Determining Unsaturated Soil Properties through Parameter Estimation

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ABSTRACT: The stability of slope is known to be affected by rainfall infiltration, especially when groundwater table is relatively deep. Thus, careful analysis of slope stability requires the determination of unsaturated properties of the soil. Laboratory tests for the determination of hydraulic properties and unsaturated shear strength are tedious and time consuming. Thus, numerous fitting equations were proposed by researchers for determination of soil water characteristic curve (SWCC), permeability function and shear strength parameter of the unsaturated soil. This paper presents derivation of unsaturated soil properties from simple laboratory tests using relevant parameter estimation equations. The laboratory data include the suction measured using filter paper, and the shear strength parameters obtained from Triaxial UU test on compacted clayey soil under different gravimetric water contents. Results show that hydraulic properties and unsaturated shear strength parameters could be derived from available data and the slope stability analysis could be carried out using the estimated properties.

Keywords: Compacted soil; Suction; SWCC; permeability function; rate of strength increase with respect to suction.

1. INTRODUCTION

Soil near ground surface are naturally found in unsaturated condition, thus the stability of slope is affected by changes in climatic condition, especially rainfall infiltration (Ching et al. 1984; Toll et al. 1990; Gasmo et al. 2000; Tsaparas et al. 2002; Rahardjo et al. 2008). Field monitoring of suction response to rainfall infiltration (e.g. Gasmo et al. 2000; Tsaparas et al. 2002; Kassim et al. 2012) supported that the suctions could be lost during wet season. Previous study showed that the critical rainfall pattern causing slope failure was influenced by the coefficient of saturated permeability of soil and initial moisture condition. Gofar et al. (2008) showed that the ratio of rainfall intensity to the coefficient of saturated permeability of soil determines the critical rainfall pattern causing slope instability. Rahardjo et al. (2008) showed that soil slopes with lower permeability requires more antecedent rainfall than residual soil slopes with higher permeability to lead to slope failure. In general, five days antecedent rainfall could be required to produce the worst pore-water pressure profiles in a slope.

The preceding discussion inferred that soil water characteristic curve (SWCC) and permeability function of unsaturated soil are required to obtain the variation of negative pore water pressure (suction) in the soil while the unsaturated shear strength parameter is needed for slope stability analysis. Field study by Gofar et al (2008) showed that the range of suction variation is reasonably consistent with the soil water characteristic curve (SWCC). The shear strength of the soil is usually high due to the apparent cohesion contributed by negative pore-water pressure or suction (Fredlund & Rahardjo, 1993). Once the soil becomes wet due to rainfall infiltration, the suction component of the shear strength was minimized, and the shear strength decreased significantly. Thus, a detailed analysis of rainfall-induced slope stability requires the establishment of hydraulic properties (SWCC and permeability function) and unsaturated shear strengths parameters of the soil.

Experimental works, despite imposing extra time and expenses, evidently provide the most appropriate means for measuring the properties. Several methods such as hanging column, pressure plate, Tempe cell, chilled mirror hygrometer and centrifuge were developed to establish the SWCC of the soil. The procedures are standardized in ASTM D6836. Combination of Tempe cell and Pressure plate test are the most common method used for determination of SWCC. Besides, filter paper method (Likos & Lu, 2002) is commonly used to develop SWCC curve through suction measurements. The procedure is standardized in ASTM D5298. The procedures for defining the points to develop SWCC curve were time consuming, thus numerous fitting equations were developed to form the SWCC curve based on limited number of laboratory measurements of suction and volumetric water content (e.g. van Genuchten, 1980; Fredlund & Xing, 1994).

Unsaturated permeability of soil could be measured using modified Triaxial cell or flexible wall permeameter test (ASTM D7664). The coefficient of permeability with respect to water for an unsaturated soil is a non-linear function of the volumetric water content of the soil. When the soil approaches saturation, the permeability becomes constant and equal to the saturated coefficient of permeability with respect to water, k_s. Thus, in the absence of laboratory data, the permeability function can be obtained using statistical method proposed by Childs & Collis-George (1950) based on the saturated coefficient of permeability and the SWCC curve. The procedure for the prediction can be found in Fredlund & Rahardjo (1993).

