Effect of Multilayered Geosynthetic Reinforcements on the Response of Foundations Resting on Stone Column-Improved Soft Soil

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ABSTRACT: The present paper pertains to the development of a mechanical model based on soil-structure interaction to study the effect of multilayered geosynthetic reinforcements on the behaviour of footings resting on stone column-improved soft soil. The footing is idealized as a beam. The soft soil and granular layer are idealized as nonlinear spring-dashpot and Pasternak shear layer, respectively. The geosynthetic reinforcements are modelled by elastic membranes. The stone columns are idealized by nonlinear springs. The governing differential equations are solved by finite difference method and results are presented in non-dimensional term. It is observed that multilayered-reinforced system is not effective for settlement reduction, but it is effective for bending moment and shear force reduction. However, for higher modular ratio (>40), the multilayered-reinforced system is not useful for maximum bending moment reduction. As the modular ratio increases positive bending moment at the centre of the beam decreases and the positive bending moment of the beam above middle of the stone column becomes negative. The negative bending moment of the beam above middle of the stone column increases as the modular ratio increases. The maximum shear force is observed for s/b_w ratio 3 and 5 corresponding to the modular ratio 10 and 100, respectively.

KEYWORDS: Beams, Granular layer, Multilayered geosynthetic reinforcements, Stone column, Soft soil, Soil-structure interaction

1. INTRODUCTION

Granular fills containing multilayer geosynthetic reinforcements can be placed over the soft soil to improve the settlement and bearing capacity. Use of stone column within the soft soil also increases the bearing capacity and decreases the settlement. Models have been developed to study the load-settlement behaviour of single layered geosynthetic-reinforced granular fill-soft soil system with or without beams (Madhav and Poorooshasb, 1988; Ghosh and Madhav, 1994; Shukla and Chandra, 1994; Yin, 1997; Maheshwari et al., 2004). Models have also been developed for pullout of reinforcements (Shahu and Hayashi, 2009; Patra and Shahu, 2012). Nogami and Yong (2003) studied the response of a multilayered geosynthetic-reinforced soil bed subjected to structural loading. Each soil layer has been modelled by a system of an infinite number of closely spaced one-dimensional columns connected with horizontal springs. It is observed that for higher loading intensity, multilayer reinforcements placed within the soft soil without any gravel layer is showing better response of the reinforced bed as compared to the single reinforcement layer placed within the gravel layer. Deb et al. (2005) developed a mechanical model for inextensible multilayered-reinforced granular fill resting on soft soil. A nonlinear model for extensible multilayered-reinforced soil has also been developed by Deb et al. (2007a). Significant reduction in the settlement has been observed due to the use of multilayered geosynthetic reinforcements. Deb et al (2007b) conducted a numerical study to investigate the behavior of multi layer geosynthetic-reinforced granular bed overlying a soft soil using the FLAC program. It has been observed that the settlement and distribution of vertical, lateral and shear stresses in the soil are greatly affected as the number of reinforcement layers is increased. Chakraborty and Kumar (2014a, b) proposed methodology to determine the ultimate bearing capacity of strip and circular footings placed over granular and cohesive-frictional soils reinforced with single and multiple layers of horizontal reinforcement. However, in the available models or studies on multilayered geosynthetic-reinforced system, the effect of the stone columns was not considered.

Deb et al. (2008) developed a mechanical model to study the behavior of multilayered geosynthetic-reinforced granular fill resting over stone column-reinforced soft soil. It has been observed that in case of stone column-improved ground, the multi layer reinforcement system is not much effective as compared to single layer reinforcement to reduce the total settlement as considerable amount of settlement has been reduced due to stone column itself. However, multilayered reinforcement system is effective to transfer the stress from soft soil to stone column. In the available models on multilayered geosynthetic-reinforced system with or without stone columns, only the settlement behaviour has been studied. However, in the design of foundation, bending moment and shear force are also very important design parameters in addition to settlement. Thus, it is necessary to determine the settlement as well as bending moment and shear force of the foundation resting on ground improved with multilayered geosynthetic reinforcements and stone columns. Idealizing foundation as beam, the effect of multilayered geosynthetic reinforcements and stone columns on the settlement, bending moment and shear force can be studied.

