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SPECIAL FEATURE

HIDDEN AND UN-HIDDEN PERSONALITIES AND DEVELOPMENTS OF SOFT CLAY BEHAVIOUR IN GEOTECHNICAL ENGINEERING

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INTRODUCTION

In Bjerrum's obituary, Professor Arthur Casagrande writes, " It is strange that many eminent disciplines of Karl Terzaghi have died prematurely. The following names come to my mind: Leo Rendulic, Walter Kjellman, Josef Jaky, Walter Bernatzik, Glennon Gilboy, Don Taylor, Robert Peterson, Nabor Carillo ----". Also, in their article by R. De. Boer, R.L.Schiffman and R.E. Gibson in Geotechnique, on the " Origin of the theory of consolidation: the Terzaghi-Fillunger dispute", the authors quote that "Terzaghi's stay in Vienna was a great technical success as a result of his technical collaboration with O.K. Frohlich, the work of his students, M.J. Hvorslev and L. Rendulic.

In describing Juul Hvorslev in his article "Recollections from 50-plus years of geotechnical practice or you can't do it alone", by John A. Focht, a past-President of the American Society of Civil Engineers, he described that quality exploration and laboratory testing were trademarks of WES (the then US Waterways Experiment Station). In Vicksburg, I lived in the same boarding house with Dr. Juul Hvorslev, a most remarkable gentleman and engineer. He carried on an immense correspondence with engineers all around the world and shared the technical correspondence with us.

Regretfully, our teaching of Soil Mechanics at the Undergraduate level and more so all the textbooks that we use in our teaching seem the put in separate compartments the section on consolidation in which Terzaghi is often quoted as the Father of Soil Mechanics and of course the Mohr-Coulomb concept in describing the shear strength of soils. The 1958 prize-winning paper in Geotechnique on the " Yielding of soils", by Roscoe, Schofield and Wroth tend to weld all aspects of the stress volume change behavior of soils at all states in a single 3D model in using the mean normal stress, deviator stress and the void ratio as the three coordinates. Indeed underlying in this excellent contribution are the works of Leo Rendulic and Juul Hvorslev as reported in the most outstanding conference ever held in our life time, that is the first International Conference held in Harvard University and organized by Arthur Casagrande. In two separate brief articles, the then students of Terzaghi-- Leo Rendulic and Juul Hvorslev reported on their findings on the constant void ratio contours from triaxial tests on clays and the shear strength of soils as dependent on the normal stress at failure and the void ratio at failure.

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Subsequent work done at the Imperial College by D.J. Henkel and R.H. G. Parry also confirmed the work of Rendulic and Hvorslev and gave greater insight into the behavior of normally and overconsolidated clays. Roscoe, Schofield and Wroth were the first to systematically formulate the state boundary surface in the (p, q, e) space. The work of Rendulic led to the calculation of volumetric strains from void ratio changes during the application of a stress increment. The subsequent work of Poorooshasb (1961) and Roscoe and Poorooshasb (1963), led to the way of calculating the volumetric and shear strains. The Thurairajah energy dissipation function helped to formulate the energy balance equation of Roscoe, Schofield and Thurairajah. The Roscoe and Burland contribution gave a better energy balance equation, which fitted the behavior of soft clays, and they also introduced the double yield locus concept. Pender was successful in developing a theory, which captured the essential behavior and features of overconsolidated clays using a non-associated flow rule. However, a proper understanding and modeling of overconsolidated clays need further insight and formulations, which is not yet complete.

Work of Rendulic (1936)

Rendulic performed a series of stress controlled compression and extension tests on remolded saturated clay and presented the results in the σ_1', $\sqrt{\sigma_3'}$ plot. Rendulic found that the stress paths followed by specimens sheared at constant void ratio (undrained tests) is in close agreement with the constant void ratio contours obtained from drained tests and isotropic consolidation tests. Further by assuming that these contours are reproducible in the $\sigma_1' = \sigma_3'$, and $\sigma_1' = \sigma_2'$ planes, Rendulic suggested that the constant void ratio contours form surfaces of revolution about the space diagonal $\sigma_1' = \sigma_2' = \sigma_3'$ when plotted in the three dimensional stress space. His results were based on a limited number of tests. The subsequent work of Henkel and Parry also confirmed such a finding.