Mostly adopted measuring devices for unsaturated shear strength of soil are direct shear box (Gan et al. 1988; Schnellmann et al. 2013), and triaxial cell (Bischop & Donald, 1961; Ho et al. 1982, Rahardjo et al. 1995), with several modifications to implement the effect of suction on soil samples. These procedures were also tedious and time consuming, thus fitting equation was also developed for estimation of un-saturated soil shear strength based on SWCC curve (e.g. Vanapalli et al. 1997; Goh et al. 2010).

This paper presents the estimations of SWCC, permeability function and the shear strength of un-saturated soil by employing limited data obtained in laboratory on compacted clayey soil by Arif (2005). The data was then analysed using relevant parameter estimation model to establish the unsaturated soil properties required for further analysis for example the stability of a slope subjected to rainfall infiltration.

2. THEORETICAL BACKGROUND

A fundamental property that relates the suction and water holding ability of soil is characterized by the water retention curve, which is also known as the soil water characteristic curve (SWCC). The SWCC curve can be divided into three regions as shown in Figure 1 i.e. boundary effect zone, transition zone and residual zone (Fredlund & Rahardjo, 1993). The definition of transition zone, which occurs between the air entry point (ψ_a) and residual suction (ψ_r) is very important. Hence; the shape of SWCC curve is characterized by the saturated volumetric water content (θ_s) the air entry value (ψ_a), and the residual suction (ψ_r) at the corresponding residual water content (θ_r).

Several SWCC equations were reviewed by Sillers & Fredlund (2001) using a database of laboratory measurements. They concluded that Fredlund & Xing (1994) equation provided the closest fit to the data sets. The equation is as follows:

$$\theta_w = C(\psi)\frac{\theta_s}{\left\{\ln\left[e+\left(\frac{\psi}{a}\right)^n\right]\right\}^m} \quad (1)$$

where: θ_w is the volumetric water content (VWC), θ_s is the VWC at saturation, *e* is the natural logarithmic base number, 2.71828, a, n, m are fitting parameters related to the air-entry value of the soil in kPa (ψ_a), the slope at the inflection point of the SWCC; and residual water content (θ_r) respectively; $\psi = (u_a – u_w)$ is the matric suction (kPa); and $C(\psi)$ is a correction factor given as follows:

$$(\ ) = 1 - \frac{\ln(1+—)}{\ln(1+\frac{1000000}{})} \quad (2)$$

In which C_r is a constant related to matric suction corresponding to residual water content. Fredlund & Rahardjo (1993) suggested that C_r = 1500. Alternatively when all laboratory data were in the low range of suction value (<1500 kPa), Leong & Rahardjo (1997a) suggested to use $C(\psi)$ =1 in Equation (1). The characteristic values of the SWCC curve (i.e. air entry value, residual suction and the slope of SWCC curve at the inflection point) can be calculated using Zhai & Rahardjo (2012) equations.

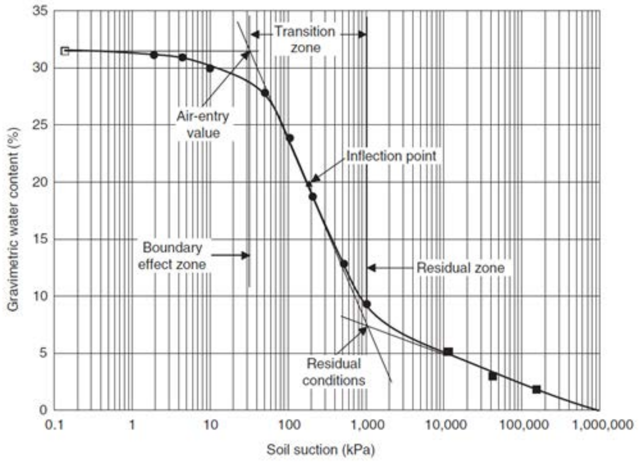


Figure 1 Typical soil water characteristic curve showing zones of desaturation (Fredlund et al, 2012).

Permeability function can be plotted based on saturated coefficient of permeability (k_s) and a few data on water coefficient of permeability (k_w) obtained from triaxial test with fitting parameters of the SWCC curve (a, n, and m). In the absence of data on the water coefficient of permeability, statistical method proposed by Childs & Collis-George (1950) and presented in Fredlund & Rahardjo (1993) could be used. Alternatively, equation (3) which took the same form as Equation (1) could be used to predict the permeability function (Leong and Rahardjo, 1997b).

$$k_w = C(\psi)\frac{k_s}{\left\{\ln\left[e+\left(\frac{\psi}{a}\right)^n\right]\right\}^{mp}} \quad (3)$$

with *p* value ranges from 2.4 to 25 depending on the type of soil. The estimation of p value could be made based on the similarity of the permeability function with the SWCC.