Balaam and Booker (1981) studied the behaviour of rigid rafts supported by stone columns or granular pile. An analytical solution using the theory of elasticity has been developed for evaluating the settlement of foundation. Expressions are also developed for determining the moment and shear distribution. Das and Deb (2014) developed a mechanical model for circular raft resting on stone column-improved ground subjected to uniformly distributed loading. Das and Deb (2017) studied the response of cylindrical storage tank foundation resting on tensionless stone column-improved soil. It is observed that as the stiffness ratio or modular ratio (ratio between the stiffness of stone column and stiffness of the soft soil) increases maximum settlement decreases whereas maximum bending moment and shear force of the raft foundation increase. Again as the flexural rigidity of the foundation increases the settlement decreases, but bending moment and shear force increase. Thus, proper modular ratio and flexural rigidity have to be chosen for design of foundation resting on stone column-improved ground to get optimum value of bending moment, shear force and settlement of the foundation system. Deb and Dhar (2013) proposed a simulation-optimization based methodology to determine the optimum value of design parameters of the system of beams (idealized as foundation) resting on stone column-improved soft soil. However, the effects of multilayered geosynthetic reinforcements on the settlement, bending moment and shear forces are not studied. Deb (2012) presented soil-structure interaction analysis for beams resting on multilayered geosynthetic-reinforced granular fill-soft soil system. It is observed that in case of very rigid beam, the use of geosynthetic reinforcement is not very effective for maximum settlement reduction. However, multilayered-reinforced system is very effective for bending moment, shear force and differential settlement reduction. In the developed model on beams resting on multilayered-reinforced system, the effect of stone column has not been considered. In this paper, based on soil-structure interaction analysis, a mechanical model has been developed to study the behavior of beams resting on ground improved with multilayered geosynthetic reinforcements and stone columns. The effect of flexural rigidity of the beam, stiffness and spacing to diameter ratio of the stone column, degree of consolidation of the soft soil due to inclusions of stone columns on the maximum settlement, bending moment and shear force of the beam has been studied. The effect of properties of soft soil and granular fill on the behavior of beam is also studied.

2. MODEL DEVELOPMENT

A multilayered geosynthetic-reinforced granular fill on soft foundation soil improved with stone columns is shown in Figure 1. In this model, the soft soil and the granular fill have been idealized by number of spring-dashpots and Pasternak shear layer, respectively. The stone columns are idealized by nonlinear springs. In the model, two stone columns are considered either side of the beam at a distance of $0.5B$ (where B is the half width of the beam) from the centre. However, model can handle any number of stone columns placed at any location. Stretched rough elastic membranes represent the geosynthetic reinforcement layers. Three geosynthetic layers with equal spacing are considered within the granular layer. However, the model is capable to predict the settlement, bending moment and shear force of the beam for unequal spacing between the reinforcements. The footing is idealized as a beam. Plane strain condition is considered for the loading and the reinforced foundation soil system. A footing load of intensity q is applied over a beam of width $2B$ resting on the multilayered geosynthetic-reinforced granular fill of width $2L$ over stone column-improved soft soil (as shown in Figure 1).

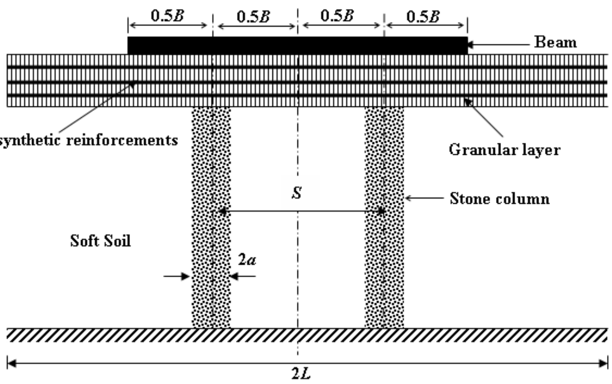

Figure 1 Beam resting on multilayered geosynthetic-reinforced granular fill on stone column-improved soft soil

The general assumptions are similar to the assumptions considered by Deb et al. (2005) and they are: (1) stone columns and the surrounding soil settle only in the vertical direction; (2) equal amount of strain is considered at the interface between the stone column and soft soil at any depth; (3) immediate settlement of the soil has been ignored as it is very small as compared to the subsequent primary consolidation settlement of the soft soil (4) soft foundation soil is fully saturated; (5) geosynthetic reinforcements are linearly elastic. No slippage between the soil and geosynthetics is considered as it is assumed that the geosynthetics are rough enough to develop full frictional resistance even at a negligibly small displacement. Thickness of reinforcement is neglected; (6) modulus of subgrade reaction of soft soil and stone columns are constant irrespective of depth and time; (7) self weight of the soil is ignored; (8) geosynthetic reinforcements are assumed to be inextensible in nature with stiffness greater than or equal to 4000 kN/m; as beyond this value the stiffness of the reinforcement has no effect on the settlement response (Han and Gabr, 2002) (9) creep in geosynthetics has been ignored.

