Parameters of the HS model

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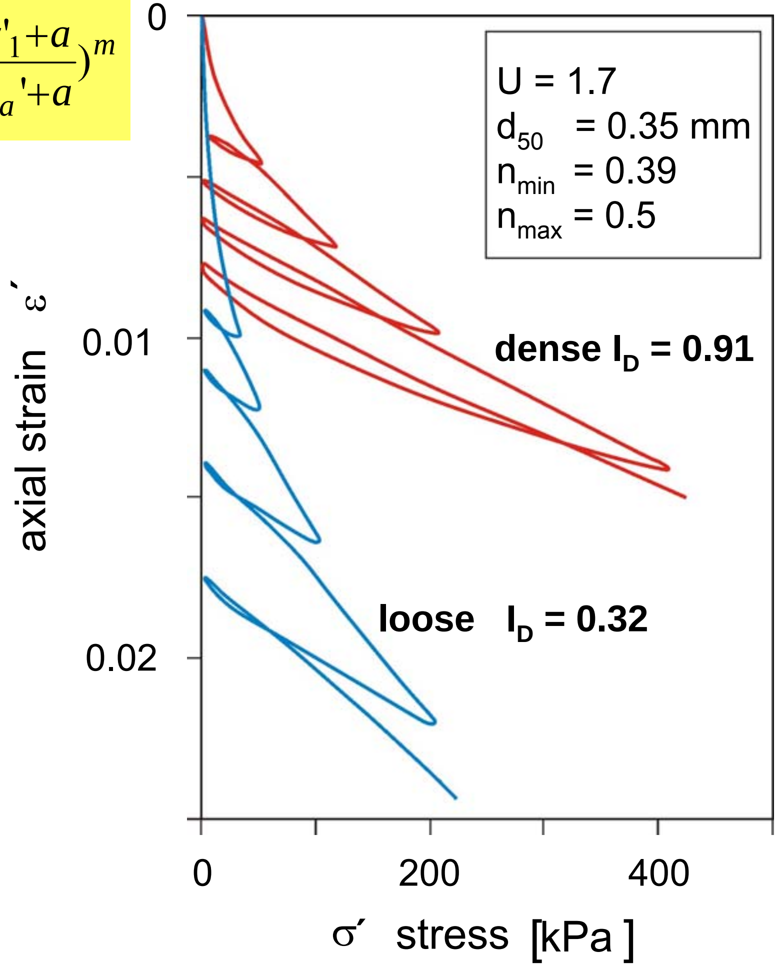
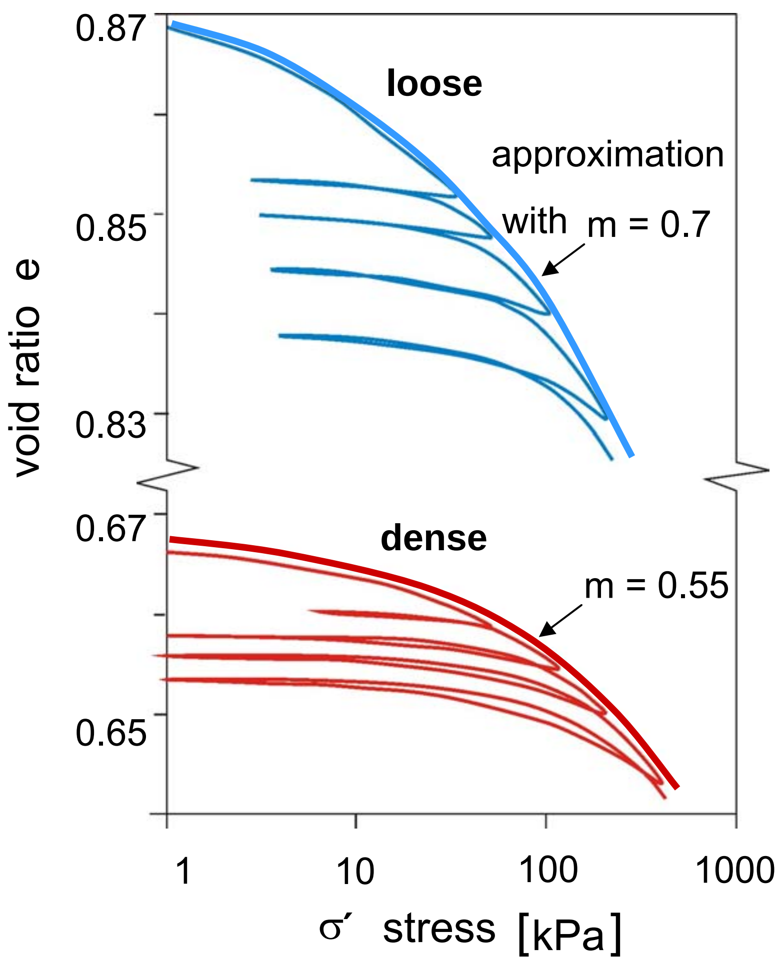
Parameters of the HS model

Parameters	Expression	Description
Deformation	E_{50}^{ref}	Reference modulus for primary loading in drained triaxial test
	E_{oed}^{ref}	Reference modulus for primary loading in oedometer test
	E_{ur}^{ref}	Reference modulus for unloading/reloading in drained triaxial test
	m	Modulus exponent for stress dependency
	ν_{ur}	Poisson's ratio for loading/unloading
Strength	c' ϕ' ψ	Effective cohesion at failure Effective friction angle at failure Dilatancy angle at failure
Special	σ_p POP: $(\sigma_p - \sigma_{vo}')$ OCR: σ_p/σ_{vo}'	Initial preconsolidation stress
	$K_o = K_o^{nc}\sqrt{OCR_o}$ $K_o^{nc} = 1 - \sin\phi'$	Earth pressure coefficient at rest

