CG 11

HARDENING SOIL MODEL

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CONTENTS

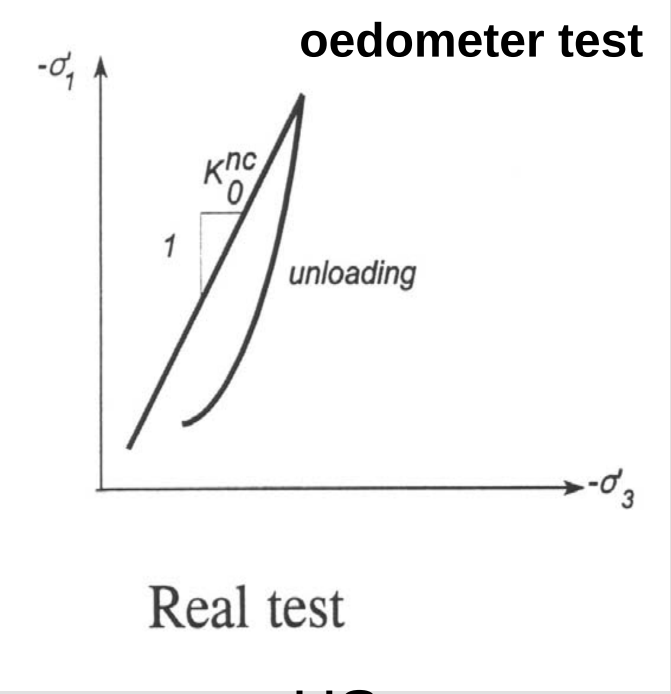
INTRODUCTION

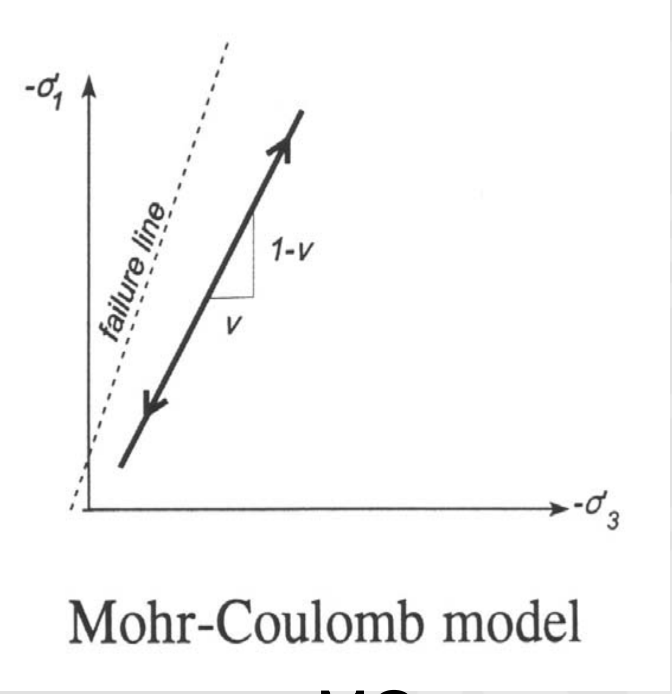
Soil behaviour includes:

- **difference in behaviour for primary loading – reloading/unloading**
- **nonlinear behaviour well below failure conditions**
- **stress dependent stiffness**
- **plastic deformation for isotropic or K_0-stress paths**
- **dilatancy is not constant**
- **small strain stiffness**
  (at very low strains and upon stress reversal)
- **influence of density on strength and stiffness**

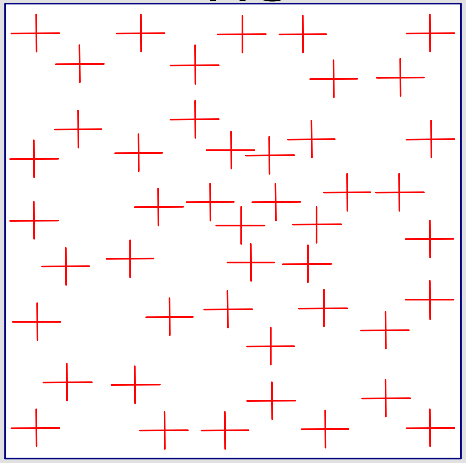
↳ **cannot be accounted for with simple elastic-perfectly plastic constitutive models**

INTRODUCTION

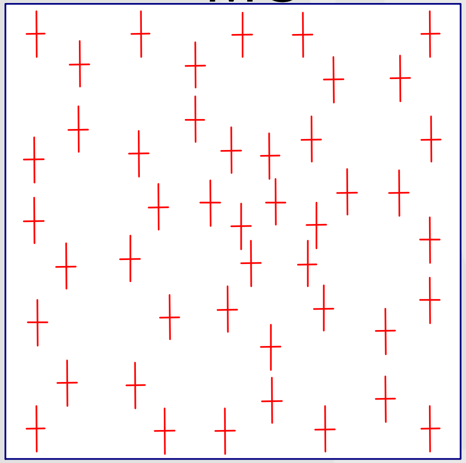




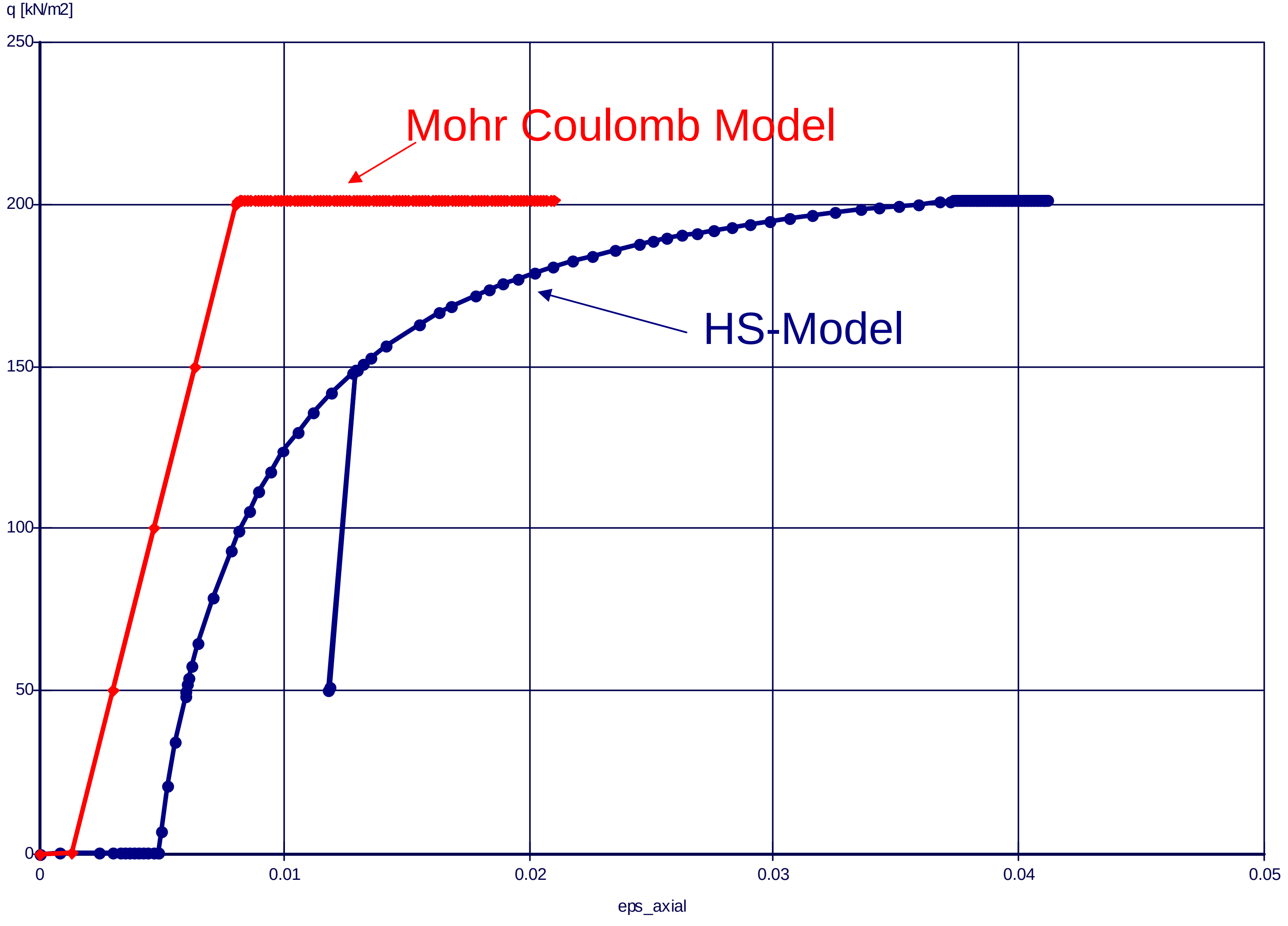
HS



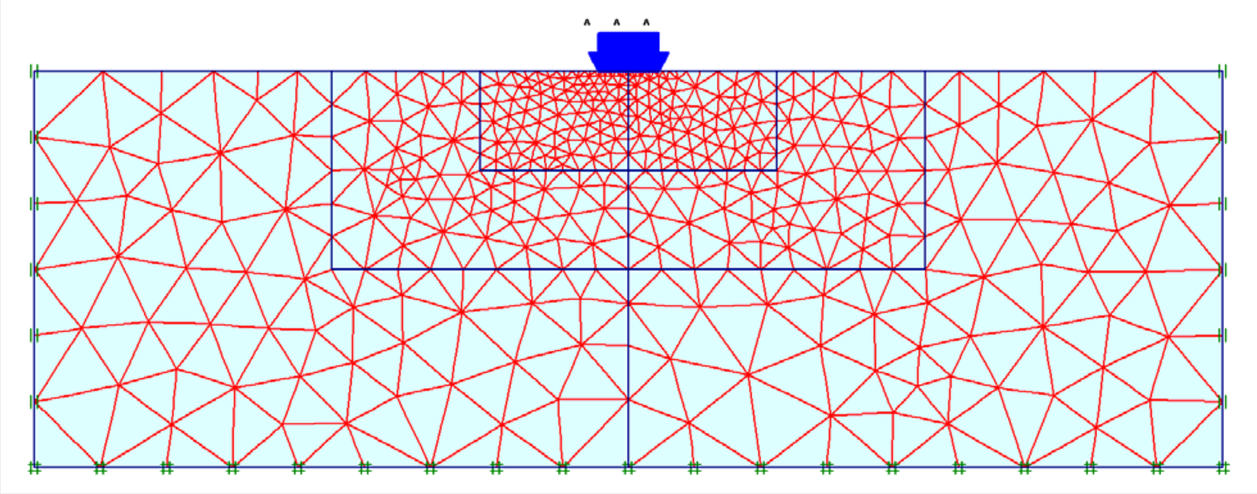
MC



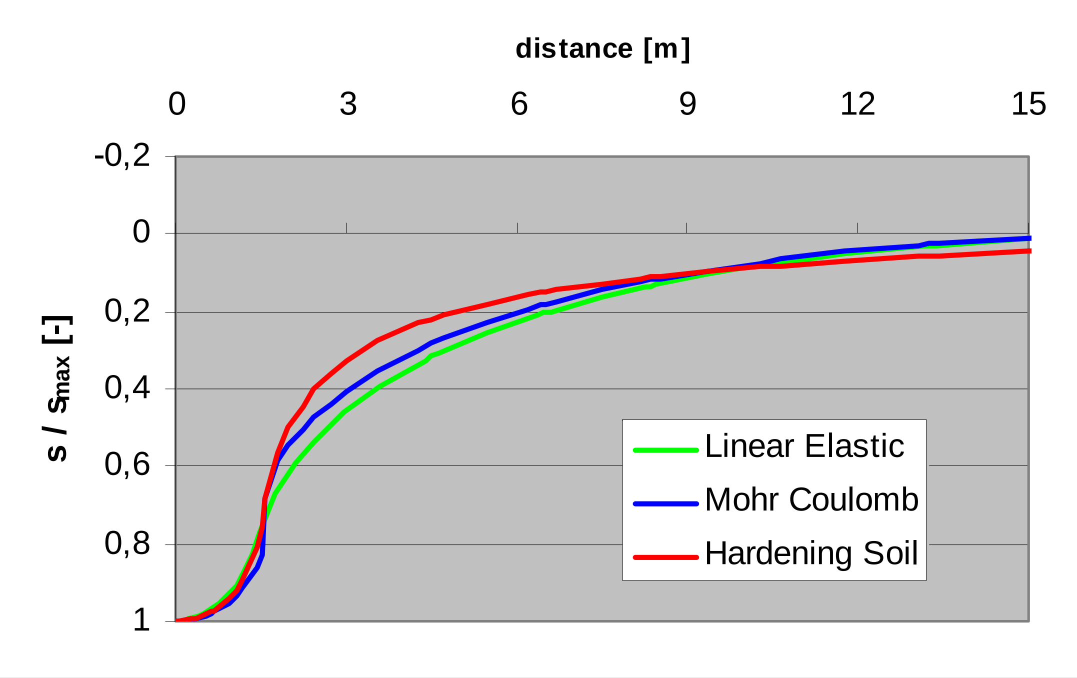
INTRODUCTION



INTRODUCTION



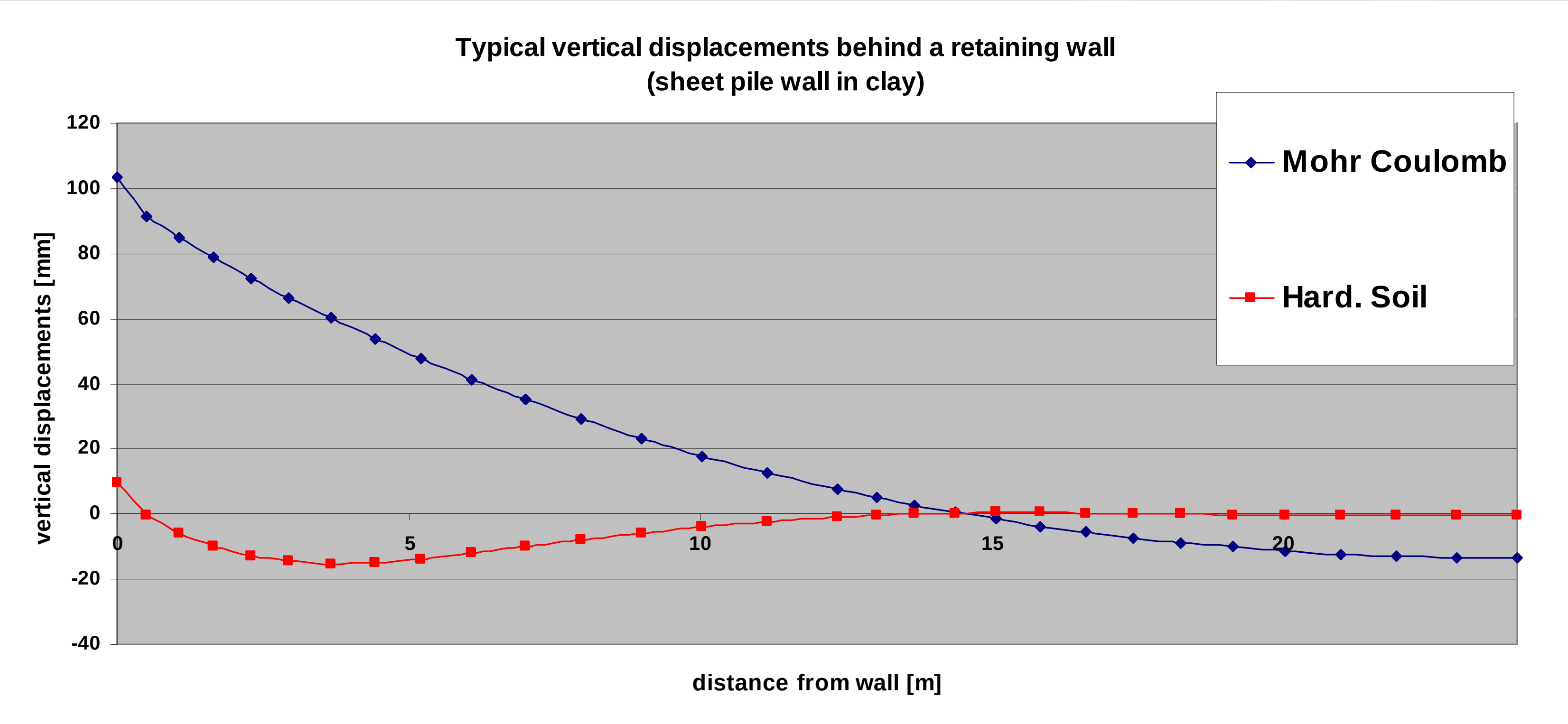
model	s_{max} [mm]
LE	33
MC	36
HS	60



- All models calculate settlements
- Differences in shape of trough and maximum values

