Duncan-Chang - Parameters for Hyperbolic Stress Strain Behaviour of Soft Bangkok Clay

Duncan-Chang - Paramètres de comportement contrainte-déformation hyperbolique d'argile molle de Bangkok

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ABSTRACT: This paper is on the analyses of the stress strain data of soft and stiff Bangkok Clays carried out at the Asian Institute of Technology. A comprehensive series of triaxial tests both in compression and extension was carried out and the results were compared with a number of stress-strain theories as based on critical state concepts. The finite element software PLAXIS contains the Hardening Soil Model as an extension of the Duncan-Chang hyperbolic stress strain model. In this paper, the parameters for the hyperbolic stress stain model are evaluated from the data gathered previously in testing soft and stiff Bangkok clays. The testing program includes two series of undrained and drained tests performed on isotropically consolidated triaxial samples both under compression and extension conditions. These testing results can be used to determine the undrained and drained shear strength parameters for soft and stiff Bangkok Clays. Finally, two set of undrained and drained shear strength and stiffness parameters for the Hardening Soil Model of soft and stiff Bangkok Clays are presented.

RÉSUMÉ: Cet article porte sur les analyses des données de contrainte-déformation des argiles molles et rigides Bangkok menées à l'Institut asiatique de technologie. Une série complète d'essais triaxiaux portent à la fois sur la compression et l'extension a été réalisée, et les résultats ont été comparés avec un certain nombre de contraintes-déformations, théories basées sur des concepts d'état critique. Le logiciel PLAXIS contient le modèle Hardening Soil comme une extension du modèle de contrainte hyperbolique de Duncan-Chang. Dans le présent document, les paramètres du modèle hyperbolique de contraintes sont évalués à partir des données recueillies pour tester les argiles douces et raides Bangkok. Le programme comprend deux séries d'essais non drainés et drainés, effectués sur des échantillons triaxiaux isotropes consolidés à la fois dans des conditions de compression et d'extension. Ces résultats d'essai peuvent être utilisés pour déterminer les paramètres non drainés et drainés de résistance au cisaillement des argiles molles et raides de Bangkok. Enfin, deux résultats de la résistance au cisaillement, non drainée et drainée, et des paramètres de rigidité pour le modèle des argiles molles et rigides Bangkok sont présentés.

KEYWORDS: shear strength parameters, hardening soil model, triaxial tests, Bangkok clay, finite element analysis

1. INTRODUCTION

Bangkok subsoils are one of the most well-known sedimentary soils and have been studied extensively in the past by many research students at the Asian Institute of Technology under the supervision of the fourth author. The experimental work was on isotropically and anisotropically consolidated triaxial tests both in compression and in extension. The results were primarily used to verify the critical state theories as developed for normally and overconsolidated clays (Balasubramaniam & Chaudry, 1978; Balasubramaniam *et al.*, 1978, 1992; Balasubramaniam & Hwang, 1980). Recently, soil models used in PLAXIS such as the Hardening Soil Model (HSM), and the Hardening Soil Model with Small Strain Behaviour (HSS) were studied by Surarak (2010) on the applications in the design and performance of deep excavations and tunnelling works in Bangkok MRT project. The HSM have been developed under the framework of the Duncan-Chang hyperbolic stress strain theory. This paper presents the work on stiffness and strength parameters based on Duncan-Chang theory.

2. GEOLOGICAL CONDITIONS OF BANGKOK SUBSOIL

The Bangkok subsoil forms a part of the larger Chao Phraya Plain and consists of a broad basin filled with sedimentary soil deposits. These deposits form alternate layers of sand, gravel and clay. While the depth of the bedrock is still undetermined, its level in the Bangkok area is known to vary between 400 m to 1,800 m depth. Based on extensive field and laboratory studies carried out in the past by numerous researchers at AIT, the following descriptions have been proposed for the Bangkok clays: (1) Weathered Crust – the upper most layer (1 – 3 m), (2) Very Soft to Soft Bangkok Clays (3 – 12 m), (3) Stiff to Very Stiff Bangkok Clays (15 – 35 m). The index properties for Weathered, Soft and Stiff Bangkok Clays are summarised in Table 1.

