Assuming an exponential correlation between hydraulic gradient and flow velocity, $v = \kappa i^n$, the consolidation equation becomes (Hansbo, 1997a-b):

$$\overline{U}_h = 1 - \left[1 + \frac{\lambda t}{\alpha D^2} \left(\frac{\Delta \overline{h}_0}{D} \right)^{n-1} \right]^{\frac{1}{1-n}} \tag{6}$$

where $\Delta \overline{h}_0 = \overline{u}_0 / \gamma_w$ ($\overline{u}_0$ = initial excess pore water pressure, equally distributed), $\lambda = \kappa_h M / \gamma_w$, t = time of consolidation, M = compression modulus determined by oedometer tests (= $1/m_v$, where m_v is the volume compressibility), γ_w = unit weight of water, $\alpha = n^{2n}\beta^n / \left[4(n-1)^{n+1} \right]$ with

$$\beta = \frac{1}{3n-1} - \frac{n-1}{n(3n-1)(5n-1)} - \frac{(n-1)^2}{2n^2(5n-1)(7n-1)} +$$

$$+ \frac{1}{2n} \left[\left(\frac{\kappa_h}{\kappa_s} - 1 \right) \left(\frac{D}{d_s} \right)^{(1/n-1)} - \frac{\kappa_h}{\kappa_s} \left(\left(\frac{D}{d_w} \right)^{(1/n-1)} \right) \right] -$$

$$- \left(\frac{1}{2n} - \frac{1}{3n-1} \right) \left[\left(\frac{\kappa_h}{\kappa_s} - 1 \right) \left(\frac{D}{d_s} \right)^{(1/n-3)} - \frac{\kappa_h}{\kappa_s} \left(\frac{D}{d_w} \right)^{(1/n-3)} \right] +$$

$$+ \frac{\kappa_h}{2q_w} \pi z (2l - z) \left(1 - \frac{1}{n} \right) \left(\frac{D}{d_w} \right)^{(1/n-1)} \left(1 - \frac{d_w^2}{D^2} \right)^{1/n}$$

Excess pore pressure variation. Replacing the initial excess pore water pressure $\overline{u}_0$ with $\gamma_w \Delta \overline{h}_0$, where $\Delta \overline{h}_0$ is the initial average hydraulic head increase, the variation of the hydraulic head increase Δh outside the zone of smear $(D/2 \geq \rho \geq d_s/2)$, based on validity of Darcy's law, becomes:

$$\Delta h = \frac{\Delta \overline{h}_0}{\mu D^2} \left(1 - \overline{U}_h \right) \left[\begin{array}{l} D^2 \ln \dfrac{2\rho}{d_s} - \dfrac{4\rho^2 - d_s^2}{2} + \dfrac{k_h}{k_s} \left(D^2 \ln \dfrac{d_s}{d_w} - \dfrac{d_s^2 - d_w^2}{2} \right) + \\ + \dfrac{k_h}{q_w} \pi z (2l - z) \left(D^2 - d_w^2 \right) \end{array} \right] \tag{7}$$

Inside the zone of smear $(d_s/2 \geq \rho \geq d_w/2)$ we have:

$$\Delta h = \frac{\Delta \overline{h}_0}{\mu D^2}\left(1-\overline{U}_h\right)\left[\frac{k_h}{k_s}\left(D^2 \ln\frac{2\rho}{d_s} - \frac{4\rho^2 - d_s^2}{2}\right) + \frac{k_h}{q_w}\pi z(2l-z)\left(D^2 - d_w^2\right)\right] \quad (8$$

The variation of hydraulic head increase outside the zone of smear in exponential flow ($D/2 \geq \rho \geq d_s/2$), assuming that $i \leq i_l$, becomes:

$$\Delta h = \frac{\Delta \overline{h}_0}{2^n \beta n^2}\left(1-\overline{U}_h\right)\begin{Bmatrix} F\left(\frac{2\rho}{D}\right) - F\left(\frac{d_s}{D}\right) + \frac{\kappa_h}{\kappa_s}\left[F\left(\frac{d_s}{D}\right) - F\left(\frac{d_w}{D}\right)\right] + \\ + \frac{\kappa_h}{q_w}\pi z(2l-z)\left(1-\frac{1}{n}\right)\left(1-\frac{d_w^2}{D^2}\right)^{1/n}\left(\frac{d_w}{D}\right)^{(1-1/n)} \end{Bmatrix} \quad (9)$$

where

$$F(x) = x^{1-1/n}\left[1 - \frac{1-1/n}{3n-1}x^2 - \frac{(1-1/n)^2}{2(5n-1)}x^4 - \frac{(1-1/n)^2(2n-1)}{6n(7n-1)}x^6 - \cdots\right],$$

in which the variable x represents $2\rho/D$, d_s/D and d_w/D.

Inside the zone of smear ($d_s/2 \geq \rho \geq d_w/2$) we have:

$$\Delta h = \frac{\Delta \overline{h}_0}{2^n \beta n^2}\left(1-\overline{U}_h\right)\begin{Bmatrix} \frac{\kappa_h}{\kappa_s}\left[F\left(\frac{2\rho}{D}\right) - F\left(\frac{d_w}{D}\right)\right] + \\ + \frac{\kappa_h}{q_w}\pi z(2l-z)\left(1-\frac{1}{n}\right)\left(1-\frac{d_w^2}{D^2}\right)^{1/n}\left(\frac{d_w}{D}\right)^{(1-1/n)} \end{Bmatrix} \quad (10)$$

Hydraulic gradient. The hydraulic gradient in Darcian flow outside the zone of smear ($D/2 \geq \rho \geq d_s/2$) becomes:

$$i = \frac{\Delta \overline{h}_0}{D}\left(1-\overline{U}_h\right)\frac{1}{\mu}\left(\frac{D}{\rho} - \frac{4\rho}{D}\right) \quad (11)$$

and inside the zone of smear ($d_s/2 \geq \rho \geq d_w/2$):

$$i = \frac{\Delta \overline{h}_0}{D}\left(1-\overline{U}_h\right)\frac{k_h/k_s}{\mu}\left(\frac{D}{\rho} - \frac{4\rho}{D}\right) \quad (12)$$

The hydraulic gradient outside the zone of smear ($D/2 \geq \rho \geq d_s/2$) in exponential flow, i.e. assuming that $i \leq i_l$, becomes:

$$i = \frac{\Delta \overline{h}_0}{D}\left(1-\overline{U}_h\right)\left[\frac{1}{4\alpha(n-1)}\left(\frac{D}{2\rho} - \frac{2\rho}{D}\right)\right]^{1/n} \quad (13)$$

while inside the zone of smear ($d_s/2 \geq \rho \geq d_w/2$):

$$i = \frac{\Delta\bar{h}_0}{D}\left(1-\bar{U}_h\right)\left[\frac{\kappa_h/\kappa_s}{4\alpha(n-1)}\left(\frac{D}{2\rho}-\frac{2\rho}{D}\right)\right]^{1/n} \tag{14}$$

Comments. The consolidation equations based on exponential flow yield the same result as the consolidation equations based on validity of Darcy's law when the exponent $n \to 1$, e.g. by inserting n = 1.0001 (Hansbo, 2001).

Correlation between λ and c_h. The ratio of λ to c_h will depend on the hydraulic gradient prevailing in the horizontal direction during the consolidation process. The consolidation equation in exponential flow presupposes that the hydraulic gradient i does not exceed the limiting value i_l. However, the equation can be applied with good approximation also for values of $i > i_l$. Since $c_h = k_h M/\gamma_w$ and $\lambda = \kappa_h M/\gamma_w$, in which the parameters M and γ_w are independent of the flow conditions, we have $c_h/\lambda = k_h/\kappa_h$. An approximate correlation between c_h and λ can be found by equalising the areas created below the flow vs. hydraulic gradient curves in the two cases non-Darcian and Darcian flow. This yields the correlation:

$$\lambda/c_h = \frac{n+1}{2i_l^{n-1}} \text{ when } i \leq i_l \tag{15}$$

and

$$\lambda/c_h \approx \frac{i^2}{2}\left[\frac{i_l^{n+1}}{n+1} + n i_l^{n-1}\left(i-i_l\right)\left(\frac{i-i_l}{2}+\frac{i_l}{n}\right)\right]^{-1} \text{ when } i \geq i_l \tag{16}$$

The maximum hydraulic gradient according to equations (11) and (13) is obtained for $\bar{U}_h = 0$ and $\rho = d_s/2$. Assuming, for example, that the maximum gradients reached during the consolidation process are, respectively, 2, 5, 15, 25 and 75 and that the exponent n = 1.5 and the limiting gradient i_l = 8, we find in due order $\lambda/c_h = \kappa_h/k_h \approx$ 0.88, 0.56, 0.34, 0.29 and 0.25. Thus, the higher the value of $\Delta\bar{h}_0$ and the smaller the drain spacing, the lower the ratio of λ to $\tilde{c}_h$

Assuming n = 1.5 and $i_{\max} \leq 1.5 i_l$, the ratio of λ to c_h can be obtained by the relation:

$$\lambda/c_h = 1.25/\sqrt{i_{\max}} \tag{17}$$

where $i_{\max} = \dfrac{\Delta\bar{h}_0}{D}\left[\dfrac{1}{2\alpha}\left(\dfrac{D}{d_s} - \dfrac{d_s}{D}\right)\right]^{2/3}$

If $i_{\max} > 1.5 i_l$, this correlation should be replaced by equation (16), inserting $i = i_{\max}$.

Influence of one-dimensional consolidation. Vertical drains are generally installed in places where the thickness of the fine-grained soil layer is so large that the influence on the consolidation process of one-dimensional consolidation by pore water escape in the vertical direction between the drains can be ignored. Since the difference in result in the beginning of the consolidation process between the analytical results based on Darcian and non-Darcian flow can be ignored (cf. Fig. 10), the contribution to the average consolidation by vertical pore water escape between the drains can be based on the classical Terzaghi solution with Darcian flow $\bar{U}_v = (2/l)\sqrt{c_v t/\pi}$, valid when $\bar{U}_v \leq$ 50% (l according to Fig. 1).

. According to Carillo's relation $\bar{U}_{\text{tot}} = \bar{U}_v + \bar{U}_h - \bar{U}_v \bar{U}_h$, the total average consolidation, including the effect of vertical drainage (horizontal pore water flow) and one-dimensional consolidation (vertical pore water flow) can be expressed by the relation:

$$\bar{U}_{\text{tot}} = 1 - \left(1 - \frac{2}{l}\sqrt{\frac{c_v t}{\pi}}\right)\exp\left(-\frac{8 c_h t}{\mu_{\text{av}} D^2}\right) \tag{18}$$

in Darcian flow, and

$$\bar{U}_{\text{tot}} = 1 - \left(1 - \frac{2}{l}\sqrt{\frac{c_v t}{\pi}}\right)\left[1 + \frac{\lambda t}{\alpha_{\text{av}} D^2}\left(\frac{\bar{u}_0}{D\gamma_w}\right)^{n-1}\right]^{1/(1-n)} \tag{19}$$

in exponential flow.

A large number of case histories have been publicised [see, for example, the special issue of Géotechnique of March 1981 on vertical drains and the publications by Lo (1991) and Hansbo (1994, 1997)]. In the following, results from well instrumented test areas in different parts of the world will be analysed on the basis of both Darcian and non-Darcian flow.

Equivalent diameter of band drains. The first type of band drains, the so-called cardboard wick, 100 mm in width and 3 mm in thickness, invented and introduced on the market by the Swedish Geotechnical Institute, was assumed by Kjellman (1947) to have an equivalent diameter of 50 mm. The author (Hansbo, 1979) showed that the process of consolidation for a circular drain and a band drain is very nearly the same if the circular drain is assumed to have a circumference equal to that of the band drain, i.e.

$$d_w = 2(a+b)/\pi \tag{20}$$

where a = the thickness of the drain and b = the width of the drain.

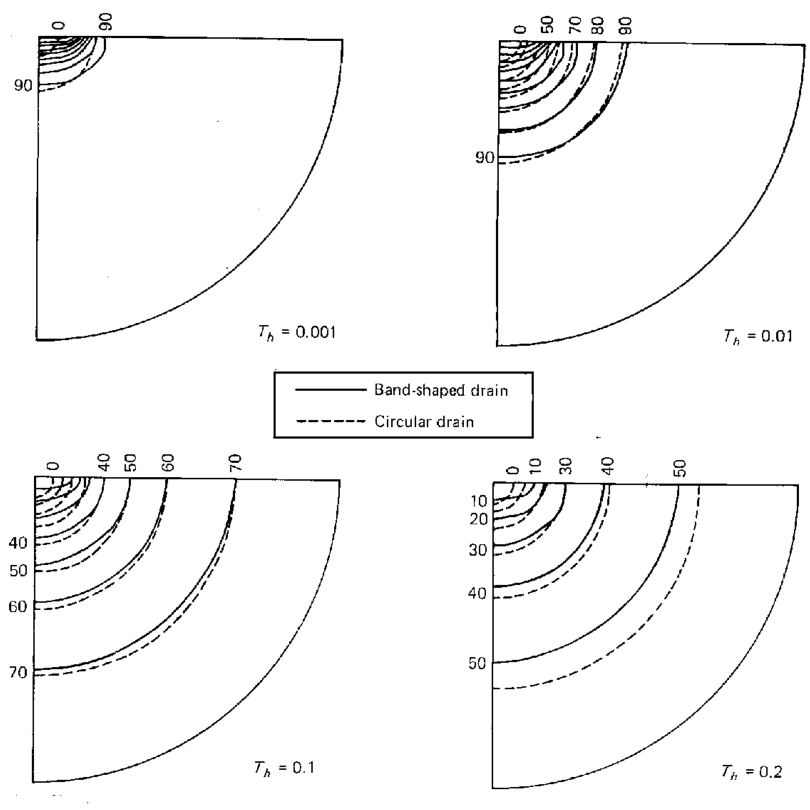

Fig. 3. Comparison of consolidation effects (remaining excess pore water pressure *u* in % of initial excess pore water pressure u_0) caused by a band drain (100 mm in width and 4 mm in thickness) and a circular drain with the same circumference (d_w = 66 mm). *D* = 1 m. $T_h = c_h t/D^2$. No effect of smear or well resistance. (Hansbo, 1979)

A comparison of the degree of consolidation obtained in the two cases band drains and circular drains is shown in Fig. 3. This comparison indicates that the equivalent diameter ought to be put somewhat smaller than according to equation (20). Therefore, Rixner *et al.* (1986) propose that the equivalent diameter should be put equal to $d_w = (a+b)/2$. However, the difference in result decreases with increasing time of consolidation and is insignificant in comparison with the influence on the result exerted by the choice of other consolidation parameters to be applied in the design.

The zone of smear. The effect on the consolidation parameters of disturbance caused by the installation of drains, expressed in the terms of zone of smear, depends very much on the method of drain installation, the size and the shape of the mandrel, and the soil structure (Sing & Hattab, 1979; Bergado *et al.*, 1993). Two problems exist: to find the correct diameter value d_s of the zone of smear and to evaluate the effect of smear on the permeability.

