3. One-dimensional model

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Constitutive modelling 1

- Stress and strain variables
- Basic features of soil response: stiffness
- Basic features of soil response: critical states
- Basic features of soil response: strength

Stress and strain variables

- triaxial apparatus: two degrees of freedom
- soil = rigid particles + voids
- importance of volume (density) changes (or their prevention)
- separate volumetric (size) and distortional (shape) effects

Stress and strain variables

- soil behaviour controlled by effective stresses
- $\sigma'_{ij} = \sigma_{ij} - u\delta_{ij}$
- axial and radial strain increments $\delta\varepsilon_a$ and $\delta\varepsilon_r$
- axial and radial effective stresses σ'_a and σ'_r

Stress and strain variables

- volumetric effects
- volumetric strain increment $\delta\varepsilon_p = \delta\varepsilon_a + 2\delta\varepsilon_r$
- work conjugate volumetric stress $p' = (\sigma'_a + 2\sigma'_r)/3$
- volumetric work increment $\delta W_v = p'\delta\varepsilon_p$

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Stress and strain variables

- distortional effects
- distortional stress $q = \sigma_a - \sigma_r = F/A$
- work conjugate distortional strain increment $\delta\varepsilon_q = 2(\delta\varepsilon_a - \delta\varepsilon_r)/3$
- distortional work increment $\delta W_d = q\delta\varepsilon_q$

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Stress and strain variables

- work done in a small increment of strain
- $\delta W = \delta W_v + \delta W_d$
- $\delta W = p' \delta \varepsilon_p + q \delta \varepsilon_q$
- $\delta W = \sigma'_a \delta \varepsilon_a + 2\sigma'_r \delta \varepsilon_r$
- separation of volumetric and distortional effects

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Stress and strain variables

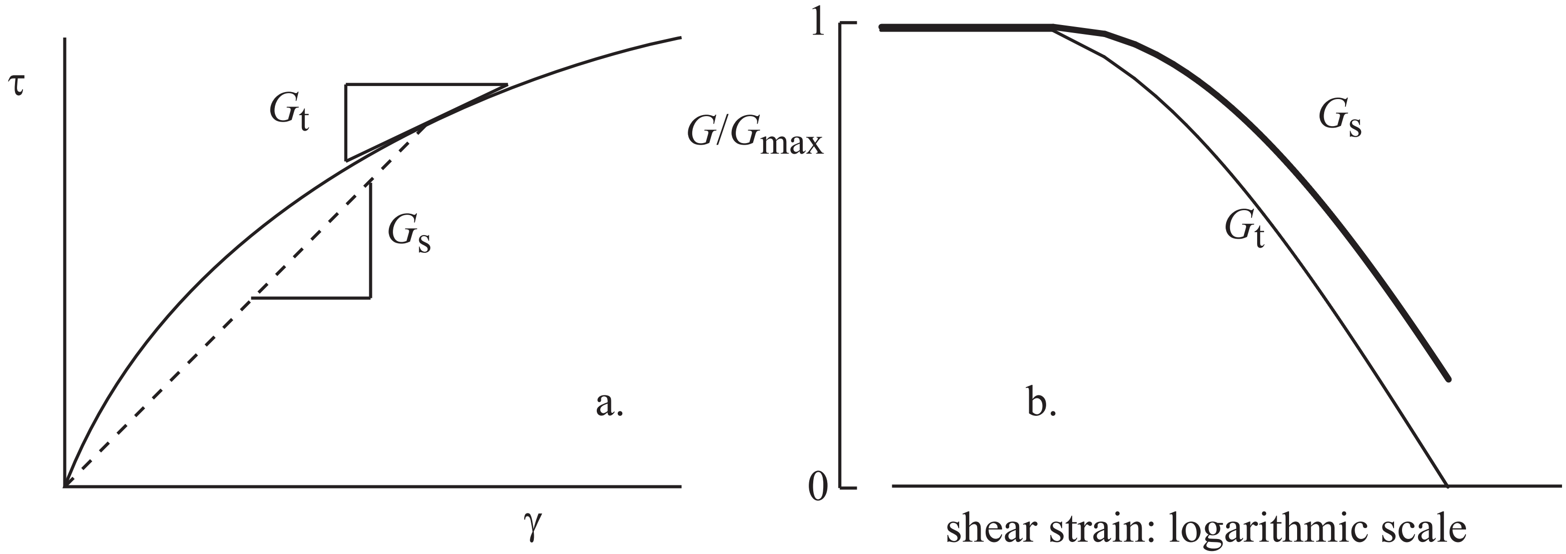
- stress ratio $\eta = q/p'$
- equivalent to a mobilised friction ϕ'_m
- triaxial compression

$$\sigma'_a/\sigma'_r = (1+\sin\phi'_m)/(1-\sin\phi'_m) = (3+2\eta)/(3-\eta)$$
$$\sin\phi'_m = (\sigma'_a-\sigma'_r)/(\sigma'_a+\sigma'_r) = (3\eta)/(6+\eta)$$
$$\eta = 6\sin\phi'_m/(3-\sin\phi'_m)$$

- triaxial extension

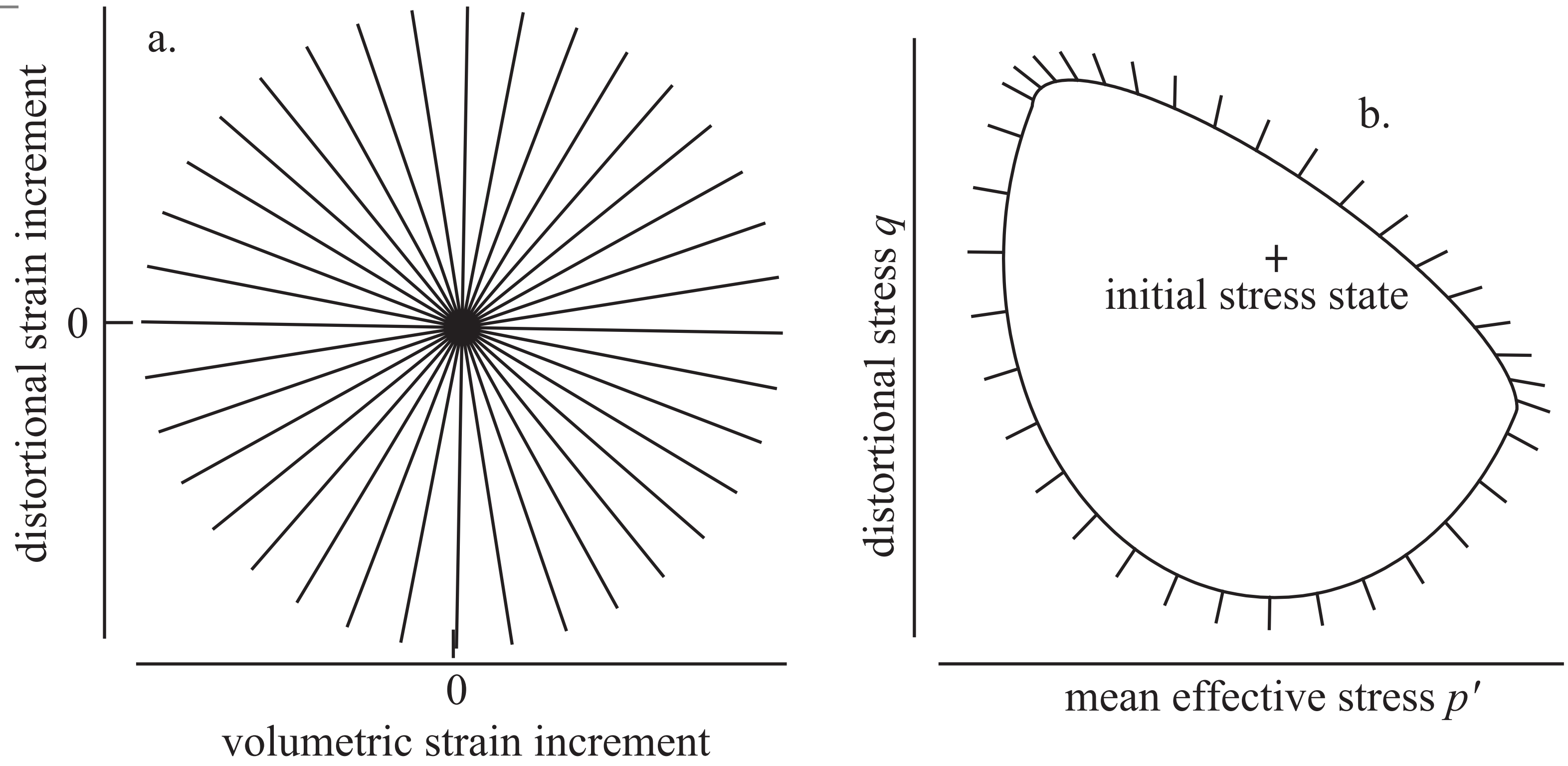
$$\sigma'_a/\sigma'_r = (1-\sin\phi'_m)/(1+\sin\phi'_m) = (3+2\eta)/(3-\eta)$$
$$\sin\phi'_m = (\sigma'_r-\sigma'_a)/(\sigma'_a+\sigma'_r) = -3\eta/(6+\eta)$$
$$\eta = -6\sin\phi'_m/(3+\sin\phi'_m)$$

Basic features of soil response: stiffness



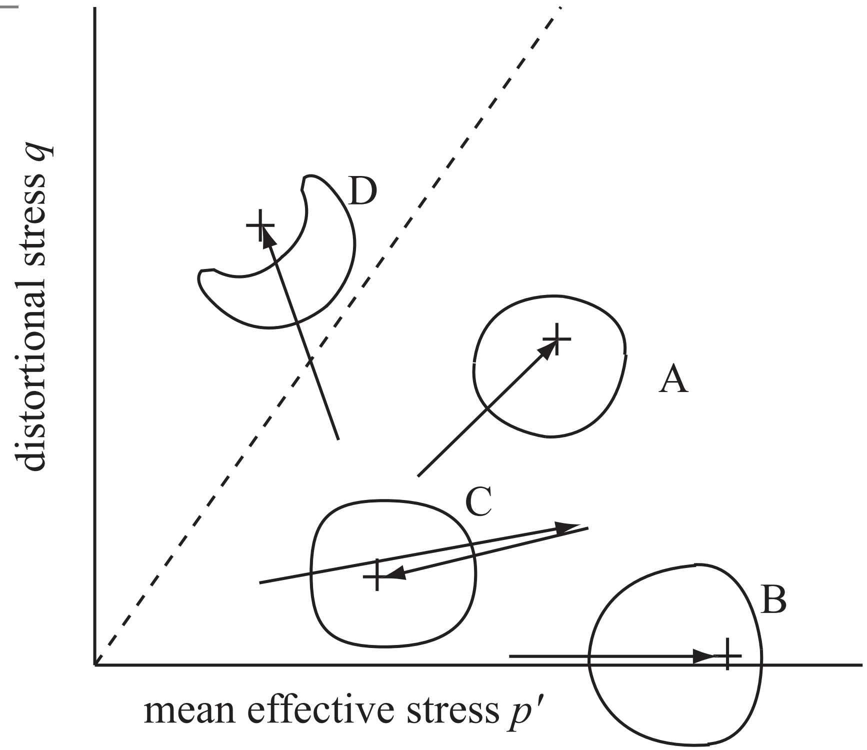
- tangent and secant stiffness
- secant: past: $G_s = \tau/\gamma$
- tangent: future: $G_t = \delta\tau/\delta\gamma$
- tangent (incremental) stiffness falls faster than secant (average) stiffness