Table 1. Index Properties of Bangkok Clays

Properties	Weathered Clay	Soft Clay	Stiff Clay
Natural water content (%)	133 ± 5	122 – 130	20 – 24
Natural voids ratio	3.86 ± 0.15	3.11 - 3.64	1.10 - 1.30
Grain size distribution			
Sand (%)	7.5	4.0	23
Silt (%)	23.5	31.7	43
Clay (%)	69	64.3	34
Specific gravity	2.73	2.75	2.74
Liquid limit (%)	123 ± 2	118 ± 1	46 ± 2
Plastic limit (%)	41 ± 2	43 ± 0.5	19 ± 2
Dry unit weight (kN/m^3)	15.8 ± 0.3	16.5	15.5 -16.5
Consistency	Soft	Soft	Stiff

Colour	Dark grey	Greenish grey	Greenish grey
Degree of saturation (%)	95 ± 2	98 ± 2	94 – 100

3. HARDENING SOIL MODEL

The PLAXIS finite element software became popular in geotechnical analysis and design. Constitutive models used in PLAXIS are in line from the linear and non-linear elastic models until the hardening double surface plasticity models (Schweiger, 2009). One of the most well-known hardening type models is the Hardening Soil Model (HSM). The HSM was introduced in the PLAXIS program as an extension of the Mohr-Coulomb model to allow for the pre-consolidation pressure to be taken into account. Indeed, the HSM has been developed under the framework of the plasticity theory. The hyperbolic formulation (Duncan & Chang, 1970) is used to define the stress-strain relationship. The total strains are calculated using a stress-dependent stiffness, which is different for both loading and unloading/reloading. The hardening is assumed to be isotropic, depending on the plastic shear and volumetric strains. A non-associated flow rule is adopted when related to frictional hardening and an associated flow rule is assumed for the cap hardening. The following explanation provides a brief summary of the hyperbolic stress-strain and stiffness response of HSM.

The stress-strain relationship, due to the primary loading, is assumed to be a hyperbolic curve in the HSM. The hyperbolic function, as given by Duncan & Chang (1970), for the drained triaxial test can be formulated as:

$$\varepsilon_1 = \frac{q_a}{2E_{50}}\frac{q}{q_a - q}, \quad \text{for } q < q_f \tag{1}$$

where ε_1 is the axial strain, and q is the deviatoric stress. The ultimate deviatoric stress (q_f) is defined as:

$$q_f = \frac{6\sin\phi'}{3-\sin\phi'}\left(\sigma_3' + c'\cot\phi'\right) \tag{2}$$

and the quantity (q_a) is the asymptotic value of the shear strength, in which $q_a = q_f/R_f$. The R_f is the failure ratio. Figure 1 shows the hyperbolic relationship of stress and strain in primary loading.

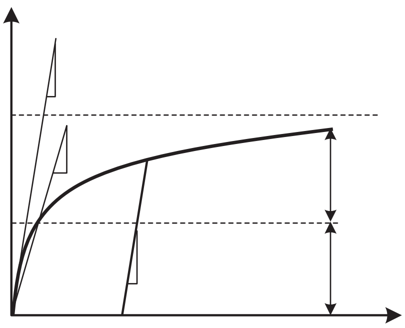


Figure 1. Hyperbolic stress-strain relationship in primary loading for a standard drained triaxial test (Schanz *et al.*, 1999)

The stress strain behaviour for primary loading is highly non-linear. The parameter E_{50} is a confining stress dependent stiffness modulus for primary loading. E_{50} is used instead of the initial modulus E_0 for small strain which, as a tangent modulus, is more difficult to determine experimentally, and is given as:

$$E_{50} = E_{50}^{ref}\left(\frac{c'\cos\phi' - \sigma_3'\sin\phi'}{c'\cos\phi' + p^{ref}\sin\phi'}\right)^m \tag{3}$$

where E_{50}^{ref} is a reference stiffness modulus corresponding to the reference stress p^{ref} (100 kN/m^2). The actual stiffness depends on the minor effective principal stress σ_3', which is the effective confining pressure in a triaxial test. The amount of stress dependency is given by the power m.