The extent of the zone of smear and the disturbance effects depend on the type of soil and the geometrical dimensions of the installation mandrel. Remoulding will take place inside a volume equal to the volume displaced by the mandrel. Outside the remoulded zone, disturbance of the subsoil will also occur owing to distortion. The extent of the zone of distortion is a function of the stiffness of the soil. The stiffer the soil, the larger the zone of influence, and vice versa. Investigations of the extent of the zone of smear in the case of displacement-type circular drains indicate that the diameter d_s of the zone of smear can be assumed equal to 2 times the diameter of the drain (Holtz and Holm, 1973; Akagi, 1976; Bergado *et al.*, 1992). Recent investigations on a laboratory scale (Indraratna & Redana, 1998) indicate that the extent of the smear zone can be put equal to 3–4 times the cross-sectional area of the band drain. If the installation mandrel is non-circular, which is the normal case, the diameter d_s according to this result would yield an area corresponding to about 4 times the cross-sectional area of the mandrel. In many cases the cross-sectional area of the installation mandrel is typically about 7000 mm^2 in size, which yields $d_s \approx 0.19$ m, i.e. about 3 times the equivalent diameter of the band drain.

Several authors have treated the other problem, the choice of permeability in the zone of smear. Of course, the permeability in the zone of smear will vary from a minimum nearest to the drain to a maximum at the outer border of the zone. The most conservative solution to the problem is to assume that horizontal layers in the undisturbed soil are turned vertical in the zone of smear, resulting in the quotient k_h/k_s being equal to the quotient c_h/c_v (see also Bergado *et al.*, 1992).

Onoue *et al*. (1991) and Madhav *et al.* (1993) divide the zone of smear into two sub-zones: an inner, highly disturbed zone, and an outer transition zone in which the disturbance decreases with increasing distance from the drain. Madhav *et al.* conclude that the author's solution based on axisymmetric smear conditions and the assumption of only one smear zone is “reasonably accurate for all practical purposes”. Chai *et al.* (1997) use a linear variation of the permeability in the zone of smear on one hand and a bilinear variation on the other and conclude that the assumption of one single average value of permeability will under-evaluate the effect of smear. Hird & Moseley (1998) conclude, on the basis of laboratory tests, that the ratio k_h/k_s for layered soil, assuming $d_s = 2d_w$, can be much larger than that mentioned in the literature but that the assumption of $k_s = k_v$ in these cases is much too severe.

Effect of well resistance. Because drains nowadays are frequently installed to great depths, well resistance has become a matter of increasing interest. This is understandable since well resistance in such cases can cause a serious delay in the consolidation process.

Before a certain drain make is accepted for a job, evidence ought to be presented that the drain fulfils the requirements on discharge capacity assumed in the design. Generally, appropriate laboratory testing can give sufficient evidence of the discharge capacity to be expected under field conditions, but in case of a drain make never before used in practice, full-scale field tests are recommended. Full-scale field tests also serve the purpose of showing that the drains are strong enough to resist the strains subjected to them during installation. Moreover, compression of the soil in the course of consolidation settlement entails folding of the drain, which may bring about clogging of the channel system. The latter effect is difficult to discern in a laboratory test (cf. Lawrence & Koerner, 1988).

Discharge capacity tests on band drains installed in soil on a laboratory scale and subjected to increasing lateral pressure have resulted in the values presented in Fig. 4. In the ENEL tests (Jamiolkowsky *et al.*, 1983) the drains were tested in full scale. In the CTH tests (Hansbo, 1983) and in the KU tests (Kamon, 1984) the drains were tested with reduced width (40 and 30 mm, respectively). The results obtained reveal a great influence on the discharge capacity of the lateral consolidation pressure.

There are several reasons why the discharge capacity of a drain may become low: siltation of the channels in the core of band drains; unsatisfactory drain makes with too low a discharge capacity; necking of drains; etc. Back-calculated values of discharge capacity of drains under field conditions have been reported to be quite low for certain makes of band drains without filter (Hansbo, 1986; Chai *et al.*, 1996). If the drain installation is taking place in clay deposits, most of the band drains marketed today have high enough a discharge capacity (q_w > 150 m^3/year) to become negligible in the design (*cf.* Hansbo, 1986; 1994). However, drain installation in more high-permeable soils than clay may require a much higher discharge capacity (cf. Fig. 5). The influence of well resistance decreases with increasing time of consolidation.

The filter sleeve surrounding the core must be strong enough to resist the tension and the wear and tear that takes place during drain installation. Moreover, the filter pore size should be small enough to prevent intrusion of fine material into the channels of the core, leading to clogging effects. Filter deterioration in organic soils will cause a time-bound reduction of the discharge capacity (Koda *et al.*, 1986; Hansbo, 1987).

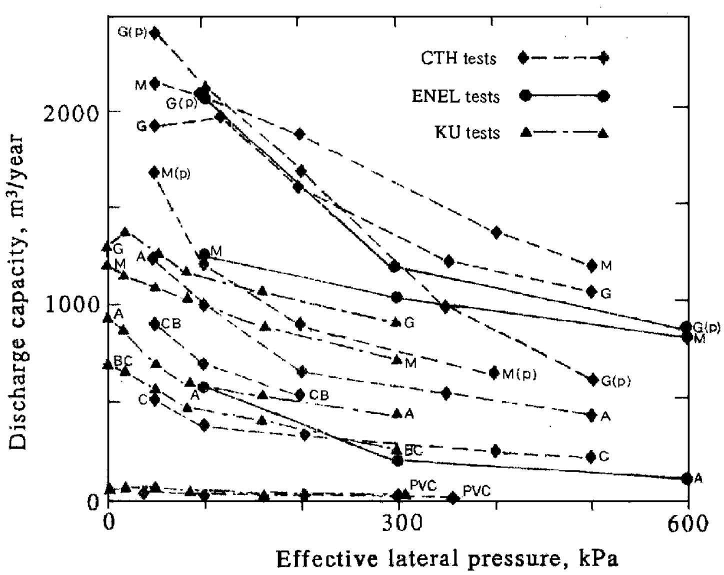

Fig. 4. Results of discharge capacity tests for different band drains carried out on a laboratory scale. Drains enclosed in soil. Legend: A = Alidrain, BC = Bando Chemical, CB = Castle Board, C = Colbond, G =Geodrain. (p) indicates filter sleeve of paper.

According to equation (5), an acceptable delay Δt (%) in time of average consolidation requires a discharge capacity of minimum [1]

$$q_w = 200\pi l^2 k_h \Big/ \left\{ 3\Delta t \left[\ln\left(D/d_w \right) - 0.75 \right] \right\}. \tag{21}$$

For drains with a discharge capacity of 500 m^3/year, the delay in the time of consolidation at depth $z = l$ (cf. Fig. 2) in various kinds of soil according to equation (5) is exemplified in Fig. 5 (Cortlever & Hansbo, 2004).

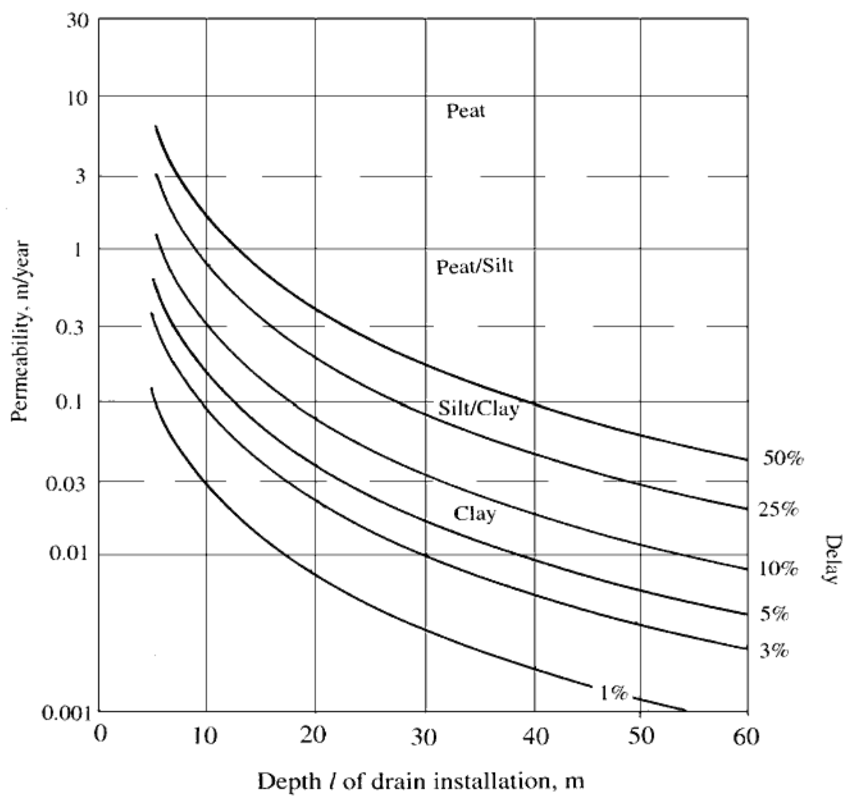

Fig. 5. Delay in time of consolidation at depth *l* of drain installation (cf. Fig. 2) for drains with a discharge capacity of 500 m^3/year (16 cm^3/s). Drain spacing 0.9 m (equilateral triangular pattern; *D* = 0.945 m), drain diameter d_w = 0.065 m.

[1] The difference in result obtained on the basis of equation (6) is negligible.

For partially penetrating drains, consolidation of the lower part of the layer that is penetrated by the drains will be somewhat delayed by inflow of pore water from the underlying part where the excess pore water pressure is higher. The delay will depend on the discharge capacity, the drain length and the drain spacing (Runesson *et al*., 1985). It can be assumed to affect the degree of consolidation to a height above the drain tips approximately equal to the drain spacing.

As a rule, investigations on a laboratory scale of the discharge capacity of a drain ought to be carried out in a way that simulates field conditions as closely as possible, preferably embedded in the soil and at the soil temperature in question. Laboratory testing devices, which are presented in the Eurocode on vertical drainage under preparation (probably accepted and published in 2005/2006 as EN), will most probably be accepted as international standard. One of these devices (Apparatus No. 2, EN/ISO 12958) and also a device for testing the discharge capacity of buckled drains are illustrated by Cortlever &Hansbo (2004).

RESULTS OBTAINED IN TEST AREAS

In the following, the results obtained in a number of test areas will be compared with the results obtained on one hand according to the classical assumption that Darcy's law is valid and on the other hand according to the assumption of non-Darcian exponential flow.

As regards test areas that have no drains, the importance of a long-term follow-up of the results obtained is crucial. Therefore, only two test areas situated in Sweden, installed in 1947 and 1957, are included in the analysis.

Test areas provided with vertical drains exist all over the world and are of great interest because of the variations in soil condition. In this paper, the results obtained in test areas situated in Sweden, Italy and Thailand are included.

1. ONE-DIMENSIONAL CONSOLIDATION

1.1 The Skå-Edeby test field

The test areas constructed at Skå-Edeby in 1957 and the depth of the soft clay deposits are shown in Fig. 6. Among the different test areas, Area IV has no vertical drains while the others are provided with fully penetrating vertical sand drains, 0.18 m in diameter.

The soil consists of postglacial clay to a depth of about 6 m underlain by glacial, varved clay to a depth of about 12.5 m. The groundwater table was situated at a depth of approximately 1 m but varies with the time of the year. The dry crust has a thickness of about 1.5 m. The geotechnical characteristics of the clay are shown in Fig. 7. According to the results of oedometer tests, the clay below the dry crust is normally consolidated. The compression ratio[2] $CR = \varepsilon_2 / \log 2 = C_c / (1 + e_0)$ according to the oedometer tests (Hansbo, 1960) was found equal to about 0.24 at 2 m depth, 0.35 (0.48) at 5 m depth, 0.32 (0.63) at 8 m depth (the values in brackets are modified with regard to sample disturbance).

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[2] In Sweden, the compression characteristics for a normally consolidated clay were previously expressed by the compression index ε_2, which means the additional deformation achieved along the virgin curve due to doubled load.

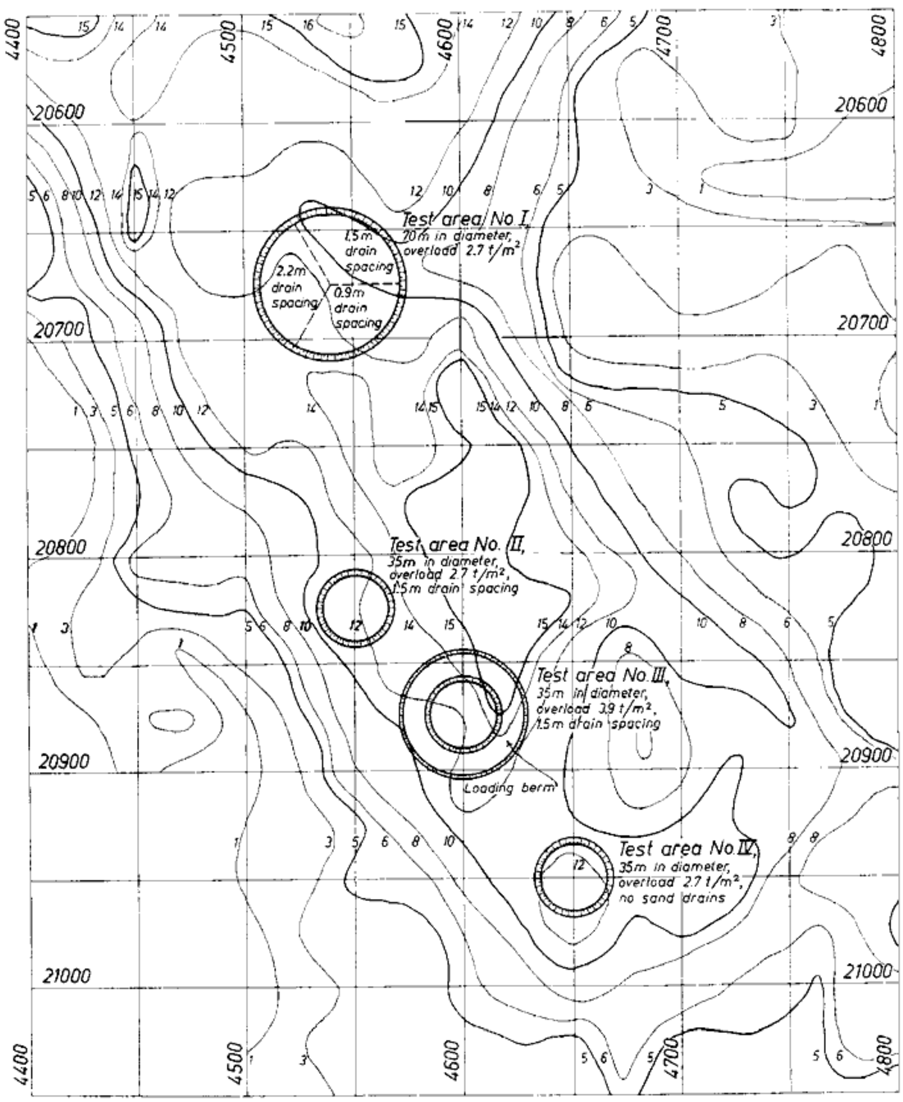

Fig. 6. The test areas and depth from ground surface to firm bottom, in m, in the test field at Skå-Edeby. Coordinate system gives horizontal distances , in m.

