

# Advances in Numerical Modelling of Different Ground Improvement Techniques

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**ABSTRACT:** Common ground improvement techniques like deep vibration compaction, vibro replacement stone columns, dynamic compaction or vibratory rolling are still a subject of research. Research fields are for example optimized layouts or estimation of main influencing parameters. A lot of successful scientific research is conducted on piles and piling using various numerical methods. Therefore, it is assumed that numerical models can be used to improve ground improvement methods. In this contribution, different ground improvement techniques and numerical models to simulate the influence of these techniques on the surrounding soil are presented. Furthermore, optimization methods and potentials of ground improvement techniques are shown.

**KEYWORDS:** Ground improvement, Numerical modeling, Mathematical optimization, Deep vibration compaction, Vibro replacement stone columns, Dynamic compaction, Vibratory rollers

## 1. INTRODUCTION

Often soils present on construction sites are not suitable for structure loads. To increase the bearing capacity of those soils and to reduce settlements as well as the risk of liquefaction, ground improvement methods are used. Different techniques for ground improvement exist. These methods are suitable for different soils. Deep vibration compaction is widely used for noncohesive soils, whereas vibro replacement stone columns are utilized in soft, cohesive soils.

Due to the variety of existing soils and ground improvement techniques and a possibly large extent of the construction site it is necessary to choose the right ground improvement method and to plan an optimized layout during the design process of a construction. Scientific research works can provide methods and data to obtain main influencing parameters as well as optimized design layouts regarding for example costs or construction time.

For different fields of applications like piles, spudcans, dikes and banks numerical methods are used to investigate influencing parameters as well as bearing capacity and loading behavior (Qiu, 2012; Henke, 2013; Rudolph, 2015; Hamann, 2015). Therefore, it is assumed that ground improvement methods and the processes taking place in the soil during improvement can be analyzed using numerical simulations.

This paper analyzes four different ground improvement methods: deep vibration compaction, vibro replacement stone columns (VRSC), dynamic compaction and vibratory rollers. These methods are investigated with different techniques for numerical modelling: Finite Element Method (FEM) and Material Point Method (MPM). Additionally, a numerical optimization is carried out for the design of a VRSC layout.

With these simulations and additional mathematical optimization demonstrated exemplarily on the design of stone columns the potential of ground improvement methods is shown.

## 2. MODELLING TECHNIQUES AND NUMERICAL METHODS

Ground improvement methods involve large deformations of the soil. To analyse these methods using numerical simulations the chosen numerical methods have to be able to process large deformations. Furthermore, constitutive models, which are able to model the highly nonlinear behaviour of soils, are needed, and also analysis procedures, which can take the hydro-mechanical coupling into account, should be developed and applied.

### 2.1 Finite Element Method (FEM)

FEM involves two formulations to describe the movement of a continuum: The Eulerian and the Lagrangian formulation.

The continuum movement as a Lagrangian formulation is described as a function of time and material coordinates, which means that a FE-node is always fixed to the same material node. A precise identification of surfaces is possible. Mesh deformations may occur.

A function of time and spatial coordinates characterizes the movement of a continuum as part of an Eulerian formulation. The mesh is considered to be fixed. Each element can be filled either completely with material, contain no material at all (void) or be partially filled. Therefore, each element can contain more than one material. The Eulerian material moves through the mesh. Hence, no mesh distortion occurs.

One technique to model large deformations numerically is the Coupled Eulerian Lagrangian (CEL) approach. Here the advantages of both formulations of FEM are used. An object described with Lagrangian formulation penetrates into an Eulerian region. The contact formulation starts to act once the contact between Eulerian material and Lagrangian object is established.

### 2.2 Material Point Method (MPM)

The Material Point Method is an extension of the classic FEM and was originally developed as the Particle-In-Cell method for fluid dynamics (Harlow, 1964). An application of MPM to geotechnical problems was performed by Więckowski and can be found in Więckowski et al. (1999) and in Więckowski (2004). To simulate large deformations of saturated and partially saturated soil, a two- and three-phase formulation was discretized using the MPM (Jassim et al., 2013, Abe et al., 2014).

In the MPM each time step is calculated in the well-known frame of the Lagrangian FEM. In addition the mesh is reset to its initial position after each Lagrangian step. The deformed material is presented by so called material points which are following the deformation through the mesh and contain their position in contrast to the mesh after a time step. This approach allows to simulate large deformations without the drawbacks of integration issues due to mesh distortions.

For this paper the joint MPM code of the MPM Research Community is used, which is being developed by the University of Cambridge, UPC Barcelona, Hamburg University of Technology, University of Padova, Delft University of Technology and Deltares.

### 2.3 Mathematical Optimization

Mathematical optimization can be used to determine most efficient geotechnical designs and can be used in combination with numerical simulations. Applications of mathematical design optimizations are shown for a piled raft foundation (Kim et al., 1999; Grabe and Pucker, 2011), for a pile grillage (Čiegis et al., 2006; Kinzler, 2011) for a quay wall construction (Grabe et al., 2010) and for an underpinning (Seitz et al., 2015).

