CRITICAL STATE MODELS AND THEIR COMPARISON WITH TRIAXIAL BEHAVIOR OF SOFT BANGKOK CLAY

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ABSTRACT

Recently, the stress-strain behavior and the strength characteristics of Soft Bangkok Clay is studied in the triaxial apparatus under a variety of applied stress paths. The data are used to evaluate the potentials of the existing Critical State Theories in predicting soft clay behaviors and to determine the limitations of these which need improvements. In doing so, comparisons of the experimental behavior was also made with the Modified Cam Clay Theory (Roscoe & Burland, 1968; Pender's Theory, 1978; Dafalias Model, 1987; and Atkinson, 1987). Brief description of the theory are also given. Most of the behavior relates to the behavior below the State Boundary Surface and as such Pender's Model is used extensively in this paper for its capabilities in predicting soft clay behavior.

INTRODUCTION

The Critical State Concept and associated stress-strain theories were developed based on experimental and analytical research undertaken from 1950 to 1970. These theories captured the bulk of the behavior of normally and overconsolidtaed clays as sheared from the isotropic pre-shear stress states with various degrees of accuracy.

The research is primarily aimed in undertaking an experimental study to evaluate the theoretical predictions for specimens sheared from K_o conditions both for normally and overconsolidated states of Bangkok clay. Thus the research work specially aims to:

1. experimentally study the behavior of Bangkok clay in the overconsolidated state (i.e., within the State Boundary Surface, SBS) and compare this behavior with the predictions from the existing stress-strain theories; and

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2. experimentally study the behavior of Bangkok clay in the normally consolidated state but sheared from K_o pre-shear consolidation conditions.

Pender (1978) extended the Cam Clay Theory based on the Critical State Concepts to predict the volumetric shear strains both below the SBS and on the SBS as sheared from isotropic and K_o pre-shear consolidation conditions. Thus, the experimental observations in this study will be compared extensively with Pender's Model (Pender, 1978) for the prediction of strains and pore pressures.

In making such comparisons with the theories, the Ko-consolidated behavior will also be studied in relation to the anisotropic stress-strain models of Pender (1977, 1978), Atkinson, et al., (1987) and Dafalias (1987), which all belong to the same family of Critical State Theories with varying degrees of modification. This study also proposes a conceptual modification of Pender's Model for the undrained behavior of lightly overconsolidated clays.

CAMBRIDGE STRESS-STRAIN THEORIES FOR WET CLAYS BASED ON THE CRITICAL STATE CONCEPT

The Cambridge Stress-Strain theories assume that the soil is isotropic, follow the Critical State Concept, and that there is no recoverable shear strain. The state of the stress inside the SBS must remain on the elastic wall which is a vertical plane above an isotropic swelling line. The plastic deformation is assumed to occur only when the state of the sample changes on the SBS.

The Cam-Clay model was developed for normally consolidated and lightly overconsolidated clay. The authors assumed that the energy dissipated at any infinitesimal increment of plastic work is only a function of the plastic shear strain. The proposed expression for energy dissipation with an assumption that the principal axes of stress and plastic strain increment coincide is,

$$dW = pd\varepsilon_{vp} + qd\varepsilon_{sp} = Mpd\varepsilon_{sp} \qquad (1)$$

where dW = energy dissipated per unit volume of soil

p , q = mean effective principal stress, deviator stress

$d\varepsilon_{vp}$, $d\varepsilon_{sp}$, = increments of plastic strains

M = Stress ratio at critical state

CRITICAL STATE MODELS

Burland (1965) proposed a modified work equation which considers the work dissipated in plastic volume change. The energy dissipated in the Modified Cam-Clay Model is,

$$dW = p\,[\,(d\varepsilon_{vp})^2 + (Md\varepsilon_{sp})^2\,]^{1/2} \tag{2}$$

The flow rule and yield locus are given by Eqs. (3) and (4) respectively.

$$(d\varepsilon_{vp} / d\varepsilon_{sp}) = (M^2 - \eta^2) / 2\eta \tag{3}$$

$$p = (p_o M^2) / (M^2 + \eta^2) \tag{4}$$

Hence, the shape of the volumetric yield locus was changed from the earlier log spiral to an elliptic form.

The incremental shear and volumetric strains are as follows :

$$d\varepsilon_s = [(\lambda - \kappa) / (1 + e)]\,[2\eta / (M^2 - \eta^2)]\,[(2\eta d\eta / (M^2 + \eta^2) + (dp / p)] \tag{5}$$

$$d\varepsilon_v = [1 / (1 + e)]\,[2\eta\,(\lambda - \kappa)\,d\eta / (M^2 + \eta^2) + \lambda(dp / p)] \tag{6}$$

The State Boundary Surface described in this theory is

$$p / p_o = [M^2 / (M^2 + \eta^2)]^{(1-\kappa/\lambda)} \tag{7}$$

The Modified Theory of Roscoe and Burland (1968) captured many of the behavior of normally consolidated and lightly overconsolidated clays. Eq. (3) can make accurate predictions of K_o. Also Eq. (7) can predict the undrained stress paths and hence the pore pressures in normally and lightly overconsolidated clays.

The volumetric strains experienced by stress paths with states on the State Boundary Surface can be accurately determined using Eq. (6), while the shear strains as calculated from Eq. (5) are perfectly correct for the radial type of stress paths for which the stress ratio remain constant.

PENDER'S MODEL (1977, 1978)

This model employed the Critical State Concept and a constitutive relationship for a work hardening plastic material as the general framework, and deals with two distinct behavior of clay, one for an overconsolidated behavior (Pender, 1978) and the other for the normally consolidated behavior (Pender, 1977).

Assumptions and Hypotheses

While retaining most of the assumptions made in the Critical State Theory, the following aspects deviate from those.

1. Plastic strains occur within the State Boundary Surface. However, no unique State Boundary Surface is defined.

2. Two yield loci are engaged simultaneously when the stress path is directed outside the normally consolidated yield locus.

Hence, there exists three strains within the State Boundary Surface, i.e., the recoverable volumetric strain and the irrecoverable strains both volumetric and shear.

Summary of Pender's Model

Whereas the Cambridge theories are concerned with behavior for stress states on the State Boundary Surface, Pender's Model is concerned not only with the behavior for stress paths beneath the State Boundary Surface but also with the behavior for the normally consolidated states. This model's great benefit is that it requires only four soil parameters which are obtainable from common laboratory tests. This model predicts fairly well the strength in the normally consolidated state, and hence, the undrained stress path as well because of the nature of the assumption of undrained stress paths. However, for the heavily overconsolidated state, the undrained stress paths are not supported by the real behavior. Inspite of the drawbacks inherent in this model, it still provides the qualitative prediction of stress strain behavior of clays under the various initial stress conditions such as their behavior both in compression and in extension under the isotropic and anisotropic stress states. This model can also be extended to include the prediction of cyclic behavior of soils.

DAFALIAS' MODEL (1987)

This model can be considered as an extended version of the classical Critical State Theory from an isotropic case to an anisotropic one. To include the effects of anisotropy induced by one dimensional stress states or other factors, Dafalias proposed a work equation including one additional term in that adopted by the Modified Cam Clay Theory of Roscoe and Burland (1968) as follows:

$$dW = p\left[(d\varepsilon_{vp})^2 + (Md\varepsilon_{sp})^2 + 2\alpha\, d\varepsilon_{vp}\, d\varepsilon_{sp}\right]^{1/2} \qquad (8)$$

where α is the triaxial non-dimensional anisotropic variable accounting for the effect of internal residual stresses.

Summary of Dafalias' Model

This model deals with the behavior of normally consolidated clay under anisotropic stress conditons. The yield surface proposed in this model consists of a rotated and distorted ellipse. The degree of rotation and distortion are determined by the value of α.

A necessary condition of the work equation suggested is that:

$$2\alpha = 2\eta + (\eta^2 - M^2)(d\varepsilon_{sp} / d\varepsilon_{vp}) \tag{9}$$

If normality is assumed, α is given by

$$\alpha = f(M, p, q, \beta) \tag{10}$$

where β is the memory function for the initial condition.

Thus, any assumption on the algebraic form of α value has a direct impact on the shape of the yield surface. For instance, $\alpha = 0$ (Modified Cam Clay Theory, Eq. 2) is a special case of a more general form of α..

Dafalias did not extend this model to any type of incremental form of stress-strain behavior as is usually expected in the stress-strain theories.

SOME FORMULATION OF ANISOTROPIC BEHAVIOR OF CLAYS

Atkinson, et al. (1987) proposed a State Boundary Surface for one-dimensionally normally consolidated clays based on the experimental data on reconstituted Speswhite Kaolin. These authors normalized all the test data with respect to the equivalent pressure, p_e, and the equivalent specific volume, v_λ, in order to take account of the different initial states and the different stress paths. The boundary surface which is normalized by v_λ, for one-dimensionally normally consolidated stress history is given as:

$$\eta - \eta_o = (\pm M - \eta_o)(2 \exp[(\Gamma - v_\lambda) / (\lambda - \kappa)])^{1/2} \tag{11}$$

where

η_o	=	stress ratio during one-dimensional consolidation
Γ	=	specific volume intercept of the Critical State line plotted as v against $ln\, p'$ at $p' = 1.0$

v_λ	=	Equivalent specific volume, $v_\lambda = v + \eta \ln p'$ (v =specific volume)
M	=	stress ratio, q/p, at Critical State

The State Boundary Surface was simply obtained by replacing η with $(\eta - \eta_o)$ and M with $(M - \eta_o)$ as the procedures adopted by the Modified Cam Clay Theory, when $\eta_o = 0$.

COMMENTS ON VARIOUS MODELS

The energy balance equation proposed by Dafalias (1987) gives the most expression for the volumetric yield loci which lie on the State Boundary Surface. The Cam Clay Theory is the special version perhaps when the volumetric yielding is small, as it happens inside the State Boundary Surface. Therefore, the Cam Clay type of volumetric yield loci seem more reasonable to apply when the volumetric yield is small.

The volumetric yield loci, as obtained by Roscoe and Burland (1968), capture most of the behavior of normally consolidated clays as sheared from the isotropic stress state. With the suitable selection of α, possibly the Dafalias' Model (1987) can apply for the anisotropic behavior of clays for stress states on the State Boundary Surface.

Pender's Model (Pender, 1977, 1978) based on experimentally observed undrained stress paths as parabolic idealization and the Critical State value of p_{cs} (the value of p at the point on the critical state line). When the model combined with an empirical flow rule, most of the type adopted for the Cam Clay Theory seems rather more complicated in its formulation. However, this model seems to capture the bulk of the clay behavior exhibited when stress paths lie below the State Boundary Surface.

These works indicate a need for the stress zone below the SBS to be investigated more closely for the existence of sub-sets of volumetric yield loci which can cause compressive volumetric strain on the wet side of the Critical State and dilatant volumetric strain on the dry side of the Critical State. It would then be necessary to seek appropriate flow rules in the stress regions where such sub-sets of volumetric yield loci exist. This is the major aim of the experimental program.

EXPERIMENTAL INVESTIGATIONS

The behavior of normally and lightly overconsolidated clays was studied

through a comprehensive series of triaxial tests. All test series were conducted on samples with OCR values of 1.00, 1.24, 1.50, 1.78, 2.15 and 2.75.

Triaxial Consolidation Tests

In this study, the idealized K_o-swelling line was used for the preparation of overconsolidated specimens. It was defined in the p-q stress space as the straight line in which the stress increment ratio is 0.95 passing through the normally consolidated stress point. Eventually, all of the final consolidation stress points of overconsolidated specimens fell on this idealized straight K_o-line.

In order to justify the idealized actual K_o-swelling line, four swelling tests (as shown in Table 1) were carried out with the different swelling stress increment ratio, $|dq/dp|$ = 0.49, 0.82, 0.85, and 0.95 after K_o-consolidation. The schematic diagram of consolidation procedures is shown in Fig. 1. The slope of the consolidation and the swelling line in the e-$ln\ p$ plot were 0.357 and 0.081, respectively. These values were used for the preparation of overconsolidated clay specimens in the laboratory.

Triaxial Test Series

A systematic triaxial test consisted of three test series, namely, Test Series I, II and III. Test Series I and II were the anisotropic tests, while the remaining one was isotropic tests. All initial stress points of each test series are illustrated in Table 2 and Fig. 2.

Detailed testing schedule is tabulated in Table 3 and typical stress paths are shown in Fig. 3.

Table 1 Triaxial Consolidation Test

Test Name	Number of Test	Consolidation Stress Ratio η	Swelling Stress Ratio Increment $\lvert dq/dp \rvert$
CA-0.49	1	0.49	0.49
CA-0.82	1	0.49	0.82
CA-0.85	1	0.49	0.85
CA-0.95	1	0.49	0.95
CI-1,2,3	3	0	0
Total No. of Test	7		

Note :
CA = anisotropic Consolidation
CI = isotropic Consolidation

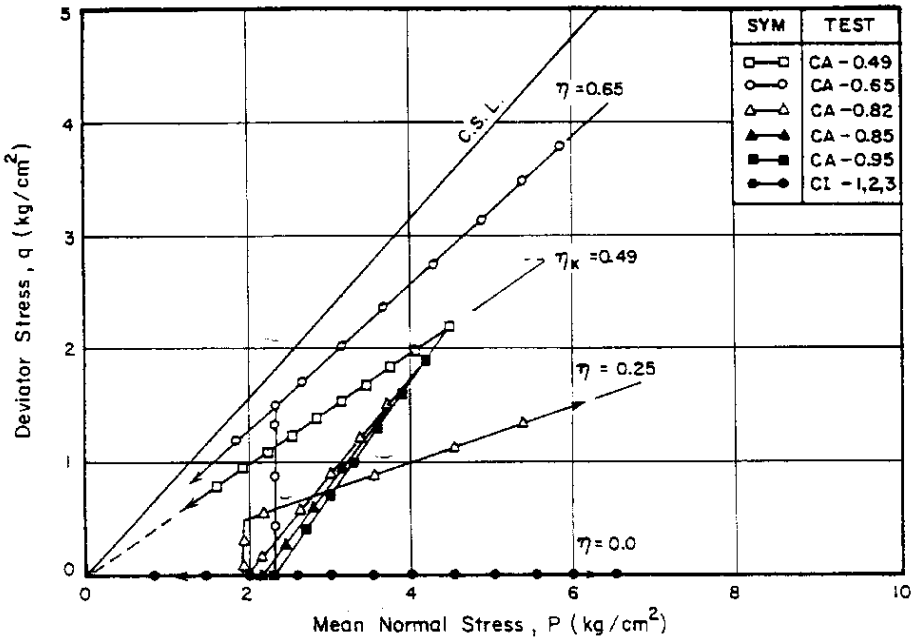

Fig. 1 Schematic Diagram of Consolidated Process

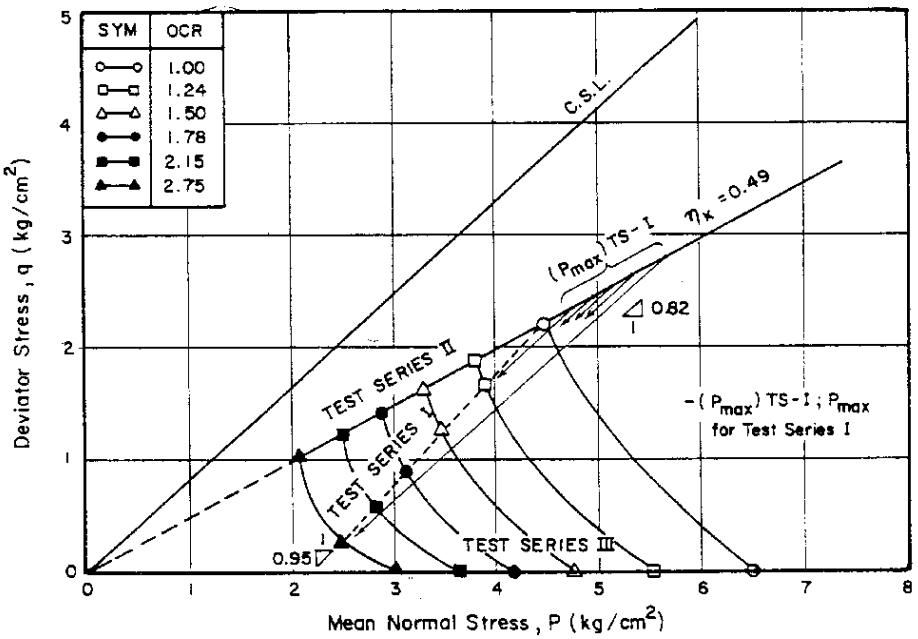

Fig. 2 Initial Stress Points of Test Series I, II, III

Table 2 Conditions for the Overconsolidated Samples in *q-p* Stress Space

OCR		TEST SERIES I					
OCR $(\bar{\sigma}_{vm} / \bar{\sigma}_{vo})$	$(OCR)_p$ p_{max} / p_o	p_{max} (kg/cm^2)	q_{max} (kg/cm^2)	p_o (kg/cm^2)	q_o (kg/cm^2)	K_o	η
1.00	1.00	4.500	2.205	4.500	2.205	0.63	0.49
1.24	1.20	4.690	2.298	3.908	1.657	0.67	0.42
1.50	1.40	4.857	2.380	3.469	1.242	0.71	0.36
1.78	1.60	5.006	2.453	3.129	0.914	0.76	0.29
2.15	1.85	5.174	2.535	2.797	0.586	0.82	0.21
2.75	2.20	5.382	2.637	2.446	0.230	0.91	0.09