Oedometer tests on Hostun Sand

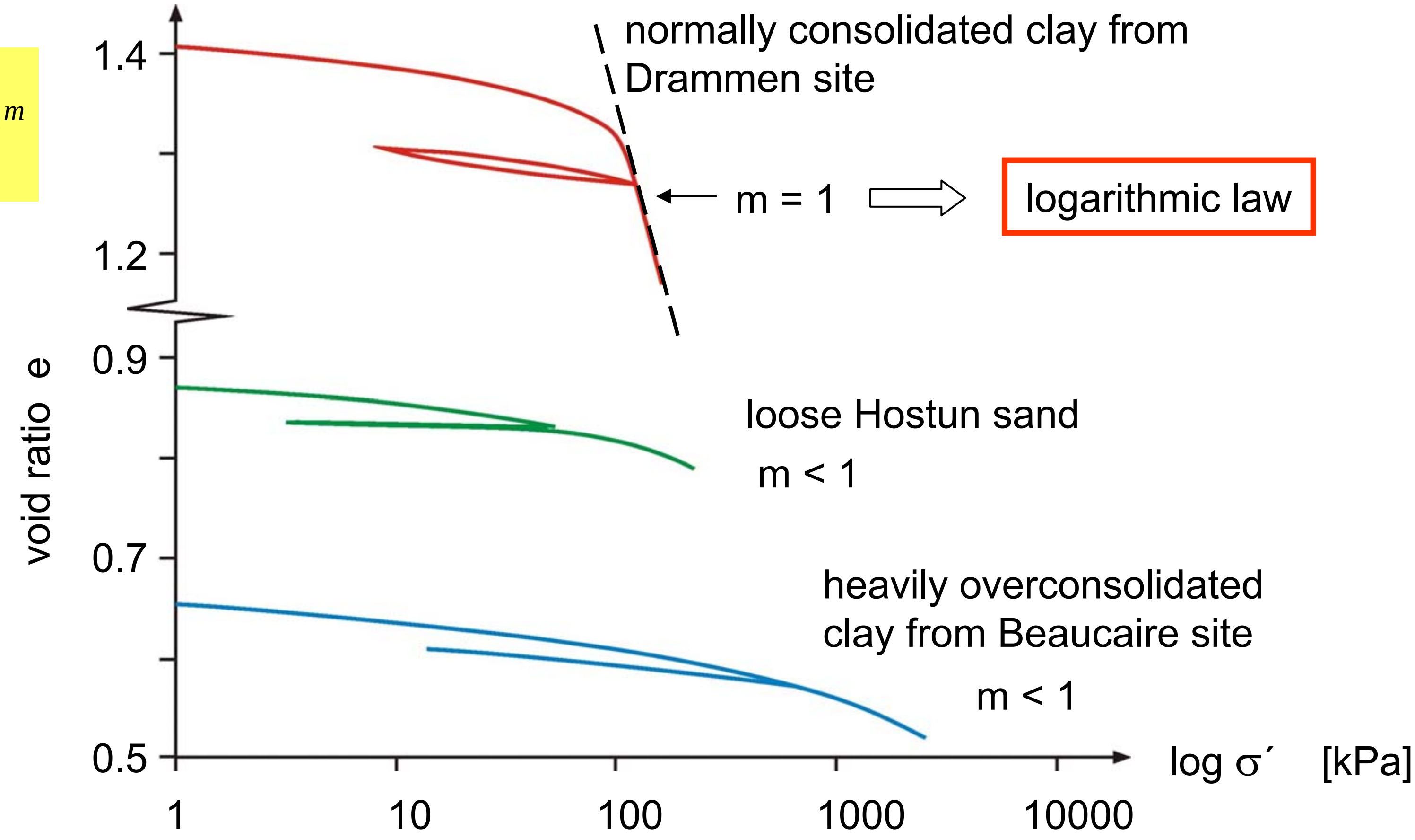
$$E_{oed} = E_{oed}^{ref} \left(\frac{\sigma'_1 + a}{p_a' + a} \right)^m$$





Modulus exponent m

$$E_{oed} = E_{oed}^{ref} \left(\frac{\sigma'_1 + a}{p_a' + a}\right)^m$$



clays: $m \approx 0.8 - 1$

sands: $m \approx 0.5 – 0.6$

Stiffness moduli

- Oedometer stiffness

$$E_{oed} = E_{oed}^{ref} \left(\frac{\sigma'_1 + a}{p_a' + a}\right)^m$$

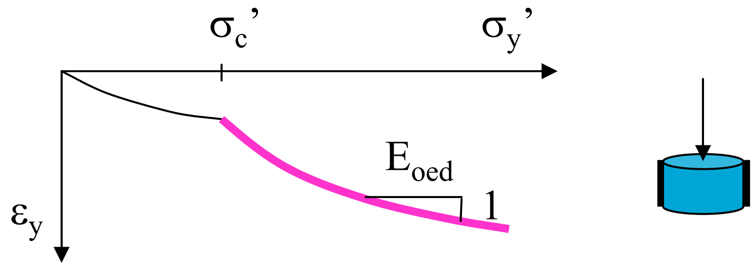
- Secant stiffness

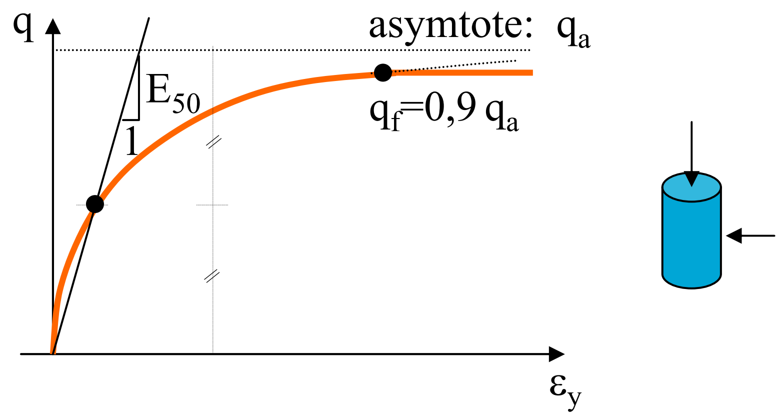
$$E_{50} = E_{50}^{ref} \left(\frac{\sigma_3' + a}{p_a' + a}\right)^m$$

$$\sigma_3 = K_0 \, \sigma_1$$

$$\frac{E_{oed}}{E_{50}} = \frac{E_{oed}^{ref} \, (\sigma_1 + a)^m}{E_{50}^{ref} \, (\sigma_3 + a)^m}$$