The stress dependent stiffness modulus for unloading an reloading stress paths is calculated as:

$$E_{ur} = E_{ur}^{ref}\left(\frac{c'\cos\phi' - \sigma_3'\sin\phi'}{c'\cos\phi' + p^{ref}\sin\phi'}\right)^m \tag{4}$$

where E_{ur}^{ref} is the reference modulus for unloading and reloading, which corresponds to the reference pressure p^{ref}.

Another input parameter, the reference oedometer modulus (E_{oed}^{ref}), is used to control the magnitude of the plastic volumetric strains that originate from the yield cap. In a similar manner to the triaxial moduli, the oedometer modulus (E_{oed}) obeys the stress dependency law.

Schanz *et al.* (1999) explained in detail, the formulation and verification of the HSM. A total of 10 input parameters are required in the Hardening Soil Model, as tabulated in Table 2.

Table 2. Hardening Soil Model Input Parameters

Parameter	Description	Parameter evaluation
ϕ'	Internal friction angle	Slope of failure line from MC failure criterion
c'	Cohesion	y-intercept of failure line from MC failure criterion
R_f	Failure ratio	$(\sigma_1-\sigma_3)_f / (\sigma_1-\sigma_3)_{ult}$
ψ	Dilatancy angle	Function of ε_a and ε_v
E_{50}^{ref}	Reference secant stiffness from drained triaxial test	y-intercept in log(σ_3/p^{ref})-log(E_{50}) space
E_{oed}^{ref}	Reference tangent stiffness for oedometer primary loading	y-intercept in log(σ_1/p^{ref})-log(E_{oed}) space
E_{ur}^{ref}	Reference unloading/reloading stiffness	y-intercept in log(σ_3/p^{ref})-log(E_{ur}) space
m	Exponential power	Slope of trend-line in log(σ_3/p^{ref})-log(E_{50}) space
ν_{ur}	Unloading/reloading Poisson's ratio	0.2
K_o^{nc}	Coefficient of earth pressure at rest (NC state)	1-sinϕ'

3. EVALUATION OF STRENGTH AND STIFFNESS PARAMETERS FOR BANGKOK CLAYS

All the test results analysed in this study were determined for undisturbed samples taken at the appropriate depths for soft clay, medium stiff clay and stiff clay. The 25.4 mm diameter thin walled sample tubes were used for soft and medium stiff clays are used for triaxial tests in weathered, soft and medium stiff clays.

3.1 *Triaxial Tests*

The results of several series of compression and extension tests carried out on weathered, soft and stiff clays are analysed. Test specimens were approximately 72 mm in height and 36 mm in diameter. Several series of isotropically consolidated drained and undrained compression (CID, CIU) and extension (CIUE, CIDE) tests carried out at the Asian Institute of Technology were re-analysed in this study. Most of the CID, CIU, CIDE and CIUE tests were carried out under strain controlled conditions (Hassan 1976; Balasubramaniam & Uddin, 1977). In addition, some load controlled CID and CIU tests were also considered (Balasubramaniam & Chaudhry, 1978).