INTRODUCTION

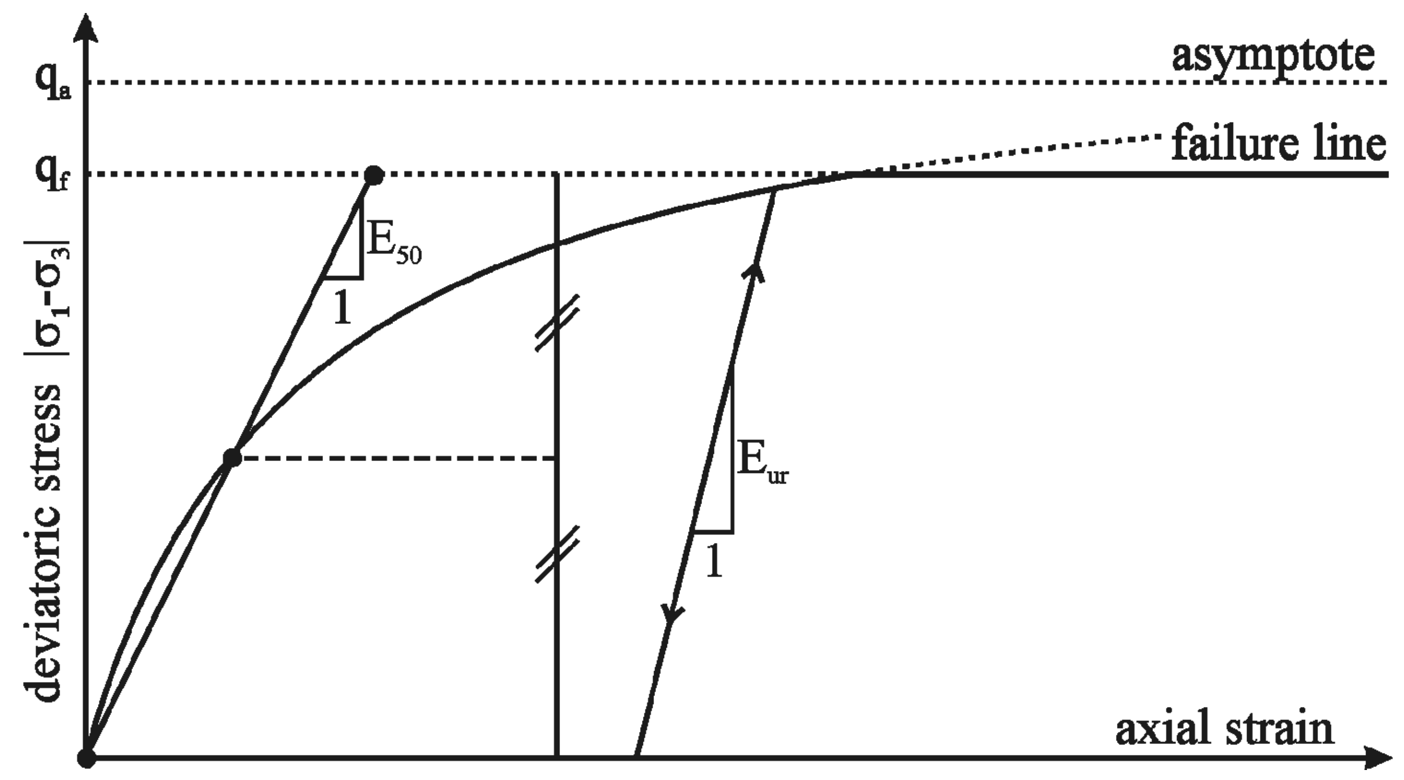
Example for vertical displacements behind a retaining wall



> Hardening Soil Model calculates **Settlements**

> Mohr-Coulomb Model calculates **Heave**

DESCRIPTION OF HARDENING SOIL MODEL



DESCRIPTION OF HARDENING SOIL MODEL

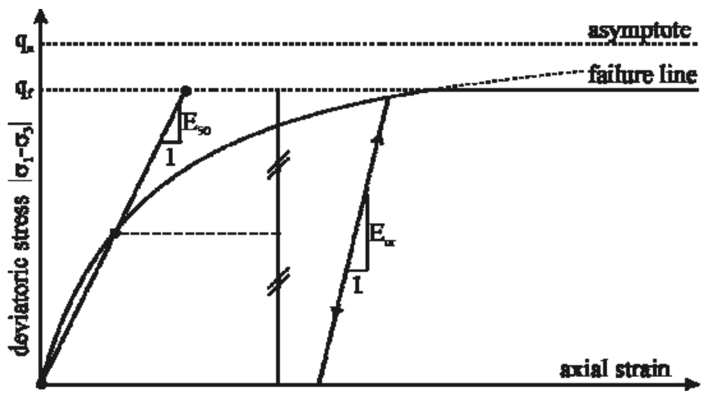
$$-\varepsilon_1 = \frac{1}{2\ E_{50}} \cdot \frac{q}{1 - q / q_a}$$

for q < q_f

q_a .... asymptotic value for shear strength

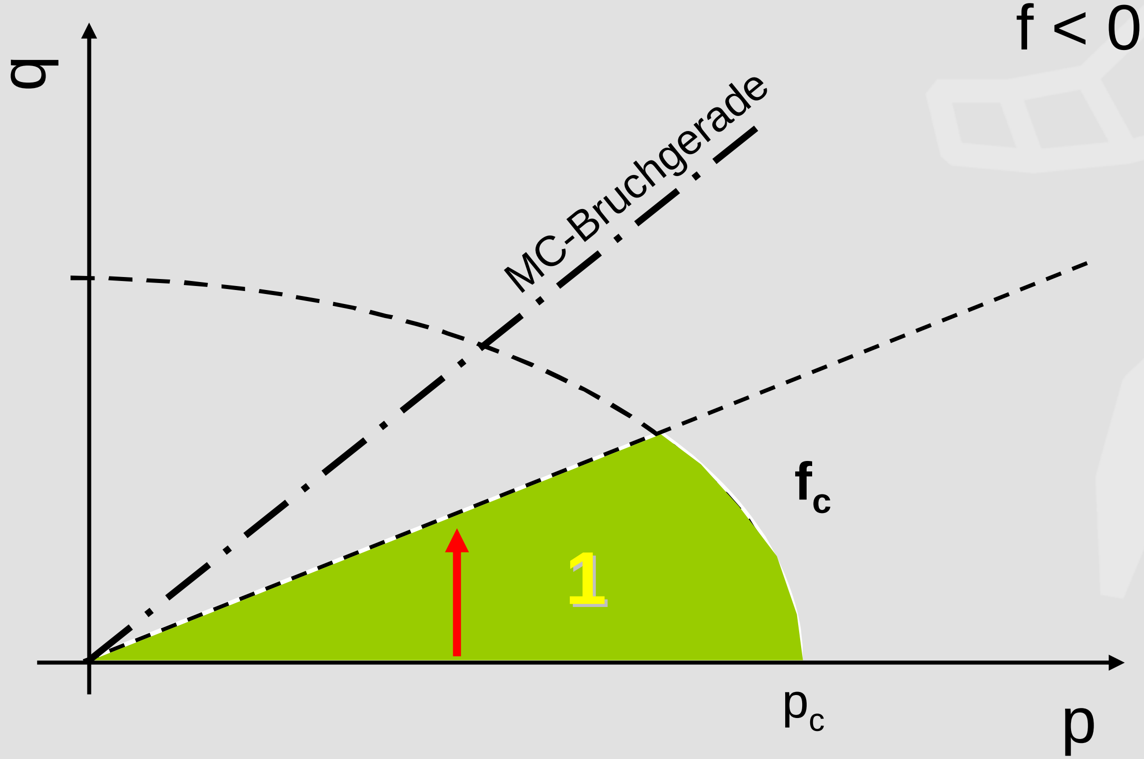
$$q_a = \frac{q_f}{R_f}$$

$$q_f = (c \cdot \cot\varphi - \sigma_3^{'}) \frac{2 \sin\varphi}{1 - \sin\varphi}$$



