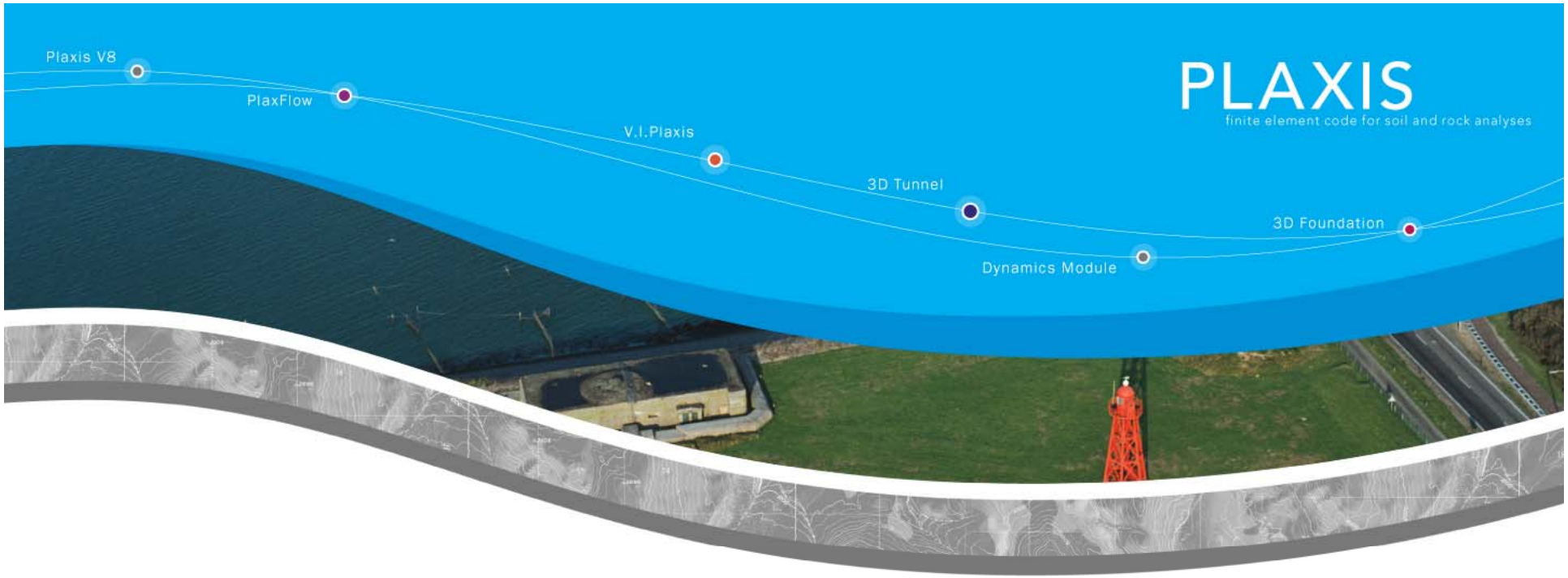


Non-linear computations

Dennis Waterman

Plaxis BV

Contents

- Calculation
- Local level
- Global level
- Load advancement

Calculation

- Initial situation
  - Geometry (mesh, loads, boundary conditions)
  - Material parameters
  - Initial stresses and pore pressures
- Calculation phases
  - Calculation types
    - Plastic
    - Consolidation
    - Phi-c reduction (safety analysis)

Calculation

- Calculation phases (continued)
  - Loading input
    - Staged construction
      - Switch on/off parts of geometry (soil, structural elements)
      - Switch on/off loads (change input values)
      - Change pore pressures
    - Total Multipliers (L.A. Ultimate level)
    - Incremental Multipliers (L.A. no. of steps)
- Start calculation

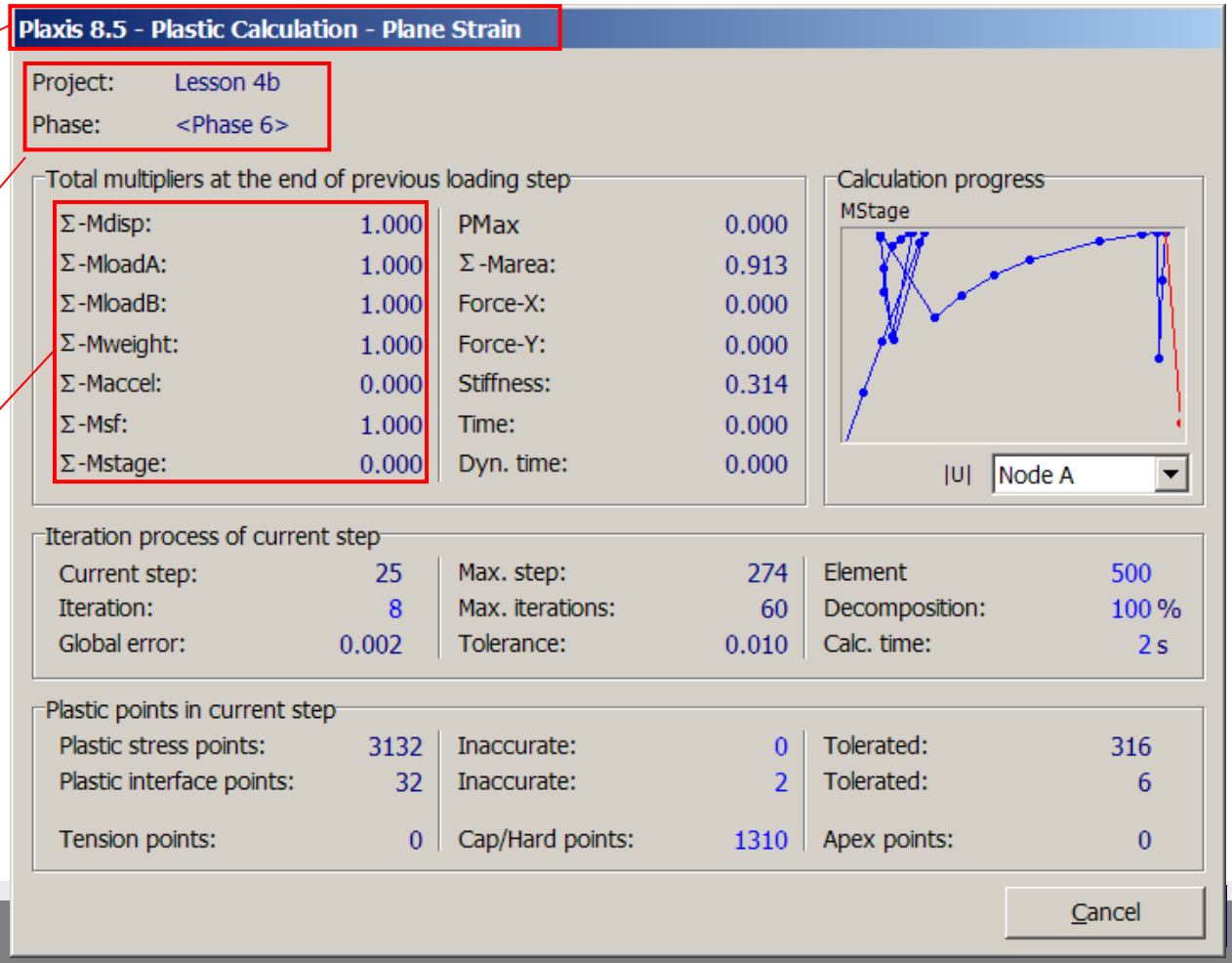
PLAXIS
finite element code for soil and rock analyses