3.2 *Mohr-Coulomb Strength Parameters*

Table 3 presents a summary of the Mohr-Coulomb strength parameters of the Bangkok subsoils (i.e. weathered clay, soft clay, stiff clay and hard clay) obtained from consolidated isotropically drained and undrained triaxial compression (CID and CIU) and extension (CIDE and CIUE) tests reported in the literature. The notations for the triaxial tests identified in Table 3 are explained. It can be seen that the differences in the applied stress path have the most significant effect on the Mohr-Coulomb strength parameters. Initial conditions at the consolidation state (i.e. isotropic or anisotropic), as well as the drainage conditions during shear (i.e. drained or undrained), also have an effect on the strength parameters, but to a lesser magnitude. Therefore, it needs to be emphasised that the strength parameters should be carefully selected according to the applied stress path, resulting from the construction sequences.

Table 3. Summary of Mohr-Coulomb Strength Parameter of Bangkok Subsoils

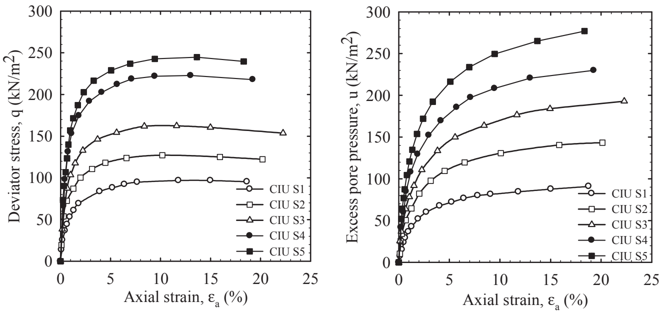
Reference	Depth (m)	Test type	ϕ' (o)	c' (kN/m^2)
Weathered Clay				
Balasubramaniam & Uddin (1977)	2.5 to 3.0	$CIUE_U$	28.9	0
Balasubramaniam *et al.* (1978)	2.5 to 3.0	CIU	22.2	0
		CID	23.5	0
		$CIUE^U$	29	0
Soft Clay				
Balasubramaniam & Chaudhry (1978)	5.5 to 6.0	CIU	26	0
		CID	21.7	0
Balasubramaniam *et. al.* (1978)	5.5 to 6.0	CIU	24	38
		CID	23.5	0
		CID^P	23.7	0
		$CIUE^L$	26	0
		$CIUE^U$	21.1	58.7
		$CIDE^L$	26.2	0
		$CIDE^U$	23.5	31.8
Stiff Clay				
Balasubramaniam *et al.* (1978)	16.0 to 16.6	CID	26	30
		$CIUE^L$	18	54
		$CIUE^U$	25	54
		$CIDE^U$	16.6	11
Hassan (1976)	17.0 to 18.0	CIU	28.1	11.4
		CID	26.3	32.8

3.3 *Stiffness and Strength Parameters of Soft and Stiffness Bangkok Clays*

3.3.1 *Soft Bangkok Clay*

Two series of isotropically consolidated triaxial compression tests, CIU and CID, conducted by Balasubramaniam & Chaudhry (1978) on soft and stiff Bangkok clay, were analysed in this study. The soil samples were taken from a depth of 6.0 m below the ground surface. The confining pressures, σ'_3 used for both CIU and CID series were 138, 207, 276, 345 and 414 kN/m^2 for tests S1 to S5, respectively. The angle of the internal friction (ϕ') obtained from the CIU and CID tests were 27^o and 23.6^o; whereas, the cohesion (c') was zero for both series. The drained strength parameters are summarised in Table 4.

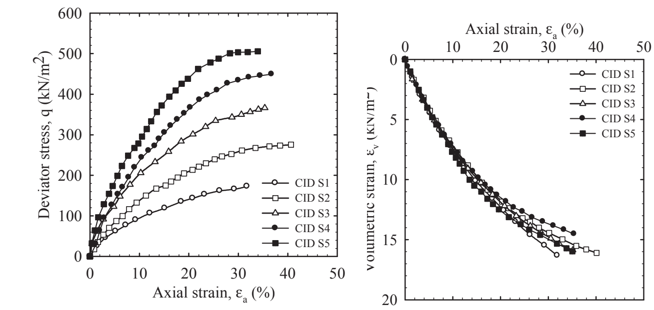
The results of the CIU triaxial tests carried out on the soft clay are plotted in Figure 2. The (q, ε_a) and (u, ε_a) relationships are shown in Figures 2(a) and 2(b), respectively. The deviator stress and excess pore pressures versus the axial strain relationships show typical normally to lightly overconslidated clay behaviour, where the deviator stress and excess pore pressure reaches their ultimate values at a relatively large strain. Moreover, all the excess pore pressure plots were located in the positive range.



