Saturated soil modelling: Griffith University: February 2010

11.Modelling particle breakage and erosion of fine particles

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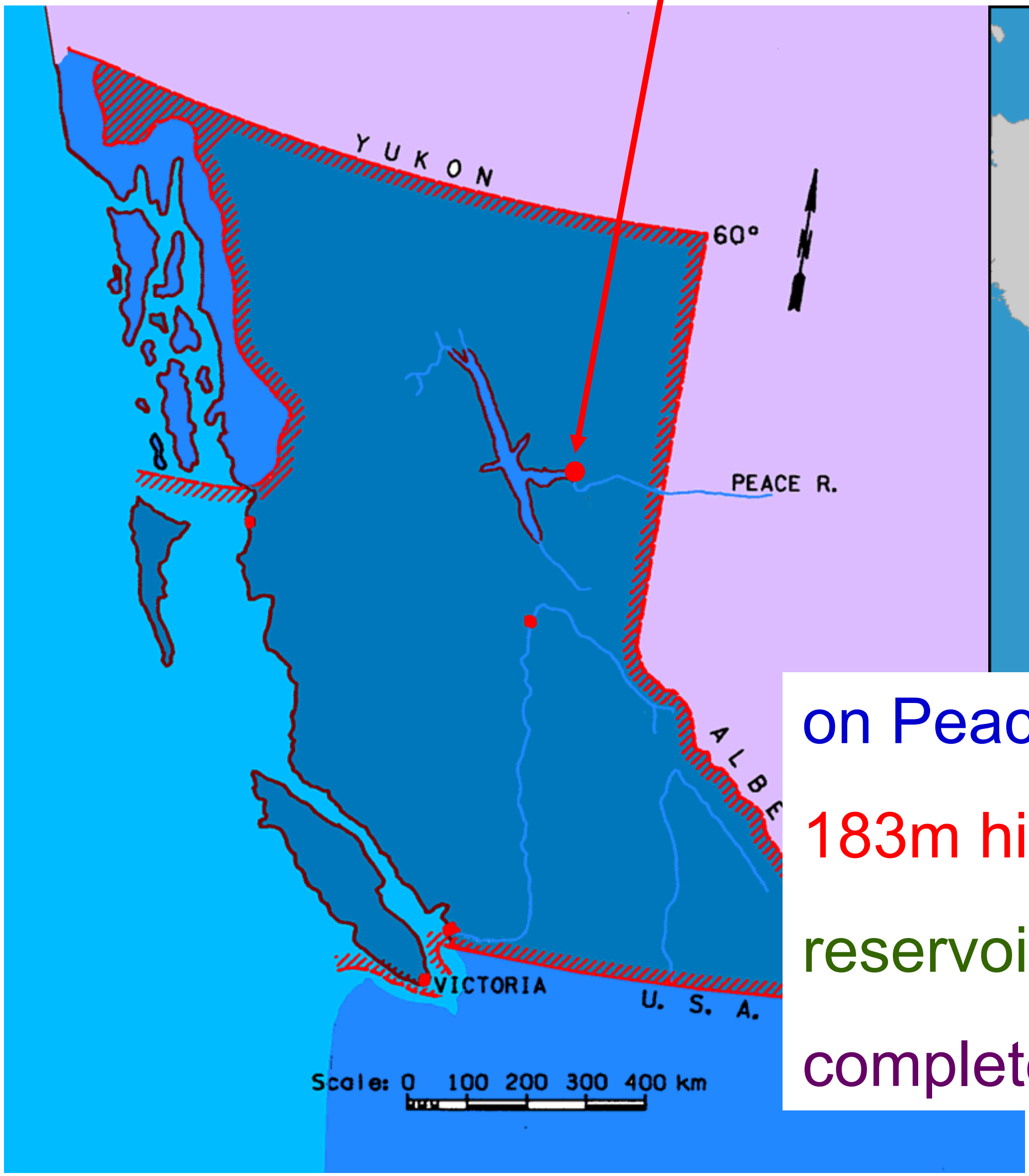
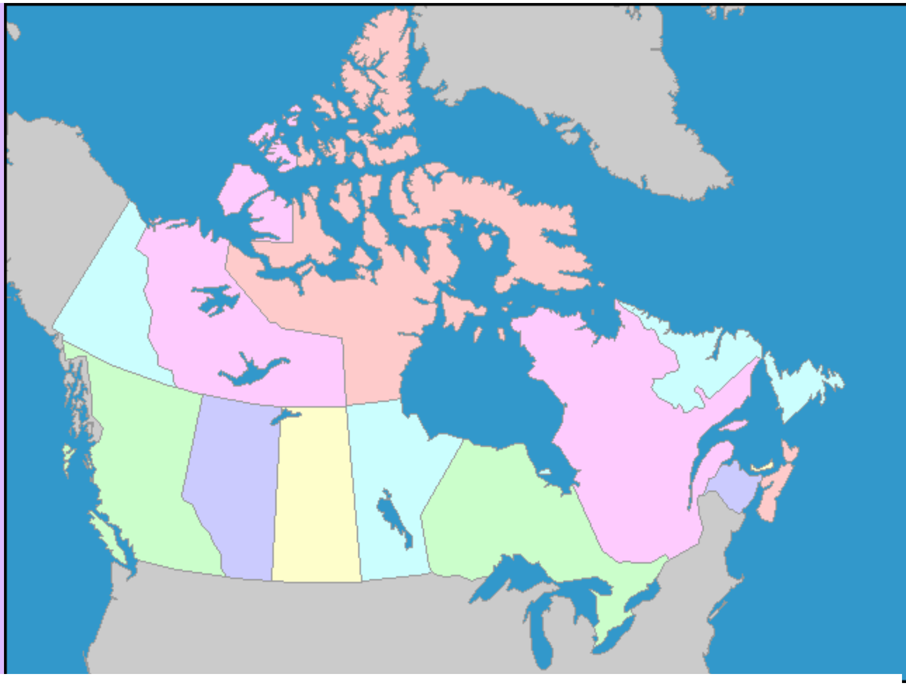
WAC Bennett Dam





on Peace River, NE British Columbia

183m high, 2km long

reservoir $70 \times 10^9 m^3$;   2.73GW

completed 1968

BC hydro

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WAC Bennett Dam

sinkhole 1: June 1996

BC hydro

WAC Bennett Dam: sinkhole incident 1996

spillway flow 3000m^3/s (> Canadian Niagara Falls)

fall in reservoir level: 2m in 7 weeks

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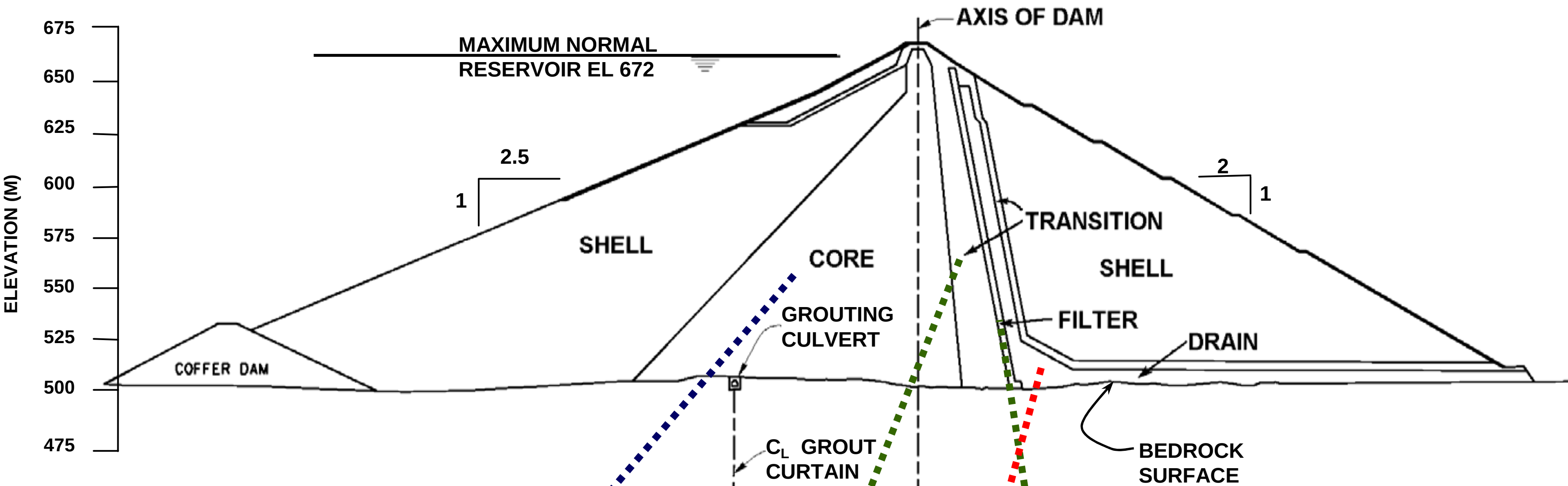
WAC Bennett Dam: cross section



zoned earthfill dam

blended 'till-like' core

well-graded transition and filter

high capacity drain

BChydro

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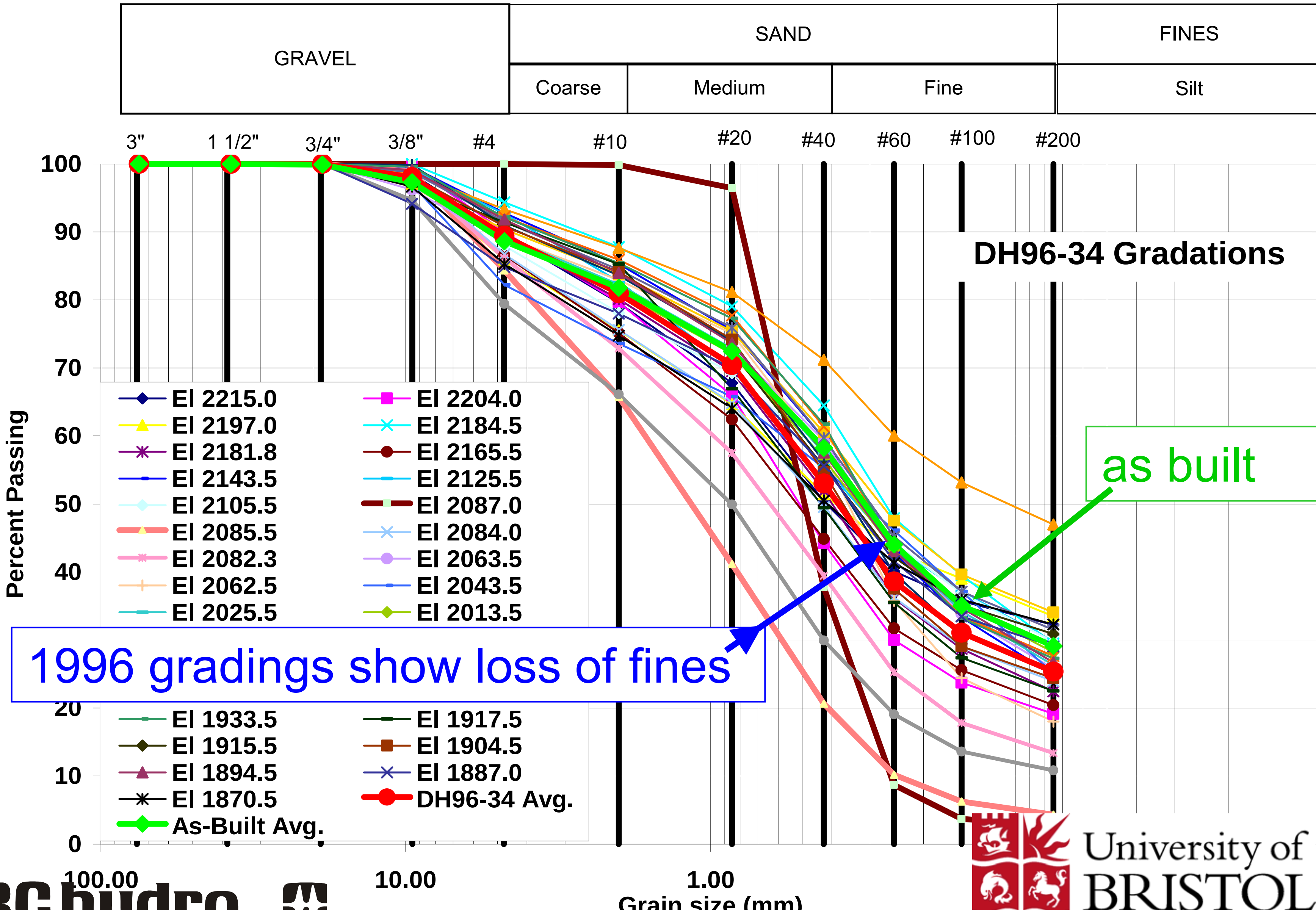
Core material (Sinkhole 1)



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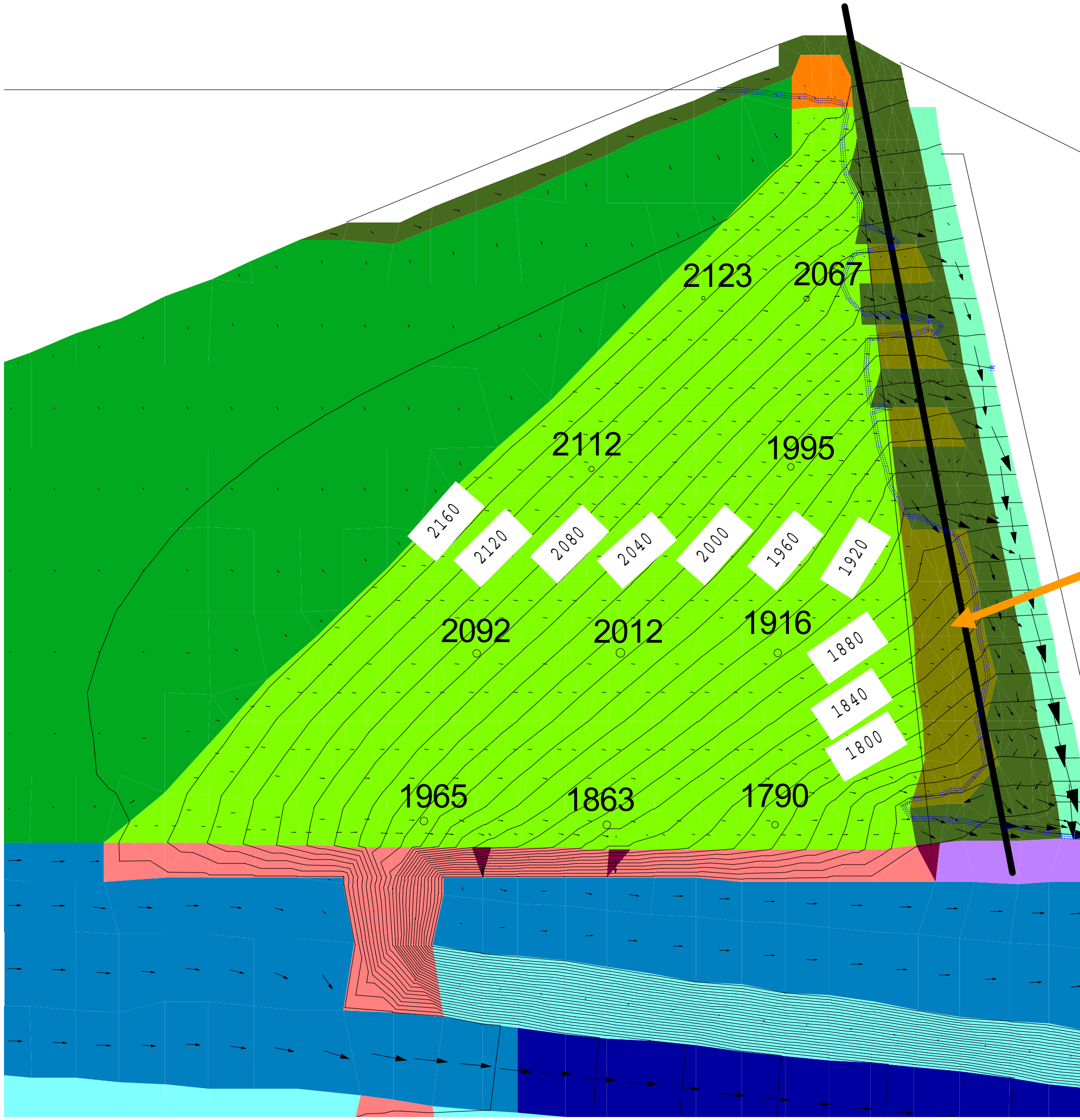
Canyon section: 2003 seepage model



pore pressures consistent with permeability changes linked with fines movement

silty transition material: reduced permeability

Bennett Dam: statement of problem

non-plastic material with changing granulometry and density

- what are consequences for mechanical response of dam?
- potential for future deformations?

need model of soil behaviour which can incorporate changes in density and grading of the soil

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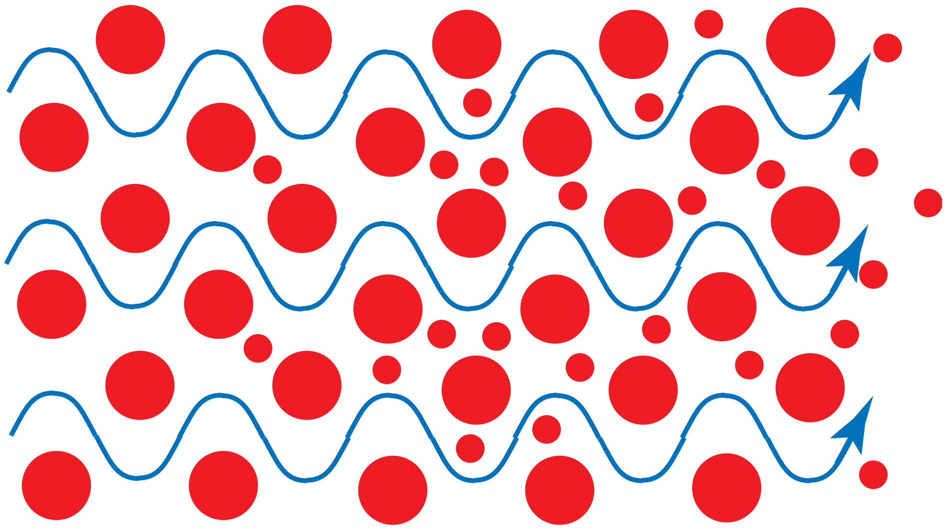
internal erosion



- grain transport by internal seepage
- suffusion
- increase in permeability
- potential consequences?
- concentrate on mechanical effects...
- ...not concerned with process of particle transport

internal erosion

Teton Dam, Idaho
5 June 1976

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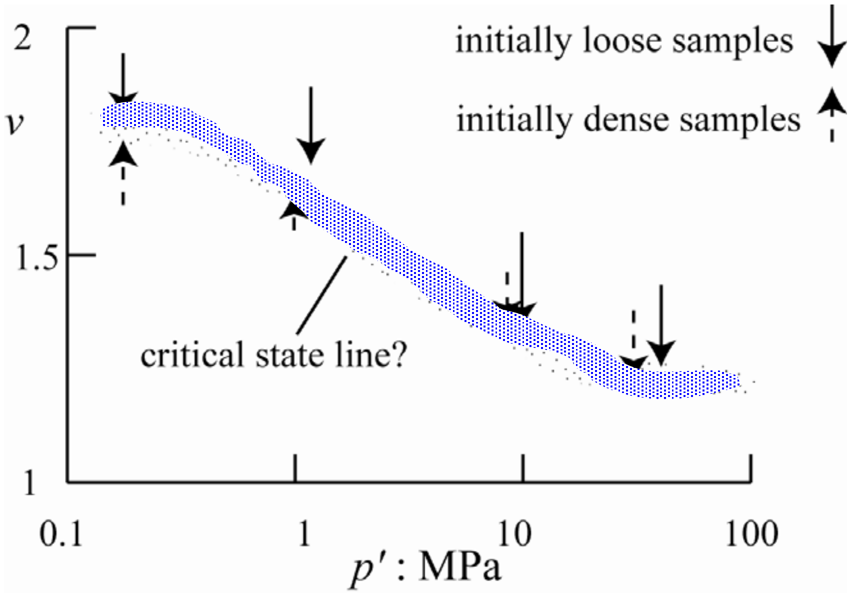
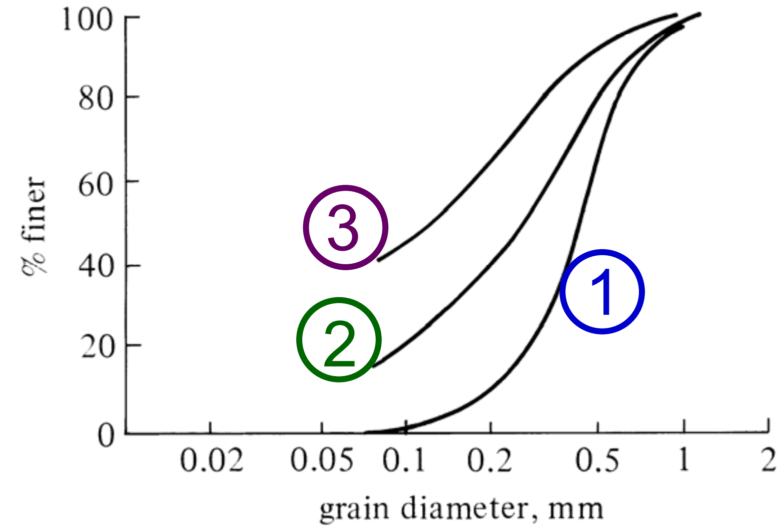
Grading state index



particle size distribution



occurrence of crushing

change in grading

irreversible

1: before testing

2: after compression to 6.21MPa

3: after triaxial compression

Chattahoochee River sand

Vesic & Clough, 1968

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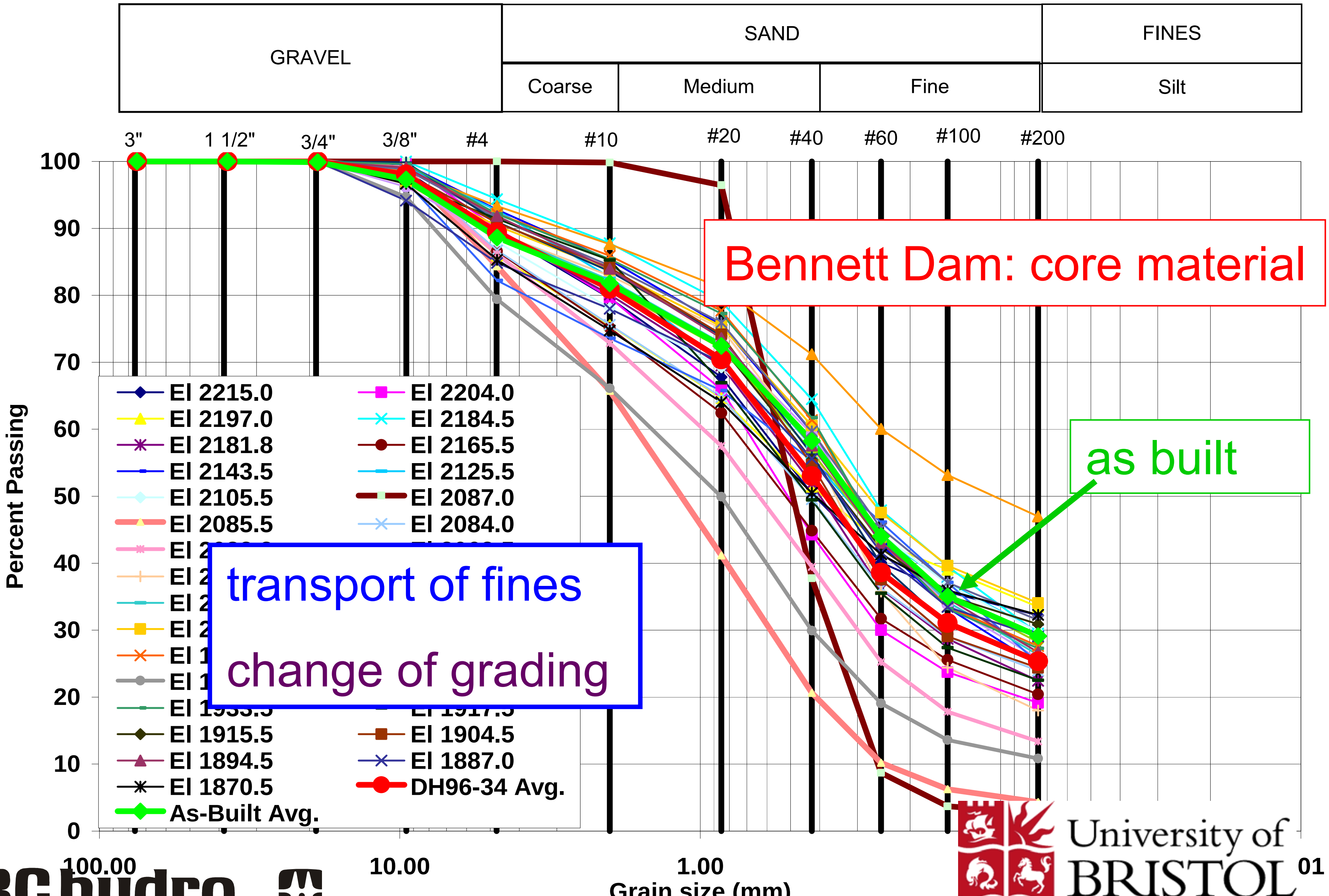
Grading state index



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Grading state index

two examples of general question:

- soil grading may change through erosion/transport or crushing induced by compression/shearing
- what effect does this have on mechanical behaviour?
- material changing (irreversibly) while being studied

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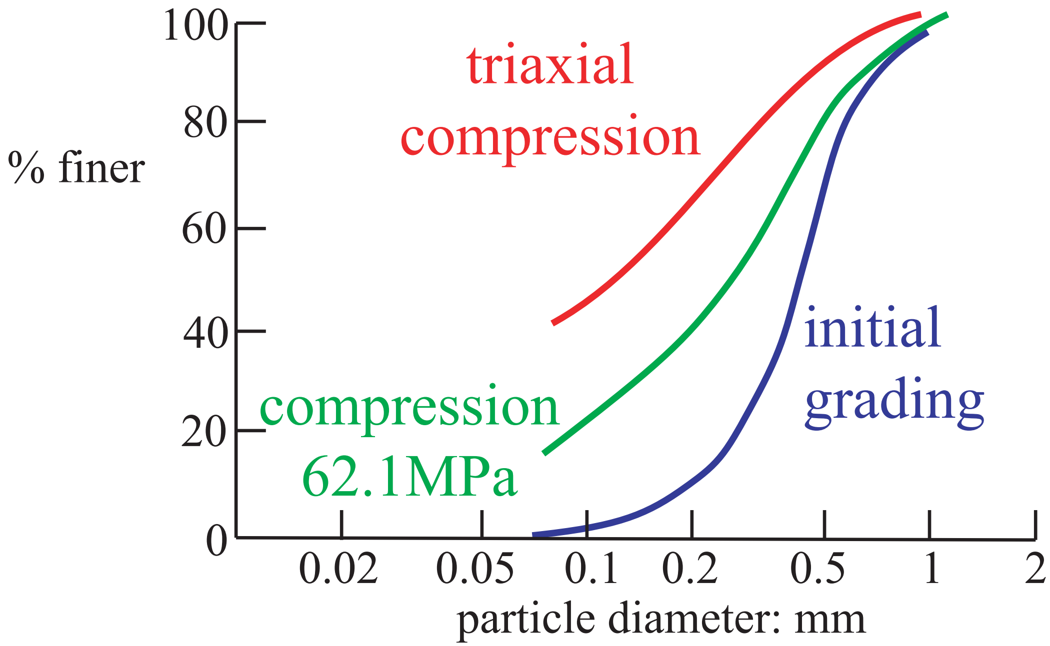
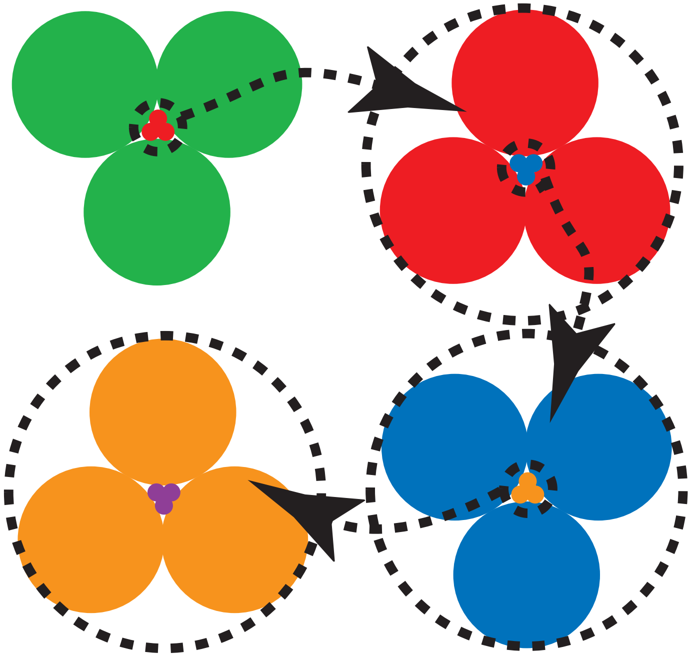
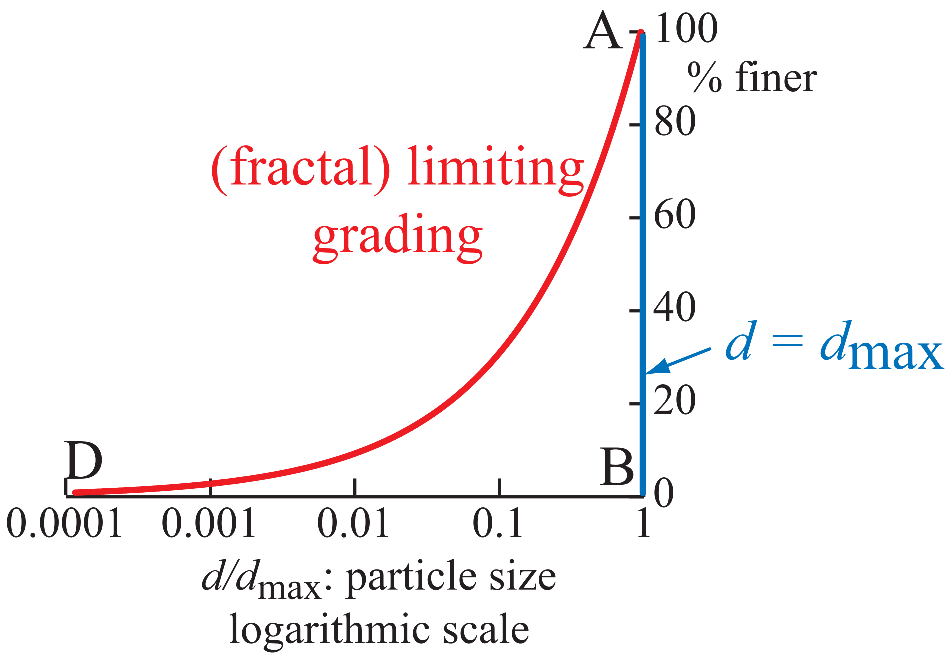
Chattahoochee River sand



- evolving particle size distribution: particle breakage
- triaxial compression with confining pressure 62.1MPa
- (Vesić & Clough, 1968)

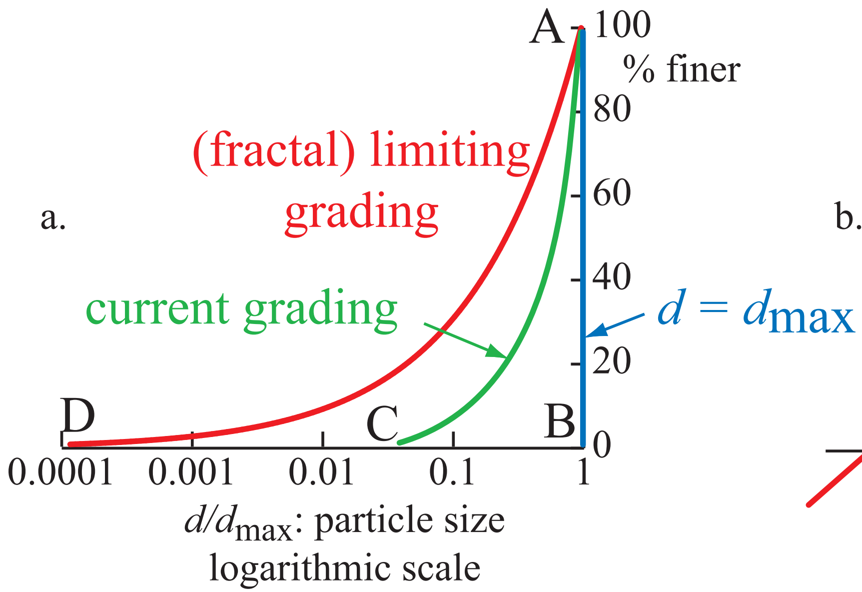
limiting gradings...