Calculation progress window (1)

Plaxis version
Analysis type
Model type

Project data

Multipliers



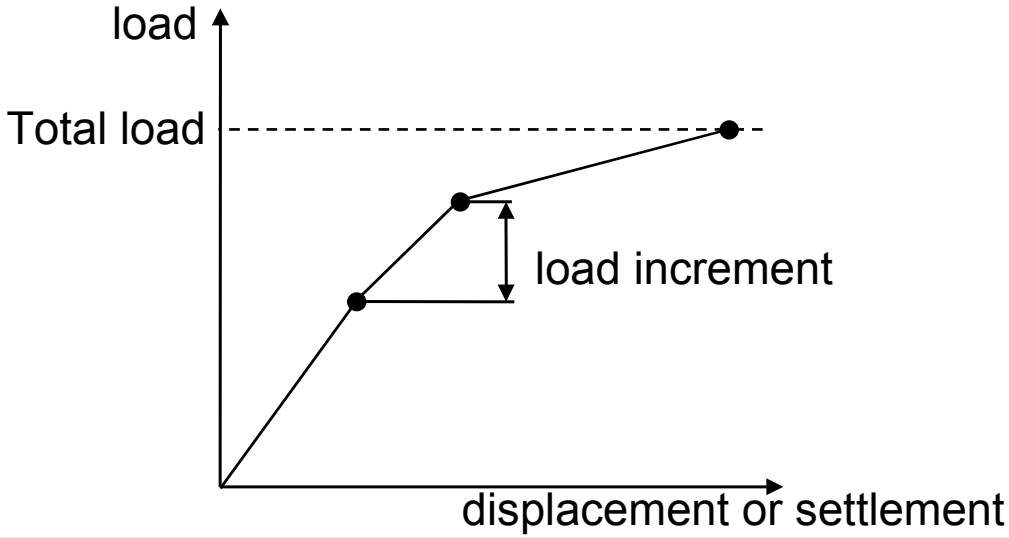
Loads and multipliers

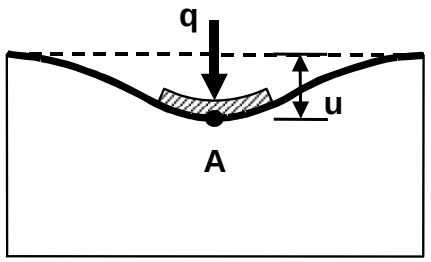
- Applied load = Load multiplier x Input load
  - Standard setting:
    - Input load = 1 stress unit
    - Load multiplier = 1
- Actual load is specified before each calculation
  - Staged construction : Change Input load
  - Total/Incr. Multipliers : Change Load multipliers

The value of the load multipliers is shown
in the Calculation window

Calculation - multipliers

A calculation phase is divided in steps and in every step the load is increased with a certain load increment.
A new load increment can only be applied if the previous load increment was calculated with sufficient accuracy. In order to reach this accuracy several iterations are needed.





Calculation – Loading Input

Most calculations: **Staged Construction**

- Total load in last calculation step

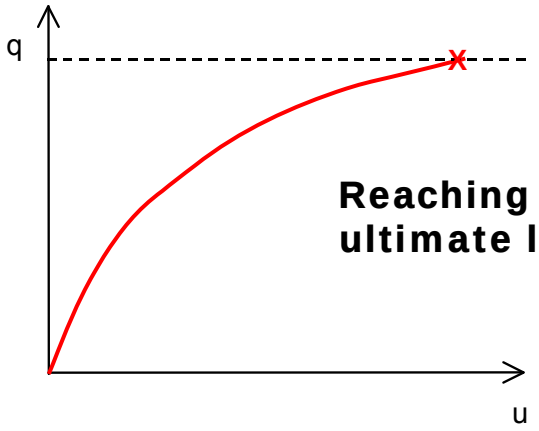
Some calculations: **Total multipliers**

- Predefined ultimate level
- Total load in last calculation step
- ΣMdisp, ΣMloadA, ΣMloadB, ΣMweight

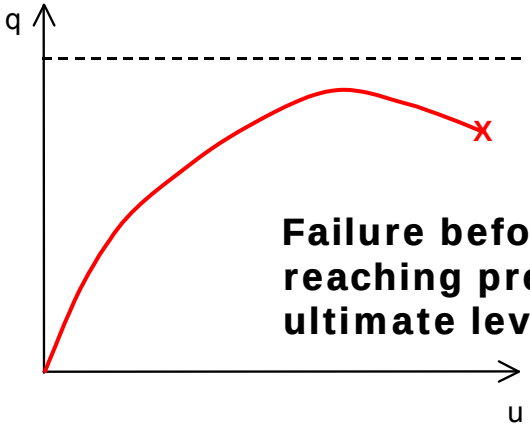
Very few calculations: **Incremental multipliers**

- Predefined number of steps
- Additional load in one calculation step
- Total load not known a-priori
- Mdisp, MloadA, MloadB, Mweight, Msf

Calculation – Loading Input

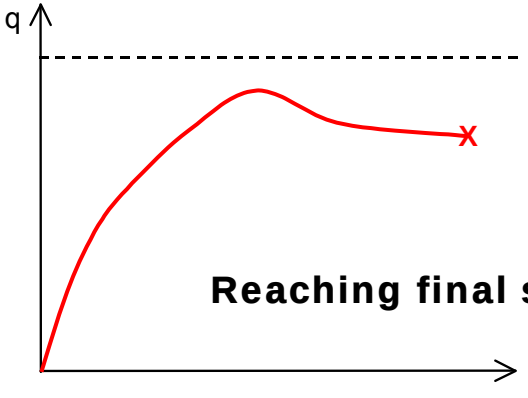


Reaching prescribed ultimate level



Failure before reaching prescribed ultimate level

Ultimate level procedure - Plastic calculation: **Staged construction**
Total Multipliers



Reaching final step

No. of steps procedure:

Plastic calculation, incr. Multipliers
Phi/c reduction (safety analysis)

Calculation - Staged Construction, remarks

- Multipliers calculation using Σ-Mstage = 0->1
- Load applied is the unbalance due to the changes made (switch on/off soil and structural elements, change groundwater level etc.)
- This Load is applied in load increments = part of the unbalance
- Such a load increment has no physical meaning! Hence, results from intermediate steps have no meaning since they are the result of a partially solved unbalance.
- Only the final result of a SC phase has a physical meaning

- Total/Incr. multipliers: load increment is increment of real load, displacement etc. Hence, intermediate results are physically valid.
- This makes a Total/Incr. multipliers calculation more suitable for the determination of a failure load than a SC calculation.