Work of Hvorslev (1937)

Systematic study on the shear strength of remolded saturated clay was done by Hvorslev (1937) at the Vienna Institute of Technology under Karl Terzaghi and was also presented in the First International Conference on Soil Mechanics and Foundation Engineering held at the Harvard University. He showed that the peak shear stress at failure τ_f of a soil is a function of the effective stress σ_f' on and the voids ratio e_f at the plane of failure. Hvorslev tests data were from the direct shear tests, and lacked the pre-shear stress and state paths in the (e, σ', τ) plot. Hvorslev work also led to the use of the mean equivalent stress, p_e, defined as:

$$p_e = p_o\left[\exp\left(e_o - e\right)/\lambda\right] \quad (1)$$

The mean equivalent pressure p_e is used in normalizing the stress strain behavior under various degrees of drainage, overconsolidation and stress paths in compression and in extension.

Roscoe, Schofield and Wroth (1958)

The award winning paper of Roscoe, Schofield and Wroth in 1958 in Geotechnique combined the work of Rendulic and Hvorslev in the (p,q,e) plot and rationalized the state boundary surface for normally consolidated clays and the Hvorslev type of surface for overconsolidated clays hinged along the critical state line. The similarity of the undrained stress paths in the normally consolidated state enabled the surface to be transformed into a two dimensional curve with the Hvorslev parameter, p_e.

Work of Poorooshasb (1961) and Roscoe and Poorooshasb contribution

The work of Poorooshasb (1961) as well as Roscoe and Poorooshasb (1963) for the first time established a procedure by which the stress strain behavior under different degrees of drainage can be correlated through an incremental relationship of the form:

$$[d\varepsilon_s]_{drained} = ([d\varepsilon_s]_{undrained} + \left\{\left(\left[\frac{d\varepsilon_s}{d\varepsilon_v}\right]_{anisotropic}\right).([d\varepsilon_v]_{drained})\right\}) \quad (2)$$

The above incremental stress strain relation when expressed in terms of the incremental plastic strains led to instances where an associated flow rule can be used with volumetric yield loci as proposed by Roscoe, Schofield and Thurairajah (1963) or a non-associated flow rule as proposed by Roscoe and Burland (1968) with double yield loci.

Work of Taylor and Thurairajah

Professor Andrew Schofield always quote the work of D.W. Taylor on the analysis of shear box tests on dry sand. This, Prof. Schofield interpreted in the form of a work equation as:

$$\tau dx = \mu\sigma dx + \sigma dy \quad (3)$$

Thurairajah followed the contribution of Taylor and described the energy dissipation term during plastic deformation as

$$\left[dE\right]_{dissipated} = Mpd\varepsilon_s \quad (4)$$

This led Roscoe, Schofield and Thurairajah to complete the energy balance equation from a thermodynamics point of view as

$$pd\varepsilon_v^p + qd\varepsilon_s^p = Mpd\varepsilon_s^p \quad (5)$$

This will then lead to the plastic dilatancy ratio $\frac{d\varepsilon_v{}^p}{d\varepsilon_s{}^p}$ as:

$$\frac{d\varepsilon_v^p}{d\varepsilon_s^p} = M - \eta \quad (6)$$

Modifications in energy balance equation

There are many forms of this equation depending on the terms adopted for energy dissipation. Roscoe and Burland have a modified version and further modifications were done by others including Dafalias. The version of Roscoe and Burland seem to fit the behavior of normally consolidated clay well and also the K_0 value. This expression is of the form:

$$\frac{d\varepsilon_v^p}{d\varepsilon_s^p} = \frac{M^2 - \eta^2}{2\eta} \quad (7)$$

This came from the energy balance equation:

$$pd\varepsilon_v^p + qd\varepsilon_s^p = p\sqrt{\left[\left(d\varepsilon_v^p\right)^2 + \left(Md\varepsilon_s^p\right)^2\right]} \quad (8)$$

Drucker, Gibson and Henkel (1957) tried to associate the plastic strain rate vector to the Mohr Coulomb failure envelope, while the Cambridge soils group and Calladine followed the normality concept of Drucker (1959) for stable materials, given as:

$$\frac{dq}{dp} = -\left[\frac{d\varepsilon_v^p}{d\varepsilon_s^p}\right] \quad (9)$$

Elastic wall concept and volumetric yield locus

Calladine (1963) then paved the way with the elastic wall concept of obtaining the volumetric yield locus from the state boundary surface of normally consolidated clays. This then helped to derive the equation of the volumetric yield locus from the relation.

