CG 03
CONCEPTS OF PLASTICITY
MOHR COULOMB MODEL

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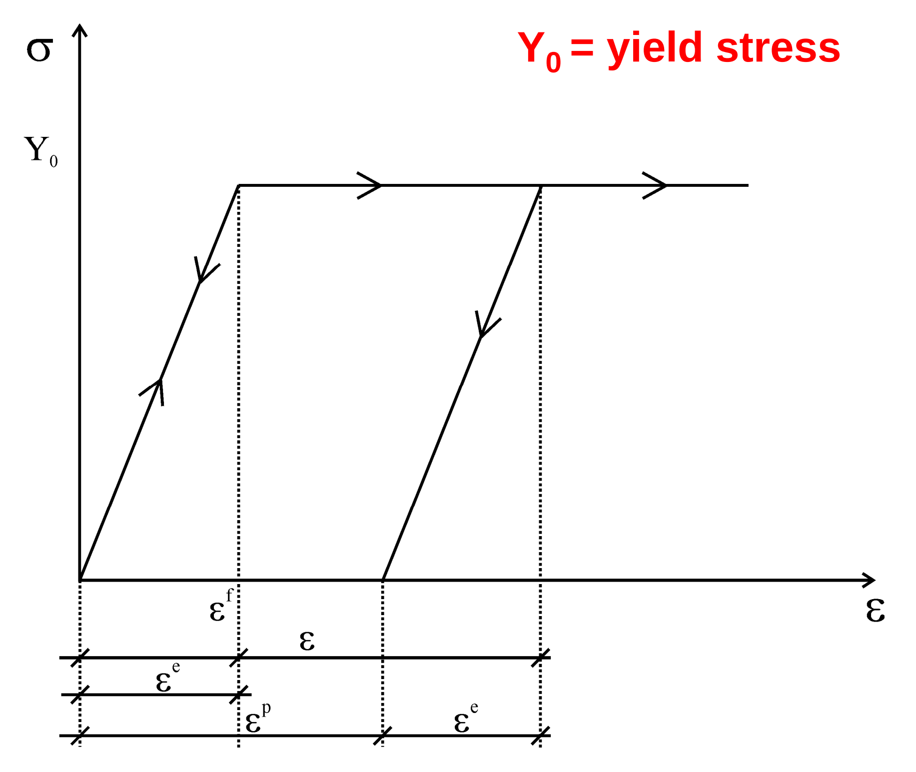
LINEAR ELASTIC - PERFECTLY PLASTIC

One-dimensional



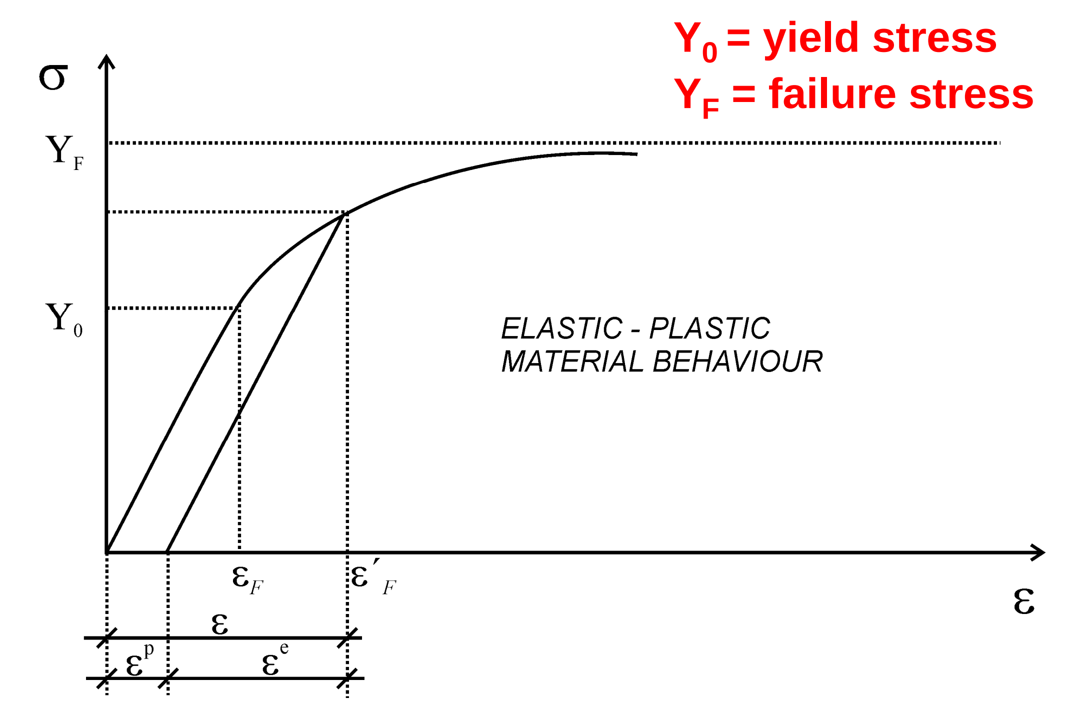
IMPORTANT: yield stress = failure stress for perfect plasticity

$$\varepsilon = \varepsilon^{e} + \varepsilon^{p}$$

General three-dimensional stress state $\{\varepsilon\} = \{\varepsilon\}^{e} + \{\varepsilon\}^{p}$