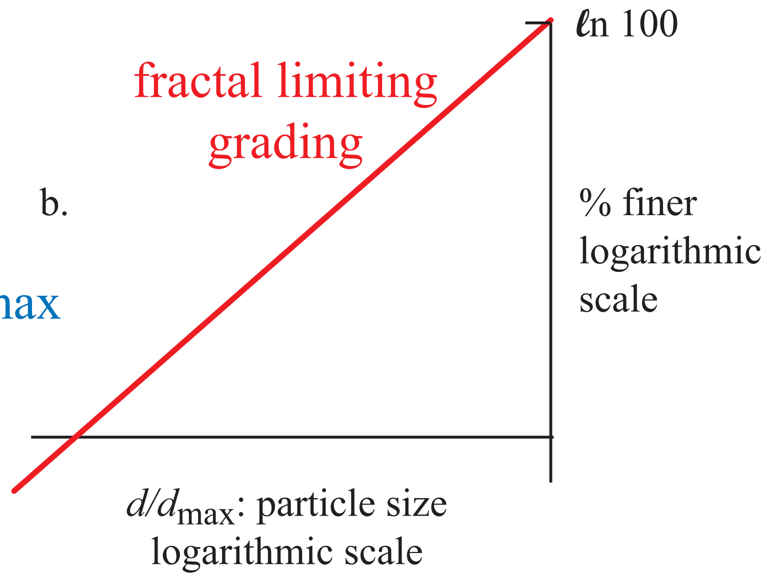




- define limiting gradings
- self-similar fractal grading: one extreme
- single size material: other extreme

define grading state index I_G





- particle crushing → self-similar 'fractal' grading (McDowell)
- fractal distribution linear in log:log plot
- define I_G as ratio of areas ABC and ABD

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Grading state index

three aspects to the problem:

•*characterisation* of evolving grading curve – additional grading state index

•*evolution law* for grading state index

•*influence* of grading state index on constitutive properties

research in progress

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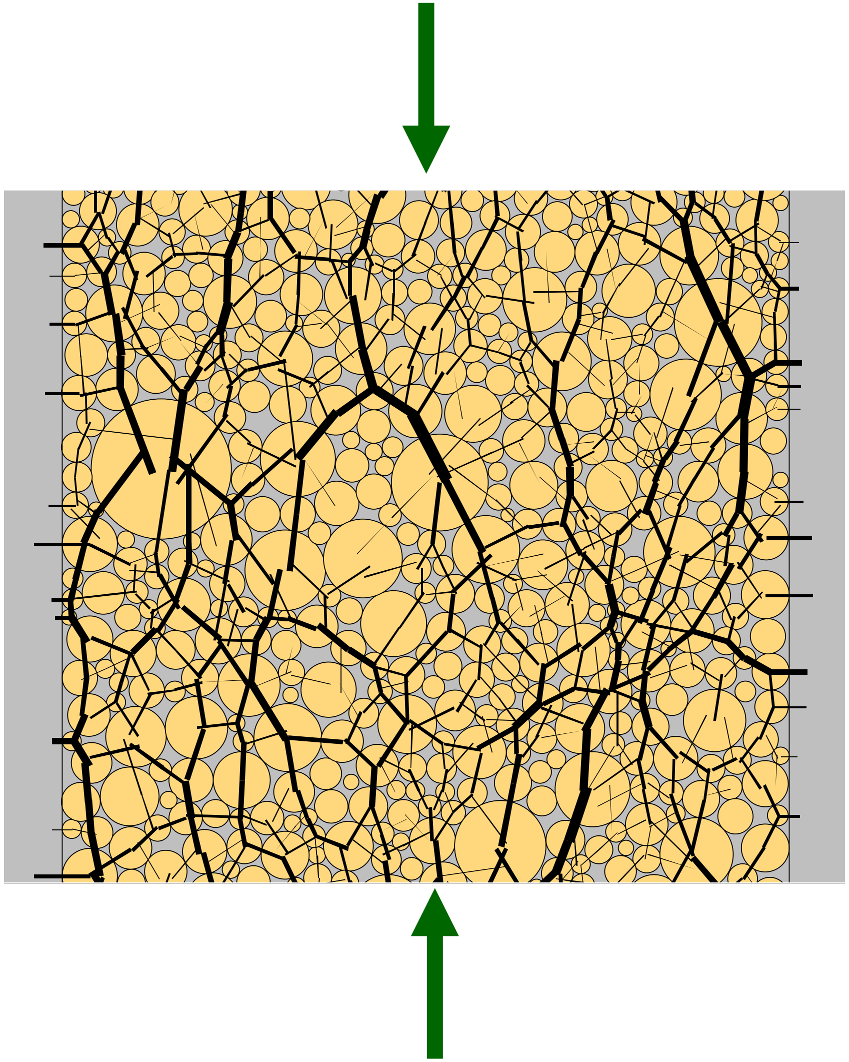
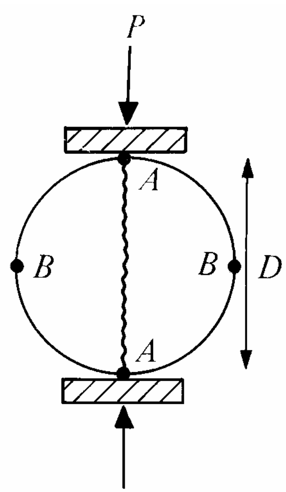
Grading state index: crushing



crushing?

coordination number (number of contacts) larger for larger particles

smaller particles tend to crush



Brazil cylinder test: *tensile* strength of concrete

Maeda (2005)

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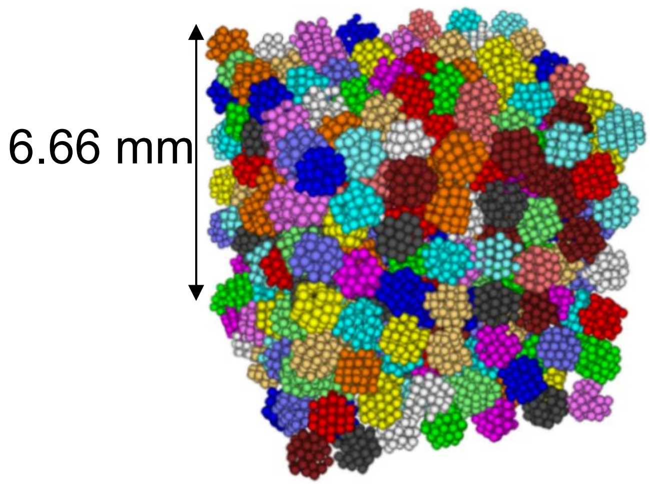
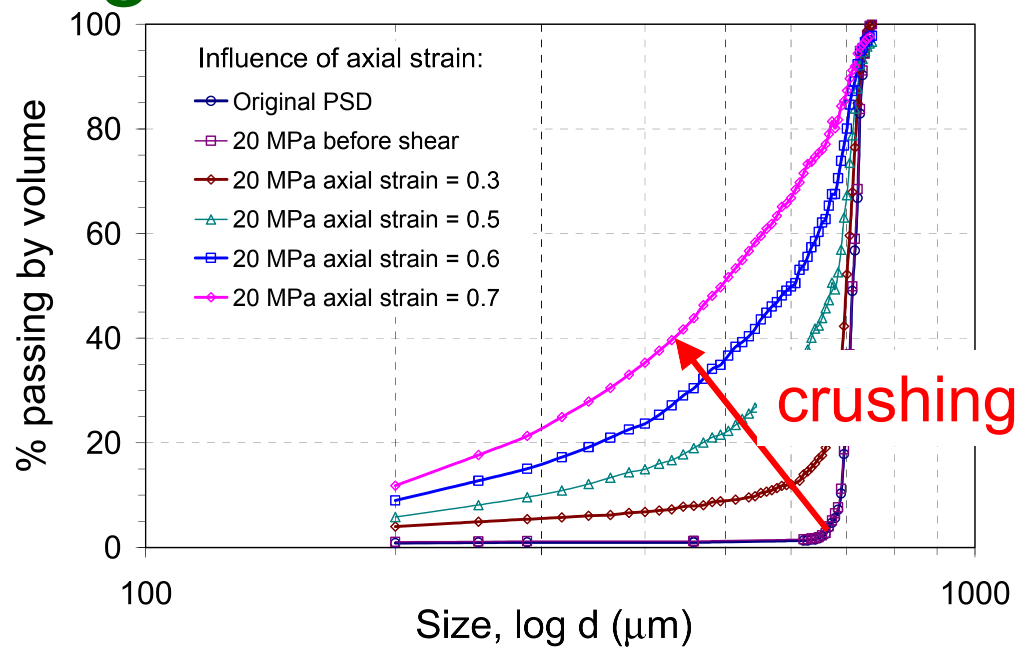
Grading state index



Cheng 389 agglomerates



numerical simulations – compression and shearing of assembly of agglomerates

gradings tend to *self similar* 'fractal' grading

continuous 'fractal' grading: every void space filled with progressively smaller particles

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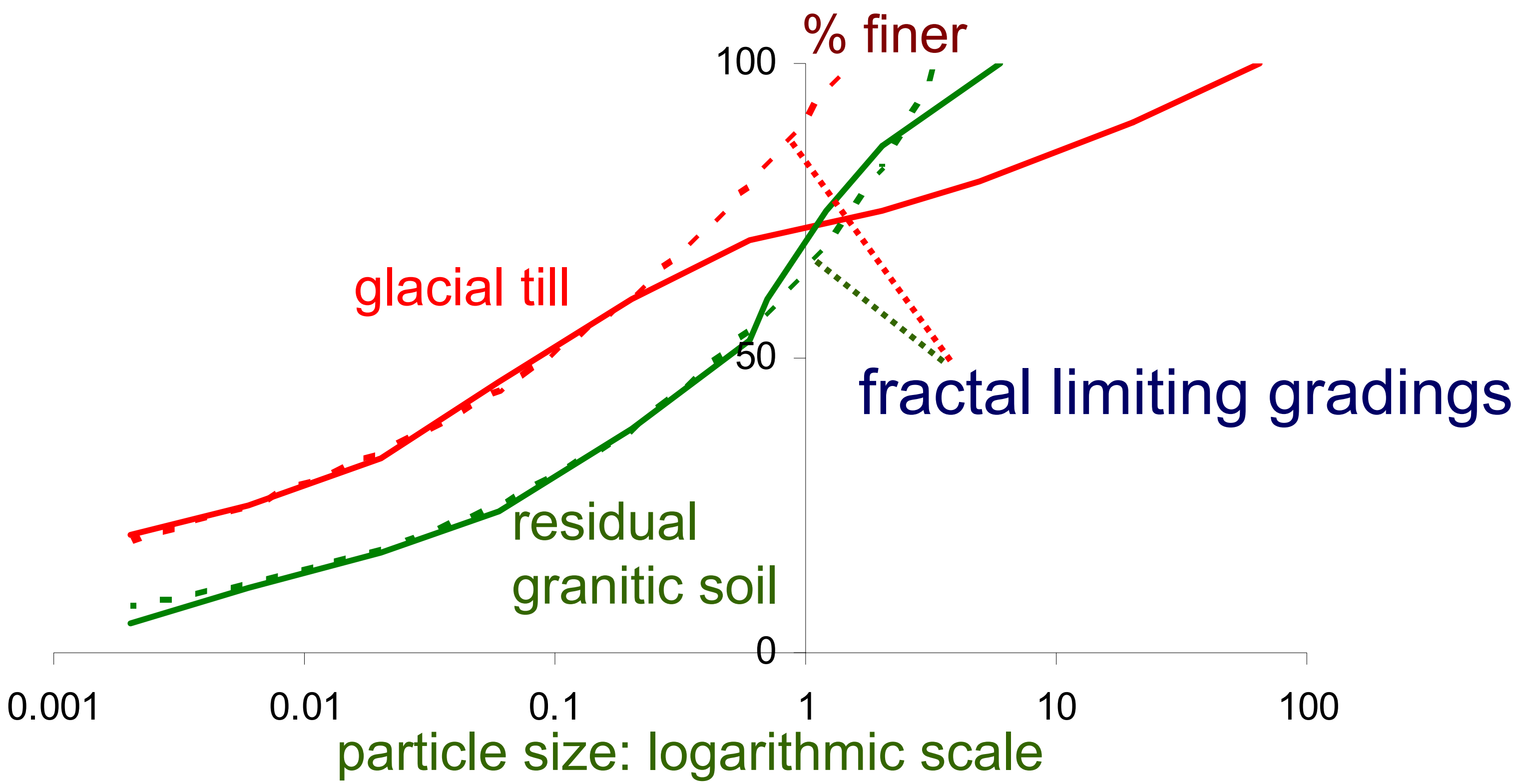
Grading state index



natural soils:

discovering fractal limiting gradings?

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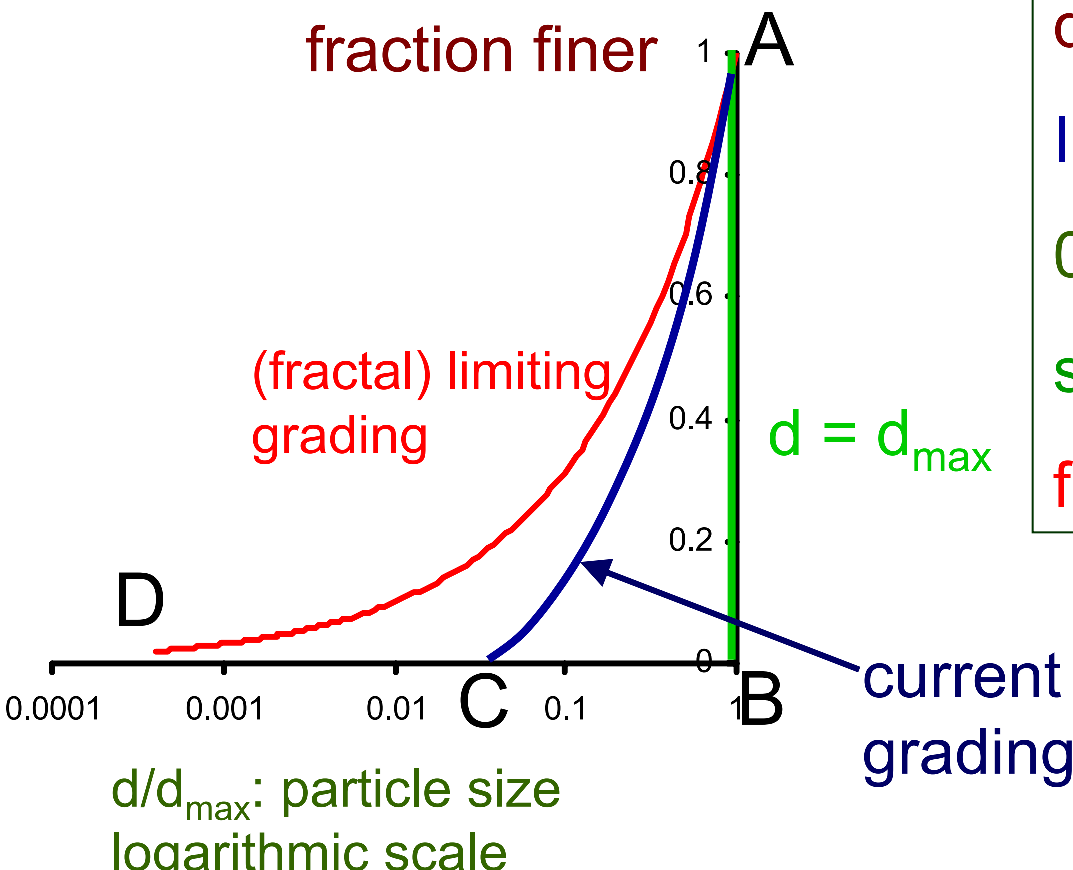
Grading state index I_G: definition



definition of I_G?

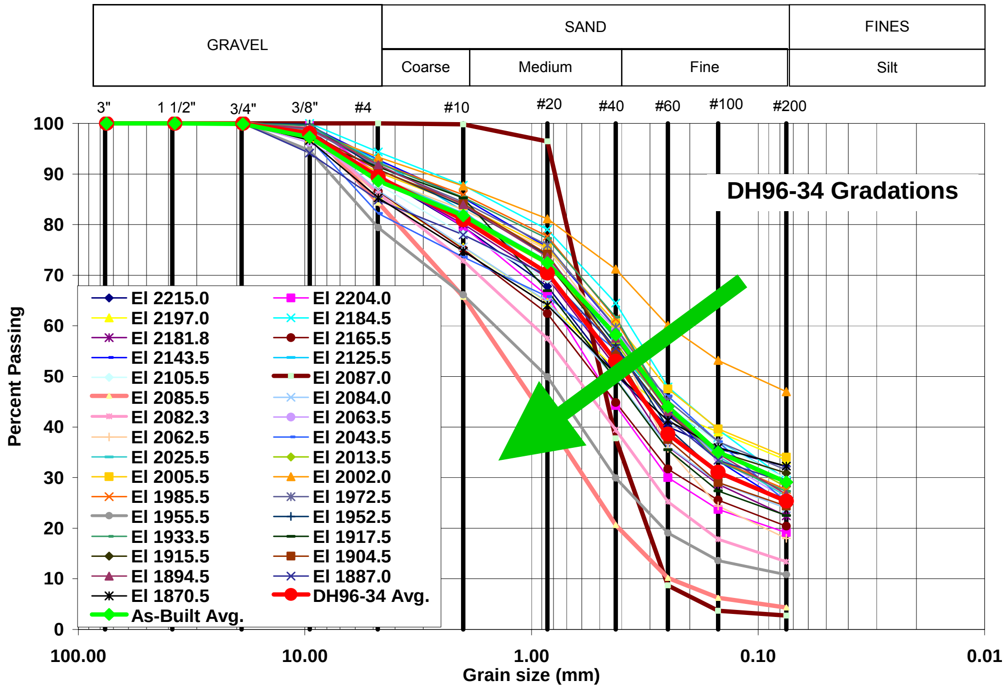
I_G = area ABC/area ABD

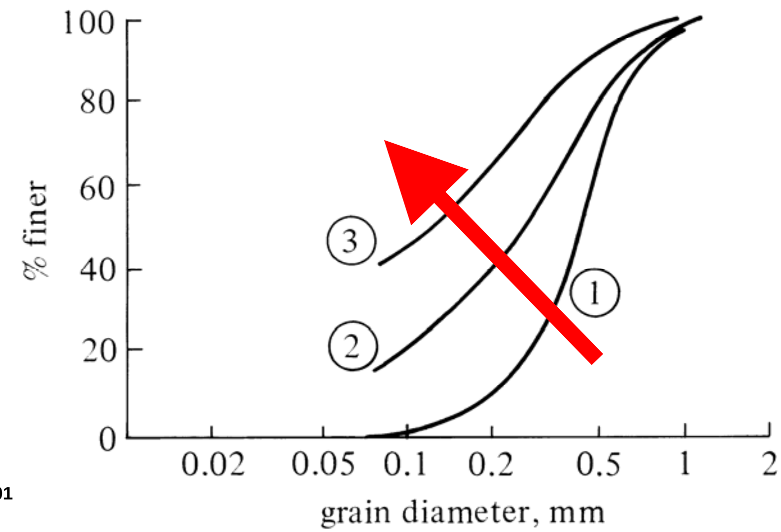
$0 < I_G < 1$

single size AB: $I_G = 0$

fractal limit AD: $I_G = 1$

Grading state index I_G





Bennett Dam core

fines removal: I_G falling

Chattahoochee River sand

grain crushing: I_G increasing

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Grading state index I_G: evolution

Bennett Dam:

•fines migration modelling (transport/conservation)

•smaller particles preferentially removed: $I_G \downarrow$

crushable particles:

•smaller particles preferentially crush: $I_G \uparrow$

•link with mineralogy, angularity, packing, stress level, mobilised friction (Hardin)

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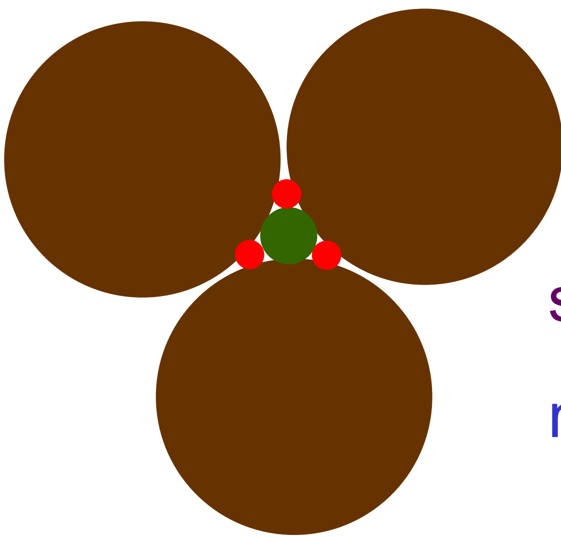
Grading state index I_G: influence

influence of grading state index on constitutive properties

•elastic properties – unchanged (first order)?

•friction/strength – unchanged (first order)?



smaller particles tending to fill gaps

maximum and minimum void ratios $\downarrow$ as I_G $\uparrow$

•critical state line – expected to change!

evidence?

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critical states (grading)

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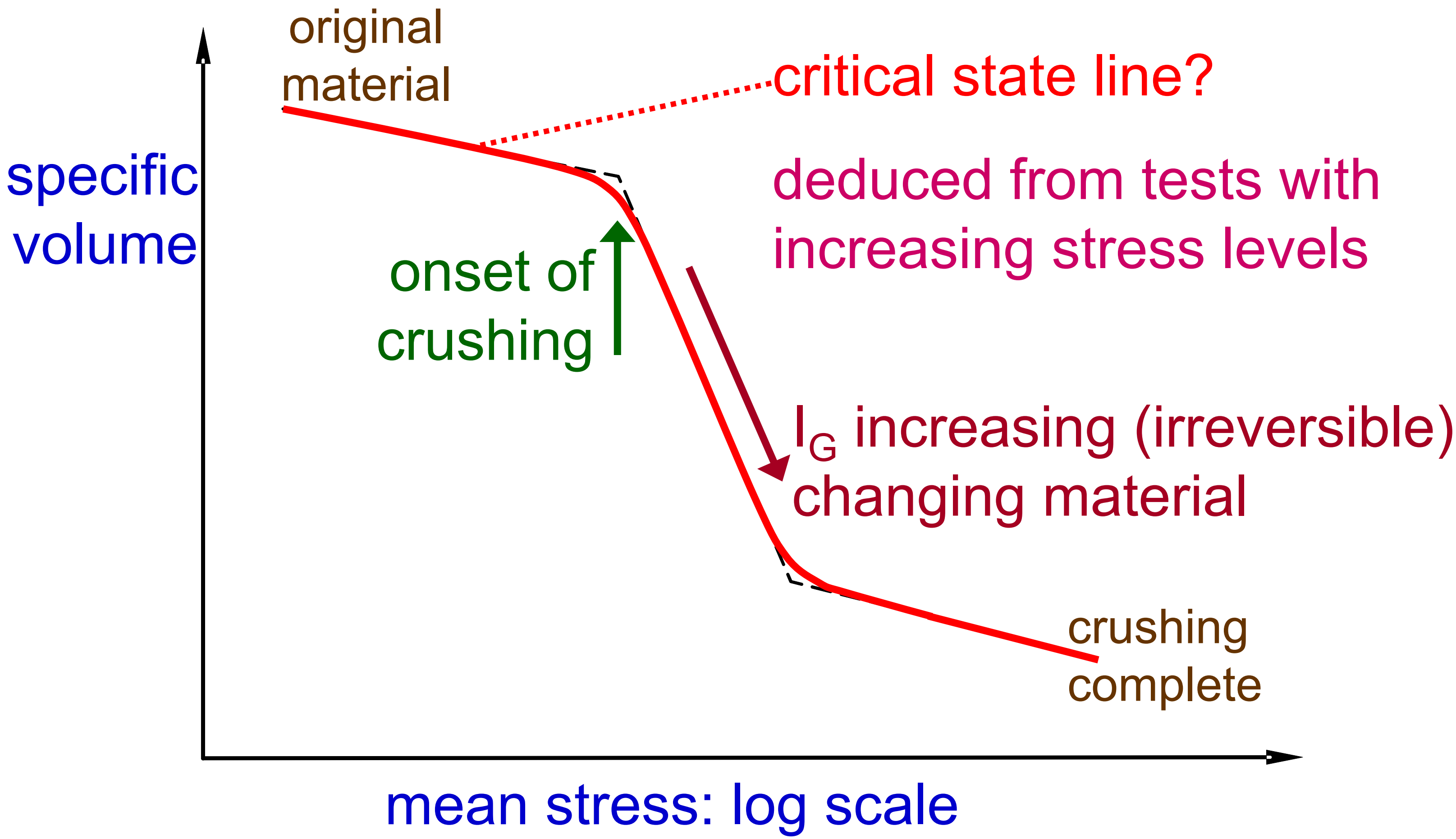
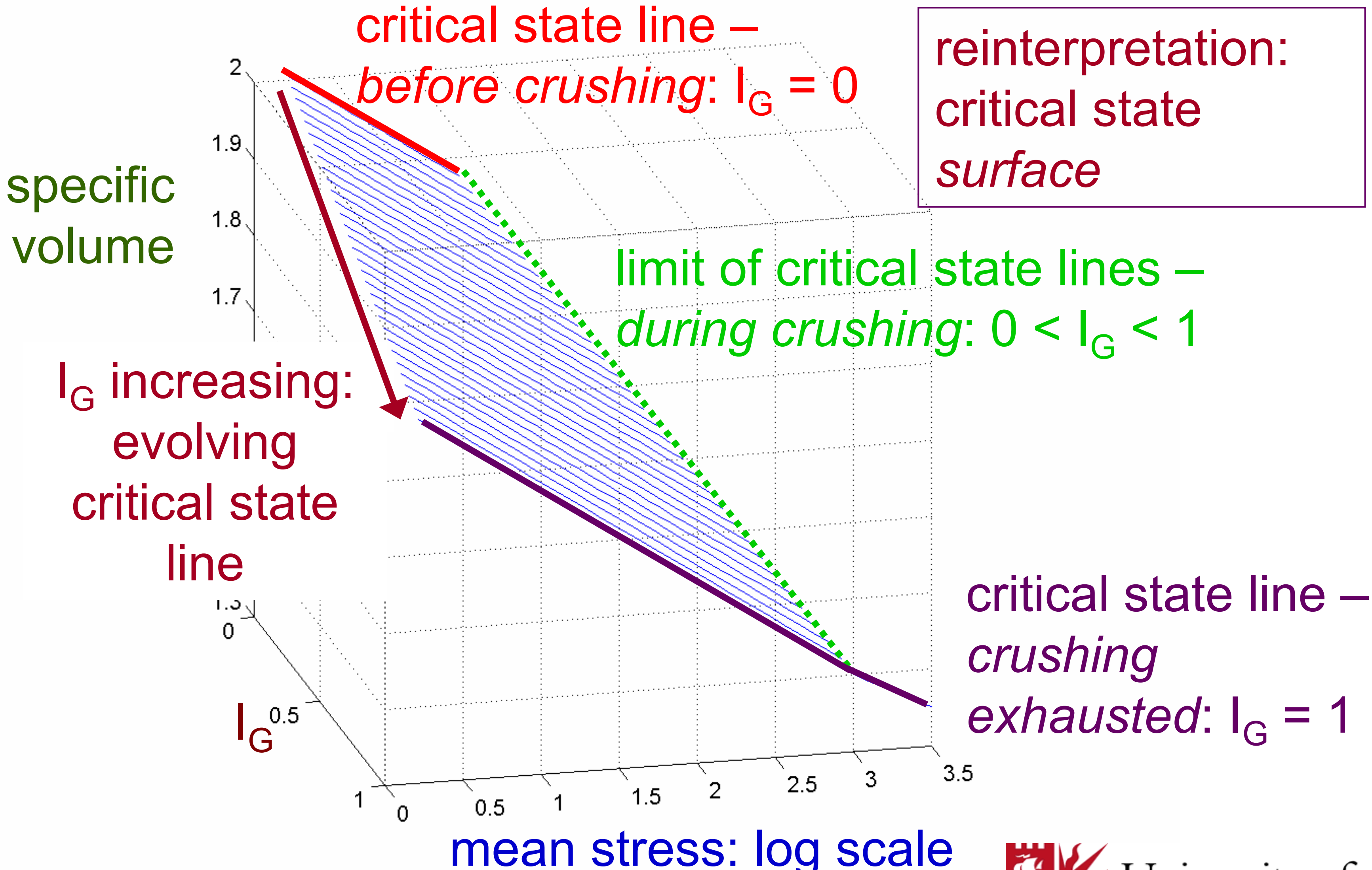
critical states: fabric, grading