The differential equation of bending of a beam is written as:

$$EI\frac{\partial^4 w}{\partial x^4} = q - p \tag{1}$$

where EI is the flexural rigidity of the beam, E is the elastic modulus and I is the moment of inertia of the beam, q is surface loads (uniformly distributed load) and p is the foundation bearing pressure. The upward-acting shearing force to the left is considered as positive and the corresponding clockwise bending moment acting from the left is considered as positive bending moment.

The normal stresses and the mobilized tension for elements of the different geosynthetic reinforcement layers are obtained as proposed by Deb et al. (2005). The three reinforcement layers divide the shear layer into four parts. According to the Pasternak shear layer concept, the vertical force equilibrium for element of shear layer 1, 2, 3 and 4 (stating from top) can be written as:

$$p = q_1 - G_1 H_1 \frac{d^2 w}{dx^2} \tag{2}$$

$$q_2 = q_3 - G_2 H_2 \frac{d^2 w}{dx^2} \tag{3}$$

$$q_4 = q_5 - G_3 H_3 \frac{d^2 w}{dx^2} \tag{4}$$

$$q_6 = q_b - G_4 H_4 \frac{d^2 w}{dx^2} \tag{5}$$

where H_1, H_2, H_3 and H_4 are the thickness of the granular layer between the geosynthetic reinforcements (stating from top); p and q_1 are the average normal stress acting on the top and bottom of the element of shear layer 1, respectively; q_2 and q_3 are the average normal stress acting on the top and bottom of the element of shear layer 2, respectively; q_4 and q_5 are the average normal stress acting on the top and bottom of the element shear layer 3, respectively; q_6 and q_b are the average normal stress acting on the top and bottom of the bottom shear layer element 4, respectively; and w is the vertical displacement. The q_b is the average normal stress acting on the soft soil (within the soft soil region) or stone columns (within the stone column region). The expression of shear modulus for the different shear layer is expressed in nonlinear form as (Ghosh and Madhav, 1994):

$$G_j = \frac{G_{j0}}{\left[1 + \frac{G_{j0}\,|\partial w/\partial x|}{\tau_{uj}}\right]^2}, \quad j = 1, 2, 3, 4 \tag{6}$$

where G_{j0} is initial shear modulus of the shear layer between the geosynthetic reinforcements (stating from top); τ_{uj} is ultimate shear resistance of the shear layer between the geosynthetic reinforcements (stating from top); $\partial w/\partial x$ is the shear strain.

The reinforcement layers are modelled according to Shukla and Chandra (1994). The normal stresses and the mobilized tension for elements of the geosynthetic reinforcement layers are obtained as:
For the top reinforcement layer:

$$q_1 = \overline{C}_1 q_2 - \overline{C}_2 T_1 \cos\theta \frac{d^2 w}{dx^2} \tag{7}$$

$$\frac{dT_1}{dx}=-\overline{D}_1 q_1-\overline{D}_2 q_2 \tag{8}$$

For the middle reinforcement layer:

$$q_3=\overline{C}_3 q_4-\overline{C}_4 T_2\cos\theta\frac{d^2w}{dx^2} \tag{9}$$

$$\frac{dT_2}{dx}=-\overline{D}_3 q_3-\overline{D}_4 q_4 \tag{10}$$

For the bottom reinforcement layer:

$$q_5=\overline{C}_5 q_6-\overline{C}_6 T_3\cos\theta\frac{d^2w}{dx^2} \tag{11}$$

$$\frac{dT_3}{dx}=-\overline{D}_5 q_5-\overline{D}_6 q_6 \tag{12}$$

Where

$$\overline{C}_1=\frac{1+K_0\tan^2\theta-(1-K_0)\mu_2\tan\theta}{1+K_0\tan^2\theta+(1-K_0)\mu_1\tan\theta}$$

$$\overline{C}_2=\frac{1}{1+K_0\tan^2\theta+(1-K_0)\mu_1\tan\theta}$$

$$\overline{C}_3=\frac{1+K_0\tan^2\theta-(1-K_0)\mu_3\tan\theta}{1+K_0\tan^2\theta+(1-K_0)\mu_2\tan\theta}$$

$$\overline{C}_4=\frac{1}{1+K_0\tan^2\theta+(1-K_0)\mu_2\tan\theta}$$

$$\overline{C}_5=\frac{1+K_0\tan^2\theta-(1-K_0)\mu_4\tan\theta}{1+K_0\tan^2\theta+(1-K_0)\mu_3\tan\theta}$$