LINEAR ELASTIC - PLASTIC

One-dimensional



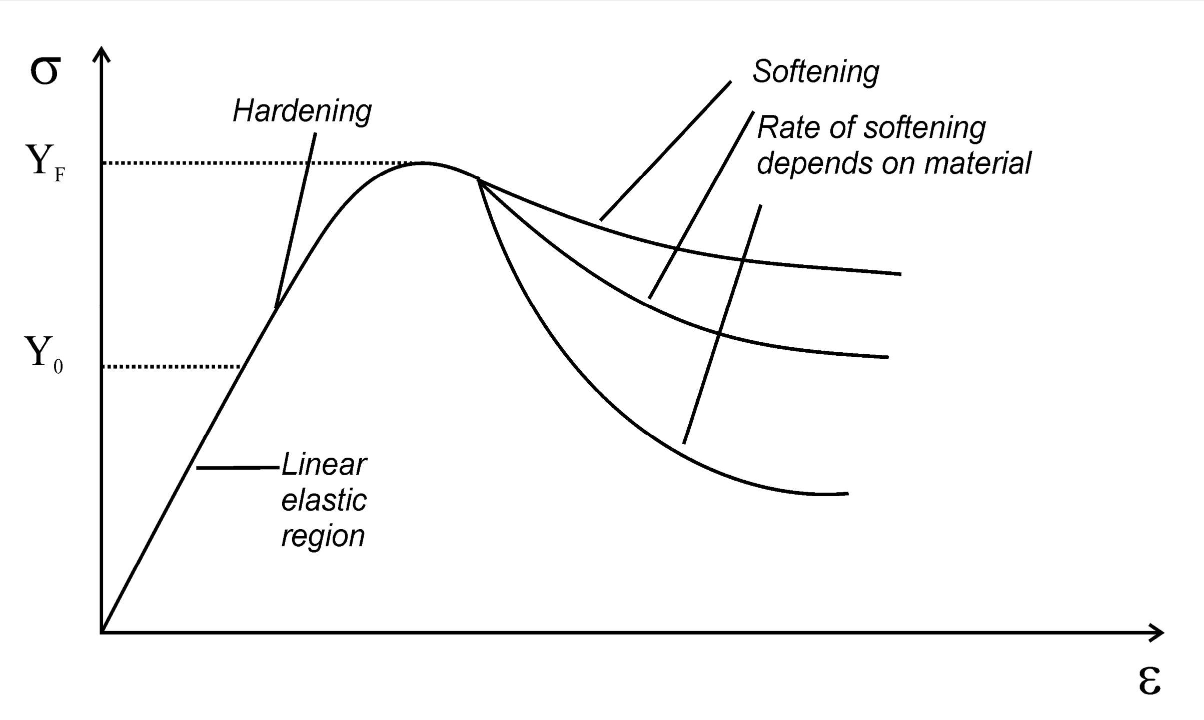
IMPORTANT: yield stress $\neq$ failure stress

$$\varepsilon = \varepsilon^e + \varepsilon^p$$

General three-dimensional stress state $\{\varepsilon\} = \{\varepsilon\}^e + \{\varepsilon\}^p$

LINEAR ELASTIC - PLASTIC WITH SOFTENING

One-dimensional



Y_0 = yield stress
Y_F = failure stress

THEORY OF PLASTICITY

For describing linear elastic - plastic material behaviour we need (for general stress states)**:**