DESCRIPTION OF HARDENING SOIL MODEL

$f < 0 + f_c < 0$

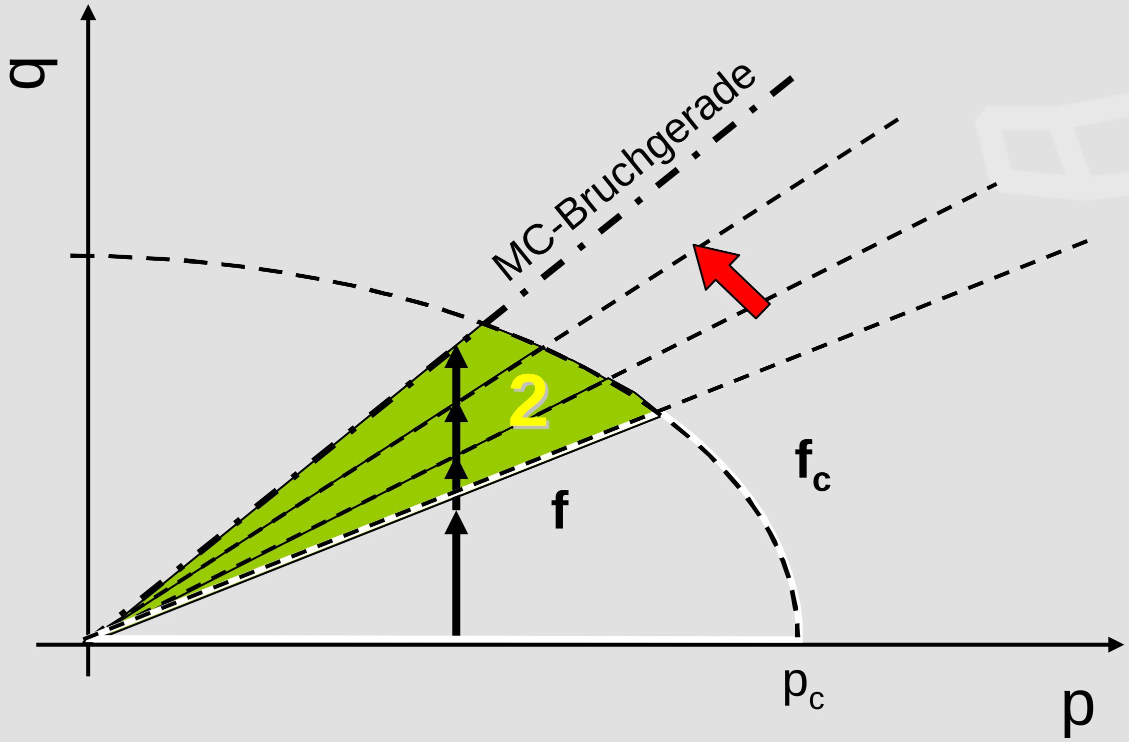


REGION 1

no yield surface active > elastic

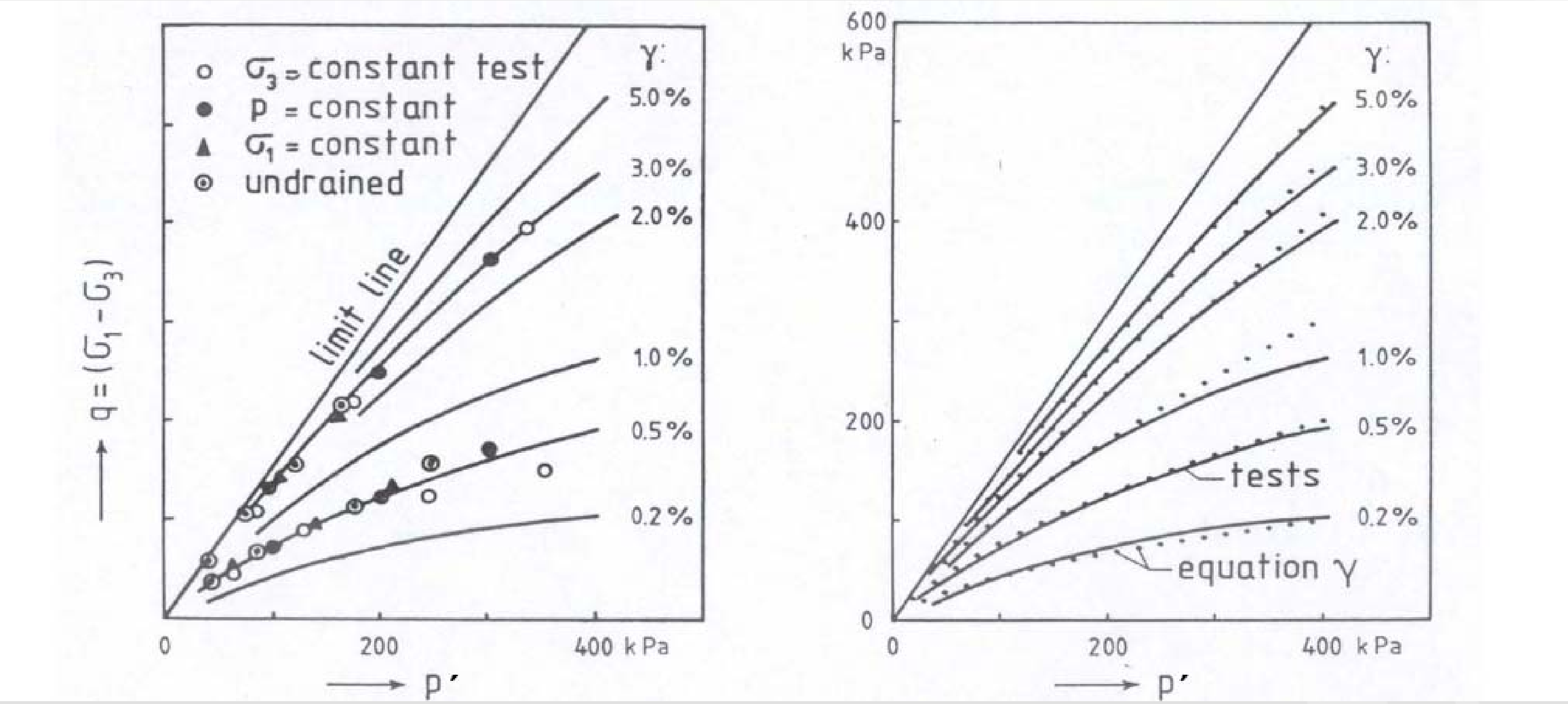
DESCRIPTION OF HARDENING SOIL MODEL

$f > 0 + f_c < 0$



REGION 2
shear hardening surface active

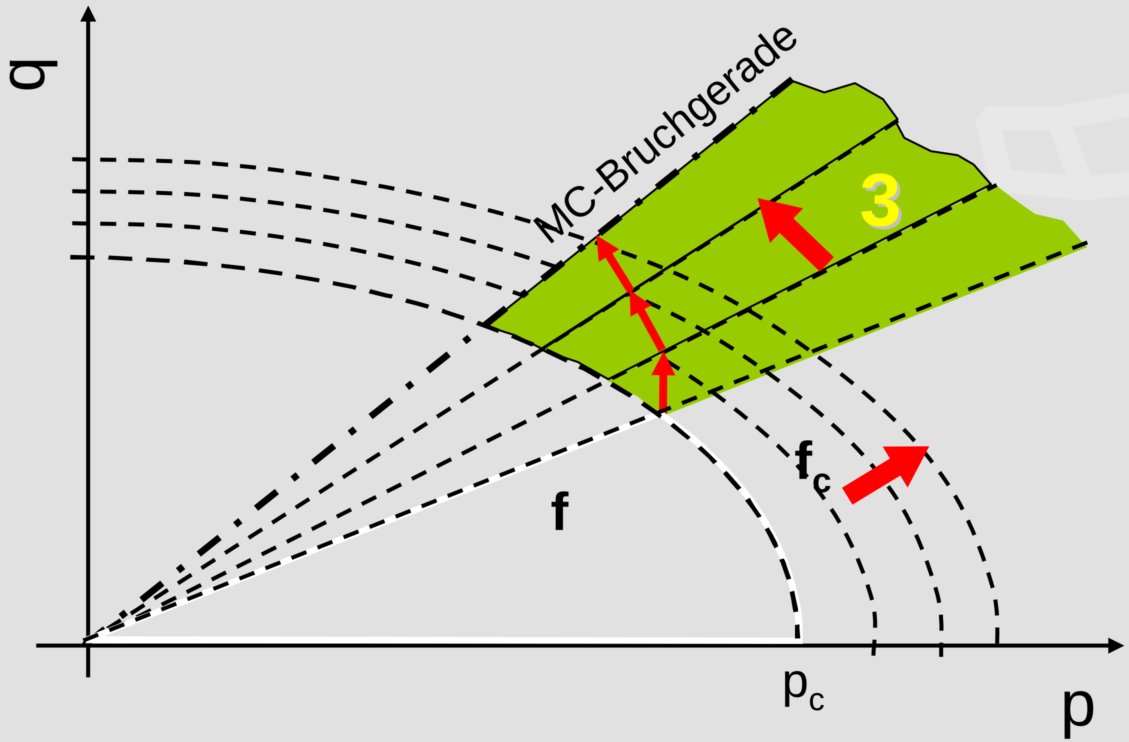
DESCRIPTION OF HARDENING SOIL MODEL



Deviatoric yield surface: lines of equal shear strains in triaxial test

DESCRIPTION OF HARDENING SOIL MODEL

$f > 0 + f_c > 0$

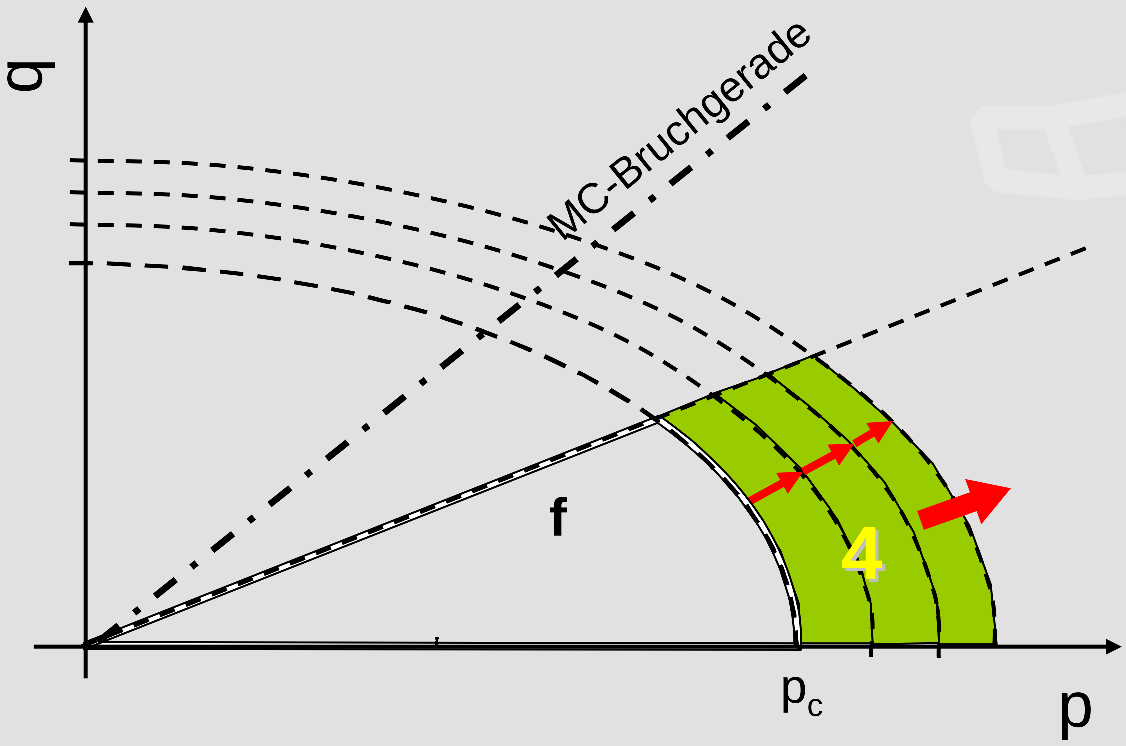


REGION 3

shear hardening and volumetric hardening surfaces active

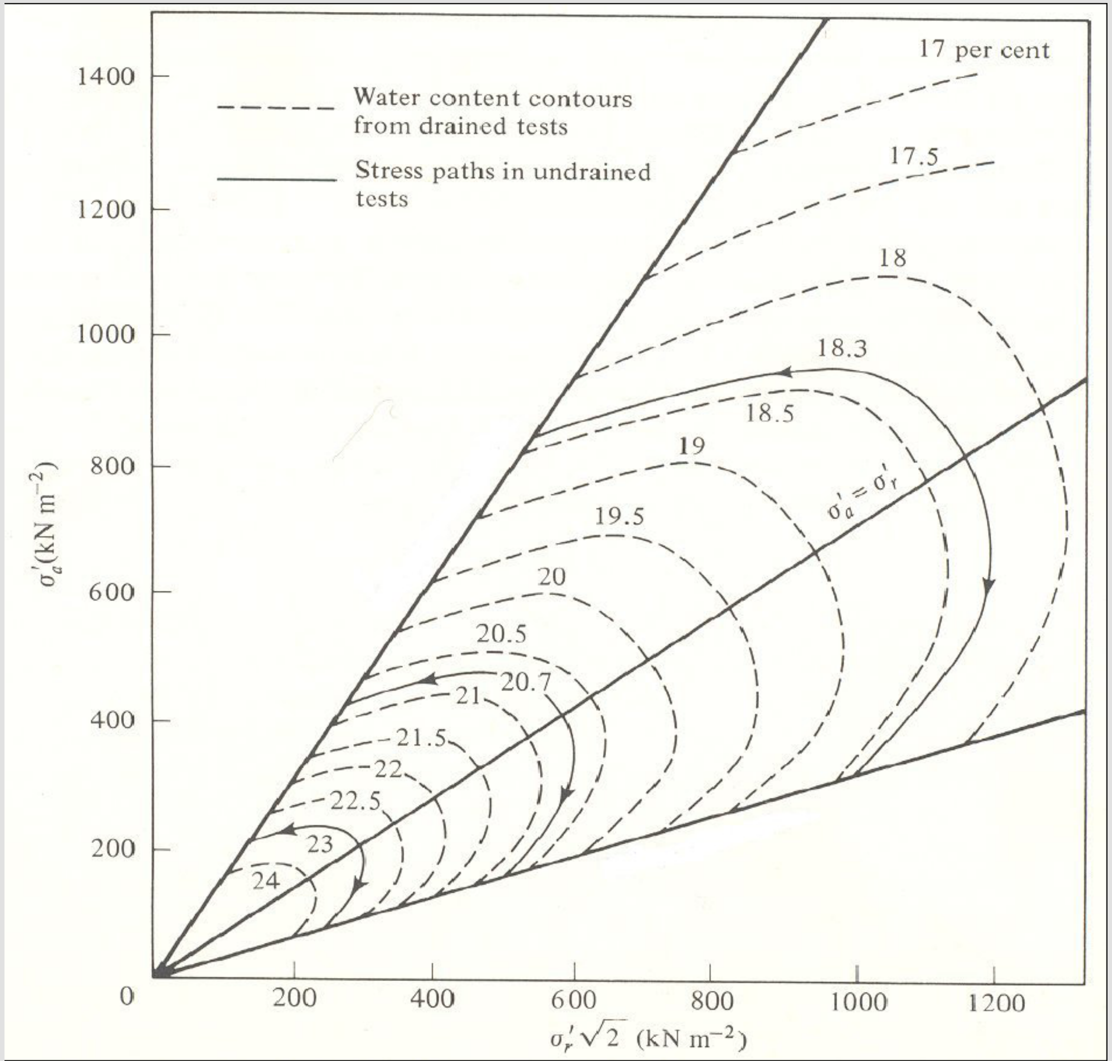
DESCRIPTION OF HARDENING SOIL MODEL

$f < 0 + f_c > 0$



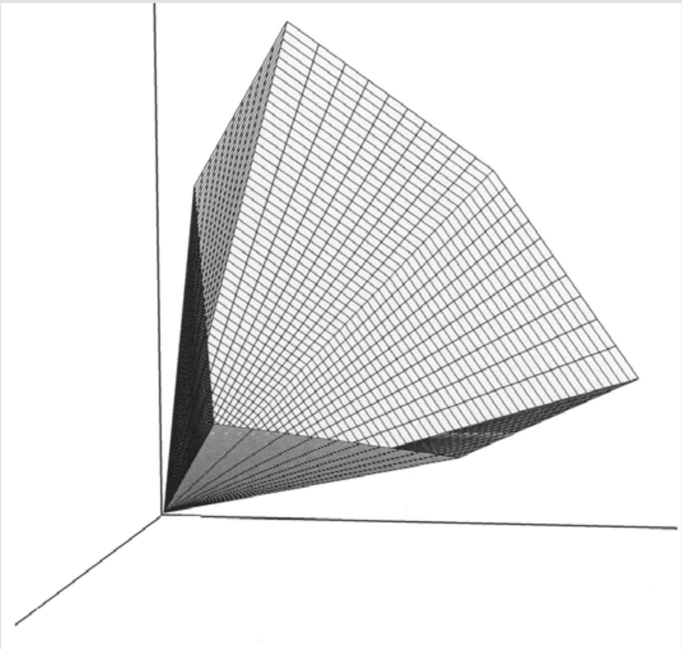
REGION 4
volumetric hardening surface active

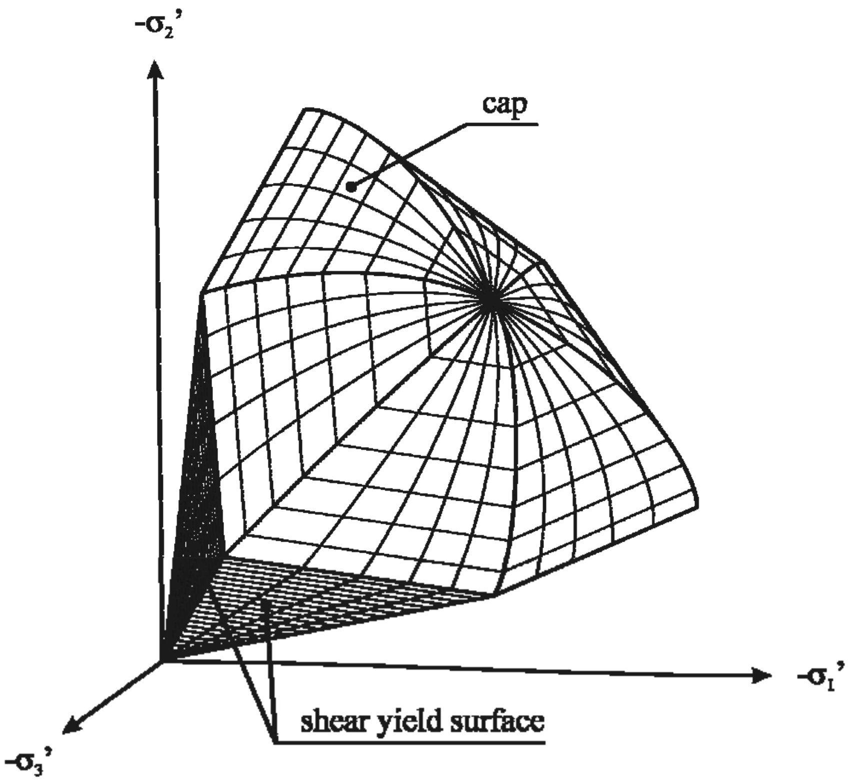
DESCRIPTION OF HARDENING SOIL MODEL



Volumetric yield surface:
lines of equal volumetric strains in triaxial test

DESCRIPTION OF HARDENING SOIL MODEL





DESCRIPTION OF HARDENING SOIL MODEL

$$E_{50} = E_{50}^{ref} \cdot \left(\frac{c \cdot \cot \varphi + \sigma_3'}{c \cdot \cot \varphi + p^{ref}} \right)^m$$

$$E_{ur} = E_{ur}^{ref} \cdot \left(\frac{c \cdot \cot \varphi + \sigma_3'}{c \cdot \cot \varphi + p^{ref}} \right)^m$$

$$E_{oed} = E_{oed}^{ref} \cdot \left(\frac{c \cdot \cot \varphi + \sigma_1'}{c \cdot \cot \varphi + p_{ref}} \right)^m$$