$$\overline{C}_6=\frac{1}{1+K_0\tan^2\theta+(1-K_0)\mu_3\tan\theta}$$

$$\overline{D}_1=\mu_1\cos\theta(1+K_0\tan^2\theta)-(1-K_0)\sin\theta$$

$$\overline{D}_2=\mu_2\cos\theta(1+K_0\tan^2\theta)+(1-K_0)\sin\theta$$

$$\overline{D}_3=\mu_2\cos\theta(1+K_0\tan^2\theta)-(1-K_0)\sin\theta$$

$$\overline{D}_4=\mu_3\cos\theta(1+K_0\tan^2\theta)+(1-K_0)\sin\theta$$

$$\overline{D}_5=\mu_3\cos\theta(1+K_0\tan^2\theta)-(1-K_0)\sin\theta$$

$$\overline{D}_6=\mu_4\cos\theta(1+K_0\tan^2\theta)+(1-K_0)\sin\theta$$

where T_1, T_2 and T_3 are mobilized tension in the top, middle and bottom geosynthetic layer, respectively; θ is slope of the membranes; μ_1 and μ_2 are interface friction at the top and bottom of the top geosynthetic layer, respectively; μ_2 and μ_3 are interface friction at the top and bottom of the middle geosynthetic layer, respectively; μ_3 and μ_4 are interface friction at the top and bottom of the bottom geosynthetic layer, respectively; K_0 is the coefficient of lateral earth pressure for the normally consolidated soil at rest is assumed to be equal to $1-\sin\phi$ (Brooker and Ireland, 1965; Alpan, 1967) and $\tan\theta = -\partial w/\partial x$. Combining Eqs. (2) to (12) one can get,

$$p=\overline{C}_1\overline{C}_3\overline{C}_5 q_b-\left\{\begin{aligned}&G_1H_1+\overline{C}_1G_2H_2+\overline{C}_1\overline{C}_3G_3H_3\\&+\overline{C}_1\overline{C}_3\overline{C}_5G_4H_4+\overline{C}_2T_1\cos\theta\\&+\overline{C}_1\overline{C}_4T_2\cos\theta+\overline{C}_1\overline{C}_3\overline{C}_6T_3\cos\theta\end{aligned}\right\}\frac{\partial^2w}{\partial x^2} \tag{13}$$

$$\frac{\partial T_1}{\partial x}=-\overline{D}_1\left(p+G_1H_1\frac{\partial^2w}{\partial x^2}\right)-\overline{D}_2\left[\begin{aligned}&\overline{C}_3\overline{C}_5q_b\\&-G_2H_2\\&+\overline{C}_3G_3H_3\\&+\overline{C}_3\overline{C}_5G_4H_4\\&+(\overline{C}_4T_2\cos\theta\\&+\overline{C}_3\overline{C}_6T_3\cos\theta)\frac{\partial^2w}{\partial x^2}\end{aligned}\right] \tag{14}$$

$$\begin{aligned}\frac{\partial T_2}{\partial x}=&-\overline{D}_3\left[\frac{1}{\overline{C}_1}\left\{p+(G_1H_1+\overline{C}_2T_1\cos\theta)\frac{\partial^2w}{\partial x^2}\right\}+G_2H_2\frac{\partial^2w}{\partial x^2}\right]\\&-\overline{D}_4\left[(\overline{C}_5q_b-G_3H_3+\overline{C}_5G_4H_4+\overline{C}_6T_3\cos\theta)\frac{\partial^2w}{\partial x^2}\right]\end{aligned} \tag{15}$$

and

$$\begin{aligned}\frac{\partial T_3}{\partial x}=&-\overline{D}_5\left[\frac{1}{\overline{C}_3}\left\{\begin{aligned}&\frac{1}{\overline{C}_1}\left\{\begin{aligned}&p+(G_1H_1\\&+\overline{C}_2T_1\cos\theta)\frac{\partial^2w}{\partial x^2}\end{aligned}\right\}\\&+(G_2H_2\\&+\overline{C}_4T_2\cos\theta)\frac{\partial^2w}{\partial x^2}\end{aligned}\right\}+G_3H_3\frac{\partial^2w}{\partial x^2}\right]\\&-\overline{D}_6\left(\begin{aligned}&q_b\\&-G_4H_4\frac{\partial^2w}{\partial x^2}\end{aligned}\right)\end{aligned} \tag{16}$$

The expression of q_b for soft soil and stone column region is given as (Kondner, 1963; Deb et al., 2007c):

$$q_b=q_s=\frac{k_{s0}w}{U[1+k_{s0}(w/q_{us})]}\quad \text{for soft soil region} \tag{17a}$$

$$q_c=\frac{k_{c0}w}{1+k_{c0}(w/q_{uc})}\quad \text{for stone column region} \tag{17b}$$

where k_{s0} and k_{c0} are the initial modulus of the subgrade reaction of the soft soil and stone column, respectively (spring constant per unit area for the spring), q_s and q_c are the average normal stress acting on the soft soil and stone columns, respectively, q_{us} and q_{uc} are the ultimate bearing capacity of the soft soil and stone column, respectively. U is the degree of consolidation of the stone column-improved ground at any time t. In the present analysis, plane-strain condition is considered for stone column-improved ground.