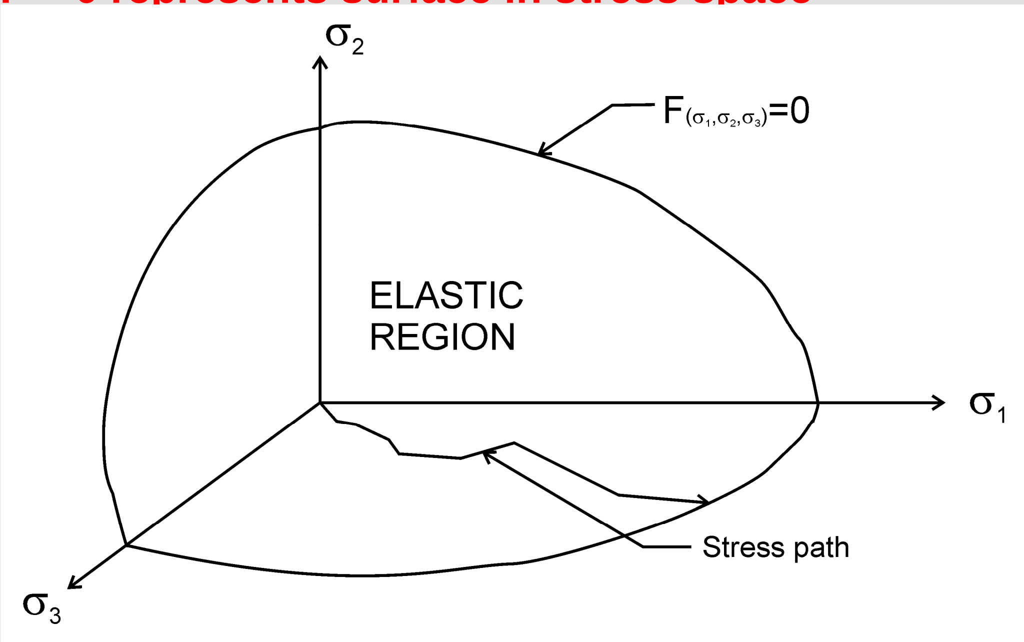
1. Stress-strain behaviour in elastic range
2. **Yield function or failure function**
   (defines onset of plastic deformation)
3. **Flow rule**
   (defines direction of plastic strain increment)
4. Definition of strain hardening (softening)
   (defines change of yield function with stress and/or strain)

For standard MC-model:

- Linear elasticity in elastic range
- No strain hardening/softening > perfect plasticity

YIELD / FAILURE FUNCTION

F = 0 represents surface in stress space



$$f_{(\{\sigma\})} = f_{(\sigma_1, \sigma_2, \sigma_3)}$$

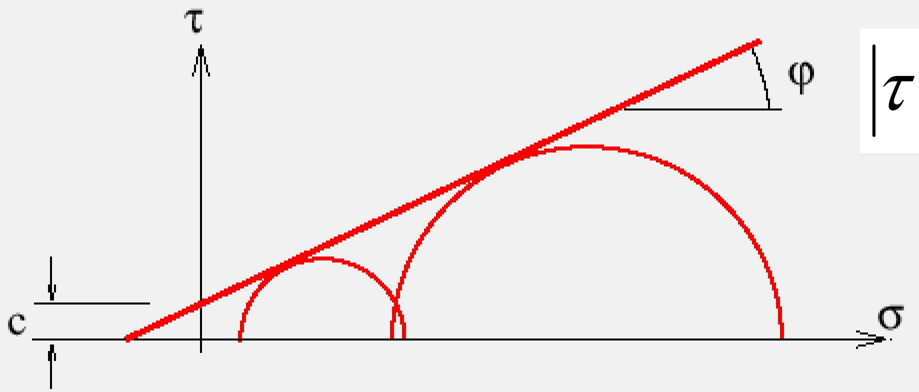
$f_{(\{\sigma\})} < 0$ **stress state is elastic**

$f_{(\{\sigma\})} = 0$ **stress state is plastic**

$f_{(\{\sigma\})} > 0$ **stress state not admissible**

MOHR COULOMB CRITERION

Mohr-Coulomb yield function



on any plane:

$$|\tau| = \sigma_n' \cdot \tan\varphi' + c'$$

yield function: $f = \frac{1}{2}(\sigma'_1 - \sigma'_3) + \frac{1}{2}(\sigma'_1 + \sigma'_3)\sin\varphi' - c'\cos\varphi'$