- can we be certain that asymptotic state has been reached?
- state = stresses + density + fabric (contacts, etc) + grading (particle size distribution)
- grading change from particle breakage or from erosion
- material changing while it is being tested
- expect all aspects of fabric *including grading* to reach steady state *on average*

Grading state index I_G: critical states



Grading state index I_G: critical states



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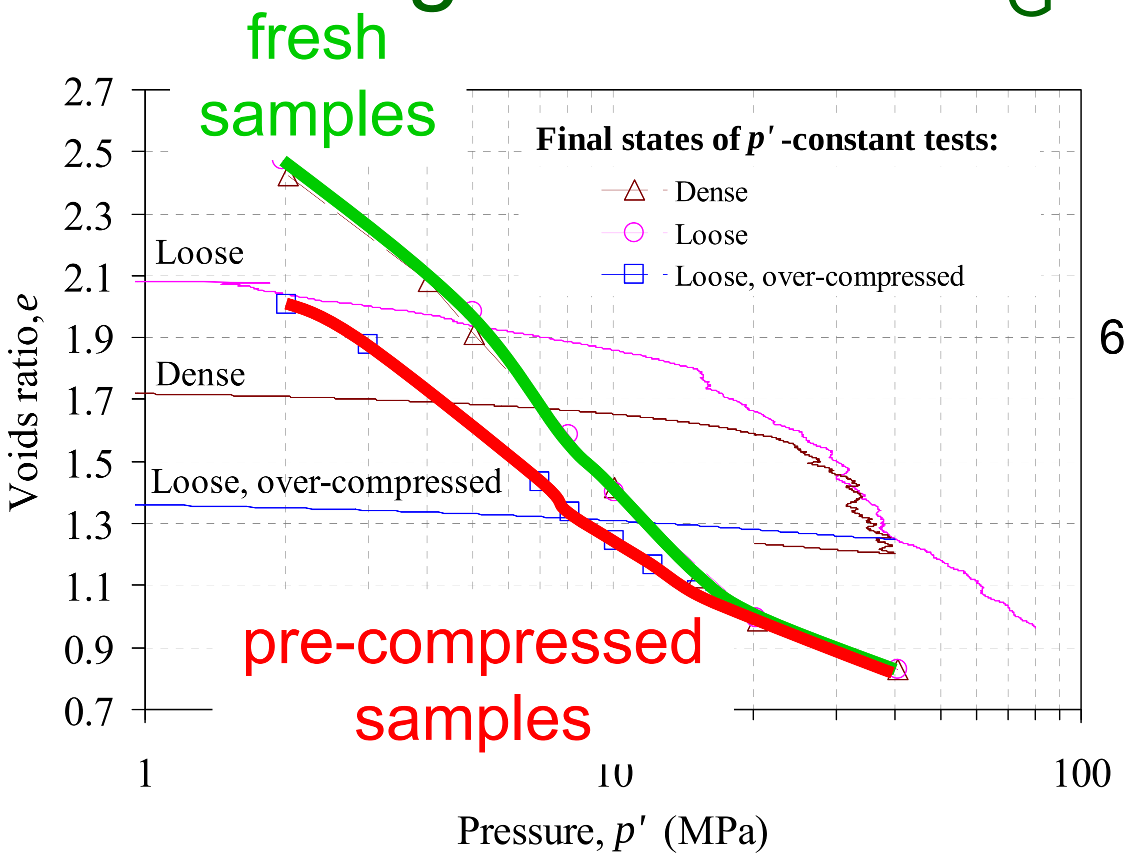
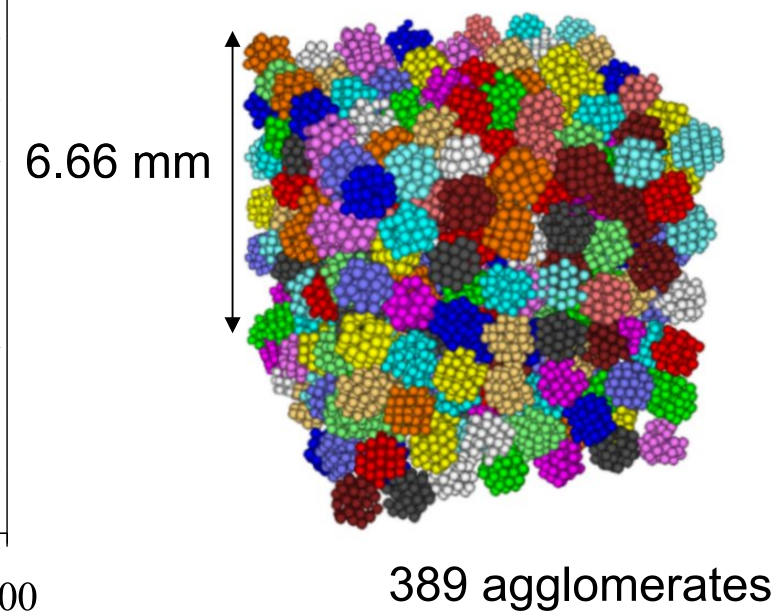
Grading state index I_G: critical states





simulations for assemblies of agglomerates

Cheng, 2005

critical state line *changes* with crushing

fresh samples – pre-compressed samples

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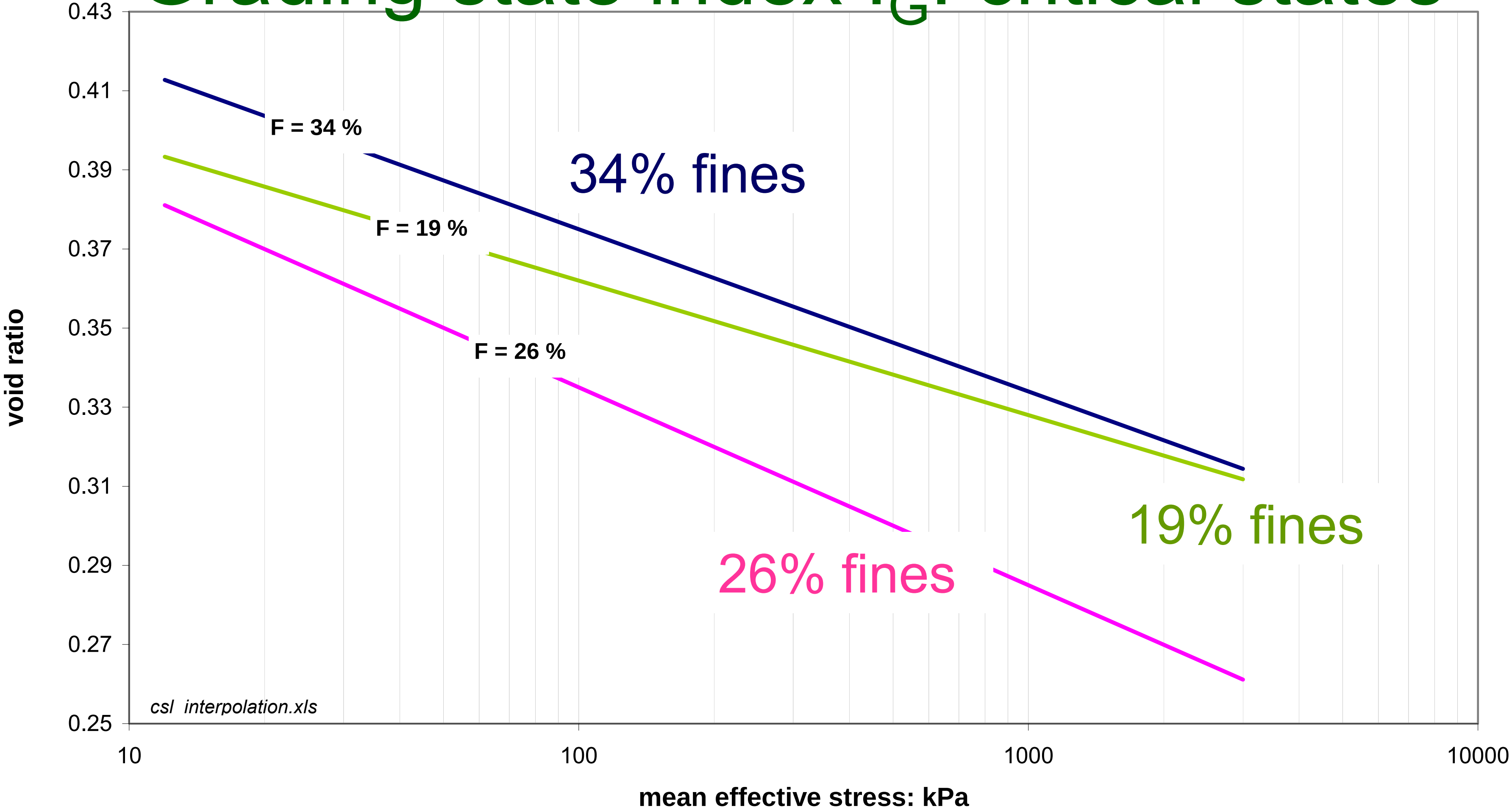
Grading state index I_G: critical states



Bennett Dam: interpretation of effect of fines content on location of critical state line (triaxial tests, artificial mixtures)

non-monotonic…!?

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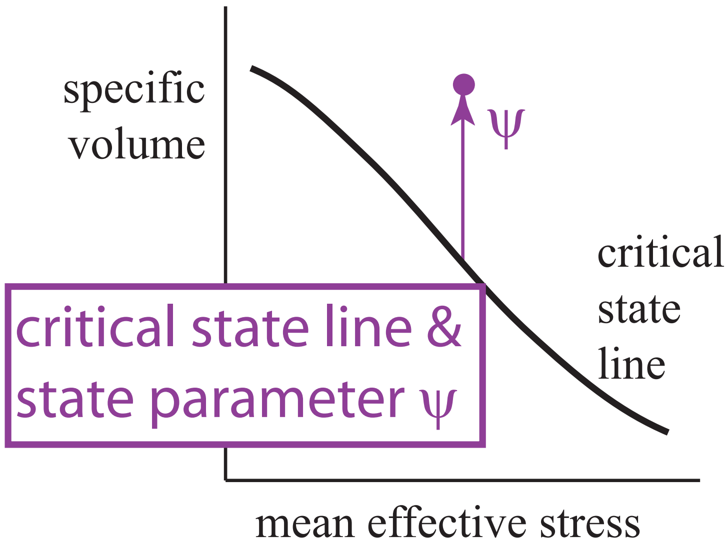
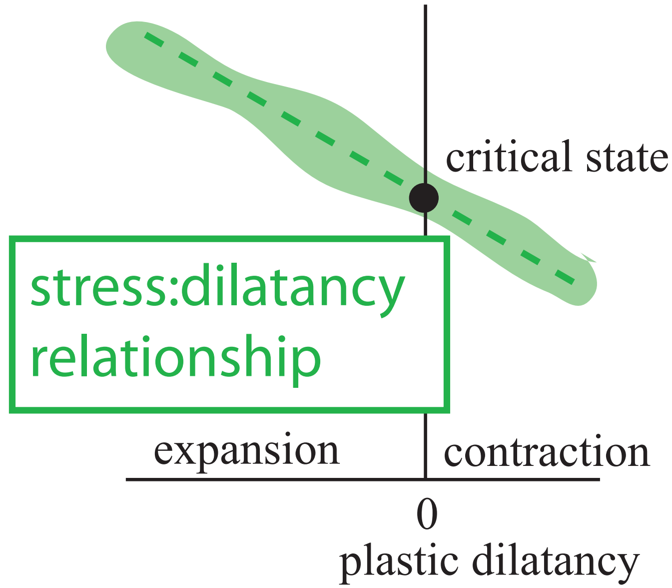
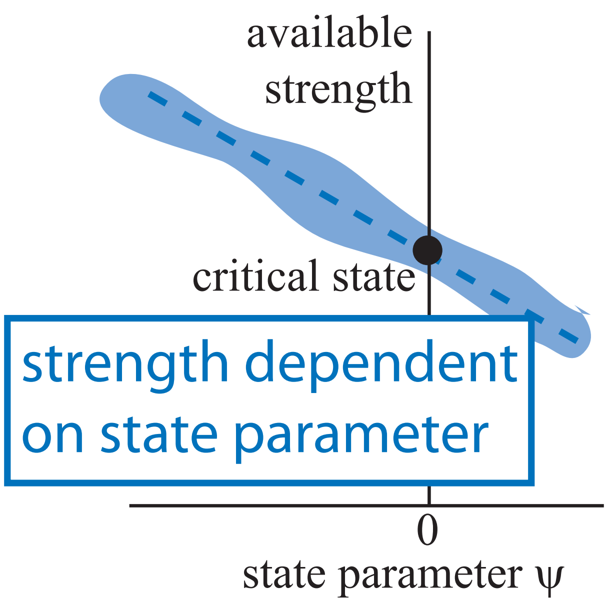
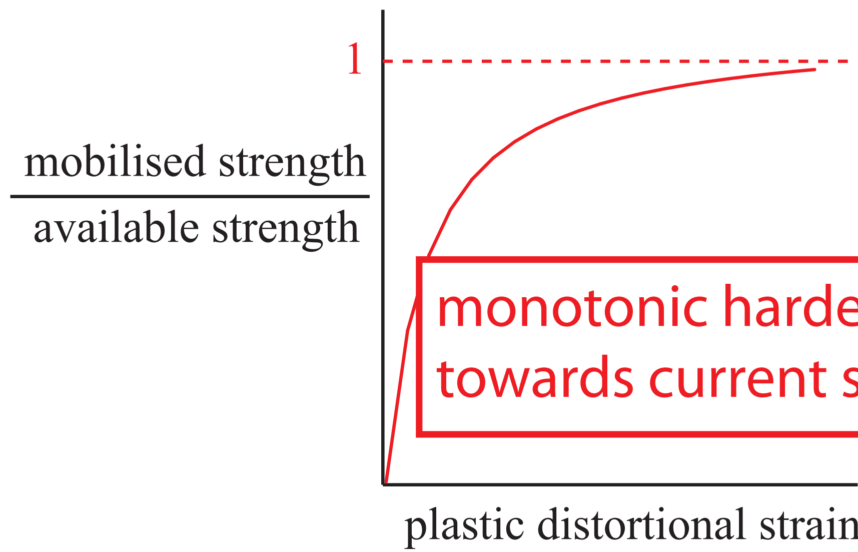
Severn-Trent sand model









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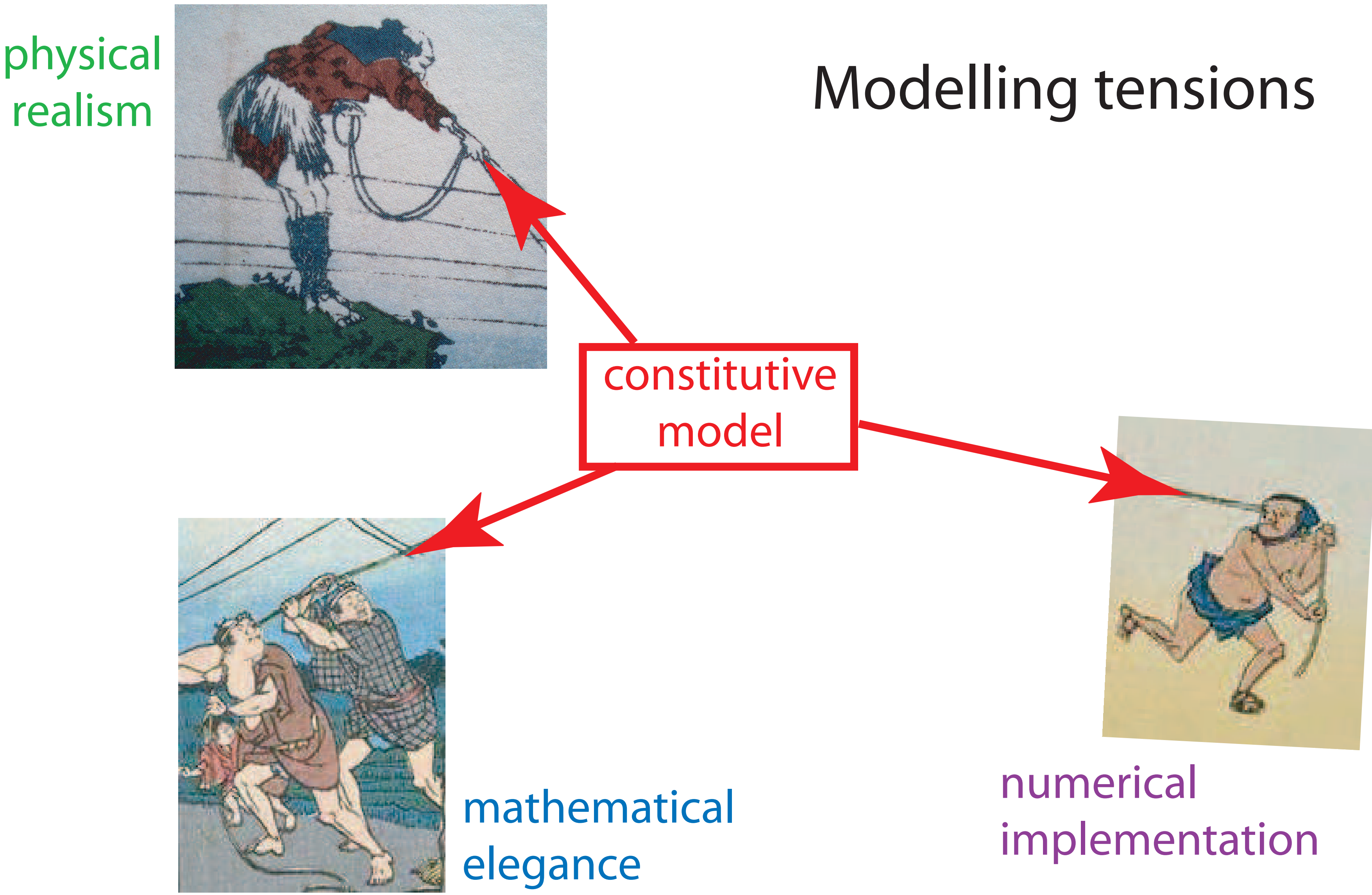
Severn-Trent sand

- extended Mohr-Coulomb model
- model built round critical state line as divider of response
- adequate complexity - effects of density, strain softening
- simple assumed relationships
- (use as basis for extended model)

- many such models exist - aesthetic judgement - mathematical expediency

Modelling tensions



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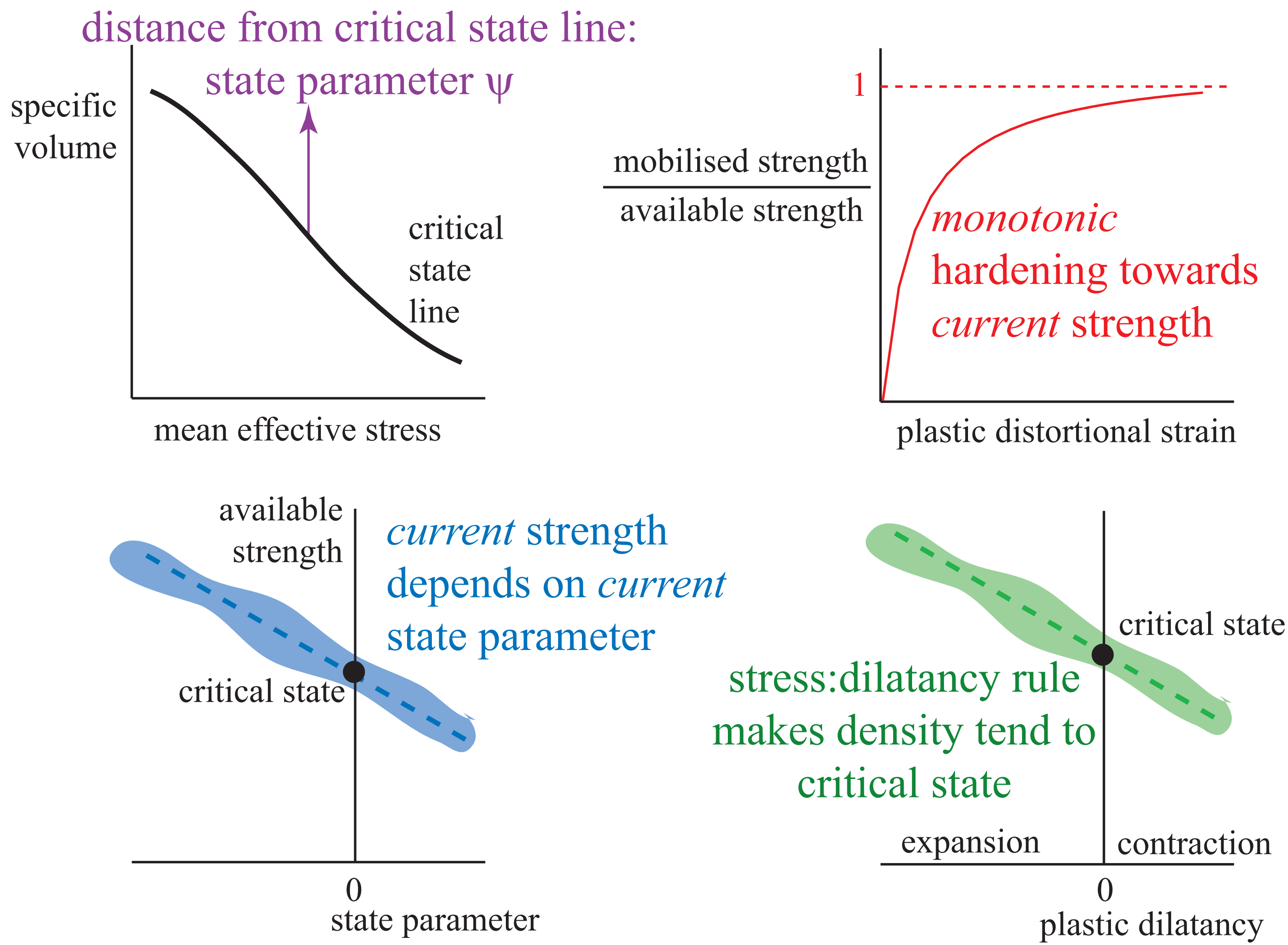
Severn-Trent sand: 4 key elements



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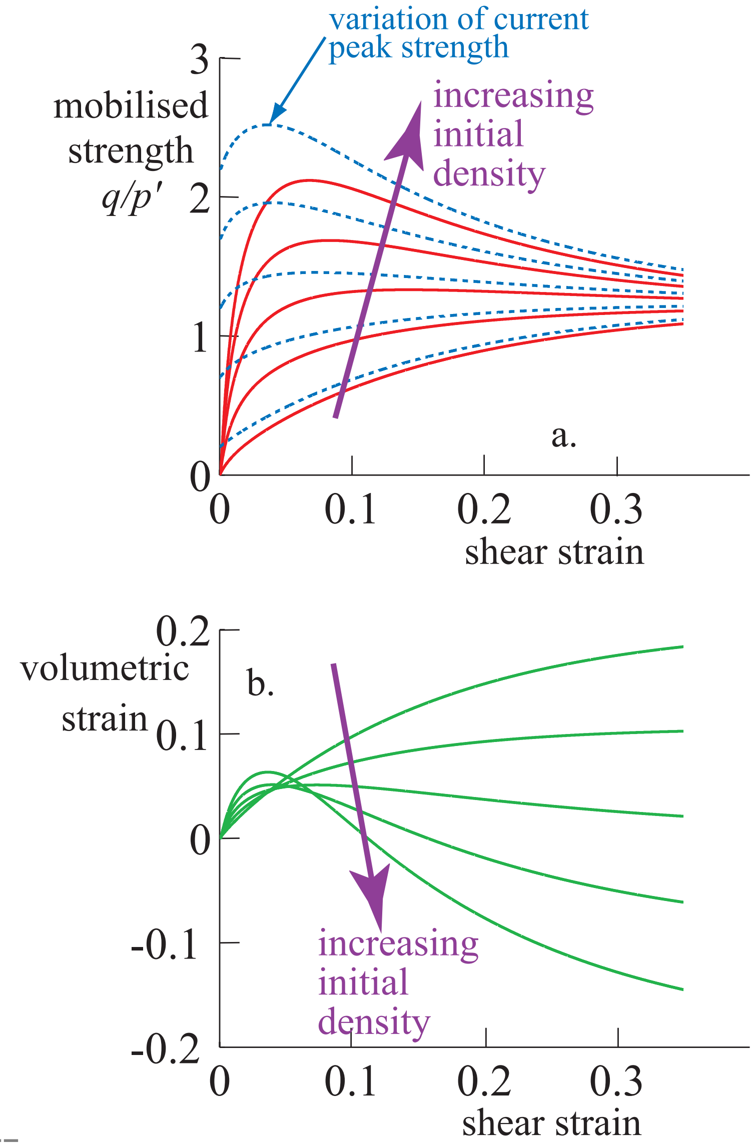
Severn-Trent sand: simulations



- drained triaxial compression tests
- different initial densities
- variation in current strength
- monotonic hardening but non-monotonic response!

