Technical Paper by P.V. Long, D.T. Bergado and A.S. Balasubramaniam

STABILITY ANALYSIS OF REINFORCED AND UNREINFORCED EMBANKMENTS ON SOFT GROUND

ABSTRACT: General solutions are developed for the rotational stability analysis of an embankment with and without reinforcement constructed on soft ground that has an undrained shear strength varying with depth. The general cases of an embankment having a berm, dry or wet tensile cracks, and multiple layers of reinforcement are considered. The additional friction caused by the reinforcement force normal to the slip plane is incorporated into the solution. The solutions are presented in the form of simple equations. The relationship between the critical slip circles through the embankment with and without reinforcement is expressed explicitly. The analysis procedures and supporting graphs allow the user to obtain the solutions using hand computations. A simple computer program in the form of a worksheet was developed to quickly obtain the final results. Applications and illustrative examples of embankments with one- and two-step berms, and distributed loads are given. The results obtained using the approach presented in this paper compare very well to the solutions obtained from the method of slices using the computer programs SB-SLOPE and PC-STABL6.

KEYWORDS: Stability analysis, Limit equilibrium, Soft ground, Reinforced embankment, Unreinforced embankment.

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PUBLICATION: *Geosynthetics International* is published by the Industrial Fabrics Association International, 345 Cedar St., Suite 800, St. Paul, Minnesota 55101-1088, USA, Telephone: 1/612-222-2508, Telefax: 1/612-222-8215. *Geosynthetics International* is registered under ISSN 1072-6349.

DATES: Original manuscript received 17 May 1996, revised version received 21 October 1996 and accepted 22 October 1996. Discussion open until 1 July 1997.

REFERENCE: Long, P.V., Bergado, D.T. and Balasubramaniam, A.S., 1996, “Stability Analysis of Reinforced and Unreinforced Embankments on Soft Ground”, *Geosynthetics International*, Vol. 3, No. 5, pp. 583-604.

1 INTRODUCTION

Limit equilibrium analyses using the method of slices are widely used for solving slope stability problems. The main advantage of this method is that a solution can be obtained for an irregular slip surface with complicated cases of boundary load, embankment configuration, and nonhomogeneous foundation soil. However, this method requires a large amount of computation and an arbitrary iteration process for obtaining the overall minimum factor of safety. Some well known slope stability computer programs using limit equilibrium method of slices are available such as, SB-SLOPE (Von Gunten Software Inc.) and PC-STABL6 (U.S Department of Transportation 1986). However, these programs are based on the simplified Bishop method (Bishop 1955), and usually require an experienced user for reliable solutions. Also, some characteristics of embankments on soft clay are not fully incorporated into the programs. The SB-SLOPE program can only be used for rotational stability analysis of unreinforced embankments without wet tensile cracks. The PC-STABL6 program incorporates the simplified Bishop method (Bishop 1955) with modifications to consider the effect of reinforcement layers that provide a resisting moment. PC-STABL6 can be applied to both reinforced and unreinforced embankments but not to embankments with tensile cracks. Moreover, the reinforcement force incorporated into the program is a free body force, and thus, the increase of the embankment soil strength provided by the reinforcement force perpendicular to the slip surface is included.

Embankments constructed on soft ground are typically not very high and are often built quickly. Conventionally, to determine embankment stability at the end of construction, a total stress analysis assuming a circular slip surface is carried out. The factor of safety of a reinforced embankment can be obtained from the moment equilibrium of the circular sliding block considering the additional resisting moment provided by the reinforcement (Milligan and Busbridge 1983; Kaniraj 1994; Holtz et al. 1995).

Based on the moment equilibrium of the circular slip block of unreinforced embankments without a berm, Low (1989) has shown that the factor of safety can be expressed as a function of the coordinates, x_C and y_C , of the center of the slip circle. Partial differentiation of the factor of safety expression with respect to x_C and y_C led to the solutions for the location of the critical slip circle and the minimum factor of safety, for a given limiting tangent, in the form of simple equations. The overall minimum factor of safety can be calculated directly by considering different limiting tangents.

The work of Low (1989) was extended by Kaniraj (1994) for the cases of unreinforced and reinforced embankments having a berm and a dry tensile crack. An excavation outside the berm was also included in the study by Kaniraj; however, the solution is valid only if the entire excavation is located inside the slip circle. For reinforced embankments, three separate solutions are presented by Kaniraj for three distinct cases of the mobilized reinforcement force direction, namely: the horizontal, bisectional, and tangential directions. Also, only one layer of reinforcement is considered and the additional friction caused by the reinforcement force normal to the slip surface was not accounted for.

Based on the unreinforced embankment studies performed by Low and Kaniraj, the solutions for the general case of an embankment having a berm, a dry or wet tensile crack, and multiple layers of reinforcement are developed further in this paper. The generalized solution for embankments with and without reinforcement are derived. The solution for unreinforced embankments is obtained by assuming a reinforced embankment with a reinforcement force value of zero. For reinforced embankments, any orientation of the reinforcement force, from the horizontal to the tangential direction of the slip circle, is applicable. The additional friction caused by the reinforcement force normal to the slip surface is also included. Embankments with a two-step berm and/or a uniform distributed load acting on the embankment crest can be applied to the generalized solution. The solutions are presented in the form of simple equations that can be easily used to produce tables or graphs by hand calculation. The relationship between critical slip circles through embankments with and without reinforcement is expressed explicitly. Analysis procedures and supporting graphs allow the user to obtain the solution using hand computations. Furthermore, a small computer program requiring simple data entry in the form of a worksheet has been developed to quickly obtain the final results.

The general embankment on soft clay configuration to be analyzed is illustrated in Figure 1. The method of analysis and systematic derivations are given in the following sections. The applications are extended to derive solutions, analysis procedures, and illustrative examples for the case of an embankment having a two-step berm (Figure 2) and distributed load. Finally, comparisons are made between the results obtained in this study and the method of slices using the computer programs PC-STABL6 and SB-SLOPE.

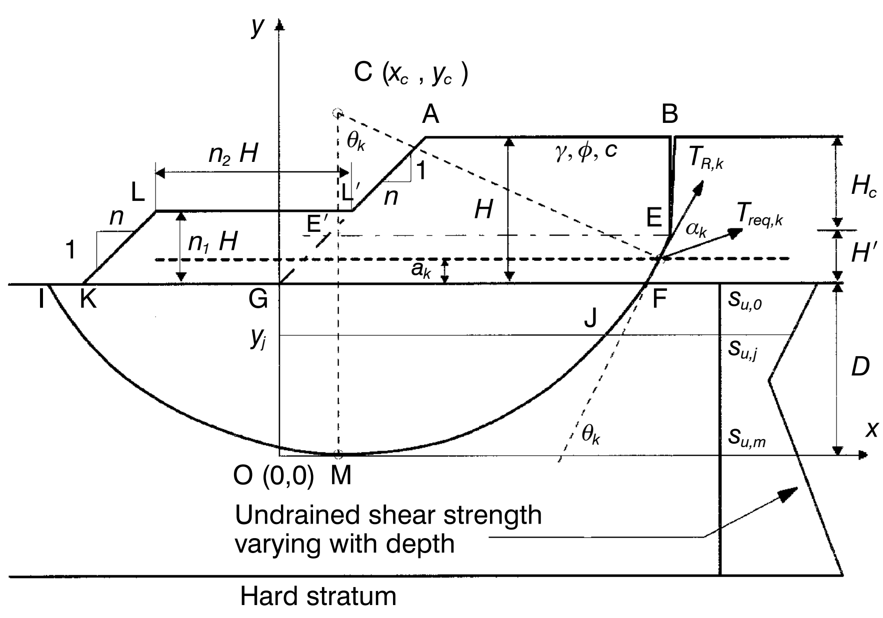


Figure 1. General configuration of embankment on soft ground.