However, the stone columns are circular in nature and are installed in triangular or square patterns. Thus, it is necessary to convert the consolidation equation under axi-symmetric condition to an equivalent plane strain condition (2-D consolidation). Similar 2-D plane-strain analysis has been carried out for vertical drain or stone columns beneath embankments (Hird et al., 1992; Chai et al., 1995; Indraratna and Redana, 1997).The degree of consolidation of the soft soil due to stone column inclusions under plane-strain condition can be determined by using the procedure presented by Deb et al. (2007c), where the simplified consolidation equation proposed by Han and Ye (2001) for stone column-reinforced soil has been modified to an equivalent plane strain equation by using the procedure suggested by Hird et al. (1992). Tan et al. (2008) described the conversion methods for stone column-improved ground from axi-symmetric condition to plane-strain condition by either changing the properties of soil (keeping same geometry for axi-symmetric and plane-strain condition) or changing geometry (keeping same soil and stone columns properties for axi-symmetric and plane-strain condition).

Using the non-dimensional parameters as: $X = x/B$; $W_b = w_b/B$; $W_f = w_f/B$; $I^* = EI/k_{s0}B^4$; $G_j^* = G_jH_j / k_{s0}B^2$; $G_{j0}^* = G_{j0}H_j / k_{s0}B^2$; $T_j^* = T_j / k_{s0}B^2$; $q^* = q / k_{s0}B$; $q_{us}^* = q_{us} / k_{s0}B$; $q_{uc}^* = q_{uc} / k_{s0}B$; $\tau_{uj}^* = \tau_{uj}H_j / k_{s0}B^2$; $q_s^* = q_s / k_{s0}B$; $q_c^* = q_c / k_{s0}B$; $\alpha = k_{c0}/ k_{s0}$, the governing differential equations can be written in non-dimensional form as (within the beam region, *i.e.* $X \leq 1$):

$$q^* = I^* \frac{\partial^4 W_b}{\partial X^4} + \overline{C}_1\overline{C}_3\overline{C}_5 A W_b - \left\{ \begin{array}{l} G_1^* + \overline{C}_1 G_2^* \\ + \overline{C}_1\overline{C}_3 G_3^* \\ + \overline{C}_1\overline{C}_3\overline{C}_5 G_4^* \\ + \overline{C}_2 T_1^* \cos\theta \\ + \overline{C}_1\overline{C}_4 T_2^* \cos\theta \\ + \overline{C}_1\overline{C}_3\overline{C}_6 T_3^* \cos\theta \end{array} \right\} \frac{\partial^2 W_b}{\partial X^2} \tag{18}$$

$$\frac{\partial T_1^*}{\partial X} = -\overline{D}_1 \left(\begin{array}{l} q^* - I^* \dfrac{\partial^4 W_b}{\partial X^4} \\ + G_1^* \dfrac{\partial^2 W_b}{\partial X^2} \end{array} \right) - \overline{D}_2 \left[\begin{array}{l} \overline{C}_3\overline{C}_5 A W_b - G_2^* \\ + \overline{C}_3 G_3^* + \overline{C}_3\overline{C}_5 G_4^* \\ + (\overline{C}_4 T_2^* \cos\theta \\ + \overline{C}_3\overline{C}_6 T_3^* \cos\theta) \dfrac{\partial^2 W_b}{\partial X^2} \end{array} \right] \tag{19}$$

$$\begin{aligned} \frac{\partial T_2^*}{\partial X} = & -\overline{D}_3 \left[\frac{1}{\overline{C}_1} \left\{ q^* - I^* \frac{\partial^4 W_b}{\partial X^4} + (G_1^* + \overline{C}_2 T_1^* \cos\theta) \frac{\partial^2 W_b}{\partial X^2} \right\} + G_2^* \frac{\partial^2 W_b}{\partial X^2} \right] \\ & - \overline{D}_4 \left[\left\{ \overline{C}_5 A W_b - G_3^* + \overline{C}_5 G_4^* + \overline{C}_6 T_3^* \cos\theta \right\} \frac{\partial^2 W_b}{\partial X^2} \right] \end{aligned} \tag{20}$$