OCR		TEST SERIES II					
OCR $(\bar{\sigma}_{vm} / \bar{\sigma}_{vo})$	$(OCR)_p$ p_{max} / p_o	p_{max} (kg/cm^2)	q_{max} (kg/cm^2)	p_o (kg/cm^2)	q_o (kg/cm^2)	K_o	η_o
1.00	1.00	4.500	2.205	4.500	2.205	0.63	0.49
1.24	1.24	4.725	2.315	3.811	1.867	0.63	0.42
1.50	1.50	4.934	2.418	3.289	1.612	0.63	0.49
1.78	1.78	5.129	2.513	2.882	1.412	0.63	0.49
2.15	2.15	5.354	2.623	2.490	1.220	0.63	0.49
2.75	2.75	5.661	2.774	2.059	1.009	0.63	0.49

OCR		TEST SERIES III					
OCR $(\bar{\sigma}_{vm} / \bar{\sigma}_{vo})$	$(OCR)_p$ p_{max} / p_o	p_{max} (kg/cm^2)	q_{max} (kg/cm^2)	p_o (kg/cm^2)	q_o (kg/cm^2)	K_o	η_o
1.00	1.00	6.500	0	6.500	0	1	0
1.24	1.24	6.840	0	5.516	0	1	0
1.50	1.50	7.155	0	4.770	0	1	0
1.78	1.78	7.451	0	4.186	0	1	0
2.15	2.15	7.792	0	3.624	0	1	0
2.75	2.75	8.260	0	3.004	0	1	0

Notes :

(OCR) = the ratio of the maximum past vertical stress to the pre-shear vertical stress

$(OCR)_p$ = the ratio of the maximum past mean normal stress to the pre-shear mean normal stress

Table 3 Test Conditions and Test Names With Test Series

TEST SERIES I				
Undrained Compression Tests	Undrained Extension Tests	Drained Compression Tests	Drained Extension Tests	Drained Extension Loading Tests
C *Ko* U1	C *Ko* UE1	C *Ko* D1	C *Ko* DE1	-
C *Ko* U2	C *Ko* UE2	C *Ko* D2	C *Ko* DE2	-
C *Ko* U3	C *Ko* UE3	C *Ko* D3	C *Ko* DE3	-
C *Ko* U4	C *Ko* UE4	C *Ko* D4	C *Ko* DE4	C *Ko* LE4
C *Ko* U5	C *Ko* UE5	C *Ko* D5	C *Ko* DE5	-
C *Ko* U6	C *Ko* UE6	C *Ko* D6	C *Ko* DE6	C *Ko* LE6
C *Ko* U1-2	C *Ko* UE1-2	-	-	-
Drained Compression Tests *p*-constant	Drained Extension Tests *p*-constant	Drained Loading Tests *q*-constant	Drained Unloading Tests *q*-constant	Drained Unloading Compression Tests
C *Ko* PC1	C *Ko* PE1	-	C *Ko* QU1	C *Ko* DU1
C *Ko* PC2	C *Ko* PE2	C *Ko* QL2	C *Ko* QU2	C *Ko* DU2
C *Ko* PC3	-	C *Ko* QL3	C *Ko* QU3	C *Ko* DU3
C *Ko* PC4	C *Ko* PE4	-	-	-
C *Ko* PC5	-	C *Ko* QL5	C *Ko* QU5	C *Ko* DU5
C *Ko* PC6	C *Ko* PE6	-	-	-

TEST SERIES II		TEST SERIES III	
Undrained Compression Tests	Drained Compression Tests	Undrained Compression Tests	Drained Compression Test
-	-	CIU1	CID1
CAU2	CAD2	CIU2	CID2
CAU3	CAD3	CIU3	CID3
CAU4	CAD4	CIU4	CID4
CAU5	CAD5	CIU5	CID5
CAU6	CAD6	CIU6	CID6
		*CIU7	
		CIU1-2	
		CIU1-3	

Notes :
(1) - : No Test is carried out.
(2) Numbers 1-6 in each test name denote OCRs of 1.0, 1.24, 1.5, 1.78, 2.15 and 2.75
(3) OCR for *CIU7 is 4.25 and p' for CIU1-2 & CIU1-3 are 6.0 and 5.0 kg/m^2

Compression Conditions

1. C*Ko*U, CAU, CIU = Conventional undrained compression tests on *Ko* anisotropically, and isotropically consolidated specimens, i.e., increasing the axial stress with keeping the constant cell pressure (Test Series I, II & III)
2. C*Ko*D, CAD, CID = Conventional drained compression tests on *Ko* anisotropically, and isotropically consolidated specimens. (Test Series I, II & III)
3. C*Ko*DU = Unloading drained compression tests on *Ko*-consolidated specimens, i.e., decreasing radial stress with keeping the axial stress constant. (Test Series I)
4. C*Ko*PC = *p*-constant drained compression tests on *Ko*-consolidated specimens. (Test Series I)
5. C*Ko*QL, C*Ko*QU = *q*-constant drained loading and unloading on *Ko*-consolidated specimens. (Test Series I)

Extension Conditions

6. C*Ko*UE, C*Ko*DE = Conventional undrained and drained extension tests on *Ko*-consolidated specimens. (Test Series I)
7. C*Ko*LE = Extension drained loading tests on *Ko*-consolidated specimens. (Test Series I)
8. C*Ko*PE = *p*-constant drained extension tests on *Ko*-consolidated specimens. (Test Series I)

INDEX PROPERTIES OF CLAY

The clay used in this study was retrieved from depths of between 3.0 m to 4.0 m (a narrow range) within the Asian Institute of Technology. The physical properties are summarized in Table 4.

Table 4 Index Properties

Natural Water Content (%)	78 – 85
Liquid Limit (%)	98
Plastic Limit (%)	37
Plasticity Limit (%)	61
Liquidity Index	0.67 – 0.79
Average Unit Weight (t/m^3)	1.51
Specific Gravity	2.69
Clay Content	70
Silt Content	24
Sand Content	6

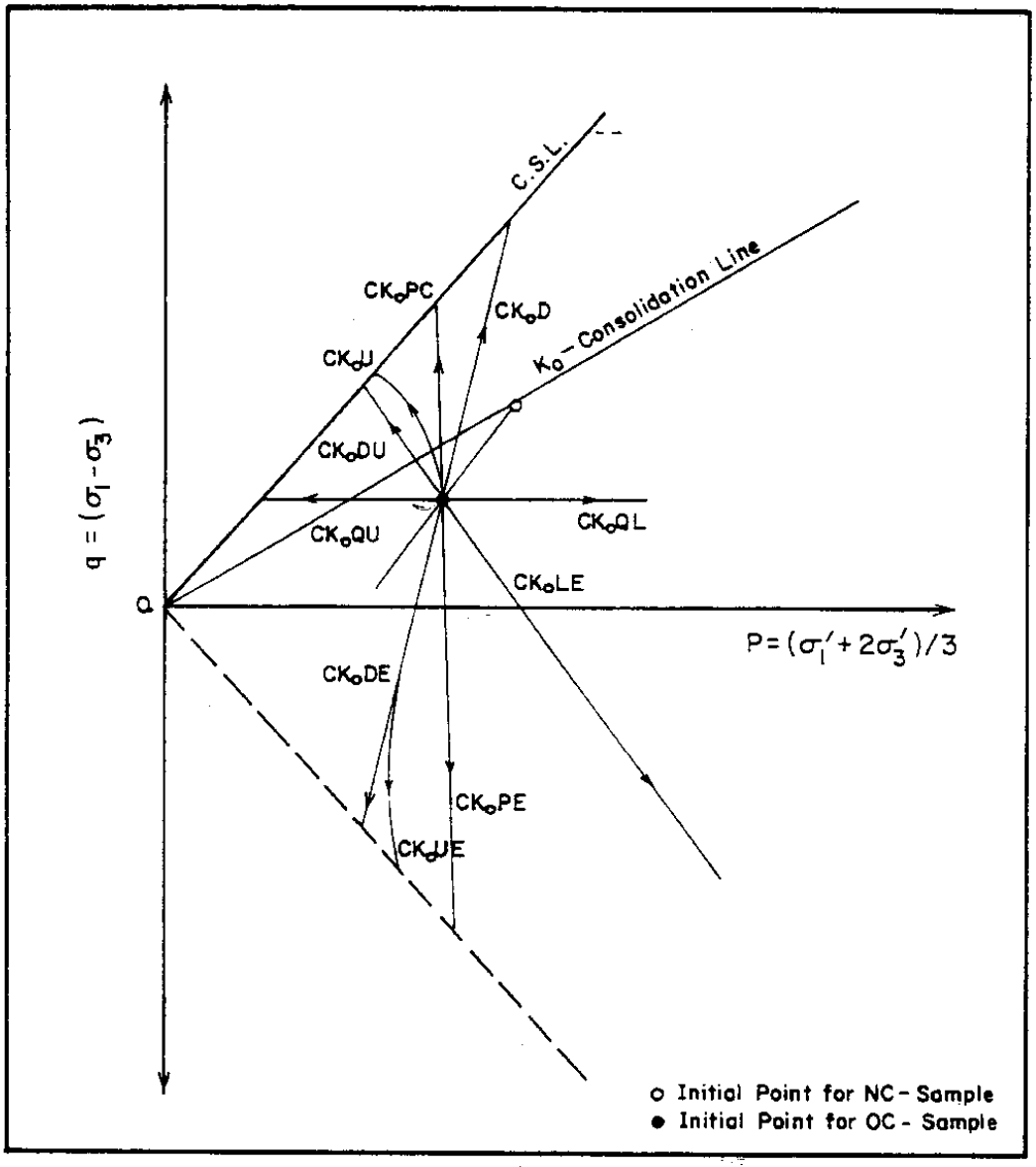

Fig. 3 Typical Stress Paths followed in this Investigation of the Behavior of Lightly Overconsolidated Clay (Test Series I)

CONSOLIDATION CHARACTERISTICS OF CLAYS

Isotropic and anisotropic consolidation and swelling tests were conducted to study the consolidation and swelling characteristics. The stress ratio, η, in p-q stress space for the K_o consolidation is 0.49 (since K_o is taken as 0.63) as determined by previous investigators at AIT (e.g., Chang, 1973; Chaiyadhuma, 1974; Wong, 1975).

Void Ratio- Pressure Relationships

The (*e, ln p*) relationships of the isotropically and K_o-consolidated specimens are shown in Fig. 4.

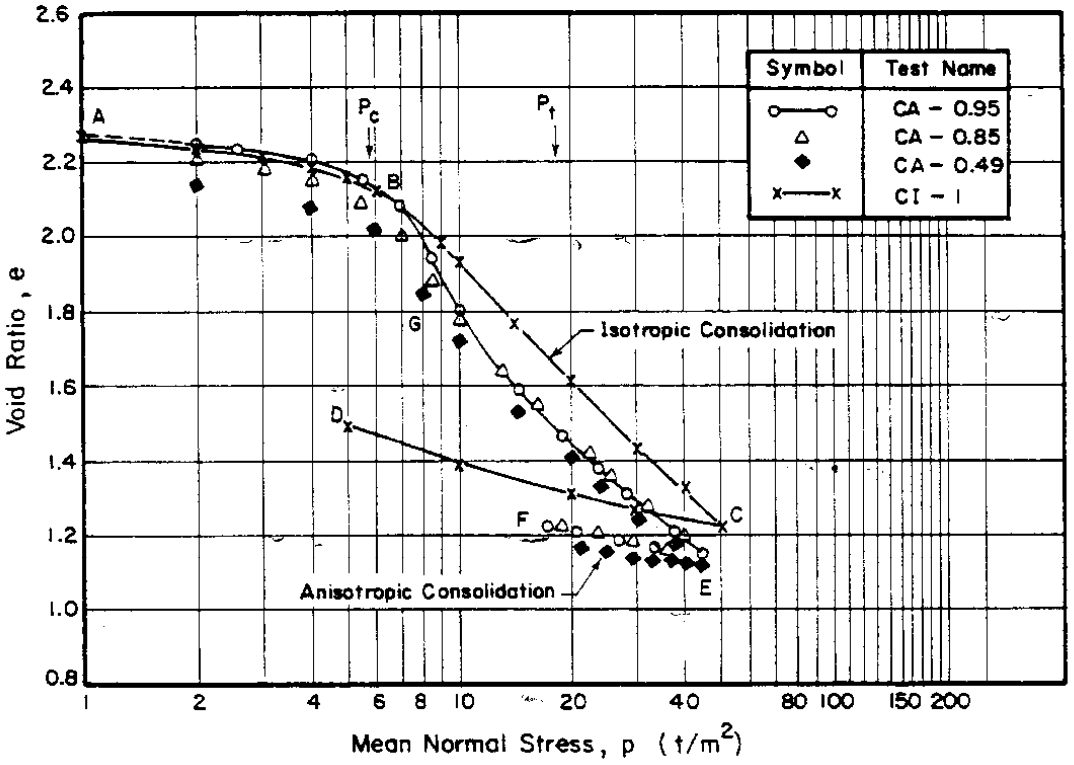

Fig. 4 Typical Consolidation and Swelling Curves

The ratio of (p_t / p_c) is about 3.3 and it seems that for consolidation pressures higher than about 3.3 times the initial pre-consolidation pressure, the effects of initial shear stress due to K_o consolidation can be erased. This value is quite close to that reported by Balasubramaniam (1969). The compression index, λ in the normally consolidated state at the higher stress level is 0.357 and this value together with the appropriate κ value of 0.081 was used in the preparation of overconsolidated samples.

Swelling Characteristics

The plastic volumetric strain ratio Λ, defined as $\Lambda = (1- \kappa/\lambda)$ will be 0.77. This value is close to the value of 0.75-0.85 and 0.78 as reported by Ladd et al. (1977).

COMPARISON OF UNDRAINED BEHAVIOR OF NORMALLY CONSOLIDATED CLAYS UNDER K_o PRE-SHEAR STRESS CONDITIONS

Effective Stress Paths

The undrained stress paths for the K_o-normally consolidated state from different stress strain models are superimposed with the experimental data in Fig. 5.