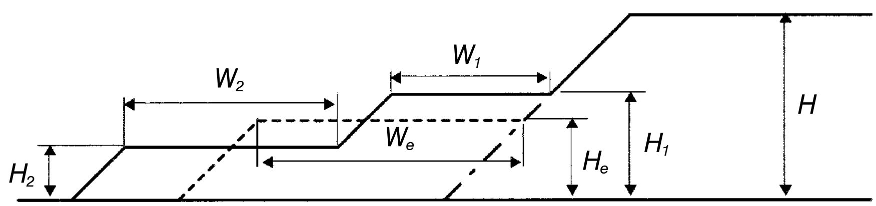


Figure 2. A two-step berm embankment and its equivalent one-step berm embankment.

2 METHOD OF ANALYSIS

2.1 General

Figure 1 shows an arbitrary slip circle tangential to a limiting tangent at depth D. The slip circle encloses a berm, and terminates at the bottom of a vertical, wet or dry crack of height H_c. The terminal point B is assumed to lie within the embankment crest. The embankment has a total height of H, a berm height of n_1H, a berm width of n_2H, and a side slope of n horizontal to 1 vertical. Multiple layers of reinforcement are assumed to be placed at heights a_k above the ground surface and within the uncracked zone of height, H'. The origin of the axes, (x,y), is assumed to be at the level of the limiting tangent, on the vertical line passing through the toe (point G) of the embankment. The coordinates of the center of the slip circle are denoted (x_c, y_c).

2.2 Equilibrium Condition and Definition of the Factor of Safety

The factor of safety, FS, is defined based on the moment equilibrium of the circular slip block IMFEB in Figure 1 as follows:

$$FS = \frac{M_R}{M_O} \tag{1}$$

The overturning moment, M_O, and resisting moment, M_R, corresponding to Figure 1, are expressed as follows:

$$M_O = M_{OE} - M_{OB} + M_{OC} + M_{OW} \tag{2}$$

$$M_R = M_{RR} - M_{RF} + M_{RE} \tag{3}$$

where: M_{OE} = overturning moment caused by the embankment within the uncracked zone GFEE$'$G; M_{OB} = overturning moment caused by the berm KLL$'$GK; M_{OC} = overturning moment caused by the cracked zone ABEE$'$A; M_{OW} = overturning moment caused by the water pressure in the wet crack; M_{RR} = resisting moment caused by the reinforcement; M_{RF} = resisting moment caused by the foundation soils; and M_{RE} = resisting moment caused by the embankment fill within the uncracked zone.

2.3 Overturning Moment

The overturning moment caused by the embankment fill (Low 1989) is re-written for the zone of no tensile crack, GFEE′G, as:

$$M_{OE} = \frac{\gamma H'}{2}\left[-x_c^{\,2} + nH'x_c + 2y_c\left(D + \frac{H'}{2}\right) - \frac{H'^2}{3}\left(n^2 + \frac{1}{4}\right) - \left(D + \frac{H'}{2}\right)^2\right] \quad (4)$$

where: γ = total unit weight of the embankment fill; H' = height of the uncracked zone of the embankment; x_c = x-coordinate of the center of the slip circle; n = cotangent of the embankment side slope angle; y_c = radius of the slip circle through the reinforced embankment; and D = tangent depth of the slip surface (Figure 1).

The overturning moment caused by the crack zone ABEE′A is presented by Kaniraj (1994) as:

$$\begin{aligned} M_{OC} = {} & \gamma H_c[-3x_c^2 + 3nx_c(H + H') + 6y_c(D + H') - 3(D + H'^2) \\ & - n^2(H^2 + H'^2 + HH')]/6 \end{aligned} \quad (5)$$

The moment caused by the berm KLL′GK is obtained from Figure 1 as:

$$M_{OB} = n_1 n_2 \gamma H^2[0.5(n_2 - n_1 n)H + x_c] \quad (6)$$

The overturning moment caused by water pressure in the tensile crack is expressed as:

$$M_{OW} = 0.5\gamma_w H_c^2[y_c - (D + H' + H_c/3)] \quad (7)$$

where γ_w equals the unit weight of water for a wet tensile crack (= 0 for a dry tensile crack).

2.4 Resisting Moment

The resisting moment provided by the reinforcement, assuming the reinforcement is placed within the uncracked zone, is calculated as follows:

$$M_{RR} = T_R y_c \quad (8)$$

where T_R is the sum of resulting reinforcement forces in the tangential direction to the slip circle, and is calculated using the following equation:

$$T_R = \sum T_{R,k} \tag{9a}$$

$$T_{R,k} = T_{req,k}\,(\cos\alpha_k + C_f \sin\alpha_k \tan\phi) \tag{9b}$$

where: $T_{R,k}$ = resulting force in the tangential direction provided by the reinforcement layer k; $T_{req,k}$ = required tensile force that is equal to the product of the factor of safety of the reinforced embankment, FS, and the mobilized tensile force in the reinforcement layer k; α_k = angle between the required reinforcement tensile force, $T_{req,k}$, and the resulting reinforcement force, T_R , for reinforcement layer k in an equilibrium state; C_f = coefficient of the behavior of the reinforcement force; and ϕ = total stress friction angle of embankment fill. In Equation 9b, the value of C_f can be equal to 0 or 1 when assuming that the reinforcement force is a free body force, or a force modifying the embankment soil strength, respectively (Bonaparte and Christopher 1987). It is noted that the reinforcement tensile force, $T_{req,k}$, can be divided into two components, namely: the normal component, $T_{req,k} \sin\alpha_k$; and the tangential component, $T_{req,k} \cos\alpha_k$. For a frictional embankment fill with multiple layers of reinforcement, the normal component of the reinforcement force will yield an additional frictional resistance (modifying the embankment soil strength), $T_{req,k}\sin\alpha_k\tan\phi$, that can be included in Equation 9b by setting the value of C_f to 1.0. However, for a single layer of reinforcement placed directly on the soft ground, some of the reinforcement force is transmitted into the soft foundation and is not available to strengthen the embankment fill. Therefore, the reinforcement tensile force should be conservatively modelled as a free body force that does not effect the strength of the embankment fill.