Basic features of soil response: stiffness



- stress response envelope
- rosette of strain probes ...
- ... and resulting stress response envelope
- discipline for evaluating models
- discipline for planning laboratory tests

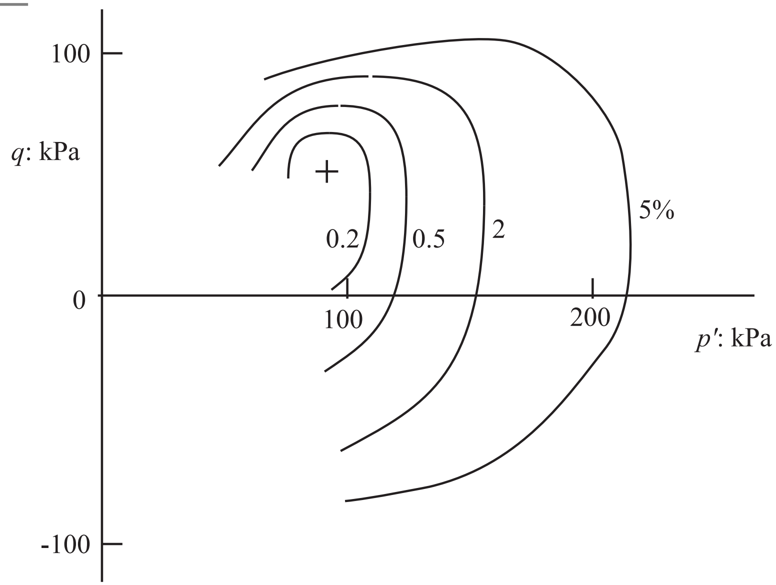
Basic features of soil response: stiffness

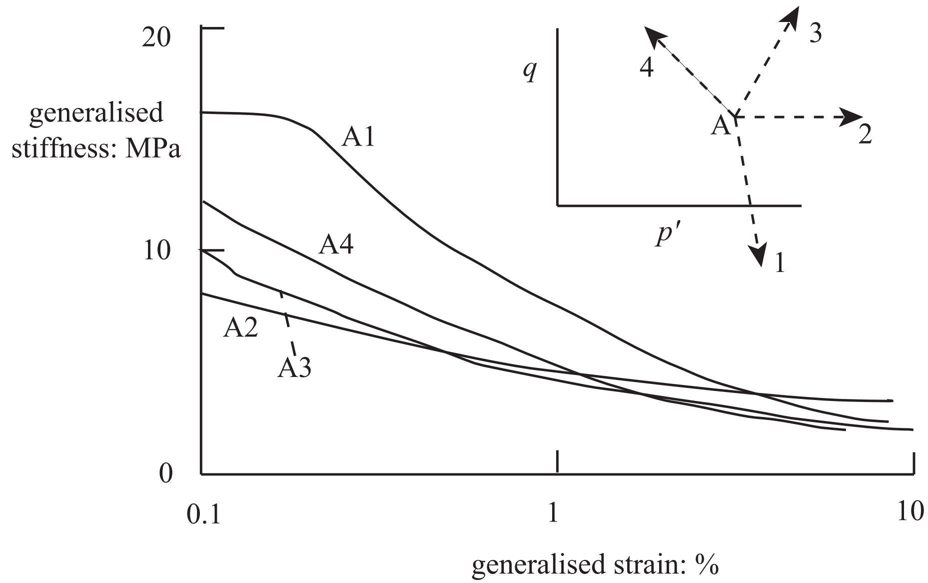


- history dependence of stress response envelopes
- points $+$ indicate initial stress states

- A, B: flattened towards loading direction
- C: unloaded - response somewhat independent of direction
- D: strain softening: fall in η for all strain increments

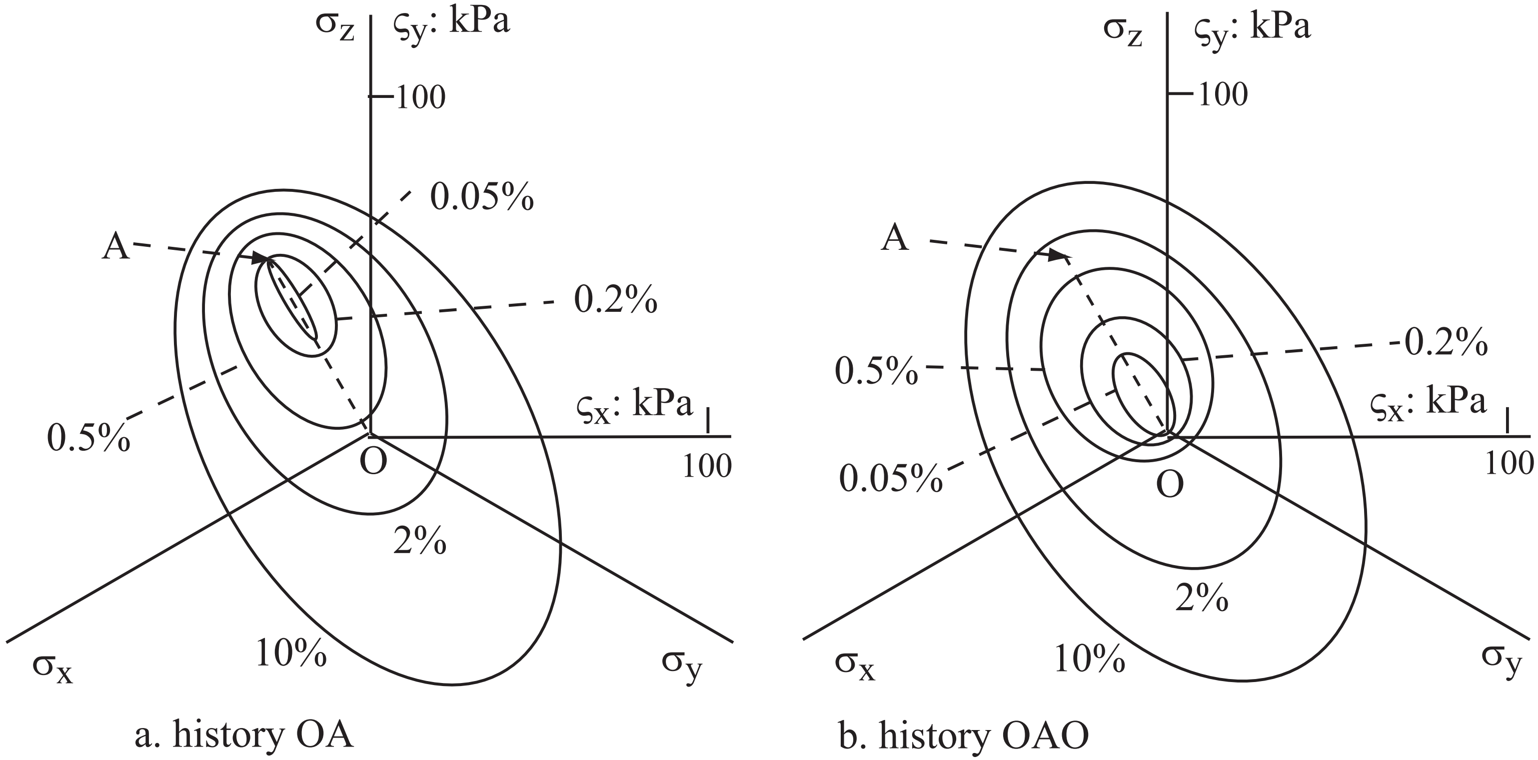
Basic features of soil response: stiffness





- response envelopes and generalised stiffness for Pisa clay (Callisto)
- effect of strain level
- erasure of memory

Basic features of soil response: stiffness



- true triaxial probing of kaolin
- schematic deviatoric stress response envelopes
- deviatoric history (a) OA and (b) OAO