A practical formulation for shear strength of unsaturated soil was proposed by Fredlund et al. (1978) as follows:

$$\tau_f = c' + (\sigma_n - u_w)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b \quad (4)$$

where c' is the effective cohesion; ϕ' is the effective angle of shearing resistance; $(\sigma_n – u_w)_f$ is the effective normal stress at failure; ϕ^b is the rate of change in shear strength relative to change in suction. For u_a equal to atmospheric, Equation (4) can be simplified as:

$$\tau_f = c' + \sigma' \tan\phi' + \psi \tan\phi^b \quad (5)$$

The ϕ^b for undrained condition can be expressed as follows:

$$\tan\phi^b = \frac{\left[\frac{(\sigma_1-\sigma_3)}{2}\cos\phi' - c' - \left(\frac{(\sigma_1+\sigma_3)}{2} - \frac{(\sigma_1-\sigma_3)}{2}\sin\phi'\right)\tan\phi'\right]}{\psi} \quad (6)$$

By extending the classical Mohr-Coulomb envelope, the three-dimensional representation of Equation 4 can be visualized in Figure 2.

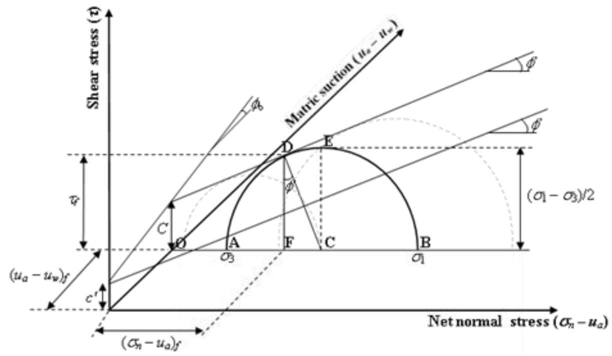


Figure 1 Extended Mohr-Coulomb failure envelope for unsaturated soil (after Fredlund et al, 2012).

Unsaturated shear strength from direct shear tests (Gan et al., 1988) indicated that the relationship between the shear strength versus and suction is not linear. However, Rahardjo et al. (1997) suggested it is possible that the relationship over a selected range of suction is linear. This is applicable for results of Undrained Unconsolidated (UU) Triaxial test performed on different water contents (Fredlund et al., 1978). As the undrained compressive strength (q_u) obtained from this test can be defined as half of the maximum deviator stress (i.e. $q_u=(\sigma_1 – \sigma_3)/2$).

Many shear strength equations were developed to predict the rate of strength increase with respect to suction (ϕ^b) (Goh et al' 2010; Abdullah et al. 2013), among them is the equations proposed by Vanapalli et al (1997) is as follows:

$$\tau_f = c' + (\sigma_n - u_w)_f \tan\phi' + (u_a - u_w)\Theta^K \tan\phi' \quad (7)$$

where Θ is normalized volumetric water content (θ_w/θ_s) and κ is an unknown fitting parameter. This equation is a fitting type of equation developed for sand and fine-grained soil. For sand, κ is equal to one, but the value increases with soil's plasticity. The equation was found to fit results of UU test by (Gofar & Lee, 2014) for both coarse and fine-grained soil for suction less than 100 kPa.

Goh et al (2010) equation is basically the improvement of Equation (7). Based on their study on eleven sets of published data as well as laboratory tests, they found that the relationship between shear strength and suction changed at suction equal to air entry value AEV = $(u_a – u_w)_b$. therefore:

$$\tau_f = c' + (\sigma_n - u_w)_f \tan\phi' + (u_a - u_w)\tan\phi^b \quad (8)$$

if $(u_a – u_w) < (u_a - u_w)_b$ or ψ <AEV

$$\tau_f = c' + [(\sigma_n - u_w) + (u_a - u_w)_b]\tan\phi' + [(u_a - u_w) - (u_a - u_w)_b]b\Theta^K \tan\phi' \quad (9)$$

if $(u_a – u_w) \geq (u_a - u_w)_b$ or $\psi \geq$ AEV

where

$$\kappa = [\log(u_n - u_w) - \log(u_a - u_w)_b]^y \quad (10)$$

where *b* and *y* are fitting parameters.