and

$$\begin{aligned} \frac{\partial T_3^*}{\partial X} = & -\overline{D}_5 \left[\frac{1}{\overline{C}_3} \left\{ \begin{array}{l} \dfrac{1}{\overline{C}_1} \left\{ \begin{array}{l} q^* - I^* \dfrac{\partial^4 W_b}{\partial X^4} + (G_1^* \\ + \overline{C}_2 T_1^* \cos\theta) \dfrac{\partial^2 W_b}{\partial X^2} \end{array} \right\} \\ + (G_2^* + \overline{C}_4 T_2^* \cos\theta) \dfrac{\partial^2 W_b}{\partial X^2} \end{array} \right\} + G_3^* \frac{\partial^2 W_b}{\partial X^2} \right] \\ & - \overline{D}_6 \left(A W_b - G_4^* \frac{\partial^2 W_b}{\partial X^2} \right) \end{aligned} \tag{21}$$

where

$$\begin{aligned} A &= \frac{1}{U[1 + (W_b / q_{us}^*)]} && \text{for soft soil region} \\ &= \frac{\alpha}{1 + (\alpha W_b / q_{uc}^*)} && \text{for stone column region} \end{aligned}$$

The subgrade modulus or spring constant ratio (α) can be expressed as (Deb et al., 2007c; 2008):

$$\alpha = \frac{(1 + \nu_s)(1 - 2\nu_s)}{(1 + \nu_c)(1 - 2\nu_c)} \frac{E_c}{E_s} \tag{22}$$

where E_c and ν_c are elastic modulus and Poisson's ratio of the stone column material, respectively; E_s and ν_s are elastic modulus and Poisson's ratio of the soft soil, respectively. The ratio E_c / E_s is called as modular ratio.

The governing differential equations beyond the beam region (*i.e.* $X > 1$) can be written in non-dimensional form as:

$$\overline{C}_1\overline{C}_3\overline{C}_5 \frac{W_f}{1 + (W_f / q_{us}^*)} - \left\{ \begin{array}{l} G_1^* + \overline{C}_1 G_2^* + \overline{C}_1\overline{C}_3 G_3^* \\ + \overline{C}_1\overline{C}_3\overline{C}_5 G_4^* \\ + \overline{C}_2 T_1^* \cos\theta + \overline{C}_1\overline{C}_4 T_2^* \cos\theta \\ + \overline{C}_1\overline{C}_3\overline{C}_6 T_3^* \cos\theta \end{array} \right\} \frac{\partial^2 W_f}{\partial X^2} = 0 \tag{23}$$

$$\frac{\partial T_1^*}{\partial X} = -\overline{D}_1 G_1^* \frac{\partial^2 W_f}{\partial X^2} - \overline{D}_2 \left[\begin{array}{l} \overline{C}_3\overline{C}_5 \dfrac{W_f}{1 + (W_f / q_{us}^*)} - G_2^* \\ + \overline{C}_3 G_3^* + \overline{C}_3\overline{C}_5 G_4^* \\ + (\overline{C}_4 T_2^* \cos\theta \\ + \overline{C}_3\overline{C}_6 T_3^* \cos\theta) \dfrac{\partial^2 W_f}{\partial X^2} \end{array} \right] \tag{24}$$

$$\begin{aligned} \frac{\partial T_2^*}{\partial X} = & -\overline{D}_3 \left[\frac{1}{\overline{C}_1} \left\{ (G_1^* + \overline{C}_2 T_1^* \cos\theta) \frac{\partial^2 W_f}{\partial X^2} \right\} + G_2^* \frac{\partial^2 W_f}{\partial X^2} \right] \\ & - \overline{D}_4 \left[\left\{ \overline{C}_5 \frac{W_f}{1 + (W_f / q_{us}^*)} - G_3^* + \overline{C}_5 G_4^* + \overline{C}_6 T_3^* \cos\theta \right\} \frac{\partial^2 W_f}{\partial X^2} \right] \end{aligned} \tag{25}$$

and

$$\begin{aligned} \frac{\partial T_3^*}{\partial X} = & -\overline{D}_5 \left[\frac{1}{\overline{C}_3} \left\{ \begin{array}{l} \dfrac{1}{\overline{C}_1} \left\{ (G_1^* + \overline{C}_2 T_1^* \cos\theta) \dfrac{\partial^2 W_f}{\partial X^2} \right\} \\ + (G_2^* + \overline{C}_4 T_2^* \cos\theta) \dfrac{\partial^2 W_f}{\partial X^2} \end{array} \right\} + G_3^* \frac{\partial^2 w_f}{\partial X^2} \right] \\ & - \overline{D}_6 \left(\frac{W_f}{1 + (W_f / q_{us}^*)} - G_4^* \frac{\partial^2 W_f}{\partial X^2} \right) \end{aligned} \tag{26}$$

where W_b is the vertical displacement of the beam and W_f is the vertical displacement of the granular layer beyond the beam in non-dimensional form. In the governing equation beyond the beam, I^* is taken as zero. The bending moment and shear force in the beam can be written in non-dimensional form as:

$$M^* = -I^* \frac{\partial^2 W_b}{\partial X^2} \quad \text{and} \quad Q^* = -I^* \frac{\partial^3 W_b}{\partial X^3} \tag{27}$$