The resisting moments, M_{RF} and M_{RE} , are given by Low (1989) and Kaniraj (1994), respectively, as follows:

$$M_{RF} = 3.06 s_u D^{0.53} y_c^{1.47} \tag{10}$$

$$M_{RE} = 1.53(c + \gamma H\lambda \tan\phi)[(D + H')^{0.53} - D^{0.53}]y_c^{1.47} \tag{11}$$

where: s_u = undrained shear strength; c = cohesion of embankment fill; and λ = averaging coefficient for the frictional stress in the embankment given by Low (1989) as:

$$\lambda = 0.19 + 0.02n/(D/H) \qquad \text{for } D/H \geq 0.5 \tag{12}$$

By substituting M_{RR} , M_{RF} , and M_{RE} from Equations 8, 10, and 11 into Equation 3, and simplifying and rearranging, the following expression for M_R is obtained:

$$M_R = \gamma H(S_F K_F + S_E K_E + S_R K_E) y_c^{1.47} \tag{13}$$

where:

$$S_F = \frac{s_u}{\gamma H} \tag{14}$$

$$S_E = \frac{c}{\gamma H} + \lambda \tan \phi \tag{15}$$

$$S_R = \frac{T_R}{\gamma H K_E y_c^{0.47}} \tag{16}$$

$$K_F = 3.06 D^{0.53} \tag{17}$$

$$K_E = 1.53[(D + \beta H)^{0.53} - D^{0.53}] \tag{18}$$

$$\beta = H'/H = 1 - (H_c/H) \tag{19}$$

In Equation 14, s_u is the constant undrained shear strength for a uniform foundation soil. If the the undrained shear strength varies with depth, the equivalent undrained shear strength of the foundation soil along the slip circle, corresponding to the limiting tangent, D, is calculated using the following equation:

$$s_u = \frac{1}{2D^{0.53}} \sum_{j=1}^{n-1} (y_j^{0.53} - y_{j+1}^{0.53})(s_{u,j} + s_{u,j+1}) \tag{20}$$

where y_j and $s_{u,j}$ are the y-coordinate and undrained shear strength, respectively, at the top of soil layer j (Figure 1).

2.5 Location of the Critical Slip Surface

From the expressions for M_O and M_R (Equations 2 and 13, respectively), the factor of safety, FS, for a given limiting tangent is expressed as a function of x_c and y_c as follows:

$$FS = f(x_c, y_c) = M_R/M_O \tag{21}$$

The location of the critical slip surface is obtained by performing partial differentiation of Equation 21 with respect to x_c and y_c and equating them to zero. The following equation is obtained by differentiating Equation 21 with respect to x_c and equating to zero:

$$\frac{\partial M_R}{\partial x_c} M_O - \frac{\partial M_O}{\partial x_c} M_R = 0 \tag{22}$$

As seen in Equation 13, M_R is independent of x_c . Thus, Equation 22 yields:

$$\frac{\partial M_O}{\partial x_c} = 0 \tag{23}$$

By substituting M_O from Equation 2 into Equation 23, the x-coordinate of the critical circle center, x_c , is obtained using the following equation:

$$\frac{x_c}{H} = \frac{n}{2} - n_1 n_2 \tag{24}$$

By substituting x_c from Equation 24 into Equations 4, 5, 6, 7, and 2, and simplifying and rearranging, the overturning moment is expressed as a linear function of y_c :

$$M_O = \gamma H(Ay_c + B) \tag{25}$$

where:

$$A = H(A_o + A_w) \tag{26}$$

$$B = -0.5H^2(B_o + B_b + B_w) \tag{27}$$

$$A_o = \frac{D}{H} - \frac{\beta^2}{2} + \beta \tag{28}$$

$$A_w = \frac{\gamma_w}{2\gamma}(1 - \beta)^2 \tag{29}$$

$$B_o = \beta^2\left(1 - \frac{D}{H} - \frac{2}{3}\beta\right) + 2\beta\left(\frac{D}{H}\right) + \left(\frac{D}{H}\right)^2 + \frac{n^2}{12} \tag{30}$$

$$B_b = n_1 n_2(1 - n_1)(n + n_2) \tag{31}$$

$$B_w = \frac{\gamma_w}{\gamma}(1 - \beta)^2\left(\frac{D}{H} + \frac{1}{3}(2\beta + 1)\right) \tag{32}$$

The following equation is obtained by performing a partial differentiation of $f(x_c , y_c)$ on Equation 21 with respect to y_c and equating to zero:

$$\frac{\partial M_R}{\partial y_c}M_O - \frac{\partial M_O}{\partial y_c}M_R = 0 \tag{33}$$

Equation 25 gives:

$$\frac{\partial M_O}{\partial y_c} = A\gamma H \tag{34}$$

Equations 13 and 16 yield:

$$\frac{\partial M_R}{\partial y_c} = \gamma H\left[1.47(S_F K_F + S_E K_E) + S_R K_E\right] y_c^{0.47} \tag{35}$$

The following equation is obtained by substituting M_R and M_O from Equations 13 and 25 and terms from Equations 34 and 35 into Equation 33, and simplifying and rearranging:

$$y_c = -3.13\frac{B}{A}\left(1 + 0.68\frac{S_R K_E}{S_F K_F + S_E K_E}\right) \tag{36}$$

Equations 24 and 36 are general expressions for the critical slip surface through an embankment on soft clay with and without reinforcement. Without reinforcement, the normalized reinforcement strength, S_R , in Equation 36 is equal to zero, thus the coordinate of the critical slip circle for the unreinforced embankment, y_0 , is derived from Equation 36 as:

$$y_o = -3.13\frac{B}{A} \tag{37}$$

2.6 Critical Slip Circle and Factor of Safety of an Unreinforced Embankment

2.6.1 *Critical Slip Circle*

Substituting A and B from Equations 26 and 27 into Equation 37, gives the following equation:

$$\frac{y_o}{H} = C_1 + \beta_n C_2 \tag{38}$$

where:

$$\beta_n = n^2/12 + n_1 n_2 (1 - n_1)(n - n_2) \tag{39}$$

$$C_1 = (B_o + B_w - n^2/12)C_2 \tag{40}$$

$$C_2 = 1.565/(A_o + A_w) \tag{41}$$

The constants C_1 and C_2 are written explicitly with the subscripts D and W for the cases of dry and wet tensile cracks, respectively, as demonstrated in the following sections.