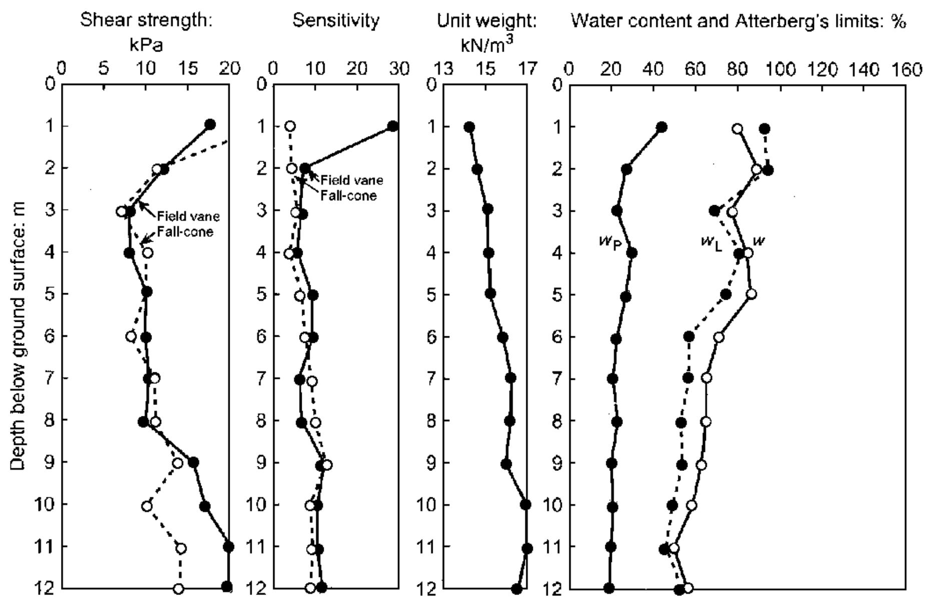

Fig. 7. Geotechnical properties of the clay subsoil in Test Area IV (after Hansbo, 1960)

The corresponding oedometer modulus M for a normally consolidated clay can be found by the relation $CR\log[(\sigma_c' + \Delta\sigma)/\sigma_c'] = \Delta\sigma/M$, where $\Delta\sigma$ is the stress increase due to loading and σ_c' is the preconsolidation pressure.

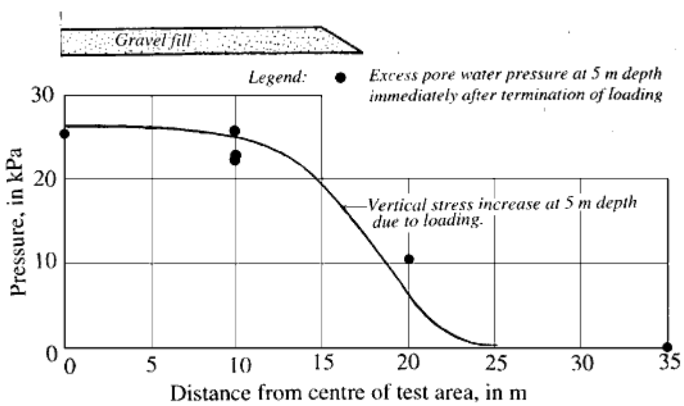

Fig. 8. Observed excess pore pressure distribution in Test area IV at 5 m depth, immediately after placement of the gravel fill.

Excess pore pressure dissipation. The excess pore pressure distribution below the test area, observed at a depth of 5 m after termination of loading, is shown in Fig. 8. This can be used in determining the parameters A and B in Skempton's pore pressure equation $u = B[\sigma_3 + A(\sigma_1 - \sigma_3)]$ where σ_1 and σ are the major and minor principle stresses induced by loading (Skempton, 1954). As we are dealing with water saturated clay, B = 1. If the soil immediately after termination of loading behaves as an elastic medium with Poisson's ratio $\nu = 0.5$, we have at 5 m depth according to the solution presented by Love (1929) $\sigma_1 = 0.98q$ and $\sigma_3 = 0.60q$ below the centre, $\sigma_1 = 0.95q$ and $\sigma_3 = 0.49q$ 10 m from the centre, and $\sigma_1 = 0.50q$ and $\sigma_3 = 0.06q$ 20 m from the centre. Inserting $q = 27$ kN/m^2 and considering the observations made this yields $A \approx 0.85$.

The analysis of the excess pore pressure dissipation has been based on the consolidation characteristics shown in Table 1. In the case of non-Darcian flow the limiting value of the hydraulic gradient i_l (where the flow turns from exponential to linear relation of the hydraulic gradient) is chosen equal to 5, Fig. 9.

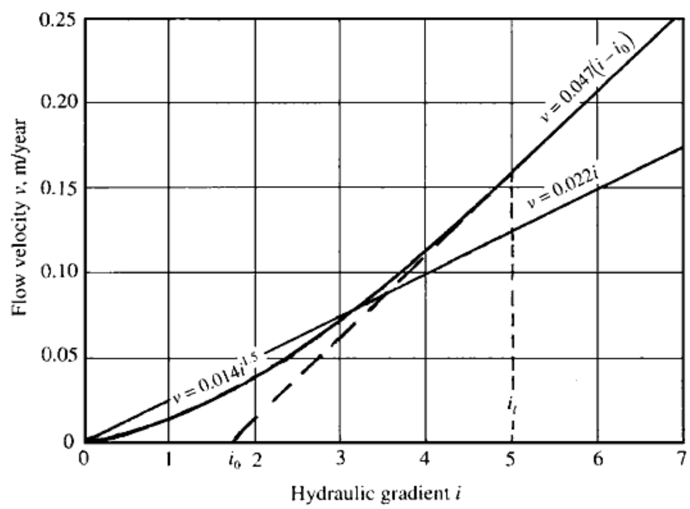

Fig. 9. Correlation between hydraulic gradient *i* and, respectively, Darcian and non-Darcian water flow *v* (m/year) at depth 3 to 5 m (cf. Table 1), used in the analysis of excess pore pressure dissipation in Test Area IV without drains, Skå-Edeby

The k and M values given in Table 1 are chosen according to Larsson (1986).Based on trial and error, the κvalue is chosen equal to about 0.65 times the k value.

Table 1. Parameters used in the consolidation analysis

Depth, m	0–1.0	1.0–1.5	1.5–3	3–5	5–7	7–9	9–11	11–12.5
$\Delta\sigma$ kPa	27	27	27	26.5	26.5	26	26	25
σ_c' kPa	OC	25	23	28	39	51	64	75
M kPa	10000	400	250	240	250	300	400	500
Δs, m	0.01	0.03	0.16	0.24	0.24	0.23	0.22	0.15
k m/year	0.031	0.031	0.025	0.022	0.018	0.018	0.017	0.016
κ m/year	0.020	0.020	0.016	0.014	0.0115	0.0115	0.011	0.0095

The remaining excess pore water pressure distribution throughout the clay layer below the centre of the circular embankment after 1.5, 14, 25 and 45 years of loading is compared with the analytical results obtained according to the two methods of analysis of one-dimensional consolidation, Figs. 10. and 11.

Initial excess pore water pressure is assumed equal to 25 kPa. Because of the width of the embankment, 35 m, the influence of two-dimensional consolidation at the centre of the embankment has been ignored.

As can be seen, the result based on non-Darcian flow is in much better agreement with the excess pore pressure observations than the result based on Darcian flow. There is a possibility that the excess pore water pressure below the centre of the load has been affected by pore pressure dissipation due to horizontal pore water flow, but this possibility only strengthens the assumption of non-Darcian flow.

The total primary consolidation settlement, based on the oedometer moduli M given in Table 1 and the stress increase σ_v due to loading, can be estimated at 1.28 m, while the total primary compression of the clay layer between 2.5 and 7.5 m can be estimated at 0.59 m. In the analysis, the effect of load reduction due to hydraulic uplift when part of the load sinks below the groundwater table has been ignored. Thus, owing to submergence of the fill during the course of settlement the load, in kN/m^2, will be reduced successively by about $\gamma' s$, where γ' = the effective unit weight, in kN/m^3, of the soil above the groundwater table and of the part of the fill that sinks below the groundwater table and s = settlement, in m, of the soil surface. However, one must keep in mind that the ground water level may vary considerably with time of the year and also that the ground water level after completed consolidation may differ from its original position. The settlement reduction caused by submergence of the fill may also be fully compensated by secondary consolidation.

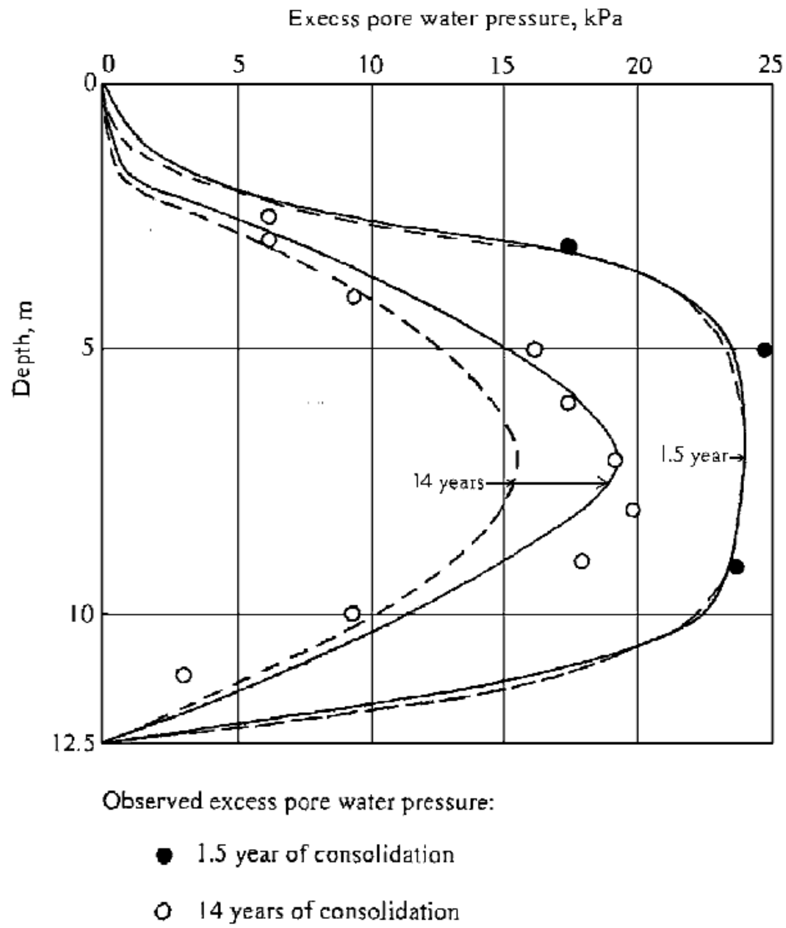

Fig. 10. Comparison between observed excess pore pressure dissipation in Test Area IV, Skå-Edeby, and analytical dissipation according to Darcian flow (broken lines) and non-Darcian flow (unbroken lines) after 1.5 and 14 years of consolidation.

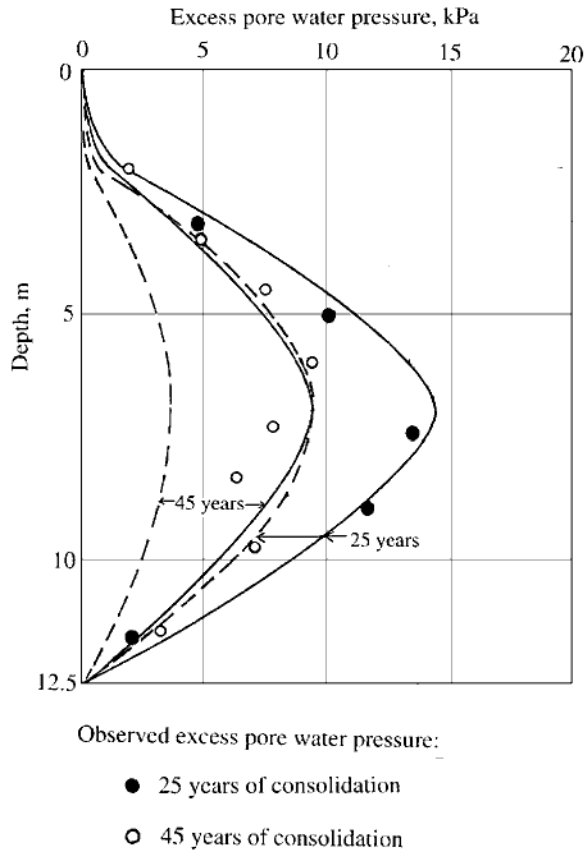

Fig. 11. Comparison between observed excess pore pressure dissipation in Test Area IV, Skå-Edeby, and analytical dissipation according to Darcian flow (broken lines) and non-Darcian flow (unbroken lines) after 25 and 45 years of consolidation.

The settlement observed in the test area is shown in Fig. 12. The settlement of the ground surface, observed after 45 years of loading, equals 1.10 m, while the compression of the clay layer between 2.5 and 7.5 m of depth equals 0.50 m. According to Asaoka (1978), the total primary consolidation settlement s_p can be estimated from the relation $s_i = 0.300 + 0761 s_{i-1}$, which yields s_p = 1.26 m ($s_{i-1} \to s_i$) and the compression Δs_p of the clay layer between 2.5 and 7.5 m of depth from the relation $\Delta s_i = 0.119 + 0.815 \Delta s_{i-1}$, which yields Δs_p = 0.64 m.

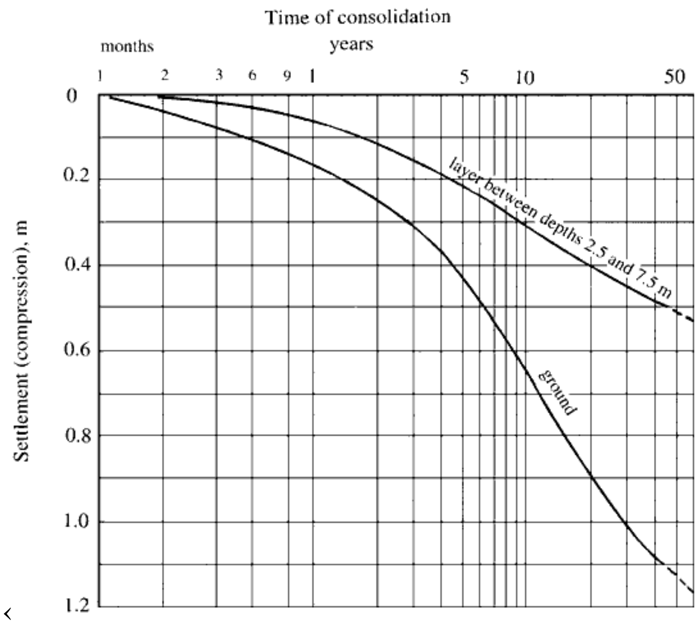

Fig. 12. Consolidation settlement of ground surface and compression of clay layer between 2.5 and 7.5 m of depth observed in Test Area IV.