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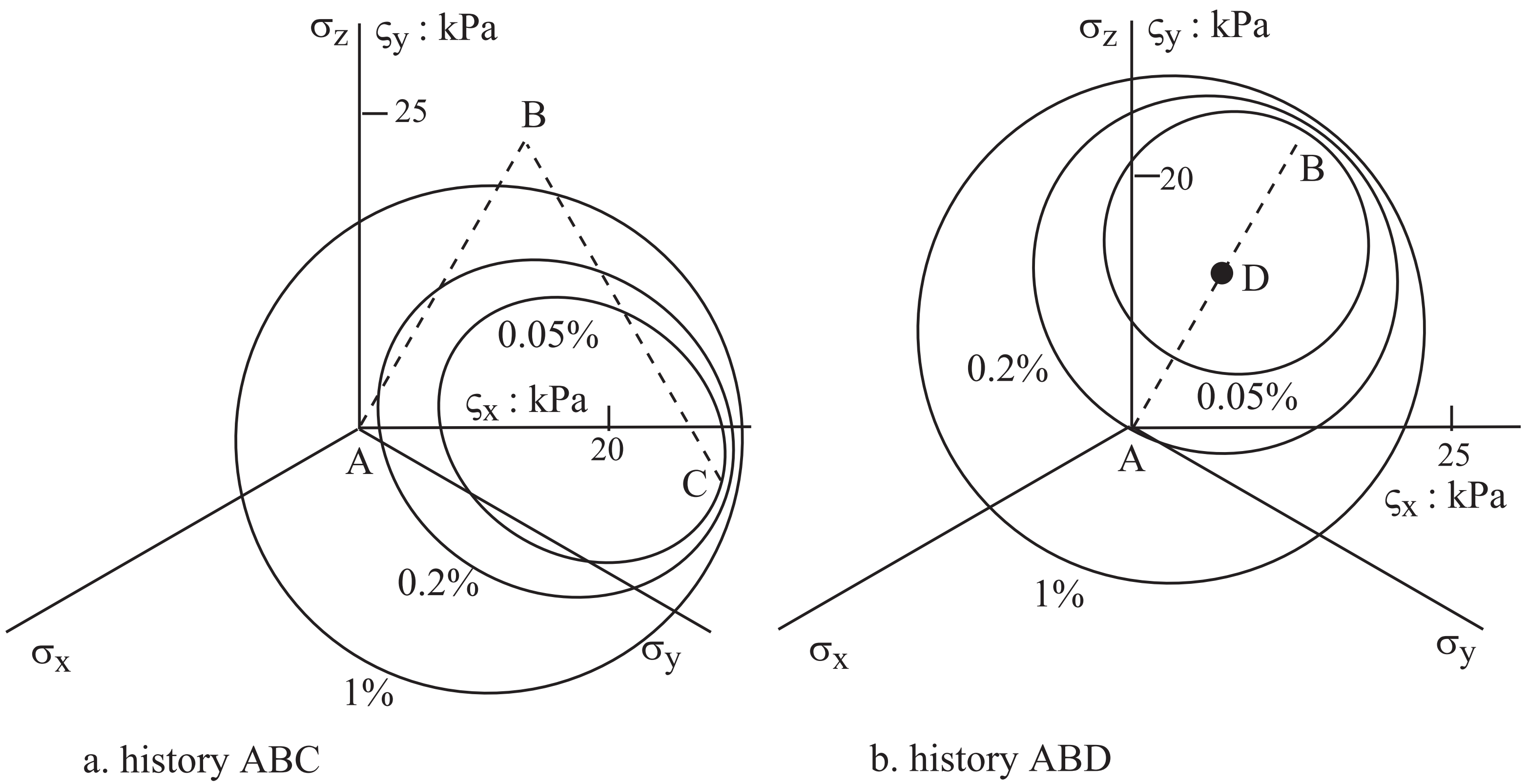
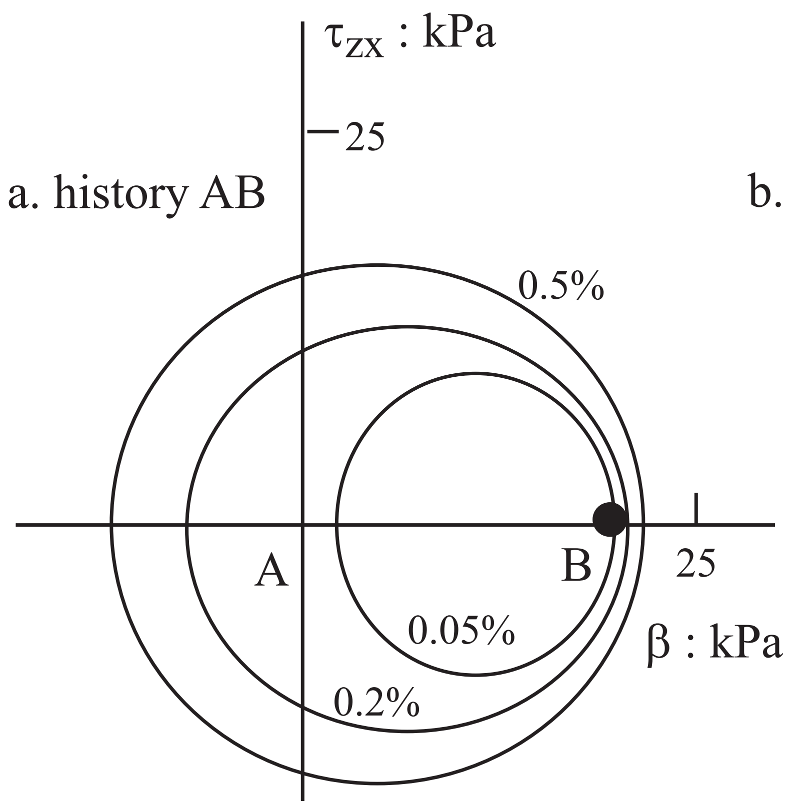
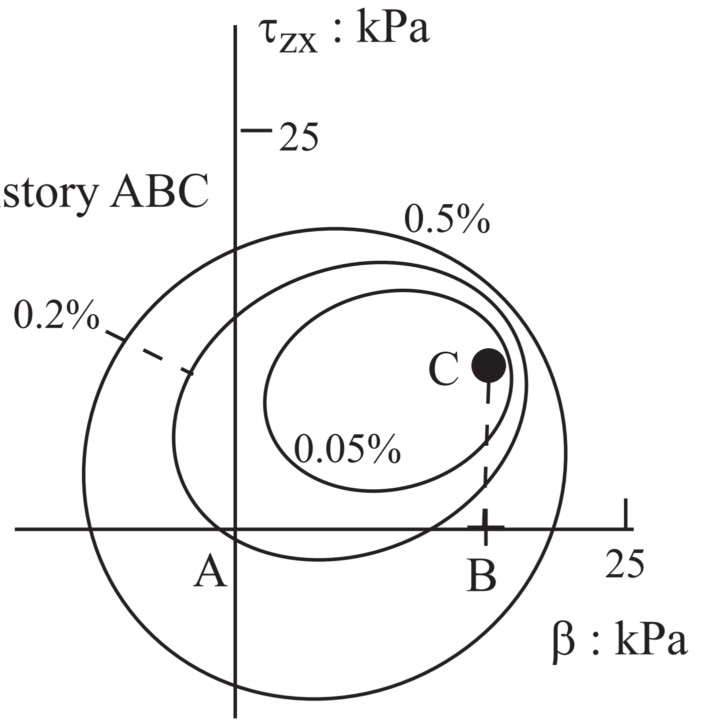
Basic features of soil response: stiffness



- true triaxial probing of Leighton Buzzard sand
- schematic deviatoric stress response envelopes
- deviatoric history (a) ABC and (b) ABD (data from Sture *et al.*, 1988)

Basic features of soil response: stiffness





- directional shear cell probing of Leighton Buzzard sand
- schematic deviatoric stress response envelopes
- deviatoric history (a) AB and (b) ABC (data from Sture *et al.*, 1988)
- rotation of principal axes

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Basic features of soil response: stiffness

- history dependence of stress response envelopes
- stiffness high for strain reversal
- stiffness low for continuing loading
- erasure of memory of past events
- messages for modelling

Basic features of soil response: critical states

- asymptotic state reached at large strain
- state = stresses, density, fabric (particle arrangement)
- evidence from experiments on soils
- evidence from discrete element modelling
- central feature of constitutive models (explicit or implicit)

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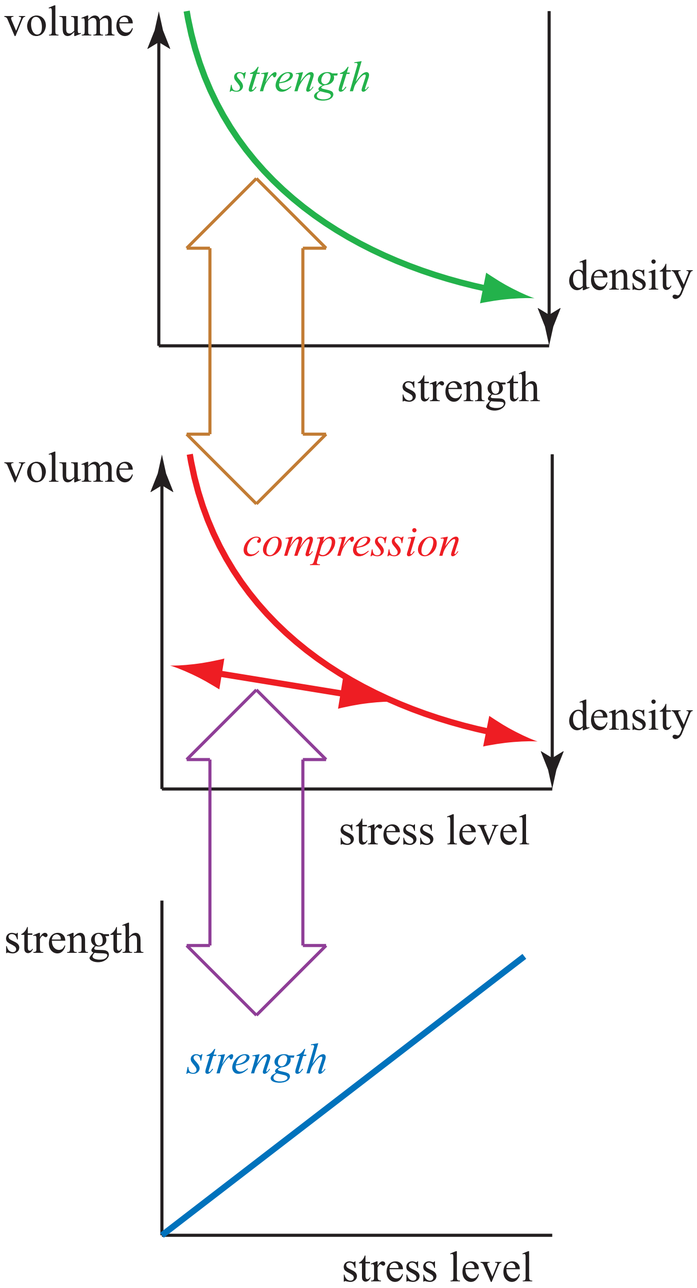
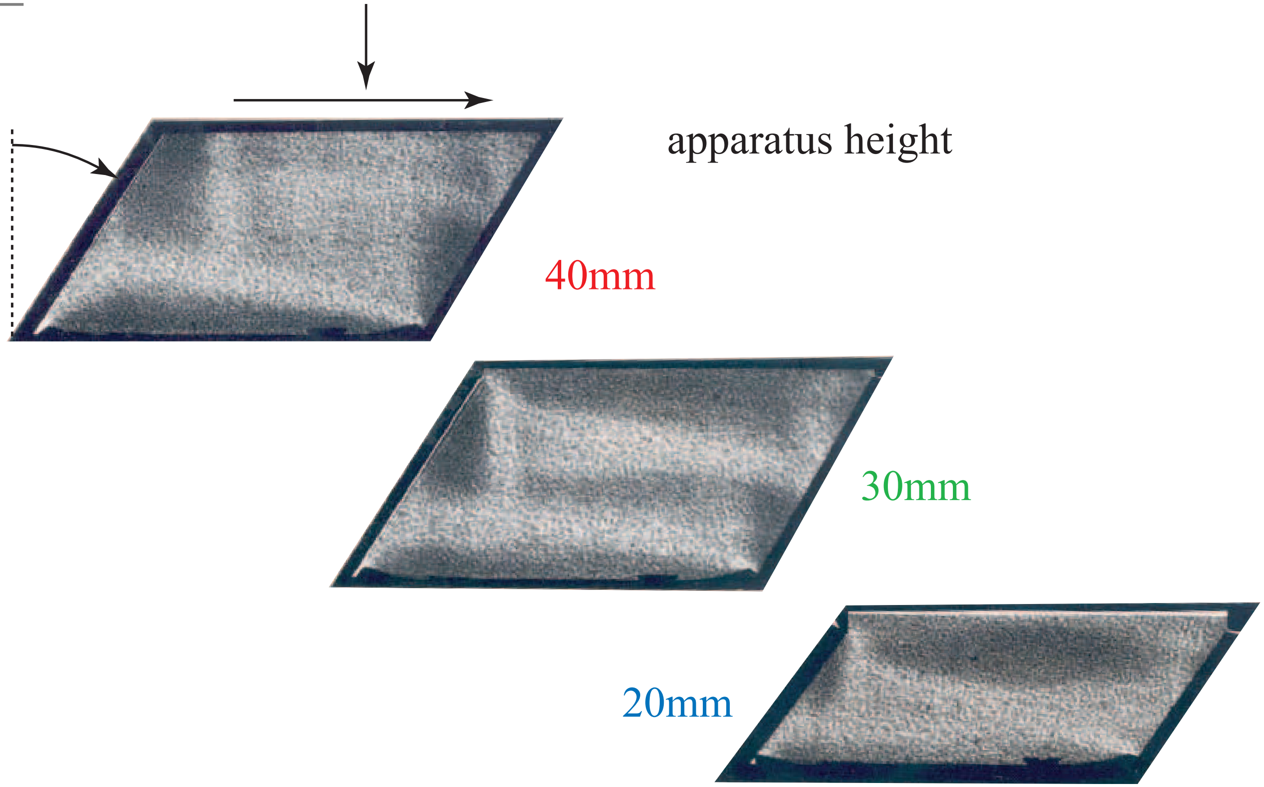
Basic features of soil response: critical states