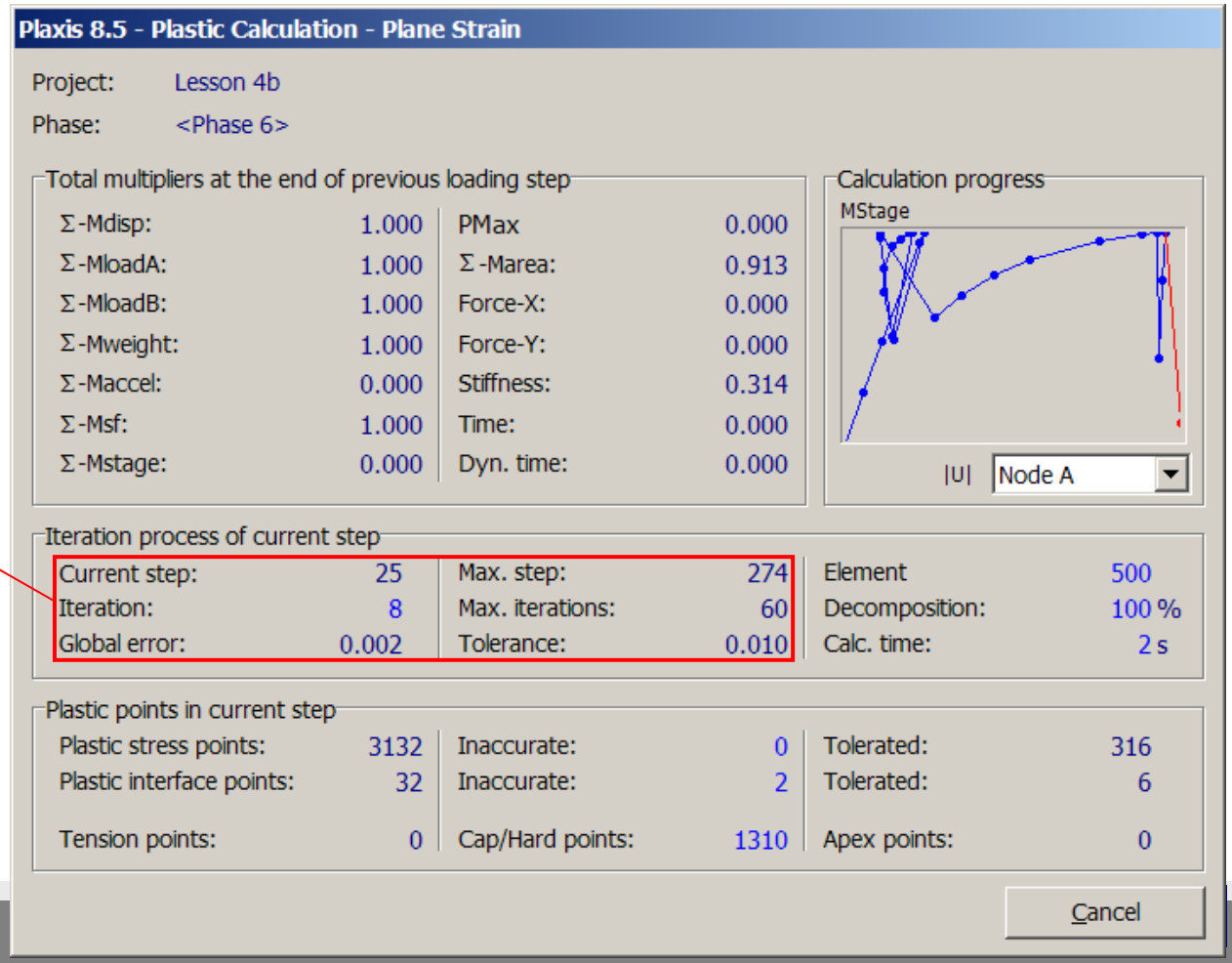
Calculation - Staged Construction, stiffness

- Change of stiffness is immediately applied at the beginning of the staged construction phase. Hence, the stiffness is NOT gradually changed during the calculation phase.
- Change of stiffness does not cause an unbalance (no stresses are changed) and has therefore no effect on the **current** situation.
- The new stiffness is used for any load **increment** applied **after** the change of stiffness.

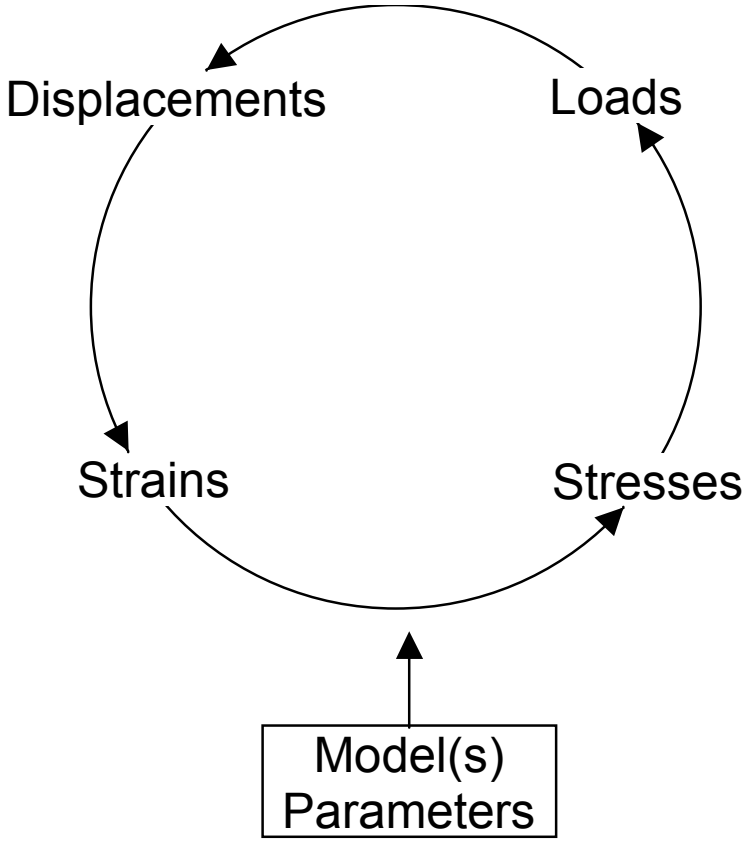
PLAXIS
finite element code for soil and rock analyses

Calculation progress window (2)