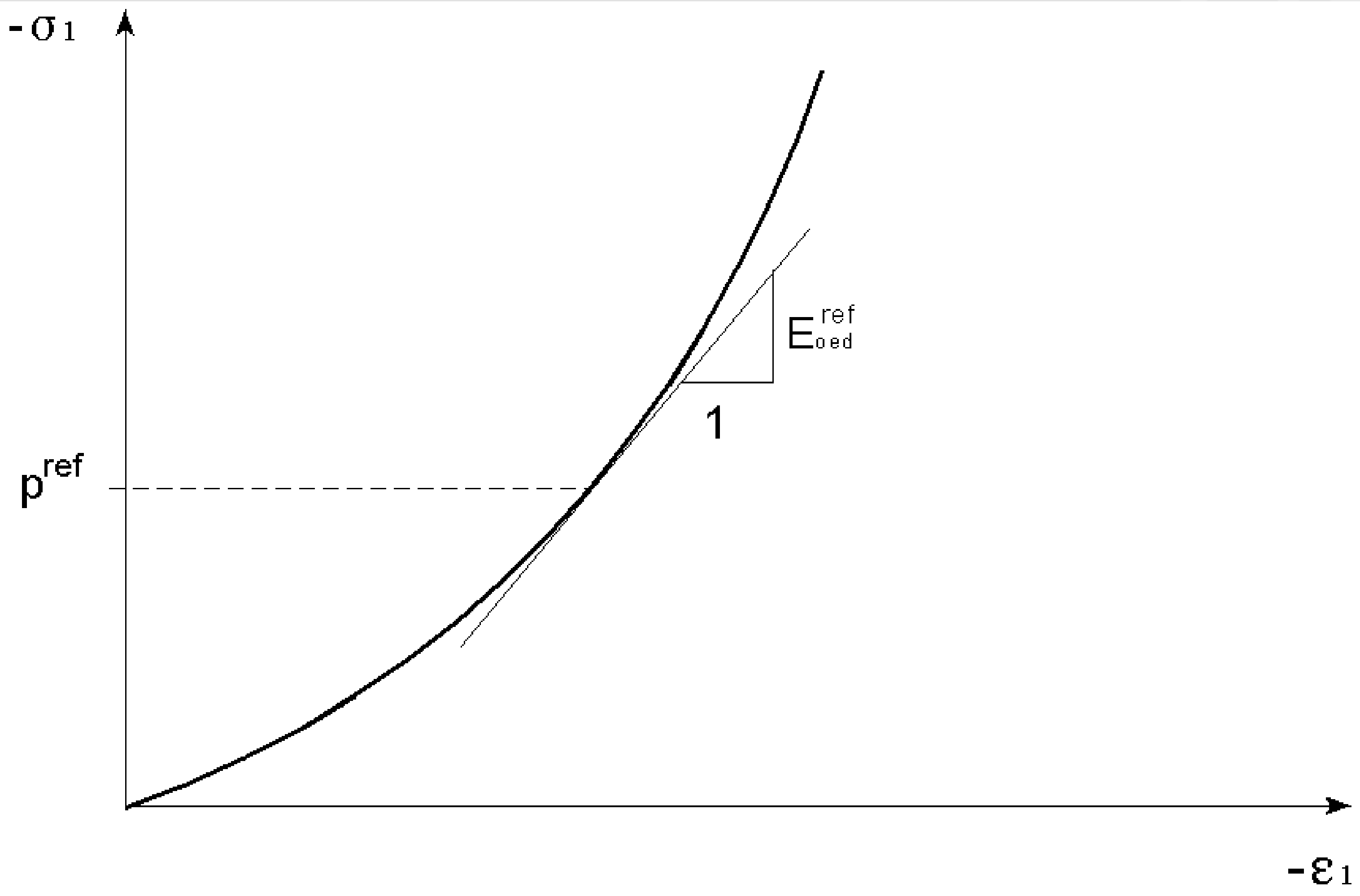
← holds strictly for K_0-stress paths only

p_{ref}:

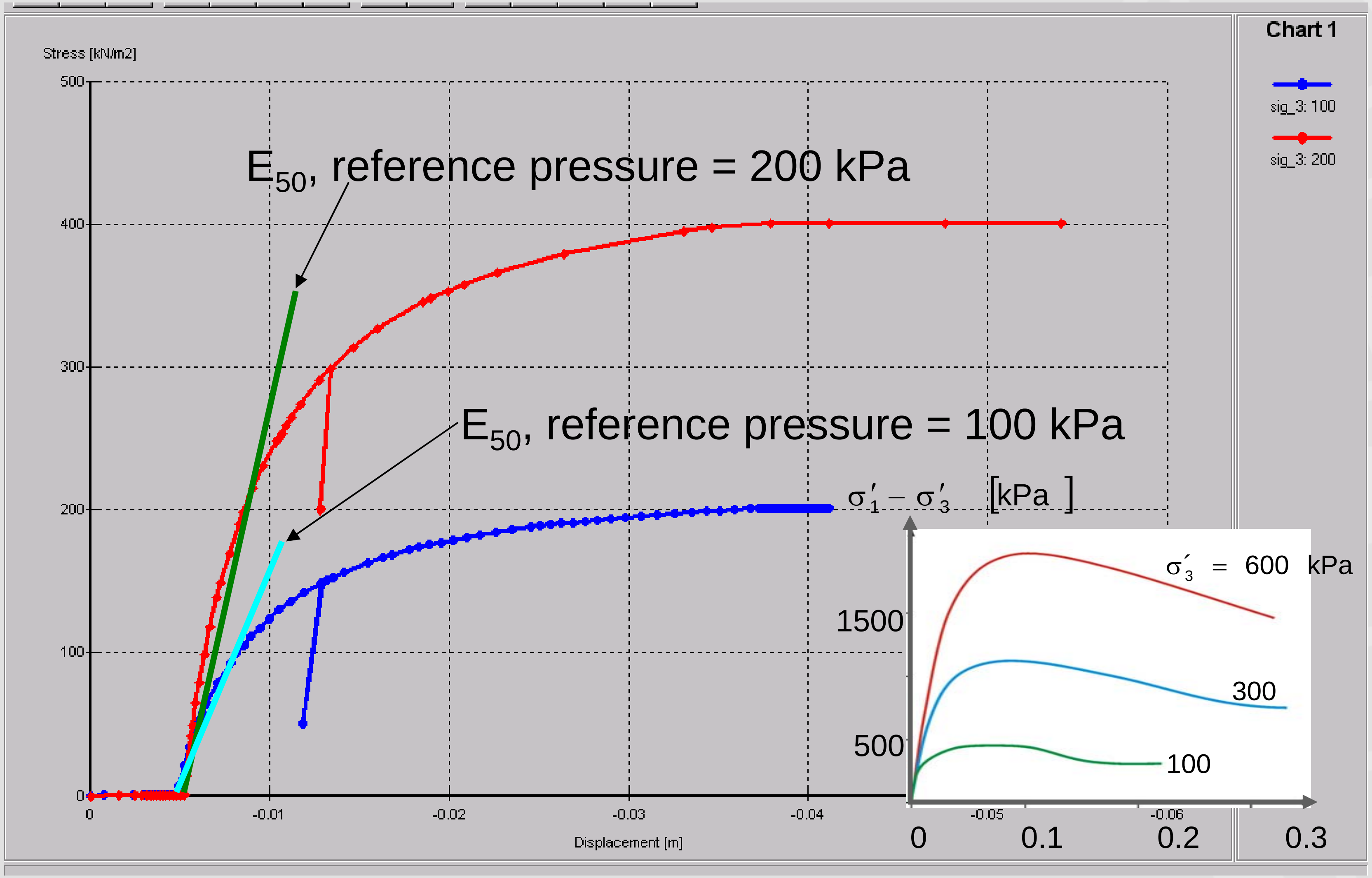
σ_3 for E_{50}

σ_1 for E_{oed}

DESCRIPTION OF HARDENING SOIL MODEL



DESCRIPTION OF HARDENING SOIL MODEL



DESCRIPTION OF HARDENING SOIL MODEL

Volumetric behaviour

"stress dilatancy theory" (Rowe, 1962)

dilatancy angle > non-associated flow rule

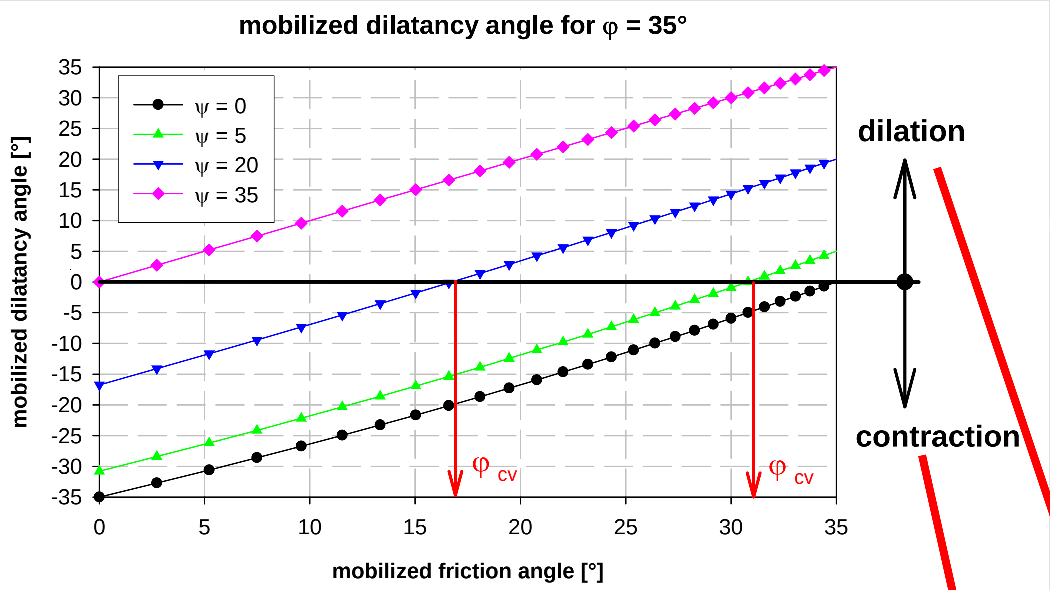
$$\dot{\varepsilon}_v^p = \sin\psi_m \dot{\gamma}^p$$

$$\sin\psi_m = \frac{\sin\varphi_m - \sin\varphi_{cv}}{1 - \sin\varphi_m \sin\varphi_{cv}}$$

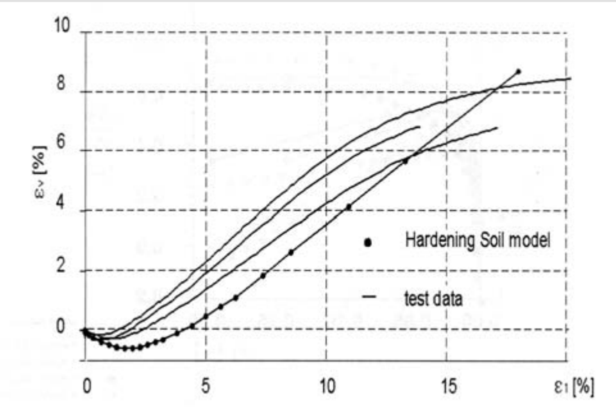
$$\sin\varphi_m = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3' - 2c \cdot \cot\varphi}$$

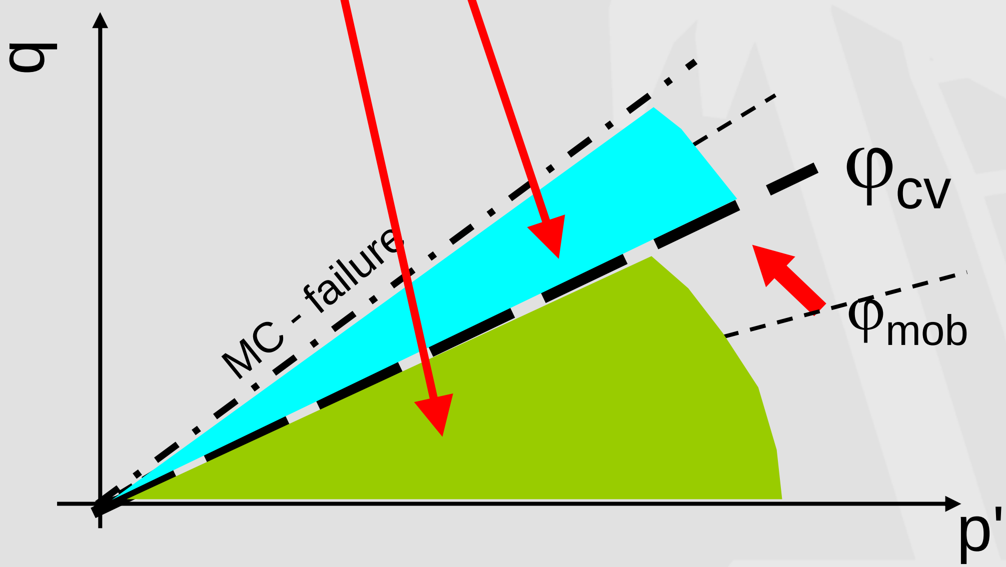
$$\sin\varphi_{cv} = \frac{\sin\varphi - \sin\psi}{1 - \sin\varphi \sin\psi}$$