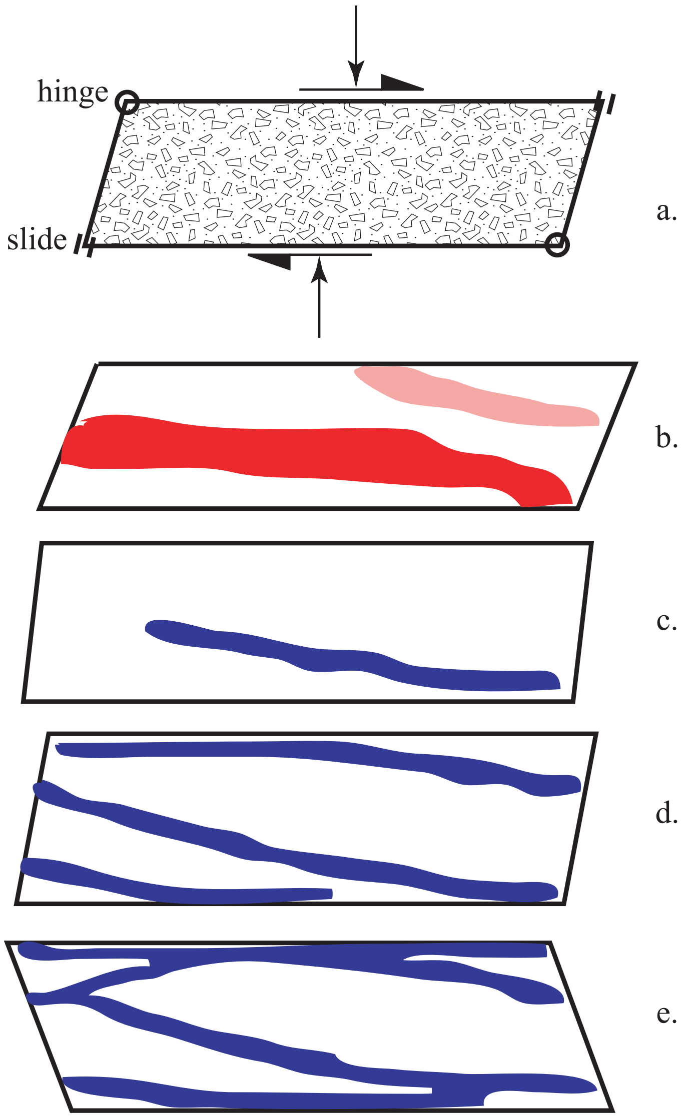
- critical state soil mechanics (weak form)
- discipline for exploring and interpreting mechanical response of soils
- importance of considering stresses *and* density

Basic features of soil response: critical states



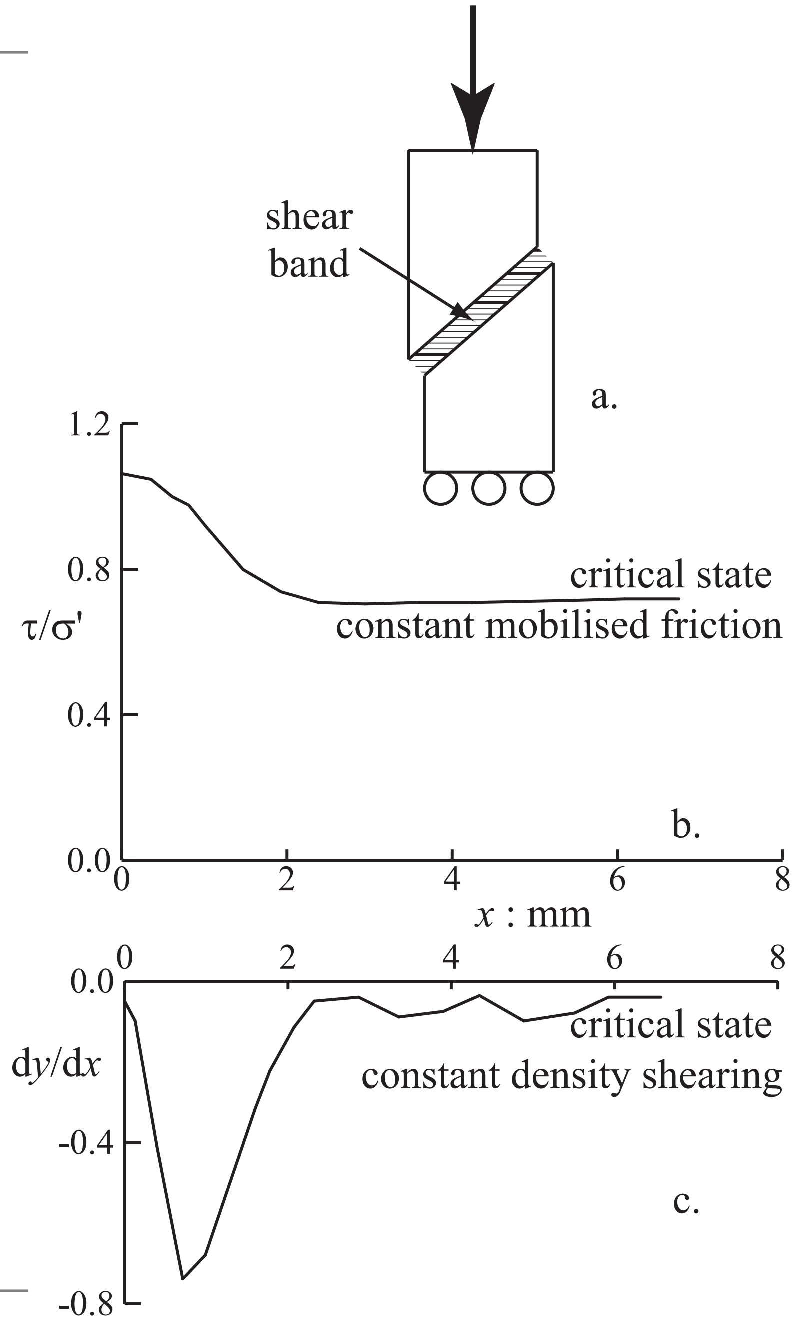
- observation of critical states? simple shear apparatus?
- internal kinematic freedom
- occurrence of localisation
- (Bassett)

Basic features of soil response: critical states



- simple shear apparatus - sample height ~ 20 mm
- coarse sand $d_{50} =\sim 1$ mm - uniform?
- fine sand $d_{50} =\sim 0.3$ mm
- (Stroud, Budhu)

Basic features of soil response: critical states



- biaxial plane strain test
- free formation of shear band
- critical state reached in shear band - not elsewhere
- free formation of shear band
- (Vardoulakis)

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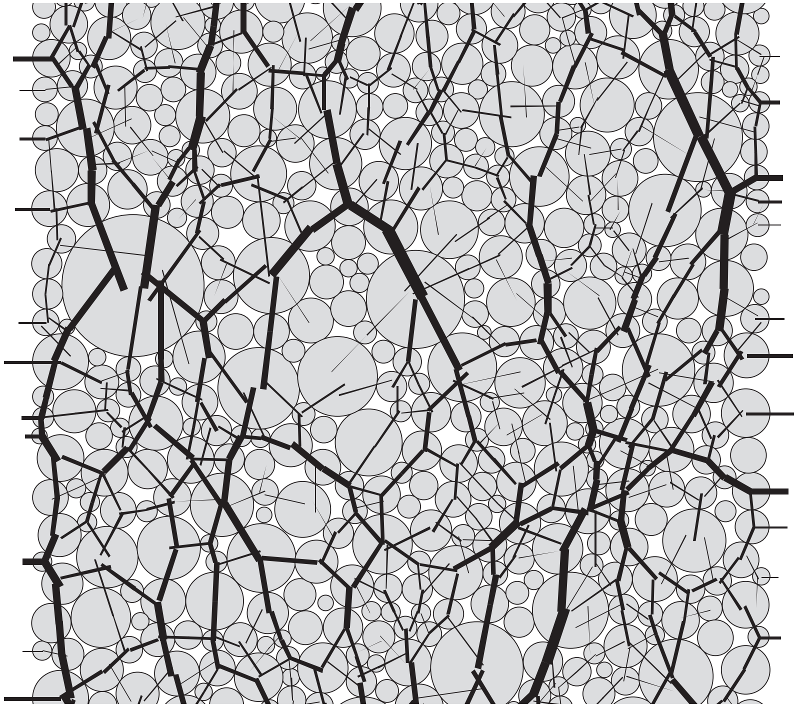
Basic features of soil response: critical states