In order to use mathematical optimization, a numerical model has to be set up. Then it is possible to automatically determine the best parameter combination for a given optimization objective. A design optimization problem usually has several objectives, e.g. minimal costs, minimal construction time, minimal resource use and maximized ground improvement. Therefore, the optimization problem is multicriterial. Additionally the relation between the variable parameters and the optimization objective is usually nonlinear.

In order to solve nonlinear multicriteria optimization problems, heuristic optimization methods are suited such as simulated annealing (Kirkpatrick et al., 1983), tabu search (Glover, 1986), particle swarm optimization (Eberhart and Kennedy, 1995), ant colony optimization (Dorigo et al., 2006) and evolutionary algorithms (Deb, 2001).

This paper presents an application of an evolutionary algorithm to a multicriterial optimization problem, which is described in section 4.3. Evolutionary optimization is an iterative procedure. All the analyzed parameter sets in one iteration form a so called generation of individuals. The objectives, such as cost minimization, are evaluated and the best individuals are identified. The best individuals persist for the genesis of the next generation. Additionally recombination and mutation of arbitrarily chosen individuals complement the generation for the next iteration. The iterative procedure of an evolutionary algorithm is shown in Figure 1.

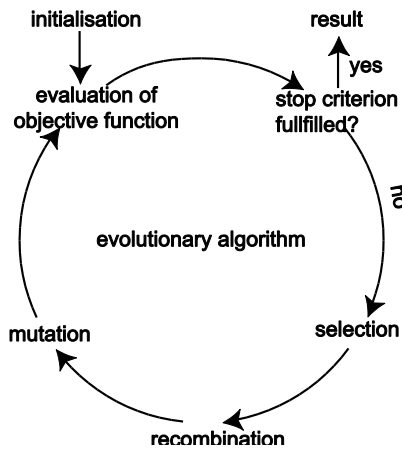


Figure 1 Iterative procedure of evolutionary multicriterial optimization algorithm (Heins et al, 2015b)

### 3. DEEP VIBRATION COMPACTION

Noncohesive soils will reach a higher relative density due to grain redistribution. This characteristic can be used for ground improvement (Witt, 2009).

For the deep vibration compaction method a deep vibrator consisting of a stay tube and a vibrator is used. The vibrator has a length of 2.0 to 4.5 m. Both tube and vibrator have a diameter of 0.3 to 0.5 m. The stay tube prolonged to its desired length is attached to a crane. The depth vibrator is installed to its designed depth. Eccentrically arranged masses inside the vibrator will cause oscillations of the vibrator and a horizontal displacement of the tip of the vibrator. These oscillations cause a horizontal force on the soil which leads to compaction of the soil. Usually a vibration frequency of 20 to 60 Hz is applied and a force of 150 to 700 kN as well as a horizontal displacement of the tip of the vibrator of 3.0 to 50.0 mm are caused (Fellin, 2000; Witt, 2009).

#### 3.1 Numerical Model

A three dimensional CEL-model, as shown in Figure 2 is used to simulate the deep vibration compaction method. The soil is modelled as

a cylinder with a height of 27.0 m and a radius of 15.0 m, where only the lower 25.0 m are filled with material to allow for soil displacement. Stress-strain behavior of the soil which is assumed to be “Mai-Liao-Sand” is described with the high class hypoplastic model according to von Wolffersdorff (1996) with the extension of intergranular strain by Niemunis and Herle (1997). Necessary material parameters taken from Qiu et al. (2011) are listed in Table 1 with additions to account for partially drained conditions for the soil as well as material parameters for hypoplast constitutive model for filling material. In addition to drained conditions of the soil partially drained conditions for the soil are considered. This is done using the approach from Hamann (2015).

The steel depth vibrator is modelled as a Lagrangian object with a diameter of 0.3 m. The vibrator has a length of 2.50 m. The stay tube is fixed for vertical and horizontal movement at its top. The depth vibrator is modelled 10.0 m preinstalled into the soil using the wished-in-place-technique. The oscillation of the vibrator is caused by a set horizontal displacement of 12.0 mm. The vibration frequency is varied. Hence, a displacement controlled simulation is conducted.

The friction between vibrator and soil is assumed to be negligible. Therefore, a frictionless contact definition is used. Further details about the numerical model can be found in Henke et al. (2012).

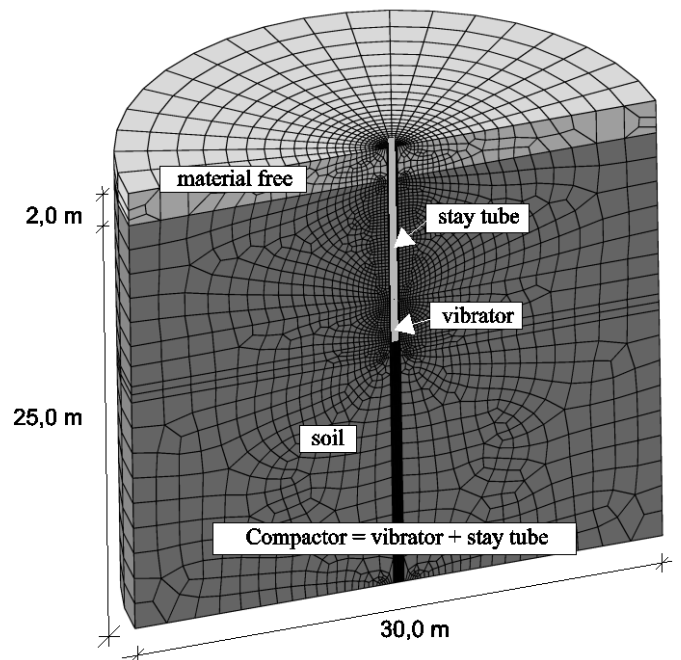


Figure 2 Vertical cut through the three dimensional CEL-model to simulate the deep vibration compaction method.