This equation was validated with published laboratory data which mostly fine grained soil (classified as CH, CL, MH, ML and SM according to USCS).

3. METHODOLOGY

The study presented in this paper made use of the available data in Arif (2005) for the evaluation of unsaturated soil properties required for slope stability analysis. The soil for his study was retrieved from Babarsari, Yogyakarta. The data were originally used for evaluation of bearing capacity of the soil. Index properties of the soil comprising grain size distribution, Atterberg limits and falling head test for determining the coefficient of saturated permeability of the soil were tested in laboratory. Compaction test was carried out to define the maximum dry density and the optimum water content of the soil. Advanced laboratory tests were performed on the compacted soil including suction measurements and shear strength tests.

The SWCC was obtained experimentally by measuring suction of the soil samples using Filter Paper method following the procedure outlined in ASTM D 5298. The calibration of the Whatman filter paper No. 42 follows the following equation:

$$= 10^{5.327-0.0779} \quad \text{for } w_f < 45.26 \qquad (11a)$$

$$= 10^{3.412-0.0135} \quad \text{for } w_f \geq 45.26 \qquad (11b)$$

where ψ is the suction (kPa) and w_f is the water content of the filter paper. However, direct correlation between the gravimetric water content of sample with suction should be derived empirically based on a number of tests performed in the study.

The shear strength parameters were obtained using conventional Triaxial testing apparatus under Unconsolidated Undrained (UU) condition. Frelund & Rahardjo (1993) suggested that the procedures outlined for conventional Triaxial test on saturated soil can be applied for undrained loading condition of unsaturated soil. In this case, the suction stress can be linearly correlated with the undrained strength of soil (Chae et al., 2010). The unsaturated soil specimen is tested at its initial water content or suction. In this case, the initial matric suction is not relaxed or changed prior to commencing the test by replacing the porous stone with metal or plastic discs on top and bottom of specimen and by closing the specimen with rubber membrane during the test. It is assumed that the variation is negligible despite the suction may decrease due to the increase of pore-water pressure during shearing stage of the soil sample. Soil specimens (38 mm dia. and 76 mm height) were prepared at the same initial gravimetric water content as for the suction measurement tests. The shearing rate was controlled at 1.50 mm/min or equivalent to a strain rate of 0.033% per second for the specimen height of 76 mm.

Fredlund & Xing (1994) equation was used to develop SWCC curve based on the suction measurement data using filter paper test. The permeability function was plotted using the coefficient of saturated permeability data and the SWCC curve. Subsequently, the rate of shear strength in-crease with respect to suction (ϕ^b) was obtained by applying non-linear failure envelope equation (Vanapalli et al., 1996 and Goh et al, 2010) on the experimental test results. The performance of both predicted types equation was evaluated for this soil by comparing with the results of Triaxial UU test.

4. RESULTS AND ANALYSIS

Following ASTM D2487, the soil used in this study was classified as low plasticity silt (ML). Specific gravity of the soil was 2.67. The grain size distribution of the soil is presented in Figure 3. The soil contains more than 60% fines with 53.5% silt and 10.1% clay fractions. Liquid limit was 47.8% while plastic limit was 29.2%, thus the plasticity index of the soil was 11.6. The coefficient of saturated permeability of the soil is quite low (i.e. 5.6×10^{-6}). Compaction test results in a maximum dry density of 1.68 Mg/m^3 and optimum water content of 18.4%. Soil samples were prepared at initial relative density of 90% or dry density of 1.51 Mg/m^3.

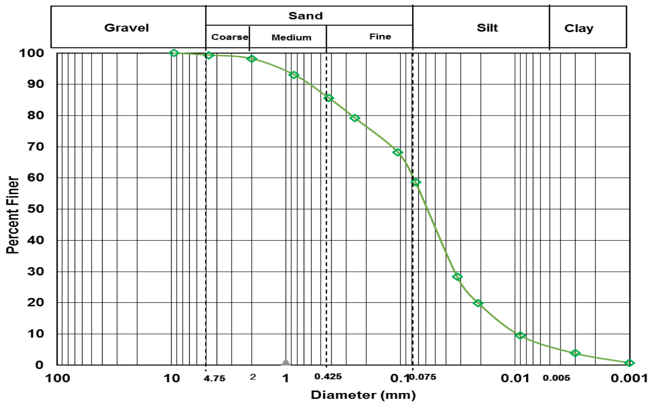


Figure 3 Grain size distribution of the soil evaluated in this study

Filter paper test was performed on two sets of samples. Each set comprises four (4) samples were prepared at gravimetric water of about 22, 19, 16, and 14% (denoted as sample A, B, C and D). In other words, two samples were prepared below the optimum water content while the other two were prepared above the optimum water content. The corresponding suction for the samples are calculated based on empirical correlation developed in this study is as follows:

$$= 10^7 \times \quad ^{0.6519} \qquad (12)$$

The corresponding suctions for the gravimetric water content for soil preparation is shown in Table 1.

Table 1. Suction corresponding to the water content of soil at preparation

Sample	Gravimetric water content	Volumetric water content	Suction (kPa)
A	22	33.26	5.9
B	19	28.73	41.8
C	16	24.18	295.2
D	14	21.17	1087.3

The resulting points from filter paper test were plotted in Figure 4 in terms of volumetric water content. The laboratory data points were fitted using Fredlund & Xing (1994) equation to form the SWCC as shown in Figure 4. The fitting parameters of the equation a, n, m are 23 kPa, 0.8 and 0.5 respectively. The air entry value (ψ_a) and residual suction (ψ_r) are 8 and 3700 kPa respectively. The saturated volumetric water content (θ_s) and the residual volumetric water content (θ_r) were 0.37 and 0.15 respectively while the slope of SWCC curve m^2_w was 0.083. The SWCC was the typical of drying SWCC of silty soil (Fredlund and Rahardjo, 1993). The permeability function was calculated based on k_s and the SWCC using statistical method (Childs & Collis-George, 1950) and Leong and Rahardjo's fitting equation is shown in Figure 5.

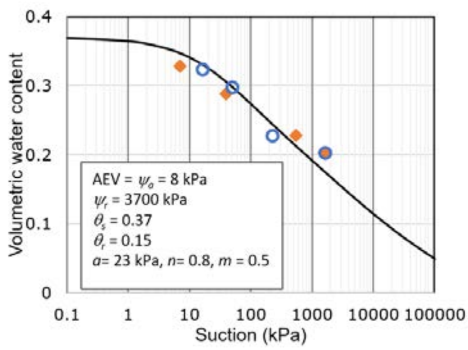


Figure 4 SWCC curve and the characteristic values.

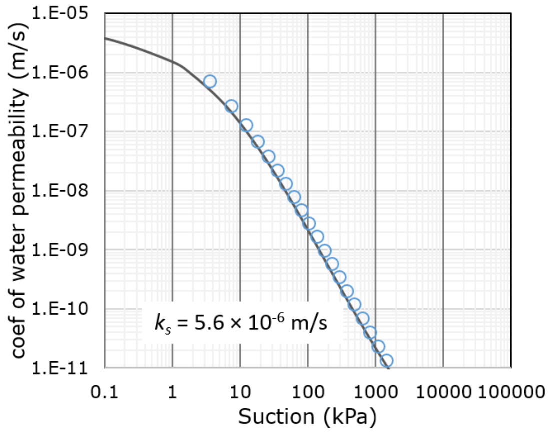


Figure 5 Permeability function.

As for the SWCC test, four samples were prepared at gravimetric water of about 14, 16, 19, and 22% (denoted as sample A, B, C and D) for shear strength test under Unconsolidated Undrained (UU) condition. The results were analyzed and plotted as Mohr circles to obtain the effective cohesion and effective angle of friction for each sample. Mohr circle derived from Triaxial UU test on sample B (16% gravimetric water content) is shown in Figure 6. The figure shows that the soil prepared at gravimetric water content of 16% had an apparent cohesion of 68 kPa and effective friction angle of 34°.