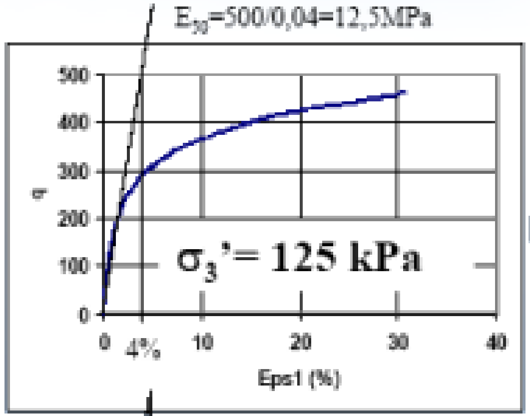
$$\text{For } c = 0: \quad \frac{1}{(K_0)^m}$$





The parameter E_{50}^{ref} for sand

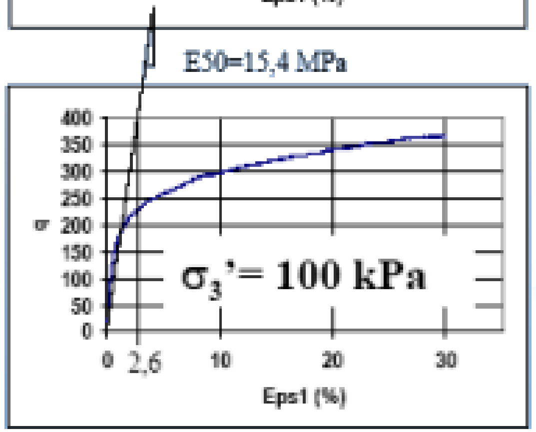
- Drained triaxial test on Hokksund sand, Dr = 20%



$$E_{50}^{ref} = E_{50}\sqrt{\frac{p_{ref}+a}{\sigma'_x + a}}$$

$$E_{50}^{ref} = 12{,}5MPa\sqrt{\frac{100+0}{125+0}} = 11MPa$$

Low stiffness for a loose sand!



$$E_{50}^{ref} = 15{,}4MPa$$

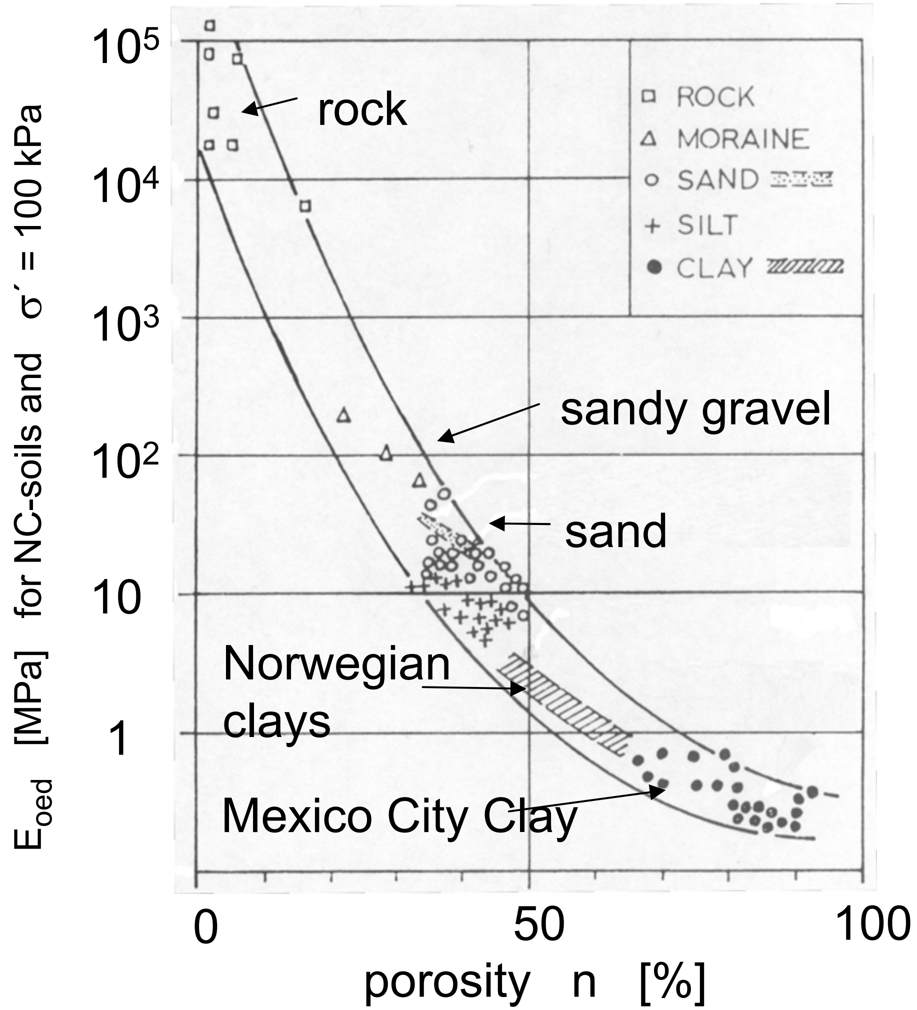
Normally:

Loose sands: E_{50}^{ref} = 15 MPa

Dense sands: E_{50}^{ref} = 50 MPa

Shaoli (2004), PhD Thesis, NTNU

Oedometer moduli for primary loading of NC-soils after Janbu (1963)



Janbu (1963) :

$$E_{oed} = E_{oed}^{ref} \cdot \left(\frac{\sigma' + a}{p_{ref} + a} \right)^m$$

with $a = c' \cot \varphi'$

Moduli for primary loading of NC-soils after von Sooss (2002)

primary loading of normally consolidated soils	γ_{sat} kN/m^3	n %	E_{oed} $[MPa]$ $\sigma' = 100$ kPa	m
very dense quartz sand very loose quartz sand	21 18	35 50	50 20	0.6
silt with liquid limit $w_L = 0.2$	19	45	5	0.75
clay with liquid limit $w_L = 0.6$	16	65	1	1.0

von Sooss (2002) provides much more detailed information on many soil types

Data from: Geotechnical Engineering Handbook, Vol. 1: (Fundamentals), Publisher: Ernst & Sons

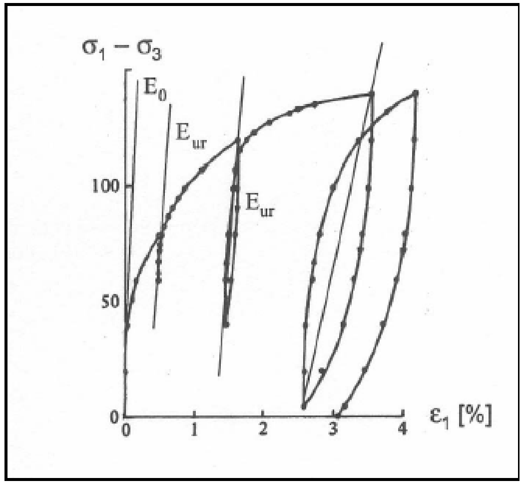
Compressibility of OC-clays and NC-clays after Tomlinson (1995)

Type	Qualitative description	Coefficient of volume compressibility m_v [m^2/MN]	E_{oed} [MPa]
Heavily overconsolidated boulder clays (e.g. many Scottish boulder clays) and stiff weathered rocks (e.g. weathered siltstone), hard London Clay, Gault Clay, and Oxford Clay (at depth)	Very low compressibility	Below 0.05	> 20
Boulder clays (e.g. Teeside, Cheshire) and very stiff ´blue´ London Clay, Oxford Clay, Keuper Marl	Low compressibility	0.05 – 0.10	10 – 20
Upper ´blue´London Clay, weathered ´brown´ London Clay, fluvio-glacial clays, Lake clays, weathered Oxford Clay, weathered Boulder Clay, weathered Keuper Marl, normally consolidated clays (at depth)	Medium compressibility	0.10 – 0.30	3 – 10
Normally consolidated alluvial clays (e.g. estuarine clays of thames, Firth of Forth, Bristol Channel, Shatt-al-Arab, Niger Delta, Chicago Clay), Norwegian ´Quick´Clay	High compressibility	0.30 – 1.50	0.7 - 3
Very organic alluvial clays and peats	Very high compressibility	Above 1.5	< 0.7

Note: Tomlinson does not indicate a stress level, but data seem to correspond to p_{ref} = 100 kPa.
Tomlinson (1995): Foundation Design and Construction, Pitman Publishing Inc.

Unloading stiffness

- Unloading-reloading stiffness

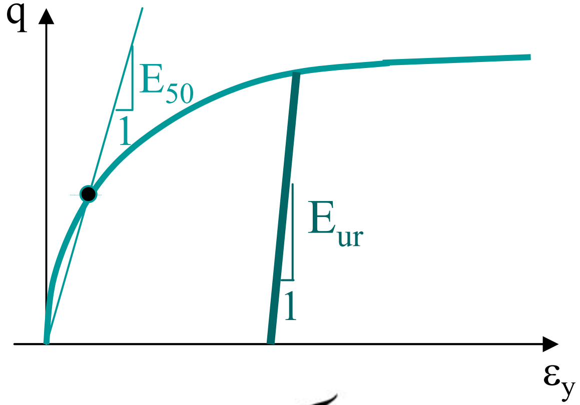


$$E_{ur} = E_{ur}^{ref} \left(\frac{\sigma_3' + a}{p_a' + a}\right)^m$$

$$\nu_{ur} = \quad low\ value$$

- Unloading is purely elastic

$$E_{ur} \approx 3\text{-}5 E_{50}$$



Unloading stiffness

Unloading stiffness in triaxial tests and oedometer tests are theoretically rather similar for $\nu_{ur} = 0.2$:

OEDO → $$E_{oed\,ur} = \frac{E_{ur}(1-\nu_{ur})}{(1-2\nu_{ur})(1+\nu_{ur})} = 1{,}11 \cdot E_{ur}$$ ← TRIAX

From experience: $E_{ur\ oed} \sim \alpha E_{oed}$

Note that σ_1' and σ_3' stress dependency complicates slightly:

$$E_{ur}^{ref}\left(\frac{\sigma_3'+a}{p_a'+a}\right)^m = \alpha \cdot E_{oed}^{ref}\left(\frac{\sigma'_1+a}{p_a'+a}\right)^m$$

$$E_{ur}^{ref} = \alpha \cdot E_{oed}^{ref}\left(\frac{\sigma'_1+a}{\sigma_3'+a}\right)^m = \alpha \cdot E_{oed}^{ref}/(K_0)^m$$

Unloading stiffness

Examples, observations from oedometer tests:

NC clay:	$E_{ur\ oed} \sim 10E_{oed}$
NC sand:	$E_{ur\ oed} \sim 3E_{oed}$
OC clay:	$E_{ur\ oed} \sim 3E_{oed}$