Iteration Process
and
Global error



Calculation cycle



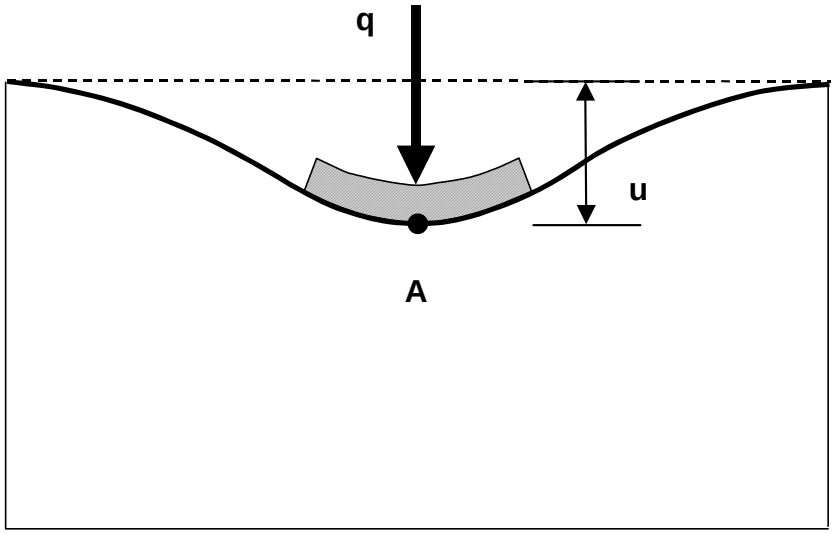
Phase

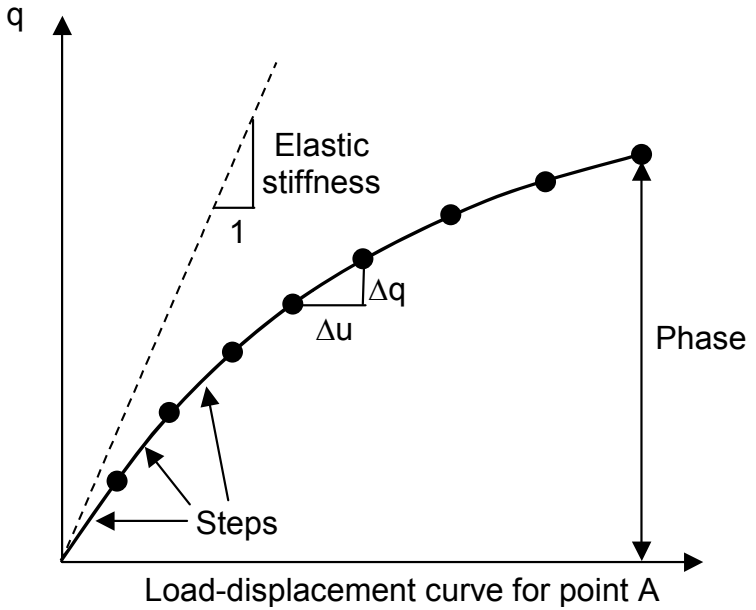
- Find equilibrium in final situation
- Subdivide in load steps

Load step

- Find equilibrium for load increment

Calculation





Global system of equations

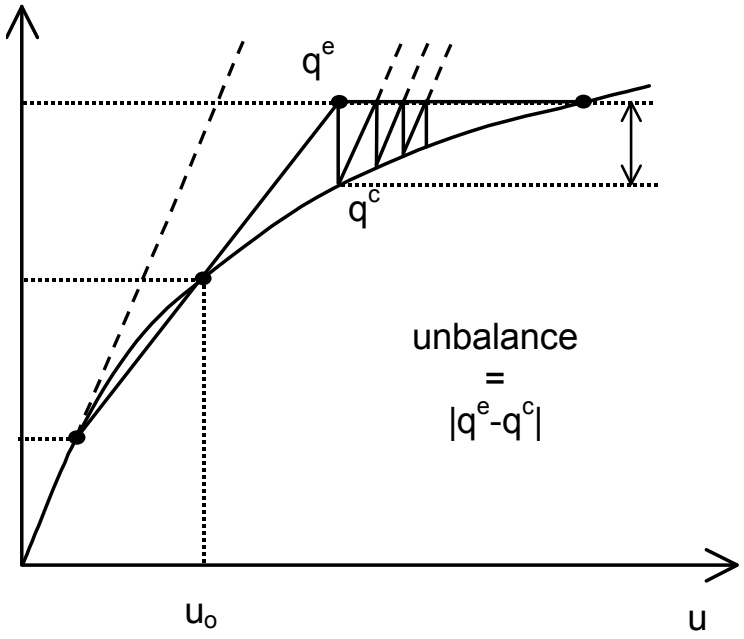
$$\underline{\underline{K}}\,\underline{u} = \underline{P} \quad \text{or} \quad \underline{\underline{K}}\,\underline{\Delta u} = \underline{\Delta P} \quad \text{or} \quad \underline{\underline{K}}\,\underline{\delta u} = \underline{\delta P}$$

Calculation

Basic algorithm of 1 iteration within a step

- additional displacements — 1: $\delta \underline{u} = \underline{\underline{K}}^{-1} \delta \underline{P}$
- total increments — 2: $\Delta \underline{u} = \Delta \underline{u} + \delta \underline{u}$
- strain increment — 3: $\Delta \underline{\varepsilon} = \underline{\underline{B}} \Delta \underline{u}$
- determine stresses — 4: $\underline{\sigma}^{c} = \underline{\sigma}^{0} + \underline{\underline{D}}^{e} (\Delta \underline{\varepsilon} - \Delta \underline{\varepsilon}^{p})$
- internal reaction — 5: $\underline{P}_{in} = \int \underline{\underline{B}}^{t} \underline{\sigma}^{c} dV$
- equilibrium ?? — 6: $\underline{P}_{ex} \approx \underline{P}_{in}$
- No: next iteration — 7: $\delta \underline{u} = \underline{\underline{K}}^{-1} (\underline{P}_{ex} - \underline{P}_{in}) \rightarrow 2$

Calculation - Global error



q^e = "external" load
(including weights)
q^c = internal reaction forces
(integral of σ^c)

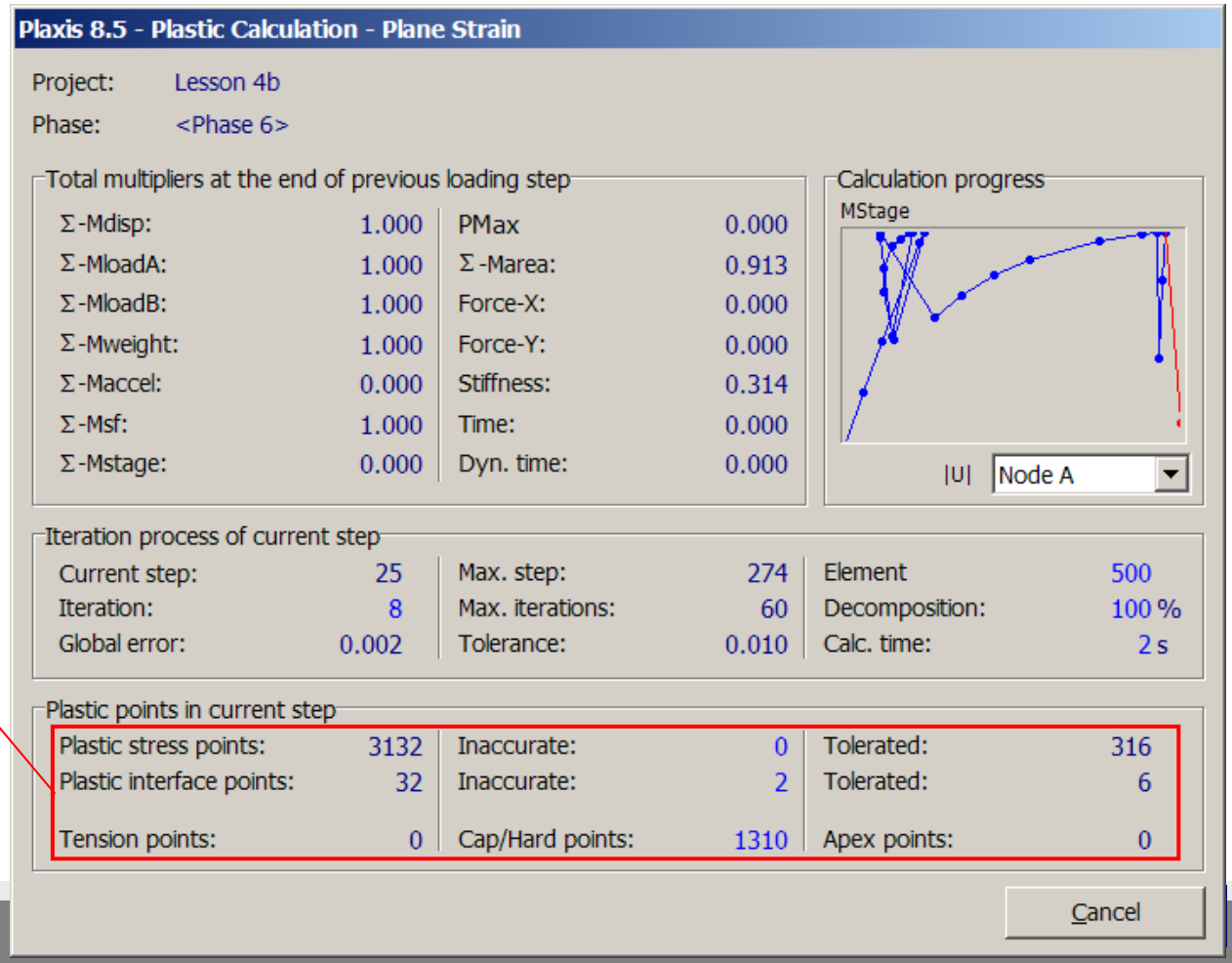
$$Global\ Error = \frac{\| unbalance \|}{\| load \|}$$