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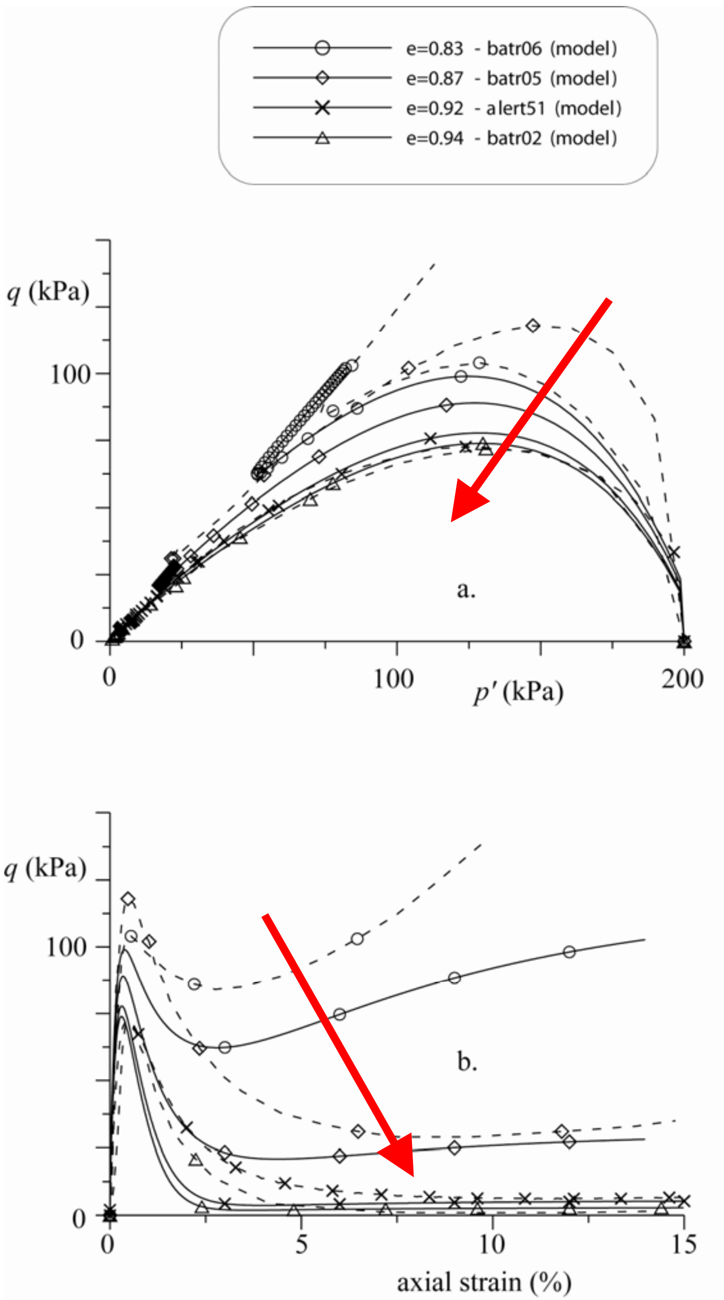
Grading state index



Bennett Dam

Severn-Trent sand

transport of fines from core

void ratio ↑

grading state index ↓

critical state line ↓??

state parameter ↑

soil feels looser ☹

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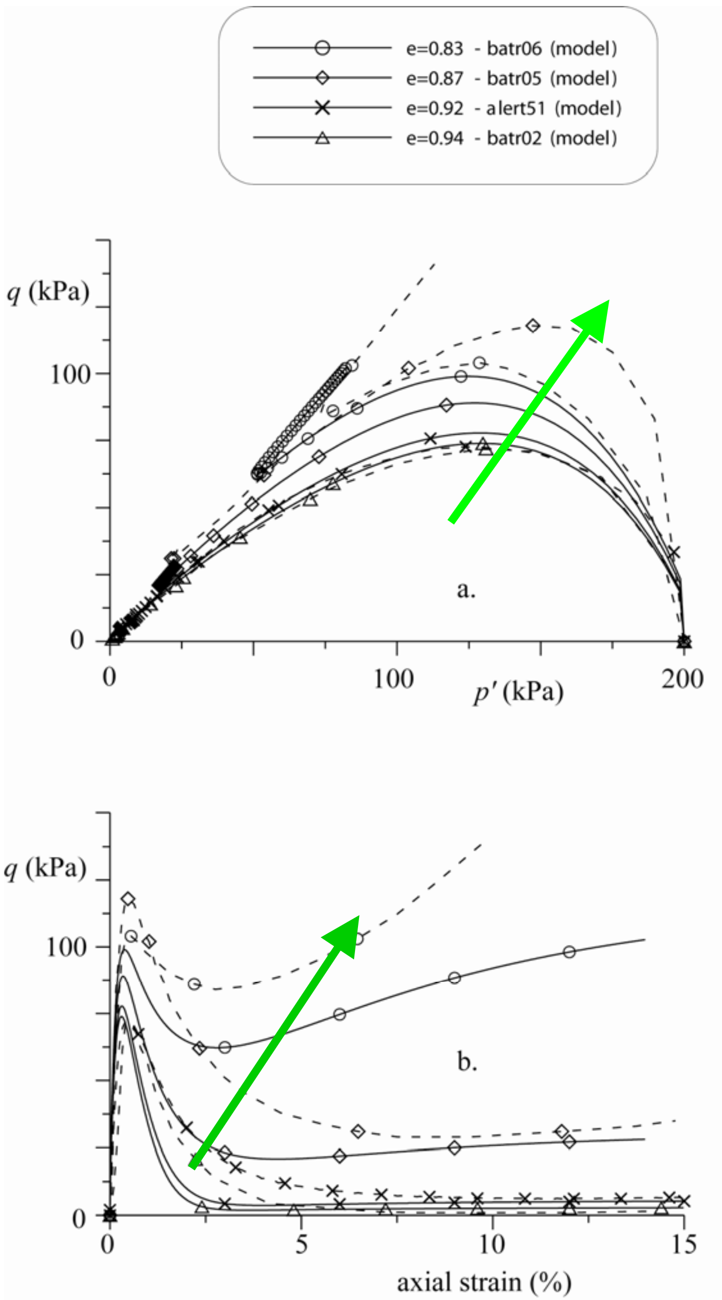
Grading state index



Bennett Dam

Severn-Trent sand

transport of fines from core

void ratio ↑

grading state index ↓

critical state line ↑??

state parameter ↓

soil feels denser ☺

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Grading state index

Bennett Dam??

benefit of simple model that systematically incorporates changes in stress level *and* density *and* grading (making up *state* of soil)

model has to be honed – subtle data requirements for calibration

most testing has used artificially prepared mixtures

Question 4: How does the changing grading of a soil affect its mechanical behaviour?

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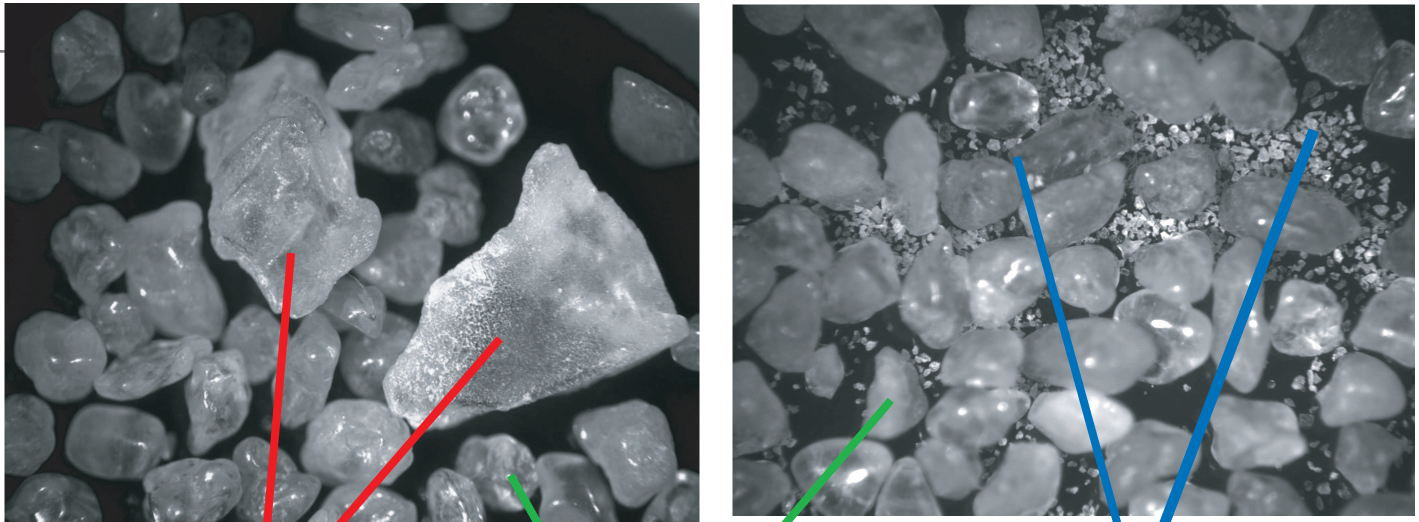
hypothesis

- erosion removes finer particles
- grading becomes narrower
- removal of particles reduces density
- narrowing of grading changes asymptotic critical states
- proximity to critical state described by state parameter
- state parameter controls response
- Severn-Trent sand - model built around critical states

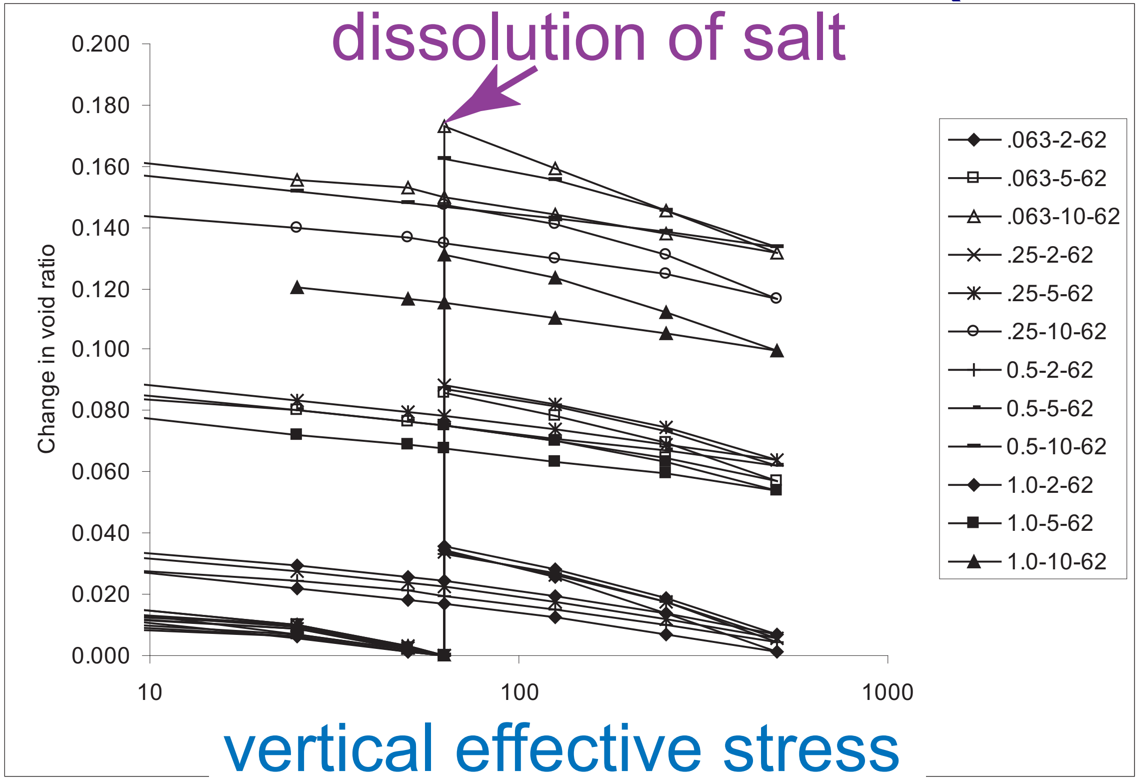
experimental evidence?



- mixtures of sand (silica) and salt (NaCl)
- oedometer tests
- dissolve salt while mixture under stress
- (tests by John McDougall)

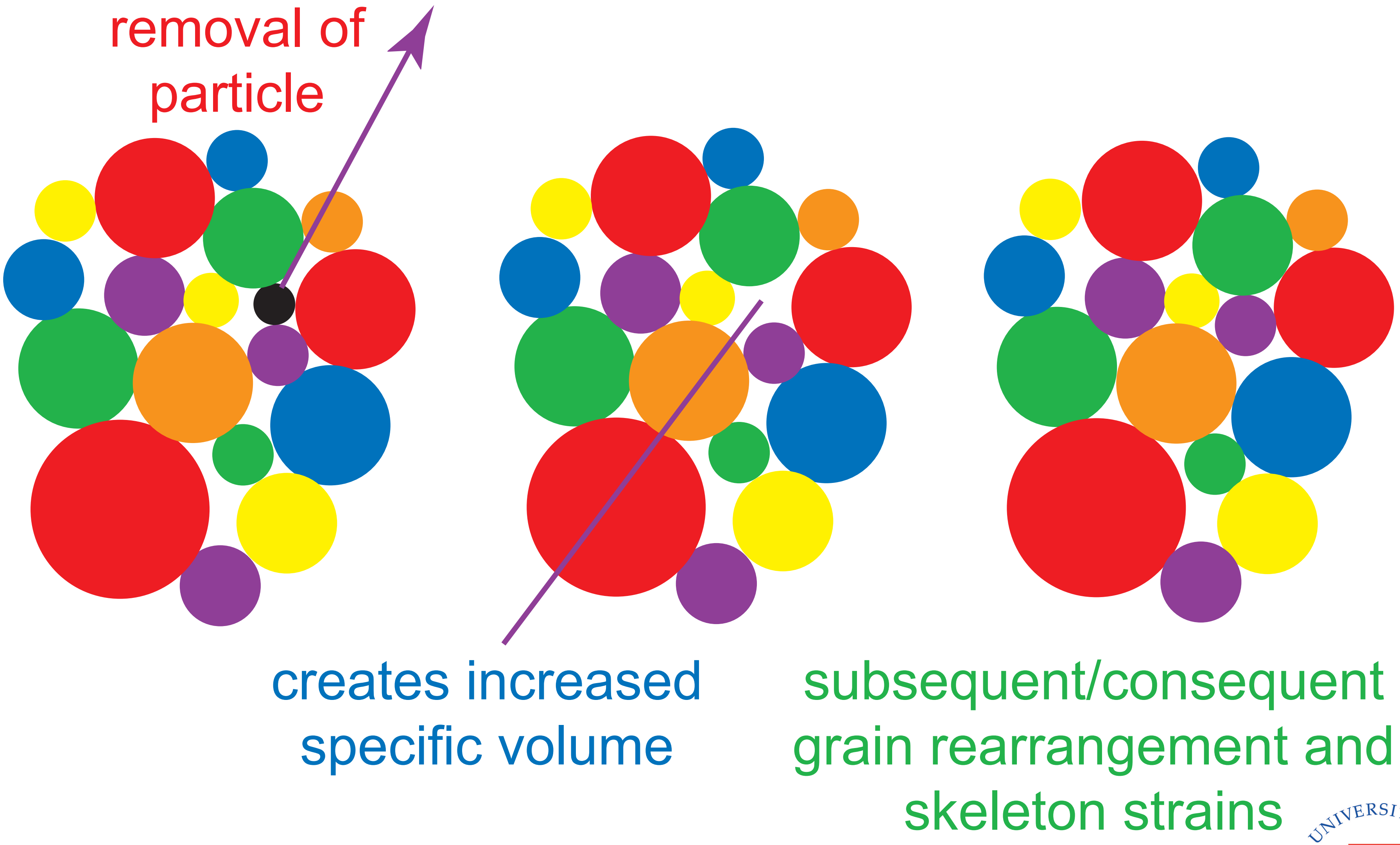
sand and salt: oedometer (McDougall)

change in void ratio



- dissolve salt under stress
- removal of salt increases specific volume (reduces density)
- resulting structure unstable - volumetric compression

effect of removal of particles



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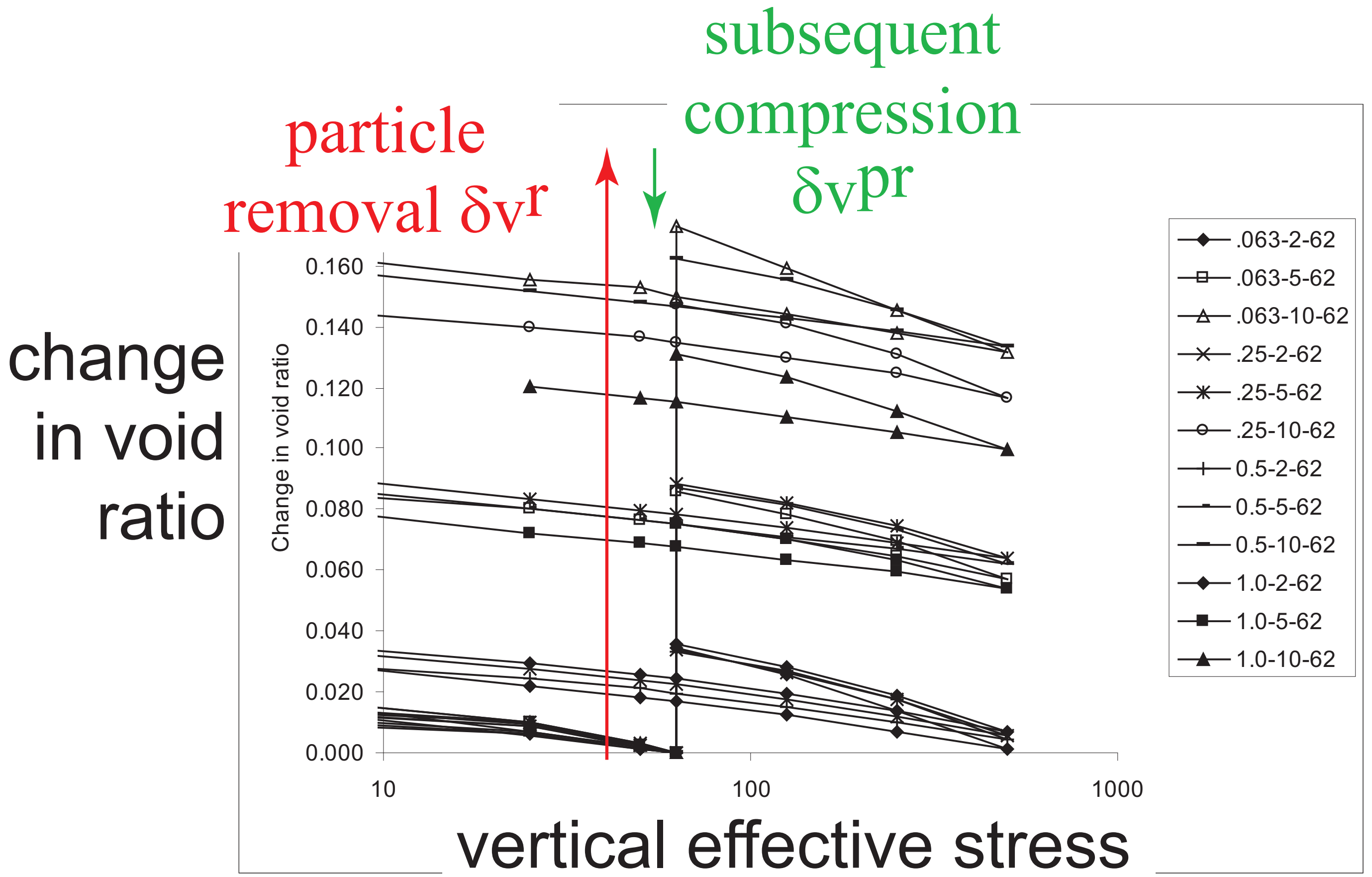
sand and salt: oedometer (McDougall)



- dissolve salt under stress
- removal of salt increases specific volume (reduces density)
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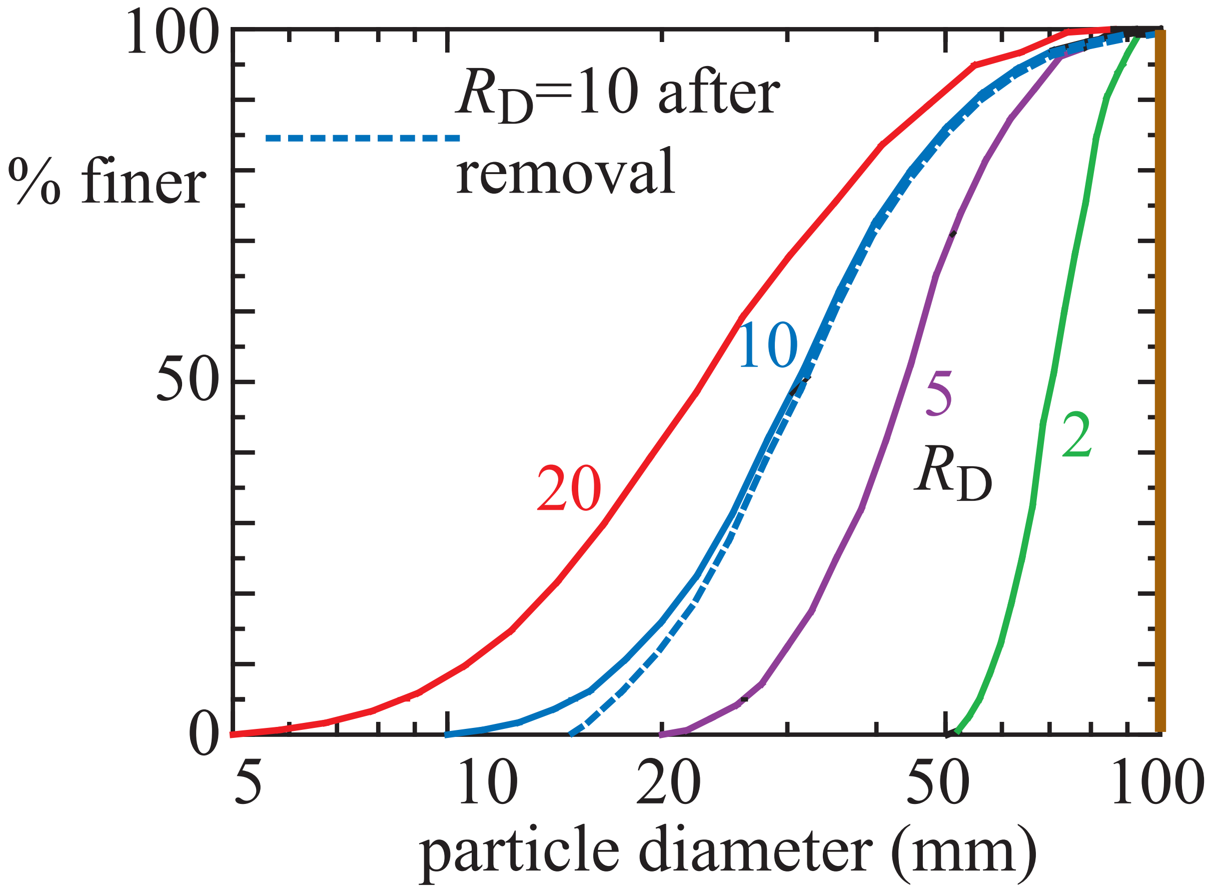
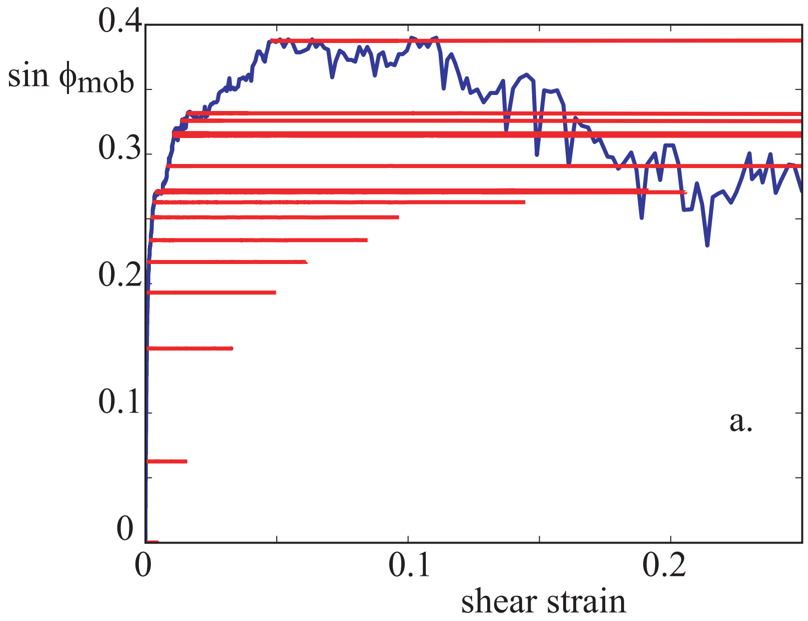
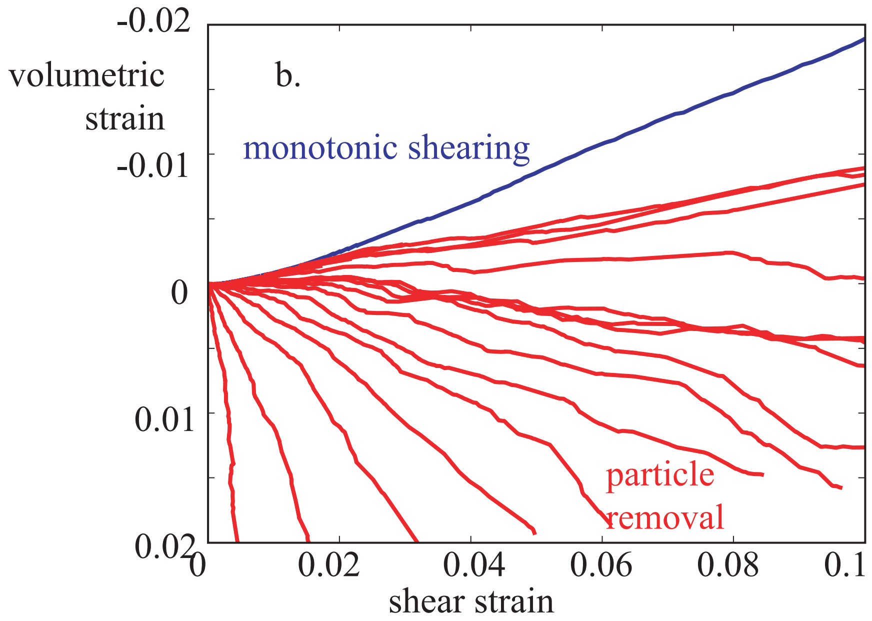
DEM: removal of particles: gradings



- gradings used for discrete element modelling (solid curves)
- grading reached by removal of particles from initial grading with $R_D = 10$ (dotted curve)

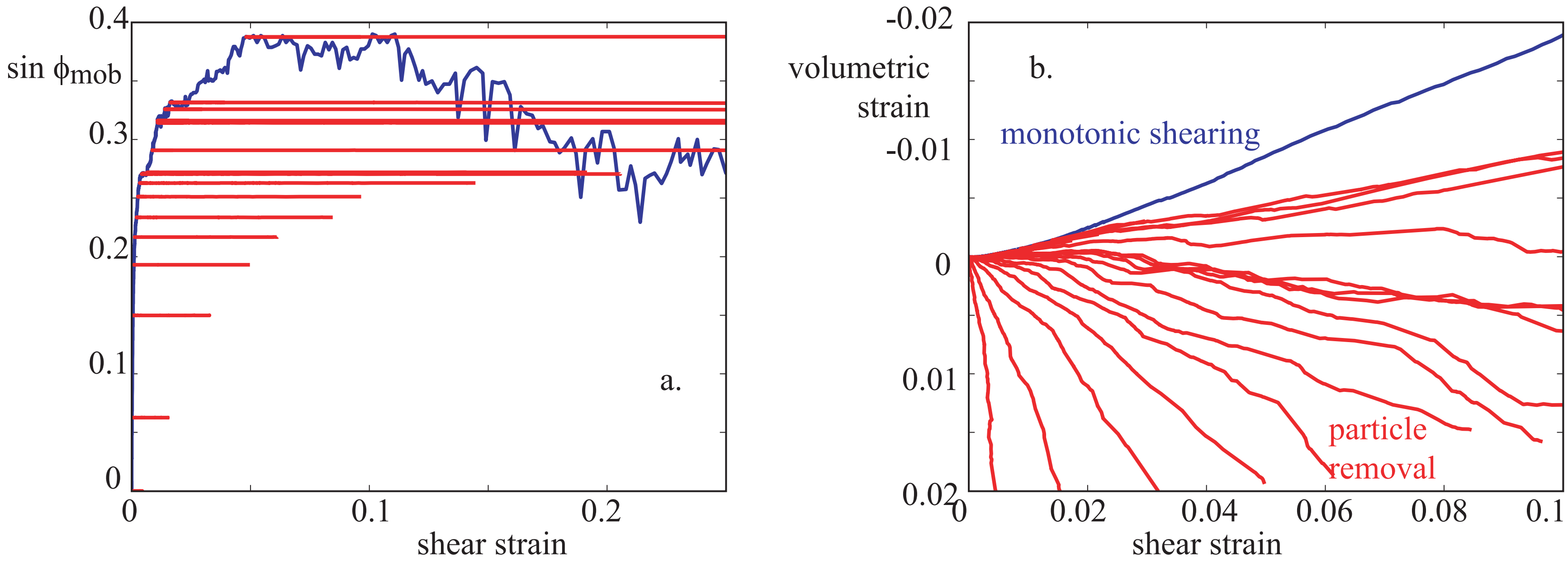
DEM: removal of particles: deformations





- fine particles plucked out by ‘deus ex machina’
- initial grading $R_D = 10$; mean stress 100kPa; constant stresses
- deformations more contractant than previous shearing

DEM: progressive removal of particles



- deformations more contractant than previous shearing
- require deformation mechanism that triggers both volumetric and distortional strains

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modelling proposals

- characterisation of grading
- link between grading and particle removal
- link between grading and critical states
- underpinning constitutive model for distortional response
- assumption concerning change of specific volume resulting from erosion (destabilisation)

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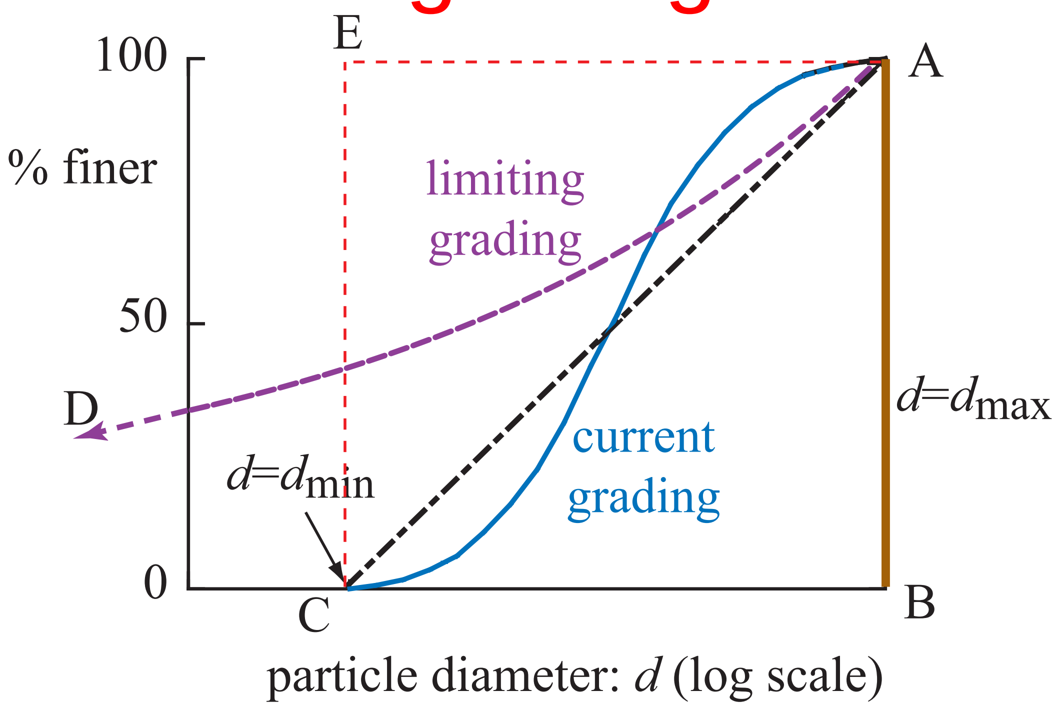
grading state index I_G