(a) Deviator stress vs axial strain (b) Pore pressure vs axial strain

Figure 2. Results of CIU triaxial tests on soft Bangkok clay.

The results obtained from the CID triaxial tests for the soft clay are shown in Figure 3, with the relationships of (q, ε_a) and (ε_v, ε_a) plotted in Figures 3(a) and 3(b), respectively. It can be seen that, during the deviator stress applied, the volume of the soil specimen gradually reduces. The volumetric and axial strain curves of all the tests seem to coincide up to 10% axial strain, after that they tend to divert slightly.



(a) Deviator stress vs axial strain (b) Volumetric strain vs axial strain

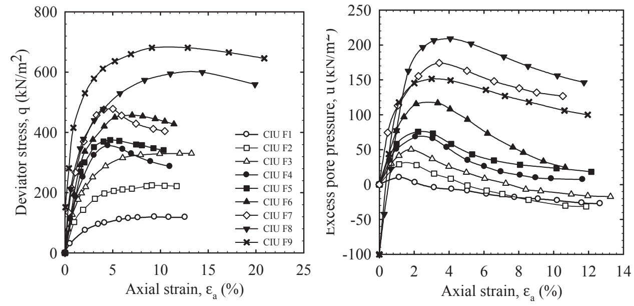
Figure 3. Results of CID triaxial tests on soft Bangkok clay

These values are also summarised in Table 4 together with the reference initial modulus (E_i^{ref}, $E_{i,50}^{ref}$), the reference moduli at 50% of strength (E_{50}^{ref}, $E_{u,50}^{ref}$), and the failure ratio (R_f) resulting from CID tests as well as the shear strength parameters (c', ϕ') for Soft Bangkok Clay.

3.3.2 *Stiff Bangkok Clay*

The two series of isotropically consolidated triaxial compression tests, CIU and CID, conducted by Hassan (1976) on stiff Bangkok clay, are re-interpreted in this study. The undisturbed soils samples were collected from a depth of 17.4 to 18 m below the ground surface. The pre-shear consolidation pressures ranged from 17 to 620 kN/m^2 and 34 to 552 kN/m^2, for the CIU and CID series, respectively. The angles of the internal friction (ϕ') from the CIU and CID series were 28.1 and 26.3 degrees; whereas, the values of cohesion (c') were 11.4 and 32.8 kN/m^2, respectively. The drained strength parameters are summarised in Table 4.

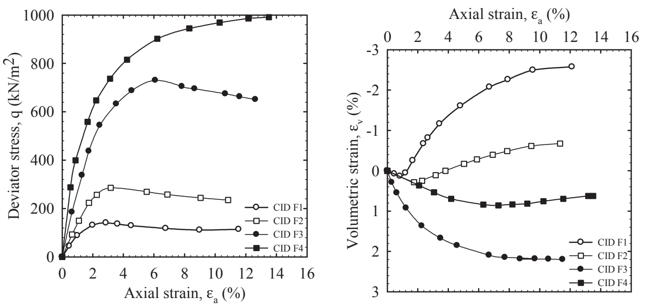
Figure 4 shows the results of CIU tests on the stiff Bangkok clay. It can be seen from Figure 4(a) that (q, ε_a) relationships, up to a pre-shear confining pressure of 138 kN/m^2 (tests CIU F1 to F3), exhibit no strain softening. At a level of confining pressure from 207 to 414 kN/m^2 (tests CIU F4 to F7), these clay samples behaved as heavily overconsolidated clay showing a clear peak deviator stress at a low axial strain, followed by a strain softening. Beyond the confining pressure of 552 kN/m^2 (tests CIU F8 and F9), these samples behaved as lightly overconsolidated clay.