#### 3.2 Simulation Results

The results of numerical simulations can give information about influencing parameters and the mechanisms taken place in the soil during compaction. Different variations have been conducted to evaluate the influence of various parameters on the reached compaction of the soil. The two main influencing aspects found from research work yet are shown below. For further variations and details compare Henke et al. (2012) as well as Heins et al. (2015a).

Figure 3 shows the excess pore water pressures and the effective radial stresses occurring in numerical simulations of the deep vibration compaction method after a compaction duration of 3.0 s. It becomes obvious that liquefaction occurs. This has an effect on the reached relative density of the soil. Therefore, it is important to consider effects of pore water on the compaction process.

Table 1 Material parameter for hypoplastic constitutive model for Mai-Liao-Sand according to Qiu et al. (2011)

Parameter	Unit	Mai-Liao-Sand	Filling Material
$\phi_c$	[°]	31.5	36
$h_s$	[MPa]	32.0	32
$N$	[-]	0.324	0.18
$e_{d0}$	[-]	0.57	0.26
$e_{c0}$	[-]	1.04	0.45
$e_{i0}$	[-]	1.20	0.5
$\alpha$	[-]	0.4	0.1
$\beta$	[-]	1.0	0.18
$m_T$	[-]	2.0	2
$m_R$	[-]	5.0	5
$R$	[-]	0.0001	0.0001
$\beta_R$	[-]	0.5	0.5
$\chi$	[-]	6.0	6.0
$S$	[%]	99	-
$k_f$	[m/s]	$1 \cdot 10^{-4}$	-
$K_w$	[kPa]	$2.02 \cdot 10^4$	-
$K_s$	[kPa]	$3.7 \cdot 10^7$	-
$\rho_w$	[t/m <sup>3</sup> ]	1.0	-
$\rho_s$	[t/m <sup>3</sup> ]	2.65	-

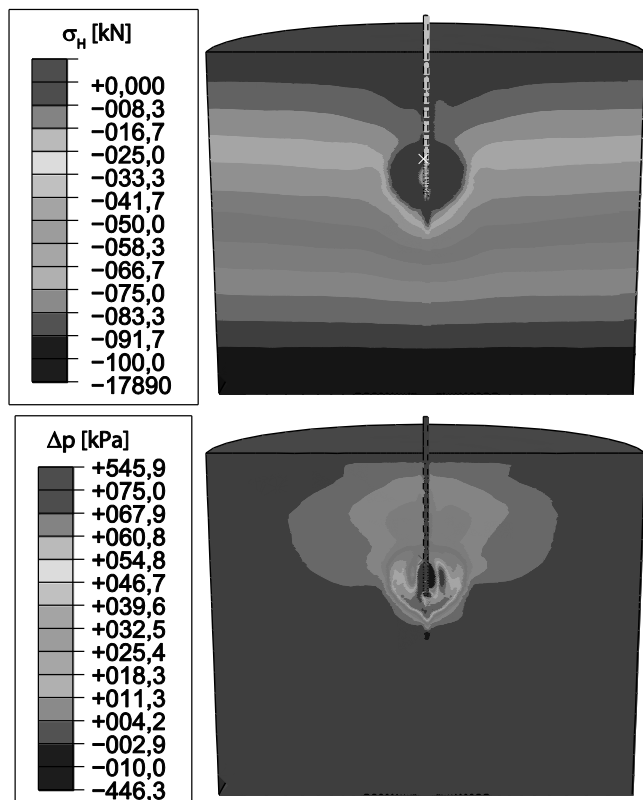


Figure 3 Excess pore water pressures (bottom) and effective radial stressed (top) of a partially drained analysis for deep vibration compaction according to Heins et al. (2015a)

During the compaction processes different horizontal forces, horizontal tip displacements as well as vibratory frequencies can occur or be applied. Varying all three of these parameters, the vibratory frequency is identified as the main influencing parameter.

Figure 4 depicts the void ratio change along a horizontal path at a depth half way down the depth vibrator. A relative density of at least 0.75 is assumed as a criterion for the impacted soil area. After a simulation duration of 3.0 s a vibration frequency of 30 Hz leads to an

impacted area with a radius of 1.72 m. In comparison a vibration frequency of 60 Hz influences only an area with a radius of 0.73 m.