1. **Triaxial Compression** - All model predictions deviate from the experimental observations, particularly, they fail to simulate the undrained stress path in the early stages of the loading. As stated earlier, these deviations may result since the sample exhibits the effects of secondary consolidation (gained during the consolidation phase in the sample preparation), while the models are not expected to include such time effects. Pender's Model seems to predict successfully the undrained stress path.

2. **Triaxial Extension** - All model predictions during the extension phase above the p-axis (i.e., $0 < \eta < \eta_\tau$) are reasonably coincident with the experimental observations. In the pure extension stress state, when η is less than zero, all model predictions deviate from the experimentally observed stress path. Pender's Model gives the best prediction for the extension condition. As reviewed studies, the mean effective normal stress for low plasticity clay during the extension phase is drastically reduced.

Stress-Strain Relationships

The stress-strain curves are shown in Figs.6a, 6b, and 7.

1. **Triaxial Compression** - The prediction provided with the approach reported by Atkinson et al. (1987) significantly underpredicts the strength as expected from an undrained stress path. Pender's Model also underpredicts the strength and overpredicts the shear strain. However, the predictions obtained from Dafalias' Model agree well with the experimental data. It seems all models fail to simulate the stress-strain relationship in the early stages of loading; probably, because the experimental observations include time effects due to secondary consolidation.

2. **Triaxial Extension** - In the case of the extension tests, all model predictions agree well with the experimental data up to the p-axis. However, there are tendencies for Pender's Model to give larger strains, while the other two models give smaller shear strains at the same deviator stress level. Except Pender's Model, the other two models are relatively rigid in distortion for stress states less than the peak deviator stress. Upon approaching the peak deviator stress, these models show an abrupt increase in shear distortion similar to those patterns seen

in the compression tests. Generally, Pender's Model, although it overpredicts the shear strains, reveals a qualitatively reasonable stress-strain relationship in the extension phase when the model parameters are determined from the peak deviator stress conditions (Fig.7).

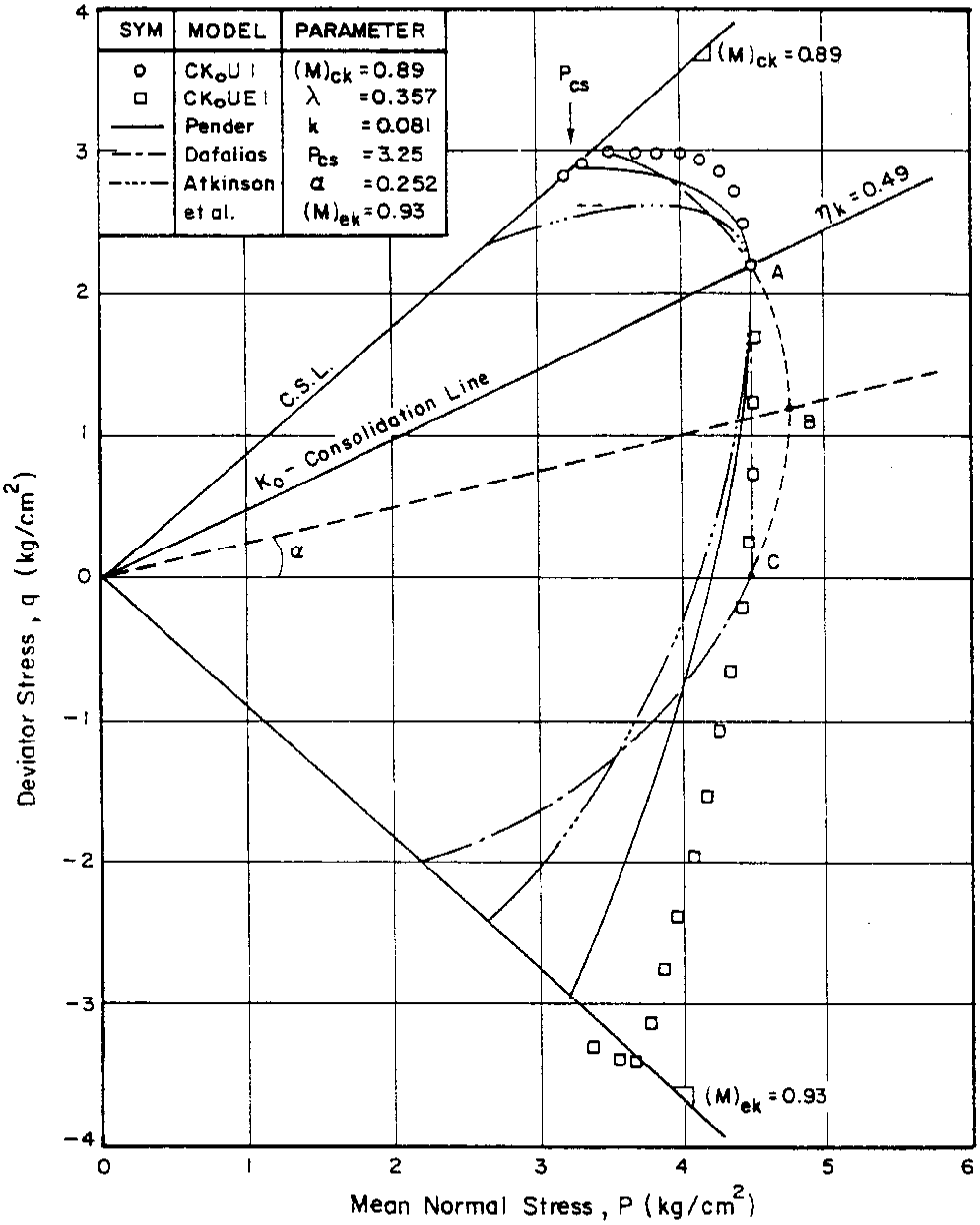

Fig. 5 (*q/p*) Plot for K_o-normally Consolidated Samples Compared with Predictions from Pender's Model (Critical State)

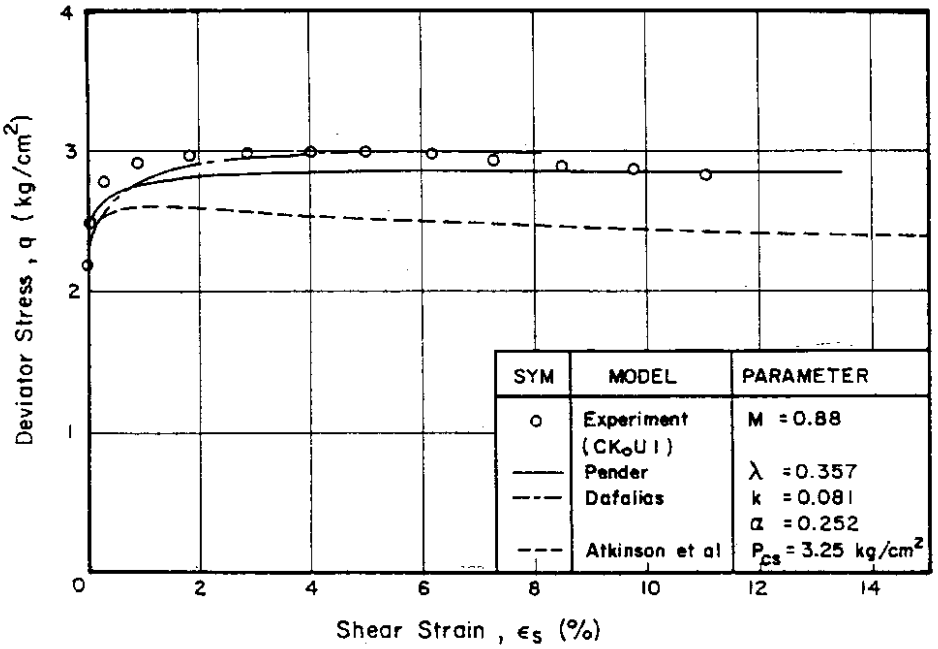

Fig. 6a (q/ε_s) Plot for CK_oU1 Sample Compared with the Model Predictions in Compression Conditions

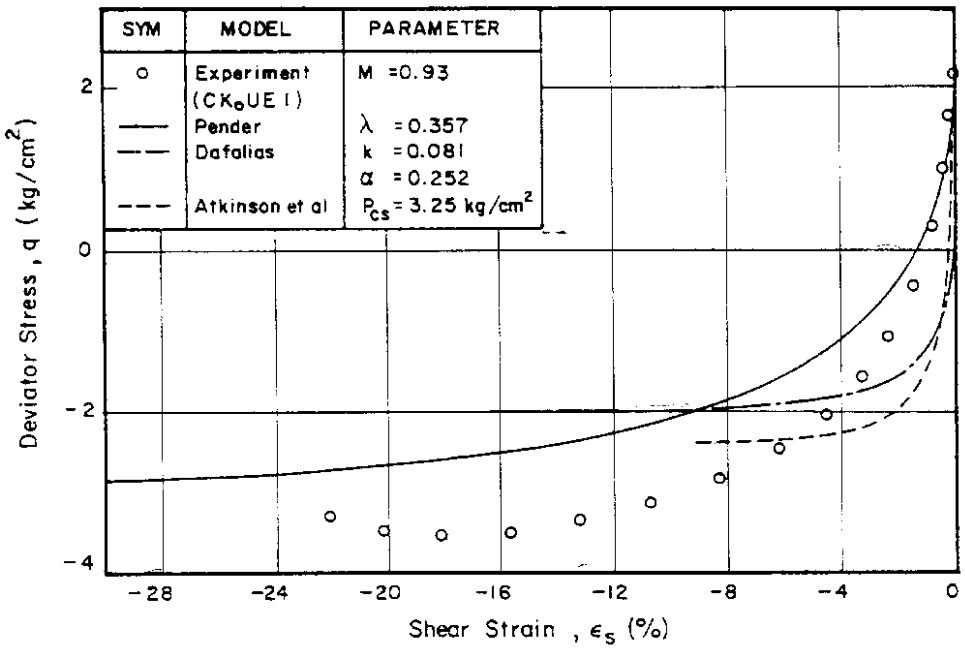

Fig. 6b (q/ε_s) Plot for CK_oUE1 Sample Compared with the Model Predictions in Compression Conditions

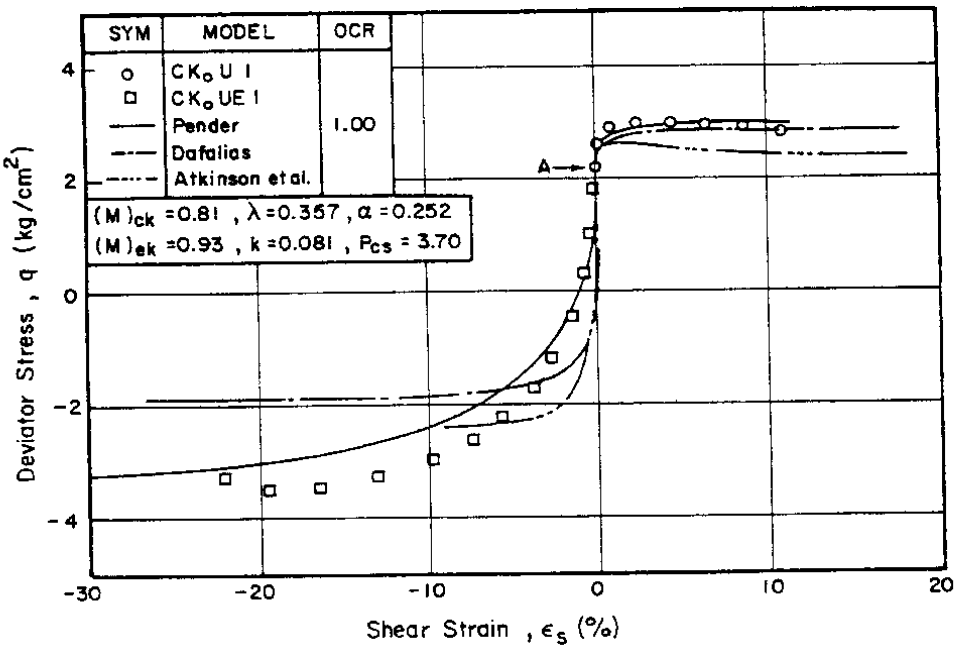

Fig. 7 (q/ε_s) Plot for K_o-normally Consolidated Samples Compared with the Model Predictions (Peak Stress State)

Excess Pore Pressure

Figures 8 and 9 present the excess pore pressure versus shear strain relationships. All models predict roughly the same (u, ε_s) relationship up to a shear strain of about 3% in the compression phase. Thereafter, they deviate from each other with increasing shear strains. All models tend to overpredict the excess pore pressures.

The patterns of model prediction in the extension phase are quite different from those in compression. Pender's Model gives a trend similar to the experimental data, although it underpredicts the negative pore pressures. This tendency is improved when the parameters are determined from the peak deviator stress condition. On the other hand, Dafalias' and Atkinson's Models predict positive pore pressures when the strain exceeds about 2%.

Variation of Parameter α in Dafalias' Model (1987)

The magnitude of α determines the shape of the volumetric yield locus and the undrained stress path for the K_o-consolidated clays. However, it is not a necessary condition that α should remain constant during the subsequent shear.