- $I_G =$ area ABC/area ABD (current and limiting gradings)
- limiting grading might be fractal (Appolonian)...
- ...scaling factor for calculation of I_G (area ABD $= B$)
- (other definitions possible)
- for linear grading $I_G = [\ln (d_{max}/d_{min})]/2B$

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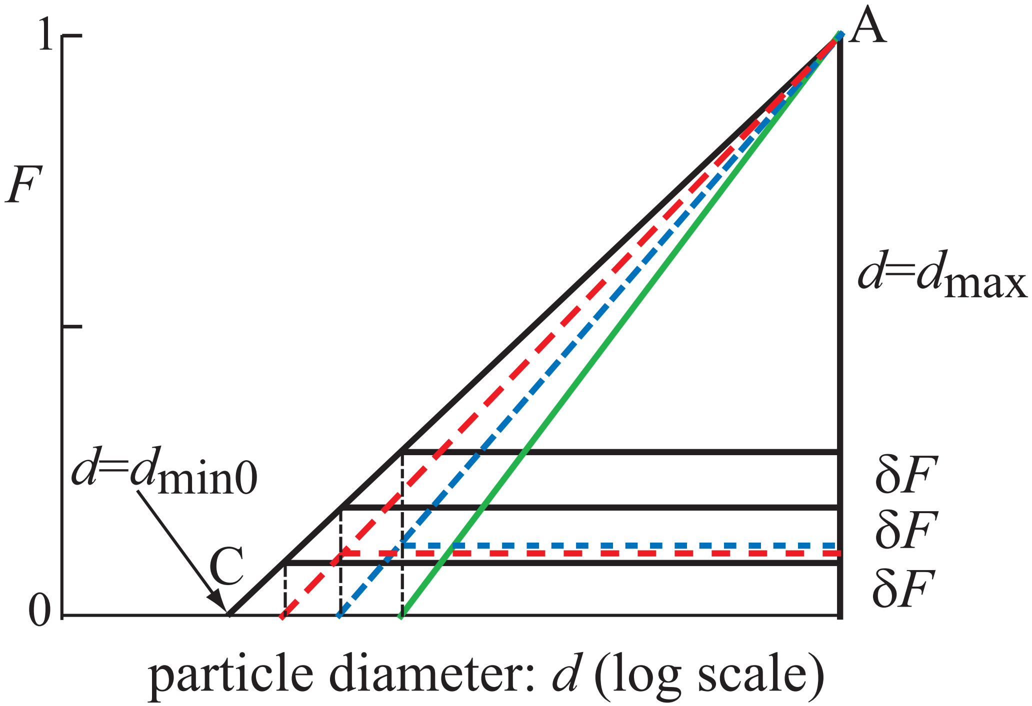
particle removal and specific volume

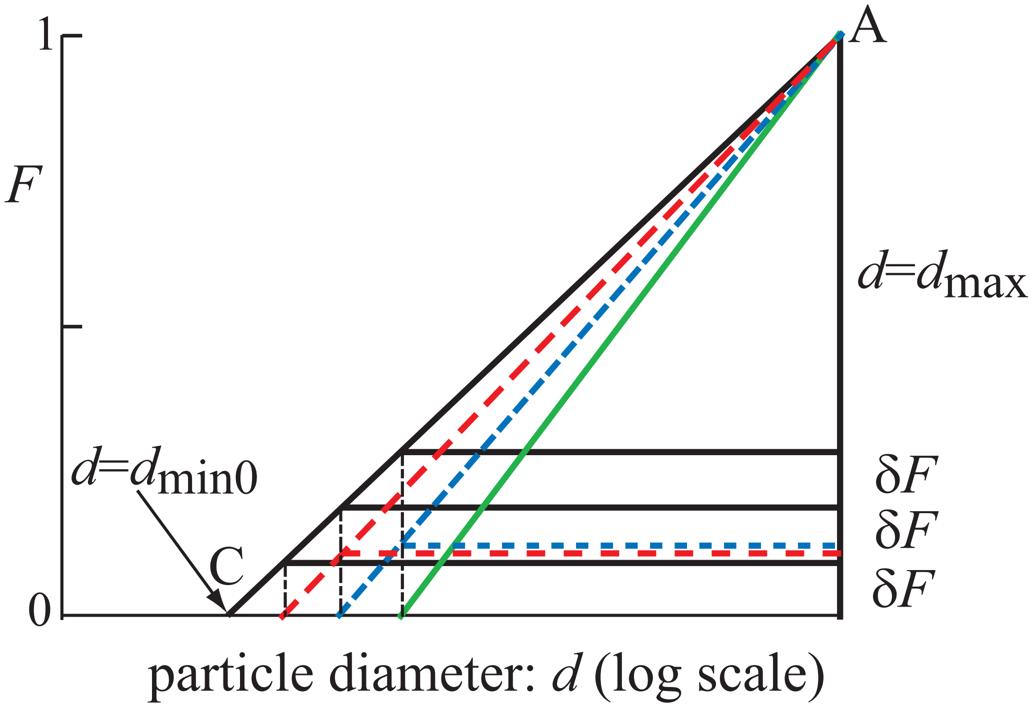
- removal of material creates void space and reduces volume of solid
- $v = (V_v + V_s)/V_s$
- $\delta v^r/v = \delta V_s/V_s$
- particle removal also changes grading state index I_G

particle removal and grading



- assume analytical (linear) grading
- $F = \ln(d/d_{min0})/\ln(d_{max}/d_{min0}) = (V_s)_{d<d}/(V_s)_{d<d_{max}} = (V_s)_{d<d}/V_{s0}$
- removing smallest fraction truncates grading

particle removal and grading



- remove δV_s of original total solid volume V_{s0} between $F = 0$ and $F = \delta F = \delta V_s / V_{s0} = (\delta v^r / v)(V_s / V_{s0})$
- modifies smallest size $\delta d_{min} = d_{min} \delta F \ln(d_{max}/d_{min0})$
- $V_s / V_{s0} = \ln(d_{max}/d_{min}) / \ln(d_{max}/d_{min0})$
- geometry of link between change in d_{min} and δV_s

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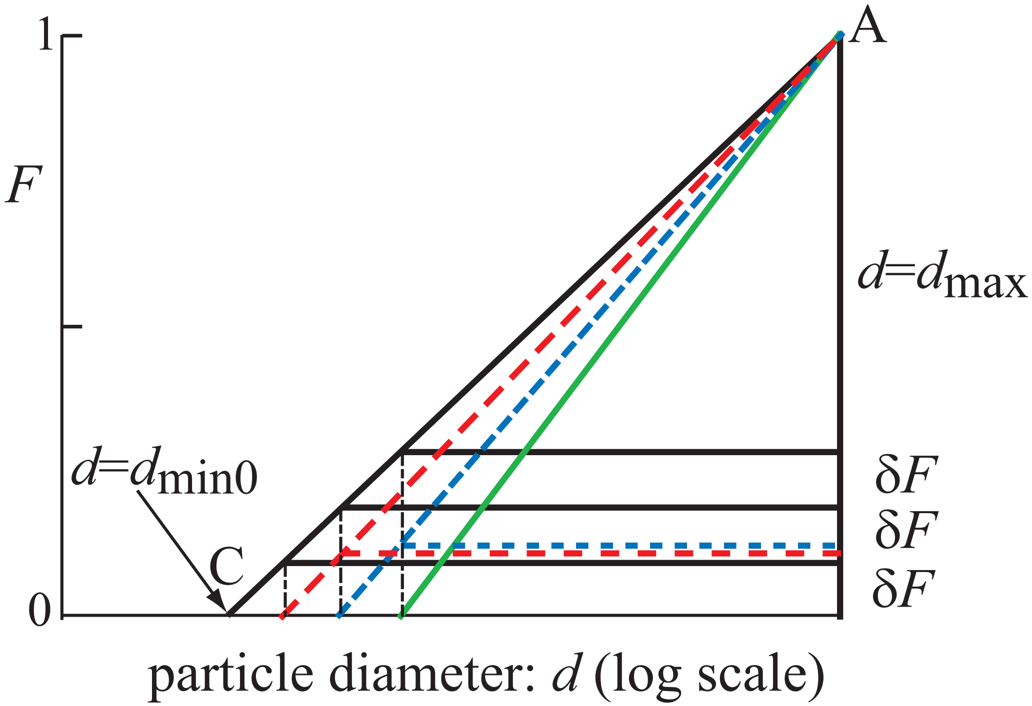
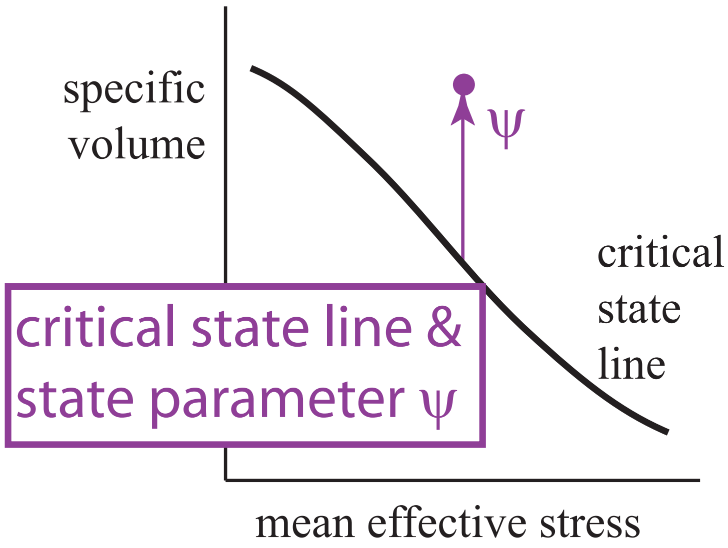
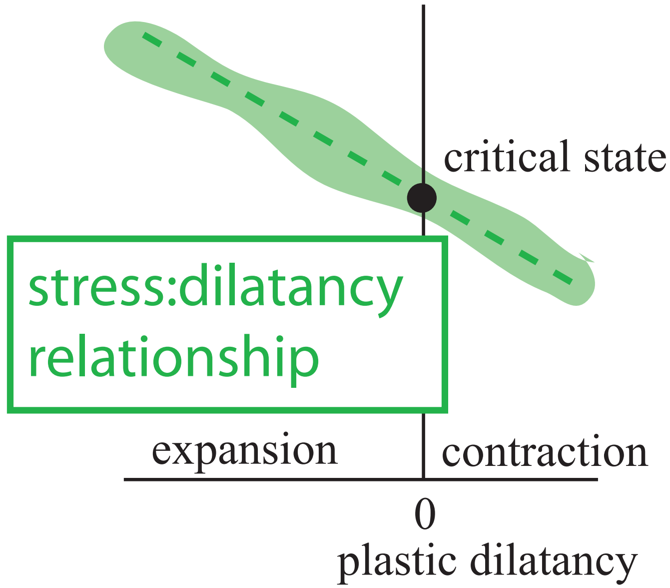
particle removal and grading

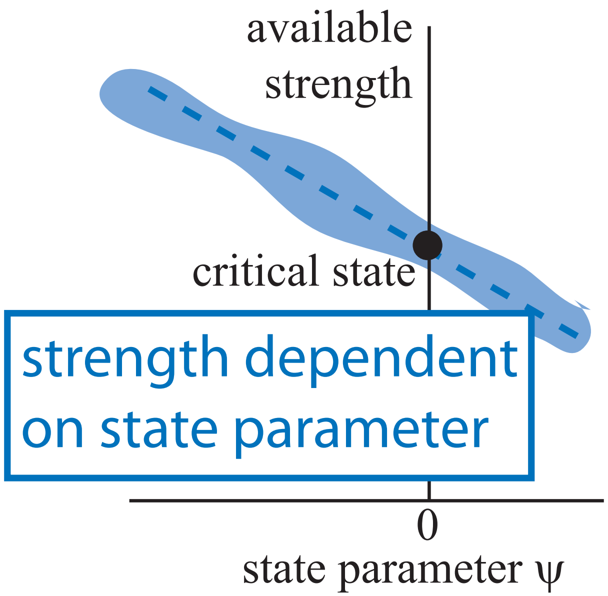


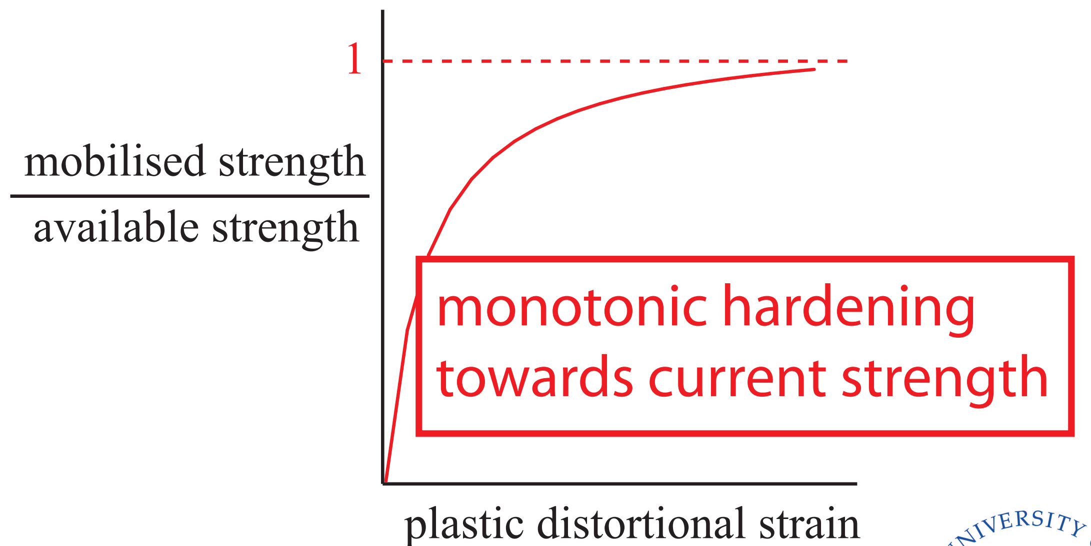
- linear distribution $I_G = [\ln d_{max}/d_{min}]/2B$
- $\delta I_G = -[\delta d_{min}/2Bd_{min}] = -[\delta F/2B]\ln[d_{max}/d_{min0}] = -[\delta v^r/2Bv]\ln[d_{max}/d_{min}] = -I_G[\delta v^r/v]$
- propose general link $\delta I_G = -k_G I_G[\delta v^r/v]$
- k_G of order 1

Severn-Trent sand model









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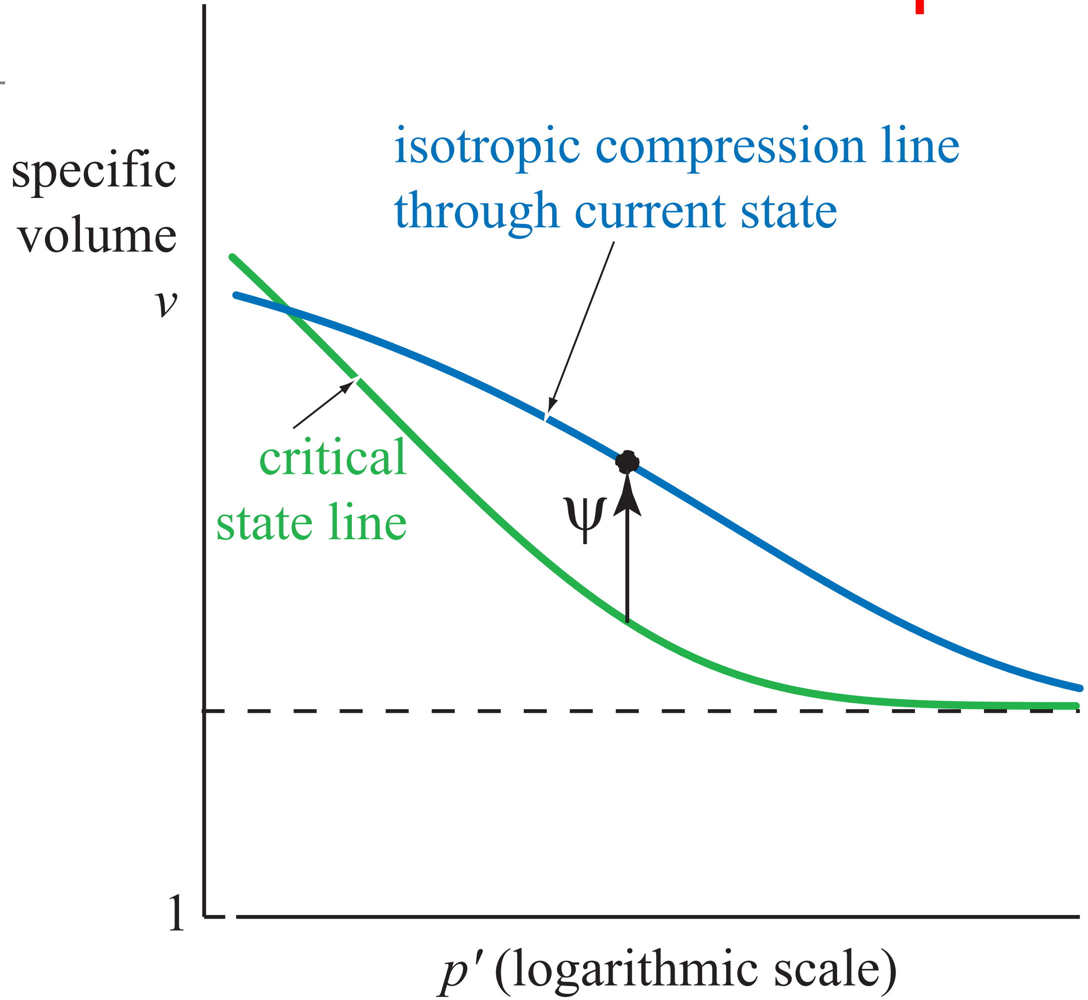
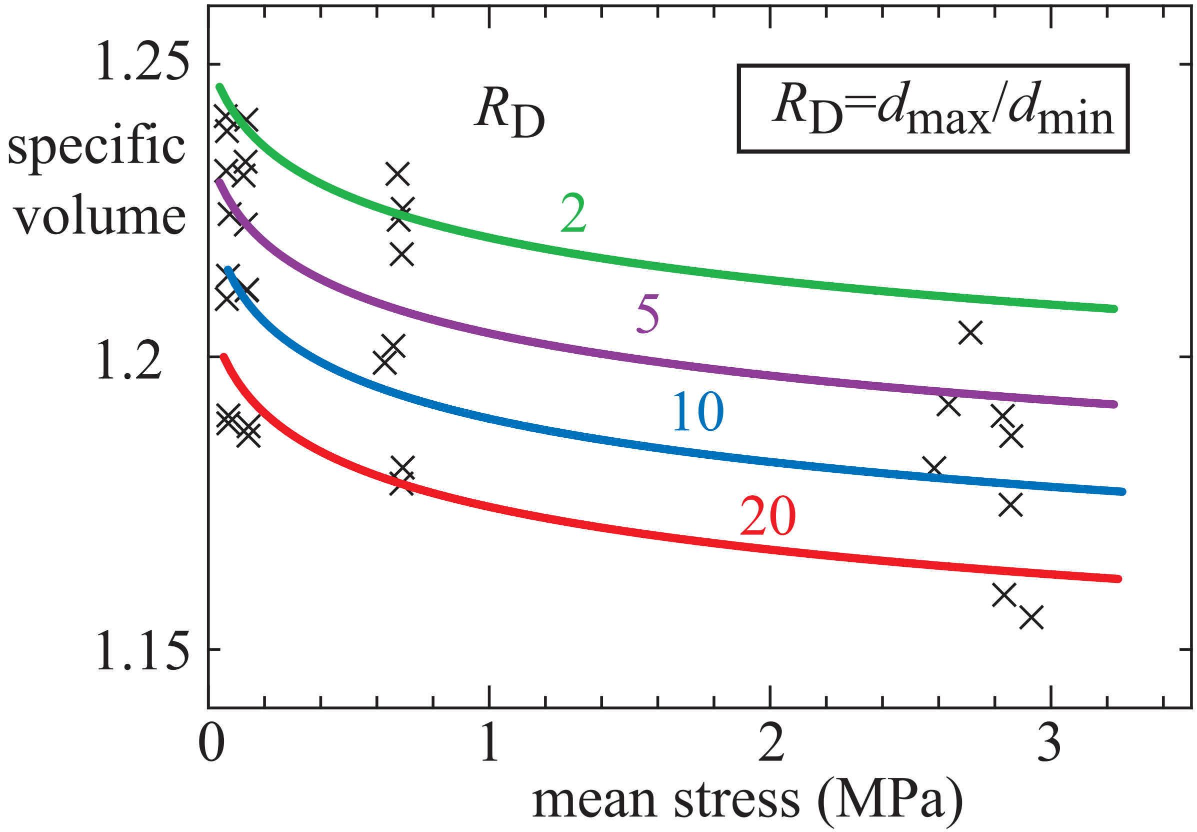
critical state and isotropic compression lines



$$v_c = \breve{v} + (\hat{v} - \breve{v}) \exp\left[-(p'/p_{cs})^{\beta}\right]$$

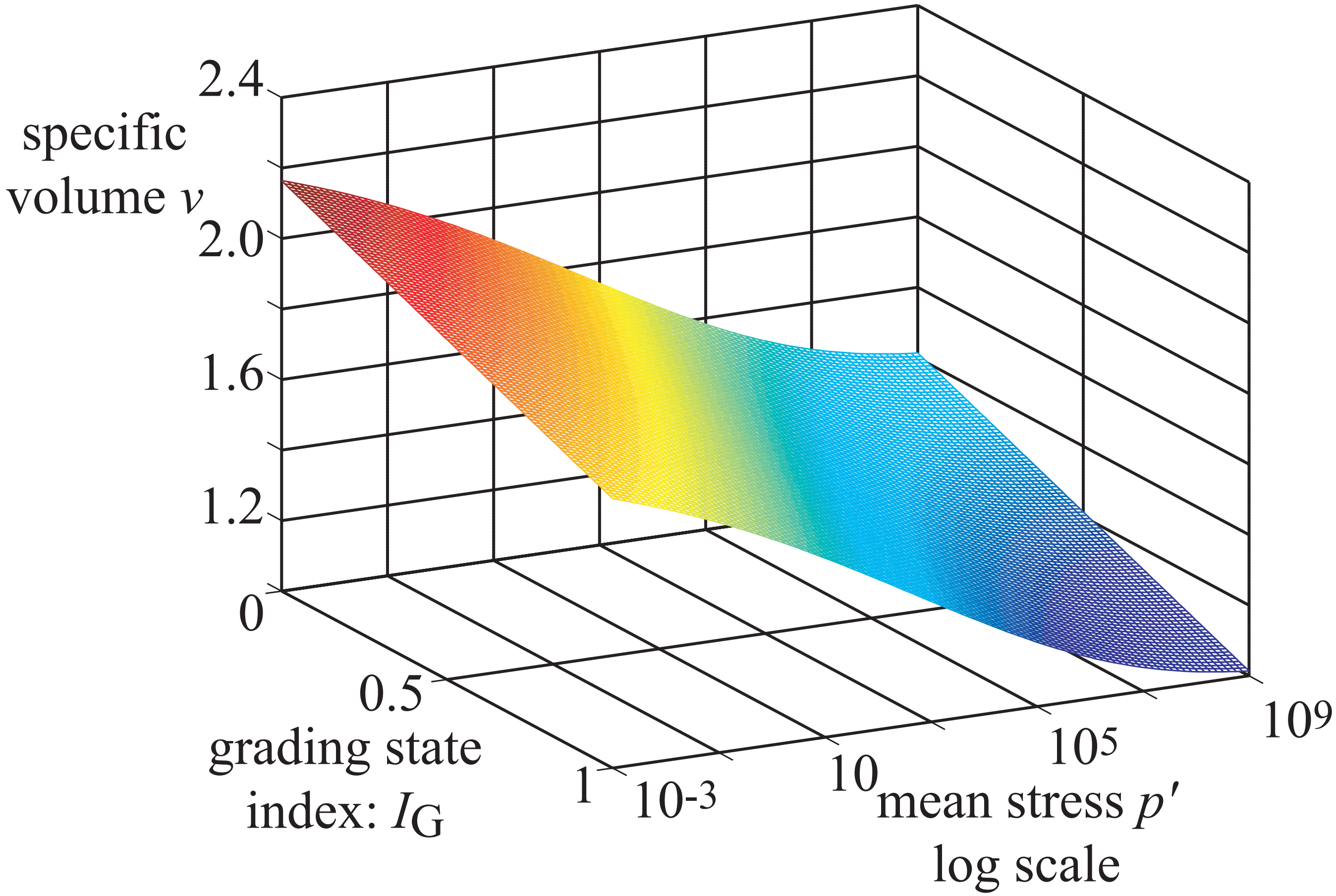
ensure realistic values at low and high stress

DEM: grading and critcal states



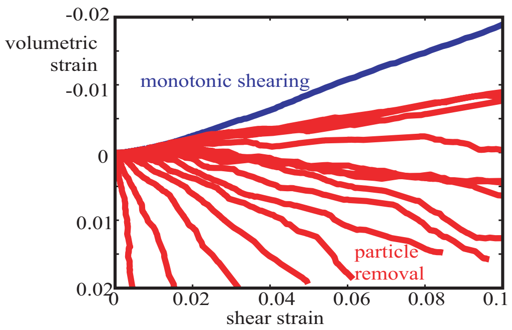
- broadening grading lowers critical state line
- broader gradings pack more efficiently

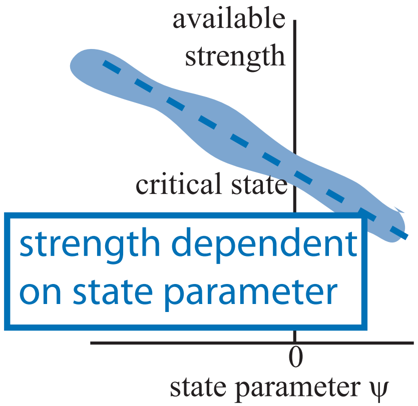
critical state surface

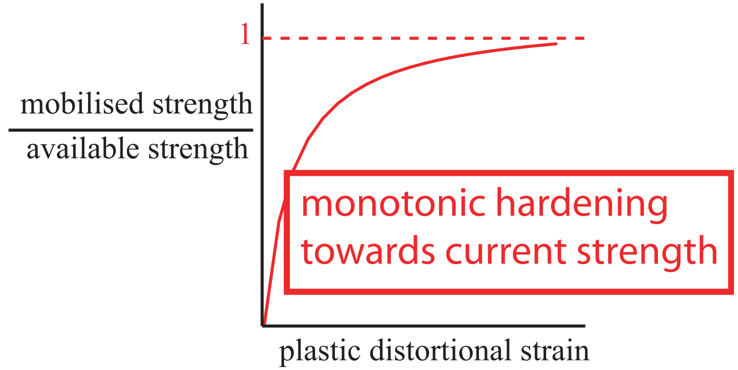


- specific volume as combined function of grading I_G and mean stress p'
- critical state line changes as particles removed

modelling particle removal







- particle removal changes volume
- change of grading changes critical state line
- change of state parameter?
- change of state parameter changes available strength
- stresses constant but mobilised strength changes
- distortional and volumetric strains from distortional mechanism