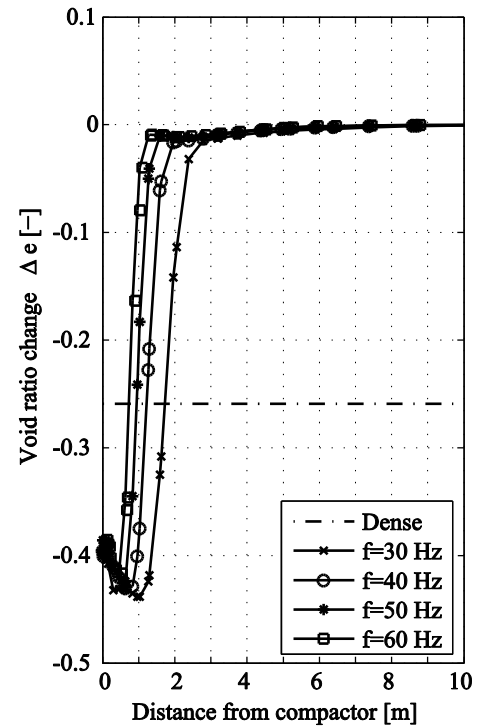


Figure 4 Void ratio change along a horizontal path halfway down the depth vibrator for a displacement controlled simulation with a horizontal displacement of the tip of the vibrator of 12 mm with varying vibration frequency according to Henke et al. (2012)

#### 4. VIBRO REPLACEMENT STONE COLUMNS

As described for deep vibration compaction a depth vibrator is installed for the vibro replacement method as well. The vibro replacement method is used for soft and cohesive soils. After installation to a desired depth, the depth vibrator is pulled upwards while at the same time a filling material is installed into the occurring cavity. Leveling of the depth vibrator causes a compaction of the filling material as well as a bracing of filling material and surrounding soft soil (Kirsch, 2004; Witt, 2009).

##### 4.1 Numerical Model

A three dimensional CEL-model as shown in Figure 5 is used to simulate the vibro replacement method. The soil is modelled as a cylinder with a height of 11.0 m and a radius of 12.0 m, where only the lower 10.0 m are filled with material to allow for soil displacement. Stress-strain behavior of the soft soil is described with the high class visco-hypoplastic model according to Niemunis (2003). Necessary material parameters are listed in Heins et al. (2015b). Drained conditions are assumed for the soft soil. The filling material is assumed to be "Hochstetten Kies", where the material parameters are taken from Herle (1997). The stress-strain behavior of the filling material is described with the high class hypoplastic model according to von Wolffersdorff (1996) with the extension of intergranular strain by Niemunis and Herle (1997). The steel depth vibrator is modelled as a Lagrangian object with a diameter of 0.3 m. The vibrator has a length of 2.70 m. The stay tube is fixed for horizontal movement at its top. The depth vibrator is modelled 1.0 m preinstalled into the soil using the wished-in-place-technique. The installation of the depth vibrator is modelled by pushing the depth vibrator and the filling material down for 0.5 m. Afterwards only the depth vibrator is pulled upwards 0.3 m. This is followed by levelling the depth vibrator and the filling material inside the vibrator for 0.2 m.

A friction contact definition including Coulomb's friction law with a friction coefficient of  $\mu = \tan(2/3\phi)$  is used. Further details about the whole numerical model can be found in Heins et al. (2015b). The values for the material parameters are listed in Table 1 and Table 2.

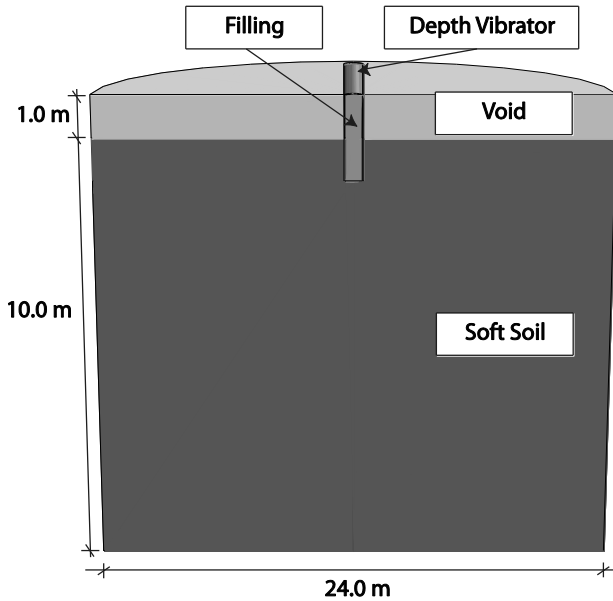


Figure 5 Vertical cut through the three dimensional CEL-model to simulate the vibro replacement method

Table 2 Material parameter for visco-hypoplastic constitutive model for the soft soil

Parameter	Unit	soft soil
$e_{por0}$	[°]	1,15
$C_c$	[MPa]	0,2
$C_s$	[-]	0,125
$\beta_s$	[-]	0,85
$I_v$	[-]	0,031
$D_r$	[-]	$1,00 \cdot 10^{-7}$
$\phi_c$	[-]	18,0
$m_T$	[-]	2,00
$m_R$	[-]	5,00
$R$	[-]	0,0001
$\beta_R$	[-]	0,50
$\chi$	[-]	1,00
$OCR$	[-]	1,00

#### 4.2 Simulation Results

Void ratio distribution and horizontal stresses of the soil can show the influence of the improvement method and vertical stone columns on the soil. Figure 6 shows the distribution of void ratio and horizontal stresses in the soil after installation of the depth vibrator exemplarily. A compaction at the shaft of the vibrator corresponding with a reduction of horizontal stresses as well as a stress increase below the depth vibrator occurs.