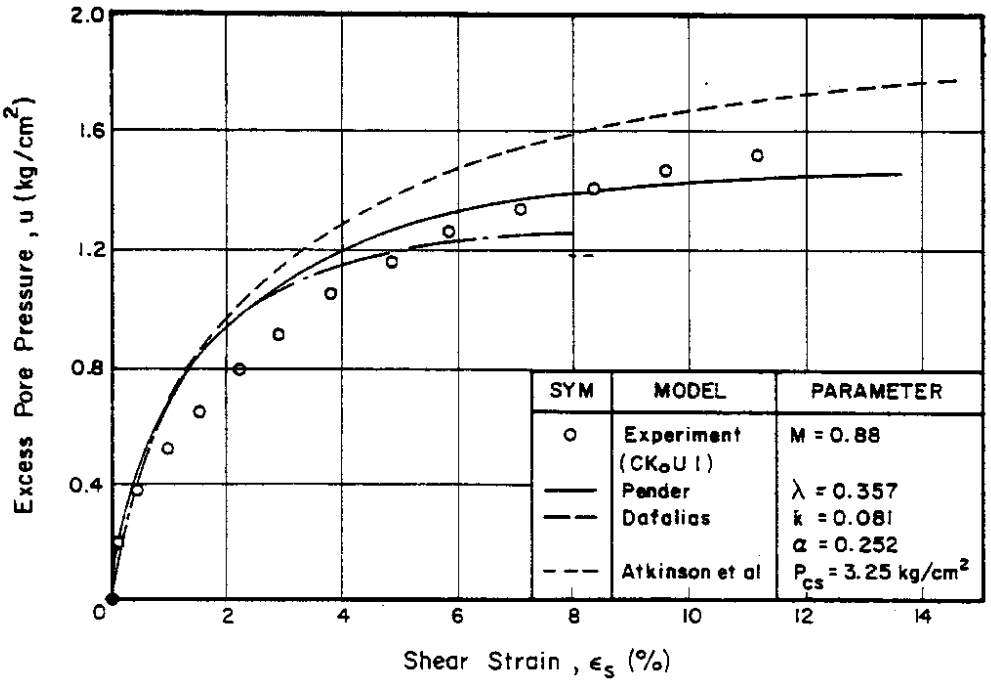

Fig. 8 (u/ε_s) Plot for CK_oU1 Sample Compared with the Model Predictions

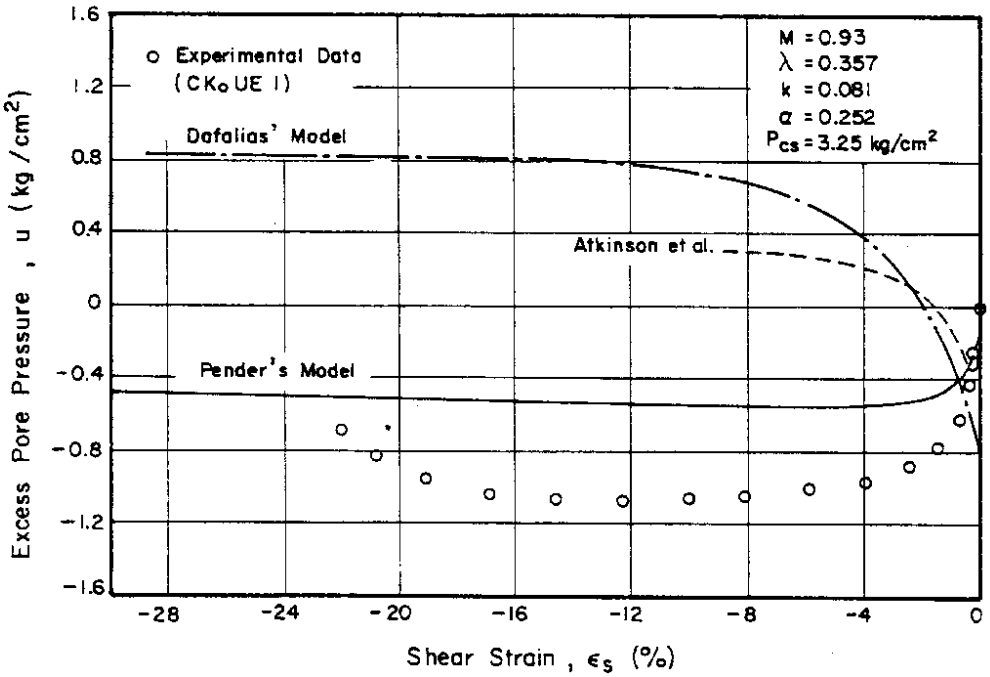

Fig. 9 (u/ε_s) Plot for CK_oUE1 Sample Compared with the Model Predictions

The α values are plotted with respect to the stress ratio in Fig. 10. The α values for the K_o-normally consolidated samples remain constant up to a stress ratio of about 0.7, thereafter, it increases rapidly as the stress ratio is increased. The variation of α values with stress ratio for the K_o-overconsolidated samples during the undrained compression tests are plotted in Fig 11 It can be concluded from these figures that the value of α varies during shear.

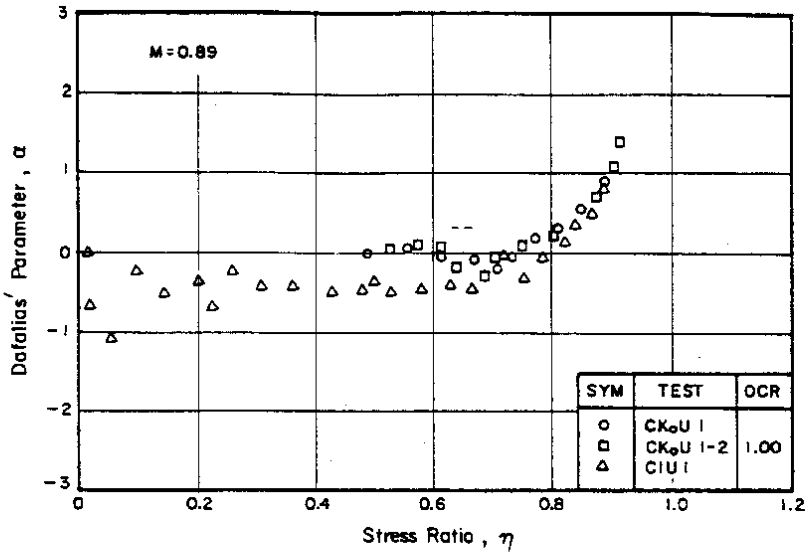

Fig. 10 Variations of Dafalias' α-parameter with the Stress Ratio for K_o-normally Consolidated Samples

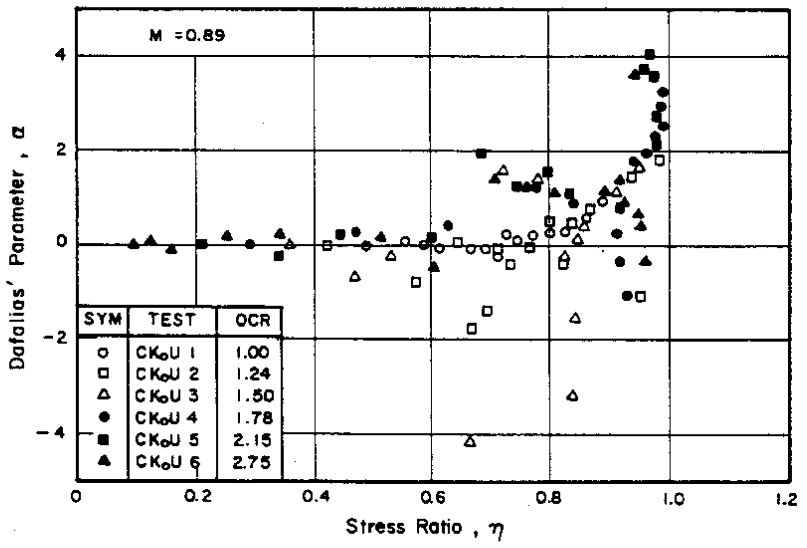

Fig. 11 Variations of Dafalias' α-parameter with the Stress Ratio for K_o-overconsolidated Samples

COMPARISON OF UNDRAINED BEHAVIOR OF OVERCONSOLIDATED SAMPLE SHEARED FROM *Ko* PRE-SHEAR STRESS CONDITIONS

In this section, Pender's Model on overconsolidated undrained behavior is examined both in compression and in extension.

Triaxial Compression

1. **Undrained Stress Paths** - The undrained stress paths predicted by Pender's Model for the K_o-overconsolidated samples are superimposed With the test data in Fig. 12. Model parameters used for the calculations are determined from the Critical State condition. The deviations are pronounced as the overconsolidation ratio is increased.

2. **Stress-Strain Relationships** - The stress-strain relationships are compared with test data in Fig. 13. Pender's Model overpredicts the strain. This trend becomes pronounced as the overconsolidation ratio is increased.

3. **Excess Pore Pressure-Strain Relationships** - Figure 14 shows the pore pressure-shear strain relationships. The figure indicates that the Pender's Model underpredicts the excess pore pressure at certain levels of shear strains. The deviations are particularly substantial for the samples sheared from the dry side of the Critical State. This may indicate that the assumption made on the plastic strain increment ratio in the theory is not realistic.

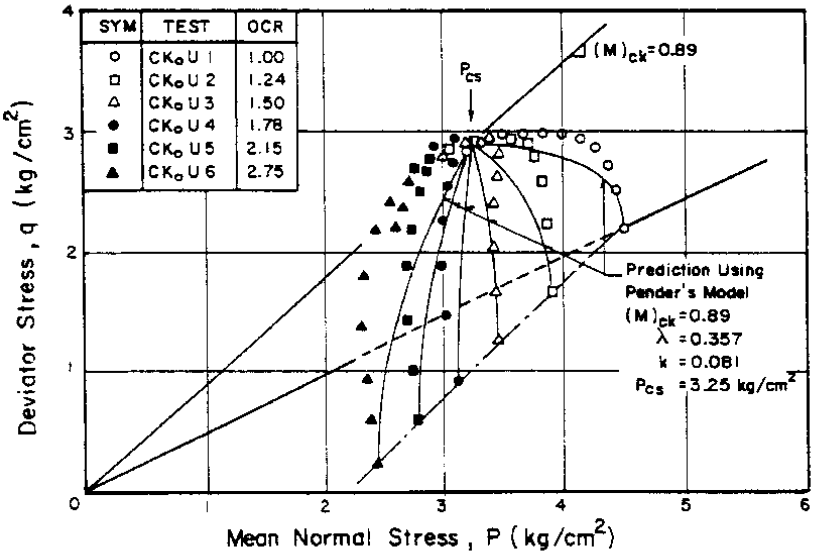

Fig. 12 (q, p) Plot for K_o-overconsolidated Samples Compared with the Predictions from Pender's Model (Critical State)

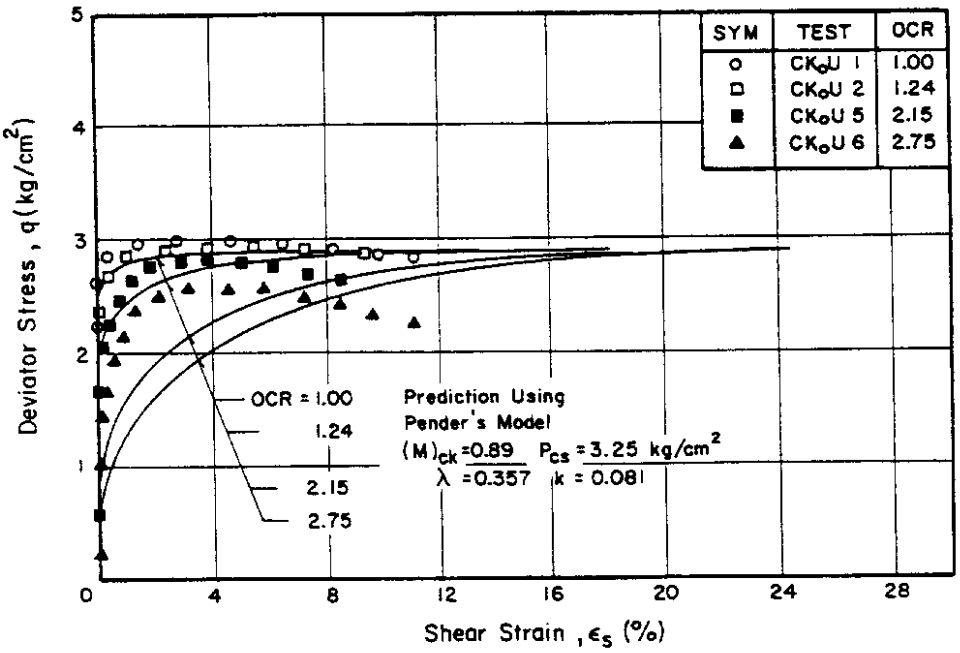

Fig. 13 (q, ε_s) Plot for K_o-overconsolidated Samples Compared with the Predictions from Pender's Model (Critical State)

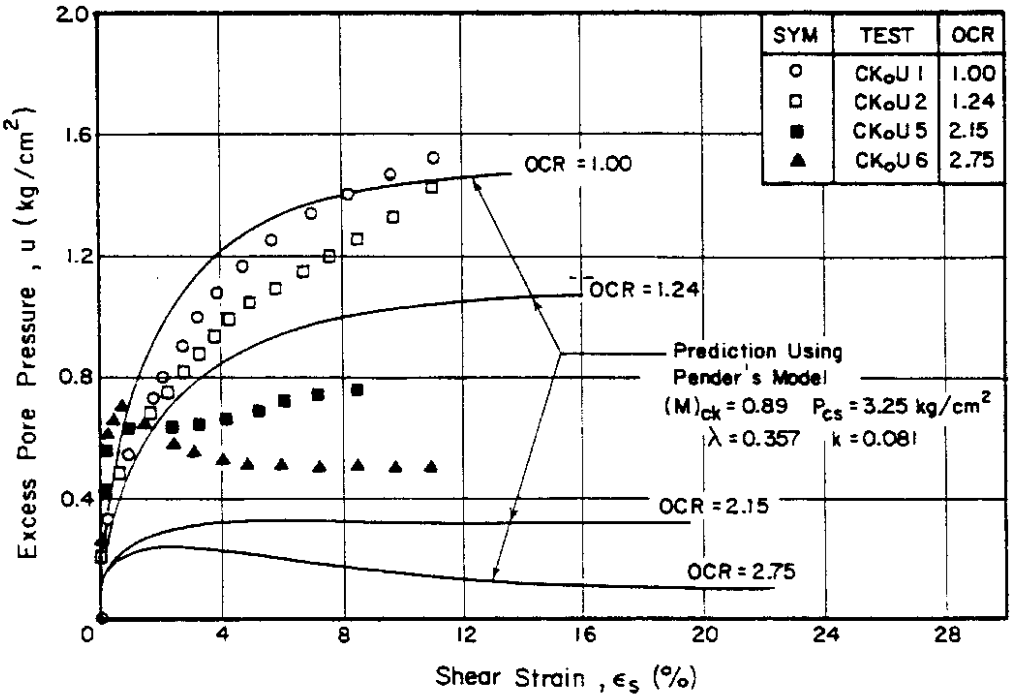

Fig. 14 (u, ε_s) Plot for K_o-overconsolidated Samples Compared with the Predictions from Pender's Model (Critical State)

Triaxial Extension

1. **Undrained Stress Paths** - Figure 15 displays the undrained stress paths in the extension tests together with those in the compression tests. The parameters used in the predictions are determined from the peak deviator stress condition The model predictions for the extension conditions is better than those for the compression conditions, especially, for the samples sheared from the dry side of the Critical State.

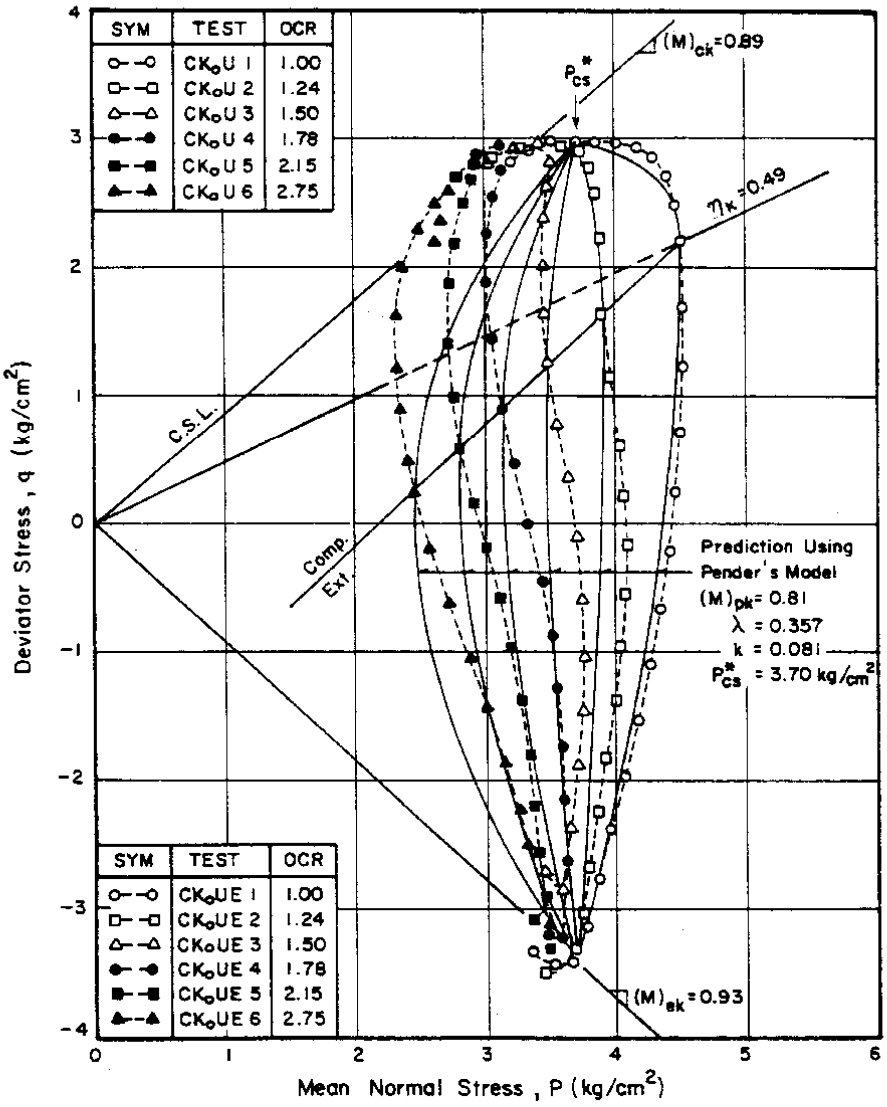

Fig. 15 (q, p) Plot for K_o-overconsolidated Samples both in Compression and in Extension Conditions Compared with the Predictions from Pender's Model (Peak Stress State)

2. **Stress-Strain Relationships** - The stress-strain relationships for the extension conditions are compared in Fig. 16. The model predicts the stress-strain relationships well.