Thus, the following differential equation for the volumetric yield locus emerged:

$$\left(\frac{dq}{dp}\right)_y = -\left[\frac{\left\{M^2 - \eta^2\right\}}{2\eta}\right] \quad (10)$$

This equation when integrated will give the volumetric yield locus as:

$$\left[\frac{p}{p_o}\right]=\left\{\frac{M^2}{\left(M^2+\eta^2\right)}\right\} \tag{11}$$

And the state boundary surface as:

$$\left[\frac{p}{p_o}\right]=\left\{\frac{M^2}{\left(M^2+\eta^2\right)}\right\}^{\left(1-\frac{k}{\lambda}\right)} \tag{12}$$

A correct selection of the energy dissipation function is important to obtain the correct plastic dilatancy ratio. Also, if the correct energy dissipation function is used then the plastic volumetric strain obtained with the expansion of this yield locus will be the correct magnitude in drained conditions. Then, the state boundary surface obtained is also correct and can predict the correct pore pressure in undrained conditions.

Stress-strain equations

The dilatancy ratio, and the incremental strains derived from the above formulation are as follows:

$$\frac{d\varepsilon_v}{d\varepsilon_s}=\left[\frac{1}{(1-\frac{k}{\lambda})}\right]\left\{\frac{\left(M^2-\eta^2\right)}{2\eta}\right\} \tag{13}$$

$$d\varepsilon_v{}^p=\left(\frac{\lambda-k}{1+e}\right)\left\{\frac{2\eta d\eta}{(M^2+\eta^2)}+\frac{dp}{p}\right\} \tag{14}$$

$$d\varepsilon_v=\left(\frac{1}{(1+e)}\right)(\lambda-k)\left[\left(\frac{2\eta d\eta}{M^2+\eta^2}\right)+\left(\lambda\frac{dp}{p}\right)\right] \tag{15}$$

$$d\varepsilon_s=\left(\frac{\lambda-k}{1+e}\right)\left(\frac{2\eta}{M^2-\eta^2}\right)\left[\left(\frac{2\eta d\eta}{M^2+\eta^2}\right)+\left(\frac{dp}{p}\right)\right] \tag{16}$$

Experience from Laboratory and Field Studies

In using the above expressions for the prediction of pore pressures and strains in both undrained and drained conditions, the following points are noted:

1. Using Eq. (13), the dilatancy ratio can be predicted accurately and also the K_0 value.

2. Using Eqs. (15) and (16), both the volumetric and shear strains can be predicted for radial type of stress paths in which the stress ratio remains constant. This will also include the one dimensional consolidation path.

3. For non-radial type of paths in which the stress ratio increases and failure conditions are sought both in compression and in extension. Only the volumetric strain can be predicted accurately by Eq. (15), while the shear strain computed is only equal to the anisotropic component as evaluated from Eq. (2). Thus, the drained component of the shear strain needs to be added separately and that was the reason of the use of a set of constant q yield loci in the Roscoe and Burland (1968) contribution.

4. Thus, naturally, for undrained conditions the pore pressures can be successfully predicted, but not the shear strains.

Application of Cambridge Theories

James Michael Duncan in his State of the Art paper, " On the role of advanced constitutive relations in practical applications", presented in the Thirteenth International Conference on Soil Mechanics and Foundation Engineering in New Delhi in 1994, defined for his presentation an advanced constitutive relationship as " A relationship among stresses, stress increments, and strain increments that models nonlinear stress strain behavior, plastic deformation and volume changes caused by changes in shear stress. It may include anisotropy and viscous behavior".

Thus, Duncan considered all theories, which explain dilatancy of soil media, are in a sense advanced models. Additionally, the behavior of soft clays can successfully be modeled as an elasto-plastic material, with approximately eighty percent of the volumetric strain being plastic and only about 20 percent include elastic and viscous components. The shear strain, however, is wholly considered as plastic in most simplified models. Indeed the Casagrande maximum past pressure and the Bjerrum preconsolidation pressure is a type of volumetric yield stress lying on the volumetric yield loci. Duncan cited several case histories in which the Cambridge style elasto-plastic theories are used in finite element analysis and this included the following:

- Soil structure interaction (26 cases)
- Soil reinforcement and anchorage (10 cases)
- Dams, embankments and settlements due to fluid extraction (42 cases)
- Tunnels (15 cases)
- Natural and unbraced cut slopes (7 cases)

Modeling of overconsolidated clays

Cambridge stress strain theories are developed for normally consolidated clays. In addition to the limitation in predicting the undrained shear strain in normally consolidated clays; they also indicate that within the state boundary surface where the behavior is overconsolidated there is no volumetric yielding while the yielding in shear is accommodated with the constant q yield loci. Balasubramaniam (1969) indicated that volumetric yielding can take place inside the state boundary surface for overconsolidated states and Pender developed a stress stain theory for clays in the overconsdolidated clays undergoing volumetric and shear yielding with a non-associated flow rule.