Dry Tensile Crack Using $C_1 = C_{1D}$ and $C_2 = C_{2D}$. Substituting A_o, A_w, B_o and B_w from Equations 28 to 31 into Equations 40 and 41 and using $\gamma_w = 0$, the equations for C_{1D} and C_{2D} are derived as follows:

$$C_{1D} = \left[\beta^2\left(1 - \frac{D}{H} - \frac{2}{3}\beta\right) + 2\beta\frac{D}{H} + \left(\frac{D}{H}\right)^2\right]C_{2D} \qquad (42)$$

$$C_{2D} = \frac{1.565}{\left(\frac{D}{H} - \frac{\beta^2}{2} + \beta\right)} \qquad (43)$$

Wet Tensile Crack Using $C_1 = C_{1W}$ and $C_2 = C_{2W}$. Assuming $\gamma_w = 0.5\gamma$, Equations 40 and 41 lead to:

$$C_{1W} = \left[\beta^2\left(\frac{7}{6} - \frac{1}{2}\frac{D}{H} - \beta\right) + \left(\beta + \frac{1}{2}\right)\frac{D}{H} + \left(\frac{D}{H}\right)^2 + \frac{1}{6}\right]C_{2W} \qquad (44)$$

$$C_{2W} = \frac{1.565}{\left(\frac{D}{H} - \frac{\beta^2}{4} + \frac{\beta}{2} + \frac{1}{4}\right)} \qquad (45)$$

2.6.2 *Factor of Safety*

The factor of safety is obtained from Equations 1 and 33 as follows:

$$FS = \frac{\dfrac{\partial M_R}{\partial y_c}}{\dfrac{\partial M_O}{\partial y_c}} \qquad (46)$$

Substituting Equations 34 and 35 into Equation 46, using $S_R = 0$ and $y_c = y_o$ for the case of no reinforcement, and substituting from Equations 17, 18, 26 and 41, rearranging and simplifying, the factor of safety of the unreinforced embankment, FS_0, is derived as:

$$FS_o = 2.874C_2(S_F + C_3S_E)\left(\frac{D}{H}\right)^{0.53}\left(\frac{y_o}{H}\right)^{0.47} \qquad (47)$$

where:

$$C_3 = 0.5\left[\left(1 + \frac{\beta}{\frac{D}{H}}\right)^{0.53} - 1\right] \tag{48}$$

2.7 Critical Slip Surface and Required Tensile Force for a Reinforced Embankment

2.7.1 *Relationship Between Critical Slip Circles of Reinforced and Unreinforced Embankments*

For a given limiting tangent, *D*, let *RF* be the ratio, FS/FS_o , where *FS* and FS_o are the factors of safety of the corresponding reinforced and unreinforced embankments, respectively, then the following equation is derived from Equations 1, 13 and 25:

$$RF = \left(1 + \frac{S_R K_E}{S_F K_F + S_E K_E}\right)\left(\frac{y_c}{y_o}\right)^{1.47} \frac{A y_o + B}{A y_c + B} \tag{49}$$

Combining Equations 36, 37 and 49 gives the following:

$$\frac{y_c}{y_o} = RF^{0.68} \tag{50}$$

The value of the radius ratio, y_c/y_o , of the critical slip circles with and without reinforcement increases as a power function with increasing factor of safety ratio values, *RF*. Equation 50 implies that higher reinforcement forces yield larger critical slip surfaces for a limiting tangent.

2.7.2 *Required Resulting Force of the Reinforcement, T_R*

From Equations 36, 37 and 50, the normalized reinforcement strength, S_R , is written in the following form:

$$S_R = \frac{S_F K_F + S_E K_E}{0.68 K_E}\left(RF^{0.68} - 1\right) \tag{51}$$

The normalized resulting reinforcement force is obtained by substituting S_R from Equation 16 into Equation 51, substituting other terms from Equations 17, 18, and 47, and then rearranging and simplifying to give:

$$\frac{T_R}{\gamma H^2} = 1.565 \frac{FS_o}{C_2}\left(RF - RF^{0.32}\right) \tag{52}$$

Equation 52 indicates that the required resulting reinforcement force in the tangential direction, T_R, is normalized as a function of the factor of safety ratio, *RF*, and the factor of safety, FS_0, of the corresponding unreinforced embankment, for a given limiting tangent.

2.7.3 *Required Tensile Force of the Reinforcement,* T_{req}

From Equations 52 and 9, the resulting force for any reinforcement layer *k*, $T_{R,k}$, can be calculated. Then, the required tensile force, $T_{req,k}$, that is equal to the product of the mobilized tensile force in the reinforcement and the design factor of safety, *FS*, of the reinforced embankment can be calculated if the inclination angles, α_k, in the equilibrium state are known. The values of α_k are often calculated from the assumed values of the inclination factor, $I_{f,k}$, and are defined as:

$$I_{f,k} = 1 - \alpha_k/\theta_k \tag{53}$$

where θ_k is the angle between the horizontal and the tangential directions at the intersection of reinforcement layer *k* and the slip surface.
From Equation 53 and Figure 1, the following equation can be derived:

$$\alpha_k = (1 - I_{f,k})\cos^{-1}\left(1 - \frac{D + a_k}{y_c}\right) \tag{54}$$

The required tensile force for any reinforcement layer *k*, $T_{req,k}$, is derived from Equation 9b as follows:

$$T_{req,k} = \frac{T_{R,k}}{(\cos\alpha_k + C_f \sin\alpha_k \tan\phi)} \tag{55}$$

3 APPLICATIONS

3.1 Two-Step Berm Embankment

A two-step berm embankment can be converted to an equivalent one-step berm embankment as shown in Figure 2. By equating the moments about the *y*-axis caused by the berms, the dimensions of the equivalent one-step berm are obtained as follows:

$$H_e = \sqrt{\delta'} - \frac{x_e}{n} \tag{56}$$

$$W_e = 2x_e + nH_e \tag{57}$$

where H_e and W_e are the height and width of the equivalent one-step berm, respectively, and the other parameters are defined as follows:

$$x_e = \frac{W_1 H_1 (W_1 - nH_1) + W_2 H_2 (2W_1 + W_2 - nH_2)}{2(H_1 W_1 + H_2 W_2)} \tag{58}$$

$$\delta' = \left(\frac{x_e}{n}\right)^2 + \frac{H_1 W_1 + H_2 W_2}{n} \tag{59}$$

3.2 Embankment With a Uniform Distributed Load

The uniform distributed load, q, acting on the crest of the embankment is approximated by an additional crack height, $\Delta H_c = q/\gamma$. The solution is then obtained as an equivalent embankment with no load, having an embankment height, H^*, and crack height, H_c^*, as follows:

$$H^* = H + \Delta H_c \tag{60a}$$

$$H_c^* = H_c + \Delta H_c \tag{60b}$$

3.3 Analysis Procedure

3.3.1 *Hand Computation*

The procedure for performing a rotational stability analysis using the solution presented in this paper is summarized in the following steps.

Step 1. From the embankment configuration (Figures 1 or 2), determine the following values: the geometry factors n, n_1 and n_2 ; the slope factor, β_n (Equation 39); and, the tensile crack factor, $\beta = H'/H$.

Step 2. For a trial tangent depth, D, use Figures 3a or 3b to obtain the values of C_1 , C_2 , and C_3 for dry or wet tensile cracks, respectively. Use Equations 14 and 15 to calculate the normalized strength parameters S_F and S_E .