Utilizing $E_{ur}^{ref} = \alpha \cdot E_{oed}^{ref} / (K_0)^m$

NC clay:	$E_{ur}^{ref} \sim 10E_{oed}^{ref}/K_o \sim 20E_{oed}^{ref}$
NC sand:	$E_{ur}^{ref} \sim 3E_{oed}^{ref}/\sqrt{K_o} \sim 4E_{oed}^{ref}$
OC clay:	$E_{ur}^{ref} \sim 3E_{oed}^{ref}/K_o \sim 3E_{oed}^{ref}$

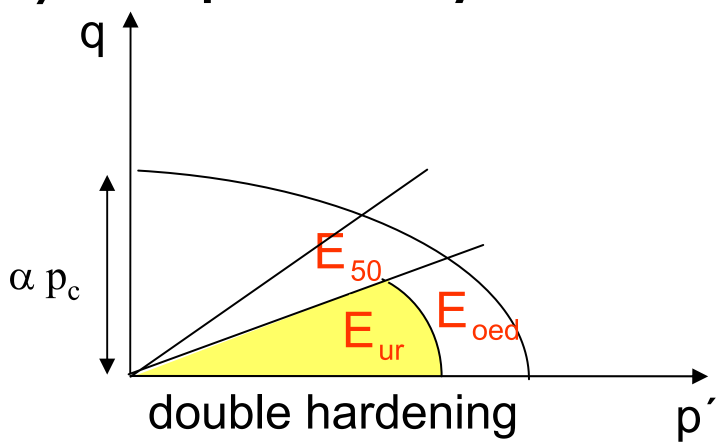
Compaction hardening in the HS model

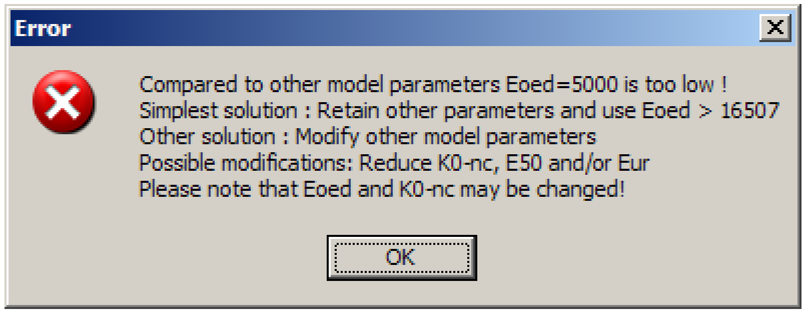
Yield function: $$f^c = \frac{\tilde{q}^2}{\alpha^2} + p^2 - p_p^2$$

Hardening rule: $$\varepsilon_v^{pc} = \frac{\beta}{1-m}\left(\frac{p_p}{p^{ref}}\right)^{1-m}$$

α is primarily determined by K_0^{nc}

β is primarily determined by E_{oed}





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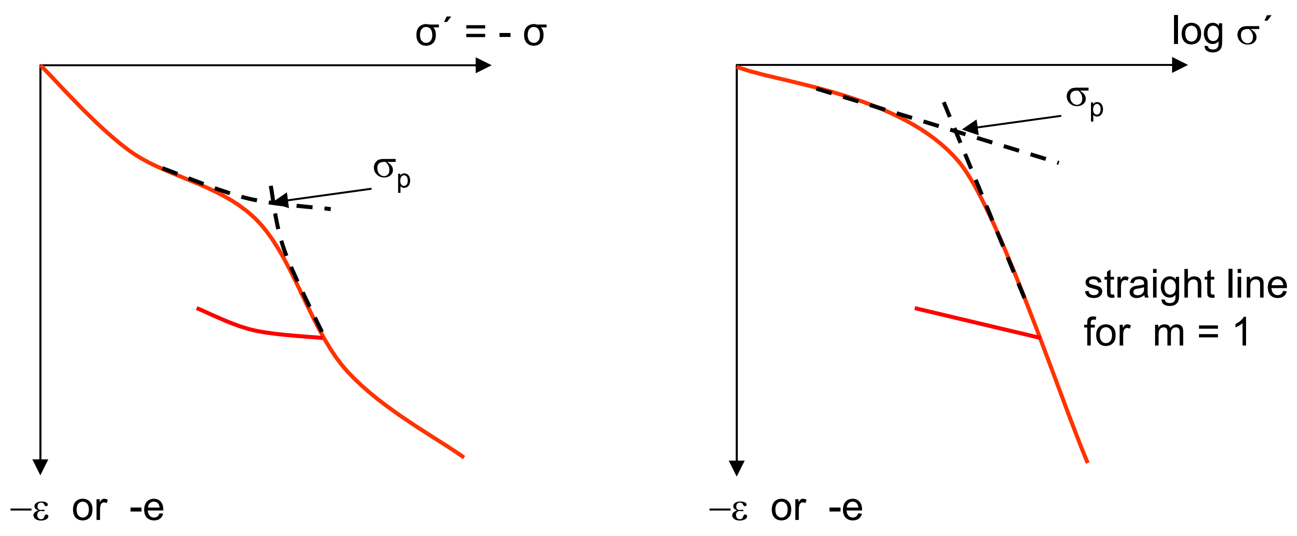
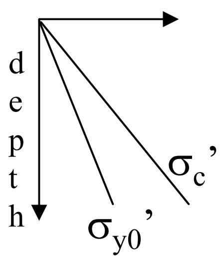
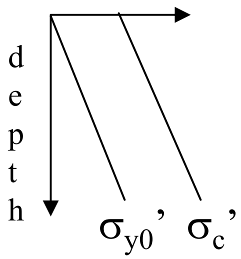
Preconsolidation pressure