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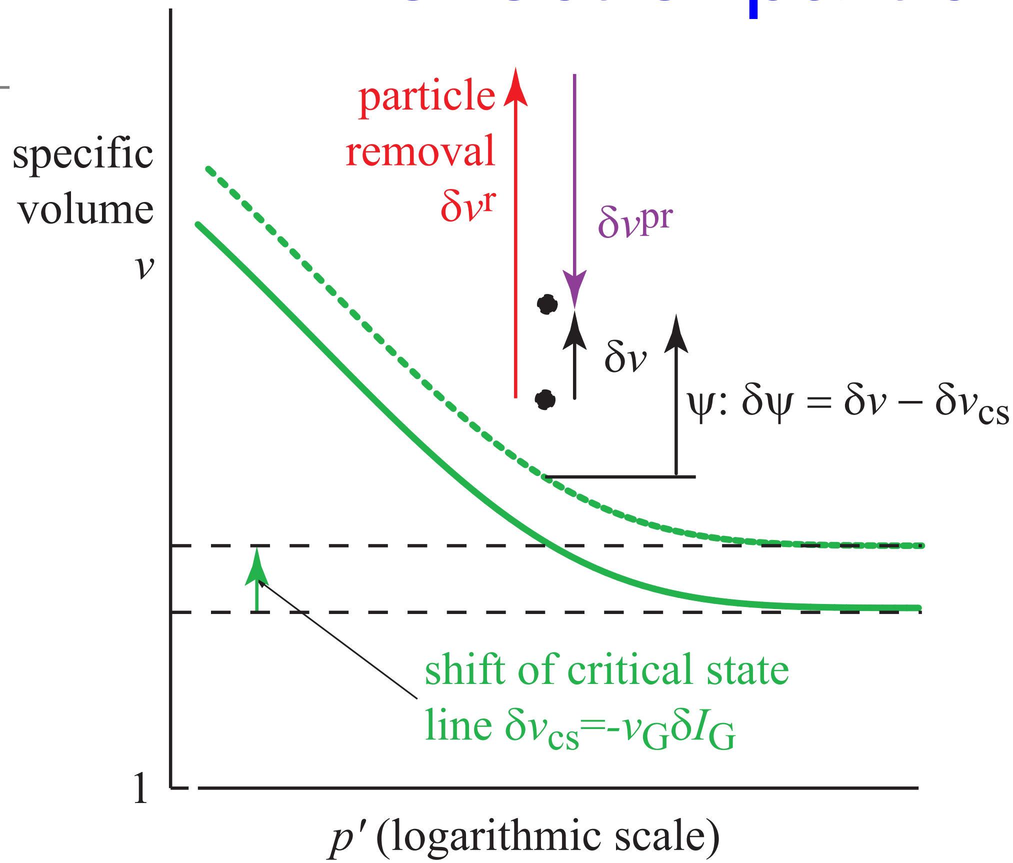
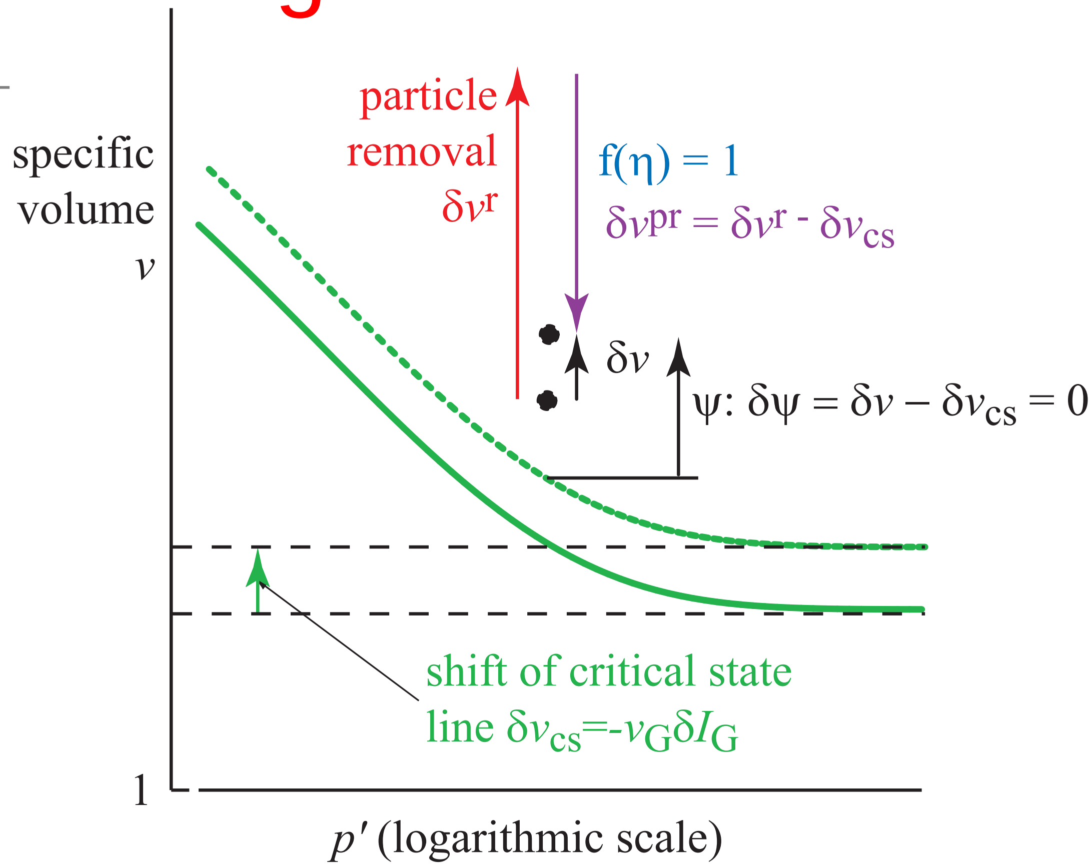
effect of particle removal



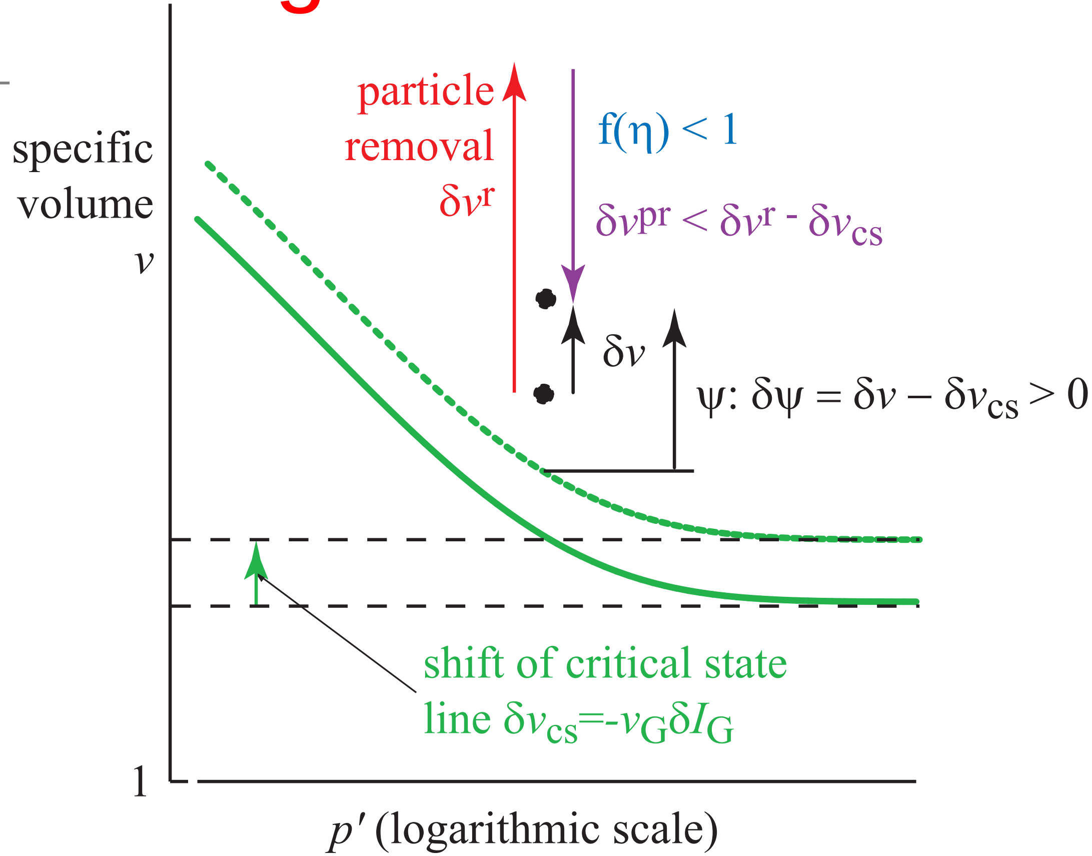
- volume increase from particle removal δv^r
- rise of critical state line δv_{cs}
- volume decrease from destabilisation δv^{pr}

changes of volume and state parameter



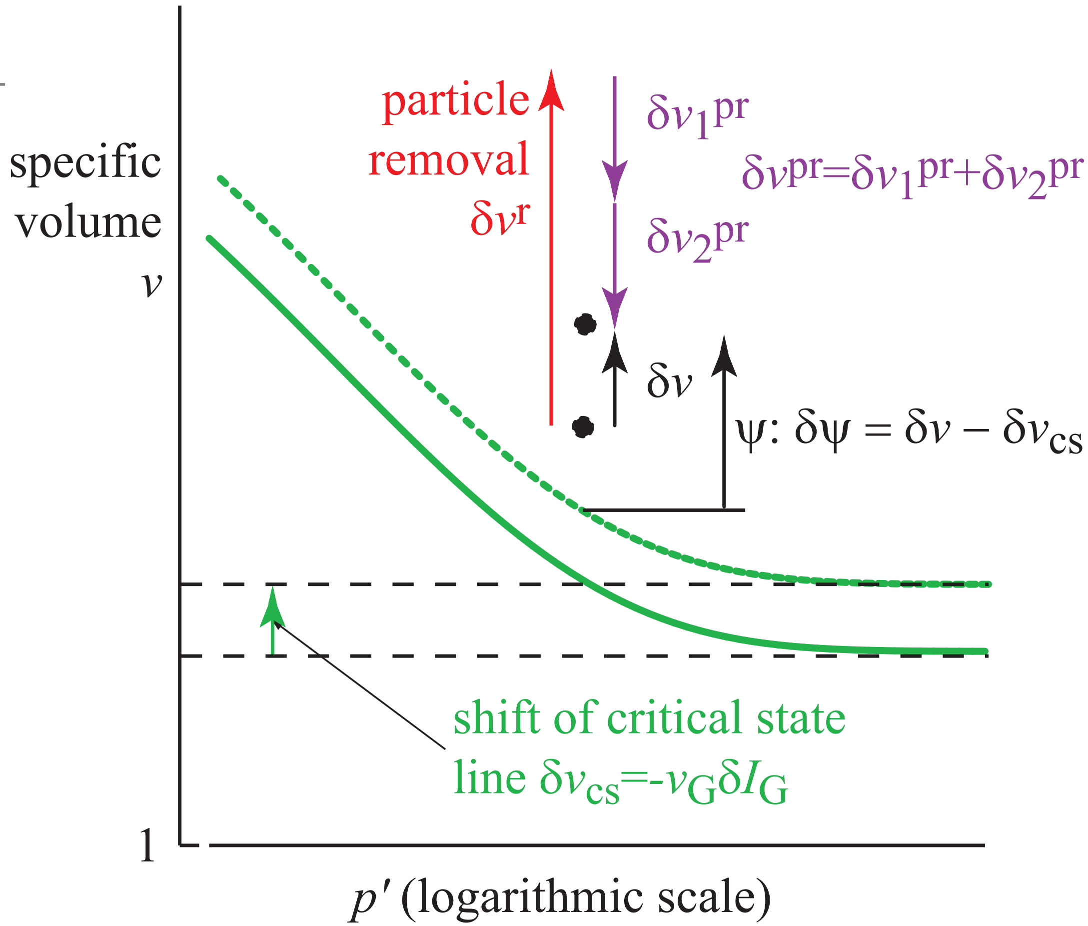
- ‘participation function’ $\delta v^{pr} = f(\eta)(\delta v^r - \delta v_{cs})$
- $f(\eta) = 1$, $\delta\psi = 0$; no change in mobilised strength

changes of volume and state parameter



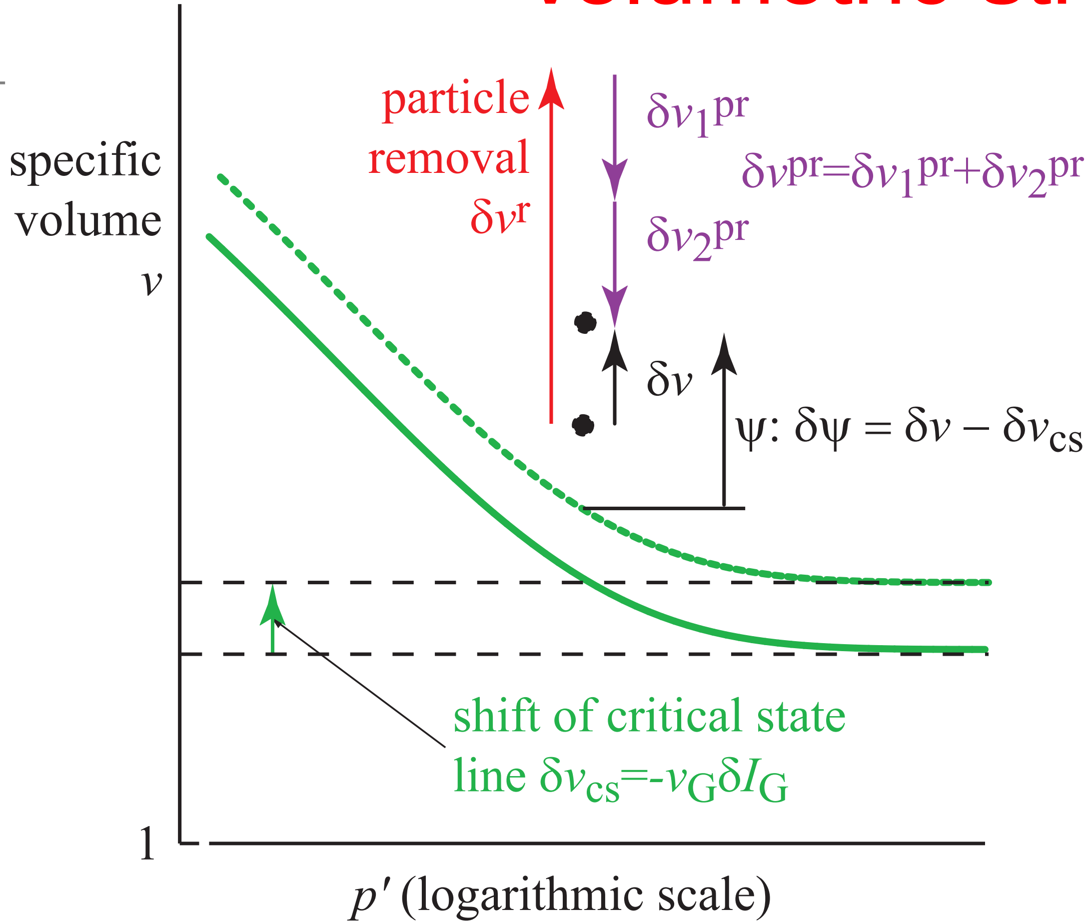
- 'participation function' $\delta v^{pr} = f(\eta)(\delta v^r - \delta v_{cs})$
- $f(\eta) < 1$, $\delta\psi > 0$ and mobilised strength increases

volume change



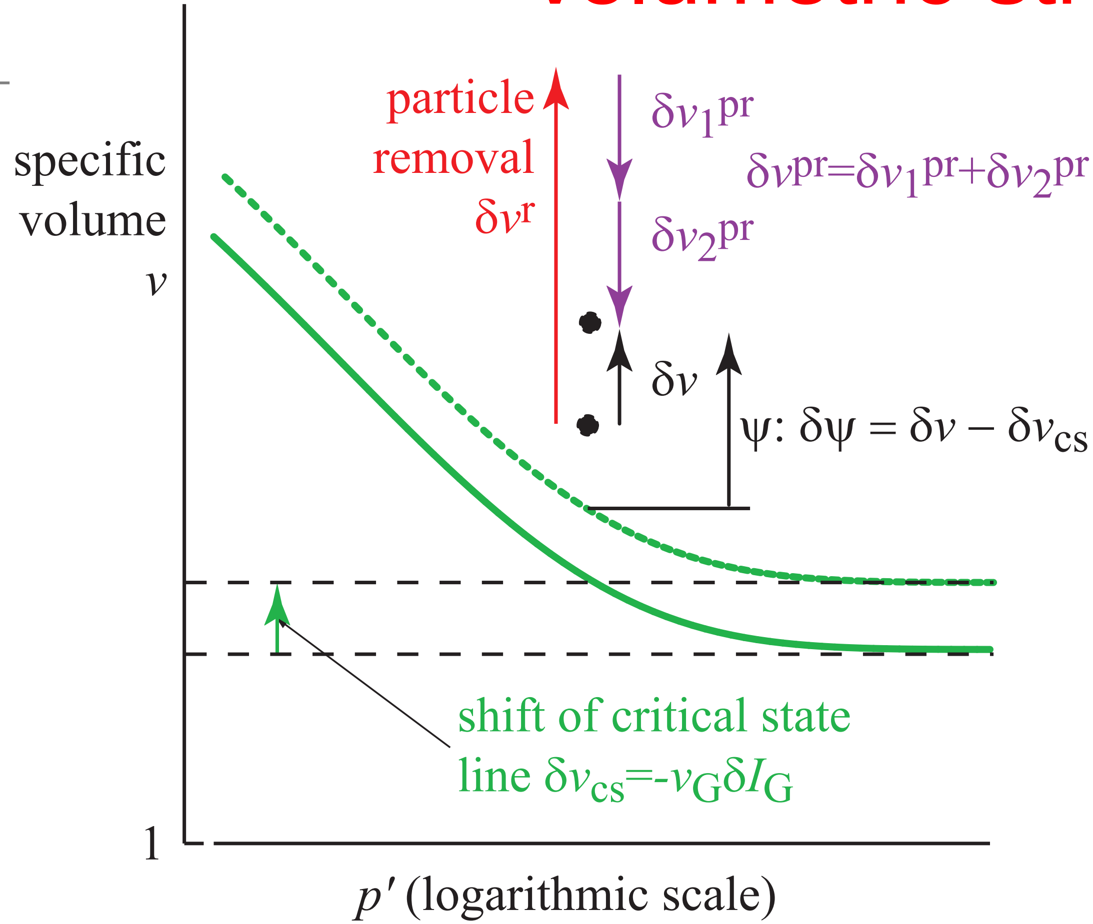
- volume decrease following particle removal $\delta v^{pr} = f(\eta)(\delta v^r - \delta v_{cs})$
- two components: $\delta v^{pr} = \delta v_1^{pr} + \delta v_2^{pr}$

volumetric strains



- volume decrease from increased mobilised strength $\delta v_1^{pr} = v\delta\epsilon_p^p = vA[(1 - k_D\psi)M - \eta]\delta\epsilon_q^p = vA[(1 - k_D\psi)M - \eta]ak_R\eta\delta\psi/(\eta_p - \eta)^2$
- stress-dilatancy and hardening relationships

volumetric strains



- volume decrease from destabilisation $\delta v_2^{pr} = \delta v^{pr} - \delta v_1^{pr} = f(\eta)(\delta v^r - \delta v_{cs}) - \delta v_1^{pr}$
- justification for participation function $f(\eta)$?

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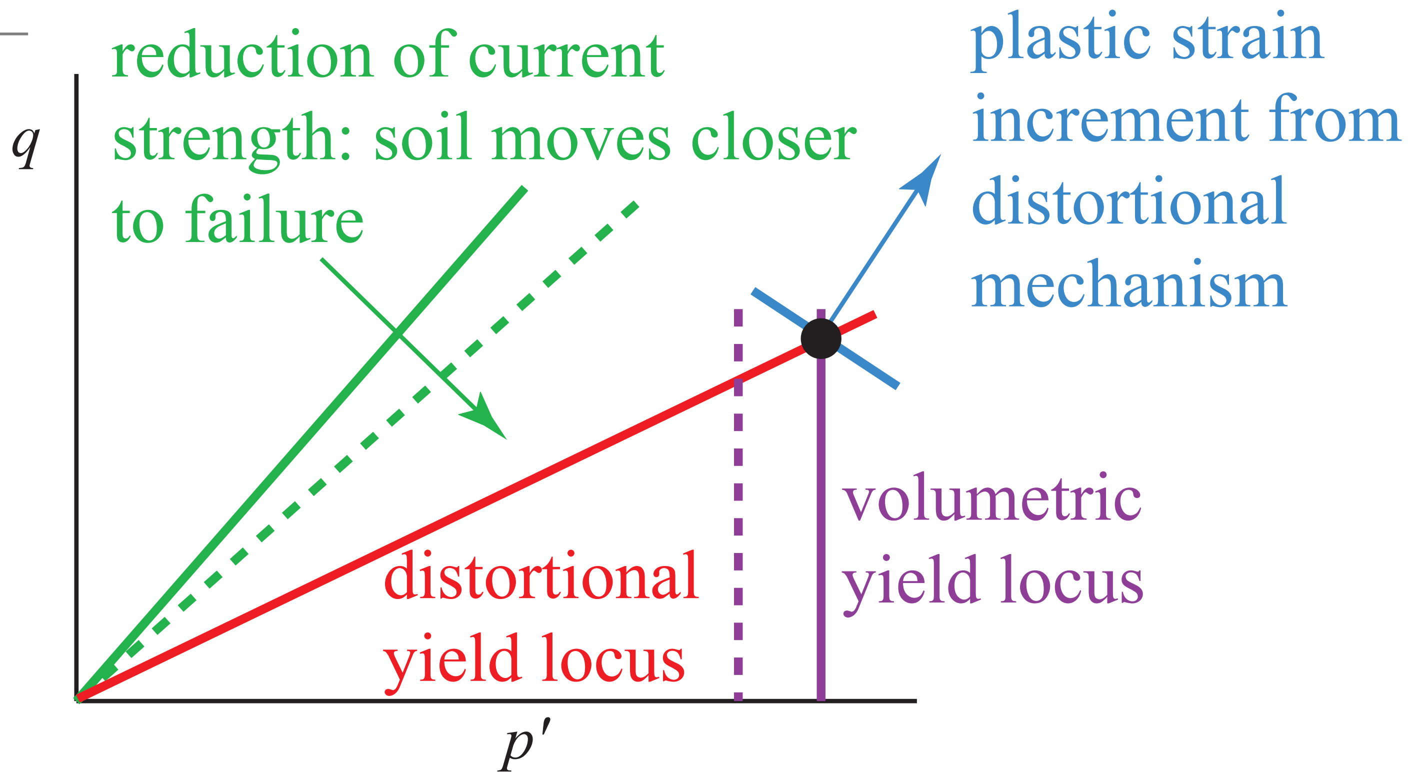
deformation mechanisms



- state parameter ↑; strength ↓; constant stresses; mobilised strength ↑; distortional (and volumetric) strains
- purely volumetric compression strains triggered by particle removal (destabilisation)

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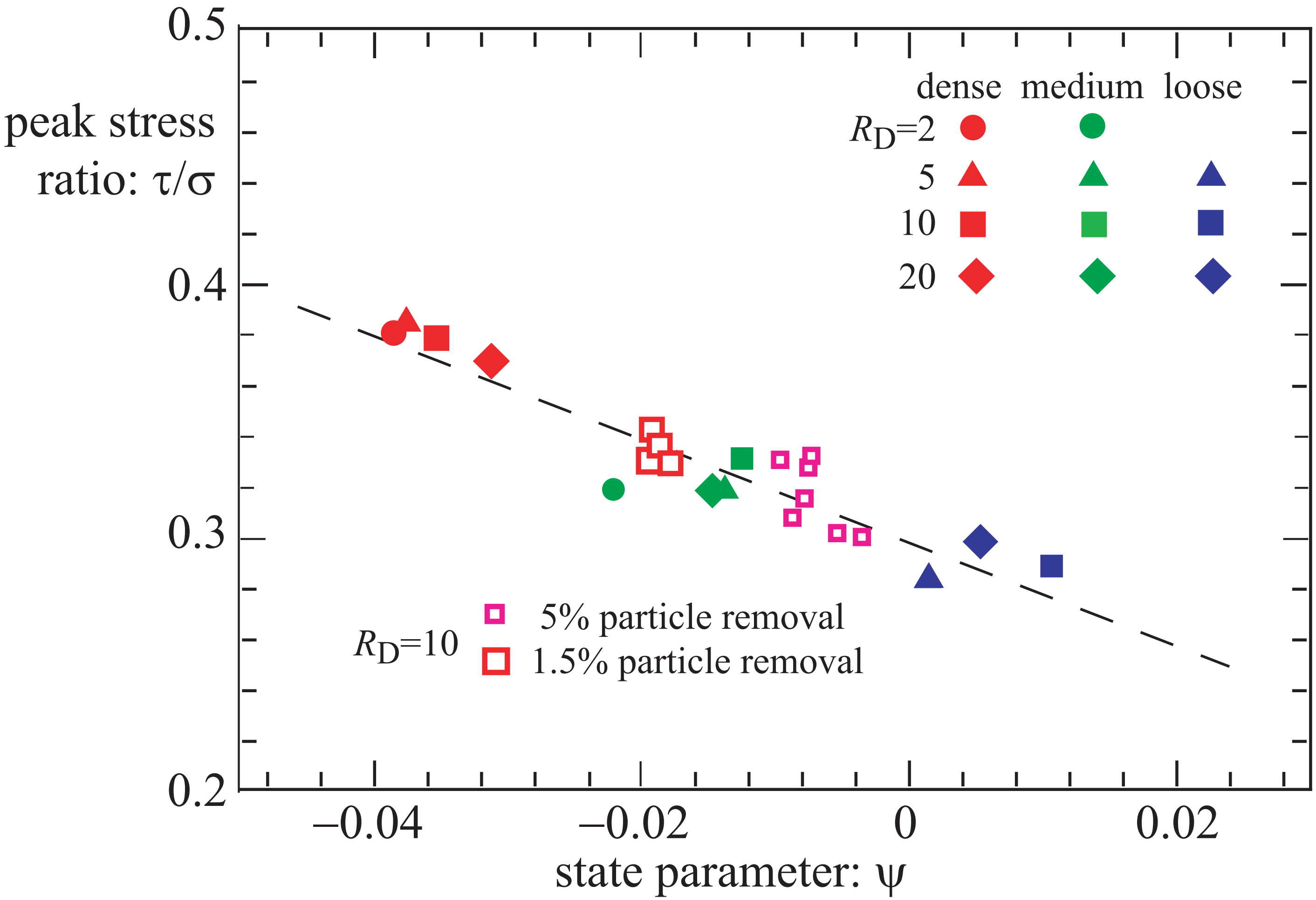
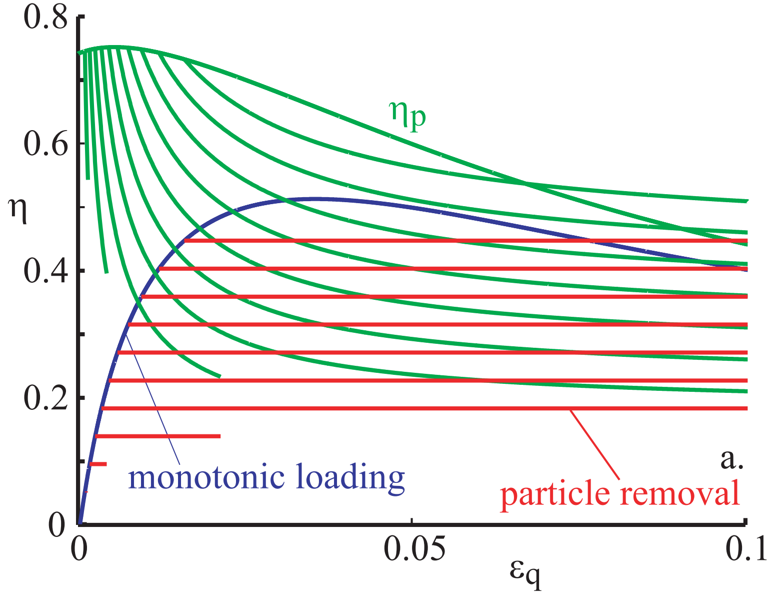
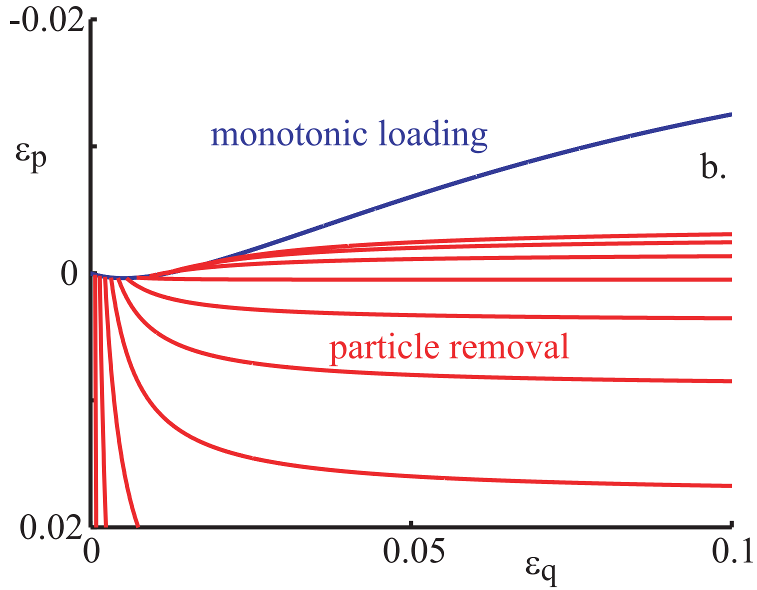
peak strengths and state parameter



- peak strength and state parameter
- tests with and without particle removal

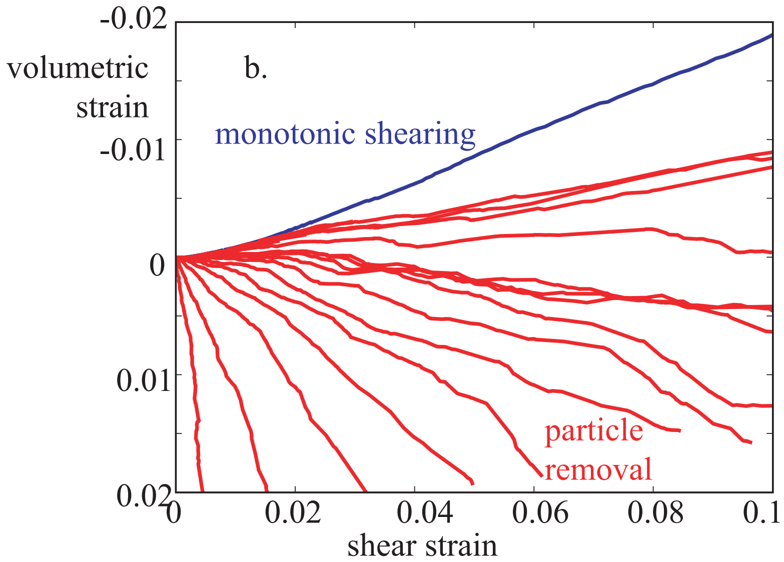
simulations: initial grading $R_D = 10$

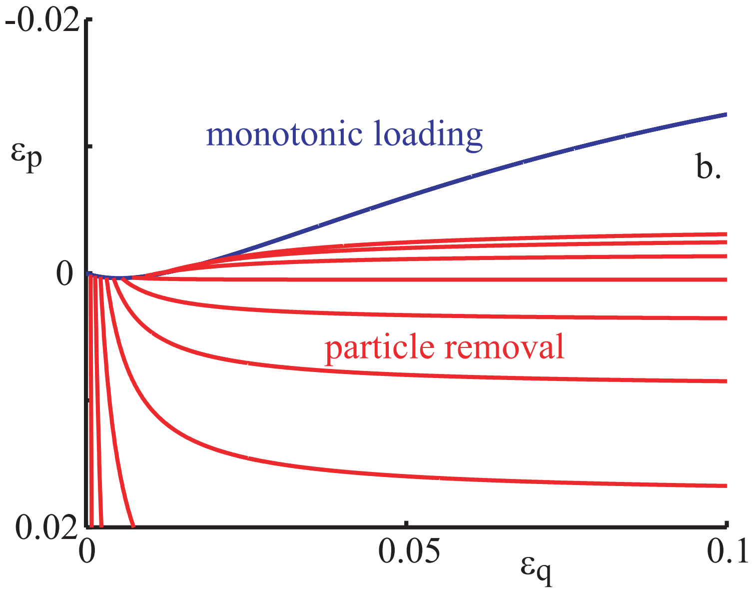




- participation function $f(\eta) = 1 - 0.8\eta/[A(1 - k_D\psi)M]$
- linked with stress-dilatancy relationship