DESCRIPTION OF HARDENING SOIL MODEL

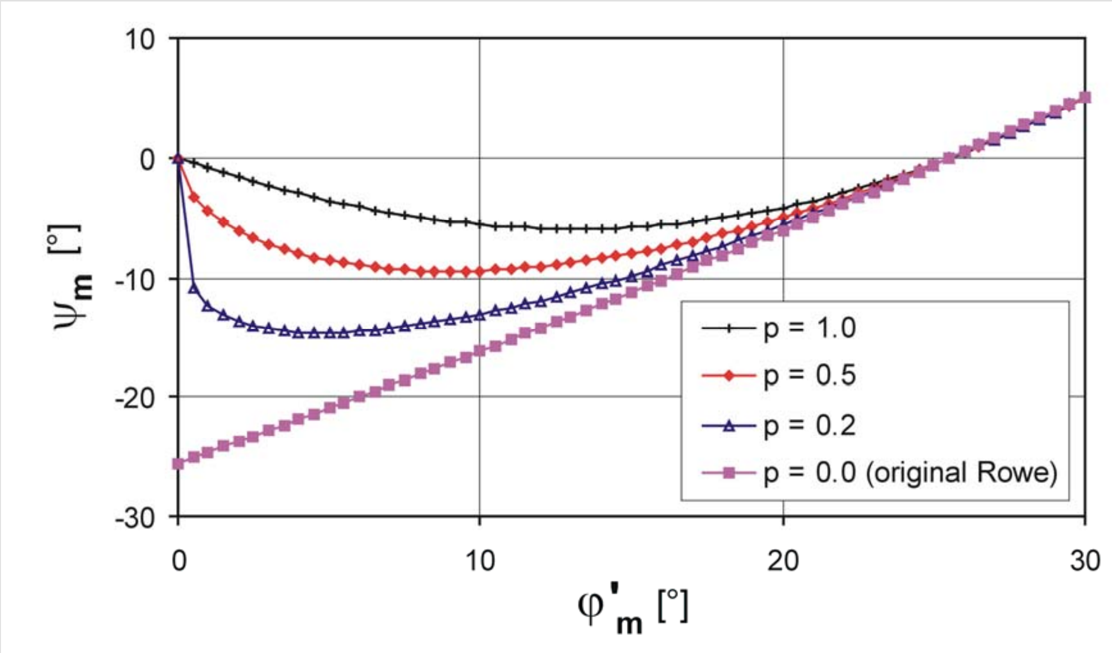


in reality negative values of ψ are cut-off in Plaxis



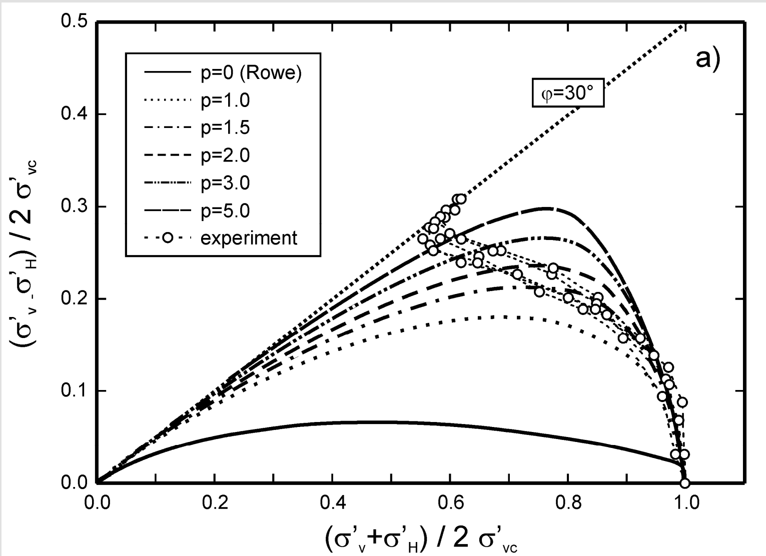


ALTERNATIVES TO ROWE



$$\sin \hat{\psi}_m = \sin \psi_m \left(\frac{\sin \varphi'_m}{\sin \varphi'} \right)^p$$

e.g. SOREIDE 2003



consequence of flow rule in undrained triaxial compression stress path

PARAMETERS OF HARDENING SOIL MODEL

φ'	friction angle
c'	cohesion
ψ'	dilatancy angle

E_{50}^{ref} **secant modulus from triaxial test** (controls deviatoric hardening)

E_{oed}^{ref} **tangential modulus from oedometer test**
(controls volumetric hardening)

E_{ur}^{ref} **unloading / reloading modulus**

default: $E_{ur}^{ref} = 3\ E_{50}^{ref}$

m **power for stress dependency of stiffness**

PARAMETERS OF HARDENING SOIL MODEL

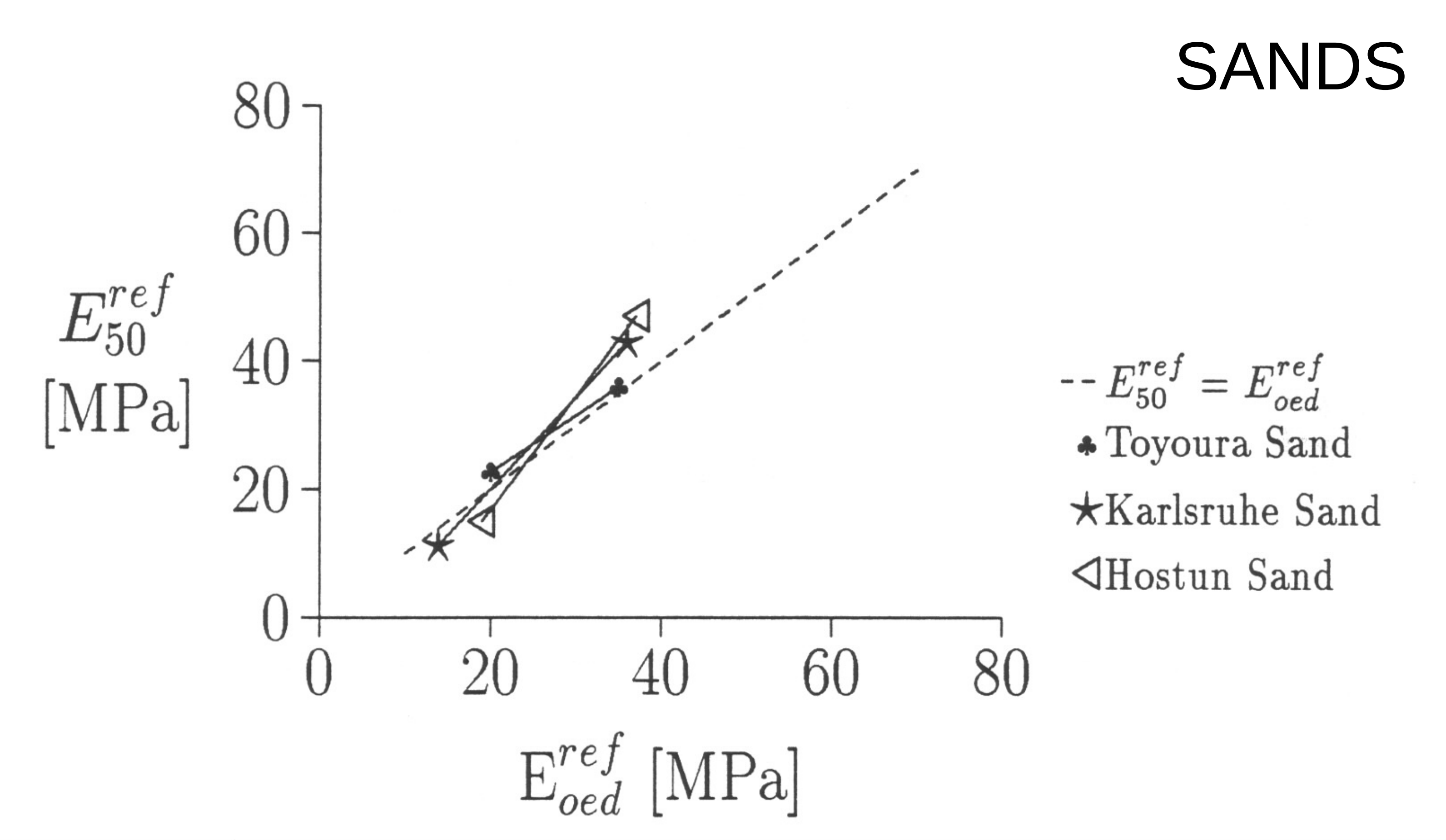
ν_{ur} **Poisson ratio for unloading / reloading** (default ν_{ur} = 0.2)

p^{ref} **reference stress**
(default p^{ref} = 100 stress units)

K_0^{nc} **K_0-value for normal consolidation** (default = 1-sinφ)
(controls volumetric hardening)

$\mathbf{R_f = q_f / q_a}$ (default R_f = 0.9)

PARAMETERS OF HARDENING SOIL MODEL

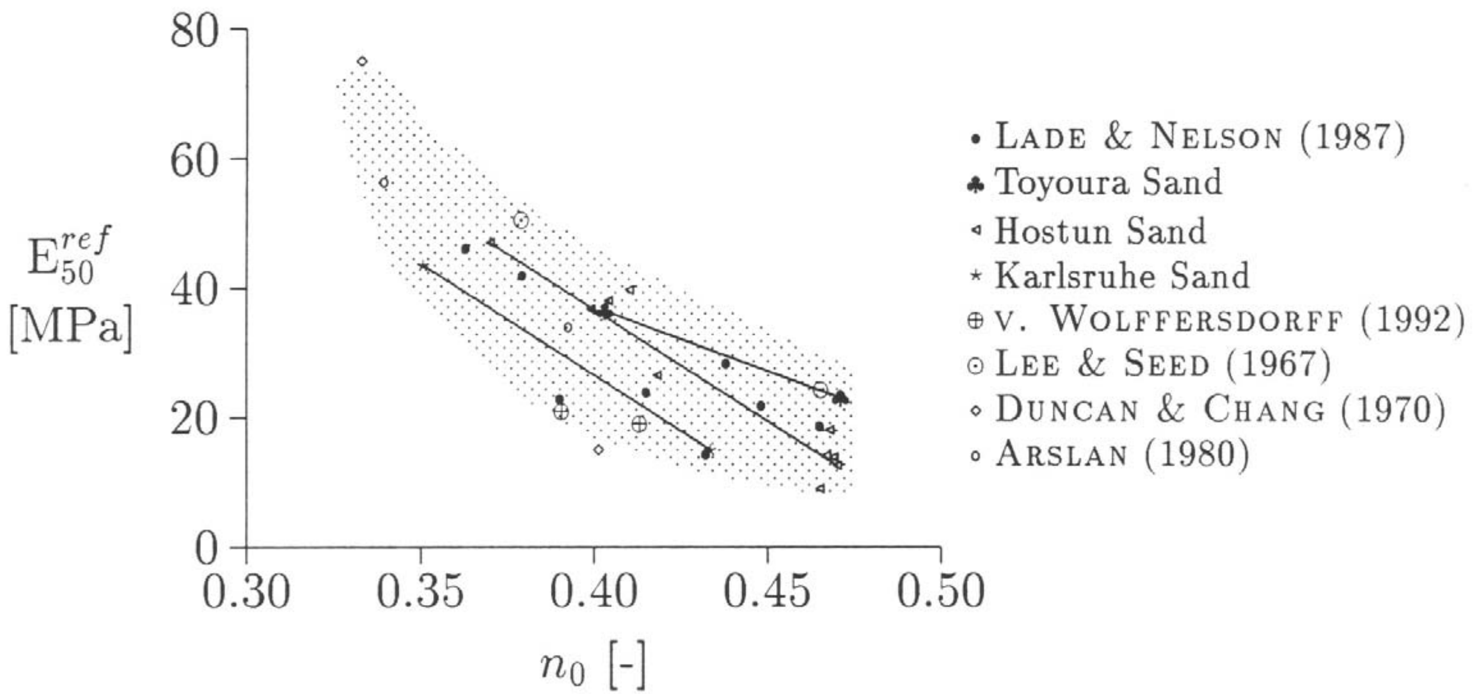


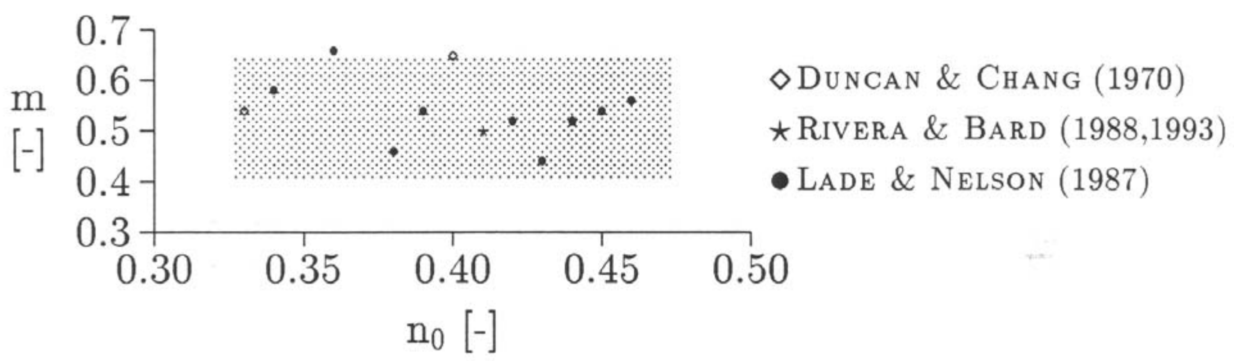
$$E_{oed}^{ref} \approx I_D \bullet 60MPa$$

Correllation for p^{ref}=100 kPa (Lengkeek)