3. **Excess Pore Pressure-Strain Relationships** - The relationship between the excess pore pressure and the shear strains are shown in Fig. 17. The Model predicts successfully the excess pore pressure for the normally consolidated sample in the extension condition. However, the predictions are not good for the overconsolidated samples.

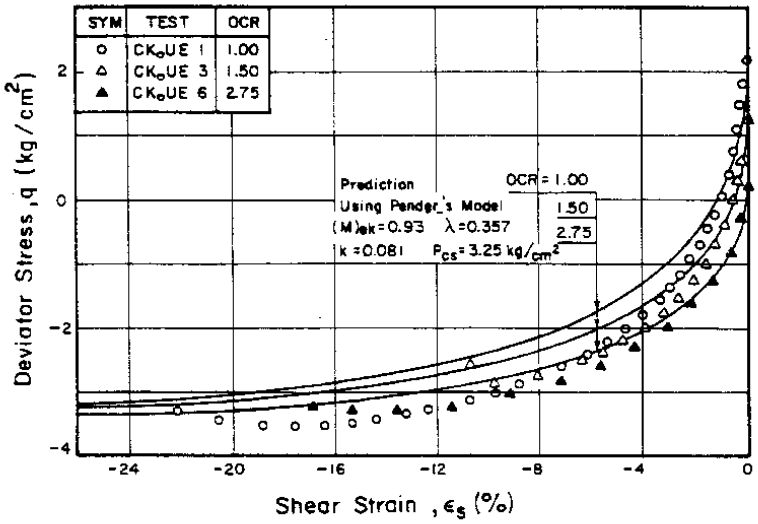

Fig. 16 (q, ε_s) Plot for K_o-overconsolidated Samples in Extension Conditions Compared with the Predictions from Pender's Model

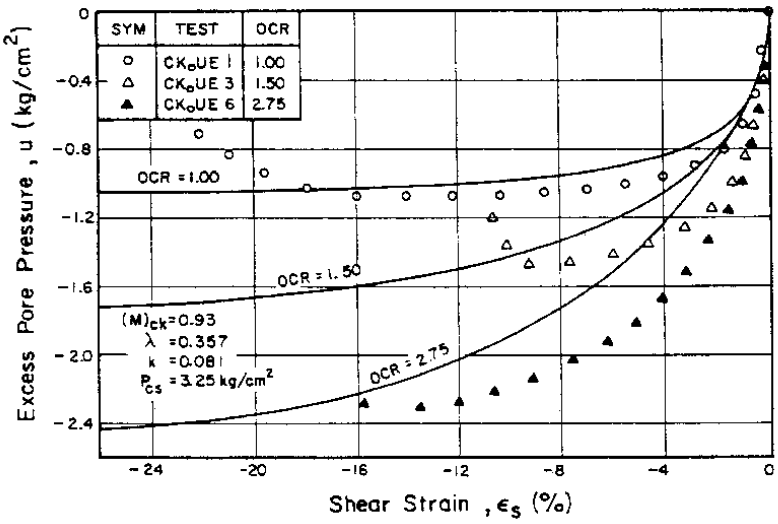

Fig. 17 (u, ε_s) Plot for K_o-overconsolidated Samples in Extension Conditions Compared with the Predictions from Pender's Model

COMPARISON OF DRAINED BEHAVIOR OF NORMALLY CONSOLIDATED CLAYS SHEARED FROM K_o PRE-SHEAR STRESS CONDITIONS

Drained test data from K_o-normally consolidated samples are compared with model predictions. The models used in the comparisons are Pender's Model (Pender, 1977, 1978), Dafalias' Model (Dafalias, 1987) and Atkinson's Model (Atkinson, et al., 1987).

Triaxial Compression

1. **Stress-Strain Relationships** - Figures 18 and 19 show the (q, ε_s) and (q, ε_v) relationships for the CK_oD1 specimen, respectively. The prediction of shear strain from all the models is good up to deviator stress of 3.2 kg/cm^2. During further increase in deviator stress, the predictions using Pender's and Atkinson's Models deviated from the test data, while those using Dafalias' Model are surprisingly close to the experimental observations. Dafalias' Model also gives an excellent prediction of volumetric strains, while the other two models overpredict the volumetric strains.

2. **Plastic Volumetric Strain-Plastic Shear Strain Relationships** - Figure 20 illustrates the (ε_{vp}, ε_s) relationship for the K_o-normally consolidated sample (CK_oD1 sample) during the conventional loading test. The prediction using Dafalias' Model agrees well with the test data. Pender's Model predicts smaller plastic volumetric strains, while Atkinson's Model predicts larger ones at the same level of shear strain.

Triaxial Extension

The behavior of normally consolidated sample becomes overconsolidated immediately after applying the conventional type of unloading. Therefore, CK_oDE1 state paths no longer lie on the State Boundary Surface (SBS). Only Pender's Model is applicable for predictions under this condition, since it deals with the overconsolidated behavior.

Stress-Strain Relationships - The comparisons of (q, ε_s) relationships between test data and model predictions during the conventional drained unloading test on K_o-normally consolidated samples are shown in Fig. 21. Pender's model makes an excellent prediction during unloading in compression. The predictions for the stress states below the p-axis begin to deviate from the test data. The other two models give smaller strain than the test data up to the deviator stress close to failure.

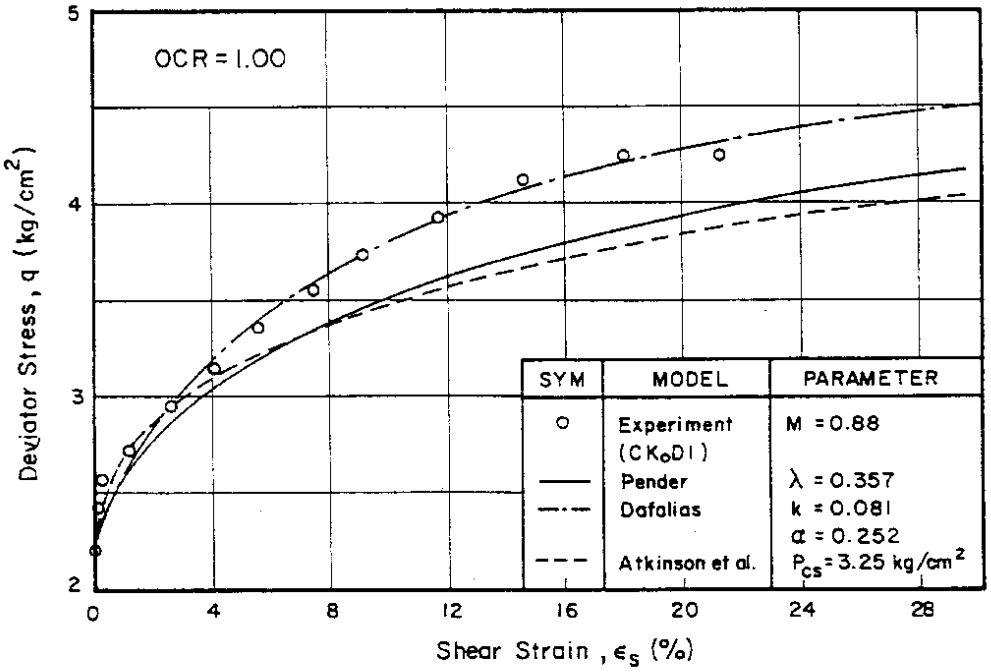

Fig. 18 (q, ε_s) Plot for CK_o1 Sample Compared with the Model Predictions

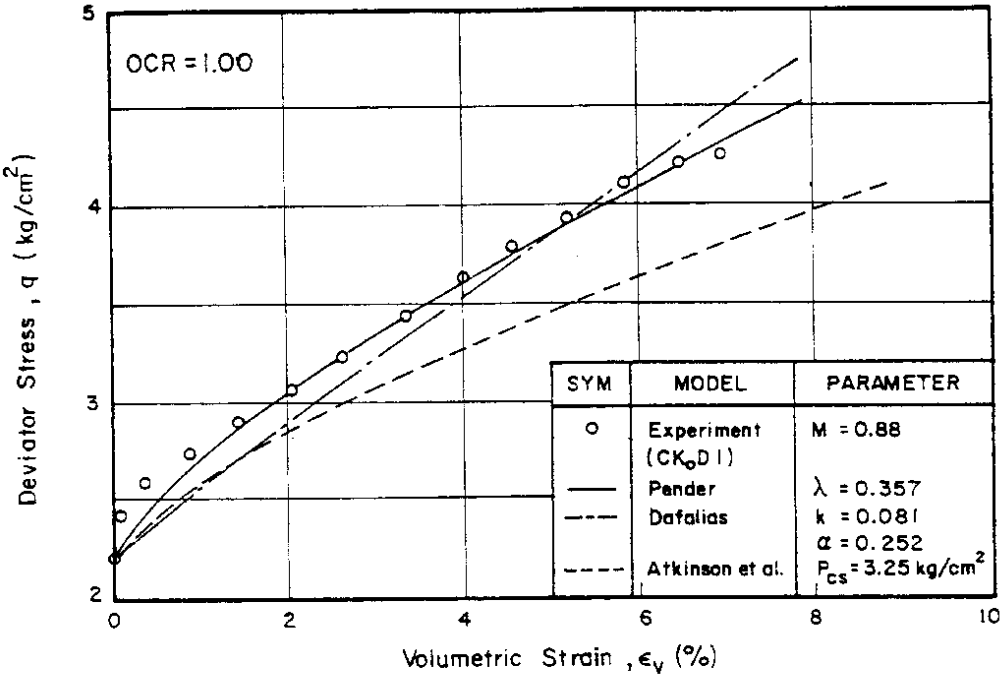

Fig. 19 (q, ε_v) Plot for CK_o1 Sample Compared with the Model Predictions

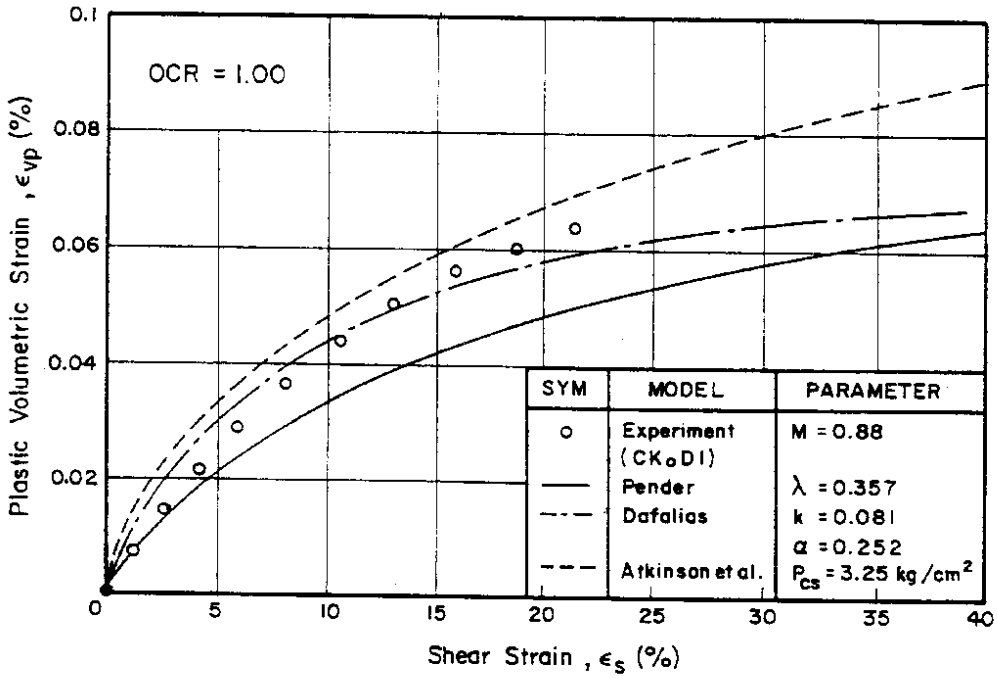

Fig. 20 (ε_{vp}, ε_{xp}) Plot for CK_o1 Sample Compared with the Model Predictions

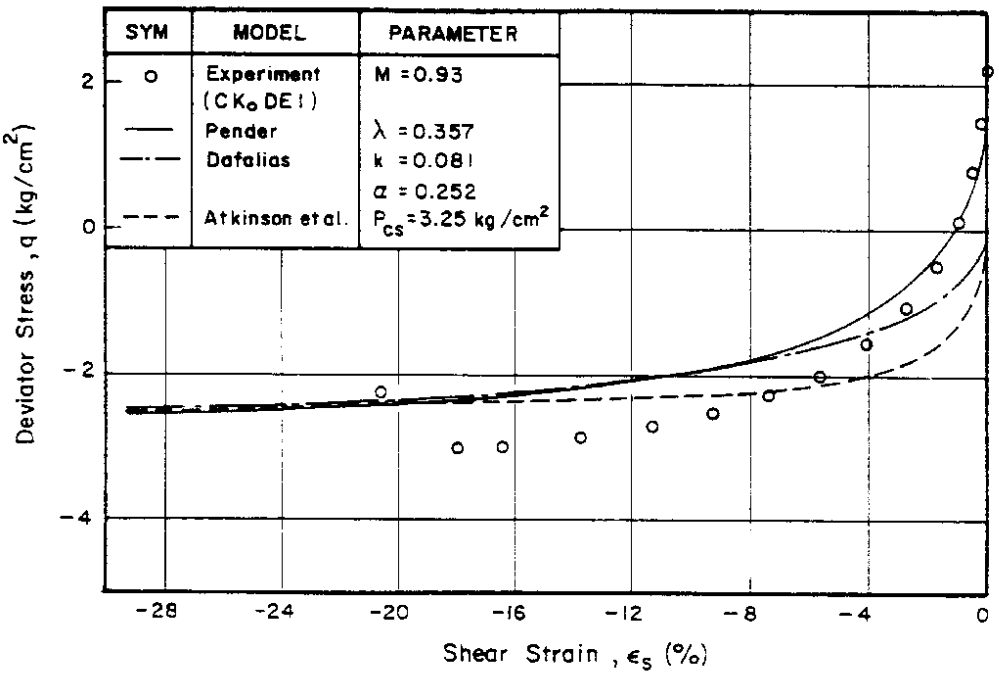

Fig. 21 (q, ε_s) Plot for CK_oDE1 Sample Compared with the Model Predictions

COMPARISON OF DRAINED BEHAVIOR OF OVERCONSOLIDATED CLAYS SHEARED FROM K_o PRE-SHEAR STRESS CONDITIONS

The behavior from a total of six series of drained tests on K_o-overconsolidated samples are compared with the predictions using Pender's Model (Pender, 1978).

Conventional Drained Compression Tests (CK_oD Tests)

Stress-Strain Relationships - Figures 22 and 23 illustrate the stress-strain relationships for CK_oD tests. Pender's Models overpredict shear strain (Fig. 23). The deviations increased with the increase in OCR values. In the case of the deviator stress-volumetric strain relationship demonstrated in Fig. 23, the prediction is improved except for the sample sheared from the dry side of the Critical State (CK_oD6).