σ_1' and σ_3' : major and minor principal stresses

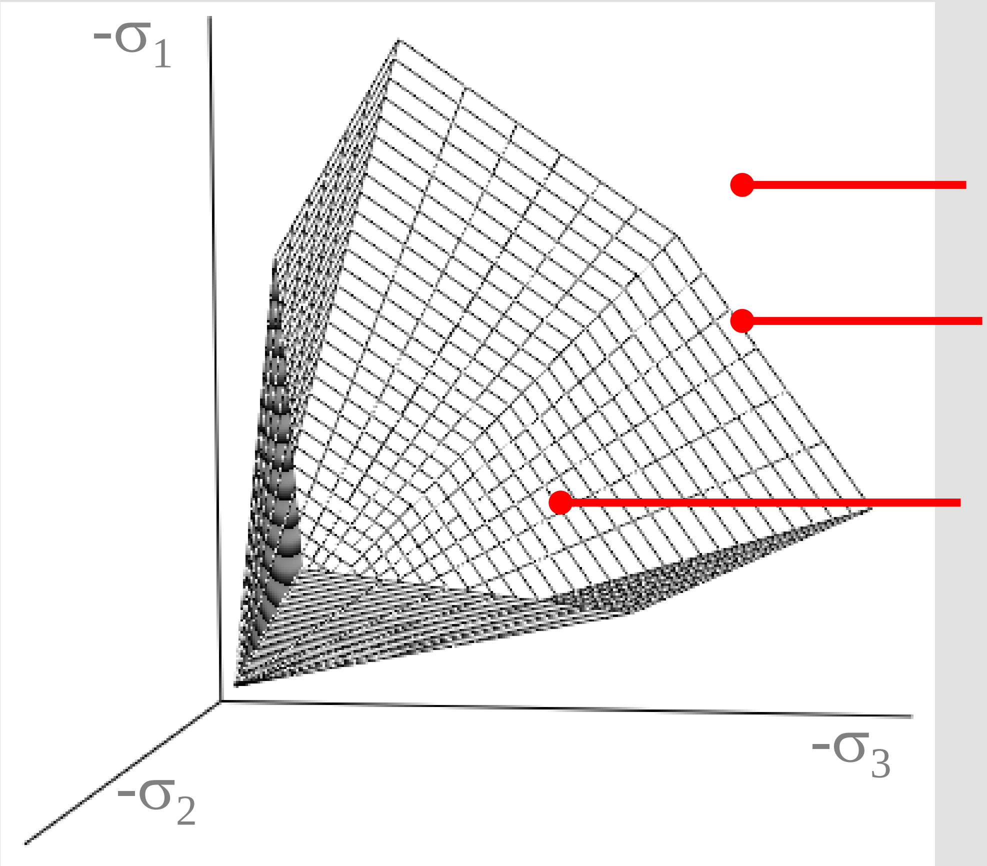
sign convention

positive: tensile stress, elongation, volume increase

negative: compressive stress, compression, volume decrease

MOHR COULOMB IN 3D STRESS SPACE

$$f = \tfrac{1}{2}(\sigma'_1 - \sigma'_3) + \tfrac{1}{2}(\sigma'_1 + \sigma'_3)\sin\varphi' - c'\cos\varphi'$$