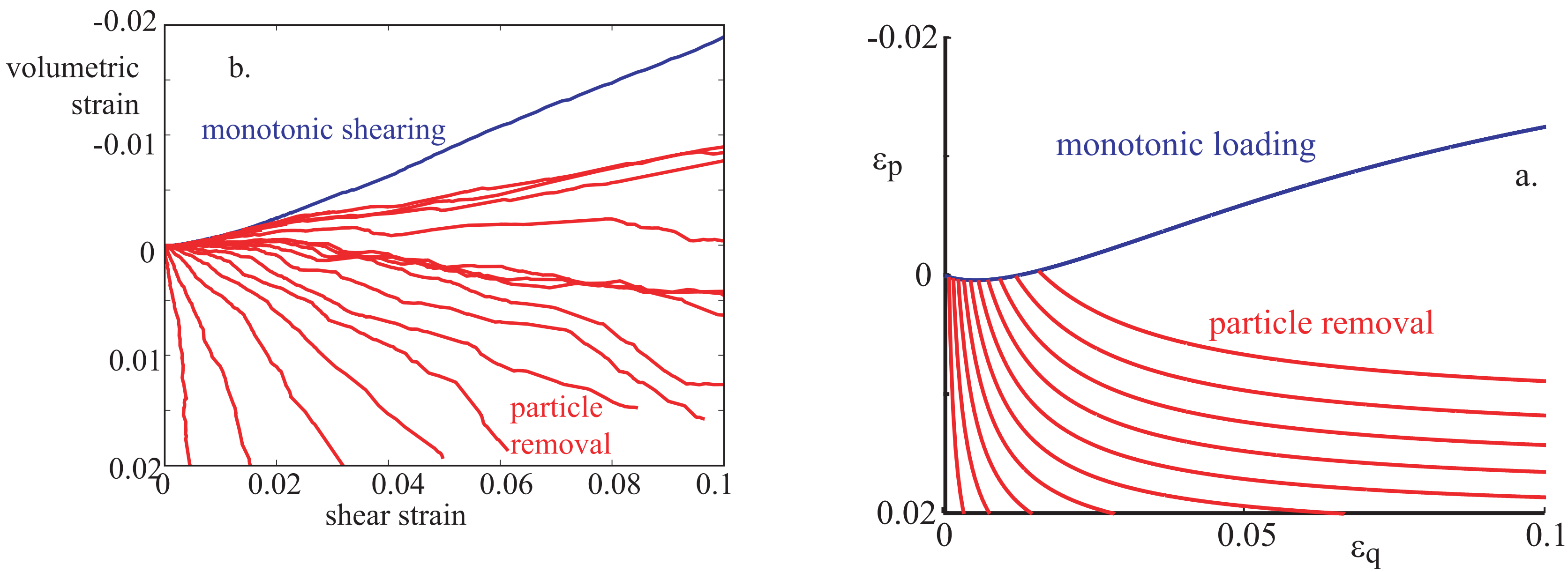
simulations: initial grading $R_D = 10$





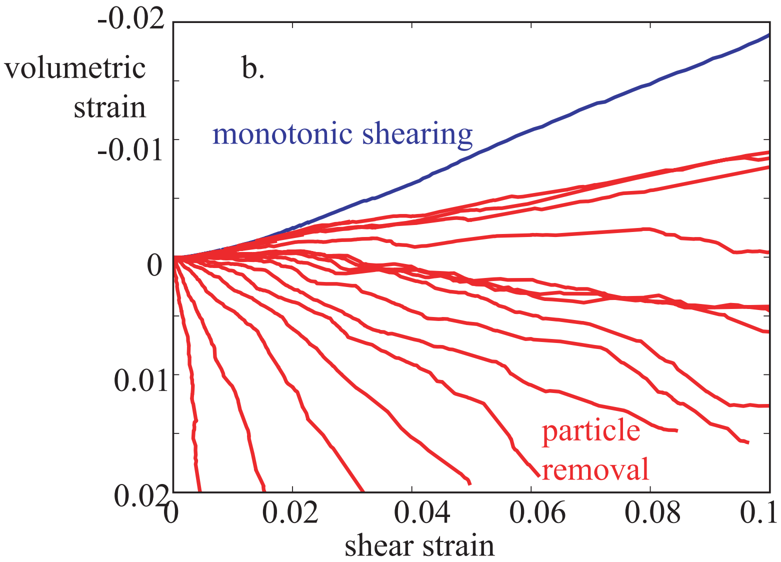
- participation function $f(\eta) = 1 - 0.8\eta/[A(1 - k_D\psi)M]$
- linked with stress-dilatancy relationship

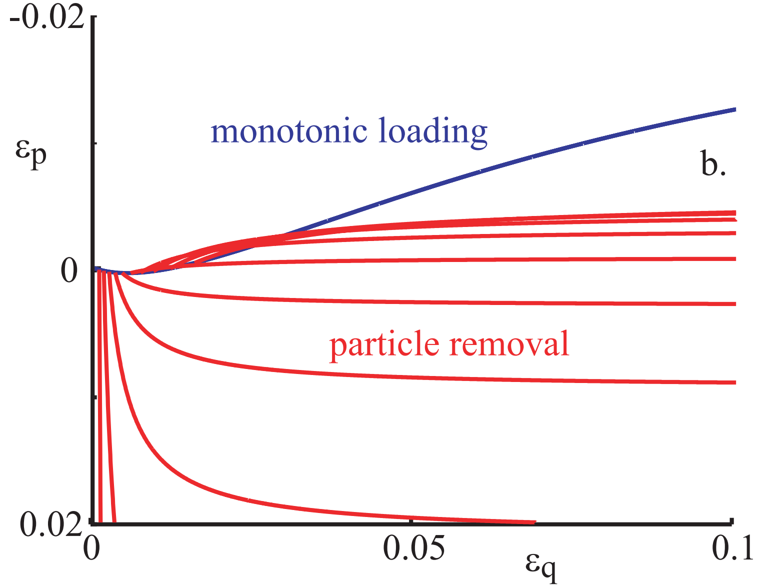
simulations: initial grading $R_D = 10$



- participation function $f(\eta) = 0.5$
- unchanging with stress ratio

simulations: initial grading $R_D = 10$





- participation function $f(\eta) = 1 - \eta/[A(1 - k_D\psi)M]$
- linked with mobilisation of current critical state stress ratio

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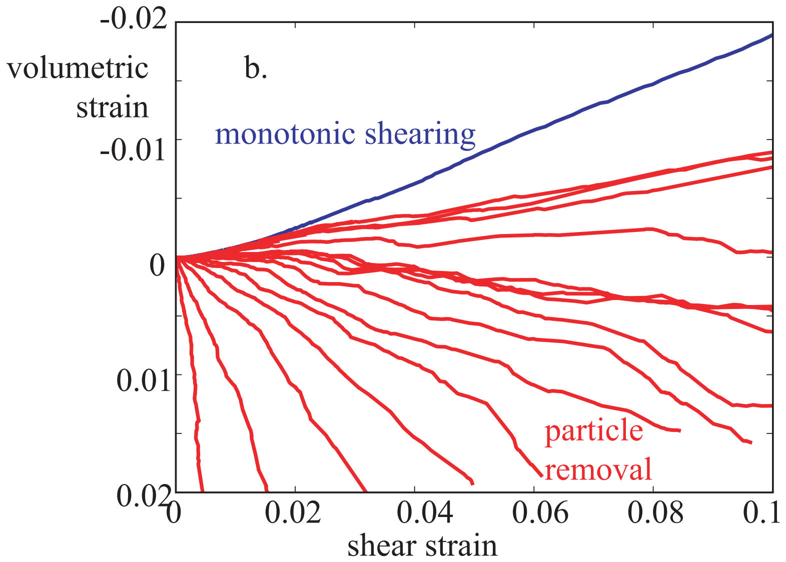
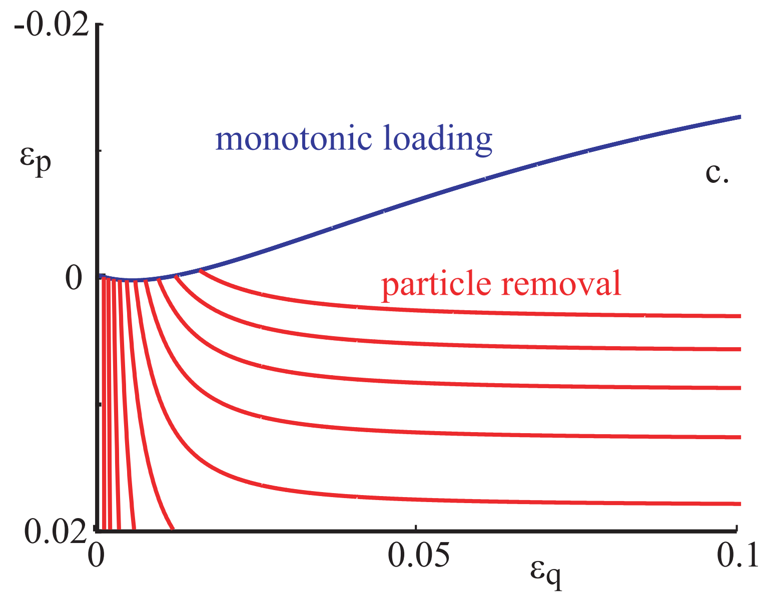
simulations: initial grading $R_D = 10$





- participation function $f(\eta) = 1 - \eta/\eta_p$
- linked with mobilisation of current peak strength

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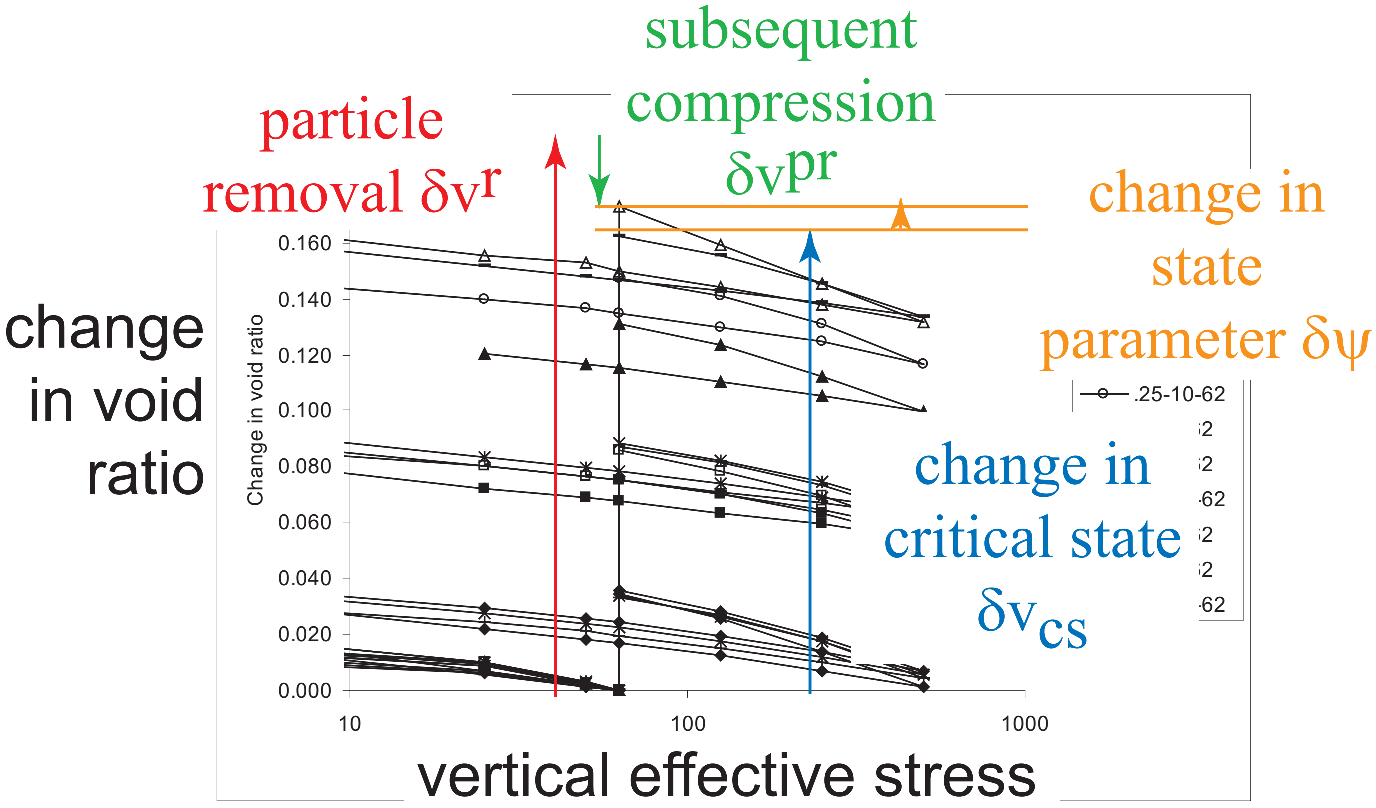
sand and salt (McDougall)

- oedometer: $\delta\epsilon_r = 0$; $\delta\epsilon_p = \delta\epsilon_a$; $\delta\epsilon_q = (2/3)\delta\epsilon_a$
- calculate δv^r; measure $\delta v \rightarrow \delta v^{pr} = \delta v^r - \delta v$
- vertical (and radial?) stress constant: no elastic strains
- distortional mechanism: $\delta\epsilon_q \rightarrow \delta\psi \rightarrow \delta v_{cs}$
- distortional mechanism: $\delta v_1^{pr}/v = D\delta\epsilon_q$; $D =$ dilatancy
- second mechanism: $\delta v_2^{pr} = \delta v^{pr} - \delta v_1^{pr}$
- participation function: $f(\eta) = \delta v^{pr}/(\delta v^r - \delta v_{cs})$

sand and salt (McDougall)



- dissolve salt under stress
- estimate participation function $f(\eta) \approx 0.82$
- single stress state

comments/conclusions

- grading change (crushing/erosion)
- adapt simple model: critical state line: state parameter
- separate loss of material and subsequent response
- missing link: participation function: how much collapse occurs?
- problem of validation data

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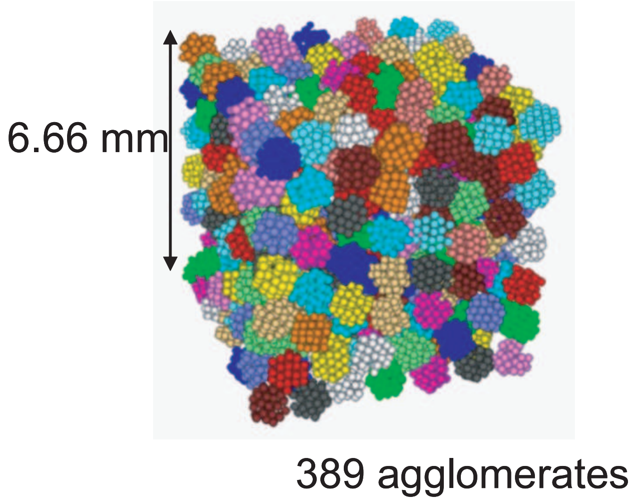
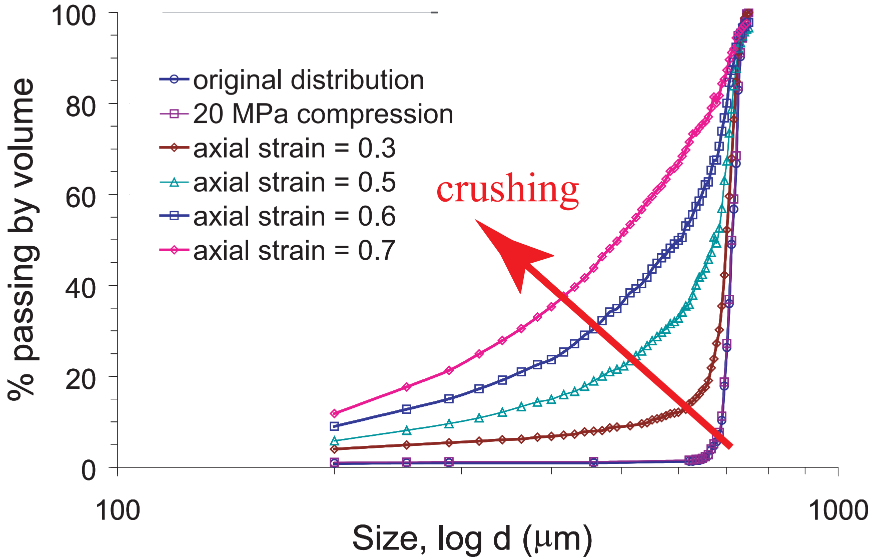
agglomerated particles: DEM: (Cheng, 2005)





- evolving particle size distribution through breakage of contact bonds within agglomerates
- isotropic compression to 20MPa (negligible change)
- shearing (axial strains indicated)
- d_{max} somewhat constant

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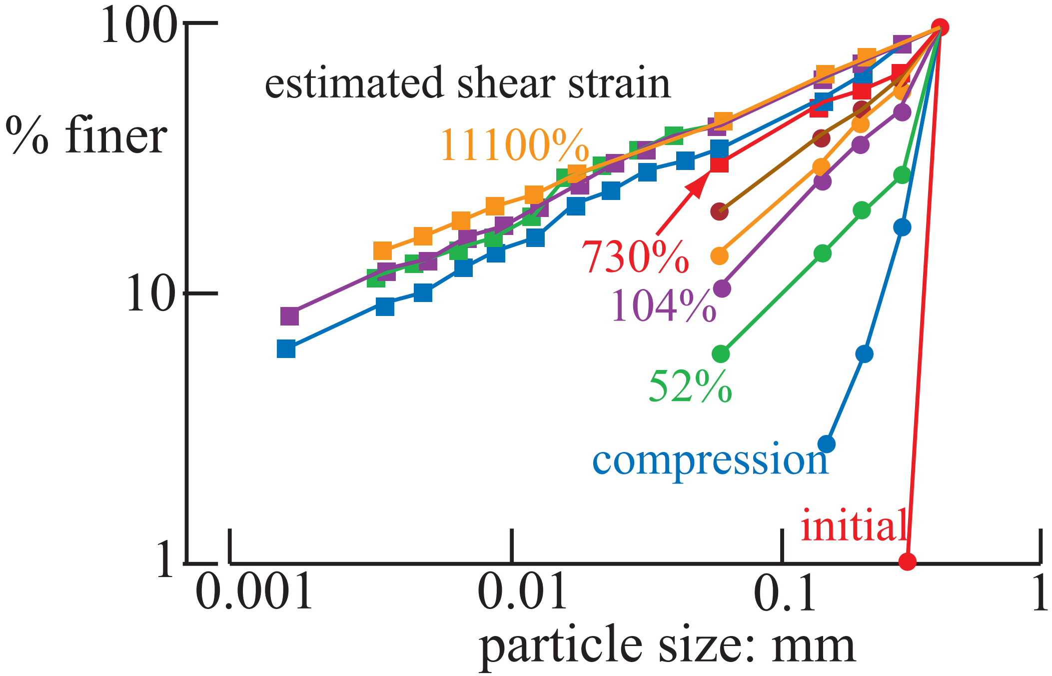
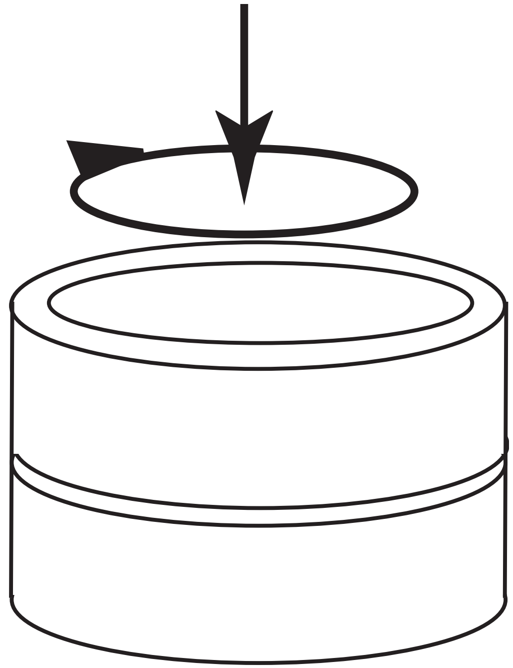
pestle and mortar

- compression produces particle breakage ...
- ... but shearing better

ring shear apparatus: Dog’s Bay sand





- evolution of particle size distribution: constant after about 730%? (definition of *strain* in ring shear?)
- double logarithmic axes
- (after Coop *et al.*, 2004)

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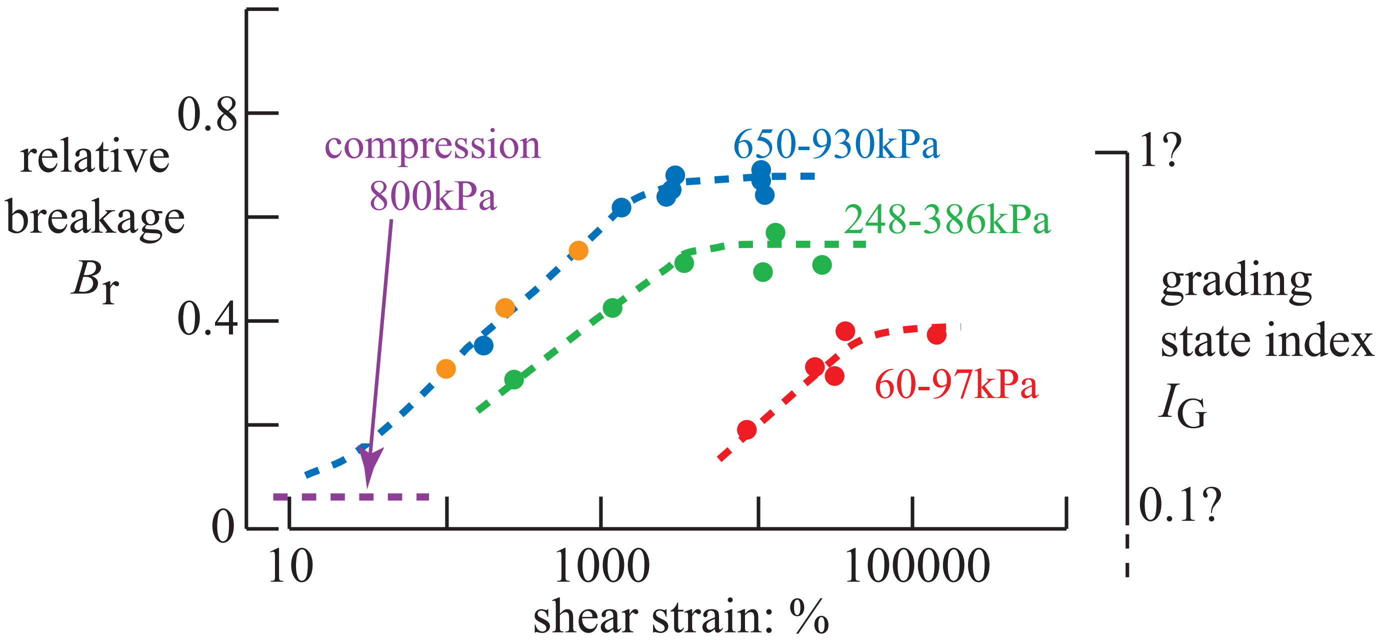
$I_G \rightarrow 1$ inevitably?



- relative breakage $B_r \propto \Delta I_G$
- different normal stresses
- crushing does not continue indefinitely
- final grading depends on stress level
- Dog's Bay sand: ring shear tests (Coop, 2004)

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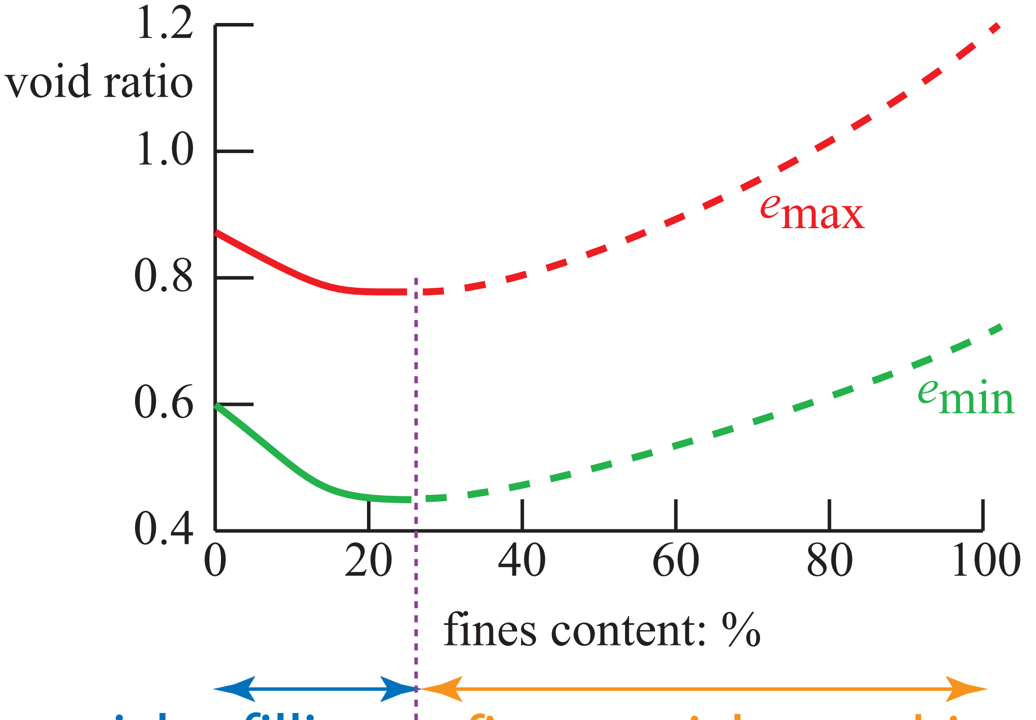
effect of addition of fine particles?



fine particles filling gaps - natural result of crushing

fine particles pushing larger particles apart - unnatural?