Constant p Compression Tests (CK_oPC Tests)

Stress-Strain Relationships - Figures 24 and 25 present the (q, ε_s) and the (q, ε_v) relationships predicted using Pender's Model together with the corresponding test data. The model predictions agree well with test data from CK_oPCl and CK_oPC2 specimens. On the other hand, larger shear strains for the sample with higher OCR values at the same level of deviator stress are given (see CK_oPC4 and CK_oPC6 samples). Deviations especially begin to occur from a very early stage of shearing. A similar contrast for the (q, ε_v) relationships is also seen in Fig. 25. This again indicates that Pender's Model can provide good predictions of volumetric strain for overconsolidated samples.

Unloading Compression Test (CK_oDU Test)

Stress-Strain Relationships - All the stress states for these overconsolidated samples lie within the State Boundary Surface. Figures 26 and 27 illustrate the (q, ε_s) and (q, ε_v) relationships, respectively. The predictions deviate substantially from the observed behavior when the OCR values increased. Pender's Model overpredicts shear strain for the overconsolidated samples. However, the predictions for the sample with a OCR value of 1.24 agree well with the observations. Figure 27 provides further evidence that Pender's Model can successfully predict volumetric strains for overconsolidated specimens.

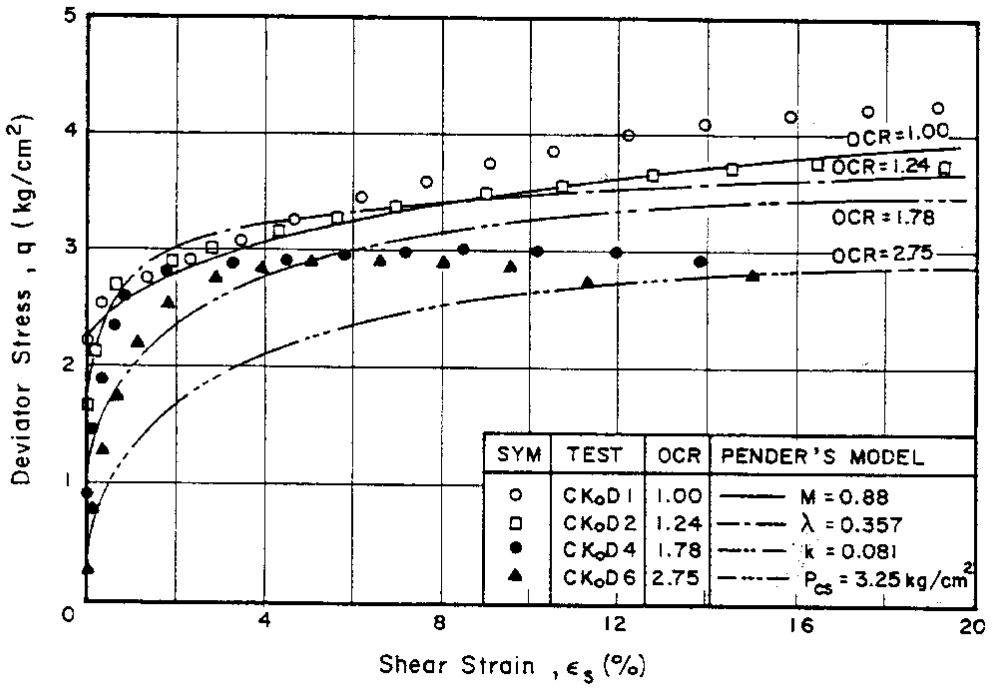

Fig. 22 (q, ε_s) Plot for CK_oD Samples Compared with the Model Predictions

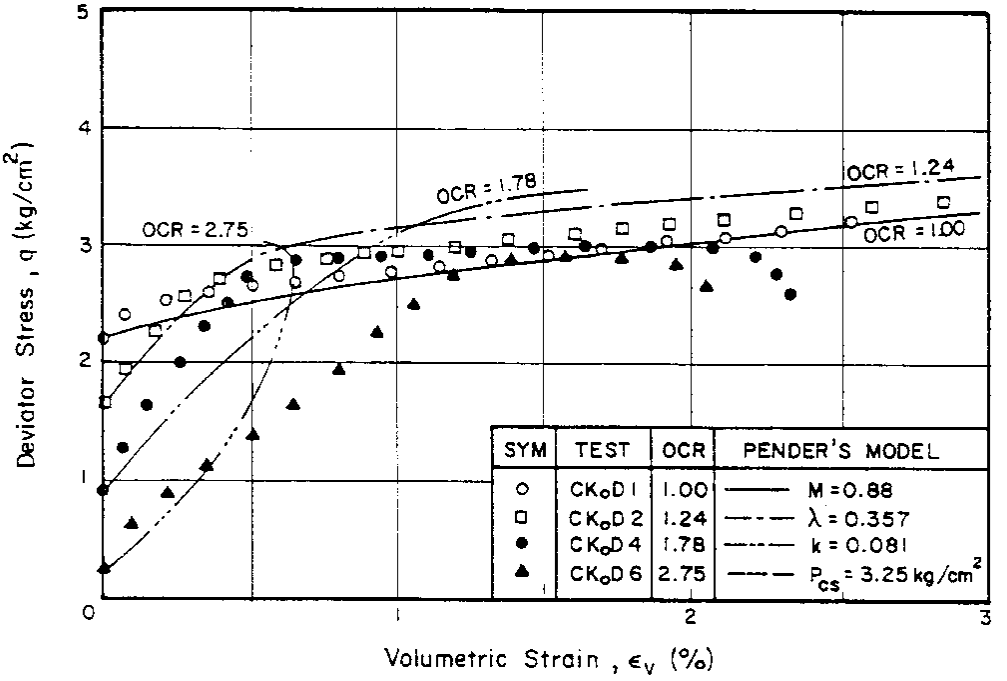

Fig. 23 (q, ε_v) Plot for CK_oD Samples Compared with the Model Predictions

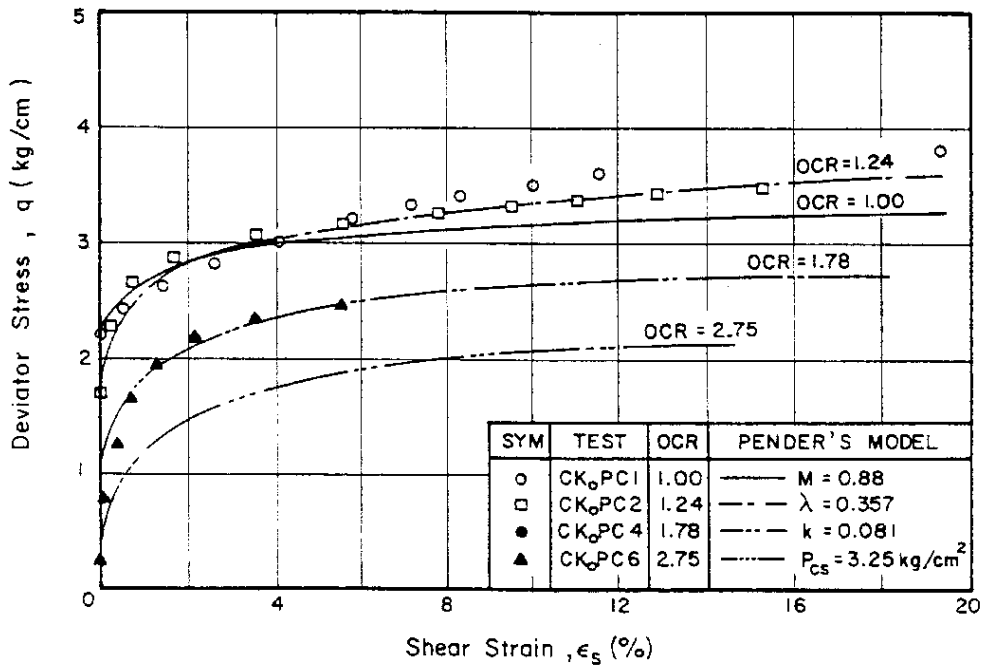

Fig. 24 (q, ε_s) Plot for CK_oPC Samples Compared with the Model Predictions

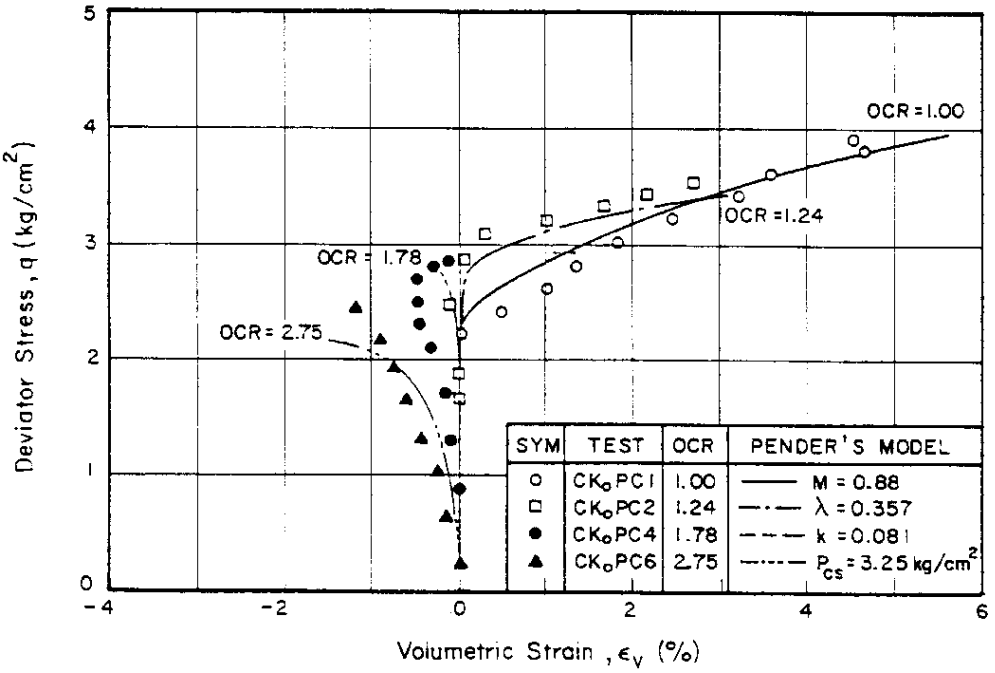

Fig. 25 (q, ε_v) Plot for CK_oPC Samples Compared with the Model Predictions

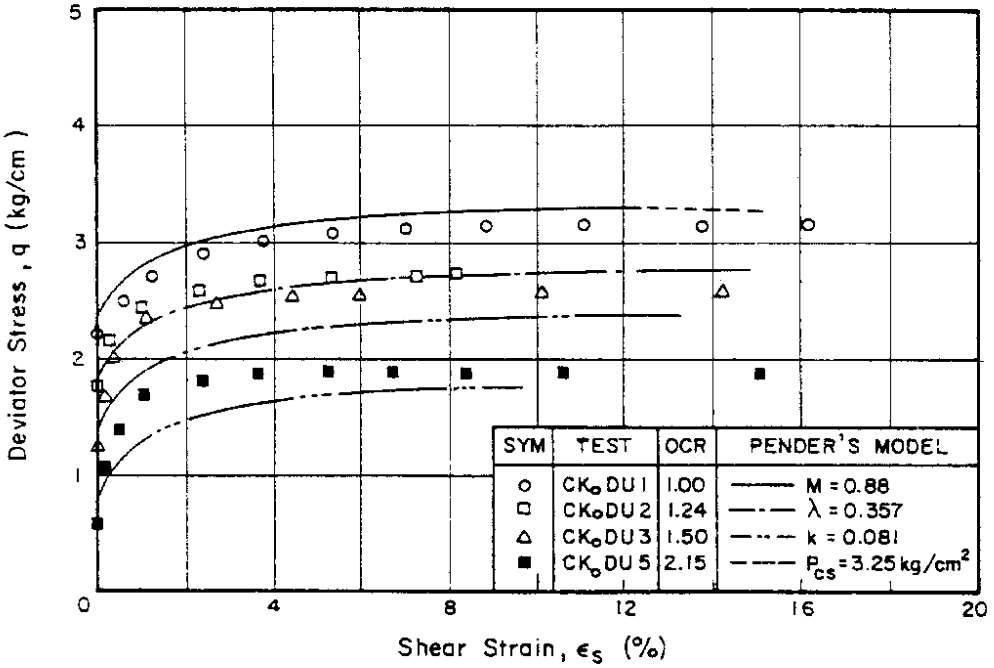

Fig. 26 (q, ε_s) Plot for CK_oDU Samples Compared with the Model Predictions

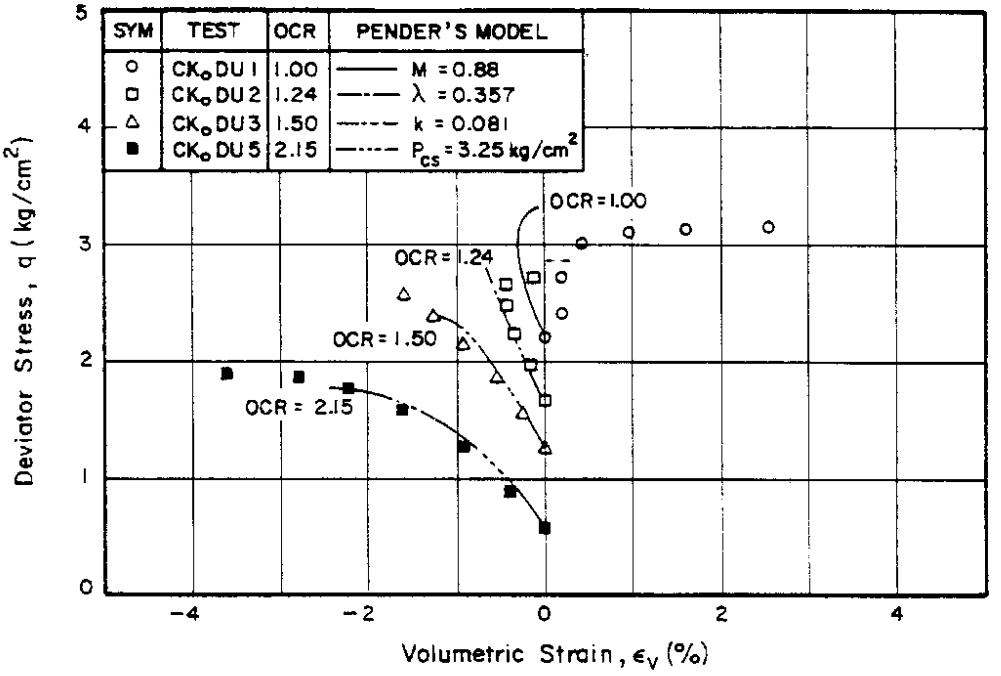

Fig. 27 (q, ε_v) Plot for CK_oDU Samples Compared with the Model Predictions

Constant q Unloading Tests (CK_oQU Tests)

All the state paths of the samples belonging to this test series lie within the State Boundary Surface. The M values used for the calculations are 1.16,1.18, 1.28 and 1.48 corresponding to samples CK_oQU1, CK_oQU2, CK_oQU3 and CK_oQU5, respectively.

Stress-Strain Relationships - Figure 28 contains the (ε_v, p) relationships of the CK_oQU test series compared with the predictions. The predictions for volumetric strains agree well with the experimental observations.

Conventional Drained Extension Tests (CK_oDE Tests)

All the state paths of these samples lie within the State Boundary Surface.

Stress-Strain Relationships - Figures 29 and 30 display (q, ε_s) and (q, ε_v) relationships, respectively. The shear strains predicted agree well with test data up to a deviator stress of -1.0 kg/cm^2. Below this deviator stress level, the predicted shear strains are larger for the CK_oDE1 and CK_oDE2 samples. For the CK_oDE4 and CK_oDE5 samples, the predictions agree well with the test data Unlike the case of the compression tests, the predictions of the volumetric strain for the extension tests deviate as shown in Fig. 30. However, the predictions are acceptable for a first order approximation.