Looking at the remaining excess pore water pressure after 25 and 45 years of consolidation, we find the average degree of consolidation equal to, respectively, 69% and 79%. This leads in both cases to a total consolidation settlement of 1.39 m, about 10% larger than the estimated primary consolidation settlement. For the layer between the depths 2.5 m and 7.5 m, the average degree of consolidation after 25 and 45 years of consolidation becomes, respectively, 62% and 73%, which in both cases would lead to a primary compression of 0.69 m, about 8% larger than the estimated compression according to Asaoka. Considering the excess pore pressure distribution after 100 years of loading, the average degree of consolidation would equal 99% in Darcian flow and 91% in non-Darcian flow. Based on excess pore pressure dissipation in non-Darcian flow, the total settlement achieved after 100 years of loading would become 1.26 m, while the compression of the layer between the depths 2.5 and 7.5 m would become 0.63 m. Ignoring the parameters of the dry crust, the input parameters given in Table 1 yield average values of the coefficient of consolidation $kM/\gamma_w = c_v \approx 0.61$ m^2/year and $\kappa M/\gamma_w = \lambda \approx 0.39$ m^2/year.

The back-calculated value of c_v is almost 4 times the value determined by oedometer tests. In a case like this with a test area, 35 m in diameter, placed on a clay layer, 12.5 m in thickness, one has to consider the contribution of pore water escape in outward radial direction. This can explain the difference between the oedometer value and the back-calculated value of c_v. Moreover, the prolonged consolidation process increases considerably the settlement contribution by secondary consolidation during the primary consolidation period.

1.2 The Lilla Mellösa test area

The test area at Lilla Mellösa that has no vertical drains was installed in 1947 and the results obtained have been followed-up ever since. Together with the test area at Skå-Edeby without vertical drains, presented above, it belongs to the world's oldest test areas.

The soil conditions at the Lilla Mellösa test field have been described in detail by Chang (1981). The soil description differs somewhat between Chang (1981) and Larsson (1986). Briefly, underneath a 0.8 m thick dry crust the soil consists of organic clay to a depth of approximately 6 m, underlain first by homogeneous grey clay to a depth of 10 m and then by grey, varved clay to a depth of 14 m, resting on a thin layer of sand on rock, Fig. 13.

The natural water content and the liquid limit decreases from about 100% to 130% at top to about 70% to 90% at bottom. The unit weight at 3 m depth is lowest, about 14 kN/m^3, and increases almost linearly with depth to about 20 kN/m^3 at 14 m depth. The undrained shear strength is lowest at 3 m depth, about 8 kPa, and increases almost linearly with depth to about 18 kPa at 14 m depth. The sensitivity increases with depth from about 10 to 20.

The ground surface level is situated at about + 7.0 m. According to the observations presented by Chang, the groundwater level has varied from about + 6.0 m to about + 6.55 m. The clay can be considered as normally consolidated.

The test area without vertical drains is square with a base width of 30 m. It was filled up with 2.5 m of gravel. The top width of the fill is 22.5 m. It has been provided with settlement meters and pore pressure meters. The unit weight of the gravel fill during the filling operation was determined and found equal to 17 kN/m^3. This yields an initial load on the test area of 42.5 kN/m^2.

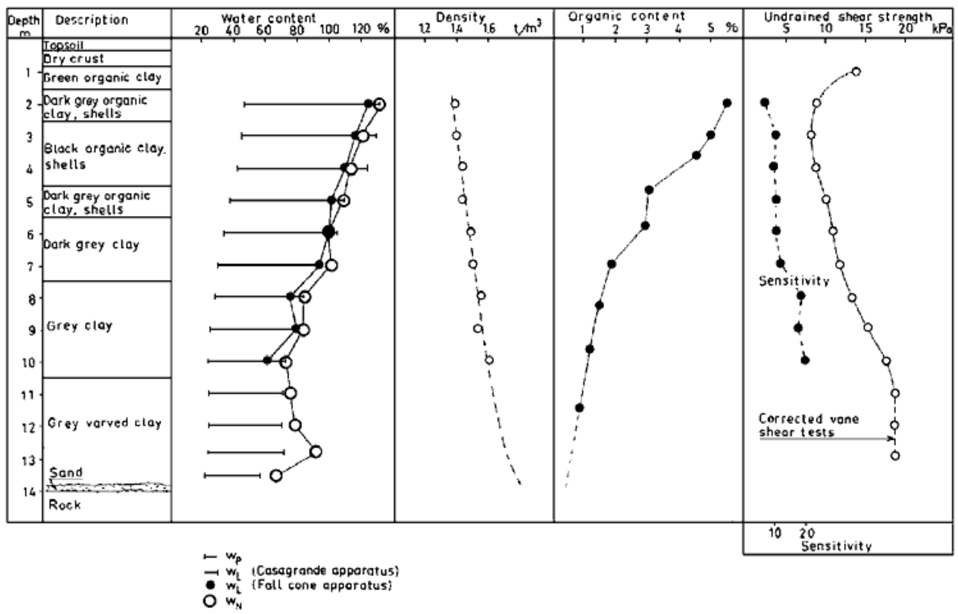

Fig. 13. Soil characteristics at the Lilla Mellösa test site (after Larsson, 1986).

Excess pore pressure dissipation

The results obtained in the two test areas included in this paper, Lilla Mellösa and Skå-Edeby, have been analysed by Larsson (1986) and the results obtained at Lilla Mellösa by Claesson (2003). In Larsson's analysis, based on the effect of creep on excess pore water pressure dissipation and settlement in the area at Lilla Mellösa that has no drains, the results show good agreement with the observations after 21 and 32 years of loading.

For the analysis it is important to apply the correct input parameters M, k, κ, n and i_l as well as the correct values of the initial excess pore pressure distribution. Since no observations of deviation from Darcy's law at small hydraulic gradients has been made on Lilla Mellösa clay, the assumption used in this analysis will be based on the findings related to Skå-Edeby clay, i.e. $n = 1.5$ and $i_l = 5$. The values of M are selected from the results of oedometer tests corrected with regard to ring friction, published by Chang (1981), taking into account the load increment obtained at various depths underneath the fill. The k values are in agreement with the values published by Larsson (1986). The ratio κ/k is chosen equal to 2/3, based on the experience from the undrained test area at Skå-Edeby.

The values chosen and the initial vertical stress increase $\Delta\sigma_v$ below the centre of the test area are presented in Table 2.

Table 2. Parameters used in the consolidation analysis

Depth: m	0 – 1.0	1.0 – 1.5	1.5 - 3	3 - 5	5 - 7	7 - 9	9 - 11	11 - 14
M kPa	9000	200	200	190	160	240	280	200
$\Delta\sigma_v$ kPa	42	42	41	41	40	39	38	33
Δs m	0.01	0.10	0.31	0.43	0.50	0.32	0.27	0.50
k m/year	0.020	0.020	0.025	0.018	0.015	0.022	0.03	0.03
κ m/year	0.013	0.013	0.017	0.012	0.010	0.015	0.02	0.02

In the analysis, the initial excess pore pressure distribution with depth has been based on the vertical stress distribution with depth below the centre of the test area, published by Chang (1981). Of course, this assumption can be questioned. We do not know the magnitude of the pore pressure coefficient A in Skempton's pore pressure equation. As in the case of the Skå-Edeby test area, we are dealing with water saturated clay, and thus $B = 1$.. In our case, A is also assumed equal to 1. In consequence, the initial excess pore water pressure has been considered equal to the vertical stress increase caused by the load, decreasing almost linearly with depth from 42 kPa at the ground surface to 32 kPa at the bottom of the clay layer. In reality, the value of A could be higher or lower depending upon the loading conditions. As previously shown, the value of A was found equal to 0.85 in test area IV at Skå-Edeby with 1.5 m of fill, corresponding to a load of 27 kN/m^2. In our case the fill height is 2.5 m and from experience of a number of observations the value of A increases with increasing load. In, for example, the Gothenburg region, A values have been noticed that increase with increasing load from 0.7 to 1.5 (Hansbo, 1994).

The result of the analysis, assuming $A = 1$, is compared with pore pressure observations, carried out 21, 32 and 56 years of loading, Fig. 14. The pore pressure observations after 21 years of loading indicate that in reality the pore pressure coefficient $A > 1$. It could also be that the groundwater level at the time of observation was higher than usual and that the oedometer modulus (representing the coefficient of consolidation) of the top layer, 1 m in thickness, is much lower than assumed. From Fig. 14 it can be concluded that the agreement between observations and the results obtained by assuming non-Darcian flow is far better than by assuming Darcian flow.

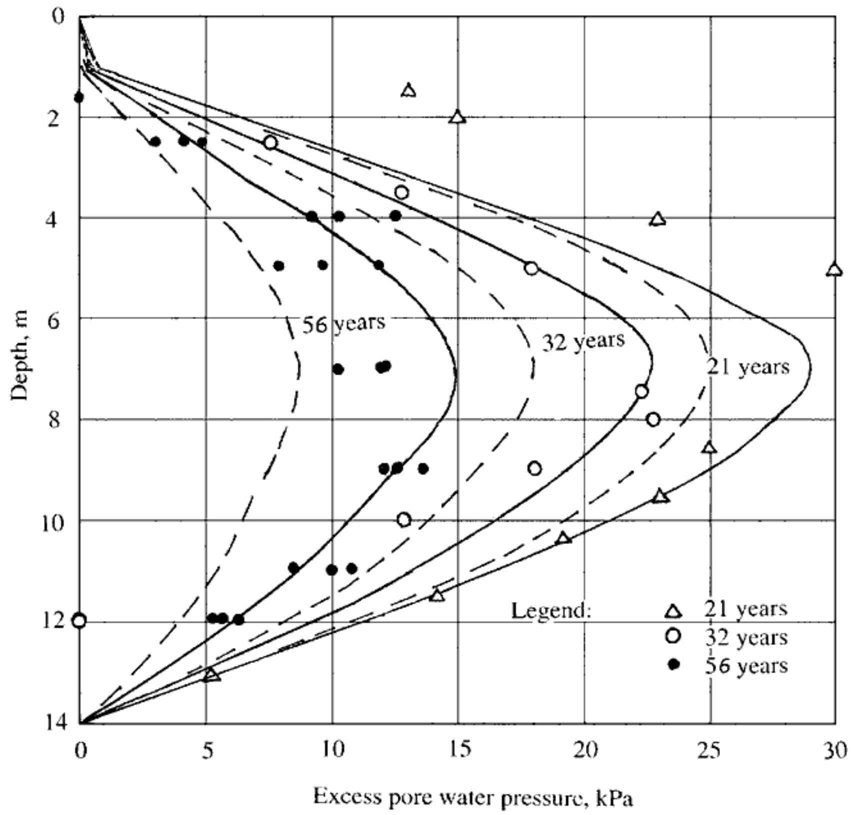

Fig. 14. Comparison between observed excess pore pressure dissipation in the undrained test area at Lilla Mellösa and analytical dissipation according to Darcian flow (broken lines) and non-Darcian flow (unbroken lines) after 21, 32 and 56 years of consolidation. Excess pore pressure observations according to Claesson (2003).

In the analysis of the pore pressure distribution, no consideration has been paid to the fact that part of the initial load will be subjected to hydraulic uplift due to settlement, causing load reduction. As previously pointed out, the ground water level may vary considerably with time of the year and the ground water level after completed consolidation may differ from its original position. In addition, the density of the fill may change with time. However, as mentioned above the excess pore water pressure is influenced by the magnitude of the pore pressure coefficient. It is therefore important to study the excess pore water pressure distribution during the loading operation to make an evaluation of the pore pressure coefficients possible. Otherwise, a direct comparison between settlement and pore pressure dissipation becomes rather intricate.

Consolidation settlement. A settlement analysis, based on the M and $\Delta\sigma_v$ values given in Table 2 (the effect of hydraulic uplift disregarded), yields a total consolidation settlement of 2.25 m. Looking at the remaining excess pore water pressure after 21, 32 and 56 years of loading according to the analytical results based on non-Darcian flow, we find the average degrees of consolidation equal to, respectively, 58%, 67% and 79%. This yields consolidation settlements after 21, 32 and 56 years of loading equal to, respectively, 1.44 m, 1.67 m and 1.97 m. These settlement values completely agree with the observed settlements, published by Claesson (2003). The primary consolidation settlement estimated according to Asaoka's method yields $s_i = 0.561 + 0.734 s_{i-1}$, from which $s_p = 2.10$ m.

The load and consequential settlement reduction due to hydraulic uplift may, as in the case of test area IV at Skåp-Edeby, have been compensated by secondary settlement. The estimated average degree of consolidation in 2024, i.e. another 20 years from now, according to non-Darcian flow becomes 84%, corresponding to a settlement of 2.1 m.

2. VERTICAL DRAINAGE

2.1 The test field at Skå-Edeby.

The Skå-Edeby test field was arranged for the main purpose of studying vertical sand drain and band drain behaviour. As shown in Fig. 6, it originally contained four circular test areas, three of which provided with sand drains, 0.18 m in diameter, with drain spacing varying from 0.9 to 2.2 m, and one without drains. Later on, a test area was arranged with band drains, type Geodrain, with drain spacing equal to 0.9 m. The test areas were loaded with 1.5 m of gravel (Δq = 27 kN/m^2) except for test area III that was loaded with 2.2 m of gravel (Δq = 39 kN/m^2) The geology at the site and the geotechnical properties of the different test areas were described in detail by Hansbo (1960). The results obtained have been presented in several publications (Holtz & Broms, 1972; Larsson, 1986).

Table 3. Compression and consolidation characteristics obtained by conventional odometer tests.

Test area	Drain spacing, m	*CR* at various depths, modified with regard to sample disturbance			Minimum c_v in m^2/year at various depths		
		2 m	5 m	8 m	2 m	5 m	8
I	0.9	0.40	0.50	0.83	0.13	0.32	0.16
I	1.5	0.37	0.63	0.76	0.09	0.16	0.22
I	2.2	0.27	0.33	0.93	0.06	0.19	0.16
II	1.5	0.27	0.50/0.56	0.66/0.70	0.19	0.09/0.13	0.16/0.44
III	1.5	0.33/0.37	0.43/0.53	0.60/0.63	0.06/0.13	0.16/0.19	0.16/0.19

The compression and consolidation characteristics of the clay in the test areas provided with vertical drains were determined on samples taken at depths 2, 5 and 8 m. The results obtained are shown in Table 3 (Hansbo, 1960).

The average value of the coefficient of consolidation c_h becomes 0.17 m^2/year. The coefficient of consolidation in horizontal pore water flow c_h, which was determined in an oedometer test where drainage was allowed only through a central drain, was 0.7 m^2/year. The corresponding permeability value k_h was estimated at 0.03 m/year.

The settlement vs. time relationship for the test areas with sand drains and no drains is presented in Fig. 15. In order to facilitate the interpretation of the test results, the settlement curves have been revised to represent a common depth of 12.2 m.