Convergence requirement:
Global error ≤ Tolerated error
(Standard setting:
Tolerated error = 0.01)

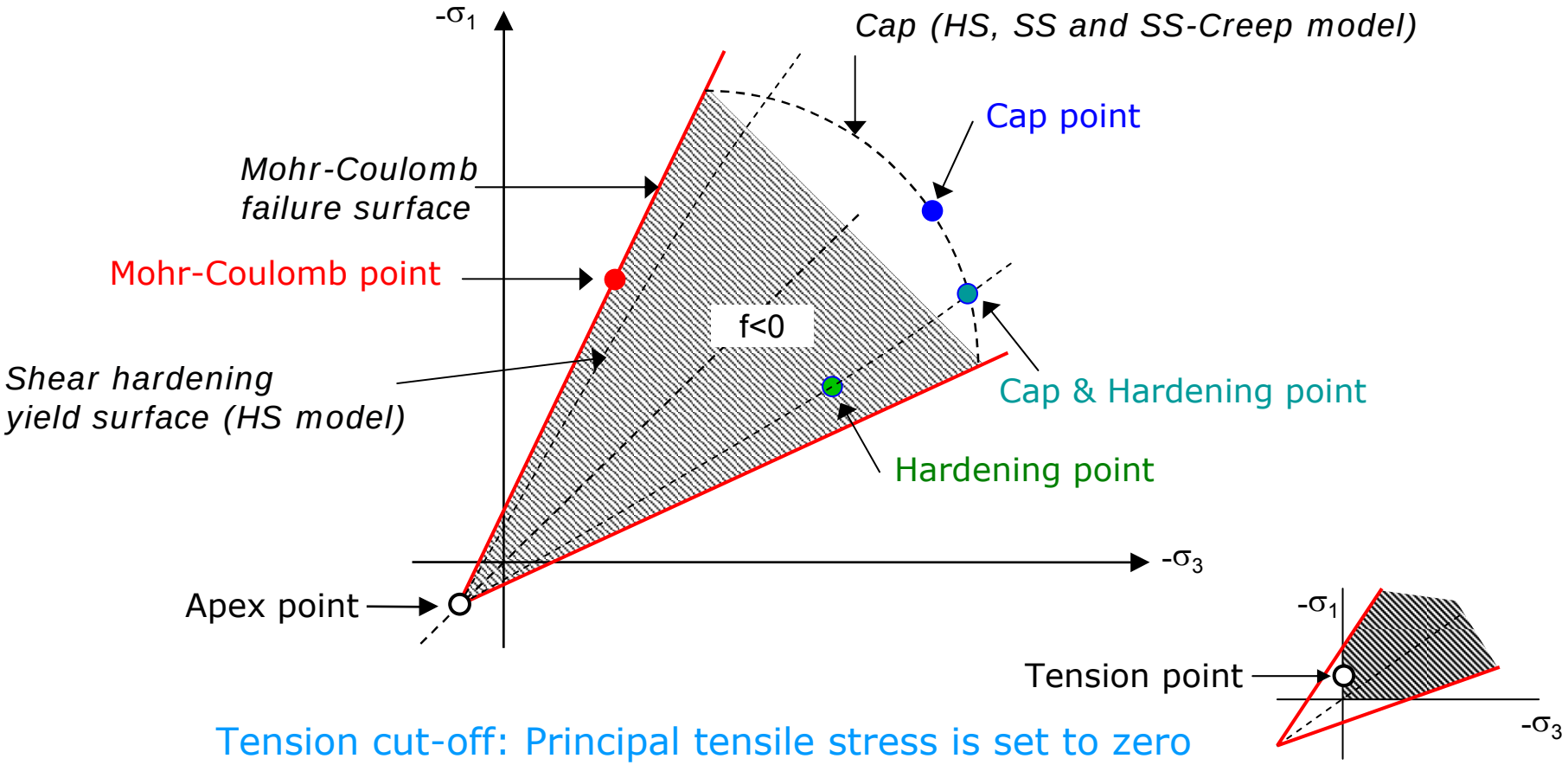
Convergence has to be reached within a maximum number of iterations.
The total load has to be applied within a maximum number steps.

Calculation progress window (3)

Plastic Points
and
Local error

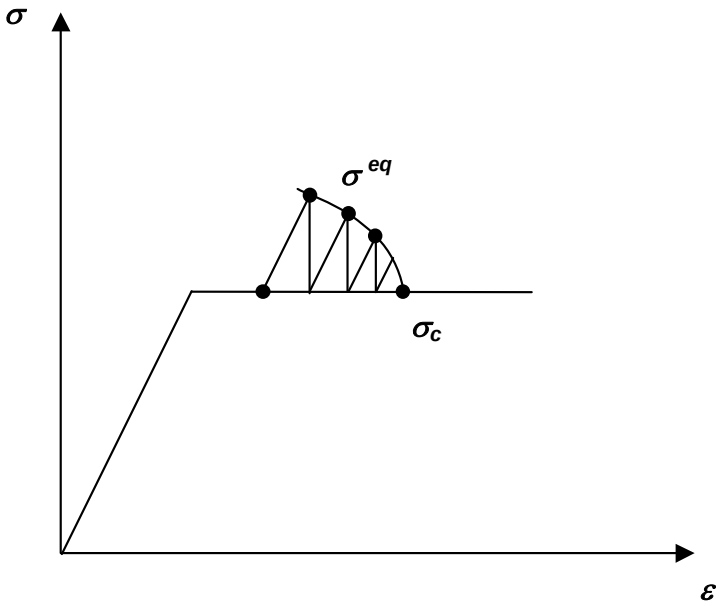


Local level – Plastic points



Tension cut-off: Principal tensile stress is set to zero

Local level – local error



Constitutive stress σ^c:

Stress that follows from the constitute model (Mohr- Coulomb)

Equilibrium stress σ^{eq}:

Stress that is in equilibrium with the external load

$$Local\ Error = \frac{\| \sigma^c - \sigma^{eq} \|}{\| \sigma^c \|}$$

Inaccurate point: Local error > Tolerated error

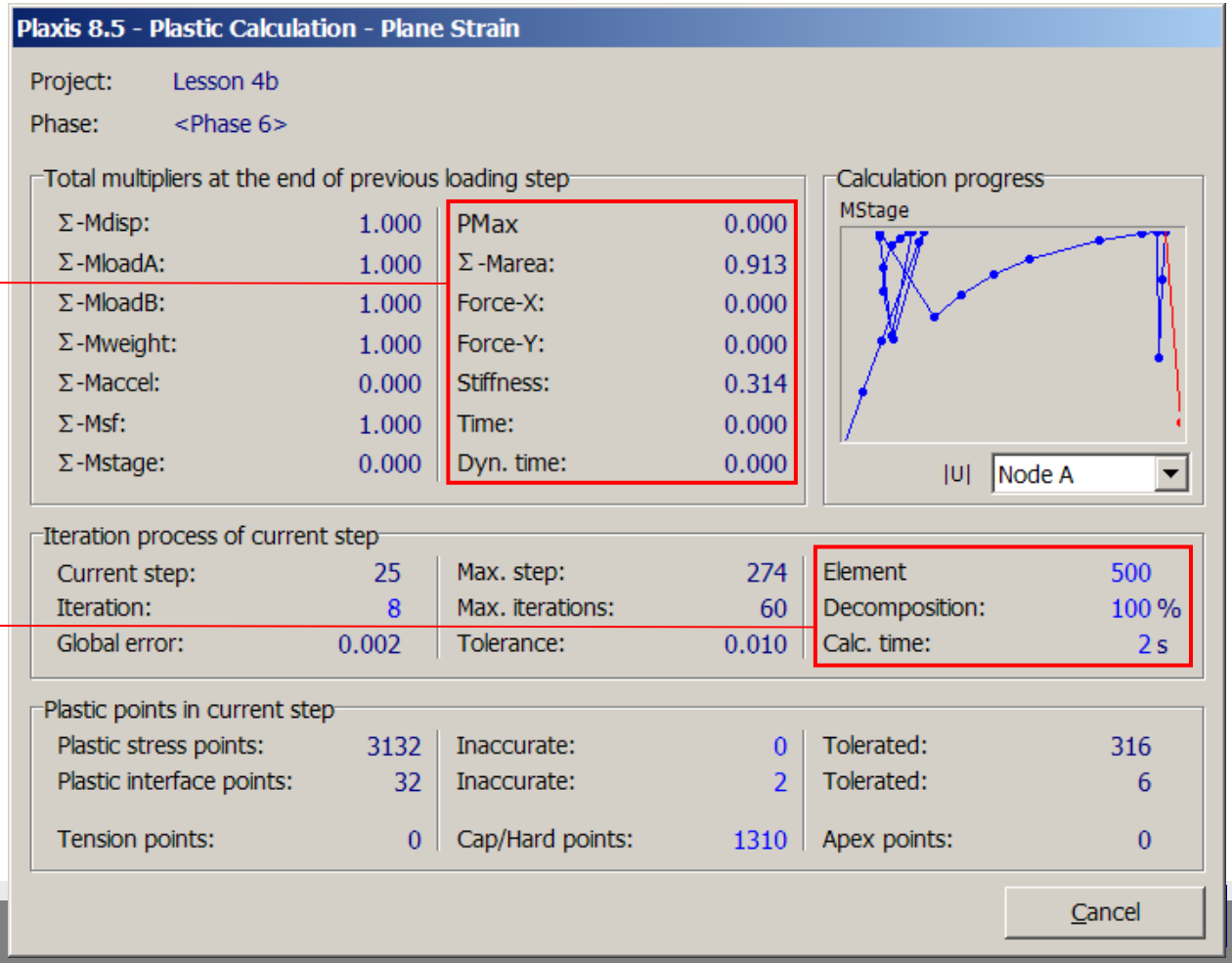
Convergence requirement:

Inaccurate stress points = 3 + (plastic soil points)/10

Inaccurate interface points = 3 + (plastic interface points)/10

Calculation progress window (4)

Miscellaneous values



Miscellaneous values

- PMax: Maximum excess pore pressure in the mesh
- Σ-Marea: part of the mesh area currently active
- Force-X: Horizontal force resulting prescr. displ.
- Force-Y: Vertical force resulting prescr. displ.
- Stiffness: Current Stiffness Parameter (see next slide)
- Time: Elapsed model time (usually in days)
- Dynamic time: Elapsed model time for dynamics (s)
- Element: number of elements in mesh
- Decomposition: stiff. matrix decomposition progress
- Calc. time: duration of the calculation

Miscellaneous – Current Stiffness Parameter

$$CSP = \int \frac{\Delta\varepsilon \cdot \Delta\sigma}{\Delta\varepsilon\, D^e \Delta\varepsilon} = \frac{Total\,elastic\,work}{Total\,work}$$

CSP is a measure of the amount of plasticity that occurs during the calculation:

- Fully elastic : CSP = 1
- Failure : CSP ≈ 0

CSP should be monitored during the calculation to check whether calculation gets very close to failure.
Calculation successful ≠Construction is safe!!

Iterative procedure - tuning

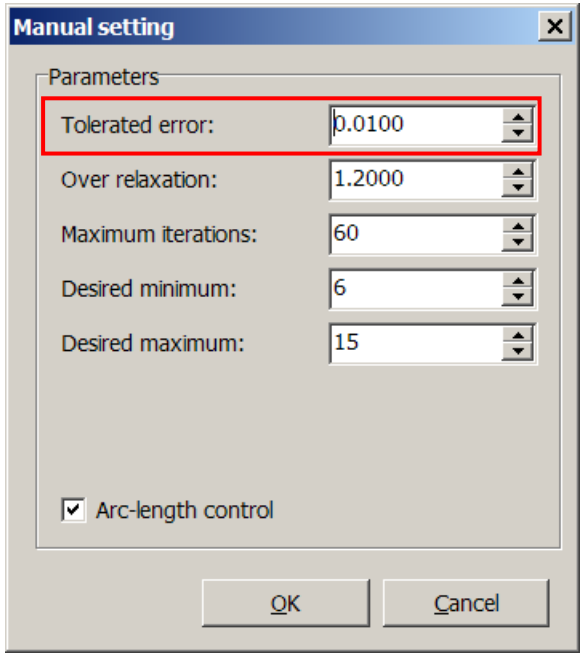
- Plaxis calculations
  - General tabsheet -> Iterative procedure