Constant p Extension Tests (CK_oPE Tests)

Stress-Strain Relationships - Figures 31 and 32 demonstrate the variations of the shear strain and the volumetric strains with respect to the deviator stress. The predictions are better for the samples with higher OCR values. This indicates that Pender's Model overpredicts shear strain as the initial state of the sample is close to the normally consolidated state. In the case of overconsolidated samples, the predicted shear strains are larger probably after the yielding took place. The (q,ε_v) relation predicted using Pender's Model deviates from all the actual observations as shown in Fig. 32.

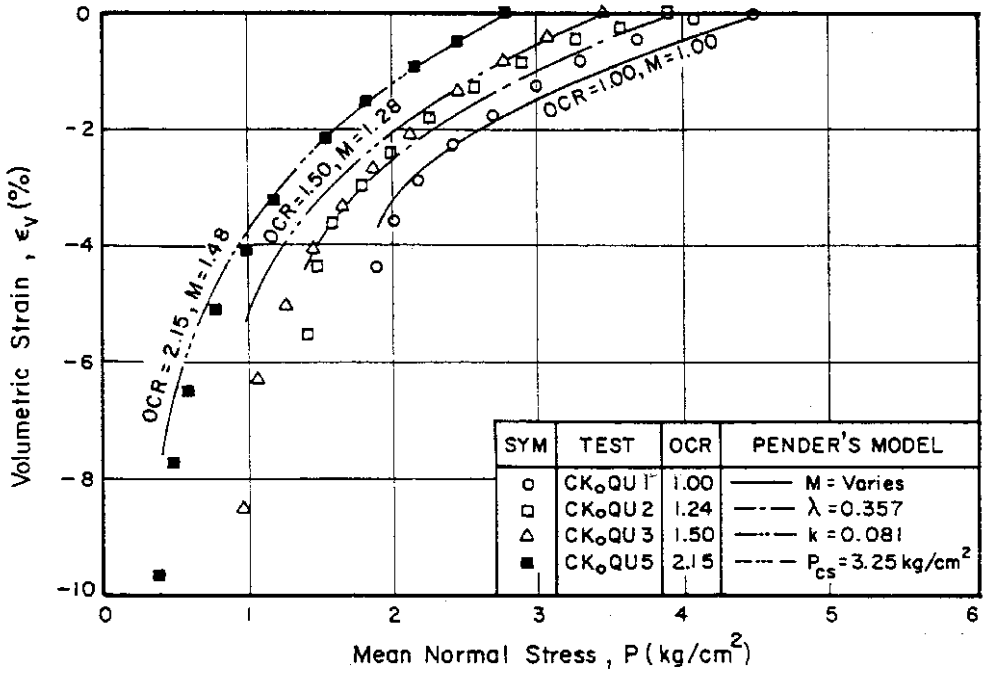

Fig. 28 (ε_v, p) Plot for CK_oQU Samples Compared with the Model Predictions

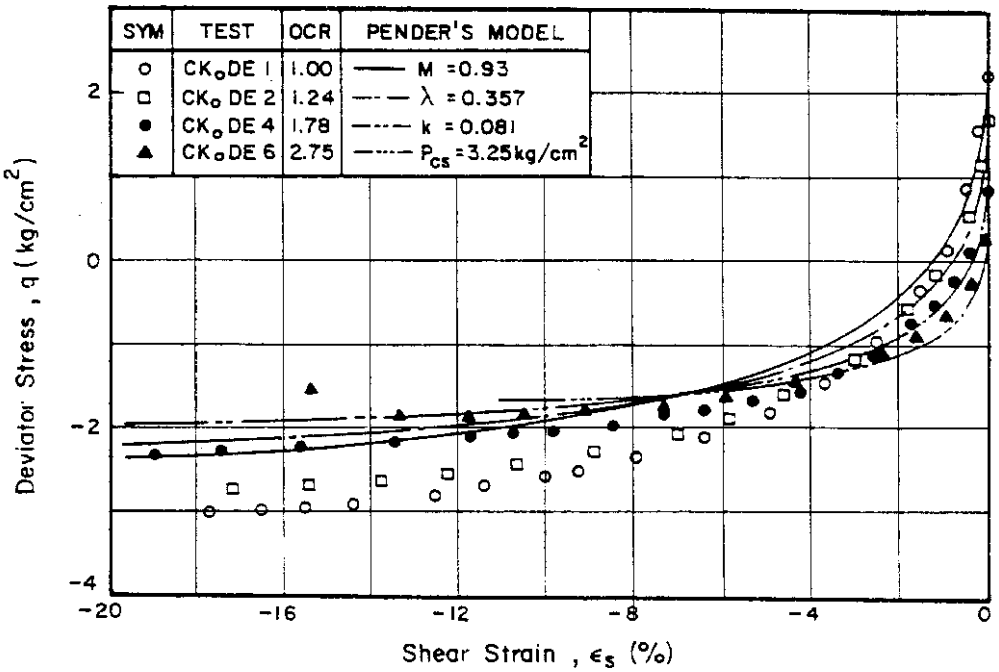

Fig. 29 (q, ε_s) Plot for CK_oDE Samples Compared with the Model Predictions

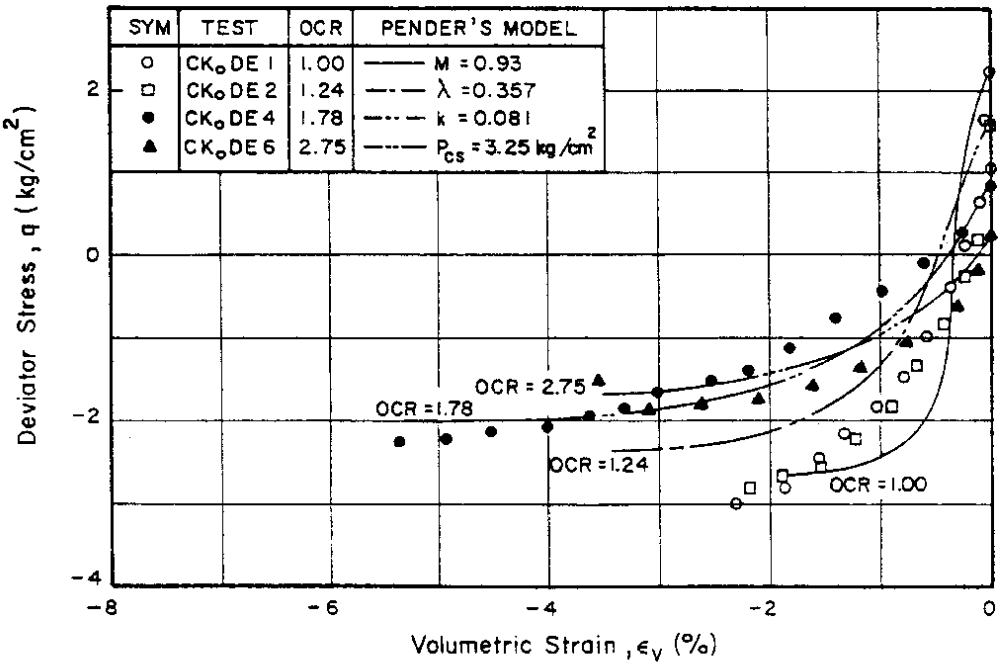

Fig. 30 (q, ε_v) Plot for CK_oDE Samples Compared with the Model Predictions

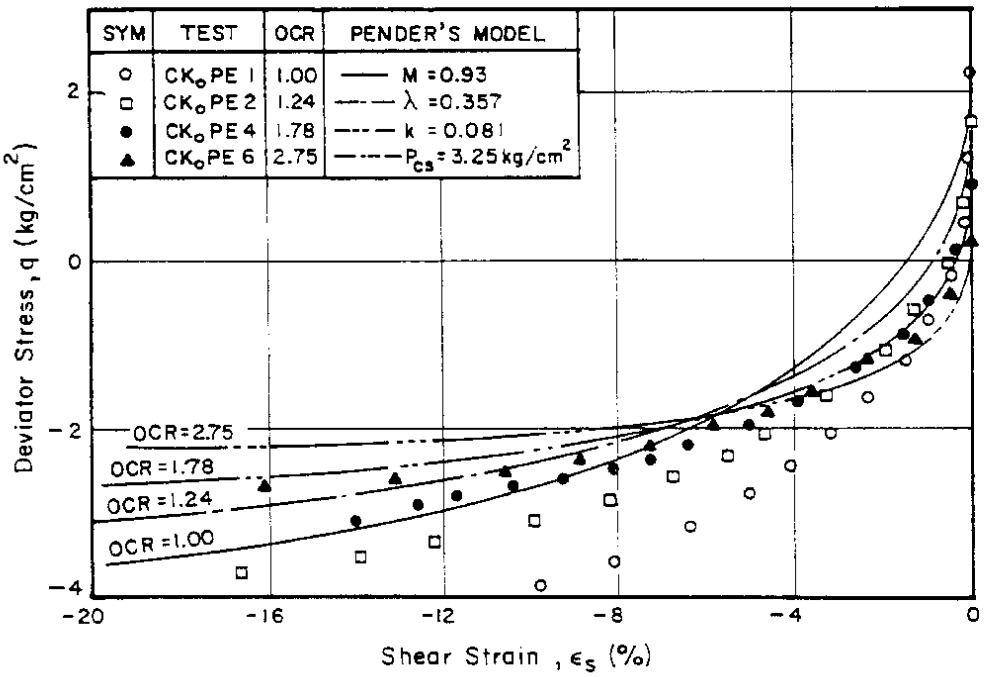

Fig. 31 (q, ε_s) Plot for CK_oPE Samples Compared with the Model Predictions

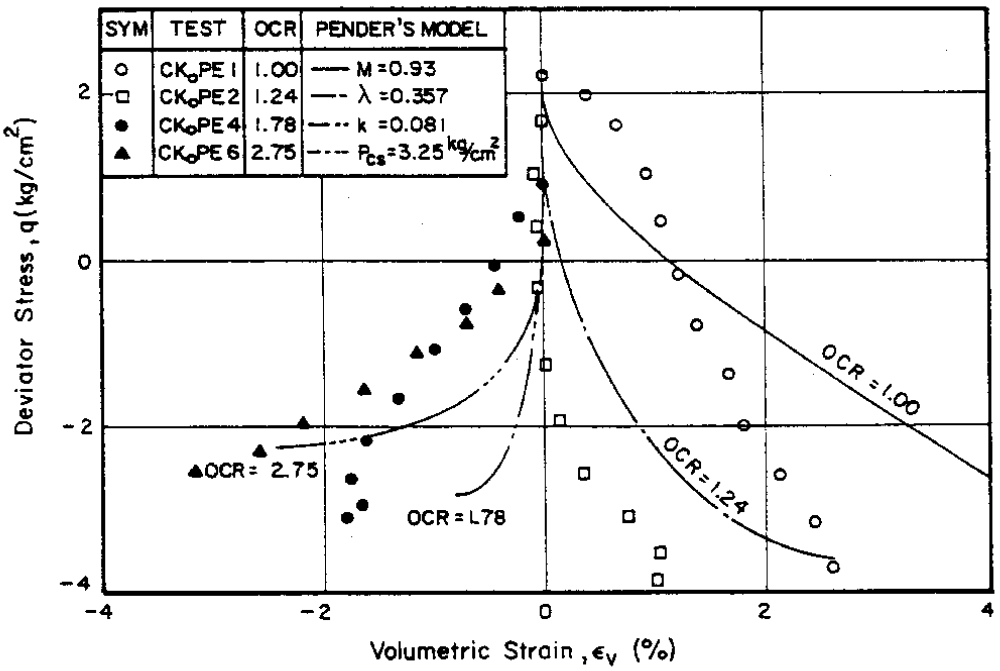

Fig. 32 (q, ε_v) Plot for CK_oPE Samples Compared with the Model Predictions

A CONCEPTUAL MODIFICATION OF PENDER'S MODEL FOR THE K_o – OVERCONSOLIDATED SAMPLES

Undrained Stress Paths

The behavior of the K_o-overconsolidated samples in the compression conditions is rigid nearly up to the failure state. A typical illustration of the rigid behavior is the vertical rise-up of the undrained stress path for almost the full range of the deviator stress. This implied that the behavior of the overconsolidated clay can be treated as pseudo-elastic at least up to a deviator stress equal to the initial deviator stress in the normally consolidated state as imposed during the sample preparation. The strain prediction will be more realistic, if Pender's Model is applied only for deviator stresses higher than the initial values imposed on the samples during the sample preparation. Figure 33 illustrates the undrained stress paths as proposed above in a modified form of prediction and the actual prediction using Pender's Model. Line OB is the base line for Pender's Model, i.e., the initial stress points of the overconsolidated sample are all shifted to this horizontal line. The region between the line OA and OB is considered as a pseudo-elastic zone. The proposed undrained stress paths appear to be promising as indicated in Fig. 34. The prediction of the undrained paths are much improved.

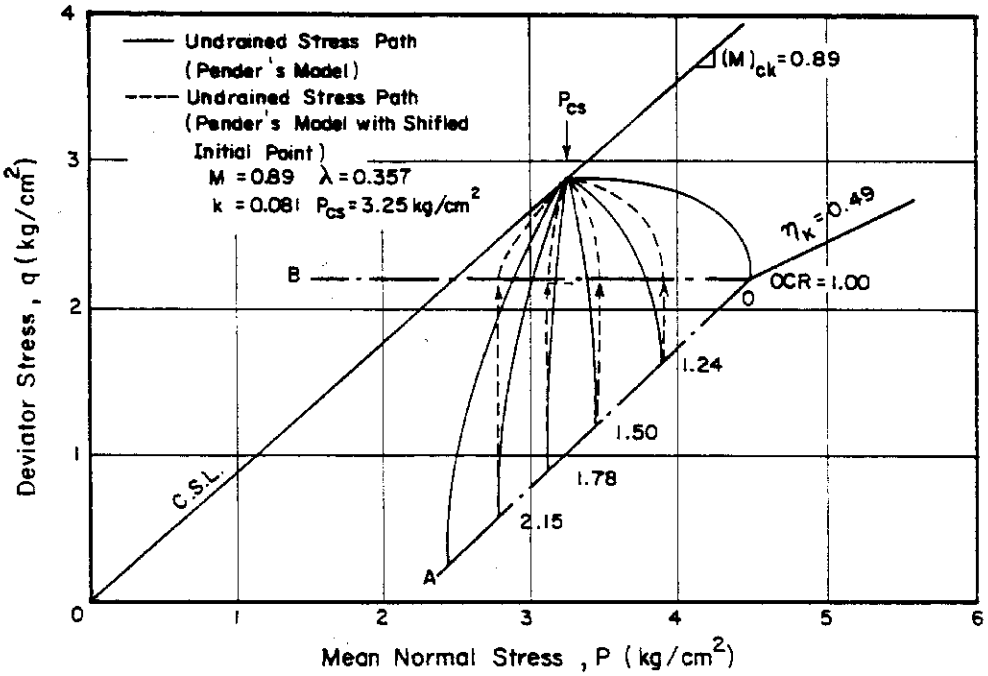

Fig. 33 (q, p) Plot from Pender's Model with and without Shifted Initial Stress Points

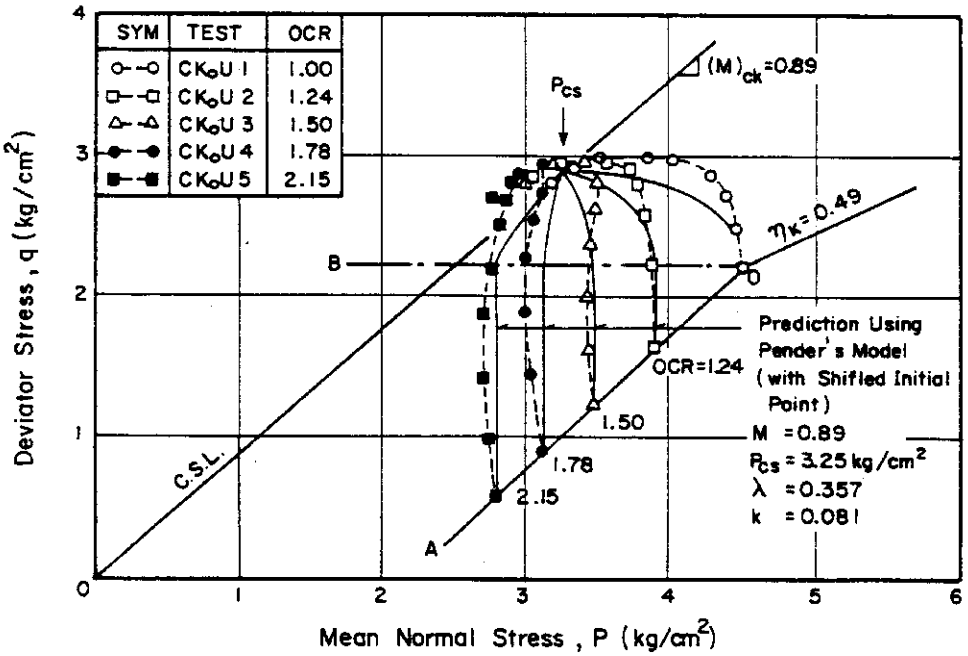

Fig. 34 (q, p) Plot for K_o-overconsolidated Samples Compared with the Predictions from Pender's Model with Shifted Initial Stress Points

Sress-Strain and Pore Pressure-Strain Relationships

Stress-strain relationships from Pender's Model as used in a modified form are plotted together with the experimental data in Fig. 35 (wet zone) and Fig. 36 (dry zone), respectively. These figures demonstrate that the proposed modified approach drastically improve the predicted stress-strain relationships. The same trends are shown in Figs. 37 and 38 for the pore pressure-strain relationships.