- effect on e_{max} and e_{min}
- all aspects of behaviour linked with void ratio range affected
- for example: location of critical state line

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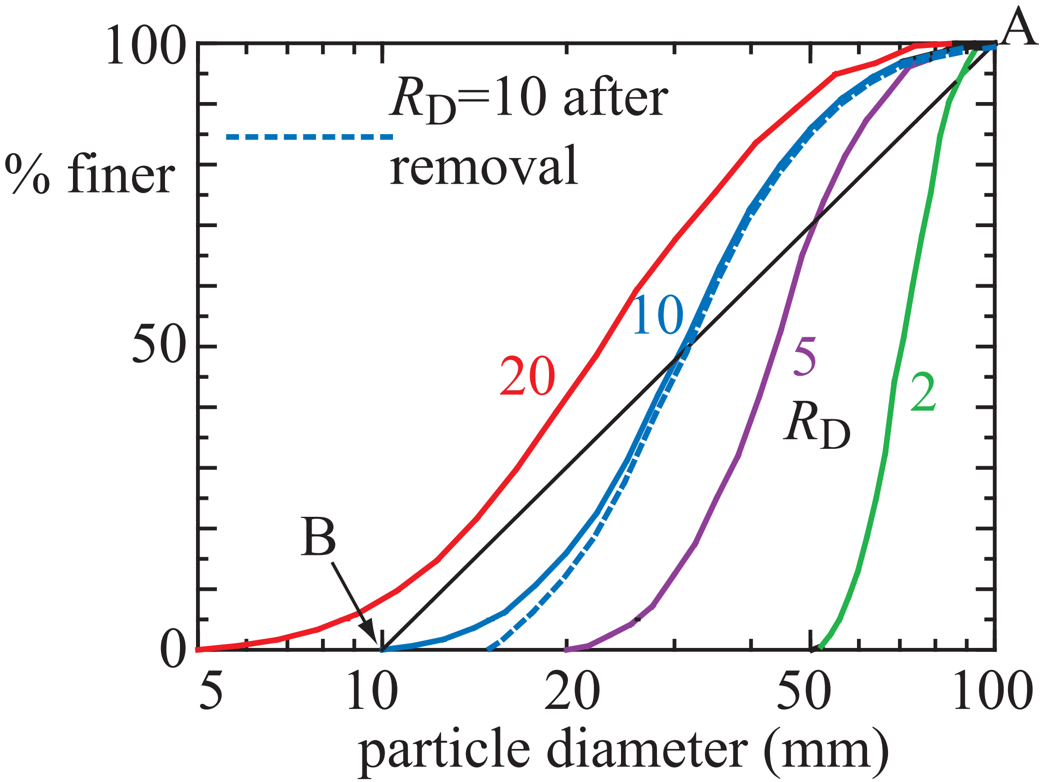
DEM analyses: different gradings of discs



- $R_D = d_{max}/d_{min}$
- tests with constant grading
- (Muir Wood & Maeda, 2007)

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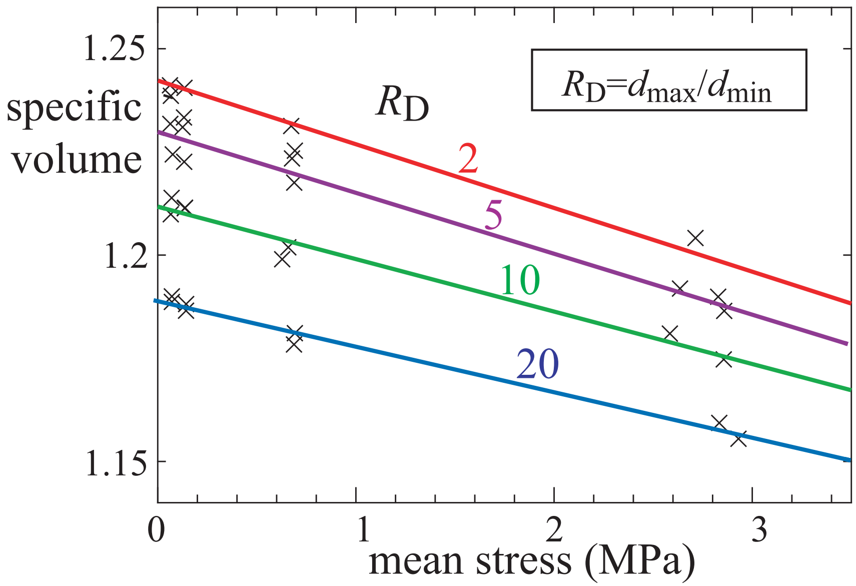
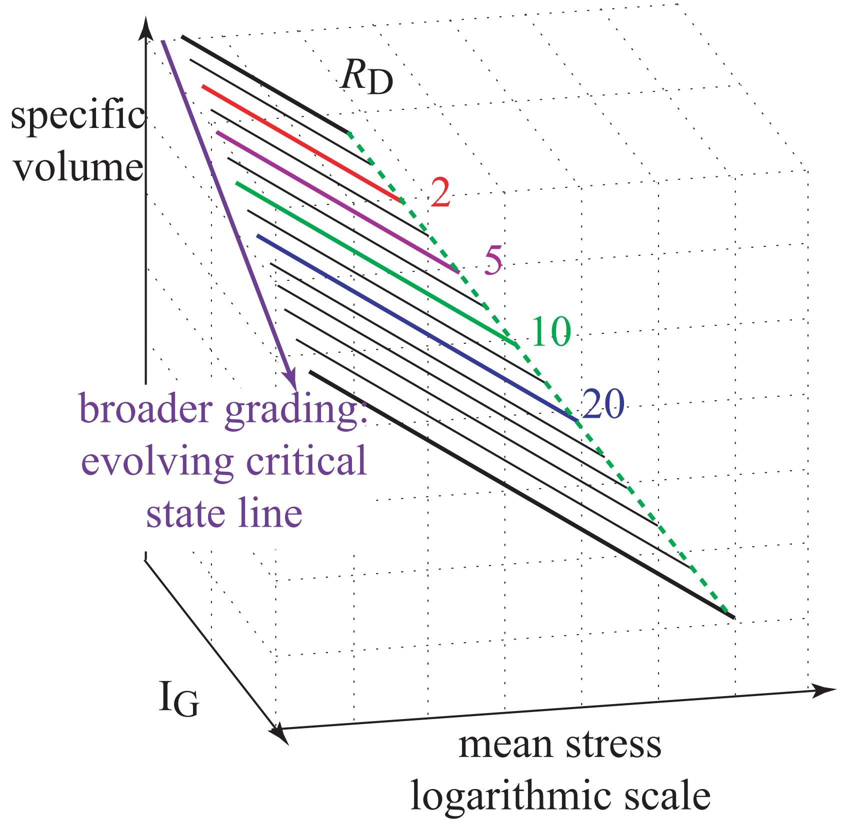
critical state *surface*





- broader gradings have lower critical state lines
- use grading index as extra dimension
- critical state *surface*: $p' : v : I_G$

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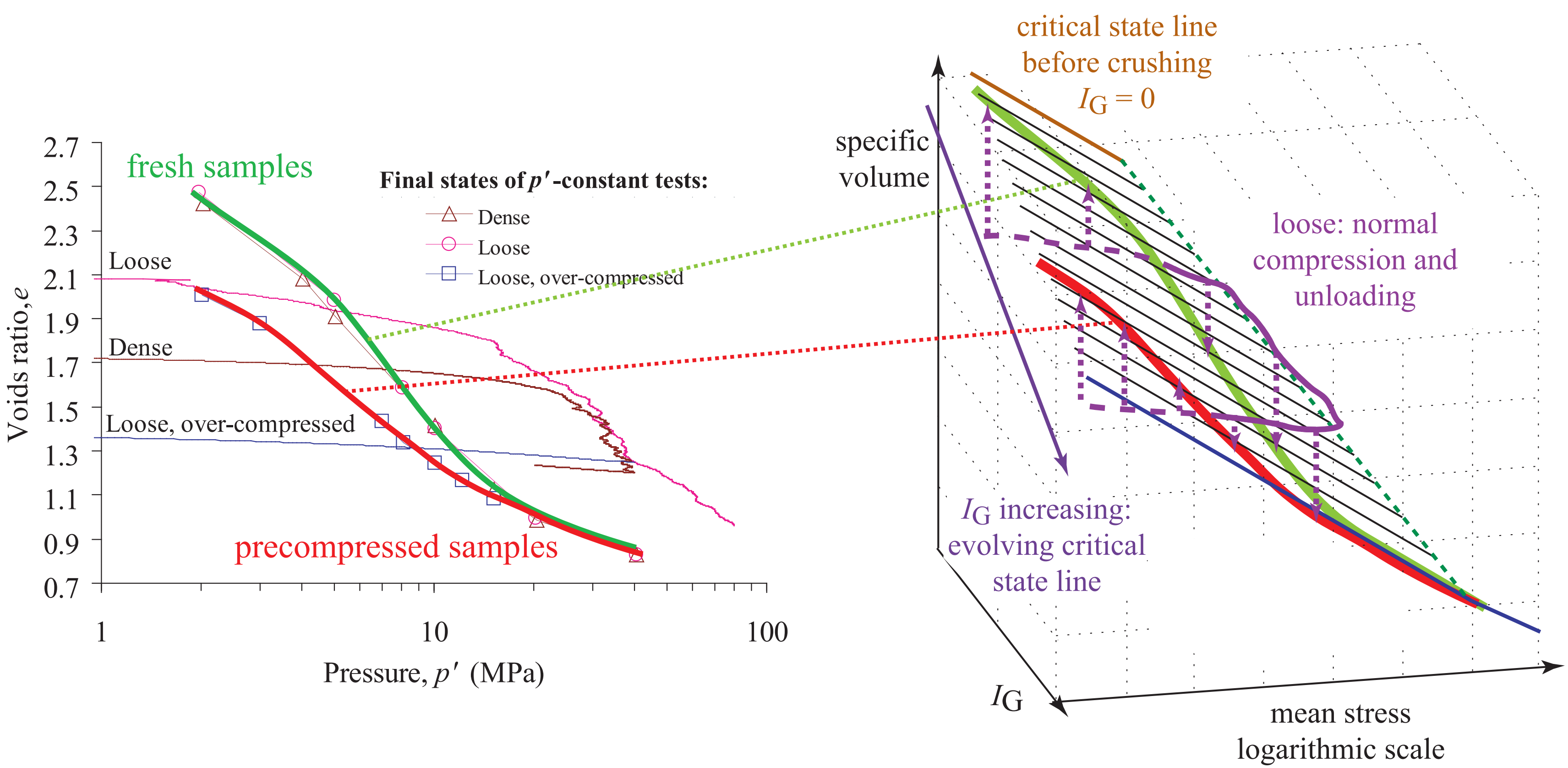
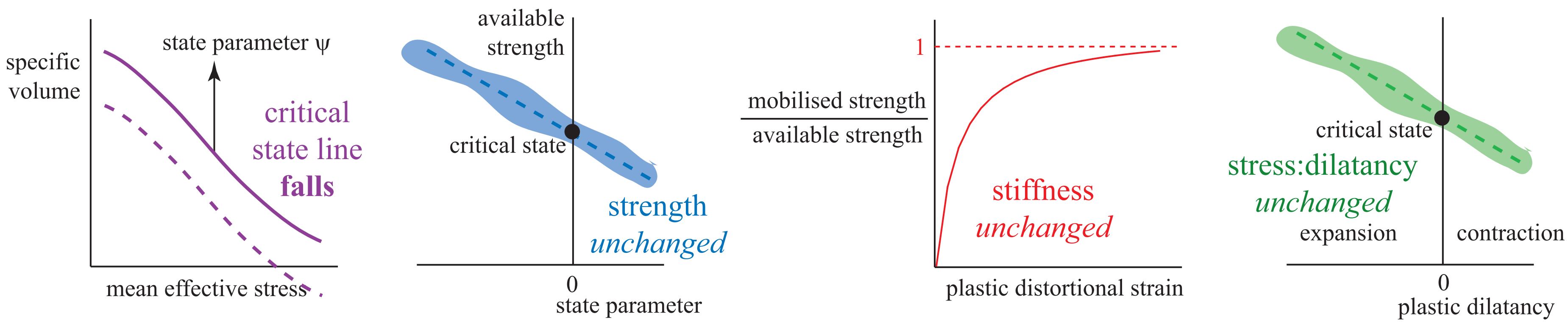
particle breakage and critical state surface



- loci of end points on critical state surface
- precompression leads to lower critical state specific volume (higher I_G)
- (Cheng)

effect of increasing I_G on response



- **lowering of critical state line** (*first order*)
- strength unchanged (*first order*)
- stiffness unchanged (*first order*)
- dilatancy unchanged (*first order*)
- slope of critical state line unchanged (*first order*)
- few data - often from artificial mixtures not naturally crushed or eroded materials

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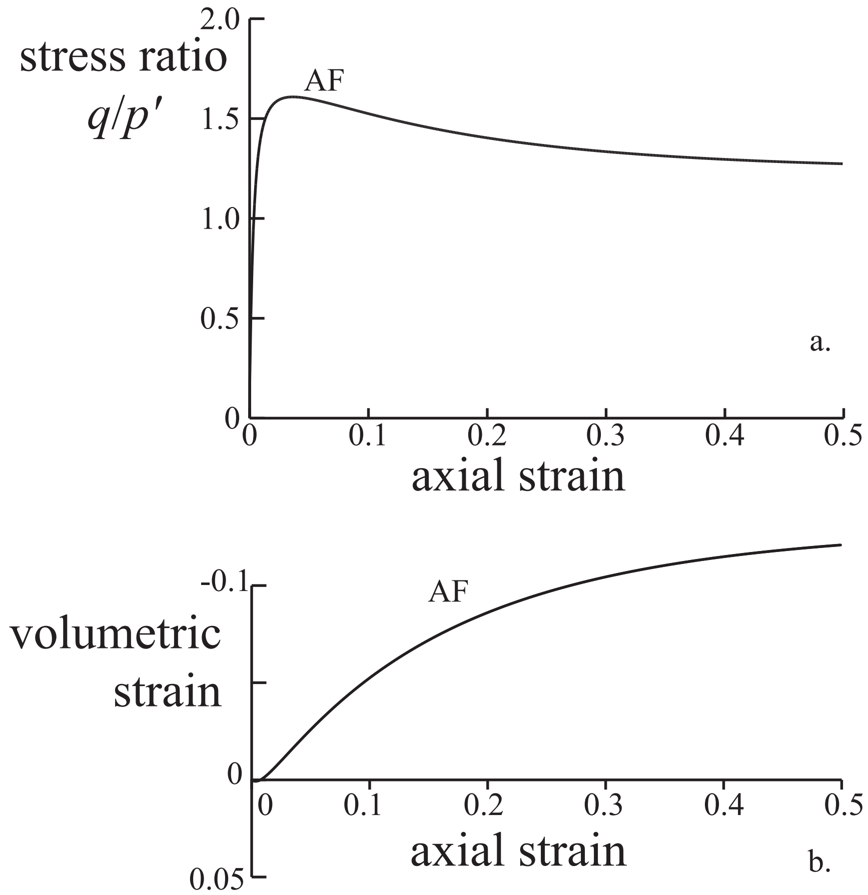
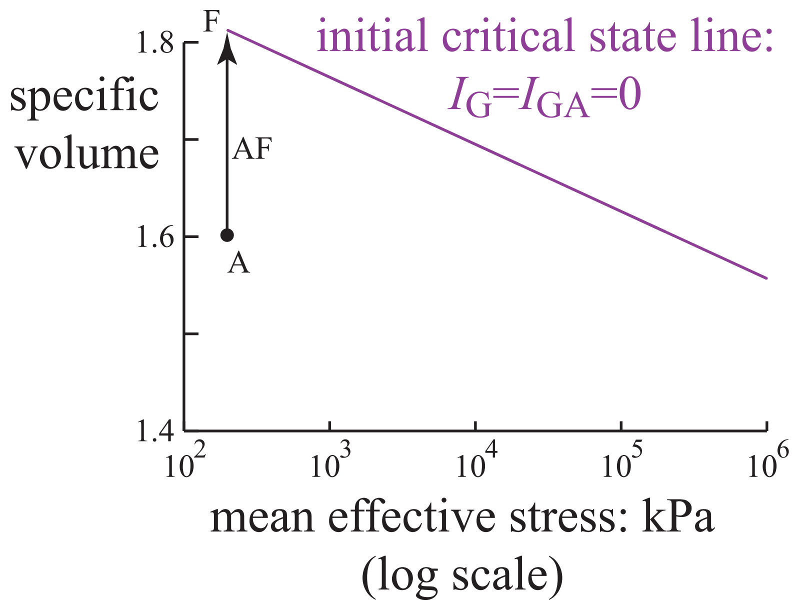
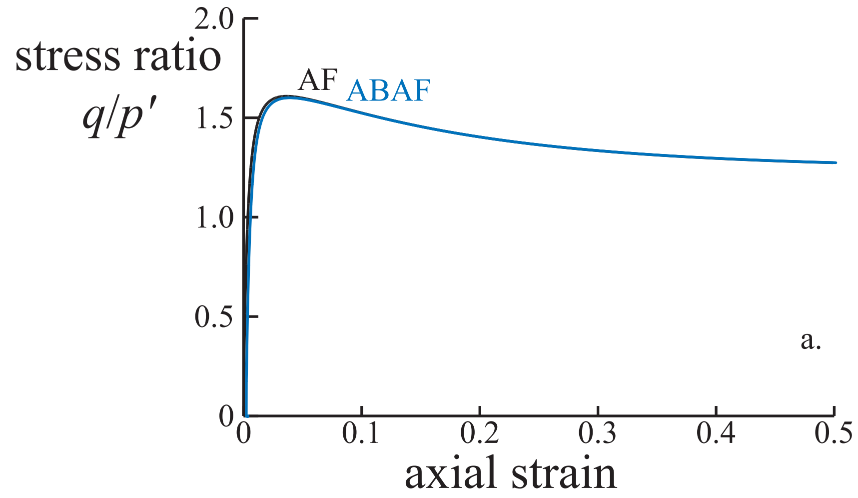
Severn-Trent sand: simulations (Kikumoto)

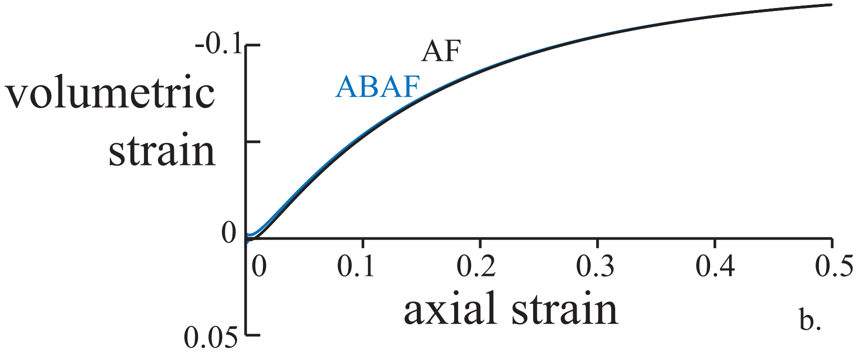


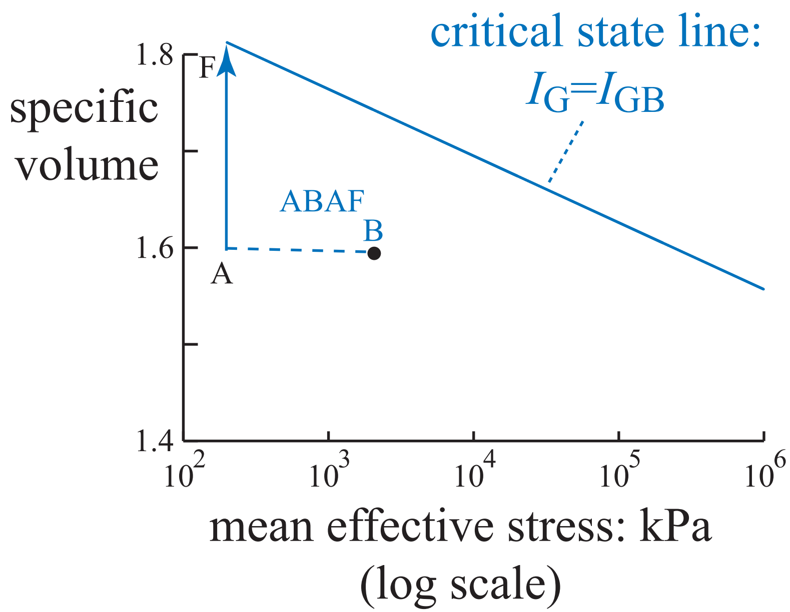


- precompression histories: A

Severn-Trent sand: simulations (Kikumoto)

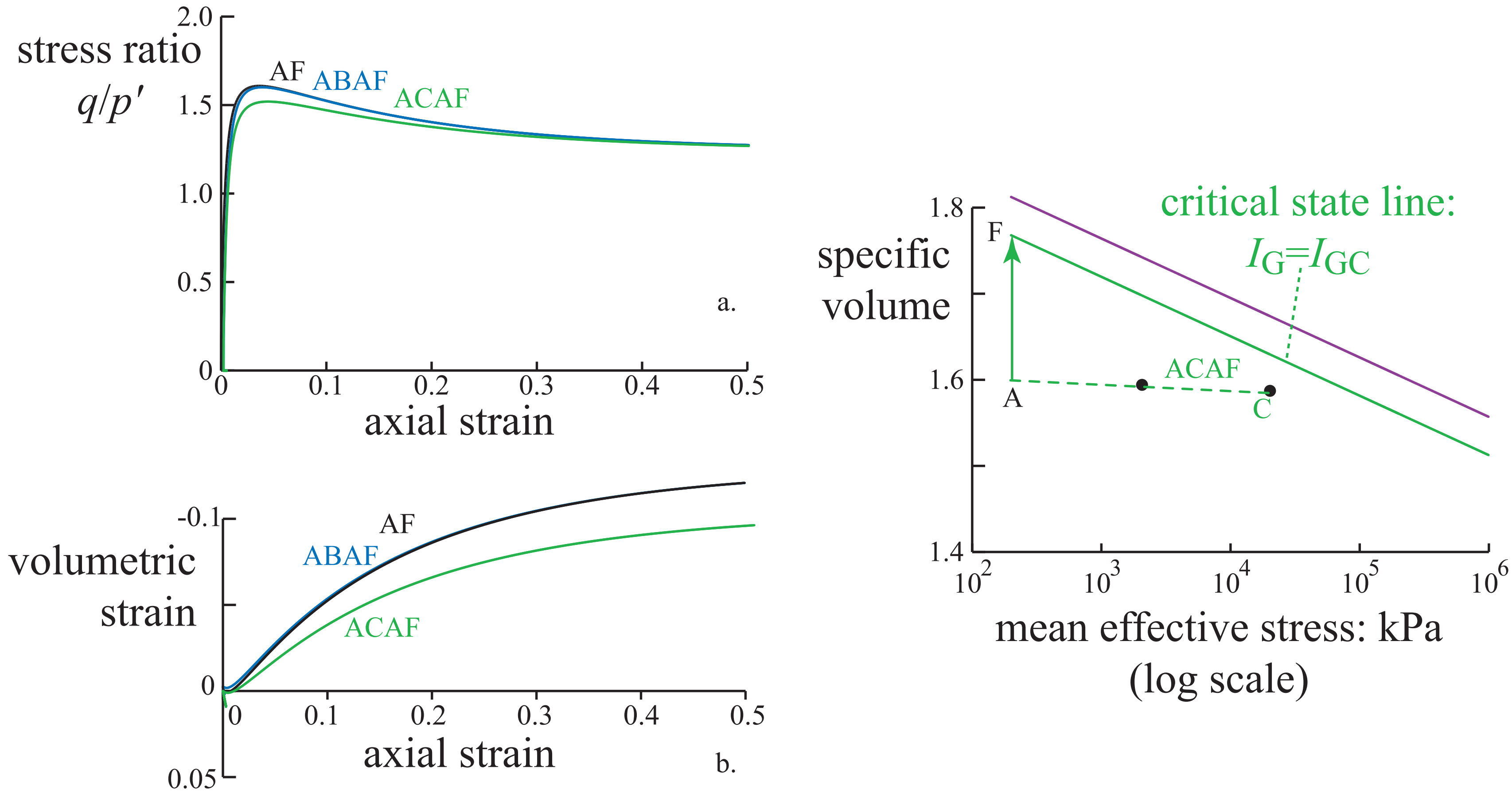






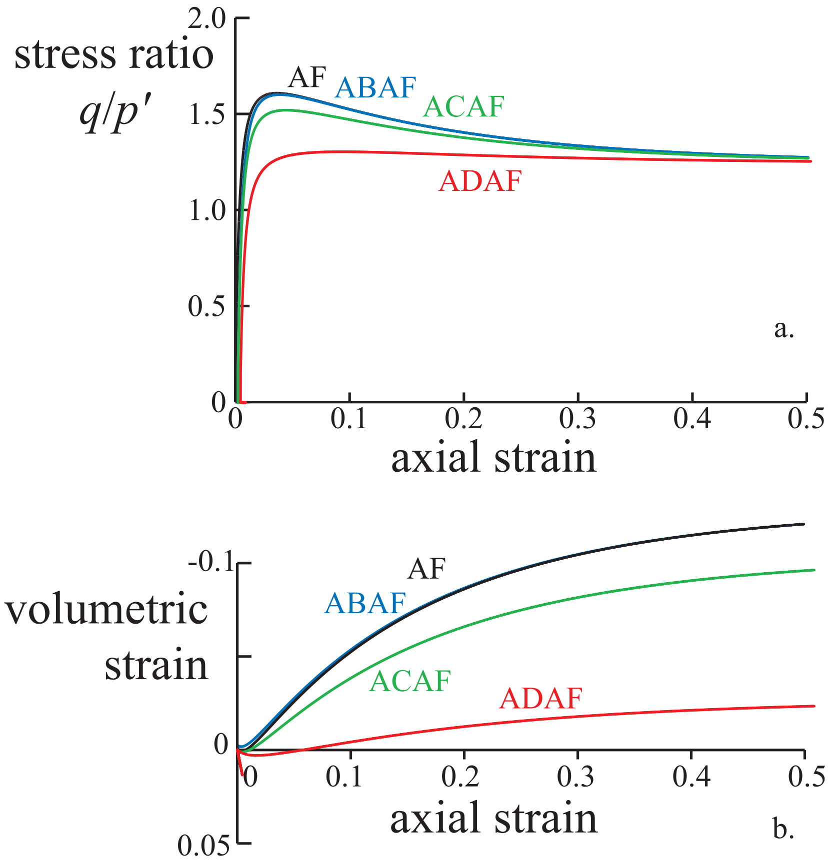
- precompression histories: A, ABA

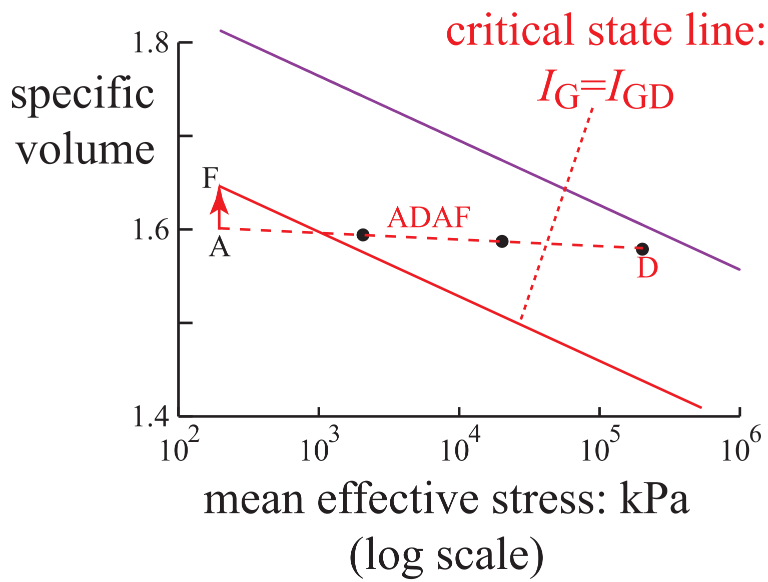
Severn-Trent sand: simulations (Kikumoto)



- precompression histories: A, ABA, ACA

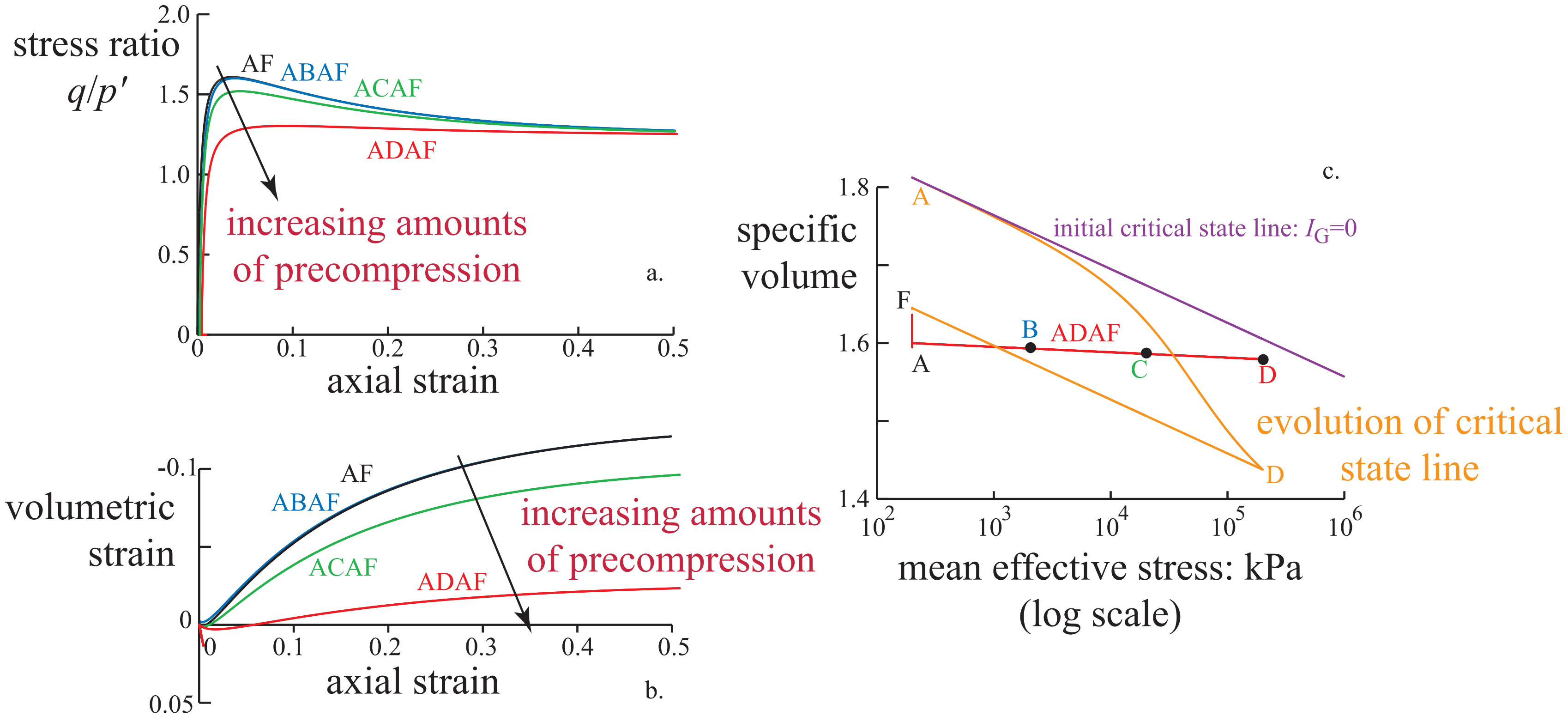
Severn-Trent sand: simulations (Kikumoto)





- precompression histories: A, ABA, ACA, ADA

Severn-Trent sand: simulations (Kikumoto)



- precompression histories: A, ABA, ACA, ADA
- precompression increases I_G, reduces peak strength, makes soil feel looser
- increases pore pressure generation ...

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