I_D = relative density

PARAMETERS OF HARDENING SOIL MODEL





PARAMETERS OF HARDENING SOIL MODEL

For normally consolidated clays (m=1):

$$E_{oed}^{ref} \approx \tfrac{1}{2} E_{50}^{ref}$$

Order of magnitude (very rough)

$$E_{oed}^{ref} \approx \frac{50000\,kPa}{I_p}$$

Correlation with I_p for p^{ref} = 100 kPa

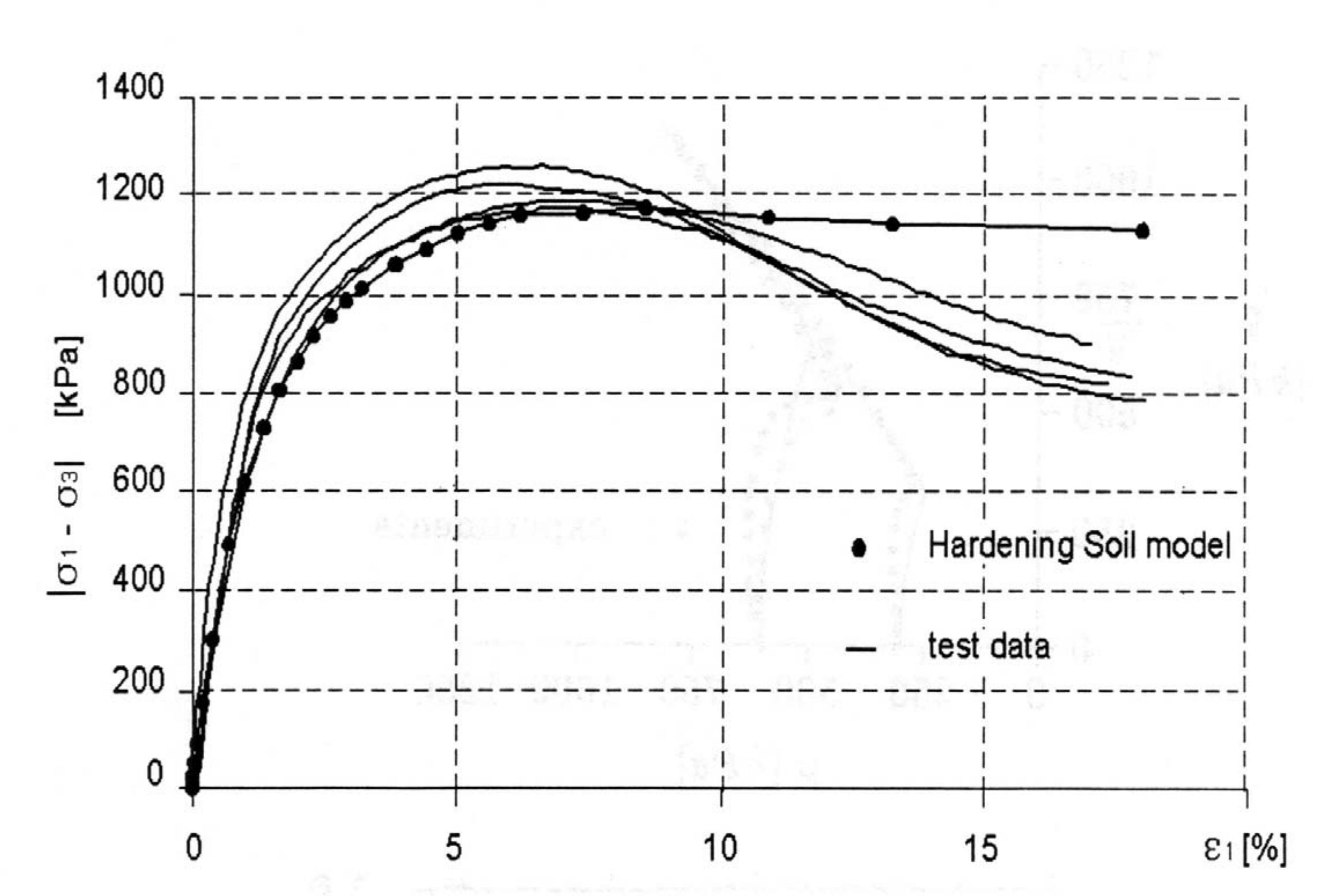
$$E_{oed}^{ref} \approx \frac{500\,kPa}{w_L - 0.1}$$

Correlation by Vermeer

$$E_{oed}^{ref} \approx p^{ref} / \lambda^*$$

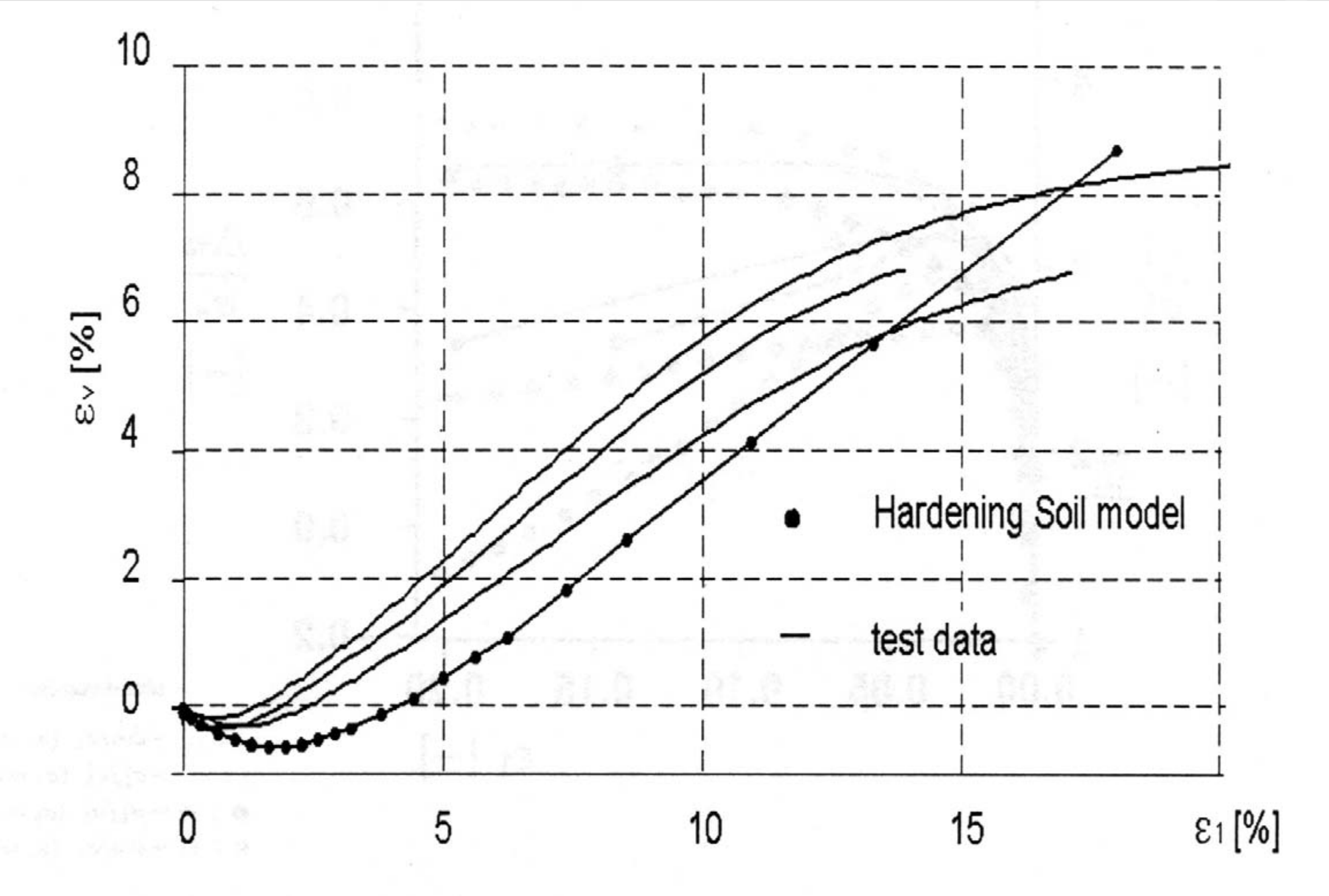
Relation with Soft Soil model

COMPARISON WITH EXPERIMENTAL DATA



dense Hostun sand

COMPARISON WITH EXPERIMENTAL DATA



dense Hostun sand

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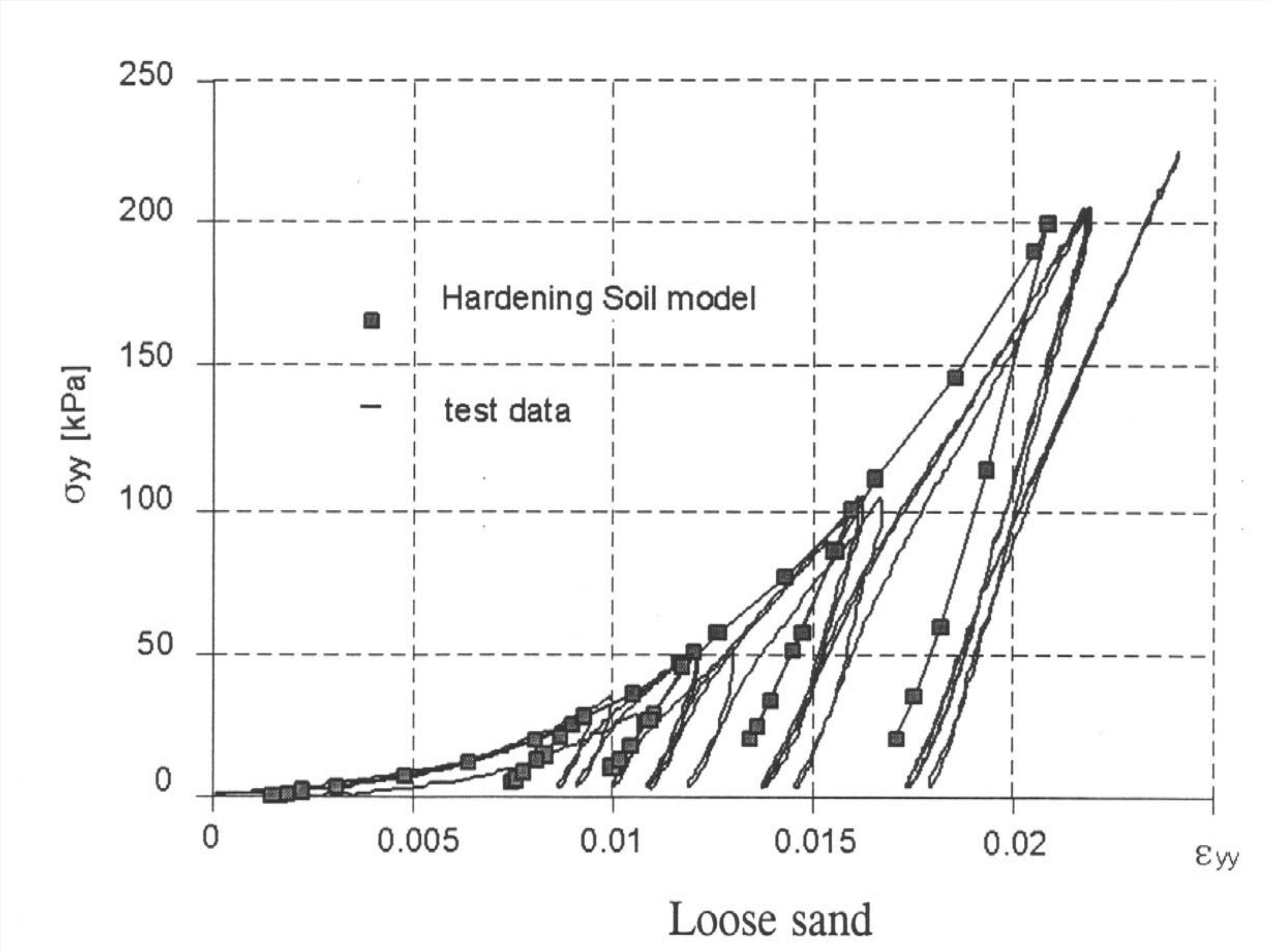