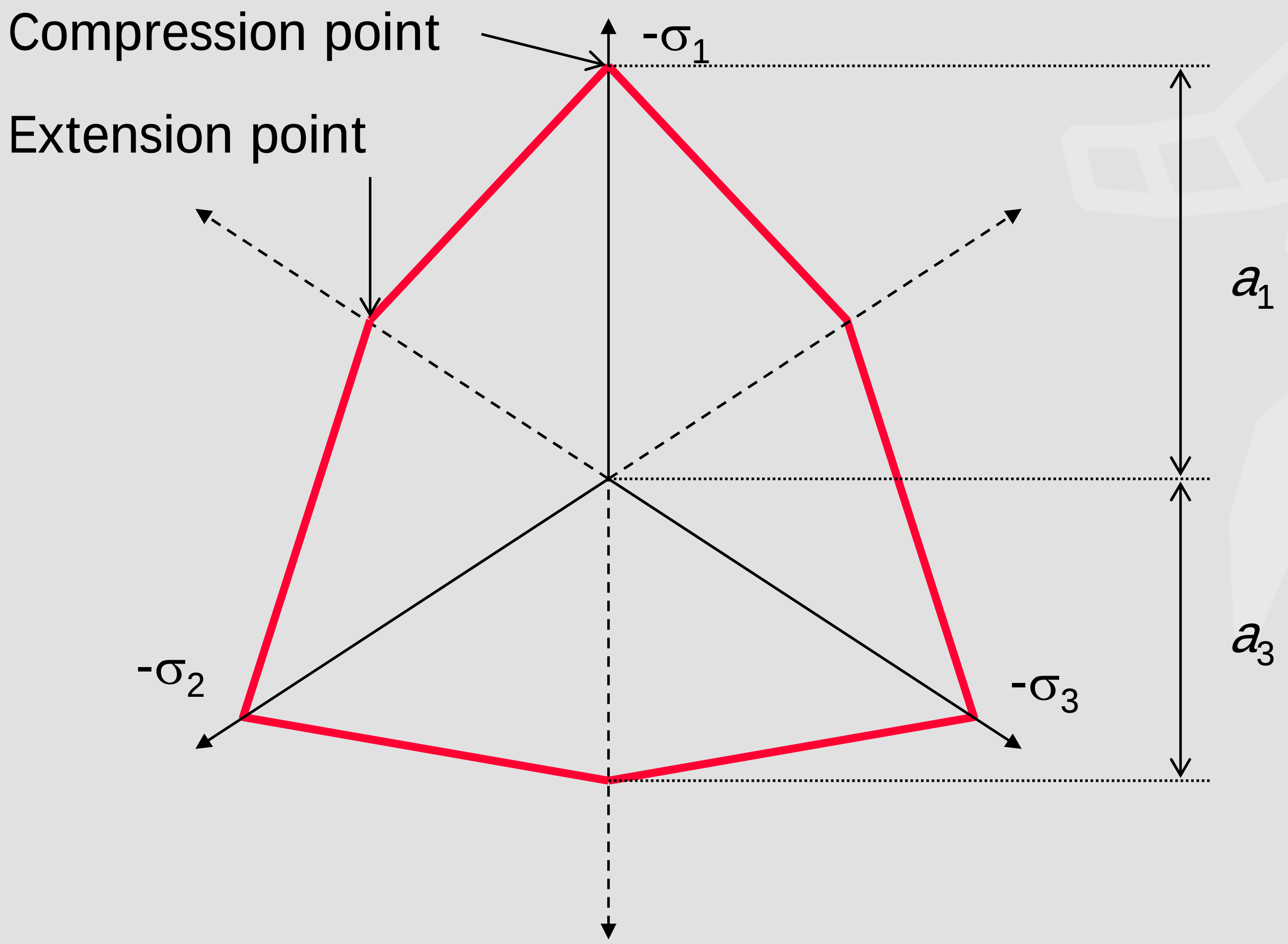


$f > 0$ Not acceptable

$f = 0$ Plasticity

$f < 0$ Elasticity

MOHR COULOMB IN DEVIATORIC PLANE



$$\frac{a_1}{a_3} = \frac{3+\sin\varphi'}{3-\sin\varphi'}$$

FLOW RULE

Check of yield / failure function tells us whether we have plastic strains but not direction and magnitude of plastic strain increment > another function required > plastic potential

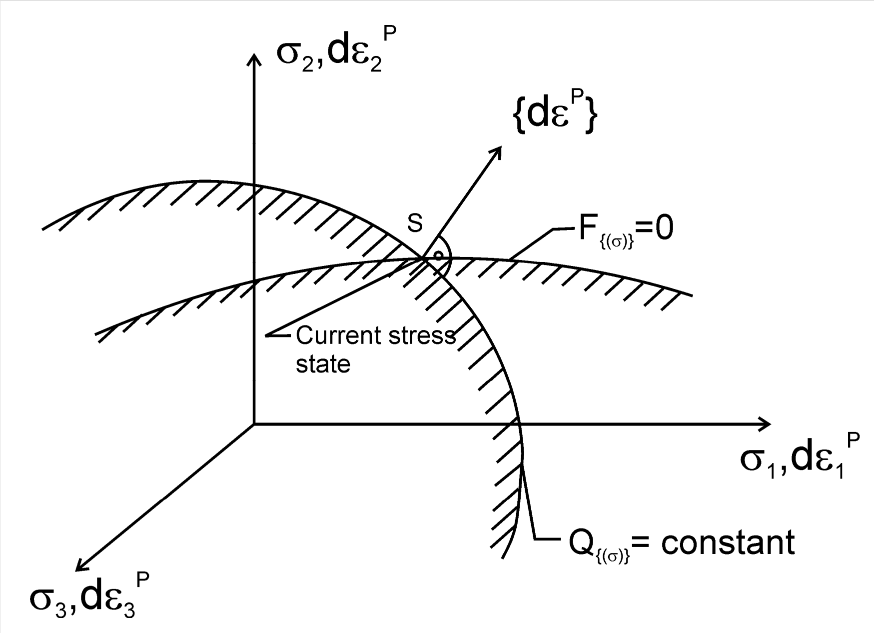
Plastic strain increments

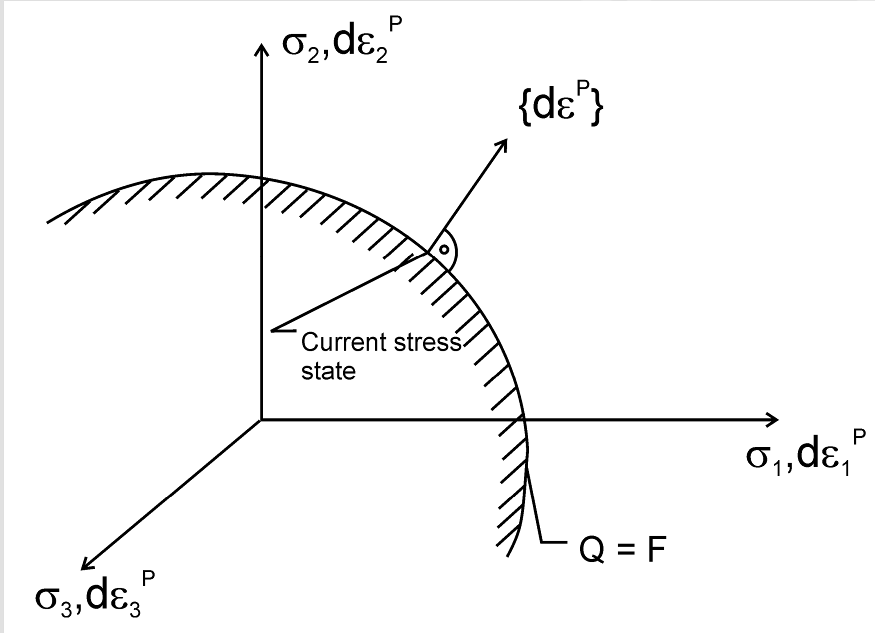
$$\{d\varepsilon\}^P = d\lambda \left\{ \frac{\partial g}{\partial \{\sigma\}} \right\}$$