Furthermore, the formation of the base of a stone column can be seen in numerical simulations of the vibro replacement technique as described here.

#### 4.3 Mathematical Optimization

The mathematical optimization is applied to a problem that has already been analyzed in Heins et al (2015b). The analysis is extended by consideration of construction costs and a novel approach to the calculation of construction time. The underground conditions (see Figure 7) are approximated to the scientifically documented construction project for a highway embankment in Kuala Lumpur, see Kirsch

(2004). These soft soils make VRSC necessary for ground improvement. The construction site may be divided into subsections. For one of these subsections the optimization is conducted exemplarily. The soil of the corresponding 370 m<sup>2</sup> area is assumed to be loaded with  $\sigma = 136 \text{ kN/m}^2$  through the highway embankment.

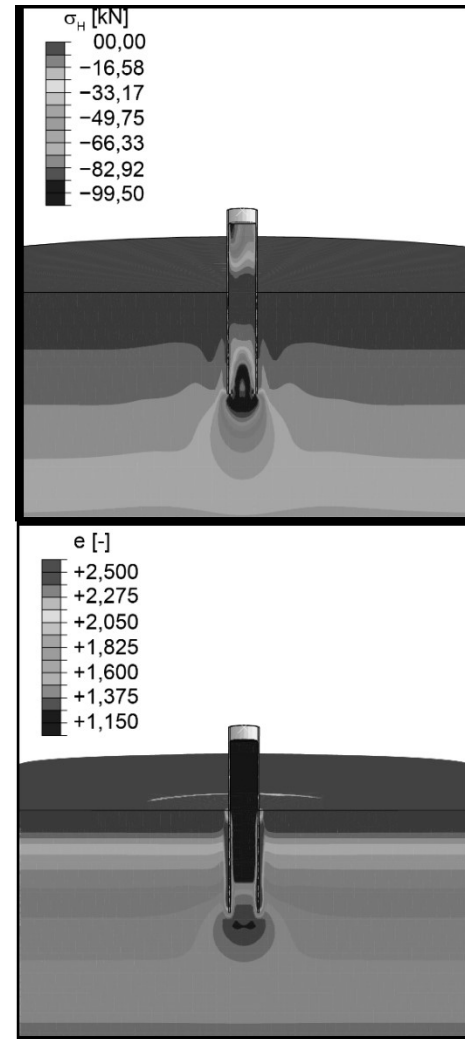


Figure 6 Distribution of void ratio (bottom) and horizontal stresses (top) in the soil after installation of the depth vibrator, according to Heins et al. (2015b)

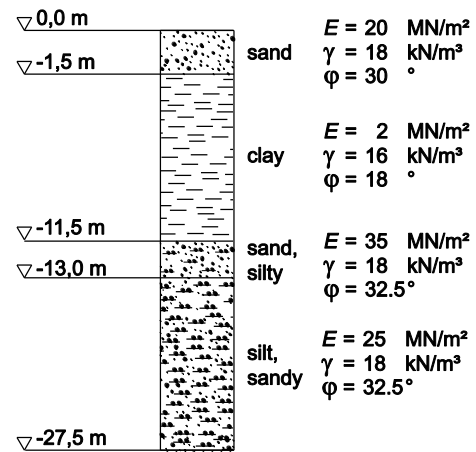


Figure 7 Underground conditions adapted from Kirsch (2004)

The design of the ground improvement measure will be optimized with respect to material demand, settlement, construction time and costs such that they become minimal. The objectives are evaluated as following:

- The settlements are calculated according to the theoretical design method presented in Priebe (1995). They depend on an improvement factor which is affected by the friction angle of the backfill material and the area ratio of the stone column and the affected soil. The improvement factor specifies how many times the compression module increases and therefore, how the settlements are reduced. The backfill is considered incompressible. For further details see Priebe (1995).
- The material demand is estimated according to the volume of the column.
- The construction time is approximated based on shift performance depending on the column diameter and length.
- The construction costs are linked to the material demand and the construction time. They may be approximated as a combination of the costs for the equipment and the material costs. The numerical model takes into account the following restrictions:
- Only designs with more than 4 columns in the construction field are considered in order to fulfill the group behavior assumption of Priebe (1995).
- For practicability the area ratio of the stone column and the affected surrounding soil shall be between 20 and 60 %, see Kirsch (2004). Other solutions will not be considered.
- An initial punch through settlement is considered for floating columns (Priebe, 2003).

Variable design parameters are: column diameter/column spacing, column length and material characteristics of the filling, which consists of gravel (friction angle, stiffness and unit weight). Table 3 summarizes the variable parameters and their possible range.