Standard settings <-> Manual settings

Usually Standard settings is sufficient.
Sometimes Manual settings needed to:

- Increase calculation speed
- Ignore local failure mechanisms
- Increase stability of the iterative procedure

Iterative procedure – Manual settings



Tolerated error

Purpose:

- Determines the accuracy of the calculation

Increase value:

- faster but less accurate results

Decrease value:

- slower but more accurate results

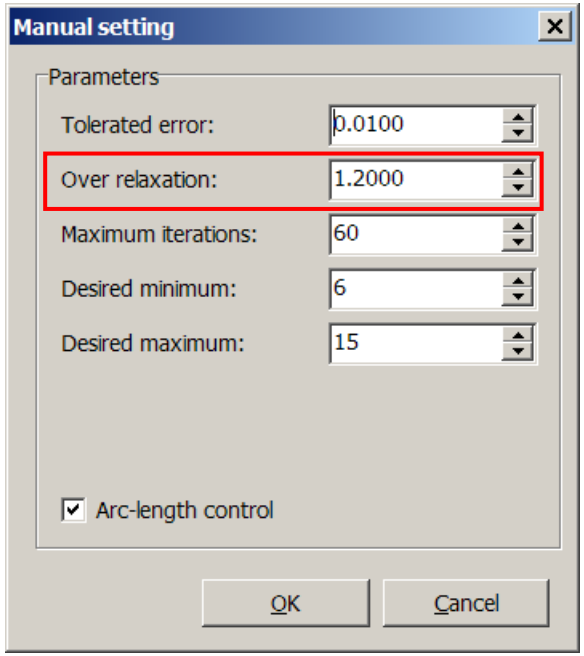
Changed by user:

- Occasionally, e.g. to overcome a local failure mechanism

Warning:

- Increasing the tolerated error often increases the FoS

Iterative procedure – Manual settings



Over relaxation

Purpose:

- Decrease number of iterations needed, and therefore increase the calculation speed

Increase value:

- faster but less stable calculation

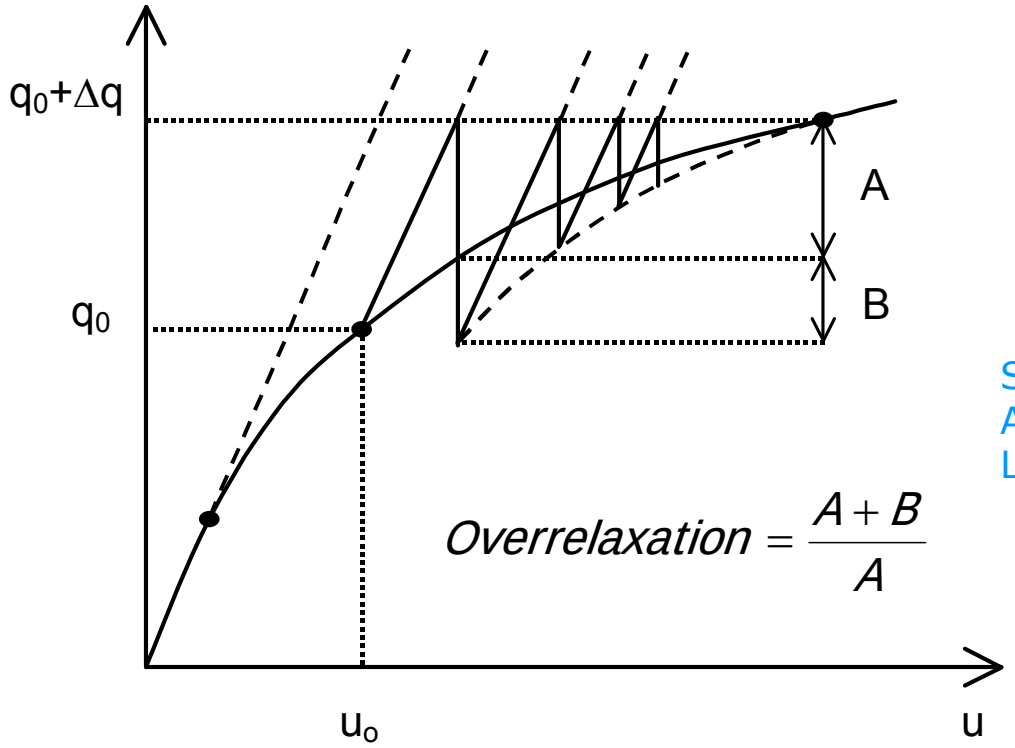
Decrease value:

- slower but more stable calculation

Changed by user:

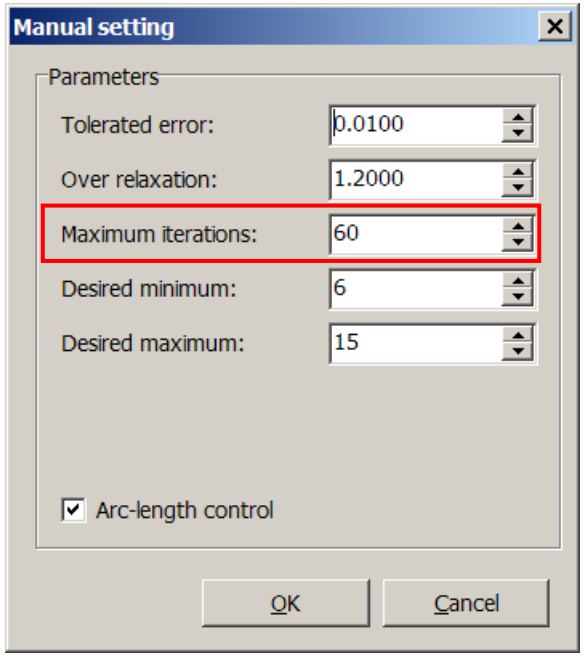
- Hardly ever

Iterative procedure – Over relaxation



Standard setting: 1.2
Absolute maximum: 2.0
Low friction angles (<20°): 1.5 acceptable

Iterative procedure – Manual settings



Maximum iterations

Purpose:

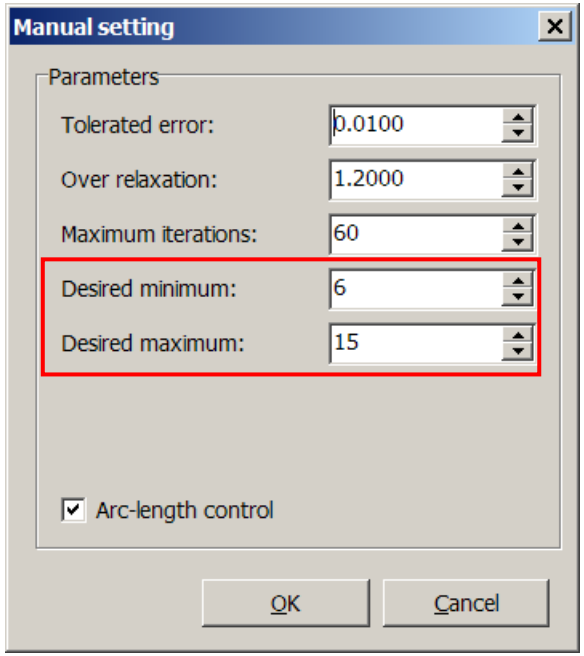
- Ensure calculation time does not get excessive due to errors in the specification.