The degree of consolidation in test area III, which was unloaded, is not as well defined as for test areas I and II where we have entered into a stage of secondary consolidation. All the same, a good estimate can be made, based on the shape of the settlement curve. Now the discharge capacity q_w of sand drains, 0.18 m in diameter, can be estimated at about 100 m^3/year based on sand permeability. According to the results of the oedometer tests $c_h / c_{v,\mathrm{av}} = 4$. Assuming a zone of smear $d_s = 2d_w = 0.36$ m and a ratio $\kappa_h / \kappa_s = k_h / k_s = 4$, an acceptable agreement is obtained if a successively decreasing value of c_h is applied as shown in Table 4.

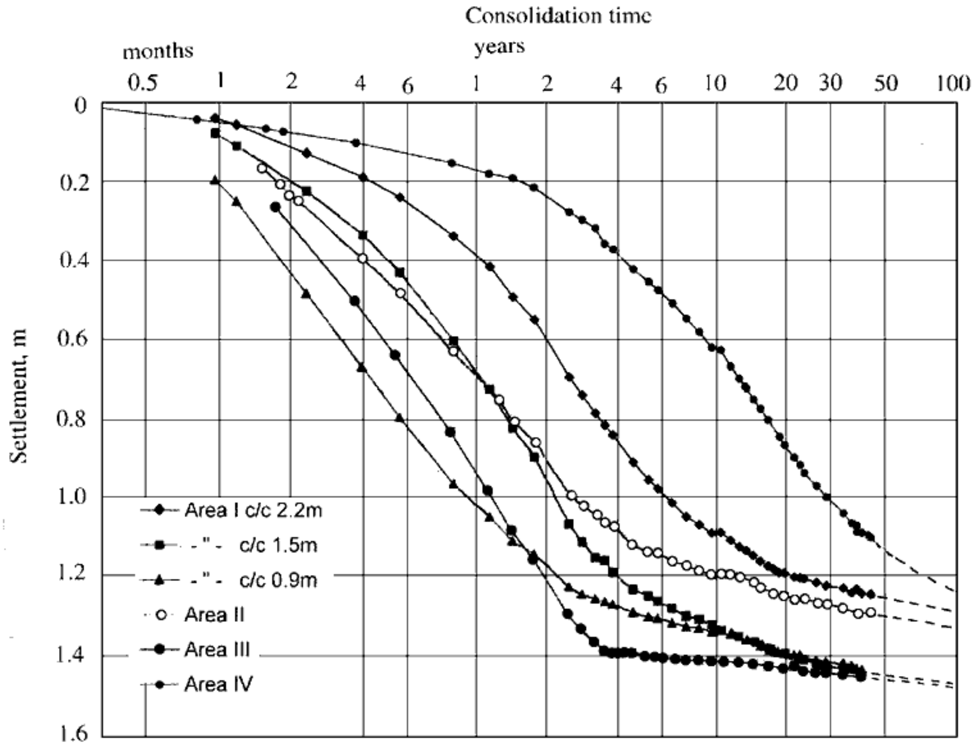

Fig. 15. Settlement *vs*. time of loading in the Skå-Edeby test areas, revised to represent a common depth of 12.2 m. After Larsson (1986). The broken lines represent an estimation of the continuation of the settlement process after the end of observations.

Table 4. Best fit of the c_h and λ values in Test Area I, based on settlement. (Load = 27 kN/m^2)

Drain spacing	0.9 m ($\bar{u}_0$ = 37 kPa)			1.5 m ($\bar{u}_0$ = 31kPa)			2.2 m ($\bar{u}_0$ = 27 kPa)		
Time, years	1/6	1	2	1/2	1	4	1	4	9
$\overline{U}_{\text{tot}}$, %	34	82	94	35	52	92	33	71	89
$\overline{U}_v$, %	6	14	17	10	14	30	14	30	46
$\overline{U}_h$, %	30	79	93	28	44	88	22	59	79
c_h, m^2/year	0.88	0.49	0.42	0.70	0.62	0.57	0.61	0.58	0.46
λ, m^2/year (n =1.5)	0.26	0.26	0.30	0.34	0.32	0.44	0.36	0.38	0.37
$\overline{U}_h$, % ($\lambda_{\text{av.}}$ = 0.34 m^2/year)	37	85	94	28	46	83	21	55	78

The $\overline{U}_h$ values given in Table 4 are determined from the measured values of $\overline{U}_{\text{tot}}$ and $\overline{U}_v$ according to Carillo's equation. The $\overline{U}_v$ values are estimated from the test area at Skå-Edeby without vertical drains, previously described, assuming a final primary settlement of 1.26 m. However, the assumption that $\overline{U}_v$ is unaffected by the disturbance due to drain installation, which undoubtedly causes a reduction of the coefficient of consolidation c_v, is most certainly overoptimistic.

The λ values are more consistent than the c_h values throughout the consolidation process. The c_h values are higher at the beginning of the consolidation process than at the end and the overall value somewhat lower than the coefficient of consolidation c_h determined in the laboratory (0.7 m^2 /year).

According to equation (19), the consolidation process in exponential flow depends on the initial excess pore water pressure $\bar{u}_0$. This is not the case in Darcian flow, equation (18). In test area III, the initial excess pore water pressure is higher than in test area II. From the test results in areas II and III, we find the best agreement by choosing the values given in Table 5. The values of c_h are higher in test area III than in test area II.

The efficiency of using occasional overload in order to eliminate secondary long-term settlement is demonstrated by the results obtained in test area III. With the load used in test area III, 39 kN/m^2, the primary settlement can be estimated at 1.5 m. As can be seen from Fig. 15, secondary consolidation did not start until several years after the removal of the occasional overload.

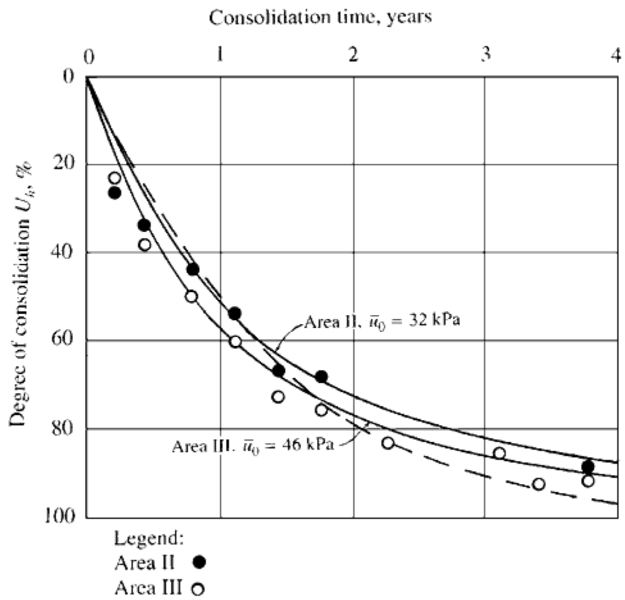

Fig. 16. Observed and calculated excess pore pressure dissipation at a depth of 5 m in the Skå-Edeby test areas II and III. Unbroken lines represent exponential flow with n = 1.5 and λ = 0.43 m^2/year, broken line Darcian flow with c_h = 0.8 m^2/year.

Table 5. Best fit of c_h and λ values in Test Areas II (load 27 kN/m^2) and III (load = 39 kN/m^2), based on total settlement.

Test Area	II ($\bar{u}_0$ = 32 kPa)					III ($\bar{u}_0$ = 46 kPa)			
Time, years	1/6	1/2	2	4	9	1/6	1/2	2	4
$\overline{U}_{tot}$, %	20	42	75	90	98	20	45	80	94
$\overline{U}_v$, %	6	10	19	30	46	6	10	19	30
$\overline{U}_h$, %	15	36	69	86	96	15	39	77	91
c_h, m^2/year	1.04	0.95	0.63	0.52	0.39	1.05	1.05	0.78	0.65
λ, m^2/year (n =1.5)	0.47	0.46	0.37	0.39	0.42	0.40	0.44	0.42	0.46
$\overline{U}_h$, % ($\lambda_{av.}$ = 0.43 m^2/year)	14	34	73	88	96	16	38	77	90

The excess pore pressure dissipation in Test Areas II and III with different loading condition is shown in Fig. 16. In the analysis based on Darcian flow, the consolidation process becomes independent of the magnitude of the load applied, which is not the case when the analysis is based on exponential flow. As can be seen, the best agreement between theory and observation is obtained by assuming exponential flow with the exponent $n = 1.5$.[3]

In Test Area V at Skå-Edeby (Fig. 17), band drains, type Geodrain with paper filter, were installed with 0.9 m spacing. The initial load, 27 kN/m^2, was doubled after 3 1/2 years of consolidation. In a case like this, the following procedure of settlement analysis is followed.

The degree of consolidation $\overline{U}_1$ and the settlement s_1 achieved under load q_1 at time t_1 when the additional load q_2 is being placed is calculated first. The part of the load q_1 that is still producing primary consolidation settlement, i.e. $\Delta q = q_1\left(1-\overline{U}_1\right)$, is added to q_2 and the primary settlement process to be added after time t_1 under load $\Delta q + q_2$ is added to the settlement s_1. According to equation (20), the equivalent diameter of Geodrain (100 mm in width and 4 mm in thickness) is equal to $d_w = 0.066$ m. Assuming a zone of smear $d_s = 0.19$ m and $k_h / k_s = \kappa_h / \kappa_s = 4$, the best fit between observations and theory is obtained by applying the values c_h = 0.45 m^2/year and $\lambda = 0.23$ m^2/year with the exponent $n = 1.5$ (Fig.18).

The λ value 0.23 m^2/year agrees fairly well with the value obtained in test area I with 0.9 m drain spacing ($\lambda = 0.26$ m^2/year) and is an indication of increasing average disturbance of the clay with decreasing drain spacing.

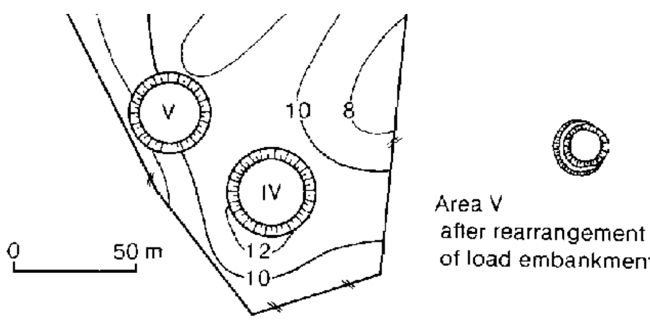

Fig. 17. The placement of test area V in relation to test area IV at Skå Edeby.

[3] Case studies and experimental evidence reported already by Terzaghi & Peck (1948) indicated that the coefficient of consolidation increases with increasing magnitude of the load that produces consolidation. For the determination of the coefficient of consolidation Terzaghi & Peck therefore recommended that the load increment "applied to the sample after a pressure equal to the overburden pressure has been reached should be of the same order of magnitude as the load per unit area of the base of the structure".

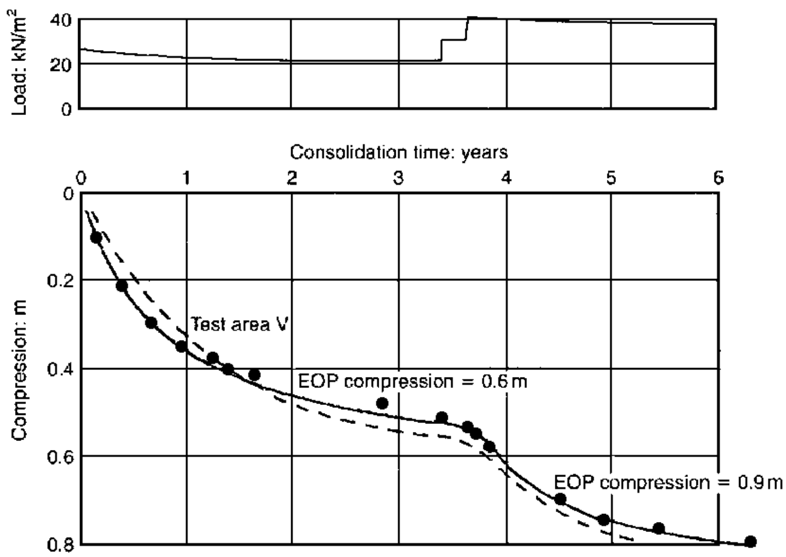

Fig. 18. Compression of the clay layer between 2.5 and 7.5 m of depths obtained at Skå-Edeby in Test Area V: 0.9 m drain spacing (*D* = 0.95 m) Broken line: Darcian flow with c_h = 0.45 m^2/year; Unbroken line non-Darcian flow with λ = 0.23 m^2/year and *n* = 1.5.

The results obtained at Skå-Edeby show that the values of the coefficients of consolidation are smaller than can be expected from the values arrived at in the undrained area. This is the case in spite of the fact that the coefficient of consolidation is normally higher in horizontal pore water flow than in vertical pore water flow due to the structural features of the soil (e.g. due to the existence of sand and silt layers). This obviously depends on the fact that the drain installation causes disturbance effects leading to a reduction of the coefficient of consolidation. The disturbance is evidenced by the fact that the sand drain installation at Skå-Edeby in itself induced excess pore water pressure of maximum 40 kPa at 0.9 m drain spacing, 35 kPa at 1.5 m drain spacing and 20 kPa at 2.2 m drain spacing (Hansbo, 1960).

2.2 The Örebro test field.

Another example of the calculated vs. observed consolidation process based on settlement is given by the results obtained at the Örebro test field, Sweden. This test field, 125 m by 45 m, was arranged just outside

Örebro for comparing the efficiency of prefabricated band-shaped drains (types Geodrain and Alidrain) with that of displacement type sand drains. The test field was divided into three sections of equal size with drain spacing 1.1, 1.4 and 1.6 m in an equilateral triangular pattern. The test area was loaded with 2.2 m of sand and gravel (load Δq = 40 kN/m^2).

The soil consists of grey, lightly overconsolidated clay, about 8 m in thickness. The clay layer is slightly organic to a depth of 1–2 m and is varved, with silt seams, in its lower part, below a depth of about 5 m. The average virgin compression ratio increases from about *CR* = 0.4 just below the dry crust to about *CR* = 0.65 in the lower part of the clay layer. The primary settlement, calculated based on the compression ratios and an overconsolidation of about 5 kPa (below the dry crust) was estimated at 1.1 m. The coefficient of consolidation according to oedometer tests varies from a minimum of c_v = 0.06 m^2/year to a maximum of 1 m^2/year. An estimated average is c_v = 0.2 m^2/year.

The coefficient of consolidation in the case of horizontal pore water flow was not determined.