g ……. plastic potential
dλ …… constant factor, NOT a material parameter

$$g = g_{(\{\sigma\})}$$

FLOW RULE





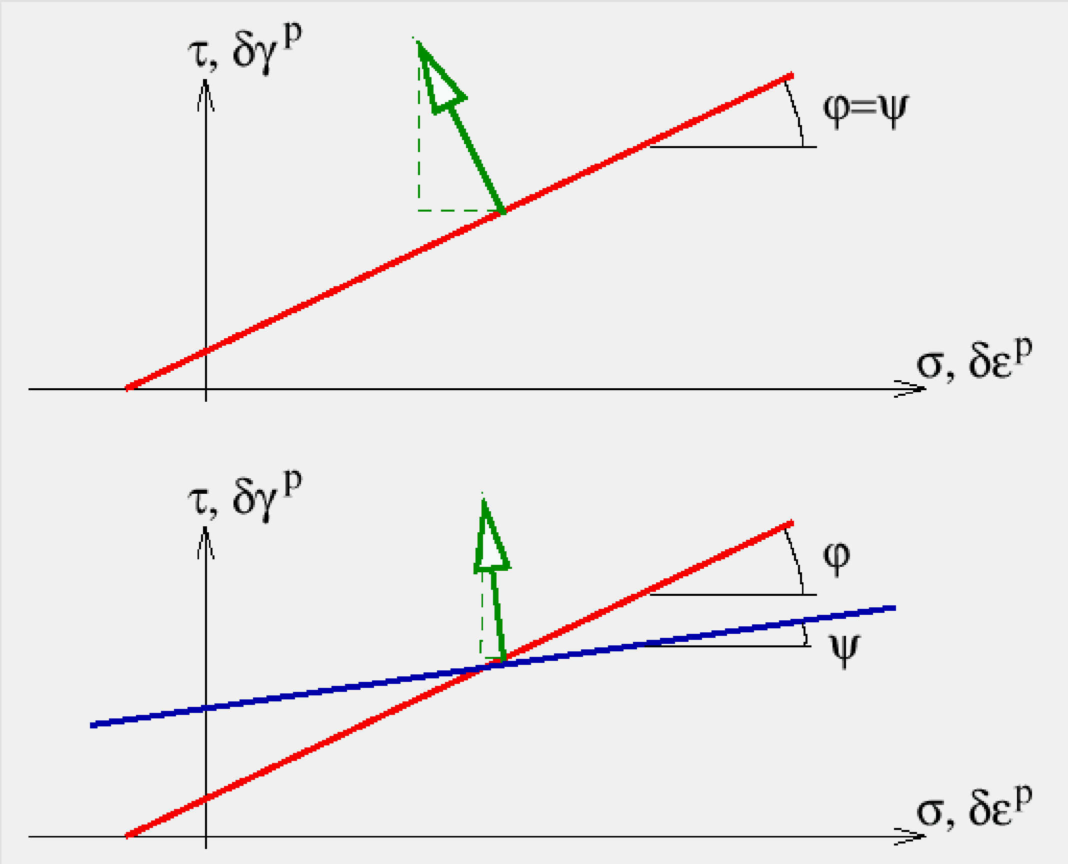
$$\{d\varepsilon\}^P = d\lambda \left\{ \frac{\partial g}{\partial \{\sigma\}} \right\}$$

$$\{d\varepsilon\}^P = d\lambda \left\{ \frac{\partial F}{\partial \{\sigma\}} \right\}$$

g ≠ f > non associated flow rule

g = f > associated flow rule

MOHR COULOMB MODEL - PLASTIC POTENTIAL



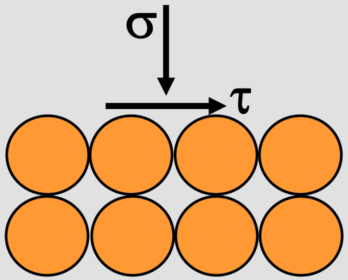
$$f = \tfrac{1}{2}\left(\sigma'_1 - \sigma'_3\right) + \tfrac{1}{2}\left(\sigma'_1 + \sigma'_3\right)\sin\varphi' - c'\cos\varphi'$$

$$g = \tfrac{1}{2}\left(\sigma'_1 - \sigma'_3\right) + \tfrac{1}{2}\left(\sigma'_1 + \sigma'_3\right)\sin\psi' + \text{const.}$$