Table 3 Variable parameters for the VRSC design (Heins et al, 2015b)

variable parameter	range
column diameter $d$ [m]	$0.4 < d < 1$
column spacing $a$ [m]	$1 < a < 5$
column length $l$ [m]	$1.5 < l < 11.6$
gravel material characteristics:	$30 < \varphi' < 32.5$ $E = 40$ $\gamma = 16 \text{ kN/m}^3$
	$32.5 < \varphi' < 36$ $E = 90$ $\gamma = 17 \text{ kN/m}^3$
	$36 < \varphi' < 45$ $E = 120$ $\gamma = 21 \text{ kN/m}^3$
	$45$ $E = 120$ $\gamma = 21 \text{ kN/m}^3$

The results of the multicriterial optimization are shown in Figure 8. Each marker cross represents a parameter set for the VRSC design. Five hundred iterations with each 250 parameter sets are conducted – 125000 parameter combinations are tested. The red crosses mark the parameter sets in the last iteration step. In order to compare the results, a reference design and an optimized design are chosen marked by the blue and green circle respectively. The reference design has been chosen prior to optimization as possible VRSC layout in order to demonstrate the optimization possibilities.

The parameter sets of the reference design and the optimized design are summarized in Table 4. The improvement is evaluated in percentage of the reference design. The settlements can be reduced (30 %), while the material demand decreases as well (87.3 %). Consequently, the construction time is reduced (81.5 %) and the costs decrease (84.7 %). The VRSC design is mainly improved by varying the design column diameter and the column spacing.

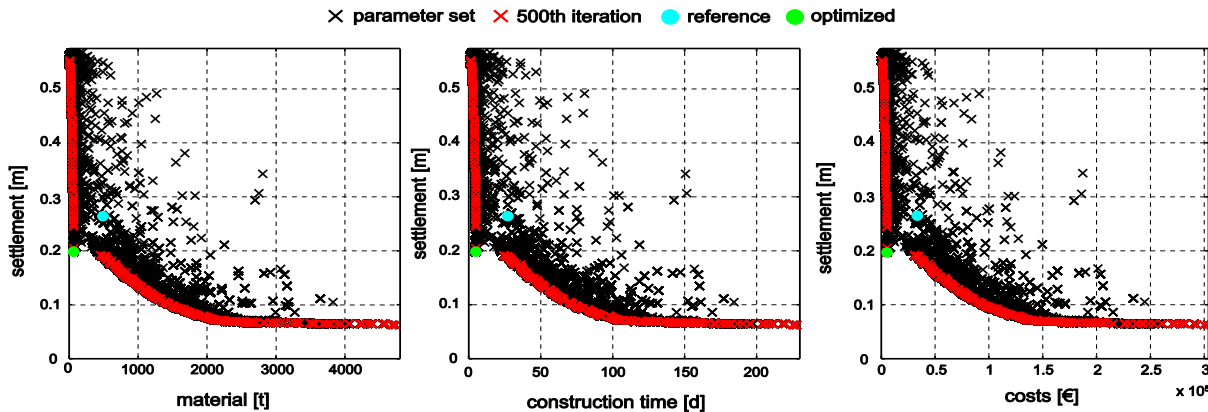


Figure 8 Results of multicriterial optimization with multiple objectives: reduction of settlements, material demand, costs and construction time; possible parameter sets are marked with crosses, the last generation is highlighted in red; the chosen reference design is marked with a blue circle and one optimized design is marked with a green circle

Table 4 Comparison of optimized design to reference design concerning their design parameters and optimization objectives (settlement, material demand, construction time, costs)

$d$ [m]	$l$ [m]	$a$ [m]	back fill material parameter	settlement [m]	material demand [m <sup>3</sup> ]	construction time [d]	costs [€]
0.7	11.5	2.5	$\varphi' = 40^\circ$ , $\gamma = 21 \text{ kN/m}^3$ , $E = 120 \text{ MN/m}^2$	0.26	495	27	33365.0
0.5	11.5	5.0	$\varphi' = 44^\circ$ , $\gamma = 21 \text{ kN/m}^3$ , $E = 120 \text{ MN/m}^2$	0.20	63	5	5121.7
improvement				30 %	87.3 %	81.5 %	84.7 %

## 5. DYNAMIC COMPACTION

For the simulation of the dynamic compaction method a two-phase formulation was used (Jassim, 2013) as the pore-water plays an important role during the deformation. In equation (1)  $w_j$ ,  $v_j$  represent the velocities of the soil and pore-water and  $p$  the pore-pressure. The system of coupled partial differential equations in equation (1) is discretized on the mesh shown in

Figure 9 and integrated explicitly by the Euler Cromer integration scheme. The soil behavior is modelled by the hypoplastic constitutive law (von Wolffersdorff, 1996). The parameters of the constitutive model are shown in Table 5.

$$\begin{aligned} \rho_w \frac{dw_j}{dt} &= \frac{dp}{\partial x_j} + \rho_w g_j - \frac{n \rho_w g}{k} (w_j - v_j) \\ (1-n) \rho_s \frac{dv_j}{dt} &= -n \rho_w \frac{dw_j}{dt} + \frac{\partial \sigma_{ij}}{\partial x_i} + \rho_{sat} g_j \\ \frac{dp}{dt} &= \frac{K_w}{n} + \left[ (1-n) \frac{dv_j}{dx_j} + n \frac{dw_j}{dx_j} \right] \end{aligned} \quad (1)$$