2.1 Method of Solution and Boundary Conditions

Finite difference method has been employed to solve the governing differential equations. In these equations, the derivative $\partial^4 W/\partial X^4$ and $\partial^2 W/\partial X^2$ have been expressed by central difference scheme while $\partial T_j^*/\partial X$ have been expressed by forward difference scheme. The length L/B may be divided into n number of the same increment length with (n+1) number of node points (i = 1, 2, 3, 4……, n). As the problem is symmetric, only half of the system is considered for the analysis. The boundary conditions chosen as:

at $X = 0$, due to symmetry, the slope, $\partial W_b/\partial X = 0$ and $Q^* = 0$.

at $X = 1$, $M^* = 0$; $Q^* = G_{tf}^* \partial W_f/\partial X - G_{tb}^* \partial W_b/\partial X$ and $W_b = W_f$, where

$$G_{tf}^* = \sum_{j=1}^{4} \frac{G_{j0}^*}{1 + \dfrac{G_{j0}^* \left|\partial W_f/\partial X\right|}{\tau_u^*}}$$

and

$$G_{tb}^* = \sum_{j=1}^{4} \frac{G_{j0}^*}{1 + \dfrac{G_{j0}^* \left|\partial W_b/\partial X\right|}{\tau_u^*}}.$$

The distance $X = L/B$ is chosen in such a way that at $X = L/B$, $W_f = 0$. As geosynthetic layers are free at the end, the mobilized tension, $T_1^* = T_2^* = T_3^* = 0$ at $X = L/B$ (or $x = L$). The continuity at the edge of the stone columns is automatically satisfied. The loading conditions are as: $q_i^*(X) = q^*$ for $|X| \le 1.0$ and $q_i^*(X) = 0$ for $|X| > 1.0$.

3. RESULTS AND DISCUSSIONS

A computer program based on the formulation has been developed and solutions are obtained using an iterative technique with a tolerance value of 10^{-4}. Figure 2 shows the comparison between the results of the present model and the results reported by Deb et al. (2007c) for single layered-reinforced system. During the comparison, the geosynthetic reinforcement is placed in the middle of the granular fill. It has been observed that as the flexural rigidity of the beam decreases, the result of the present model converges towards the result of the model presented by Deb et al. (2007c). This is due to the fact that for very low flexural rigidity of the beam, the present model is identical with the model presented in Deb et al. (2007c). The results of the present developed model are not compared with the field or laboratory test results due to unavailability of proper test data and model parameters. However, results of the similar developed model for single layer geosynthetic-reinforced granular fill over stone column-improved ground under axi-symmetric condition (without considering the beam/foundation) are compared with the laboratory test data and reasonably good agreement is observed up to 60% of the ultimate bearing capacity pressure (Deb et al., 2010).

In the parametric study, the typical values used are the angle of shearing resistance for the granular fill, ϕ = 36°; the coefficient of lateral stress, K_0 = 0.41; the interface friction coefficients equal to 0.5. The reinforcements are placed within the granular layer such that they divide granular layer into equally parts. For three layered-reinforced system, non dimensional shear modulus are equal for all the parts of the granular layers, *i.e* $G_1^* = G_2^* = G_3^* = G_4^* = G_0^*$. The Poisson's ratio of soft soil and stone column material is taken as 0.45 and 0.3, respectively.