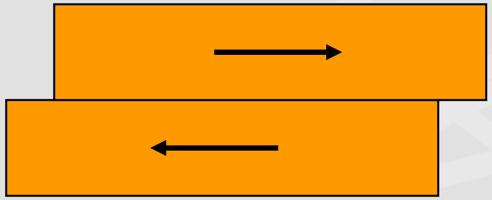
dilatancy angle

DILATANCY

Model | Mechanism

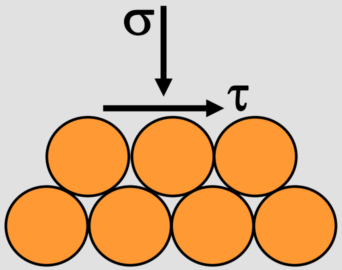


loose soils are non-dilatant or contractant

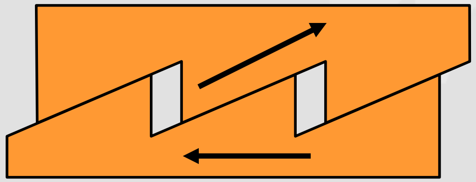


loose

$\psi = 0$



dense soils are dilatant



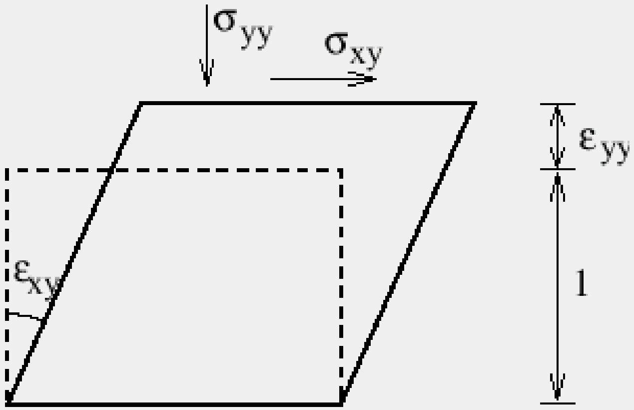
dense

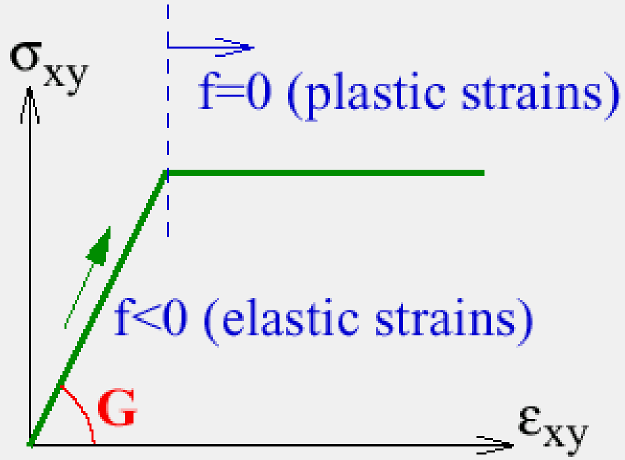
$\psi \neq 0$, but $< \varphi'$

strength = friction + dilatancy

DILATANCY

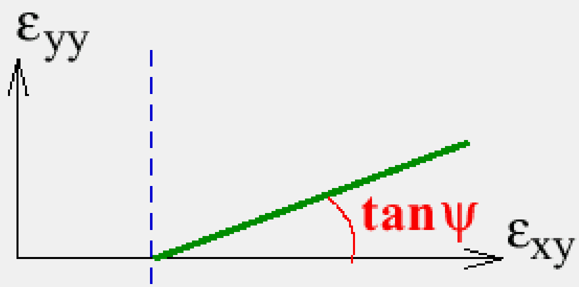
Simple shear test (drained)