Step 3. Use Equations 38 and 47 to calculate the normalized radius, y_o/H, and the factor of safety, FS_o , respectively, of the unreinforced embankment.

Step 4. For the desired factor of safety, FS, of the reinforced embankment, use Equation 52 to compute the required resulting force in the tangential direction, T_R .

Step 5. Repeat Steps 2 to 4 for several trial depths. Draw the relation of FS_o versus D and T_R versus D to obtain the overall minimum factor of safety of the unreinforced

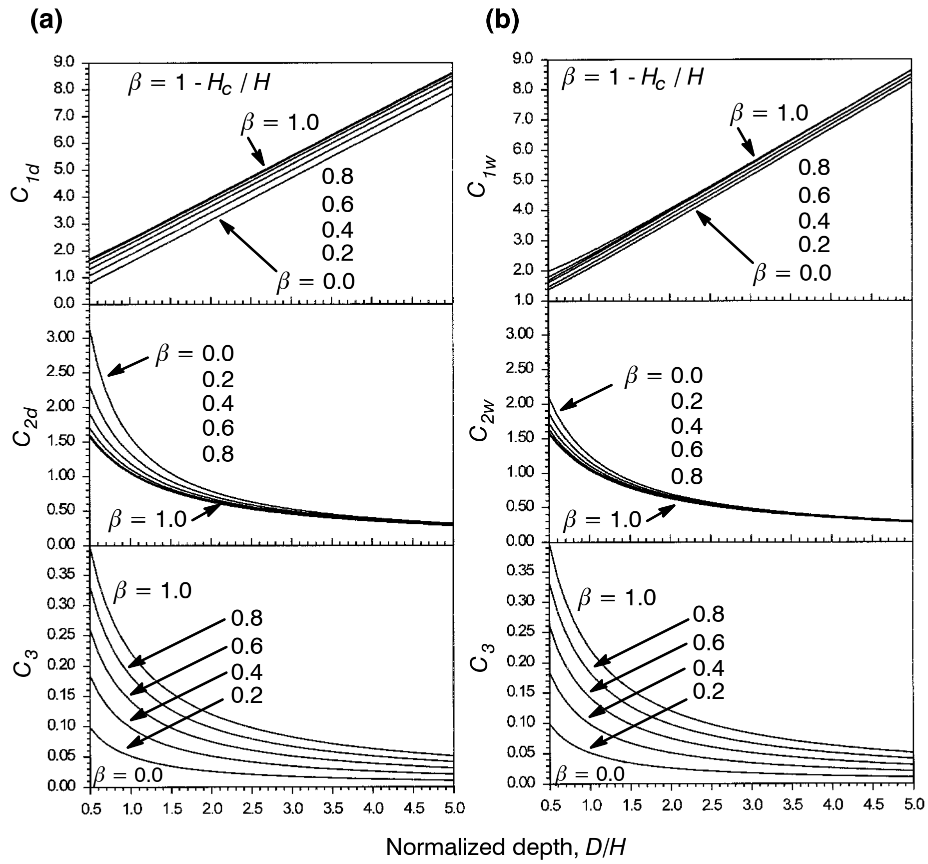


Figure 3. Coefficients C_1 , C_2 and C_3 : (a) dry tensile crack; (b) wet tensile crack.

embankment, $FS_{o,min}$, and overall maximum resulting force, $T_{R,max}$. Use Equations 38 and 50 to calculate the radius, y_c , of the critical slip circle yielding $T_{R,max}$.

Step 6. Use Equation 54 to calculate the inclination angle, α_k , for each reinforcement layer. Use Equation 9a to calculate the resulting forces, $T_{R,k}$. Finally, the required tensile force, $T_{req,k}$, for each reinforcement layer is calculated using Equation 55.

3.3.2 *Computer Program*

The solution presented in this paper is easily programmed for analysis using a personal computer. A computer program, WSTABL1, that requires simple input was developed in a QuattroPro worksheet. The user is required to input the following: the embankment configuration; the crack height; the strength parameters of the embankment fill and foundation soils; the required factor of safety for the reinforced embankment; and the number of reinforcement layers and their locations as illustrated in Figure 4.

WSTABL1
ROTATIONAL STABILITY OF EMBANKMENT ON SOFT CLAY
FORMULATED AND PROGRAMMED BY P.V. LONG
ASIAN INSTITUTE OF TECHNOLOGY

INPUT DATA

FOUNDATION SOIL	
Depth (m)	Su (kPa)
0	14.0
1	12.3
2	10.7
3	9.0
4	9.0
5	9.0
6	9.0
7	11.0
8	13.0
9	15.0
10	17.0
11	19.0
12	21.0
14	25.0

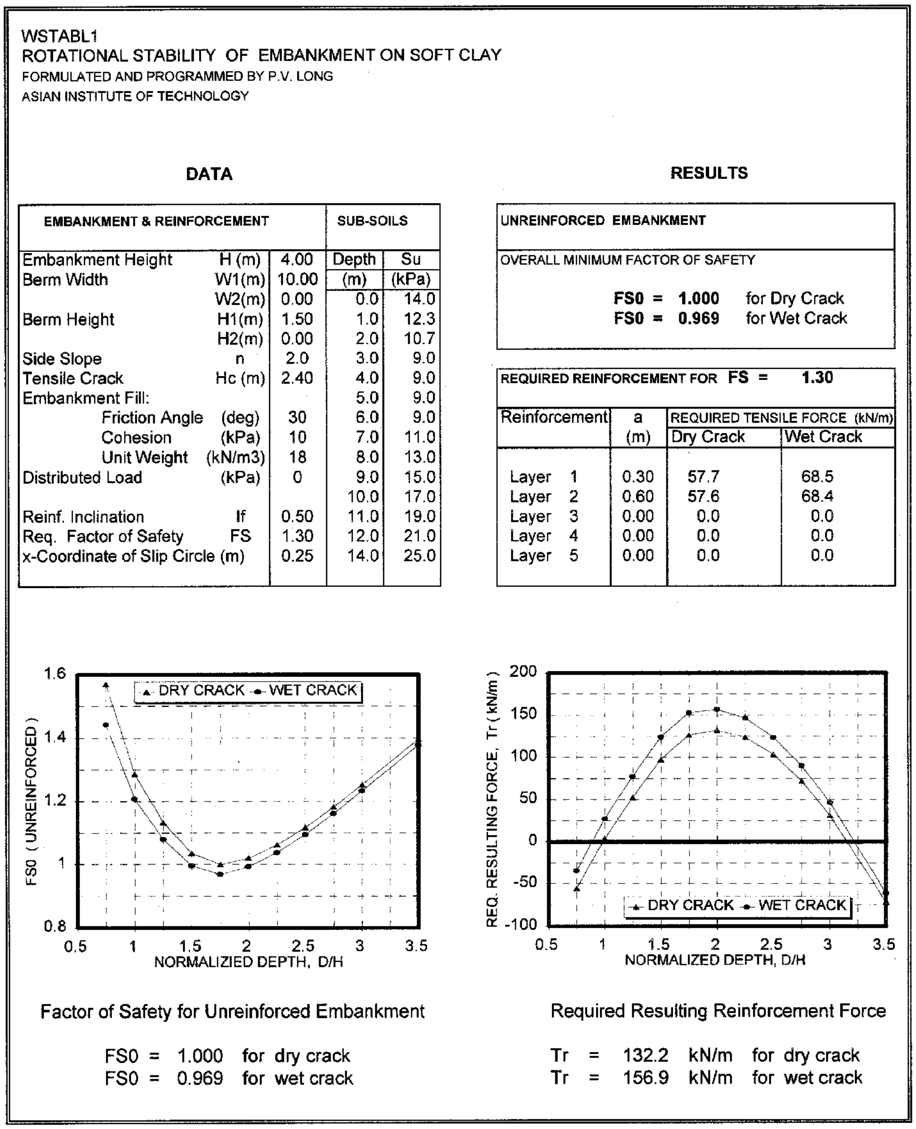
EMBANKMENT			
Emb. Height	(m)	4	
Emb. Crest Width	(m)	20	
Berm Height : H1, H2	(m)	1.5	0
Berm Width: W1, W2	(m)	10	0
Crack Height	(m)	2.4	
Side Slope	H / V	2	
Friction angle	deg.	30	
Cohesion	kPa	10	
Unit Weight	kN/m3	18	
Distributed Load	kPa	0	
REINFORCEMENT			
Required Factor of Safety		FS	1.30
Inclination Factor		If	0.50
Number of Reinf. Layers		N	2
(maximum = 5 layers)	C f	ak (m)	F. ratio
Layer 1	1	0.30	1.00
Layer 2	1	0.60	1.00
Layer 3	1	0.00	0.00
Layer 4	1	0.00	0.00
Layer 5	1	0.00	0.00