- discrete element modelling
- force chains of heavily loaded particles
- buckling of force chains encouraged by principal stress rotation

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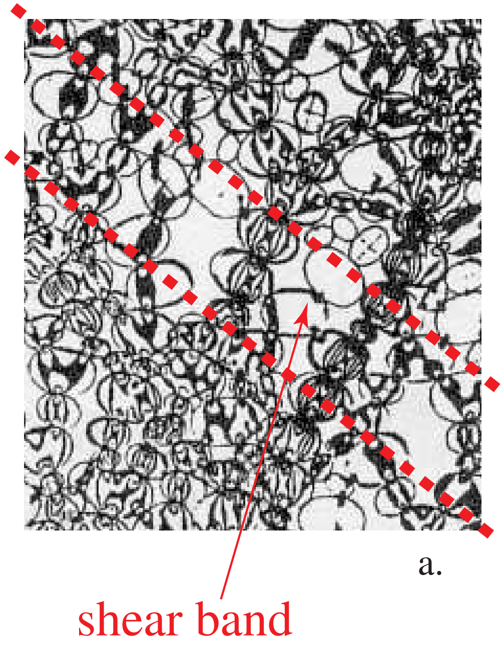
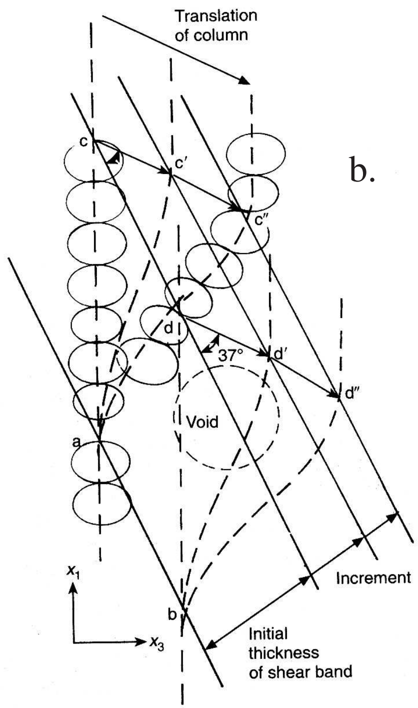
Basic features of soil response: critical states



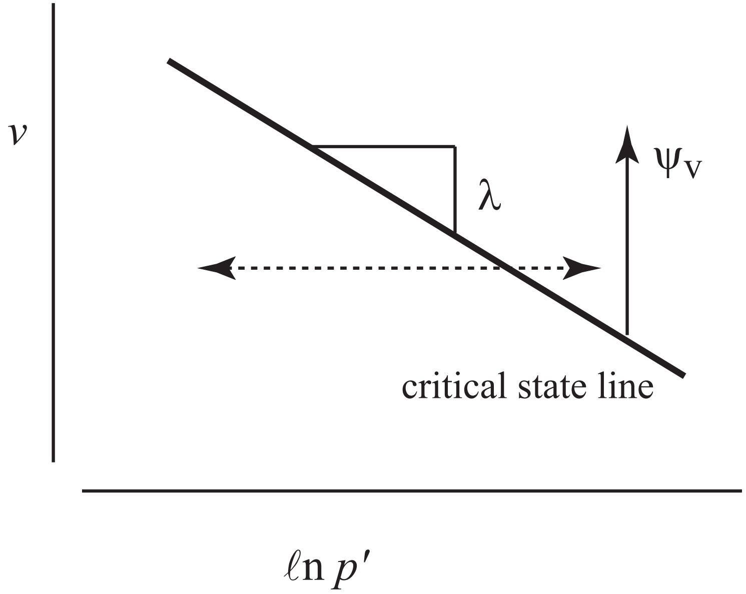


- assembly of elliptical photoelastic discs
- force chains and voids in shear band
- buckling and rotation of columns in shear band
- (from Oda and Kazama, 1998)

Basic features of soil response: critical states

- asymptotic state reached at large strain
- fabric not isotropic at critical state
- difficult to detect experimentally
- central feature of constitutive models (explicit or implicit)

Basic features of soil response: strength



- critical state line
- state parameter or state variable as generalisation of ‘dense’, ‘loose’
- $\psi = v - v_c = v - (\Gamma - \lambda \ln p')$
- semilogarithmic shape - not essential

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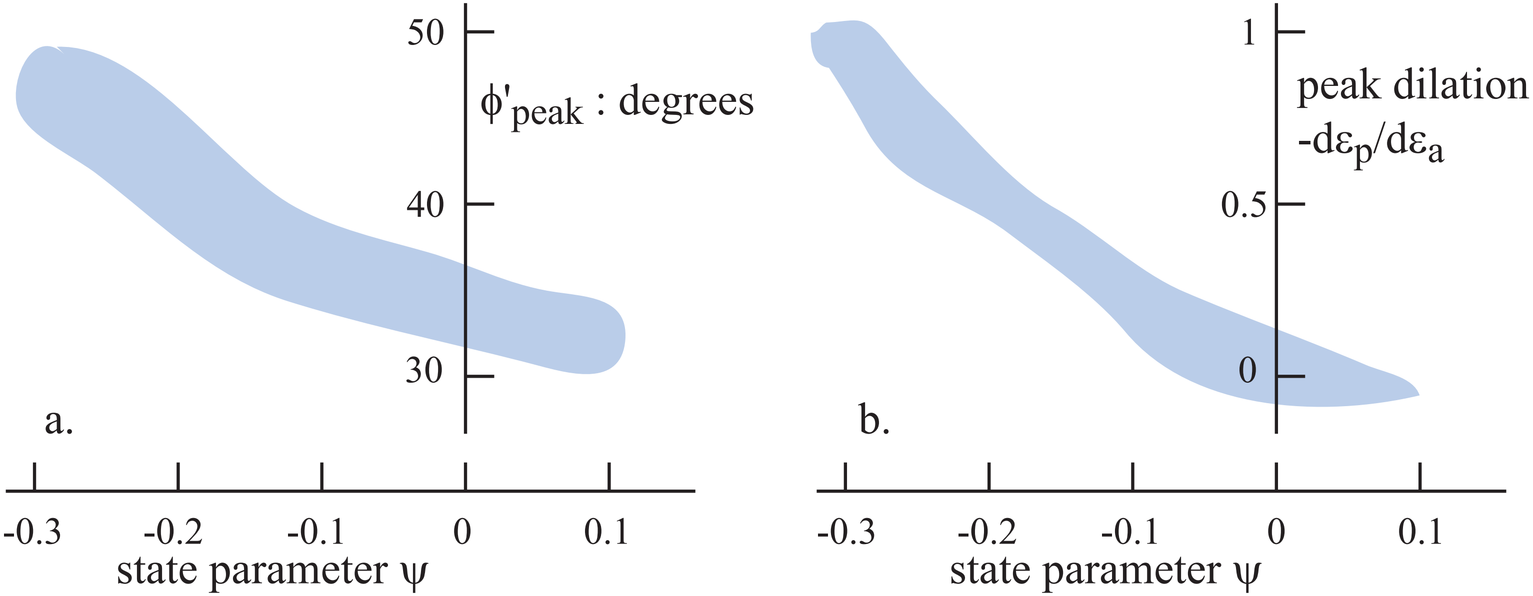
Basic features of soil response: strength



- dependence of peak strength on state parameter
- dependence of peak dilatancy on state parameter
- (Been & Jefferies)

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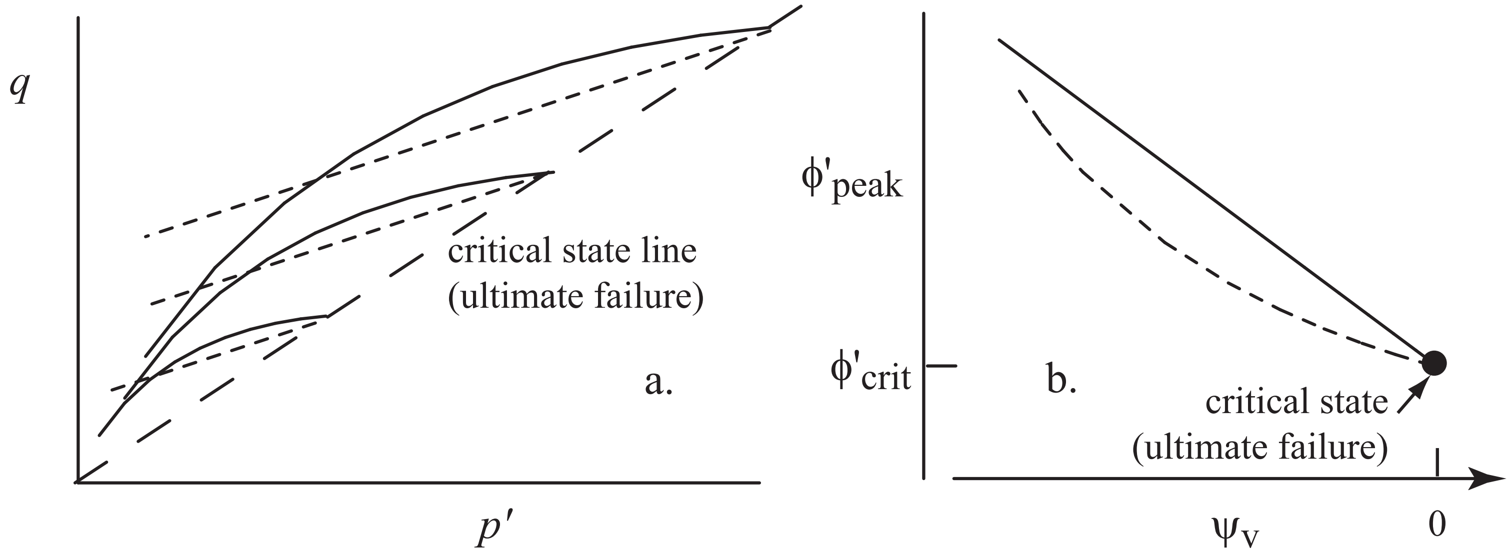
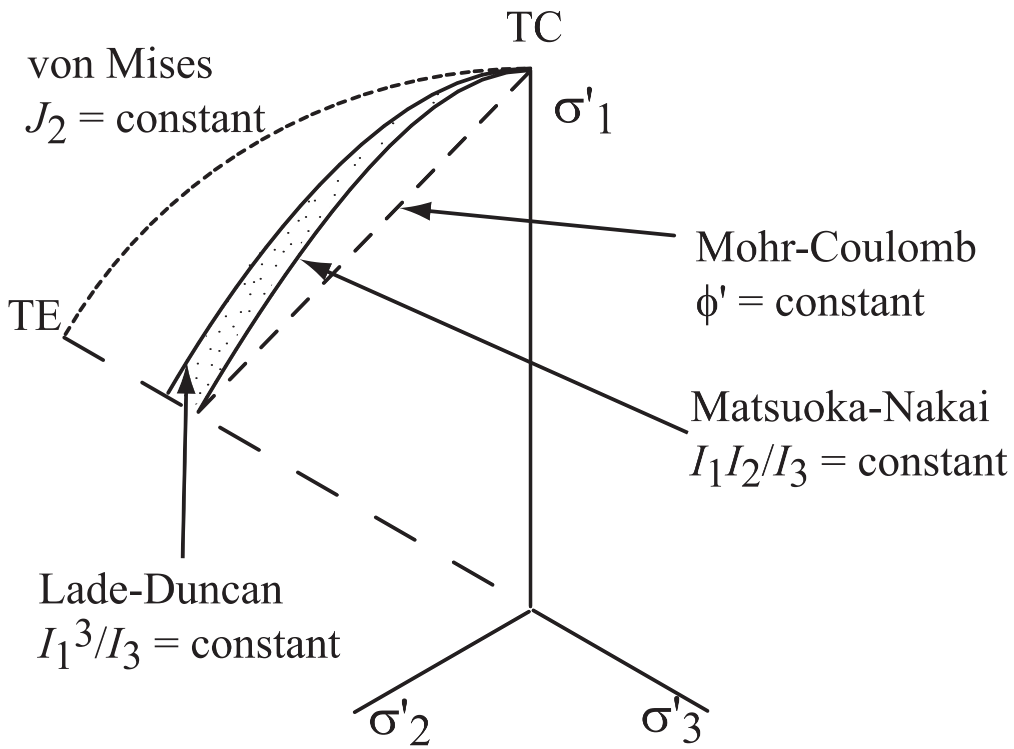
Basic features of soil response: strength