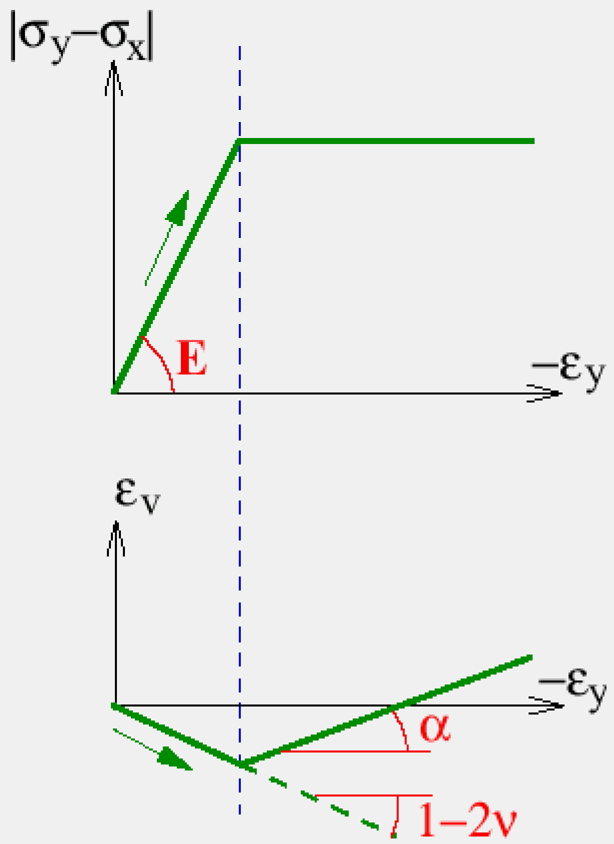
$$G = \frac{E}{2(1+\nu)}$$

$$\tan\psi = \frac{\Delta\varepsilon_{yy}}{\Delta\varepsilon_{xy}}$$



DILATANCY

Triaxial test (drained)



$$E = \frac{\Delta|\sigma_y - \sigma_x|}{\Delta|\varepsilon_y|} = 2G(1+\nu)$$

$$\varepsilon_v = 2\varepsilon_x + \varepsilon_y$$

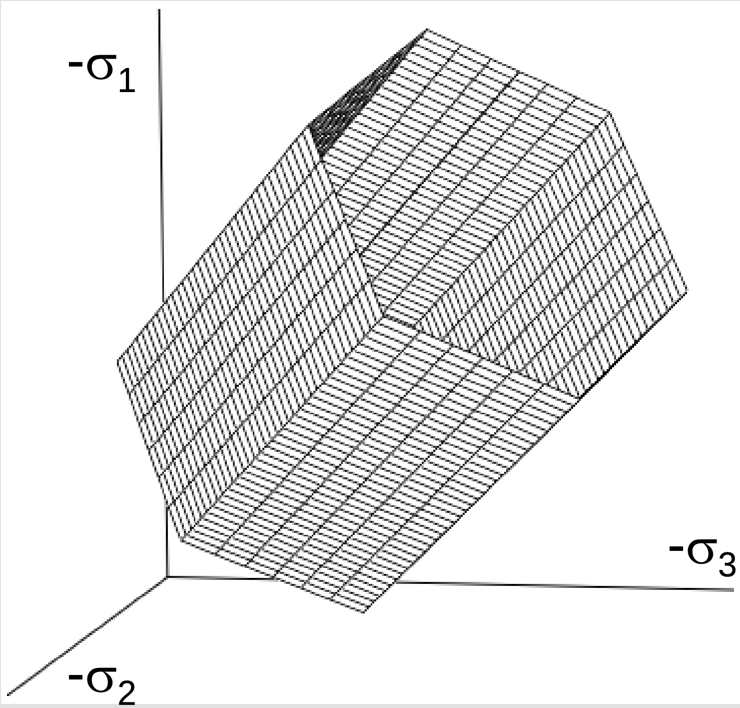
$$\tan\alpha = \frac{2\sin\psi}{1-\sin\psi}$$

MOHR COULOMB MODEL - PARAMETERS

E	Young's modulus	[kN/m²]
ν	Poisson's ratio	[-]
c'	(effective) cohesion	[kN/m²]
φ'	(effective) friction angle	[º]
ψ	Dilatancy angle	[º]

TRESCA FAILURE CRITERION

Tresca = Mohr-Coulomb with $\varphi = 0$ and $c \neq 0 = c_u$



$\psi = 0$ > no plastic volumetric strains
$\nu = 0.495$ > negligible elastic volumetric strains

> can be used for undrained analysis in terms of total stresses

MOHR COULOMB MODEL - POSSIBILITIES AND LIMITATIONS

- **Simple elastic perfectly-plastic model**
- **First order approach of soil behaviour in general**
- **Suitable for some practical applications** (not for deep excavations and tunnels)
- **Limited number and clear parameters**
- **Good representation of failure behaviour (drained)**
- **Dilatancy can be included**

MOHR COULOMB MODEL - POSSIBILITIES AND LIMITATIONS

- **Isotropic and homogeneous behaviour**
- **Linear elastic behaviour until failure**
- **No stress-dependent stiffness**
- **No distinction between primary loading and unloading or reloading**
- **No dilatancy cut off**

  (for associated flow dilatancy is significantly overpredicted)
- **Undrained behaviour not always realistic**
- **No anisotropy, no time-dependency (creep)**