Changed by user:

- Rarely.
  May be increased for slow converging problems due to:
  - High friction angle
  - Consolidation with low permeabilities
  - Consolidation + Creep + Upd. Mesh

Usually slow convergence should be solved by improving the input!

Iterative procedure – Manual settings



Desired minimum / maximum

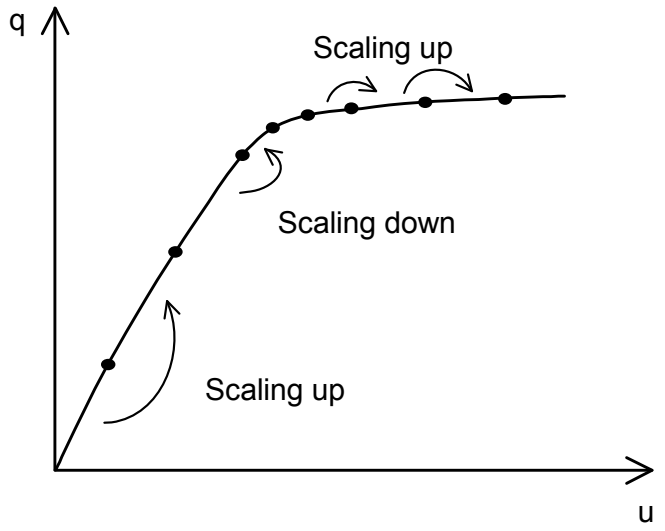
Purpose:

- To force the automatic load stepping procedure to use smaller/larger load steps after more/less iterations.

Changed by user:

- Rarely.
  May be changed for calculations that keep scaling up/down every other step. A higher desired maximum then usually increases the calculation speed.

Iterative procedure – Load advancement

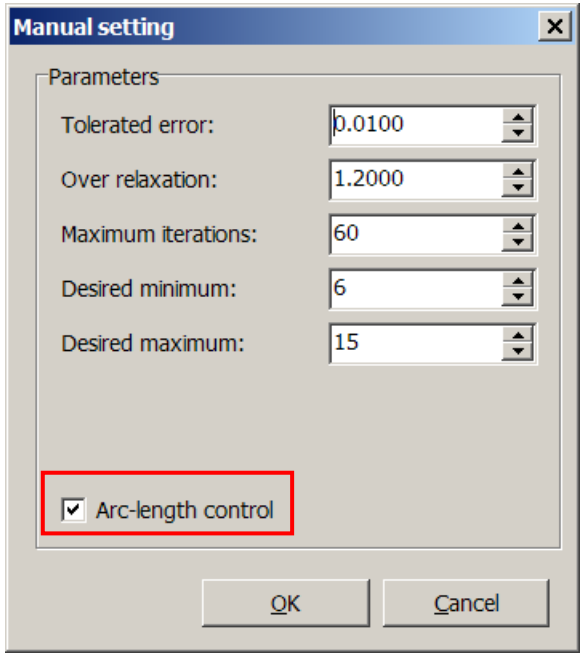


- Converged within desired minimum number of iterations:
  - Scaling up load step by a factor 2
- Not converged within desired maximum number of iterations:
  - Scaling down load step by a factor 2
- Default values:
  - Desired minimum = 6
  - Desired maximum = 15

Soils with very low φ: choose lower values to reduce step size
Soils with high φ: choose higher values to improve convergence

Special case: Desired minimum=2, Desired maximum=max. iterations
No scaling will be done: constant load step.
Load step size can be controlled using Incremental Multipliers phase

Iterative procedure – Manual settings



Arc-length control

Purpose:

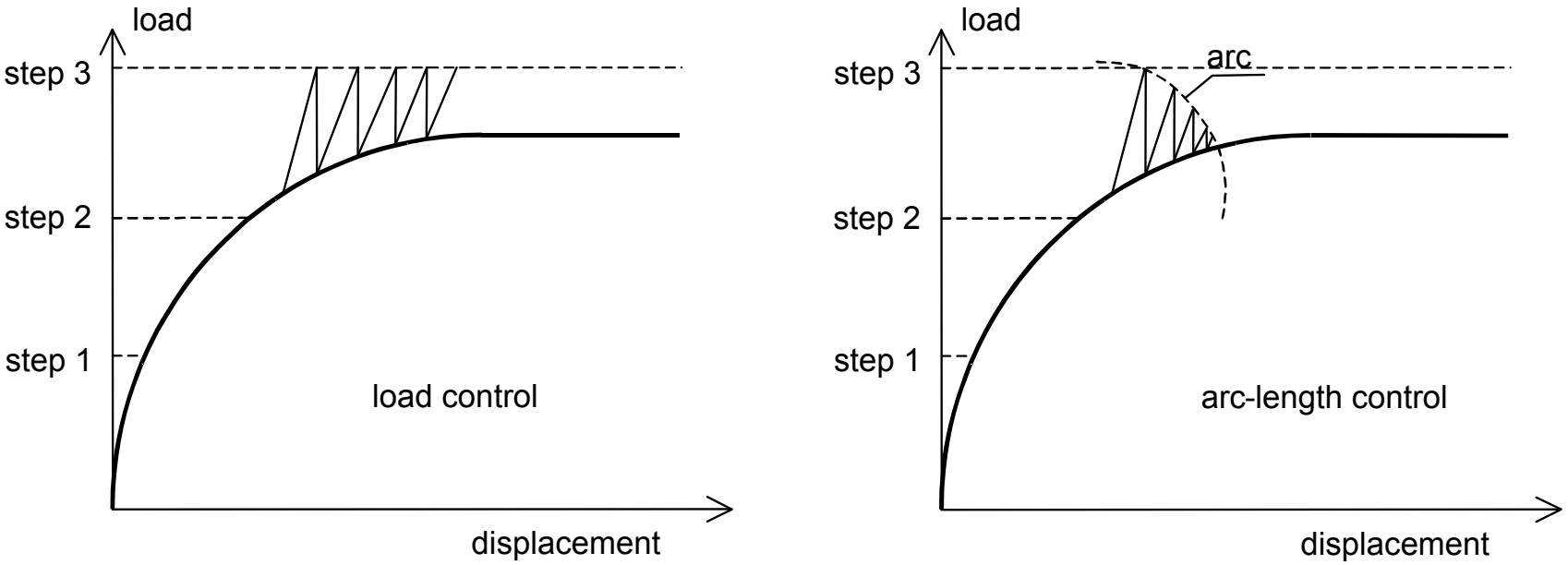
- To obtain reliable collapse loads for load-controlled calculations.

Changed by user:

- Occasionally.
  Generally to overcome a local failure mechanism

Arc-length control is by default switched on. However, it is not available for consolidation analysis.

Iterative procedure – Arc-length control



Recommendation

- During a calculation the different parameters of the calculation process should be monitored in order to be more confident about the calculation process and its results
- Though for the majority of the calculations the default settings of iterative procedure work fine, occasionally settings may have to be manually adjusted to ensure the best calculation performance. Knowledge about the parameters and their influence on the calculation process and the results is then required.