Figure 4. Input worksheet for Example 1 using the WSTABL1 program.

Figure 5 shows the graphical output including the overall minimum factor of safety for the unreinforced case, the maximum required tension forces, and the input information. The options in this program are: analysis of a two-step berm embankment; use of a distributed load; and a reinforcement force that acts as a free body force, or a reinforcement force that modifies the embankment soil strength.

3.4 Illustrative Examples

Three examples of an embankment on soft clay are presented in this section. The first example is an embankment with a berm, and a wet tensile crack. The second and third examples are a two-step berm embankment, and an embankment with a distributed load, respectively. For all of the examples, the embankment fill has the following properties: soil friction angle, $\phi = 30°$; cohesion, $c = 10$ kPa; and a unit weight of 18 kN/m^3. The undrained shear strength of the foundation soils vary with depth as shown in Table 1, and the reinforcement is assumed to modify the embankment soil strength ($C_f = 1$).


WSTABL1
ROTATIONAL STABILITY OF EMBANKMENT ON SOFT CLAY
FORMULATED AND PROGRAMMED BY P.V. LONG
ASIAN INSTITUTE OF TECHNOLOGY

DATA

EMBANKMENT & REINFORCEMENT			SUB-SOILS	
			Depth (m)	Su (kPa)
Embankment Height	H (m)	4.00	0.0	14.0
Berm Width	W1(m)	10.00	1.0	12.3
	W2(m)	0.00	2.0	10.7
Berm Height	H1(m)	1.50	3.0	9.0
	H2(m)	0.00	4.0	9.0
Side Slope	n	2.0	5.0	9.0
Tensile Crack	Hc (m)	2.40	6.0	9.0
Embankment Fill:			7.0	11.0
Friction Angle	(deg)	30	8.0	13.0
Cohesion	(kPa)	10	9.0	15.0
Unit Weight	(kN/m3)	18	10.0	17.0
Distributed Load	(kPa)	0	11.0	19.0
Reinf. Inclination	If	0.50	12.0	21.0
Req. Factor of Safety	FS	1.30	14.0	25.0
x-Coordinate of Slip Circle (m)		0.25		

RESULTS

UNREINFORCED EMBANKMENT

OVERALL MINIMUM FACTOR OF SAFETY

FS0 = 1.000 for Dry Crack
FS0 = 0.969 for Wet Crack

REQUIRED REINFORCEMENT FOR **FS = 1.30**

Reinforcement	a (m)	REQUIRED TENSILE FORCE (kN/m) Dry Crack	Wet Crack
Layer 1	0.30	57.7	68.5
Layer 2	0.60	57.6	68.4
Layer 3	0.00	0.0	0.0
Layer 4	0.00	0.0	0.0
Layer 5	0.00	0.0	0.0

Factor of Safety for Unreinforced Embankment

FS0 = 1.000 for dry crack
FS0 = 0.969 for wet crack

Required Resulting Reinforcement Force

Tr = 132.2 kN/m for dry crack
Tr = 156.9 kN/m for wet crack

Figure 5. Results of Example 1 using the WSTABL1 program.

Table 1. Undrained shear strength of the foundation soils.

Depth (m)	0	1	2	3	4	5	6	7	8	9	10	12	14
s_u (kPa)	14.0	12.3	10.7	9.0	9.0	9.0	9.0	11.0	13.0	15.0	17.0	21.0	25.0

Example 1. The embankment dimensions are as follows: height, H = 4 m; side slope, n = 2; berm width, W_1 = 10 m; berm height, H_1 = 1.5 m; and height of wet tensile crack, H_c = 2.4 m (Figure 6). Two layers of reinforcement are placed at 0.3 and 0.6 m above the ground surface for the purpose of obtaining a required factor of safety of 1.30 for the reinforced embankment. The inclination factor, I_f = 0.5, is assumed for both reinforcement layers. Calculate FS_o , T_R and T_{req} for the example parameters.

From the above embankment configuration the following values are obtained using Equation 39: β = (4 - 2.4)/4 = 0.40; $n_1 = H_1/H$ = 0.375; $n_2 = W_1/H$ = 2.5; and β_n = 2.97.

Using a trial depth, D = 4 m: s_u = 10.2 kPa (Equation 20); S_F = 0.141 (Equation 14); and S_E = 0.272 (Equation 15).

For D/H = 4/4 = 1 and β = 0.4, Figure 3b gives: C_{2W} = 1.110; C_{1W} = 2.341; and C_3 = 0.098.

Equations 38 and 47 give: y_o/H = 5.64; and FS_o = 1.208 for an unreinforced embankment.

The required resulting force for FS = 1.30 is T_R = 27 kN/m (Equation 52).

Repeating the above procedure for the other trial depths, one obtains FS_0 values of 0.996, 0.993 and 1.095 together with T_R values of the required resulting force of 124, 157, and 124 kN/m corresponding to tangent depths of D = 6, 8 and 10 m, respectively.