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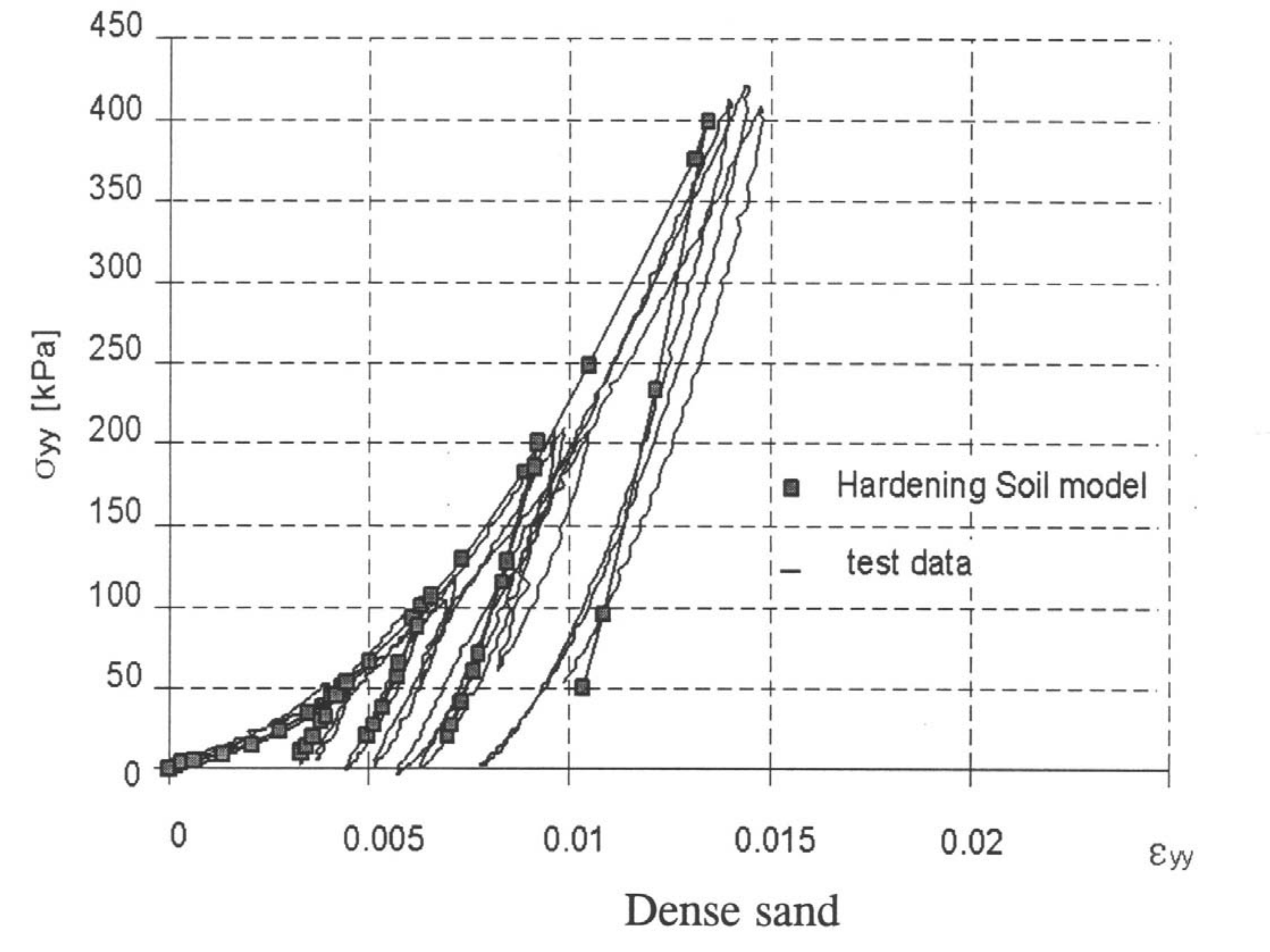
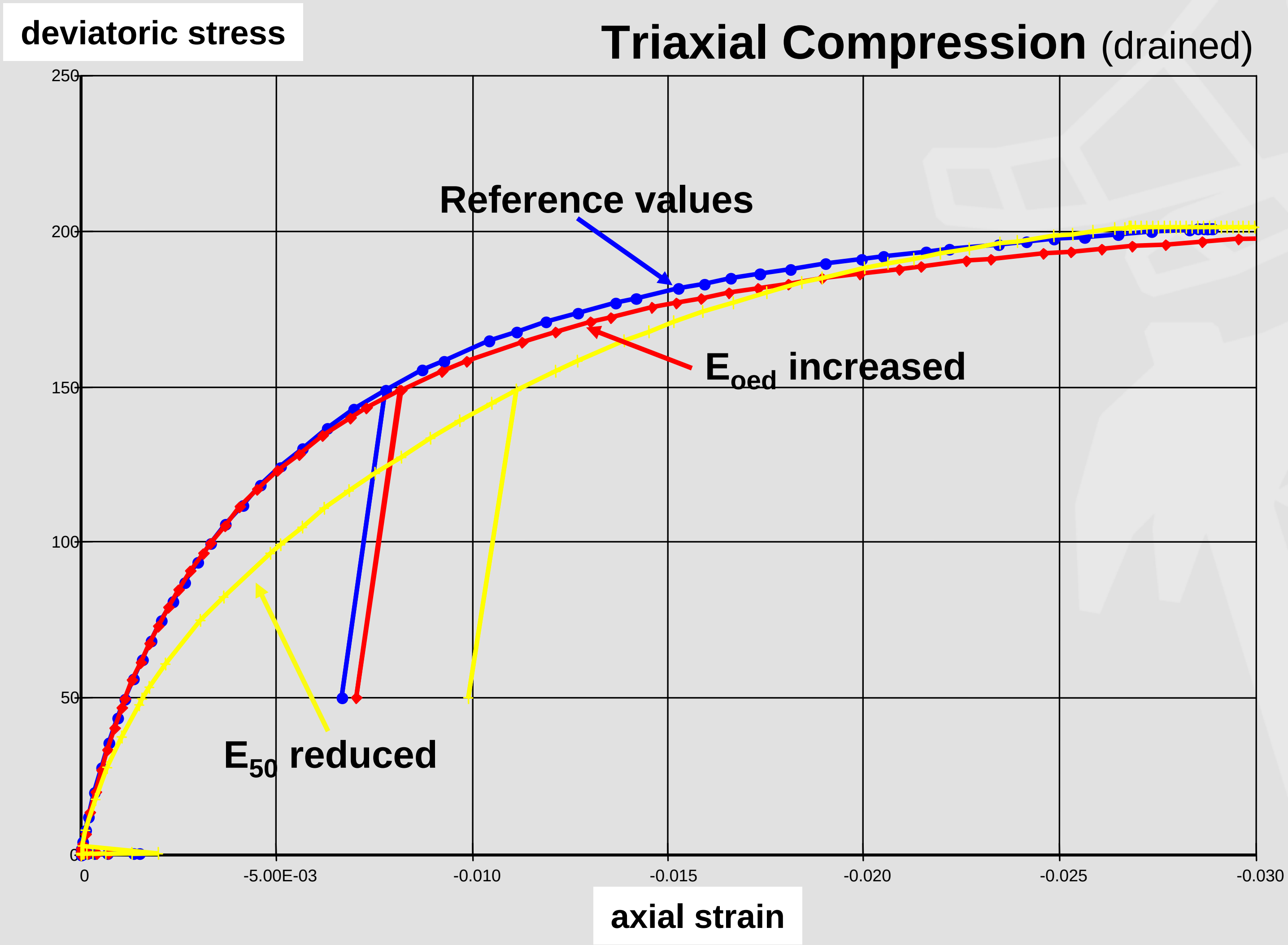
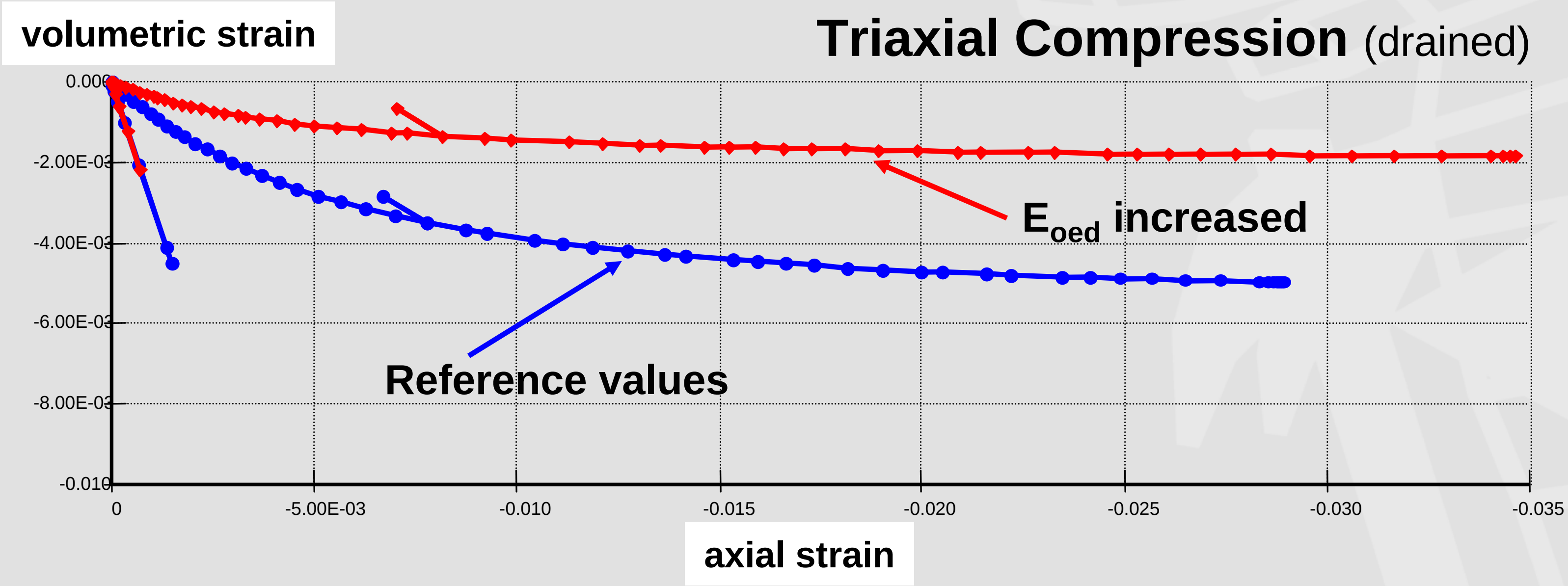


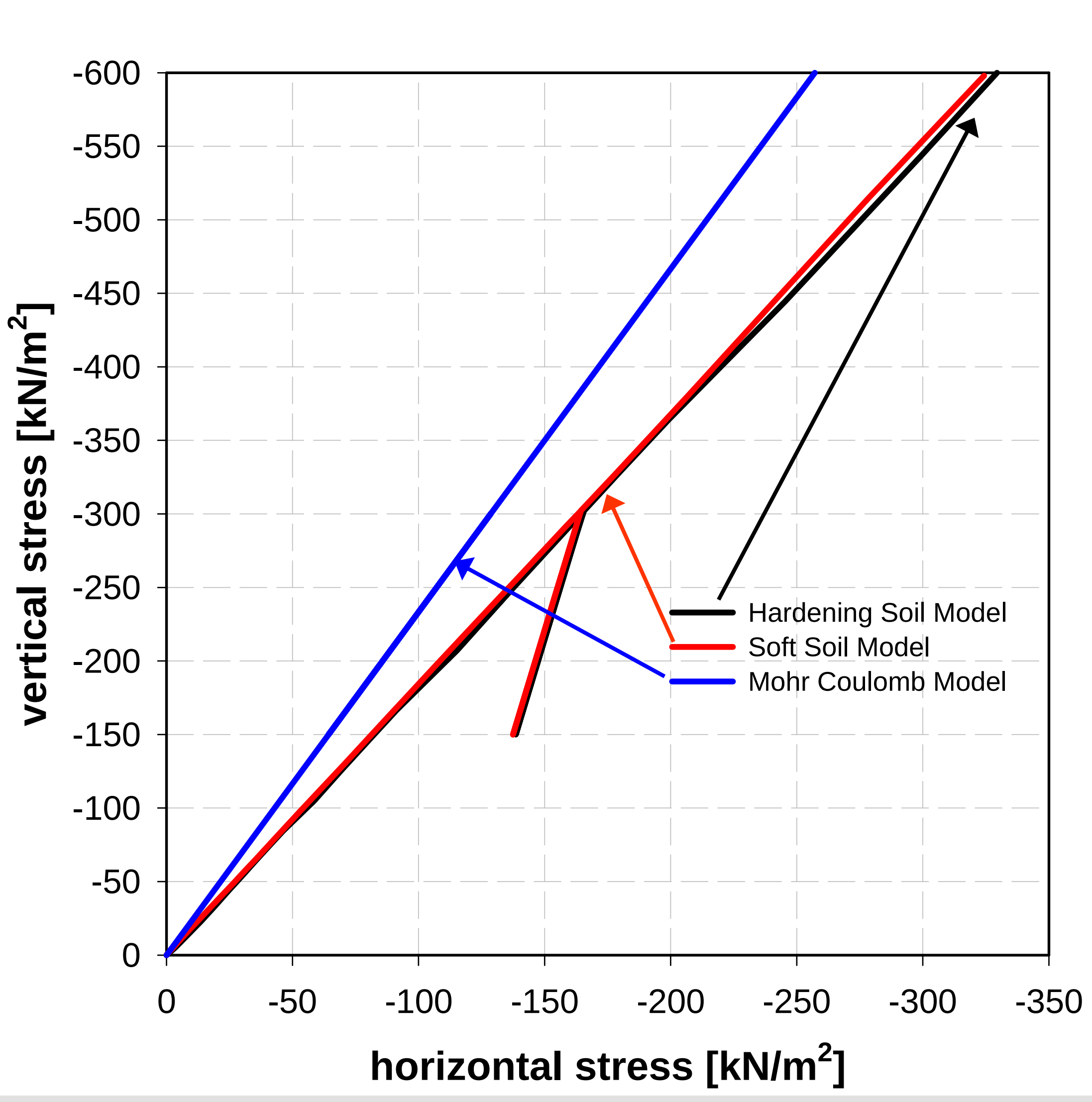
INFLUENCE E_{50} / E_{oed}



INFLUENCE E_{50} / E_{oed}



OEDOMETER TEST - COMPARISON MC / SS / HS

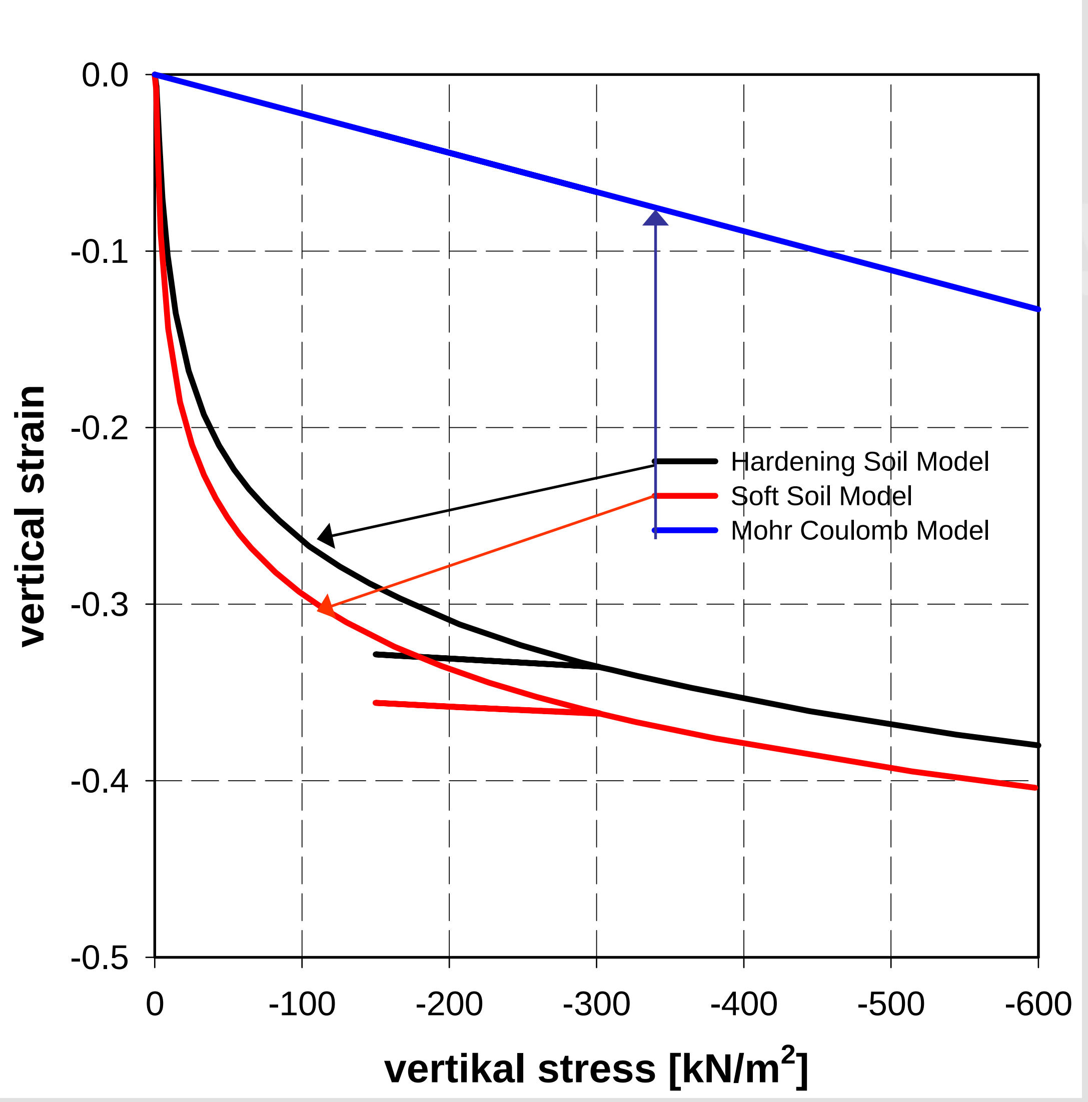


Mohr-Coulomb:
ratio σ_3/σ_1 determined by ν

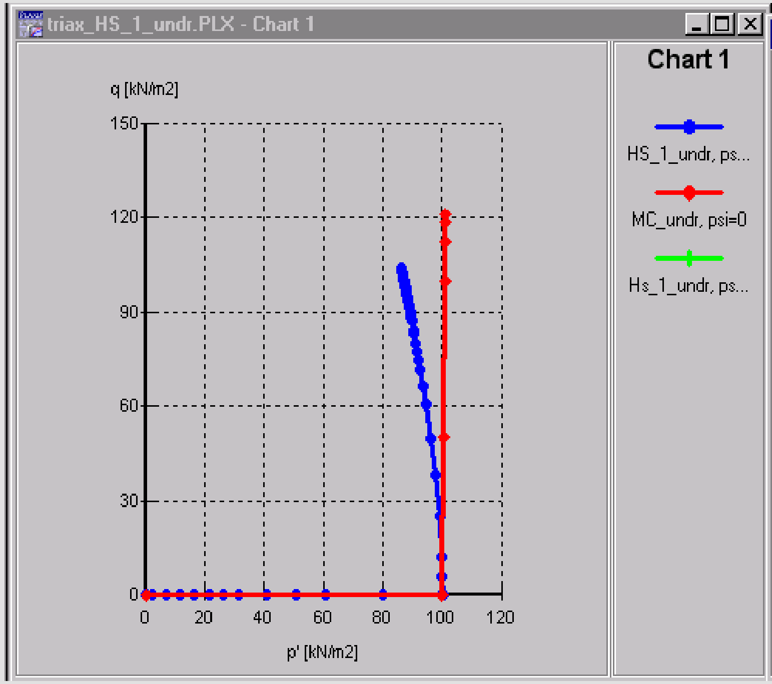
Hardening Soil:
ratio σ_3/σ_1 determined by K_0^{nc}