The Cambridge and other theories usually follow the style:

1. The volumetric strains are calculated from voids ratio changes due to a stress increment and, thus, one should be able to obtain the shape of an undrained stress path from fundamental considerations. While for normally consolidated clays the undrained stress paths are similar and can be derived from fundamental considerations, for overconsolidated clays, the shape of the undrained stress path changes as the OCR value change and yet to date there is no fundamental way in which their shapes can be formulated. Pender assumed empirically that the shape is parabolic both on the wet side for lightly overconsolidated states as well as on the dry side for heavily overconsolidated states. Even though there is a degree of empirism in this approach yet this formulation seem to simplify the modeling process.

2. The flow rule for plastic deformation is normally obtained from an energy balance equation and this again seem rather difficult for the overconsolidated clays with plastic strain rate vectors dependent on the stress ratio and the mean normal stress.

Stress-strain behavior of overconsolidated clays

Extensive laboratory tests were performed in the triaxial apparatus to study the stress stain behavior of oversonsolidated clays below the state boundary surface. These studies included tests under compression and extension conditions with isotropic, anisotropic and K_0 consolidation states. Additionally stress-probing type of tests were carried out radially from three stress states one on the isotropic axis and the other two on the compression and extension sides. The stress paths covered the full 360-degree spectrum. Several types of applied stress paths were also used from five stress states on the compression side with anisotropic and K_0 conditions. Constant p type of tests was performed both in compression and in extension to determine the variation of the plastic dilatancy ratio with the stress ratio and the mean normal stress. These data were analyzed with a view to establish a flow rule of the plastic strain increment ratio in terms of the stress ratio and the mean normal stress. Then, an incremental approach somewhat similar to the Roscoe and Poorooshasb type was used to calculate the volumetric and shear strains under drained conditions. The undrained shear strains were also modeled by empirical expression.

Various types of strain contours were plotted both in terms of the total strains and plastic strains. It appears that these contours show a clear trend in the behavior, especially for the strain $\sqrt{\left((\Delta\varepsilon_v{}^p)^2 + (\Delta\varepsilon_s{}^p)^2\right)}$, which appear to lie on an expanding concentric set of circles. Finally the state boundary surface, which is of the strain hardening volumetric yield locus in the non-failure states and the curved Hvorslev type surface for peak stress conditions tend to merge. Conformal mapping of the circular stress increment contours in the (q, p) plot to elliptical type of strain increment contours in the $(\Delta\varepsilon_v, \Delta\varepsilon_s)$ plot is realized. A similar transformation takes place when the circular strain increment contours in the $(\Delta\varepsilon_v, \Delta\varepsilon_s)$ plot is mapped on the $(\Delta q, \Delta p)$ plot. Efforts are being made to develop theories with sound fundamentals to describe the stress strain behavior of overconsolidated clays.

Micro-structural and micro-mechanical approach

Drucker (1962) pointed out that the mathematical theory of plasticity, which is often, used in developing a stress strain theory at times, is only a formalization of known experimental results and does not deeply inquire into the physical basis of the behavior. A micro-structural view of the mechanical properties of saturated clay was presented by Calladine (1971). Also, a simple micro-mechanical model for the yielding and consolidation of clays is presented by Houlsby and Sharma (1999). These concepts are still not yet fully developed for the estimation of the strains experienced by the soil in the laboratory and in the field. Such developments can only take place with a good understanding of the experimental results and is an ongoing research and challenge in Geotechnics.

Concluding Remarks

This brief note illustrates the historical contributions that led to the formalization of the state boundary surface and an incremental approach based on experimental observations helped in the development of stress strain theories based on plasticity theories with associated and non-associated flow rules. The natural deposits of soft clay are often overconsolidated and needs additional work to describe their undrained and drained mode of deformations. The experimental conclusions are in a form suitable for inclusion in computer programs for the analysis of geotechnical problems in soft clays. However, theories based on micro-structural and micro-mechanics are a long way from their completion in describing this behavior.

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