σ_p = preconsolidation pressure

Initial stress

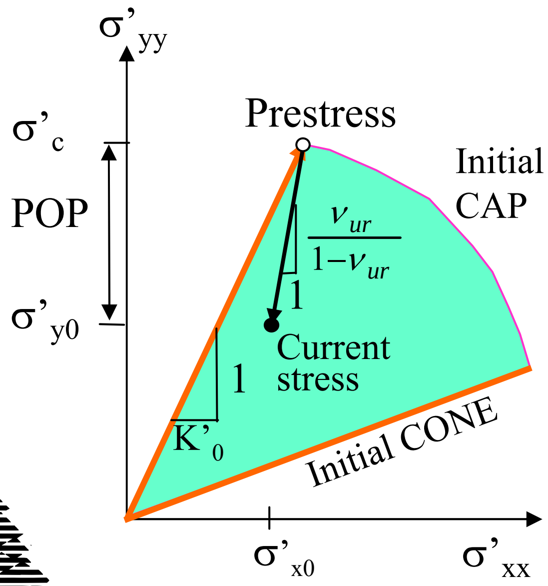




$$\sigma'_c = OCR \cdot \sigma_{y0}'$$

$$\sigma'_c = \sigma_{y0}' + POP$$

Preconsolidation is entered by OCR or POP relative to initial vertical stress. Converted to p_p by program



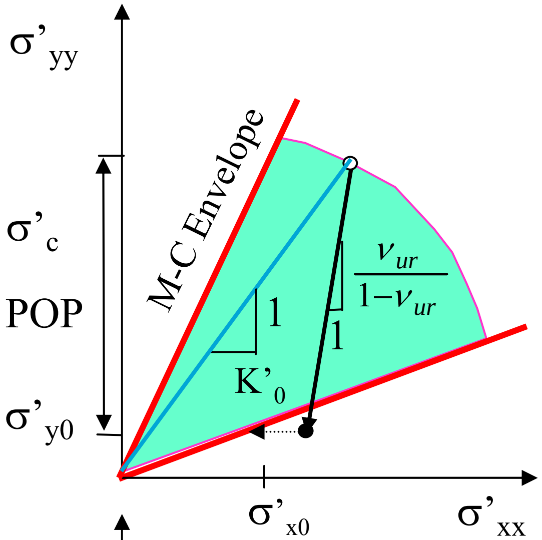
Initial horizontal stress by:

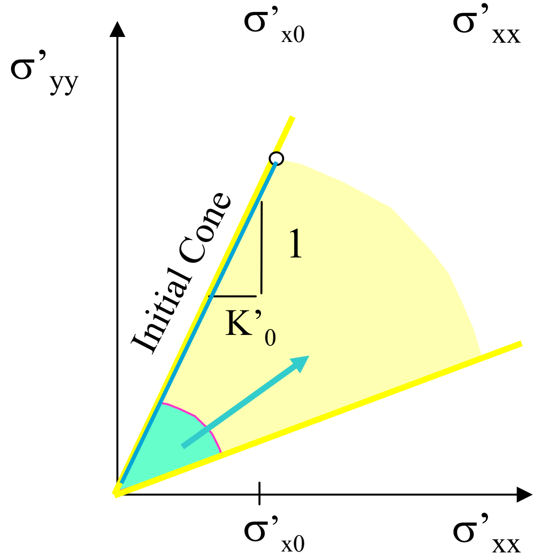
$$\sigma_{x0}' = K'_0 \cdot \sigma_c' - (\sigma_c' - \sigma_{y0}') \cdot \frac{\nu_{ur}}{1-\nu_{ur}}$$

Default:

$$K'_0 = 1 - \sin\varphi$$

Initial stress



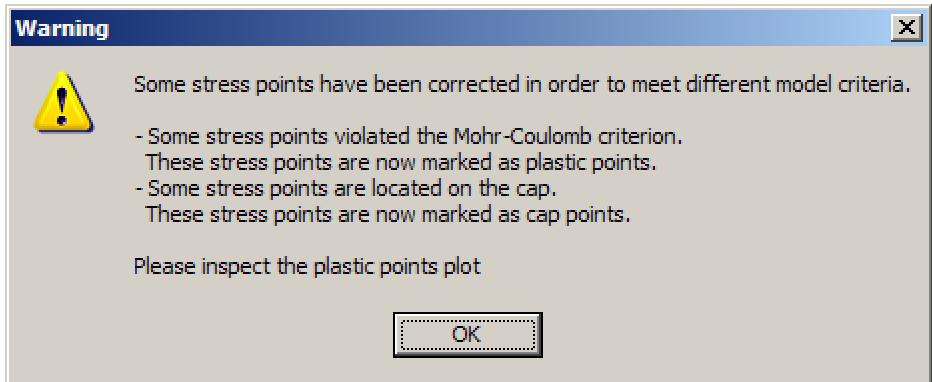


Initial horizontal stress by:

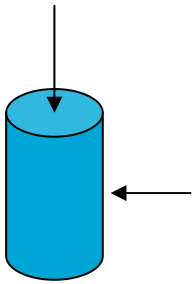
$$\sigma_{x0}' = K'_0 \cdot \sigma_c' - (\sigma_c' - \sigma_{y0}') \cdot \frac{\nu_{ur}}{1-\nu_{ur}}$$

Default:

$$K'_0 = 1 - \sin\varphi$$

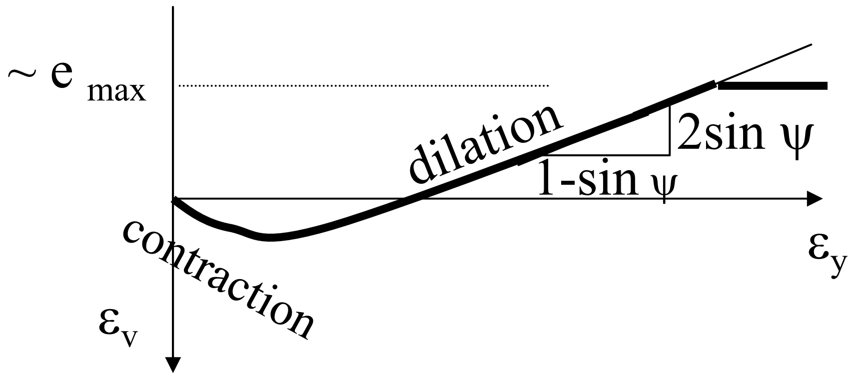


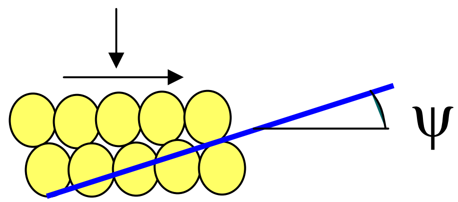
Dilatancy



Drained triaxial test

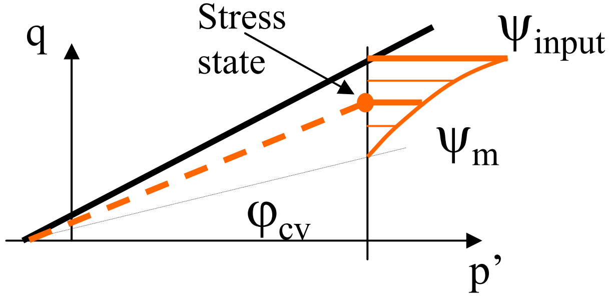
Sand: Often contraction
then dilation, Rowe (1962).





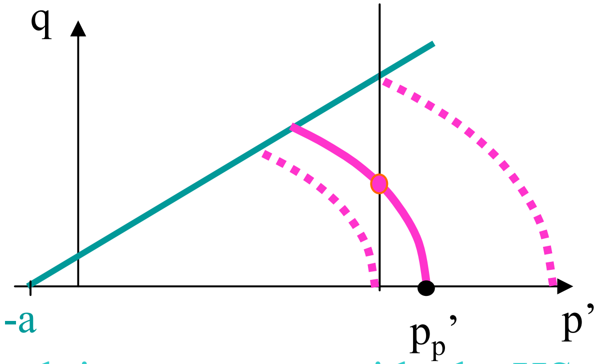
Dilatancy

- Dilatancy from cone (most important part)



Nonassociated cone flow: increasing dilatancy ψ_m from zero at φ_{cv} to input value ψ_{input} on MC line (Rowe).

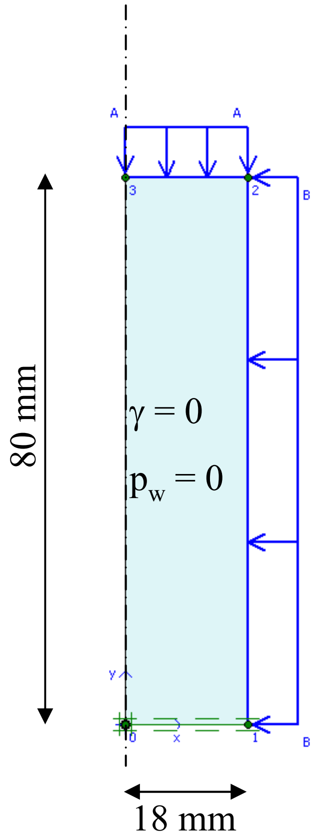
- Contractancy from cap (less dominating)



Associated cap flow: Increasing contractancy from zero to a maximum value at MC line, but only when cap moves!

Both interact to provide the HS model dilatancy behaviour

Drained triaxial test, soft clay, simulation



Axial symmetry

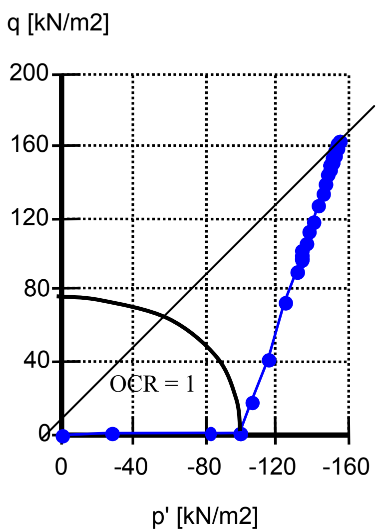
Soft clay by
Hardening soil
model:

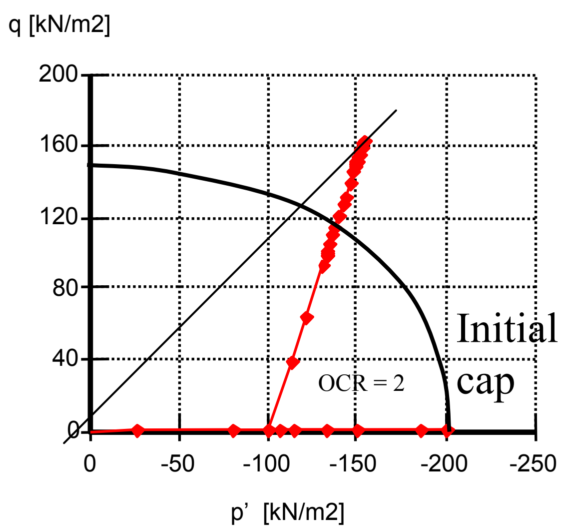
$E_{50}^{ref} = 2\ MPa$	$m = 1$	$\varphi = 25^o$
$E_{oed}^{ref} = 2\ MPa$	$\nu_{ur} = 0{,}2$	$\Psi = 0^o$
$E_{ur}^{ref} = 10\ MPa$	$c = 5\ kPa$	$K^o_{nc} = 0{,}577$