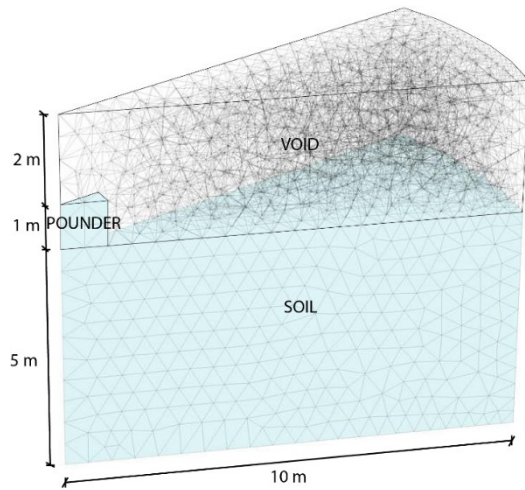


Figure 9 Computational mesh of the dynamic compaction model

Table 5 Parameters for the hypoplastic model

$h_s$	30°
$\varphi_c$	5,8 MPa
$n$	0,28
$e_{d0}$	0,53
$e_{c0}$	0,84
$e_{j0}$	1,0
$\alpha$	0,13
$\beta$	1,05

The poulder is normally made of steel and has a much higher stiffness than the soil. Therefore, we assume the poulder as a rigid body in our simulations. Additionally, the drop of the poulder is simulated by prescribing the velocity directly before the impact.

Similar to CEL a void domain of “empty elements” needs to be defined. This domain will allow the material points to deform by entering the void region. This technique allows to simulate large deformations which we can expect for the dynamic compaction as well. Assuming radial symmetry for a cylindrical poulder we only model a 30°-part of the whole three-dimensional problem. The density of the poulder was set to 10000 kg/m<sup>3</sup>. To capture the surface deformation after the impact we define the void domain only at the top of our model as shown in Figure 10. The applied boundary conditions at the

top and the bottom of the model allowing movement only in the horizontal plane. The rest of the boundaries is fixed in the normal direction. The boundaries of the void domain are treated as normal boundaries.

In Figure 10 the results of simulating one impact are shown. The void ratio distribution is one of the most interesting values to judge the soil improvement after compaction and is only accessible in numerical simulations. The initial void ratio was set to a value of 0.685. Directly under the poulder (orange) the void ratio is decreasing sharply to a value of 0.39. It can be observed that the compaction reaches to a depth of 1-2 m after one impact. Repeating dropping the poulder would increase the compaction effect.

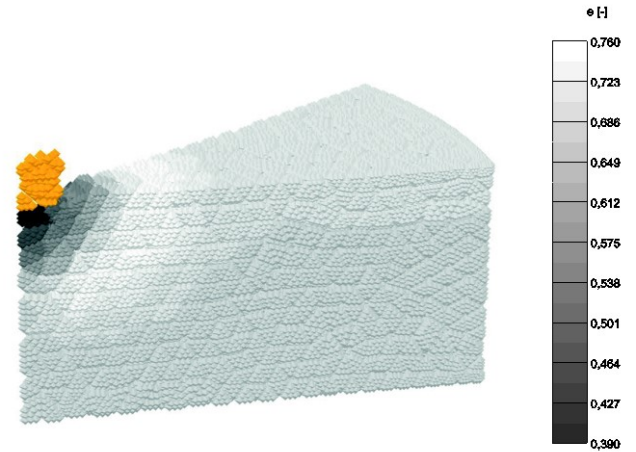


Figure 10 Void ratio distribution after one impact

## 6. VIBRATORY ROLLERS

A frequently used method for compaction in earthworks and pavement construction is the application of vibratory rollers. These construction vehicles are equipped with a vibrating drum to compact the soil layers. This method is especially effective for granular soils for which high compaction rates can be created with the help of cyclic shearing due to the vibrations.

Different mechanical phenomena characterize the soil-drum interaction in the near field of a vibratory roller. These are: the cyclically changing contact between drum and soil surface, the nonlinear material response with areas of compaction and ground loosening, the build-up of excess pore water pressures and soil liquefaction as well as different forms of dynamic wave propagation (compression, shear and different surface wave types). Further effects, influencing the soil stiffness and shear strength, are given due to capillary in the unsaturated zone at the soil surface. The combination of the named phenomena is responsible for the compactibility of the soil by means of vibratory rolling.

The impact of vibratory rollers on dry granular soil has been investigated numerically by Kelm (2003) who used a hypoplastic soil model (von Wolffersdorff, 1996, Niemunis und Herle, 1997) in dynamic FE-simulations. Starting from a static initial calculation step, where a  $K_0$ -stress state is prescribed, a vibratory roller, modelled as a rigid body, was placed on the soil surface. After application of its dead weight, a dynamic analysis step was performed in which several passages of the vibrating drum were simulated. The simulations allowed investigating the soil-drum interaction as well as the achieved compaction depth underneath the roller, see Figure 11.

By comparing his numerical results to measurements (e. g. measured spatial distributions of stress and acceleration by d'Appolonia, 1969), Kelm (2003) could demonstrate the ability of his FE-model in combination with Hypoplasticity as a soil model to simulate soil compaction by vibratory rollers in a realistic way. However, the model only considered soil as a one-phase-medium. The material parameters for the soil are listed in Table 6.