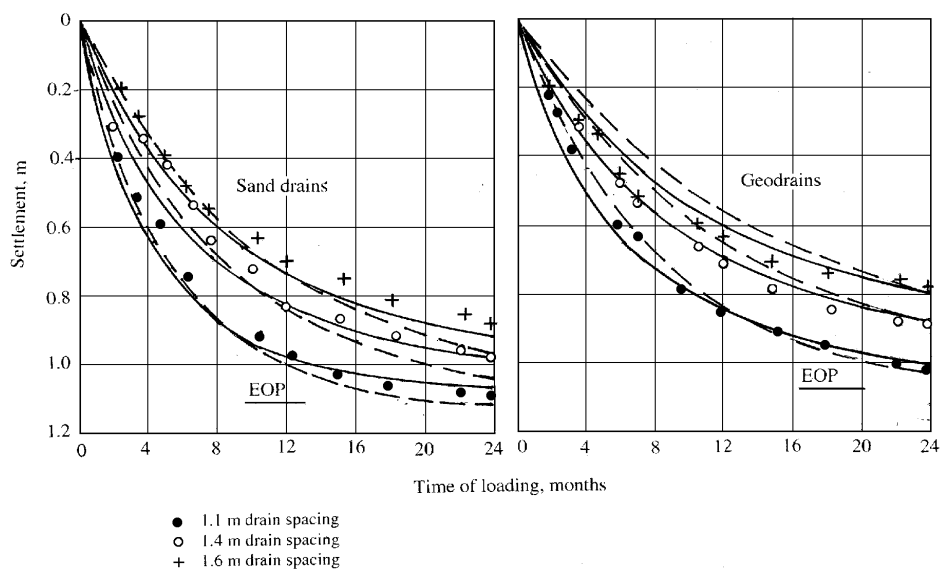

Fig. 19. Measured and calculated settlements in test field, Örebro. Sand drains, 0.18 m in diameter, and band drains, type Geodrain. Load Δq = 40 kN/m^2. EOP means end of primary settlement. Unbroken lines represent exponential flow, equation (19) (λ = 0.58 m^2/year; n = 1.5), broken lines Darcian flow, equation (18) (c_h = 1.15 m^2/year).

The settlement process observed in the test sections provided with Geodrain and Alidrain was very nearly the same. The results obtained in the test sections provided with Geodrains and sand drains are shown in Fig. 19.

Using Asaoka's method for determination of the primary consolidation settlement we find in the test area provided with Geodrains $s_i =$ 0.1809 +0.8343s_{i-1}, which yields ($s_{i-1} \rightarrow s_i$) s_p = 1.09 m and for the test area provided with sand drains s_i = 0.4312 + 0.6119s_{i-1}, which yields s_p = 1.11 m. These values are in excellent agreement with the settlement estimated on the basis of the oedometer tests.

The theoretical analysis of the consolidation process in the case of sand drains is based on the following assumptions: d_w = 0.18 m, d_s = 0.36 m. The corresponding values for Geodrain are: d_w = 0.066 m, d_s = 0.19 m. Inserting the parameters $k_h/k_s = \kappa_h/\kappa_s =$ □,□ l = 4 m and c_v = 0.2 m^2/year, the best fit of c_h and λ with observations is given in Table 6.

Table 6. Best fit of c_h and λ values in the Örebro test areas (load 40 kN/m^2).

Sand drains						
c_h m^2/year						
Loading time	4 months	8 months	12 months	16 months	20 months	24 months
c/c 1.1 m	1.01	1.02	1.02	1.02	1.03	1.03
c/c 1.4 m	0,93	1.00	1.00	0.89	0.90	0.84
c/c 1.6 m	0.99	1.13	0.97	0.94	0.87	0.79
λ m^2/year						
c/c 1.1 m	0.42	0.50	0.62	0.75	0.93	1.15
c/c 1.4 m	0.38	0.46	0.53	0.48	0.53	0.52
c/c 1.6 m	0.42	0.52	0.47	0.48	0.47	0.44
Geodrains						
c_h m^2/year						
Loading time	4 months	8 months	12 months	16 months	20 months	24 months
c/c 1.1 m	1.08	1.24	1.20	1.15	1.06	1.05
c/c 1.4 m	1.39	1.49	1.44	1.42	1.27	1.28
c/c 1.6 m	1.70	1.88	1.53	1.42	1.25	1.06
λ m^2/year						
c/c 1.1 m	0.36	0.50	0.57	0.64	0.60	0.67
c/c 1.4 m	0.66	0.74	0.67	0.65	0.59	0.52
c/c 1.6 m	0.57	0.57	0.62	0.69	0.61	0.67

The agreement between field observations is better according to Darcian flow at 1.1 m drain spacing while the agreement according to exponential flow with $n = 1.5$ is better at 1.6 m drain spacing.

The result of the back-analysis shown in Table 6 indicates that the flow condition prevailing in the soil of the Örebro test area follows Darcy's law better than the exponential correlation. However, as can be seen in Fig. 19, the theoretical correlations based on the average of the consolidation parameters shown in Table 6, i.e. c_h = 1.15 m^2/year and λ = 0.58 m^2/year, show better general agreement between observations and the exponential flow theory.

2.3 The Bangkok Test Field.

In connection with the planning of a new international airfield in Bangkok, Thailand, three test areas were arranged in order to form a basis for the design of soil improvement by preloading in combination with vertical drains. The results of the settlement observations in two of these test areas, TS 1 and TS 3, (Hansbo, 1997b) showed a better agreement with equation (6) than with equation (5). In this paper, the results obtained in test area TS 3 will be examined in detail.

The crest width of TS 3 is 14.8 m (square) and the bottom width 40 m. It is provided with an approximately 10 m wide loading berm, 1.5 m thick. The fill placed on the area amounts to maximum of about 4.2 m, corresponding to a load of about 80 kN/m^2.

The drains, type Mebradrain, were installed to a depth of 12 m in a square pattern with a spacing of 1.0 m, which yields D = 1.13 m. The equivalent drain diameter determined according to equation (20) becomes d_w = 0.066 m. The equivalent diameter of the mandrel d_m = 0.10 m. The smear zone is estimated at d_s = 0.20 m. The permeability ratios k_h/k_s and κ_h/κ_s are assumed equal to the ratio c_h/c_v.

The consolidation characteristics of the clay deposit, determined by oedometer tests, can be summarized as follows (DMJM International, 1996; Airports Authority of Thailand, 1996): average coefficient above the preconsolidation pressure c_v = 1.06 m^2/year (standard deviation = 0.061 m^2/year), c_h = 1.37 m^2/year (standard deviation = 0.050 m^2/year). This yields $k_h/k_s = \kappa_h/\kappa_s = 1.3$ (in a paper previously published by the author (Hansbo, 1997b) this ratio was assumed equal to 2). The virgin compression ratio CR varies from 0.3 to 0.55 (average 0.43; standard deviation 0.1) and the average recompression ratio RR = 0.03 (standard deviation = 0.007). The clay penetrated by the vertical drains is slightly overconsolidated with a preconsolidation pressure about 15–50 kPa higher than the effective overburden pressure. The clay below the tip of the drains is heavily overconsolidated. In this case the influence on the consolidation process of one-dimensional vertical consolidation will be ignored owing to difficulties in assessing the drainage conditions.

The monitoring system consisted of vertical settlement meters placed on the soil surface and at different depths and of inclinometers to study the horizontal displacements. Unfortunately, the results of the settlement observations at various depths are contradictory and, therefore, only the surface settlement observations can be trusted. The contribution to the vertical settlement of horizontal deformations is analysed based on the inclinometers placed 7.8 m from the centre of the test area. The total settlement observed, including the settlement caused by horizontal deformations, and the corrected settlement curve with regard to the influence on vertical settlement by horizontal deformations (representing the mere effect of consolidation), are shown in Fig. 20.

A follow-up of the course of consolidation settlement according to Asaoka's method, based on settlement observations at equal time intervals results in the following relation $s_i = 0.2625 + 0.8195 s_{i-1}$ which yields the primary settlement s_p = 1.45 m.

The settlement analysis based on equation (19) is carried out in 4 successive steps: load-step 1 with load Δq_1 = 20 kN/m^2; load-step 2 with load Δq_2 = 30 kN/m^2; load-step 3 with Δq_3 = 10 kN/m^2 and load-step 4 with Δq_4 = 20 kN/m^2. The primary consolidation settlement caused by a load intensity of 80 kN/m^2, determined by the compression characteristics, becomes equal to 1.4 m. By slightly modifying the compression characteristics to yield a final primary consolidation settlement of 1.45 m, we find Δs_1 = 0.15 m; Δs_2 = 0.6 m; Δs_3 = 0.2 m and Δs_4 = 0.5 m (in total 1.45 m).

The analysis of the consolidation process according to equation (19) has to be carried out in the following way. The degree of consolidation $\overline{U}_1$, inserting $\Delta \overline{h}_1 = \Delta q_1 / \gamma_w$, determines the course of settlement in the first load-step. When calculating the course of settlement in the second load-step we have to apply the value $\Delta \overline{h}_2 = \left(1 - \overline{U}_1\right) \Delta q_1 / \gamma_w + \Delta q_2 / \gamma_w$. The settlement at the end of the load-step is obtained from $\Delta s = \Delta s_1 \overline{U}_1 + \left[\Delta s_1 \left(1 - \overline{U}_1\right) + \Delta s_2\right] \overline{U}$, and so on.

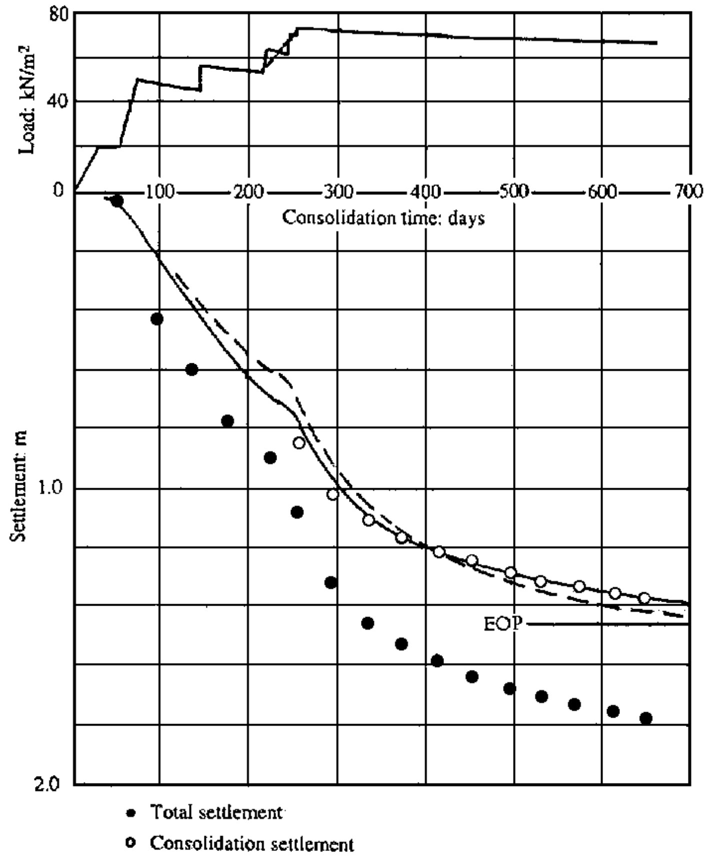

Fig. 20. Settlement of ground surface in the Bangkok test field, Thailand. Test area TS 3: 1.0 m drain spacing (D = 1.13 m). Settlement corrected with regard to immediate and long-term horizontal displacements. EOP = end of primary consolidation settlement estimated according to Asaoka's method. Full lines: analytical results according to equation (19) with λ = 0.37 m^2/year and n = 1.5. Broken lines: analytical results according to equation (18) with c_h = 0.93 m^2/year.

Now, 50 days after the start of loading (consolidation time t = 50 –15 = 35 days; $\Delta\bar{h}$ = 2 m; λ = 0.37 m^2/year, n = 1.5) we find $\overline{U}$ = 0.21 which yields s = 0.03 m. In load-step 2 the load Δq_2 = 30 kN/m^2 has to be increased by 0.79x20 = 16 kN/m^2 corresponding to $\Delta\bar{h}_{2,\mathrm{corr}}$ = 4.6 m and $\Delta s_{2,\mathrm{corr}}$ = 0.6 + 0.12 = 0.72 m. After 75 days when load-step 2 is completed we find [t = (75 – 50)/2] $\overline{U}$ = 0.12 which yields Δs = 0.09 m and s = 0.12 m. After 140 days when load step 3 is being applied we have (t =

12.5 + 65 = 77.5 days) $\overline{U}$ = 0.50 from which Δs = 0.36 m and s = 0.39 m. This yields $\Delta h_{3,corr}$ = 3.3 m and $\Delta s_{3,corr}$ = 0.2 + 0.36 = 0.56 m. After 220 days when load-step 4 is being applied we find (t = 80 days) $\overline{U}$ = 0.46 from which Δs = 0.26 m and s = 0.26 + 0.39 = 0.65 m. This yields $\Delta h_{4,corr}$ = 1.8+2.0 = 3.8 m and $\Delta s_{4,corr}$ = 0.5 + 0.3 = 0.8 m. After 250 days when load-step 4 is completed we have (t = 15 days) $\overline{U}_2$ = 0.13 which yields Δs = 0.10 m and s = 0.75 m. 100, 200 and 400 days later we have $\overline{U}$ = 0.59, $\overline{U}$ = 0.74 and $\overline{U}$ = 0.89 from which Δs = 0.47 (s = 0.47 + 0.65 = 1.12 m), 0.67 (s = 1.24 m) and 0.71 m (s = 1.36 m), respectively.

The theoretical course of settlement determined in the conventional way, equation (18), is less complicated in that the total consolidation curve can be determined for each load step separately and added to each other. Assuming c_h = 0.93 m^2/year we find, to give an example, 400 days after the start of the loading process (t_1 = 385 days, $\overline{U}$ = 0.92; t_2 = 340 days, $\overline{U}$ = 0.89; t_3 = 260 days, $\overline{U}$ = 0.82; t_4 = 170 days, $\overline{U}$ = 0.67) the settlement $s = 0.92 \cdot 0.15 + 0.89 \cdot 0.6 + 0.82 \cdot 0.2 + 0.67 \cdot 0.5 = 1.17$ m.

The results obtained by the two methods of analysis are shown in Fig. 20.

The values of $\Delta \overline{h}$ applied in the different load steps agree quite well with observations of the excess pore water pressure (cf. Hansbo, 1997). Inserting the maximum value $\overline{h}_{0v}$ = 4.6 m into equation (14) we find outside the zone of smear $i_{\max}$ = 17.7 and inserting this into equation (16), the correlation between c_h = 0.93 m^2/year and λ = 0.37 m^2/year correspond to i_l = 3.7.

2.4 The Vagnhärad vacuum test.

Torstensson (1984) reported an interesting full-scale test in which consolidation of the clay was achieved by the vacuum method. The subsoil at the test site consists of postglacial clay to a depth of 3 m and below this of varved glacial clay to a depth of 9 m underlain by silt. The clay is slightly overconsolidated with a preconsolidation pressure about 5–20 kPa higher than the effective overburden pressure. The coefficient of consolidation c_h was found equal to 0.95 m^2/year and the average virgin compression ratio CR equal to 0.7 (max. 1.0).

The vacuum area, 12 m square, was first covered by a sand/gravel layer 0.2 m in thickness, and then by a Baracuda membrane, which was buried to 1.5 m depth along the border of the test area and sealed by means of a mixture of bentonite and silt.