Zero initial stresses,
preconsolidation generated by preloading

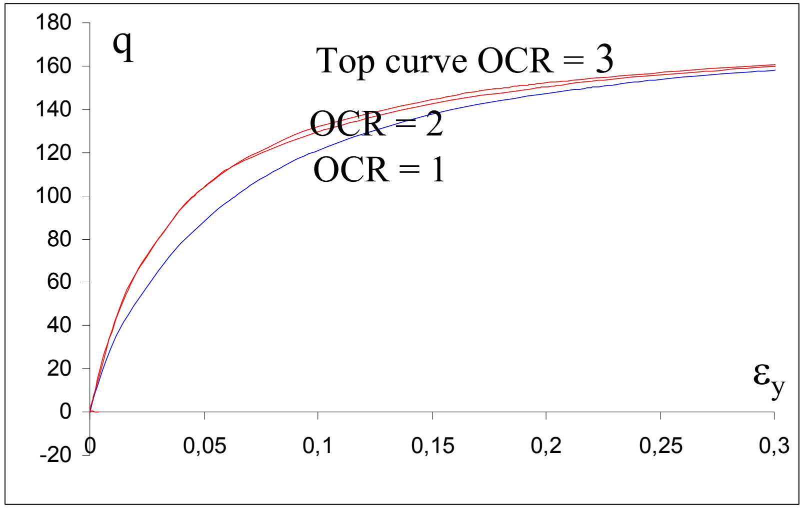
Drained triaxial test, soft clay, results

Stress paths:

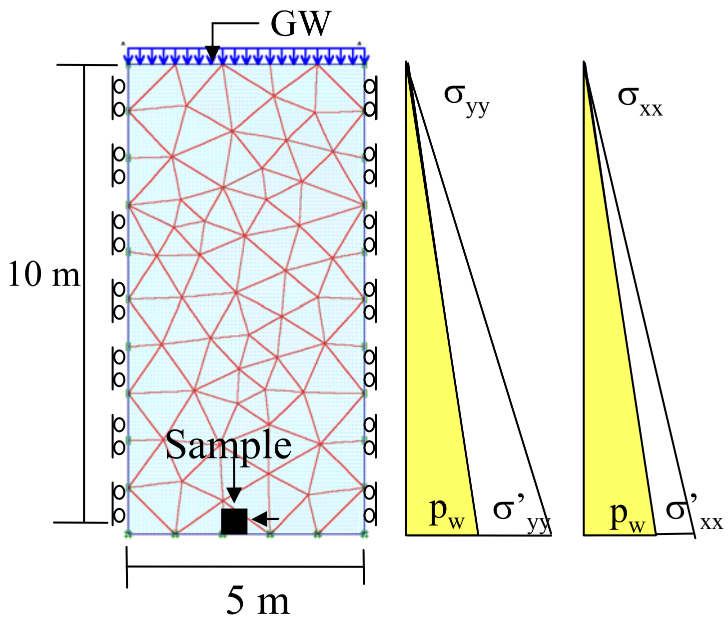




Effect of OCR:



Oedometer test simulation, soft clay



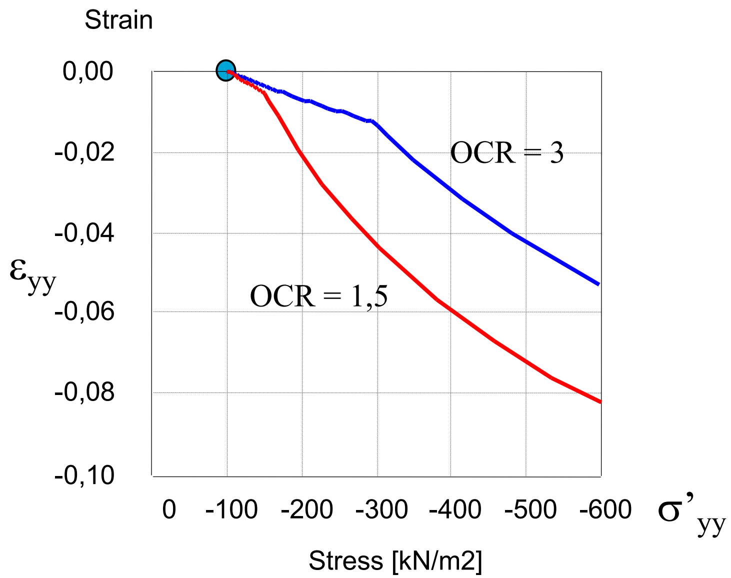
High "oedometer" cut out as a vertical column from the site in question.

Start the "test" from in situ stresses and specified preconsolidation for the sample studied.

Soft clay by Hardening soil model:

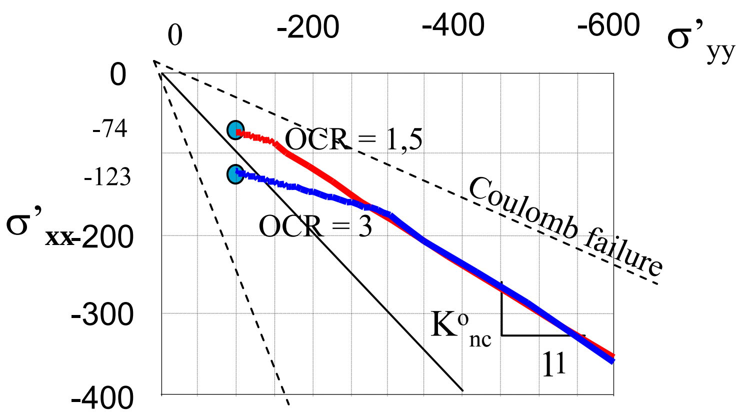
$E_{50}^{ref} = 2\ MPa$	m = 1	$\varphi = 25^{o}$
$E_{oed}^{ref} = 2\ MPa$	$\nu_{ur} = 0{,}2$	$\Psi = 0^{o}$
$E_{ur}^{ref} = 10\ MPa$	c = 5 kPa	$K^{o}_{nc} = 0{,}577$

Oedometer test results, soft clay



Note the preconsolidaton levels at:

$\sigma'_{yy} = 150$ kPa
$\sigma'_{yy} = 300$ kPa



Note how the initial state for OCR = 3 starts with $\sigma'_{xx} > \sigma'_{yy}$