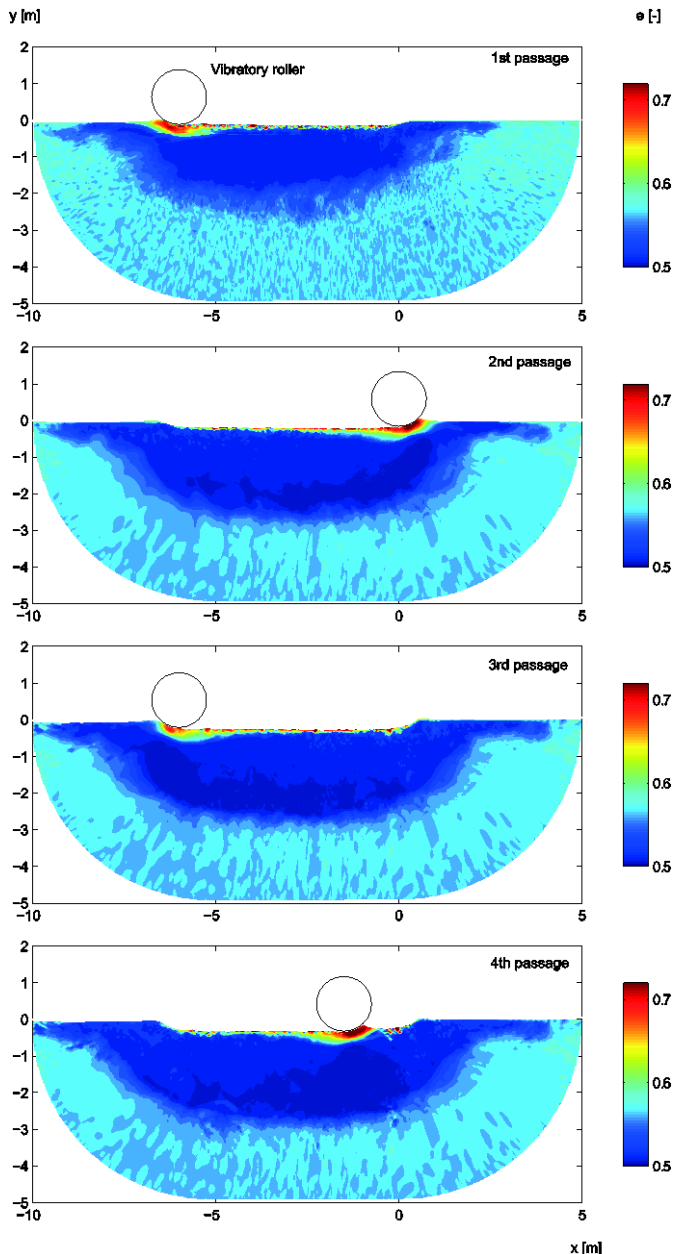


Figure 11 Calculated void ratio distributions after multiple passages of a vibratory roller on a dense sand half space in dynamic FE-analyses from Kelm (2003)

In order to consider unsaturated soil, the dynamic model by Schümann (2016) could be applied, who implemented a dynamic three-phase-model in the form of a user defined finite element which is based on a u-p-p-formulation. This model allows to include capillary effects as well as moisture transport in the unsaturated zone, where capillary pressure, influencing effective stress, consists of the difference of the pore air pressure  $p_a$  and the pore water pressure  $p_w$ , which are changed by deformations  $\mathbf{u}$  of the grain skeleton (Schümann, 2010).

Another interesting aspect could be seen in the implementation of control engineering features to the numerical models of vibratory compaction. With the help of suitable control functions, e. g. reacting to the soil stiffness derived from captured calculated accelerations, the compaction parameters, such as vibration frequency of the drum, could be optimized to warrant a good and homogeneous spatial compaction. This could be used to improve practical methods such as continuous compaction control.

Table 6 Material parameter for hypoplastic constitutive model used to model vibratory rollers

Parameter	Unit	Value
$\varphi_c$	[°]	30
$h_s$	[MPa]	5600
$N$	[-]	0.28
$e_{d0}$	[-]	0.53
$e_{c0}$	[-]	0.84
$e_{i0}$	[-]	1.00
$\alpha$	[-]	0.13
$\beta$	[-]	1.05
$m_T$	[-]	2
$m_R$	[-]	5
$R$	[-]	0.0001
$\beta_R$	[-]	0.5
$\chi$	[-]	6
$e_0$	[-]	0.6

## 7. CONCLUSION

The content and investigations of this paper shows the applicability of numerical methods to ground improvement techniques. Both utilized numerical methods (MPM as well as FEM) show realistic results which can be used to improve the method and the understanding of the processes taking place in the soil.

The potential of all introduced ground improvement techniques is shown with the conducted numerical simulations.

Furthermore the conduction of mathematical optimization can lead to a very efficient design for ground improvements. The example of a design for vibro replacement stone columns shown here leads to a reduction of settlements by 30 %, of 87.3 % less material demand, a reduction of construction time by 81.5 % and a reduction of the construction costs by 84.7 %. Obviously, this will have a huge impact on cost efficiency.

## 8. ACKNOWLEDGEMENT

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