Mebradrains (d_w = 0.066 m) were installed in a square pattern with 1.0 m spacing to a depth of 10 m. The equivalent diameter of the mandrel d_m = 0.096 m. The average under-pressure achieved by the vacuum pump was 85 kPa. After 67 days, the vacuum process was stopped and then resumed after 6 months of rest. From the shape of the settlement curve (Fig. 21) Asaoka's method yields the correlation s_i = 0.0756+ 0.9075s_{i-1} from which s_p = 0.82 m. This value is low with regard to the loading conditions and the compression characteristics. The main reason seems to be that the applied vacuum effect is not fully achieved in the drains. Thus, the primary settlement 0.82 m corresponds to a vacuum effect of about 35 kPa (Δh = 3.5 m). Another reason may be that the test area is too small as compared to the thickness of the clay layer.

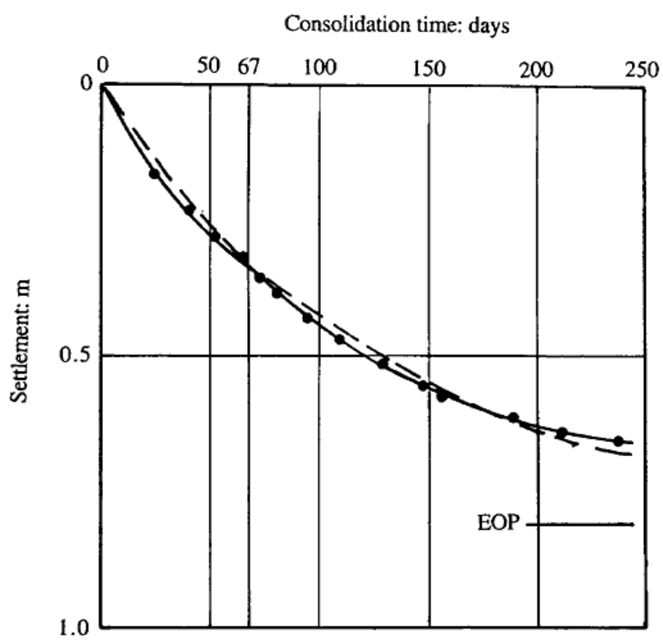

Fig. 21. Results of settlement observations at Vagnhärad, Sweden. Consolidation by vacuum. 1.0 m drain spacing (D = 1.13 m). EOP = end of primary consolidation settlement estimated according to Asaoka's method. Full lines: analytical results according to non-Darcian flow, equation (19), with λ = 0.95 m^2/year and n = 1.5. Broken lines: analytical results according to Darcian flow, equation (18), with c_h = 2.4 m^2/year.

The theoretical settlement curve in this case has to be determined in two steps, the first one up to a loading time of 67 days leading to a then settlement $s_1 = \overline{U}_{h1} s_p$. In the next load step, starting again from the time of resumption of the application of vacuum, the remaining primary settlement is obtained from the relation $\Delta s = \overline{U}_h \left(s_p - s_1 \right)$, i.e. the remaining settlement equals $s_t = s_1 + \overline{U}_{ht} \left(s_p - s_1 \right)$, where t starts from the time of resumption of the application of vacuum. In this case, where vacuum is applied to create underpressure in the drains, the effect of vertical one-dimensional consolidation is eliminated.

Inserting the values D = 1.13 m, d_w = 0.066 m, d_s = 0.19 m, $k_h / k_s = \kappa_h / \kappa_s = 4$ and $\Delta \overline{h}$ = 3.5 m into equations (18) and (19), the best agreement between theory and observations is found for c_h = 2.4 m^2/year and λ = 0.95 m^2/year (Fig. 21). Even in this case the λ theory agrees better with observations than the classical theory.

Inserting the maximum value $\Delta \overline{h}$ = 3.5 m into equation (13) yields $i_{\max}$ = 7.3, and inserting the values c_h = 2.4 m^2/year and $i_{\max}$ = 7.3 into equation (15) yields λ = 1.1 m^2/year. The value λ = 0.95 m^2/year corresponds according to equation (15) to $i_{\max}$ = 9.9. Assuming i_l = 5, the value of $i_{\max}$, derived from equation (16), becomes respectively 7.7 instead of 7.3 and 11.8 instead of 9.9.

2.6 The Porto-Tolle test site, Italy

The Porto Tolle experimental embankment had a crest width and length of 30 and 300 m, respectively, and a bottom width of 65 m. The load placed on the embankment was about 100 kN/m^2. It was divided into four sections with different types of vertical drains. Here, two types of drains will be analysed, namely one with band drains of type Geodrain and another with jetted sand drains, 0.3 m in diameter. The drains were placed in equilateral triangular pattern, Geodrains with 3.8 m spacing and sand drains with 5.0 m spacing.

The soil consists of 7 m sand and silty sand underlain by about 20 m of soft, normally consolidated light-brown silty clay resting on dense, silty sand. The consolidation characteristics of the clay were determined by means of oedometer tests and field investigations. According to the oedometer tests, the coefficient of consolidation c_v was found equal to 3–9 m^2/year and the coefficient of consolidation c_h equal to 6–16 m^2/year. From the results of a trial embankment without vertical drains, c_v was found equal to 9 m^2/year. The virgin compression ratio *CR* was not given. The primary consolidation settlement of the section with Geodrains, shown in Fig. 22, estimated according to Asaoka's method, yields $s_i = 0.249 + 0.756 s_{i-1}$, corresponding to a total primary consolidation settlement of 1.0 m, while the settlement of the section with sand drains yields $s_i = 0.259 + 0.703 s_{i-1}$, corresponding to a total primary consolidation settlement of 0.9 m. The vertical stress increase in the middle of the clay layer due to the load of the embankment can be estimated at $\Delta\sigma = 100[(30+65)/2]/[17+(30+65)/2] \approx 74$ kPa.

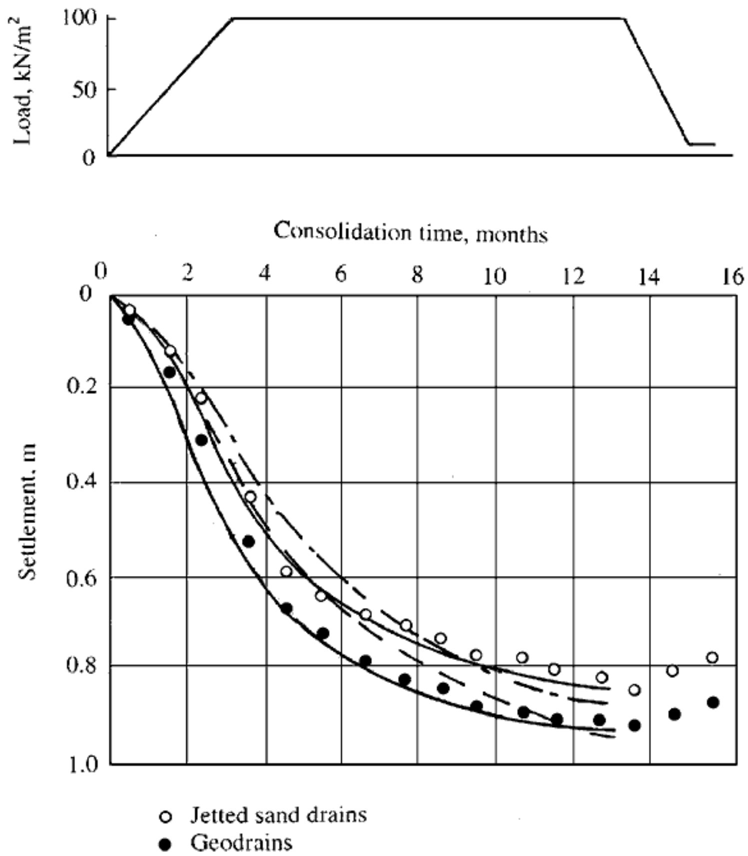

Fig. 22. Trial embankment at Porto Tolle, Italy. Measured and calculated settlements and loading conditions for test sections provided with band drains, type Geodrain and jetted sand drains. Broken line represents Geodrain, and the dash-dotted line jetted sand drains according to equation (18) with c_h = 24 m^2/year, and full lines equation (19) with λ = 14 m^2/year and *n* = 1.5.

The observations are compared with equations (18) and (19) assuming that the primary consolidation settlement is in accordance with the values given above, i.e. equal to 1.0 m in the section provided with Geodrain and 0.9 m in the section provided with sand drains. Inserting for Geodrains the values D = 3.99 m, d_w = 0.066 m, d_s = 0.19 m, $k_h/k_s = \kappa_h/\kappa_s = 2$ and for sand drains D = 5.25 m, d_w = 0.3 m, d_s = 0.4 m, $k_h/k_s = \kappa_h/\kappa_s = 2$ and an initial hydraulic head $\Delta\bar{h}_0$ = 7 m (assuming $\Delta\bar{u}_0 = \gamma_w \Delta\bar{h}_0 = \Delta\sigma$) into equations (18) and (19), the best agreement between theory and observations is found for c_h = 24 m^2/year and λ = 14 m^2/year with the exponent n = 1.5 (Fig. 22). In both cases the coefficient of consolidation c_v is assumed equal to 9 m^2/year.

According to equation (18) the average degree of consolidation when the load-step leading to full load is completed (consolidation time t = 3.4 months; full load during 1.7 months) $\bar{U} = 41\%$ for Geodrains and 42 % for sand drains. At t = 6, 9 and 12 months we find $\bar{U} = 68\%$, 85% and 93%, respectively, for Geodrains, and $\bar{U} = 72\%$, 87% and 94%, respectively, for sand drains. According to equation (19) the average degree of consolidation when the load-step leading to full load is completed (consolidation time t = 3.4 months; full load during 1.7 months) $\bar{U} =$ 51% for Geodrains and 47 % for sand drains. At t = 6, 9 and 12 months we find $\bar{U} =$ 76%, 87% and 92%, respectively, for Geodrains , and $\bar{U} =$ 73%, 85% and 91%, respectively, for sand drains. Even in this case the λ theory agrees better with observations than the classical theory.

2.7 The Stockholm-Arlanda project.

At Stockholm-Arlanda Airport, different types of soil improvement techniques have been used. Amongst these, vertical drainage in combination with preloading has been used along the runway/taxiway in an area of 250000 m^2 containing clay and organic deposits.

The requirements placed on the completed project allowed a maximum settlement of 30 mm. The design of the vertical drain project aimed at creating an effective preconsolidation pressure of at least 1.25 times the effective vertical pressure reached in the soil after completion of the project. This was done in order to eliminate the risk of too large creep settlement after the primary consolidation period (cf. Test Area III at Skå-Edeby, Fig. 15). The drain spacing and the preloading conditions were selected in order to achieve 95 % primary consolidation within 12 months of loading. The Arlanda project is interesting from the point of view of the extreme loading conditions. Thus, in order to achieve the goal set up, preloading has been carried out by means of 14–20 m of fill. The load of the fill has caused a relative compression of 22–29 %, corresponding to total settlements of 1.6–2.6 m.

According to the design requirements the drains had to be installed in equilateral triangular pattern with a drain spacing of 0.9 m. Furthermore, depending on the large relative compression to be expected, the drains had to be capable of undergoing a vertical compression in the order of 35 % without loosing their function. Prefabricated drains, type Mebradrain 88, were selected for the project. The installation of the drains was made by means of steel mandrel with the dimensions 60x120 mm.

The clay at one of the sites of observation presented here, site K, was medium to highly sensitive and normally consolidated with water content of mainly 60–90 % and a liquid limit of 40–80 % (the results at other points of observation and consequential analysis are presented by Eriksson *et al.*, 2000). The undrained shear strength was fairly constant, mainly about 7–9 kPa, irrespective of depth. The ground water level was situated just below the ground level. The results of the oedometer tests, interpreted in the way shown in Fig. 23, and the settlements derived accordingly are given in Table 7.

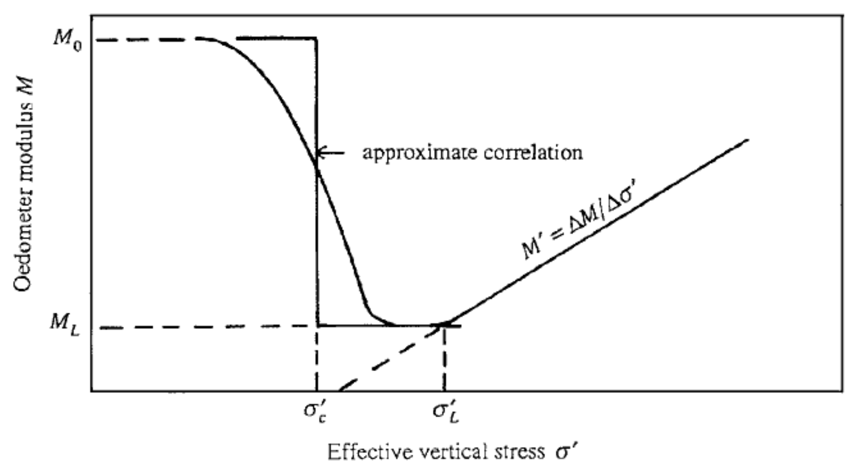

Fig. 23. Interpretation of the results of oedometer tests used for the Stockholm-Arlanda project. Oedometer modulus $M = \Delta\sigma' / \Delta\varepsilon$ **.**

The settlement values calculated from the oedometer test results were compared with those determined from the course of settlement shown in Fig. 24. Thus, according to Asaoka's method the primary consolidation settlement is obtained from the correlation $s_i = 0.590 + 0.775 s_{i-1}$, which yields s_p = 2.63 m. The discrepancy can be explained by the fact that the oedometer results given in Table 7 represent mean values of oedometer tests carried out on samples taken as far as 70–165 m away from the point of settlement observation. The settlements obtained in the various load steps were adjusted by multiplication by a factor obtained by dividing the final settlement values obtained by Asaoka's method with the final settlement values evaluated based on the oedometer tests. The adjusted values are given in Table 8.

Table 7. **Parameters used in the settlement analysis (see Fig. 18). Δh_j = hydraulic head of excess pore water pressure.**

Depth, m	M_{ij} kPa	M'_j	$\sigma'_{ij} - \sigma'_{cj}$, kPa	Δh_j, m	Load, kN/m^2	Settlement, m
2	80	12.9	9	2.5	0–80	1.8
4	165	19.3	12	2.0	80–215	0.7
6	188	15.3	21	2.0	215–390	0.4
8	279	17.5	20	2.5	0–390	2.9

Table 8. Settlement values, adjusted according to Asaoka's method.

Load step (kN/m^2):	0–80	80–215	215–390	0–390
Settlement (m):	1.63	0.64	0.36	2.63

Analysis of consolidation process. According to the oedometer tests, the coefficient of consolidation c_v varies from about 0.2–0.3 m^2/year at preconsolidation pressure to about 0.5–1.0 m^2/year (maximum 2.5 m^2/year) at the end of primary compression under the maximum load applied (390 kN/m^2). The following assumptions are applied in the analysis: D = 1.05x0.9 = 0.945 m; d_w = 0.066 m; d_s = 0.19 m; $k_h / k_s = \kappa_h / \kappa_s = c_h / c_v = 3$; l = 4.5 m; c_h = 2.6 m^2/year and λ = 0.7 m^2/year with the exponent n = 1.5.