- equivalence of failure relationships
- effective stress plane p', q
- friction angle and state parameter

Basic features of soil response: strength



- multiaxial stress conditions
- Matsuoka-Nakai: $I_1 I_2 / I_3 =$ constant
- Lade-Duncan: $I_1^3 / I_3 =$ constant

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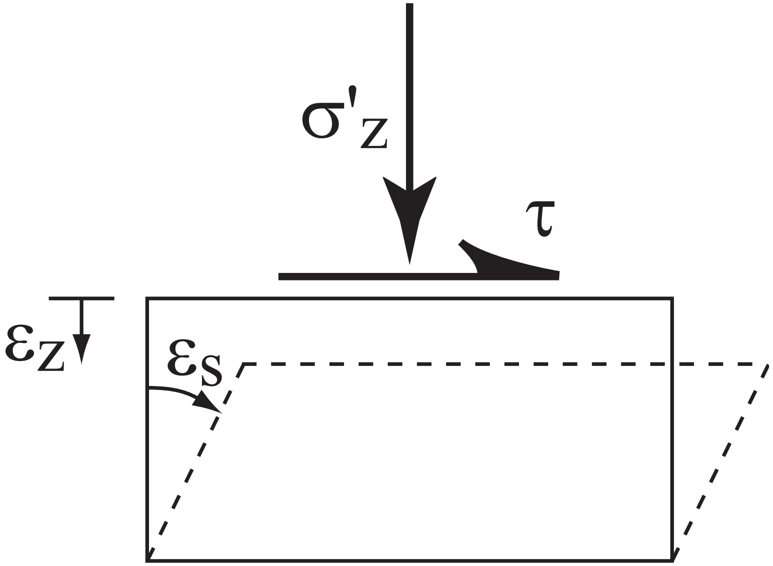
Basic features of soil response: strength

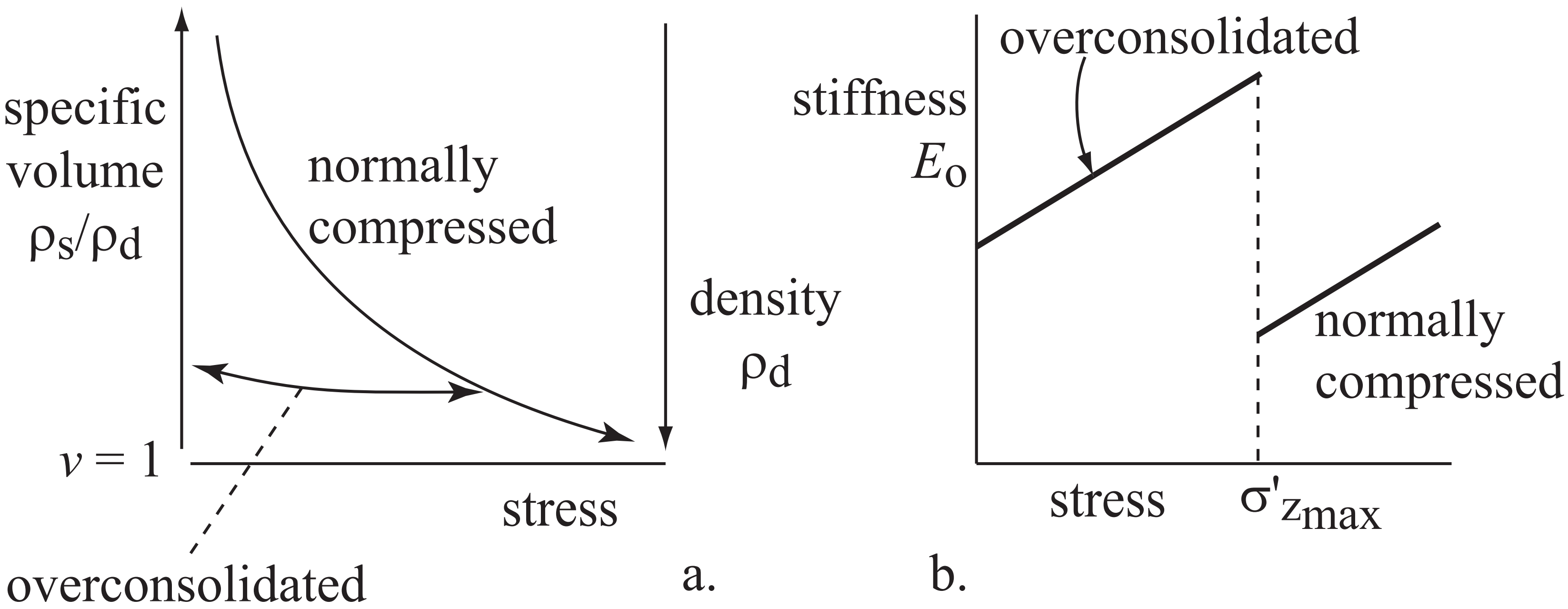
- importance of density and stress level
- peak strength is a dependent variable not a constant
- most stress conditions are not axisymmetric

Simple model of shearing



- build one-dimensional model
- simple shear/direct shear box
- separate vertical deformation (oedometer) and shearing
- stiffness - strength - dilatancy

Simple model of shearing



stiffness - normal consolidation - overconsolidation

$$\frac{E_o}{\sigma_{ref}} = \chi \left(\frac{\sigma'_z}{\sigma_{ref}} \right)^{\alpha}$$

Simple model of shearing

stiffness - normal consolidation (α_{nc}, χ_{nc}) - overconsolidation (α_{oc}, χ_{oc})

$$\frac{E_o}{\sigma_{ref}} = \chi \left(\frac{\sigma'_z}{\sigma_{ref}} \right)^{\alpha}$$

$\delta\varepsilon'_z = \delta\sigma'_z / E_o$

- If $\sigma'_z = \sigma'_{z_{max}}$ and $\delta\sigma'_z > 0$, then $n = 1$, $\alpha = \alpha_{nc}$ and $\chi = \chi_{nc}$ (normally compressed)
- If $\delta\sigma'_z < 0$, then $\delta n > 0$, $n \geq 1$, and $\alpha = \alpha_{oc}$ and $\chi = \chi_{oc}$ (overconsolidated)
- If $\sigma'_z < \sigma'_{z_{max}}$ and $\delta\sigma'_z > 0$, then $n > 1$, $\delta n < 0$, and $\alpha = \alpha_{oc}$ and $\chi = \chi_{oc}$ (overconsolidated)

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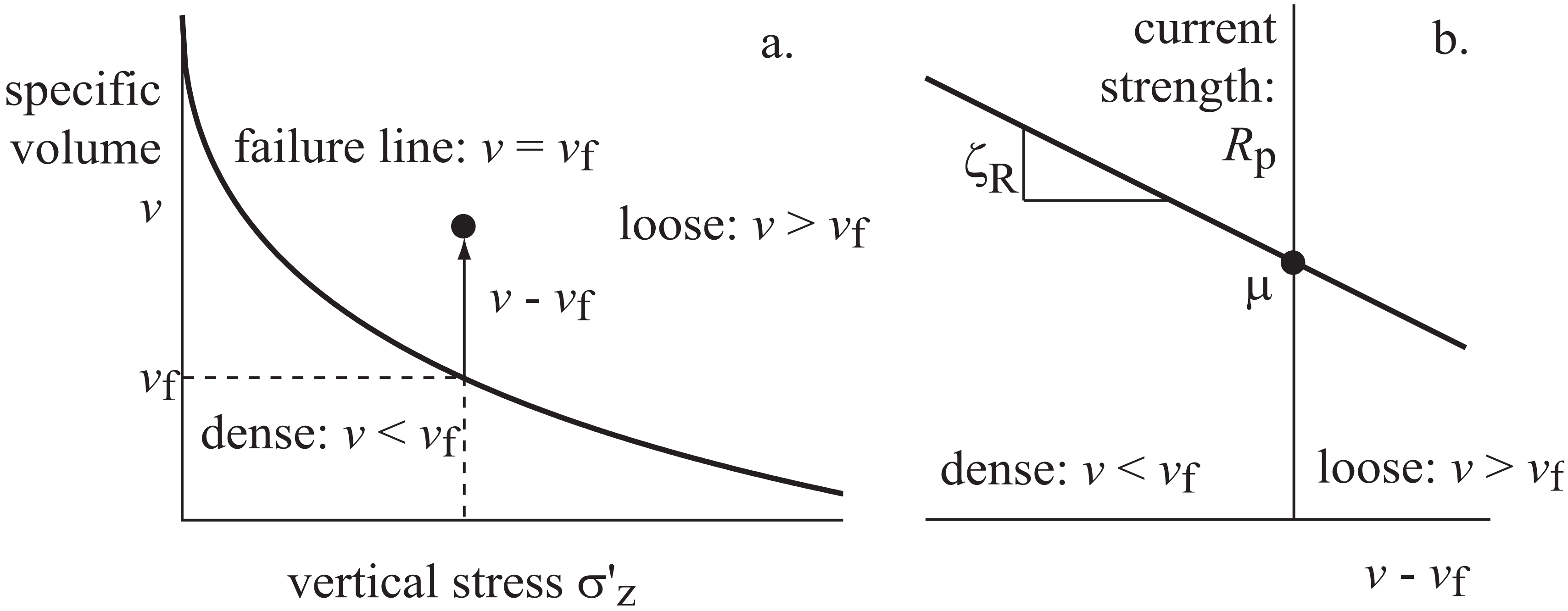
Simple model of shearing