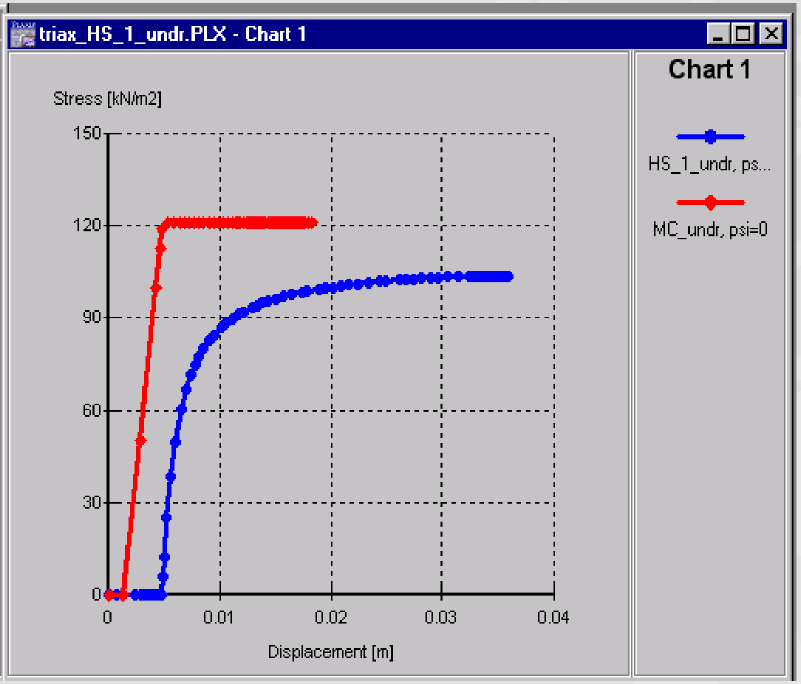
Unloading: ν_{ur}

OEDOMETER TEST - COMPARISON MC / SS / HS

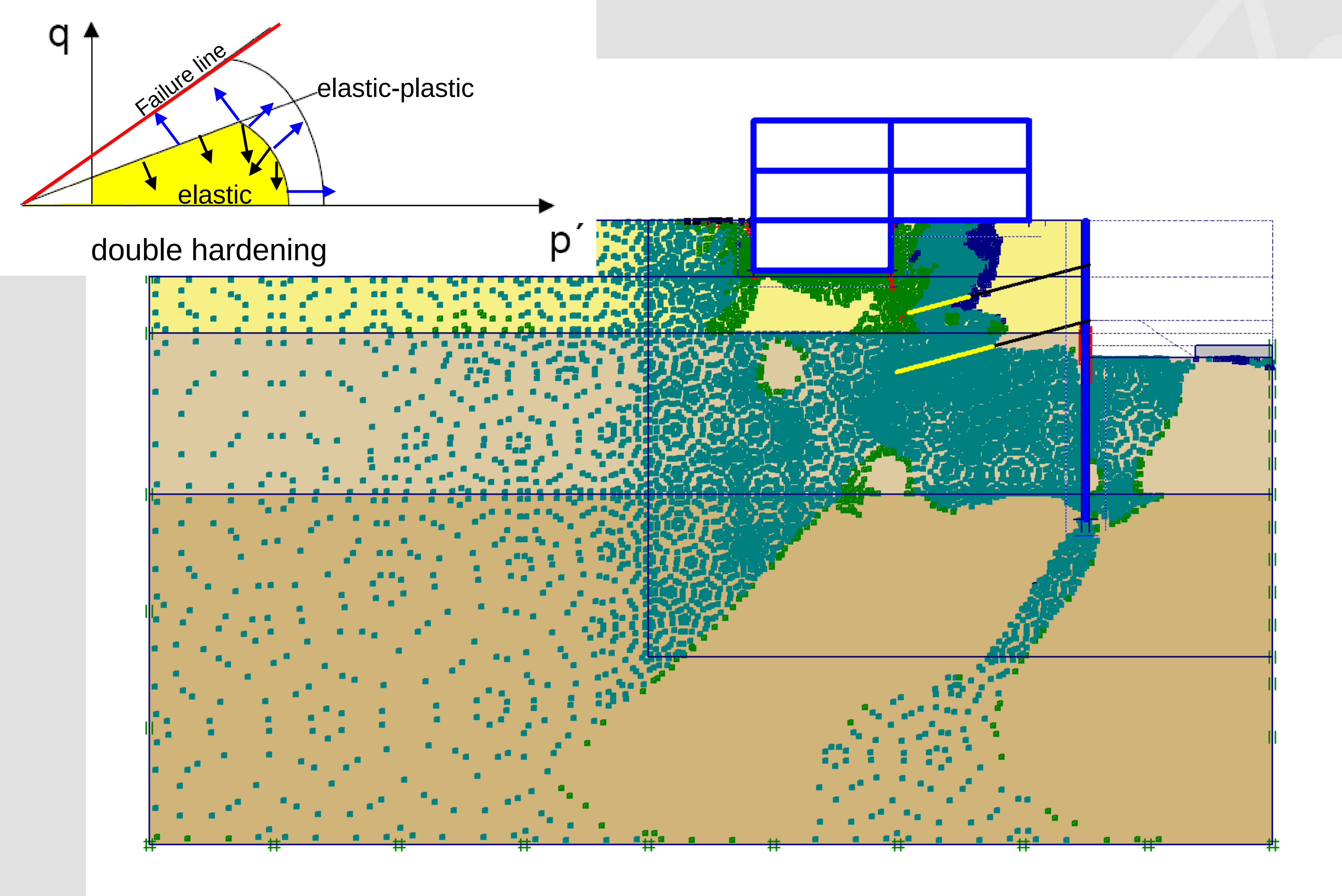


UNDRAINED BEHAVIOUR

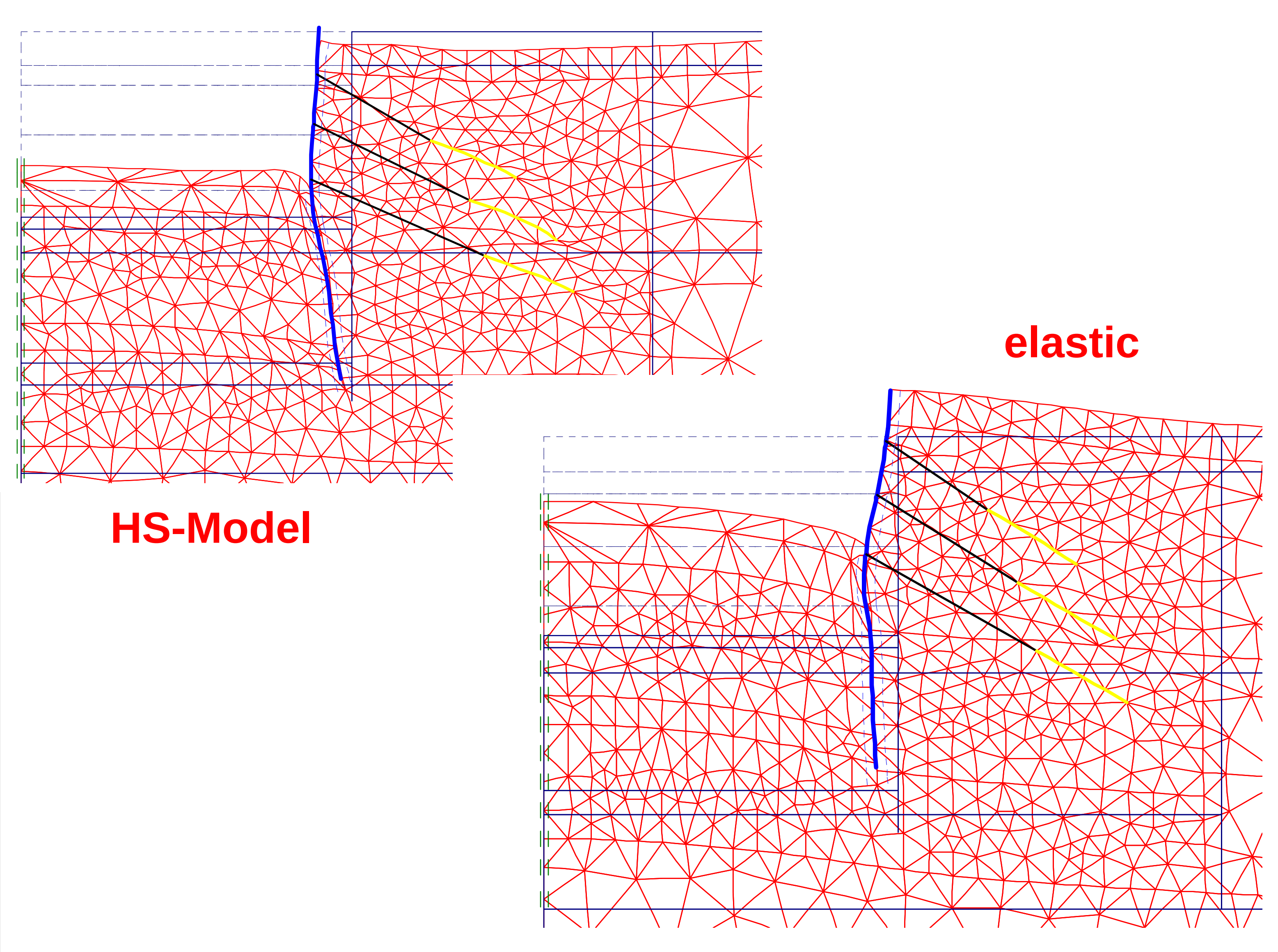




PLASTIC POINTS



SUMMARY



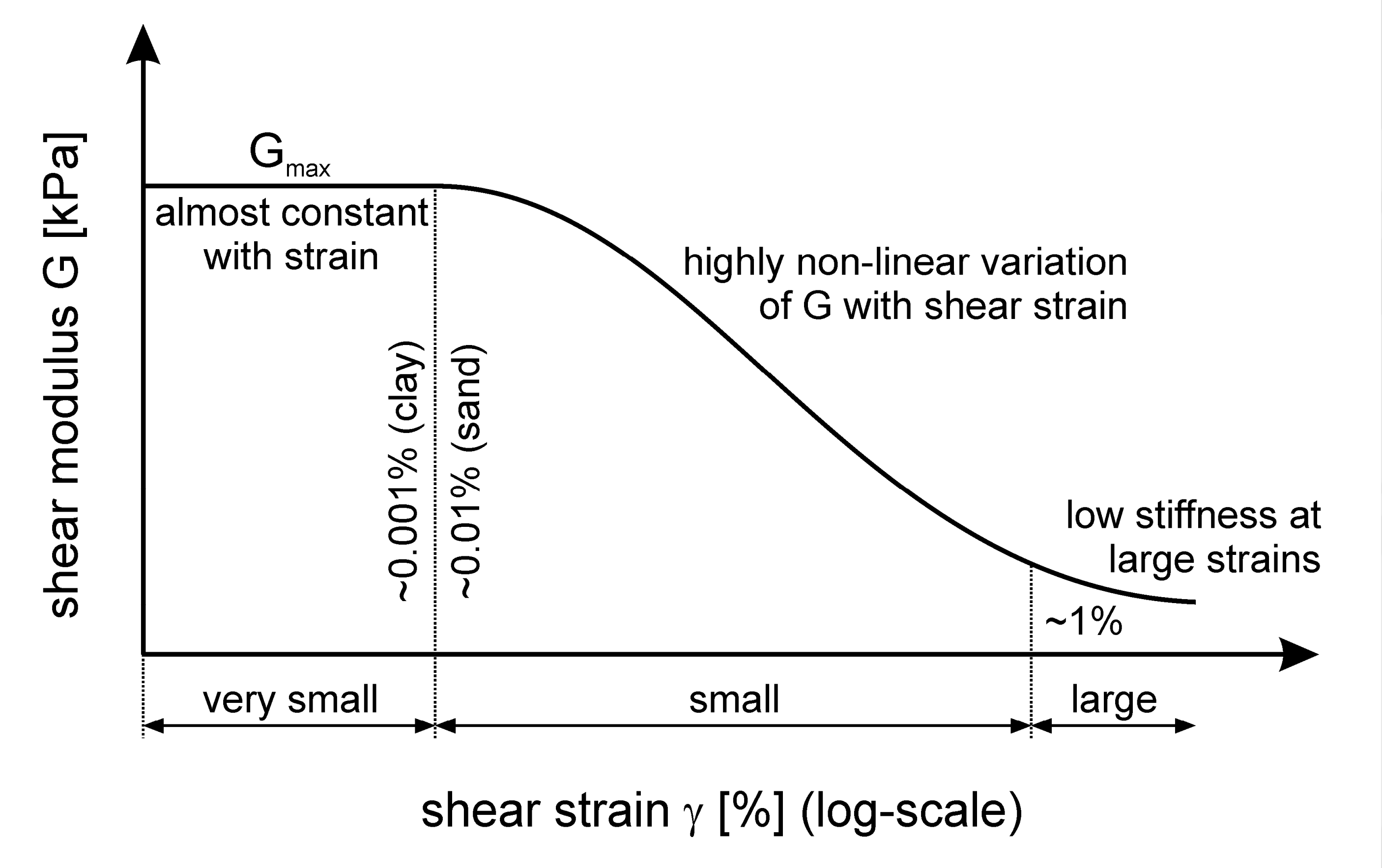
SUMMARY

	Mohr-Coulomb Model	Hardening Soil Model
stress dependent stiffness*	**NO**	**YES**
distinction in stiffness for primary loading and unloading / reloading	**NO**	**YES**
plastic strains for stress states below MC - failure line (deviatoric and volumetric hardening)	**NO**	**YES**
failure according to Mohr-Coulomb	**YES**	**YES**

*** (not only dependent on σ_0, this is possible also with MC-Model)**

FURTHER DEVELOPMENTS

HS-small: extension of HS model incorporating small strain stiffness



2 additional parameters:

- reference shear modulus at very small strains: G_0^{ref}
- shear strain at which G = 0.7 G_0: $\gamma_{0.7}$