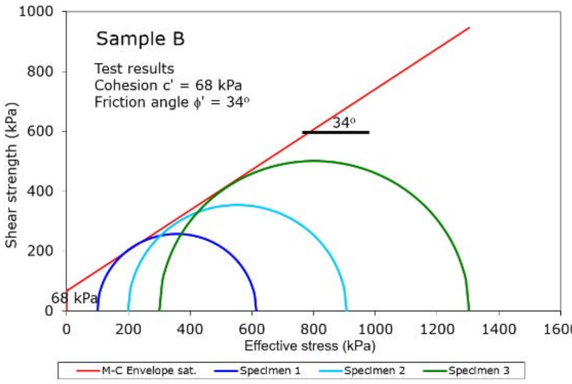
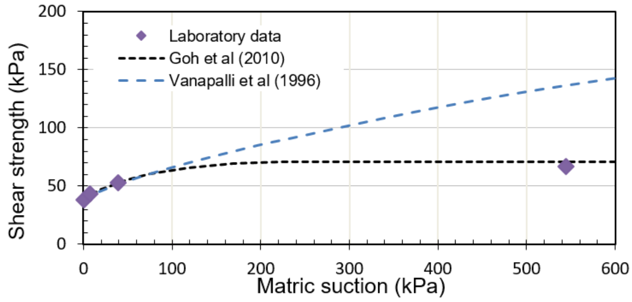
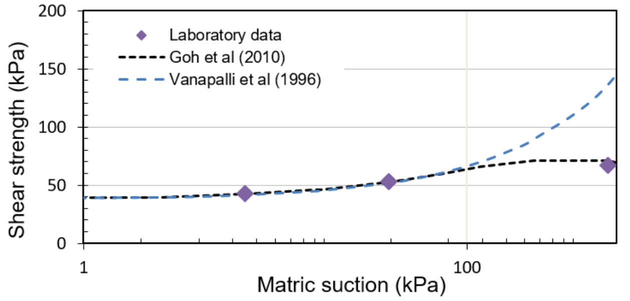


Figure 6 Mohr circle derived from Triaxial UU test on sample B.

Results of all tests (sample A, B, C, and D) in terms of apparent cohesion was plotted in Figure 7a with suction in linear scale and Figure 7b with suction in logarithmic scale. The unsaturated soil shear strength properties was predicted using equation (7) as well as equations (8) and (9). Note that equation (8) is used for $\psi < \psi_a$ while equation (9) was used for $\psi \geq \psi_a$. In this case $\psi_a = 8$ kPa. Observation of Figure 7b indicated that both equations are in good agreement with laboratory data obtained for samples A, B, and C or for suction below 100 kPa. However, when data obtained from sample D was included, the prediction using Goh et al (2010) equation showed a better agreement as compared to Vanapalli et al. (1996) equation. Extension of the data to zero suction indicated that the soil has an effective cohesion c'=38 kPa. The internal friction angle of the soil was estimated based on the results of the shear strength test on samples A and B i.e. ϕ'=34°. Thus, based on Figure 7a, the rate of increase of shear strength with suction $\phi^b = \phi' = 34°$ for suction less or equal to the air entry value. For suction between 8 and 100 kPa, $\phi^b = 0.5\ \phi' = 17°$ while for suction greater than 100 kPa, $\phi^b = 0°$. Previous publications revealed that, in tropical environments, field suction rarely increased to more than 100 kPa.



(a)



(b)

Figure 7 Rate of increase in shear strength with respect to suction

5. CONCLUSIONS

Fitting and predictive type equations were used in this study to determine unsaturated soil properties based suction measurements using filter paper method and shear strength measurements using simple Triaxial UU test. The samples were prepared under four different gravimetric water contents. Equations for Whatman No.42 filter paper was used to calculate suction for each sample. Fredlund and Xing (1994) fitting equation was used to plot the SWCC curve. Then, permeability curve was obtained using statistical method and fitting equation by Leong & Rahardjo (1997b) based on the coefficient of saturated permeability and the SWCC curve. The characteristic values from SWCC curve are $\theta_s = 0.37$, $\psi_a = 8$ kPa, $\theta_r = 0.15$ and $\psi_r = 3700$ kPa. The effective strength parameters of the soil are given by $c' = 38$ kPa and $\phi' = 34°$. The shear strength data in terms of apparent cohesion obtained from Triaxial UU tests were used to predict the rate of strength increase due to increasing suction (ϕ^b). The contribution of suction on the shear strength of the soil decreases with increasing suction. It varies from $\phi^b = \phi' = 34°$ to 0 at suction 100 kPa. This means that, for this soil, suction greater than 100 kPa is no longer a state of stress. Since the range of field suction is less than 100 kPa, both Goh et al (2010) and Vanapalli et al. (1997) equation could be used to estimate the rate of shear strength increase with respect to suction.

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