The course of settlement in Darcian flow. Inserting the consolidation time t_1 = 1 month (the length of time that corresponds to full loading condition in load step 1), we find $\overline{U}_1$ = 0.42 at the completion of load step 1, and, consequently, s =0.42x1.63 = 0.69 m. At the completion of load step 2 (t_1 = 2 months and t_2 = 0·5 months) we find $\overline{U}_1$ = 0.65 and $\overline{U}_2$ = 0.25, whence s = 0.65x1.63 + 0.25x0.63 = 1.22 m. Finally, at the completion of load step 3 (t_1 = 4.5 months, t_2 = 3 months and t_3 = 1.25 month) we find $\overline{U}_1$ = 0.90, $\overline{U}_2$ = 0.79 and $\overline{U}_3$ = 0.49, i.e. s = 0.90x1.63 + 0.79x0.63 + 0.49x0.36 = 2.15 m. Three months later we find $\overline{U}_1$ = 0.98, $\overline{U}_2$ = 0.95 and $\overline{U}_3$ = 0.89, i.e. s = 0.98x1.63 + 0.95x0.63 + 0.89x0.36 = 2.52 m. Another three months later we find $\overline{U}_1$ = 0.995, $\overline{U}_2$ = 0.99 and $\overline{U}_3$ = 0.98, i.e. s = 2.60 m.

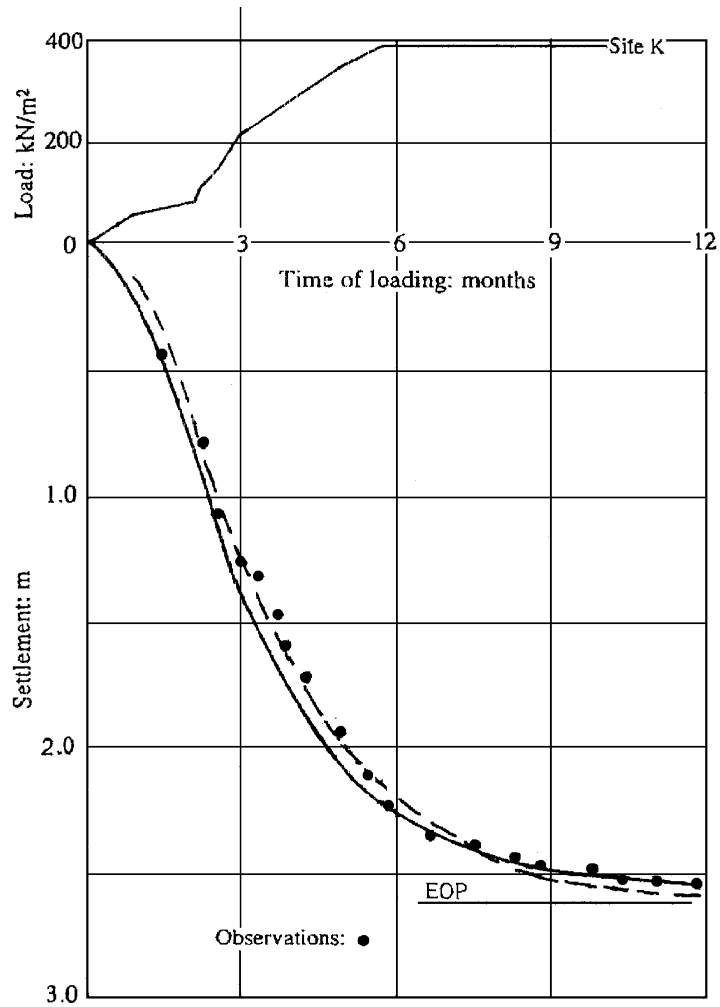

Fig. 24 Comparison between calculated course of settlement and observations at Stockholm Arlanda Airport. Broken line represents Darcian flow according to equation (18) with c_h = 2.6 m^2/year, unbroken line non-Darcian flow according to equation (19) with λ = 0.7 m^2/year and n = 1.5.

The course of settlement in non-Darcian flow: In this case, according to equation (19), the average piezometric head in each load-step has a great influence on the rate of consolidation.

Assuming that the average excess pore water pressure is equal to the load placed on the ground we have in load-step 1 (time of full loading one month) the hydraulic head $\Delta\overline{h}_1$ = 8 m, which yields $\overline{U}_1$ =0.46. This yields s = $\Delta s_1\overline{U}_1$ = 0.46x1.63 = 0.76 m and a hydraulic head $\Delta\overline{h}_2 = \left(1-\overline{U}_1\right)\mathrm{x}\,\Delta q_1/\gamma_w + \Delta q_2/\gamma_w$ = 0.54x8 + 13.5 = 17.8 m. At the completion of load-step 2, inserting a time of consolidation of half a month (the time of full loading) we have $\overline{U}_2$ = 0.38, which yields the settlement $s = \Delta s_1\overline{U}_1 + \overline{U}_2\left[\Delta s_1 + \Delta s_2 - \Delta s_1\overline{U}_1\right]$ = 0.76 + 0.38(2.27 – 0.76) = 1.33 m. The hydraulic head now becomes $\Delta\overline{h}_3$ = 0.62x17.8 + 17.5 = 28.5 m. Finally, at the completion of load-step 3, when the loading operation has been completed, we have (time of consolidation under full load 1 1/4 month) $\overline{U}_3$ = 0.70 which yields s = 1.33 + 0.70(2.63 – 1.33) = 2.24 m. Three months later we have $\overline{U}_3$ = 0.93 from which s = 2.45 m.

A comparison between analytical results and observations are made in Fig. 24.

The course of excess pore pressure dissipation. In this case, the piezometers were placed inside concrete tubes with an outer diameter of 1m before the installation of the drains. The existence of these tubes created an obstacle to keeping up the intended drain pattern. Thus, the distance from a piezometer to the nearest drain was estimated at 0.9 m. The average D value in the vicinity of a piezometer was estimated at 1.335 m instead of the normal value of 0.945 m.

The excess pore water pressure u caused by loading is analysed by Skempton´s pore pressure equation $u = B\left[\sigma_3 + A\left(\sigma_1 - \sigma_3\right)\right]$. In our case the pore pressure coefficients can be assumed equal to B = 1 (water-saturated clay) and A = 1.2 (load increase very high in comparison with the preconsolidation pressure). According to the theory of elasticity the principal stress increase becomes $\sigma_1 = \Delta q$ and $\sigma_3 = 0.9\Delta q$. This yields u_0 = 1.02Δq whence the hydraulic head increase $\Delta h_0 = u_0/\gamma_w = 0.104\Delta q$.

Now, assuming that the piezometers are placed in the centre between the drains we have $\tilde{\rho}D$ = 0.5. For $\overline{U}_h$ = 0 and D = 1.335 m we then have according to equation (7) Δh = 1.06x0.104Δq = 0.11Δq. When dealing with the excess pore pressure dissipation, the excess pore pressure developed during the placement of each load-step is assumed to correspond to the total load increase (no reduction of the pore pressure caused during the placement of the load-step). This can be justified by the dynamic disturbance effects caused by the filling operation. Therefore, the time of consolidation is chosen with reference to the time when the filling operation in a load-step is completed. The effect of the contribution by one-dimensional vertical consolidation on the excess pore pressure values is not taken into account.

Assuming Darcian flow, the value of Δh during the course of consolidation can be found by equations (7) and (5). Proceeding along the mentioned principle, taking into account only the effect of the drains ($c_v = 0$), we find, at the completion of load step 1, Δh = 0.11x80 = 8.8 m. At the completion of load step 2 we have t_1 = 1 month which yields $\overline{U}_{h1}$ = 0.19. Hence Δh = 0.81x8.8 + 0.11x135 = 22.0 m. Finally, at the completion of load-step 3 we have t_1 = 3.5 months and t_2 = 2.5 months from which $\overline{U}_{h1}$ = 0.53 and $\overline{U}_{h2}$ = 0.42; from this we obtain Δh = 0.47x8.8 + 0.58x0.11x135 + 0.11x175 = 32.0 m. An easier way of finding the value of Δh, applied in the following, is to start the analysis only from the time when load-step 2 is completed, which yields the same result: Δh = 0.58x22.0 + 0.11x175 = 32.0 m. Three months later we have $\overline{U}_{h3}$ = 0.48, whence Δh = 0.52x32.0 = 16.7 m, and another 3 months later, $\overline{U}_{h3}$ = 0.73, i.e. Δh = 8.7 m.

Assuming non-Darcian flow, we find according to equation (9) for $\overline{U}_h$ = 0 and ρ/D = 0.5 the value of Δh = 1.11x0.104Δq = 0.115Δh_0. At the completion of load-step 1 we now have Δh = 0.115x80 = 9.2 m. At the completion of load-step 2 we have according to equation (6) $\overline{U}_{h1}$ = 0.24, whence Δh = 0.76x9.2 +0.115x135 = 22.5 m. At the completion of load-step 3 we have $\overline{U}_{h2}$ = 0.59, from which Δh = 0.41x22.5 + 0.115x175 = 29.4 m. Three months later we have $\overline{U}_{h3}$ = 0.68, whence Δh = 0.32x29.4 = 9.4 m, and another 3 months later, $\overline{U}_{h3}$ = 0.84, i.e. Δh = 4.6 m.

The results of the analysis are compared with the results of pore pressure observations in Fig. 25. In reality, the observational results were obtained through measurements where the horizontal distance between the piezometers and the nearest drain was 0.8 m ($\rho = D/2 = 0.8$). The values of ρ and D, however, chosen in the analysis, are based on the average drain spacing nearest to the piezometers.

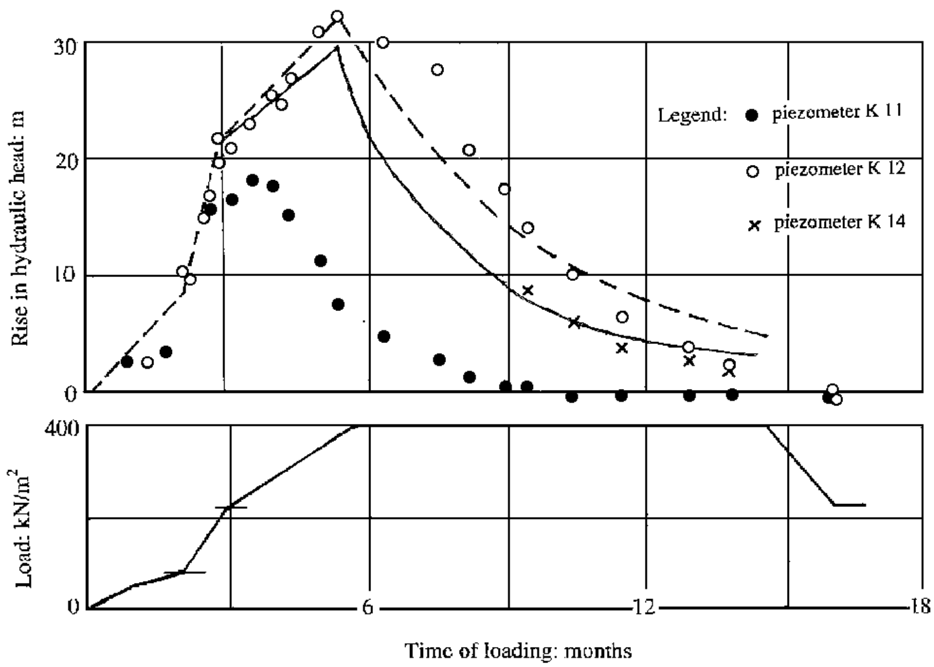

Fig. 25 Comparison between theoretical and observed excess pore pressure dissipation at Stockholm Arlanda Airport. Piezometers placed at the following levels (level of original ground surface + 21.0): K 11 + 17.0; K 12 + 15.0; K 14 (after 10 months of loading) +14.3. Broken line represents Darcian flow according to equation (18) with c_h = 2.6 m^2/year, unbroken line non-Darcian flow according to equation (19) with λ = 0.7 m^2/year and n = 1.5.

From the results of the analysis of the consolidation process, shown in Figs. 24 and 25, we find a slightly better agreement between the observations and the analysis based on validity of Darcy's flow law than the analysis based on exponential flow. This can be explained by the fact that the prevailing hydraulic gradients in this case are very high and in excess of the limiting hydraulic gradient i_l in exponential flow almost during the whole consolidation process. In this case, the maximum hydraulic gradient during the consolidation process is equal to 77 while the adapted values of λ and c_h used in the analysis correspond to a limiting hydraulic gradient i_l equal to 7.

FINAL COMMENTS

The choice of consolidation parameters is mostly based on the results of conventional oedometer tests representing one-dimensional consolidation with pore water escape in the vertical direction. The coefficient of consolidation in the case of pore water escape in the horizontal direction is usually determined by means of piezocone tests. The results thus obtained have to be checked by a follow-up of the consolidation process, preferably by careful instrumentation of a test area or of the project itself. In the case of vertical drainage, the choice of the consolidation parameters will depend on the assumptions connected with the diameter of the zone of smear and the ratio of permeability coefficients in natural soil and in the zone of smear. For example, if the ratios $\kappa\ /\kappa_s$ and k_h/k_s in the Örebro test areas are chosen equal to 2 instead of 4, the coefficient of consolidation λ, giving equally good agreement with the test results as mentioned above, will be 0.31 m^2/year instead of 0.58 m^2/year. The coefficient of consolidation c_h will be 0.70 m^2/year instead of 1.15 m^2/year. This example indicates the importance of a follow-up of the consolidation process to check the choice of consolidation parameters. Monitoring should include both settlement and pore pressure observations. From the results obtained at an early stage of the consolidation process the consolidation parameters can be estimated and the continuation of the consolidation process predicted with acceptable accuracy.

NOTATIONS

a = thickness of prefabricated band-shaped drain,
b = width of prefabricated band-shaped drain,
c_h = coefficient of consolidation valid for vertical drainage (Darcian flow),
c_v = coefficient of consolidation valid for one-dimensional consolidation (Darcian flow),
C_c = compression index,
CR = compression ratio = $C_c / (1 + e_0) = \varepsilon_2 / \log 2$,
D = diameter of soil cylinder dewatered by a drain,
d_w = drain diameter,
d_s = diameter of zone of smear,
e = void ratio,
h = hydraulic head,

i = hydraulic gradient,
k = coefficient of permeability (Darcian flow),
k_s = coefficient of permeability in the zone of smear (Darcian flow),
k_h = coefficient of permeability in horizontal pore water flow,
k_w = coefficient of permeability of the drain,
l = length of partially penetrating drain (half length of penetrating drain),
M = compression modulus determined by oedometer tests,
n = exponent of hydraulic gradient in exponential flow correlation,
q_w = discharge capacity of drain at an inner hydraulic gradient equal to one,
t = consolidation time,
u = excess pore water pressure,
z = depth coordinate,
ε = relative compression,
ε_2 = relative compression along the virgin curve caused by doubling the load,
γ_w = unit weight of water,
$\gamma' = \gamma_{\text{sat}} - \gamma_w$ = effective unit weight of soil (γ_{sat} = unit weight of water saturated soil),
κ = coefficient of permeability (exponential flow),
κ_h = coefficient of permeability in horizontal pore water flow (exponential flow),
κ_s = coefficient of permeability in the zone of smear (exponential flow),
λ = coefficient of consolidation (exponential flow),
ρ = radius vector,
σ = total stress,
σ' = effective stress

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