CONCLUDING REMARKS AND COMMENTS OF THE MODEL EVALUATION

The stress-strain theories based on the Critical State Concepts are examined by comparing the predictions with the experimental observations. For the K_o-consolidated samples, all model predictions deviate from the experimental observations, particularly, they fail to simulate the undrained stress path in the early stages of loading. Furthermore, all models are poor in their predictions of the K_o extension behavior. Brief concluding remarks for each model are made below.

Undrained Behavior of Normally Consolidated Samples

1. **Pender's Model** - For the CK_oU test, Pender's Model predicts reasonably well the undrained stress paths both in compression and in extension with parameters determined from the peak deviator stress criterion. However, it overpredicts shear strains in the compression phase, and gives poor strain predictions in extension.

2. **Dafalias' Model** - This model provides good stress-strain predictions for the compression tests. However, its prediction for the extension tests is poor, especially at the larger strains. The parameter α varies during undrained shearing and is not zero even for the isotropically consolidated sample whereas it should be according to the theory.

3. **Atkinson et al.'s Model** - Atkinson's Model predicts low undrained strength in compression, and its predictions for the extension conditions are poor specifically for the larger strain range.

Undrained Behavior of Overconsolidated Samples

Pender's Model - Pender's Model generates a successful family of undrained stress paths for the overconsolidated clays on the wet side of the Critical State both for compression and for extension conditions when the parameters are selected from the

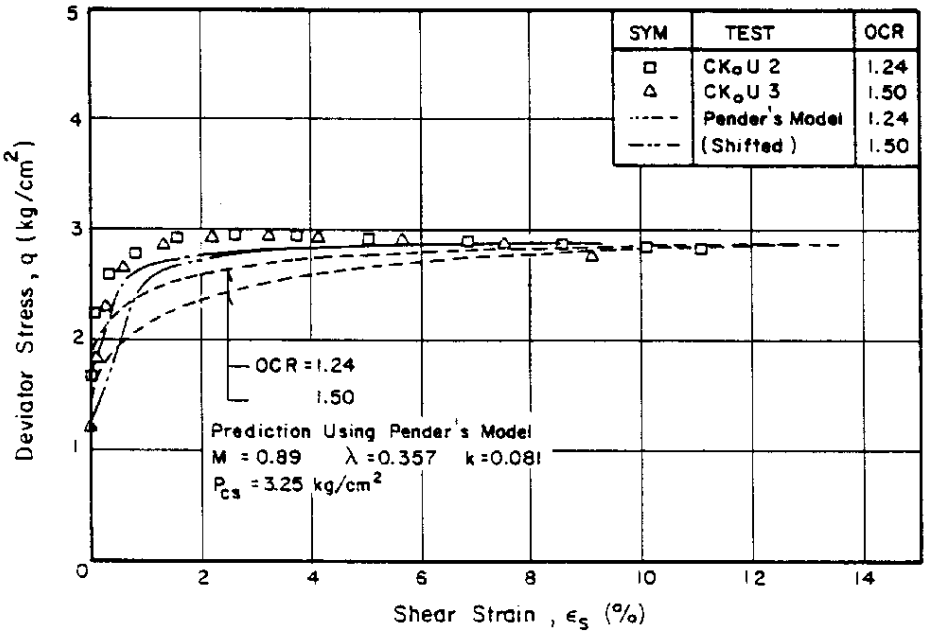

Fig. 35 (q, ε_s) Plot for K_o-overconsolidated Samples Compared with the Predictions from Pender's Model with and without Shifted Initial Stress Points (Wet Zone)

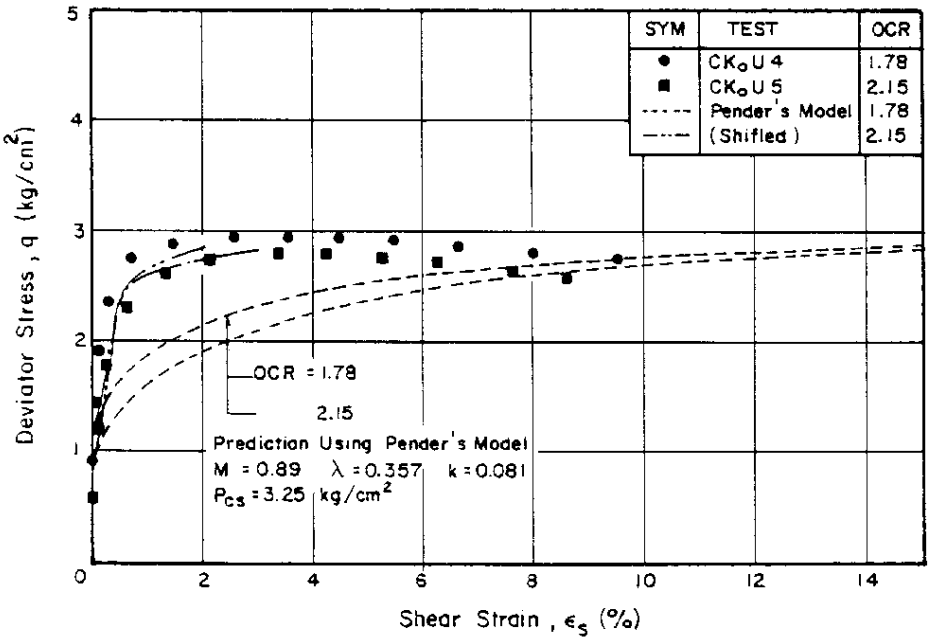

Fig. 36 (q, ε_s) Plot for K_o-overconsolidated Samples Compared with the Predictions from Pender's Model with and without Shifted Initial Stress Points (Dry Zone)

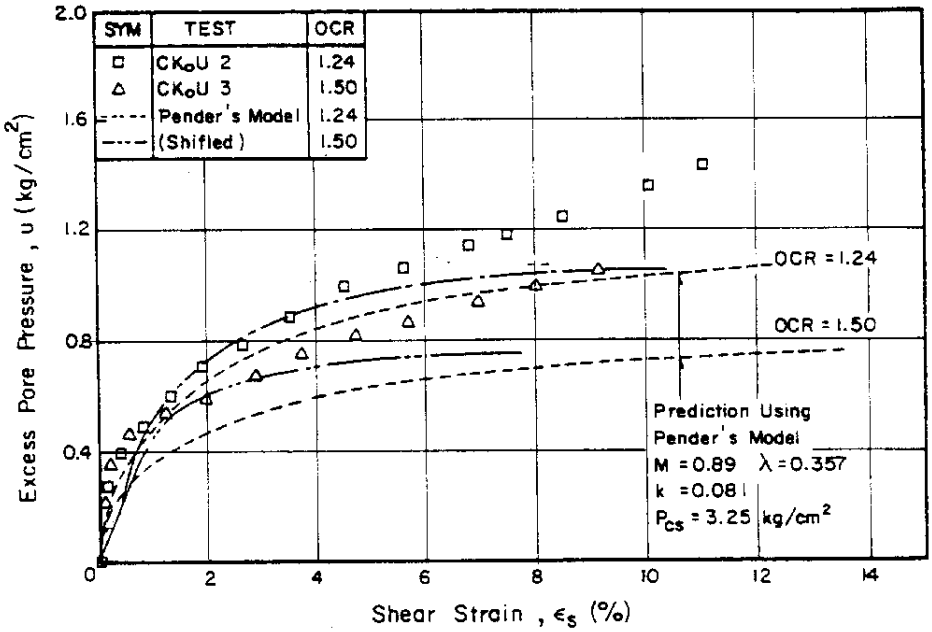

Fig. 37 (u, ε_s) Plot for K_o-overconsolidated Samples Compared with the Predictions from Pender's Model with and without Shifted Initial Stress Points (Wet Zone)

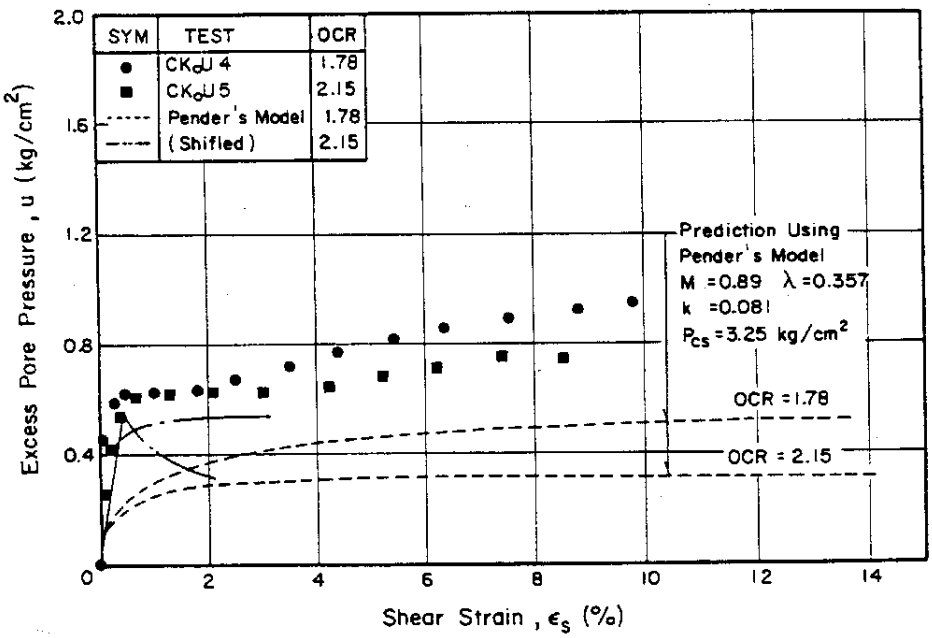

Fig. 38 (u, ε_s) Plot for K_o-overconsolidated Samples Compared with the Predictions from Pender's Model with and without Shifted Initial Stress Points (Dry Zone)

peak deviator stress conditions. However, this model overpredicts the shear strains both for the isotropically consolidated and for the K_o -consolidated samples.

Drained Behavior of Normally Consolidated Samples

1. **Pender's Model** - Pender's Model overpredicts the strains for the CK_oD and CK_oDE tests. This model, however, gives good predictions of the volumetric strain both in compression and in extension.

2. **Dafalias' Model** - For the CK_oD type of loading, this model predicted the (q, ε_s) and (q, ε_v) relationships well.

3. **Atkinson et al.'s Model** - This model overpredicts shear and volumetric strains for the CK_oD type of loading.

Drained Behavior of Overconsolidated Samples

Pender's Model - Pender's Model is found to overpredict shear strains in most cases. The deviations become more and more apparent when the OCR values are increased. However, this model gives excellent predictions of the volumetric strains for the overconsolidated samples under compression types of loading. It can be concluded that Pender's Model, for the overconsolidated behavior, overpredicts the shear strain both in undrained and in drained tests. This is attributed partially or fully to the incorrect assumption made on the plastic strain increment ratio in the formulation of the theory.

Improvement of Pender's Model

The OCR value or related parameter can be a possible reduction factor to be applied to the assumption of the plastic strain increment ratio for the lightly overconsolidated samples (OCR < 3.0). Pender's hypotheses on the shapes of the undrained stress path for the K_o overconsolidated sample is found to be an oversimplification. Thus, the prediction of Pender's Model for K_o overconsolidated samples is expected to be promising when this factor is incorporated in the expression for the plastic strain increment ratio.

REFERENCES

ATKINSON, J. H.; RICHARDSON, D.; and ROBINSON, P. J. (1987). Compression and extension of K. normally consolidated Kaolin clay, *Journal of Geotechnical Engineering Division*, ASCE, Vol. 113, No. 12, pp. 1468-148

BALASUBRAMANIAM, A. S. (1969). Some factors influencing the stress-strain behavior of clays, Ph. D. Thesis, Cambridge University, Cambridge.

BURLAND, J. B. (1965). The yielding and dilation of clay, Correspondence, *Geotechnique*, Vol. 15, No. 2, pp. 211-214.

CHANG, M. F. (1973). A rational method for determining the in-situ coefficient of earth pressure at rest, M. Eng. Thesis, No. 510, AIT, Bangkok, Thailand.

CHAIYADHUMA, W. (1974). Undrained shear strength characteristics of Nong Ngoo Hao soft clay under K_o-anisotropic consolidation, M. Eng, Thesis , No. 698, Asian Institute of Technology, Bangkok, Thailand.

DAFALIAS, Y. F. (1987). An anisotropic critical state clay plasticity model. *Constitutive Laws for Engineering Materials Theory and Applications*, Vol. 1, pp. 513-522.

LADD, C. C.; FOOTT, R.; ISHIHARA, K.; SCHLOSSER, F.; and POULOS, H. G. (1977), Stress-deformation and strength characteristics: State-of-the-art report, *Proceedings of the Ninth International Conference on Soil Mechanics and Foundation Engineering*, Vol. 2, Tokyo, Japan, pp. 421-494.

PENDER, M. J. (1977), A unified model for soil stress- strain behavior. *Proceedings of the Tenth International Conference on Soil Mechanics and Foundation Engineering*, Specialty Session No. 9, Constitutive Equation of Soils, Tokyo, Japan, pp. 213-222,

PENDER, M. J. (1978). A model for behavior of overconsolidated soil, *Geotechnique*, Vol. 28, No. 1, pp. 1-25.

ROSCOE, K. H. and BURLAND, J. B. (1968). On the generalized stress-strain behavior of wet clay, *Engineering Plasticity*, Cambridge: Cambridge University Press, pp. 535-609.

WONG, P. K. K. and MITCHELL, R. J. (1975). Yielding and plastic flow of sensitive clay. *Geotechnique*, Vol. 25, No. 4, pp. 763-782.