- critical state line $v_f = v_{min} + \Delta v \exp\left[-(\sigma'_z/\sigma_{ref})^{\beta}\right]$
- state parameter $\psi = v - v_f$
- current strength $R_p = \mu + \zeta_R(v_f - v)$
- strength is a variable

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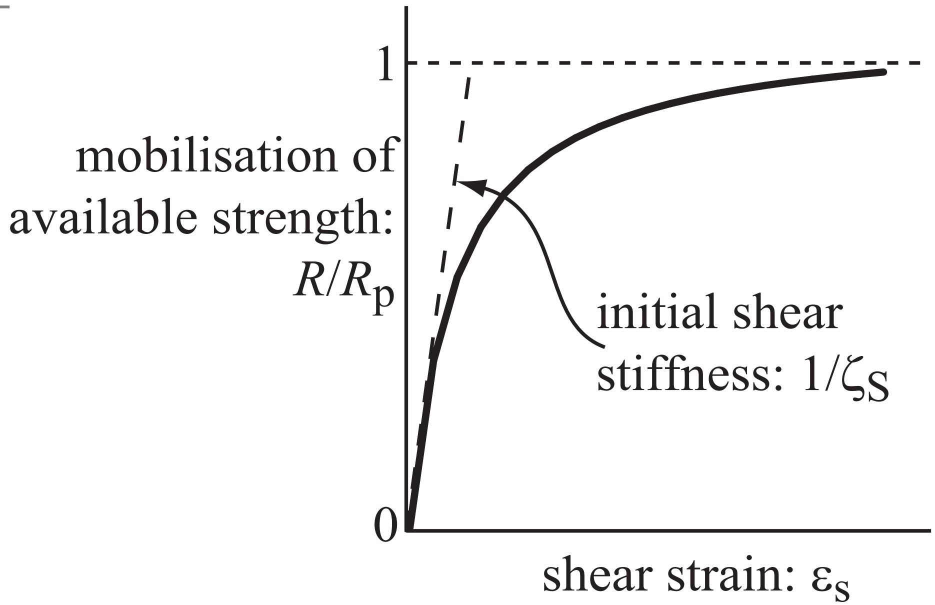
Simple model of shearing



hyperbolic mobilisation of strength

$$\frac{R}{R_p} = \frac{\varepsilon_s}{\zeta_S + \varepsilon_s}$$

$$\delta R = \frac{1}{R_p}\left[(R_p - R)^2 \frac{\delta\varepsilon_s}{\zeta_S} + R\delta R_p\right]$$

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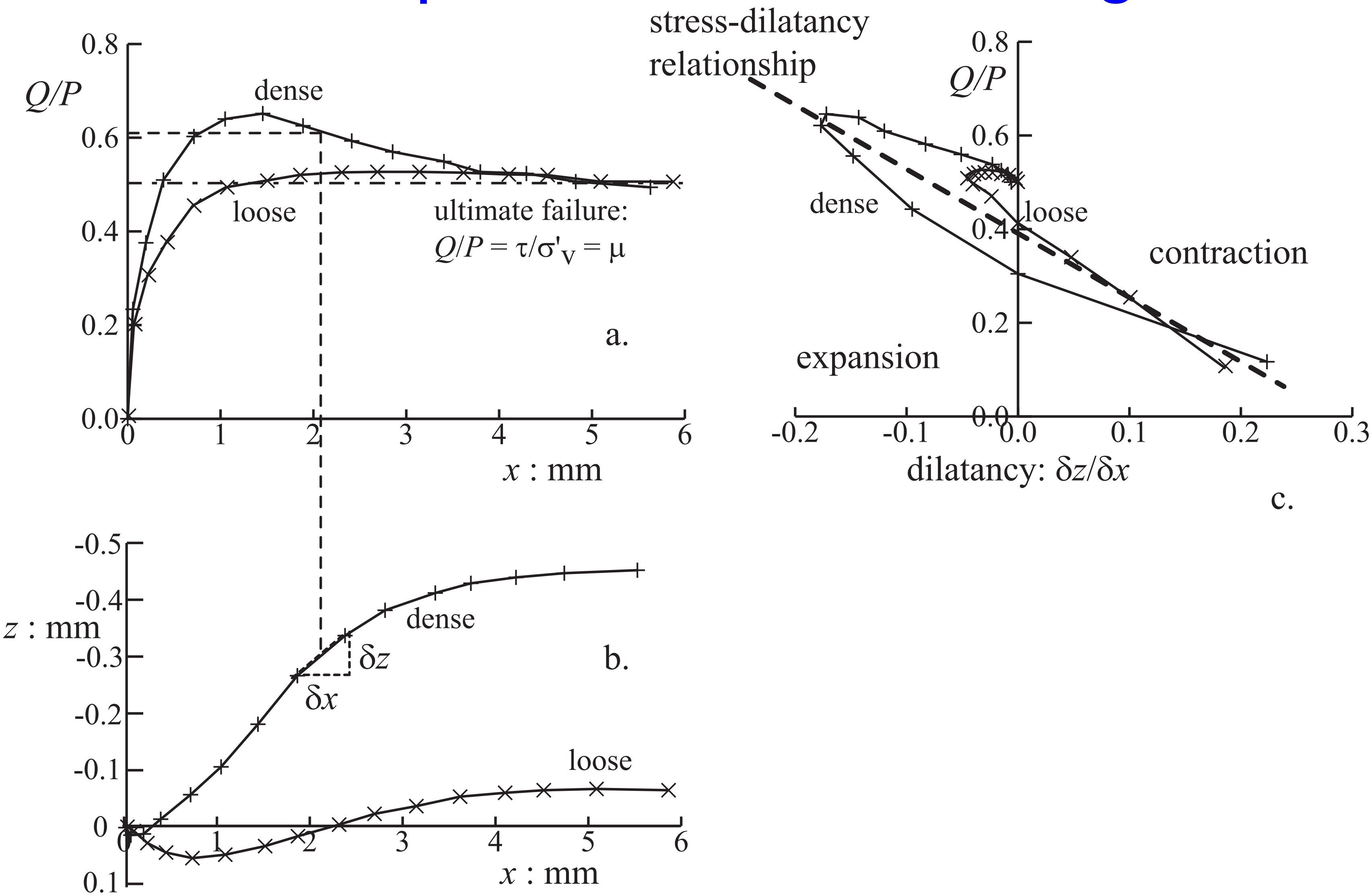
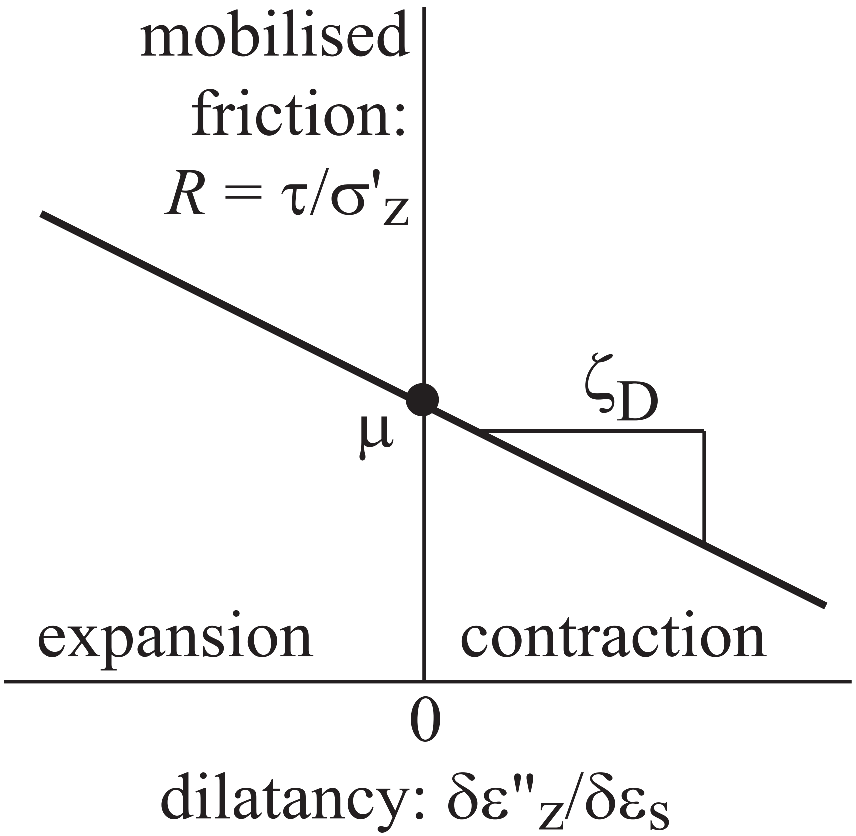
Simple model of shearing



- shear box data for dense and loose sand
- stress-dilatancy relationship (Taylor)

Simple model of shearing



stress-dilatancy relationship

$$\frac{\delta\varepsilon_z''}{\delta\varepsilon_s} = \zeta_D(\mu - R)$$

- feedback mechanism: if soil not at critical state, volume change occurs to bring it to asymptotic critical state
- total vertical strain $\delta\varepsilon_z = \delta\varepsilon_z' + \delta\varepsilon_z''$
- elasticity + dilatancy