Plotting FS_o and T_R versus tangent depth, D, the overall minimum factor of safety for an unreinforced embankment, $FS_{o,min}$ = 0.969 at D = 7 m, and the maximum required resulting force, $T_{R,max}$ = 157 kN/m at D = 8 m, are obtained.

For a limiting tangent depth, D = 8 m, the following values are calculated: y_o/H = 5.78; FS_o = 0.993; RF = 1.309; and, y_c = 27.77 m. With I_f = 0.5 and y_c = 27.77 m, Equation 54 yields α_1 = 23° and α_2 = 23.4°. Finally, Equation 55 gives the required tension forces in reinforcement layers 1 and 2 of $T_{req,1} = T_{req,2}$ = 68.5 kN/m.

If the computer program, WSTABL1, is used to perform the above calculations, the required input parameters are: strength parameter values of the embankment fill and foundation soils; embankment configuration; and reinforcement parameters as shown in Figure 4. Figure 5 gives the resulting output.

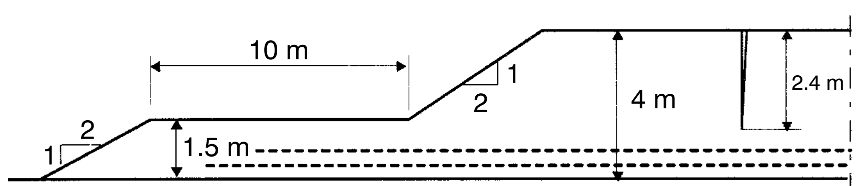


Figure 6. Embankment configuration for Example 1.

END OF EXAMPLE 1

Example 2. The two-step berm embankment dimensions are as follows: height, H = 5.5 m; no tensile cracks; side slope, n = 2; W_1 = 7.5 m; H_1 = 3.0 m; W_2 = 10 m; and H_2 = 1.5 m. The required factor of safety, FS = 1.3, there is one layer of reinforcement placed 1.5 m above ground surface, and $I_f = 0$. Calculate FS_o , T_R and T_{req} for the example parameters.

For an equivalent one-step berm embankment: Equations 58 and 59 give: $\delta' = 24.63$ m^2 and x_e = 4.85 m. Equations 56 and 57 yield the following dimensions of an equivalent one-step berm: H_e = 2.54 m and W_e = 14.78 m.

Applying the same procedure that is used in Example 1 for an equivalent embankment height of H = 5.5 m, a berm width of 14.78 m, and a berm height of 2.54 m, the following final results are obtained:

- overall minimum factor of safety for an unreinforced embankment: FS_0 = 0.931;
- maximum required resulting force: T_R = 301 kN/m; and
- maximum required tensile force: T_{req} = 269 kN/m.

END OF EXAMPLE 2

Example 3. Using the same input as in Example 1, with no tensile cracks, and a uniform distributed load of 10 kPa acting on top of the embankment, calculate FS_o , T_R and T_{req} for the example parameters.

For an equivalent embankment without an external load, the equivalent tensile crack for a 10 kPa uniform load is, $\Delta H_c = q/\gamma = 10/18 = 0.556$ m. Equation 60b then gives, $H_c* = 0 + 0.556 = 0.556$ m, $H* = 4 + 0.566 = 4.566$ m.

Using an embankment height of 4.566 m, and a crack height of 0.566 m, the final solution for Example 3 is obtained as follows:

- overall minimum factor of safety for an unreinforced embankment: FS_o = 0.925;
- maximum required resulting force: T_R = 219 kN/m; and
- maximum required tensile forces: $T_{req,1} = T_{req,2}$ = 96 kN/m.

END OF EXAMPLE 3

3.5 Comparison of Calculated Results

Three cases of an embankment constructed on soft clay are examined using the program WSTABL1, and are compared to the results calculated by the method of slices using the computer programs, PC-STABL6 and SB-SLOPE (Table 2). The first two cases deal with an embankment having one and two-step berms, with and without a uniform distributed load acting on top of the embankment. Example 3 is a re-analysis of a full-scale test embankment constructed on soft Bangkok clay, namely: Nong Ngu Hao Test Fill III (Asian Institute of Technology 1973). The test fill was built quickly to failure. When an embankment fill thickness of 3.4 m was reached, a tensile crack was observed on the crest of embankment. The embankment failed on the same day after the development of the tensile crack, without an increase in fill thickness. The embankment fill was a silty sand having a total unit weight, γ = 18 kN/m^3, friction angle, ϕ = 30°, and cohesion, c = 10 kPa. The undrained shear strength of foundation soils varied with depth (Table 1).

The results of the analyses are summarized in Table 2. It is seen that the maximum difference in the factors of safety obtained from the method presented herein and the method of slices using PC-STABL6 is less than 2 and 4% for unreinforced and reinforced embankments, respectively. Furthermore, factor of safety values of 0.98 (with a crack) and 1.03 (no cracks) were calculated using WSTABL1 for an embankment test fill with no berms (Table 2) that are close to the values calculated using SB-SLOPE. These results indicate that the method developed in this paper can yield a reliable stability calculation for embankments on soft ground.

Table 2. Comparison of stability analysis results.

Case	H (m)	n	H_c (m)	Berm 1		Berm 2		Load	Factor of safety					Rein.
									Unreinforced			Reinforced		
				W_1 (m)	H_1 (m)	W_2 (m)	H_2 (m)	q (kPa)	(1)	(2)	(3)	(1)	(2)	T_R (kN/m)
One-step berm														
No cracks	4.0	2	0.0	10.0	1.5	0.0	0.0	0.0	1.04	1.05	1.04	1.33	1.30	118
With a crack	4.0	2	2.4	10.0	1.5	0.0	0.0	0.0	*	1.00	0.95	*	1.30	132
With a load	4.0	2	0.0	10.0	1.5	0.0	0.0	10.0	0.92	0.93	0.90	1.33	1.30	219
Two-step berm														
No cracks	5.5	2	0.0	7.5	3.0	10.0	1.5	0.0	0.94	0.93	0.93	1.34	1.30	301
With a crack	5.5	2	2.5	7.5	3.0	10.0	1.5	0.0	*	0.88	0.87	*	1.30	326
With a load	5.5	2	0.0	7.5	3.0	10.0	1.5	10.0	0.86	0.85	0.84	1.35	1.30	432
No berms														
No cracks	3.4	3	0.0	0.0	0.0	0.0	0.0	0.0	1.05	1.03	0.98			
With a crack	3.4	3	1.2	0.0	0.0	0.0	0.0	0.0	*	0.98	0.95			

Notes: (1) = PC-STABL6 program; (2) = WSTABL1, the program developed in this study; (3) = SB-SLOPE program. *Option not available in the program. W_1, H_1, W_2, H_2 = width and height of berms 1 and 2; T_R = resulting reinforcement force in the tangential direction. The case with no berms is used for the Nong Ngu Hao Test Fill III embankment (Asian Institute of Technology 1973).