(a) Deviator stress vs axial strain (b) Pore pressure vs axial strain

Figure 4. Results of CIU triaxial tests on stiff Bangkok clay

The relationships between the excess pore pressure and the axial strain are shown in Figure 4(b). For all clay samples (CIU F1 to F9), the excess pore pressure increases as the deviator stress increases, until the peak values are reached at 1 to 4% axial strain, depending on the confining pressure. The peak excess pore pressure seems to be reached at a higher axial strain as the confining pressure increases. As the sample was further sheared, the excess pore pressure gradually reduced to the minimum value, at approximately 12% axial strain. Only the first three samples (tests CIU F1 to F3) reached negative excess pore pressures.



(a) Deviator stress vs axial strain (b) Volumetric strain vs axial strain

Figure 5. Results of CID triaxial tests on stiff Bangkok clay

The results of CID triaxial tests carried out on the stiff Bangkok clay are shown in Figure 5. The deviator stress versus the axial strain relationships of the stiff clay are shown in Figure 5(a). The pre-shear confining pressures of 34, 103, 414 and 552 kN/m^2 were applied. None of the stiff clay samples demonstrated a well defined peak. However, samples CID F1 to F3 (with confining pressure of 34, 103 and 414 kN/m^2) illustrate some degree of strain softening after the peak deviator stresses are reached at axial strain levels of 3 to 5%. The plots of the volumetric versus the axial strain are given in Figure 5(b). The specimens with a confining pressure of 34 and 103 kN/m^2 (tests CID F1 and 2) start to dilate at about 1.2 and 3.5% axial strain. The specimen at 414 kN/m^2 confining pressure consolidates up to an axial strain level of 8%. After that, the volumetric strain seems to be constant with an increase in axial strain. The last specimen with a confining pressure of 552 kN/m^2 consolidates up to 7% of the axial strain, and then it tends to dilate.

The values of E_{50}^{ref} and $E_{u,50}^{ref}$ together with the deformation moduli and the failure ratios resulting from the CIU and CID series are also summarised in Table 4. It can be observed from Table 4 that the failure ratio (R_f) falls in a narrow range with an average value of 0.88. The power *m* for both the initial and the 50% moduli are approximately 0.5.

Table 4. Stiffness and strength parameters from CID and CIU tests for Bangkok Clays

Parameters		CID	CIU
Soft Clay			
Confining pressure (kN/m^2)		138 – 414	138 – 414
Initial	E_i^{ref}, $E_{u,i}^{ref}$ (kN/m^2)	1343	7690
	m	1.0	1.2
50%	E_{50}^{ref}, $E_{u,50}^{ref}$ (kN/m^2)	690	4831
	m	1.1	1.0
	R_f	0.72	0.94
	ϕ'	23.6	27.0
	c' (kN/m^2)	0	0
Stiff Clay			
Confining pressure (kN/m^2)		34 – 552	17 – 620
Initial	E_i^{ref}, $E_{u,i}^{ref}$ (kN/m^2)	29676	30109
	m	0.52	0.46
50%	E_{50}^{ref}, $E_{u,50}^{ref}$ (kN/m^2)	14398	11104
	m	0.48	0.53
	R_f	0.89	0.88
	ϕ'	26.3	28.1
	c' (kN/m^2)	32.8	11.4

4. CONCLUSION

In this study, the experimental data on soft and stiff Bangkok clays available in the literature was reanalysed in order to obtain the Duncan-Chang stiffness and strength parameters required for the Hardening Soil Model. Undrained and drained behaviour of Soft and Stiff Bangkok Clays was modelled using these parameters.

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