Simple model of shearing

complete stress-strain stiffness relationship

$$\delta\sigma'_z = E_o[\delta\varepsilon_z - \Psi_3\delta\varepsilon_s]$$

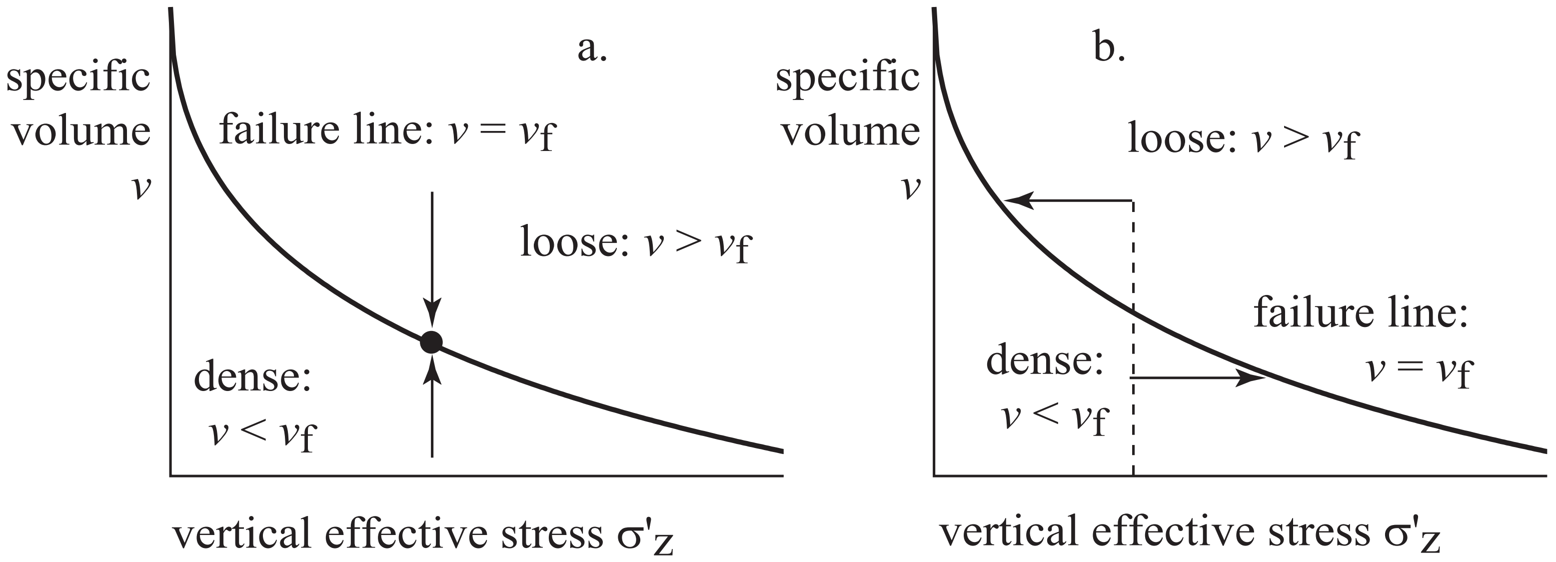
$$\delta\tau = E_o[-\Psi_1\delta\varepsilon_z + (\Psi_2 + \Psi_1\Psi_3)\delta\varepsilon_s]$$

$$\Psi_1 = \frac{\zeta_R R}{R_p}\left[\beta(v_f - v_{min})\left(\frac{\sigma'_z}{\sigma_{ref}}\right)^{\beta} - \frac{v\sigma'_z}{E_o}\right] - R$$

$$\Psi_2 = \left[\frac{(R_p - R)^2}{\zeta_S R_p} + \frac{\zeta_R v R \Psi_3}{R_p}\right]\frac{\sigma'_z}{E_o}$$

$$\Psi_3 = \zeta_D(\mu - R)$$

Simple model of shearing



- $\delta\sigma'_z = E_o[\delta\varepsilon_z - \Psi_3\delta\varepsilon_s]$
- $\delta\tau = E_o[-\Psi_1\delta\varepsilon_z + (\Psi_2 + \Psi_1\Psi_3)\delta\varepsilon_s]$
- drained (constant σ'_z): $\delta\tau = \Psi_2 E_o\delta\varepsilon_s$, $\delta\varepsilon_z = \Psi_3\delta\varepsilon_s$
- undrained (constant height) $\delta\tau = (\Psi_2 + \Psi_1\Psi_3)E_o\delta\varepsilon_s$, $\delta\sigma'_z/\delta\tau = -\Psi_3/(\Psi_2 + \Psi_1\Psi_3)$

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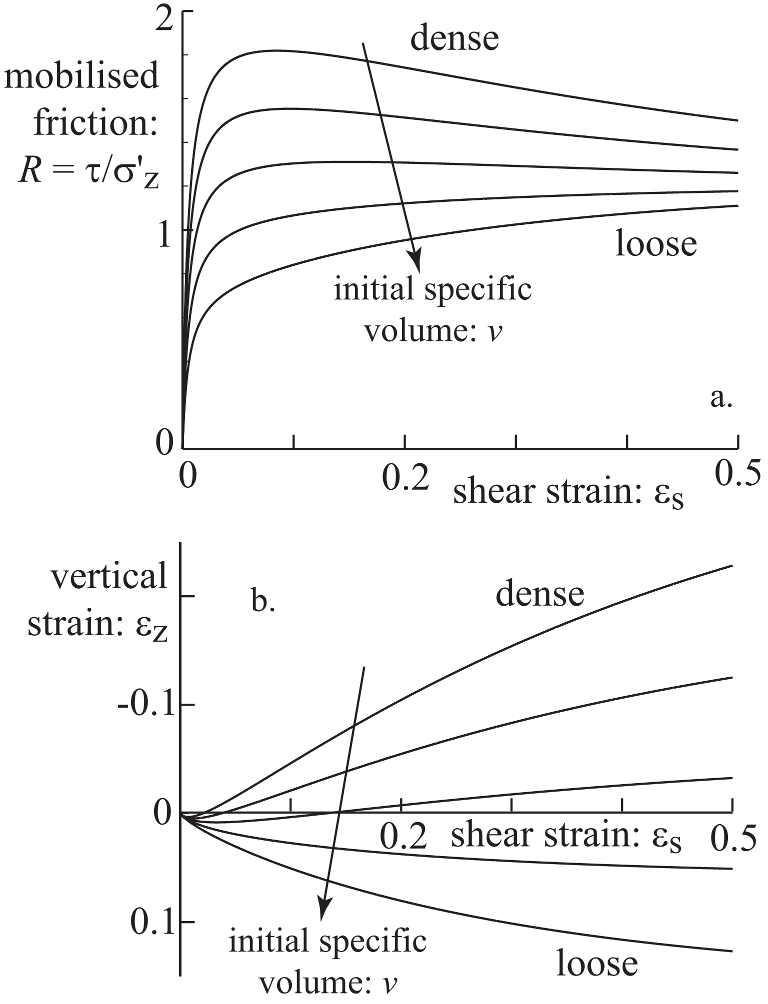
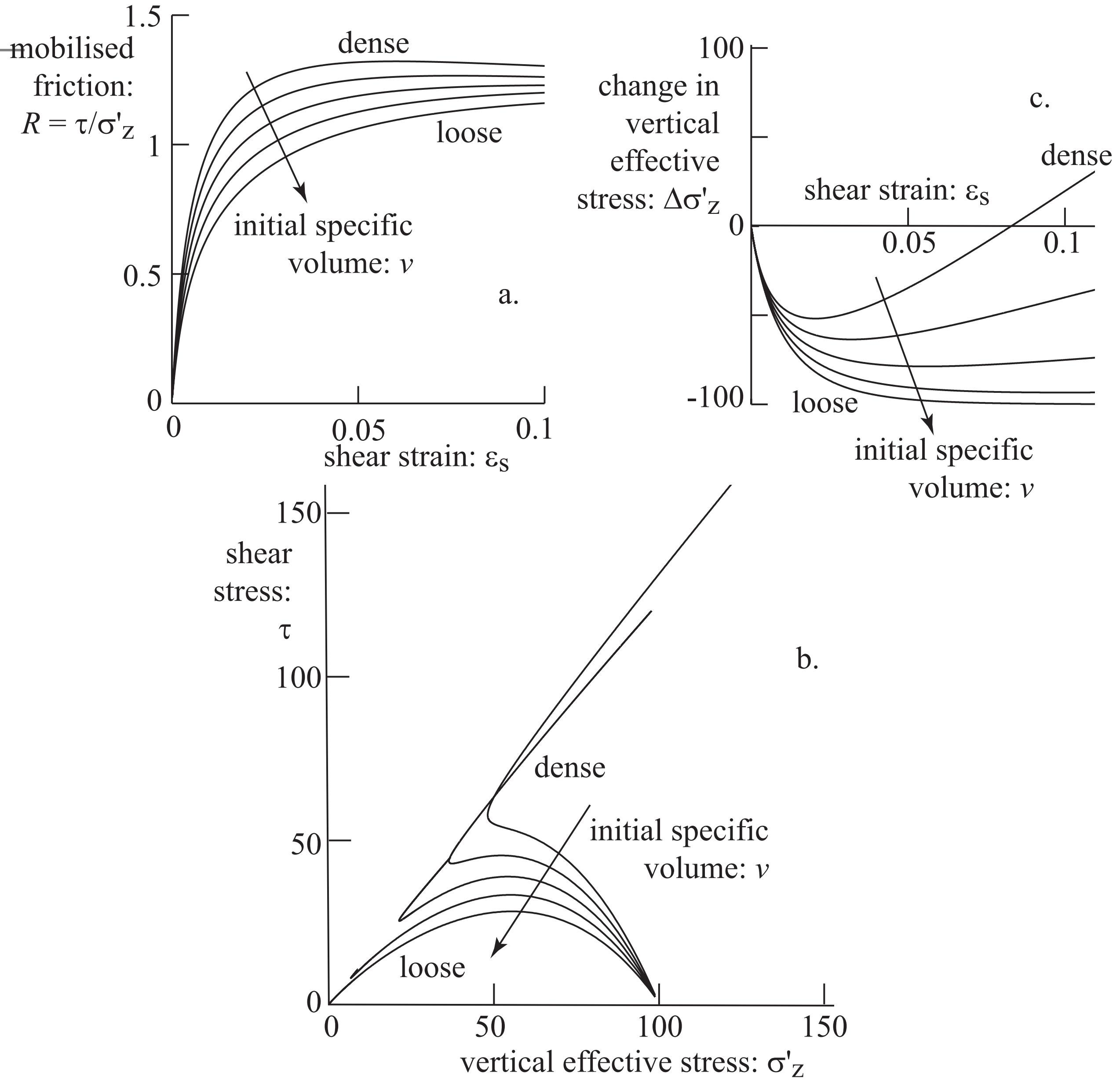
Simple model of shearing



- drained shearing with different initial densities

Simple model of shearing



- undrained shearing with different initial densities

Simple model of shearing

- one-dimensional model
- stiffness - strength - state parameter - dilatancy
- rich range of predicted responses

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