4 CONCLUSIONS

Solutions for the stability analysis of embankments with and without reinforcement, and constructed on soft ground with an undrained shear strength profile varying with depth are presented in this paper. The general case of an embankment having either a dry or wet tensile crack, a berm, and/or multiple layers of reinforcement with mobilized tensile forces acting in any orientation from the horizontal to tangential direction of the failure surface are considered. The friction component created by the reinforcement force is also considered. Different applications of the general solution (embankment with a two-step berm, and a distributed load acting on top of the embankment) are derived. Analysis procedures and illustrative examples are presented. Comparisons of the results calculated from different methods, using common embankment configurations on soft ground including a full-scale test embankment built to failure, are presented. The main results of this study are summarized as follows:

1. General solutions for rotational stability analysis of an embankment on soft ground with and without reinforcement are derived in the form of simple equations. The factor of safety for an unreinforced embankment, and the required reinforcement force for a reinforced embankment are obtained directly without iterative computations, for a given limiting tangent.
2. The analysis procedures and supporting graphs allow the user to obtain solutions using hand computations. Furthermore, a small computer program requiring minimal input in the form of a worksheet, was developed to quickly obtain the overall minimum factor of safety of an unreinforced embankment together with the maximum required tensile forces in each layer of reinforcement.
3. For a limiting tangent, the relationship between the critical slip circles through the embankment with and without reinforcement is expressed explicitly. The required resulting force of the reinforcement is derived as a function of the factors of safety of an embankment with and without reinforcement.
4. Good agreement between the proposed method of analysis with the results calculated using the method of slices computer programs, PC-STABL6 and SB-SLOPE, for common embankment configurations on soft clay are achieved. Moreover, nearly identical factors of safety are obtained in the re-analysis of a full-scale test embankment built to failure, indicating that the method presented herein can yield reliable solutions for the stability analysis of embankments on soft clay.

REFERENCES

Asian Institute of Technology, 1973, "*Performance Study of Test Sections for the New Bangkok Airport at Nong Ngu Hao*", Final Report Submitted to Northrop Airport Development Corporation, Asian Institute of Technology, Bangkok, 173 p.

Bishop, A.W., 1955, "The Use of the Slip Circle in the Stability Analysis of Slopes", *Geotechnique*, Vol. 5, No. 1, pp. 7-17.

Bonaparte, R. and Christopher, B.R., 1987, "Design and Construction of Reinforced Embankments Over Weak Foundations", *Transportation Research Record* 1153, pp. 26-39.

Holtz, R.D., Christopher, B.R. and Berg, R.R., 1995, "*Geotextile Design and Construction Guidelines*", Report No. FHWA HI-95-038, U.S. Department of Transportation, Federal Highway Administration, Washington, DC, USA, May 1995, 396 p.

Kaniraj, S.R., 1994, "Rotational Stability of Unreinforced and Reinforced Embankments on Soft Soils", *Geotextiles and Geomembranes*, Vol. 13, No. 11, pp. 707-726.

Low, B.K., 1989, "Stability Analysis of Embankments on Soft Ground", *Journal of Geotechnical Engineering*, ASCE, Vol. 115, No. 2, pp. 211-22.

Milligan, V. and Busbridge, J.R., 1983, "*Guidelines for the Use of Tensar Geogrids in Reinforcement of Fills over Weak Foundations*", Golder Associates Report to the Tensar Corporation, Mississauga, Ontario, Canada.

U.S. Department of Transportation, 1986, "PC-STABL6 Users Guide - Slope Stability Analysis Program", Federal Highway Administration, Turner-Fairbank Highway Research Center, McLean, Virginia, USA, 33 p.

Von Gunten Software, Inc., 1986, "SB-SLOPE - Simplified Bishop Slope Stability Analysis", Version 1.3, Fort Collins, Colorado, USA, 63 p.

NOTATIONS

Basic SI units are given in parentheses.

a_k	= vertical distance from reinforcement layer k to the ground surface (m)
B_1 , B_2	= width of berms (m)
c	= cohesion of the embankment fill in terms of total stress (Pa)
C_1 , C_2 , C_3	= coefficients (dimensionless)
C_{1D} , C_{2D}	= coefficients for dry cracks in an embankment (dimensionless)
C_{1W} , C_{2W}	= coefficients for wet cracks in an embankment (dimensionless)
C_f	= coefficient for the mobilized reinforcement force (dimensionless)
D	= tangent depth of the slip surface (m)
FS	= factor of safety for the reinforced embankment (dimensionless)
FS_o	= factor of safety for the unreinforced embankment (dimensionless)
H	= height of the embankment (m)
H_1 , H_2	= width of berms (m)
H_c , H'	= height of the cracked and uncracked zones, respectively (m)
H_c^*	= equivalent crack height of an embankment with a distributed load (m)
H_e	= equivalent height for a two-step berm embankment (m)
H^*	= equivalent height for an embankment with a distributed load (m)
I_f	= inclination factor (dimensionless)

M_O = overturning moment (N-m/m)
M_{OB} = overturning moment caused by the berm (N-m/m)
M_{OC} = overturning moment caused by the cracked zone in the embankment fill (N-m/m)
M_{OE} = overturning moment caused by the embankment within the uncracked zone (N-m/m)
M_{OW} = overturning moment caused by water pressure in a wet crack (N-m/m)
M_R = resisting moment (N-m/m)
M_{RE} = resisting moment provided by the embankment fill (N-m/m)
M_{RF} = resisting moment provided by the foundation soils (N-m/m)
M_{RR} = resisting moment provided by the reinforcement (N-m/m)
n = cotangent of the embankment side slope angle (dimensionless)
n_1 , n_2 = normalized values of berm height and berm width (dimensionless)
q = distributed load on the crest of an embankment (Pa)
RF = factor of safety ratio (dimensionless)
s_u = equivalent undrained shear strength of the foundation soils (Pa)
$s_{u,j}$ = undrained shear strength of subsoil layer j (Pa)
S_E = normalized strength parameter for the embankment fill (dimensionless)
S_F = normalized strength parameter for the foundation soils (dimensionless)
S_R = normalized strength parameter for the reinforcement (dimensionless)
T_R = resulting reinforcement force in the tangential direction (N/m)
T_{req} = required reinforcement tensile force (N/m)
W_e = width of the one-step berm (m)
x = horizontal coordinate (m)
x_c = x-coordinate of the center of slip circle (m)
y = vertical coordinate (m)
y_0 = radius of the slip circle through an unreinforced embankment (m)
y_c = radius of the slip circle through a reinforced embankment (m)
α_k = angle between the required reinforcement tensile force and the resulting reinforcement force for reinforcement layer k (Figure 1) (degrees)
β = crack factor (dimensionless)
β_n = slope factor (dimensionless)
γ = unit weight of the embankment fill (N/m^3)
γ_w = unit weight of water (N/m^3)
θ_k = angle between the horizontal and the tangent at reinforcement layer k (degrees)
λ = averaging coefficient for frictional stress in the embankment (dimensionless)
ϕ = total stress friction angle of the embankment fill (degrees)