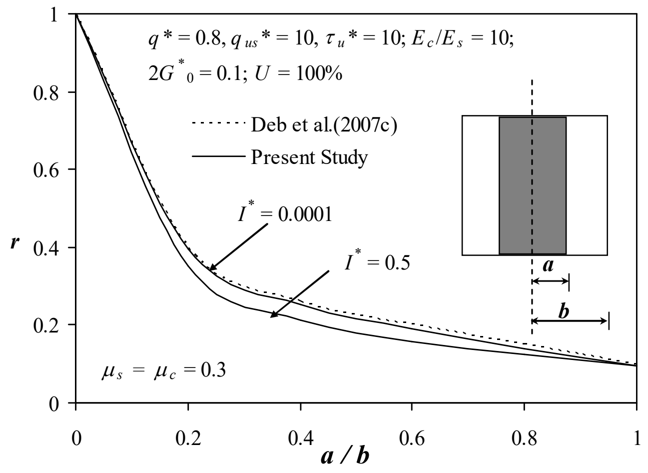

Figure 2 Comparison of the results of present study with the results presented by Deb et al. (2007c)

3.1 Effect of flexural rigidity of the beam

Figure 3 shows the effect of normalized flexural rigidity of the beam on normalized settlement at the centre of the beam. It is observed that for lower flexural rigidity of the beam, the maximum settlement decreases as the number of reinforcement (N) increases. However, the rate of reduction decreases due to addition of reinforcements. For I^* = 0.05, the settlement is reduced by 8.2%, 10.7% and 11.4% as the number of reinforcement increases from zero to one, two and three, respectively. Thus, it is observed that even for lower flexural rigidity of the beam, multilayered-reinforced system is not useful for settlement reduction. However, for lower flexural rigidity of the beam, single layered reinforcement system is useful for maximum settlement reduction. Although for higher flexural rigidity of the beam (I^* > 0.3), even single layered-reinforced system is also not very useful for maximum settlement reduction.

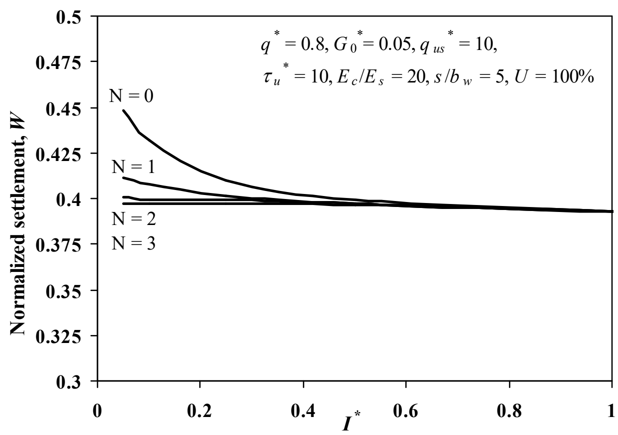

Figure 3 Effect of I^* on normalized settlement at the centre of beam

Figure 4 shows the effect of normalized flexural rigidity of the beam on normalized maximum bending moment at the centre of the beam. It is observed that as the number of reinforcement increases bending moment decreases. For I^* =0.05, the bending moment is reduced by 42%, 60% and 70% as the number of reinforcement increases from zero to one, two and three, respectively. However, for I^* =1, the reduction is 19.1%, 32.6% and 42.6%, respectively. Thus, multilayered-reinforced system is very effective for bending moment reduction. However, the reduction is more in case of lower flexural rigidity of the beam as compared to the higher flexural rigidity of the beam. Figure 5 shows the effect of normalized flexural rigidity of the beam on normalized maximum shear force of the beam at the edge of the stone column. It is observed that as the number of reinforcement increases shear force value decreases. For I^* =0.05, the shear force is reduced by 27%, 37% and 42.5% as the number of reinforcement increases from zero to one, two and three, respectively. However, for I^*=1, the reduction is 8.4%, 14% and 18%, respectively. Thus, multilayered-reinforced system is also effective for shear for reduction and the reduction is more in case of lower flexural rigidity of the beam as compared to the higher flexural rigidity of the beam. The effect of thickness of the beam is considered in the flexural rigidity (EI) of the beam. The moment of inertia I=bh^3/12, where b is the width of the beam and h is the thickness of the beam.

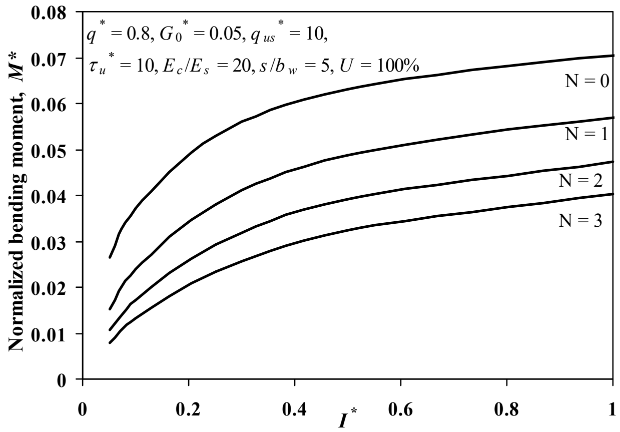

Figure 4 Effect of I^* on normalized bending moment at the centre of beam

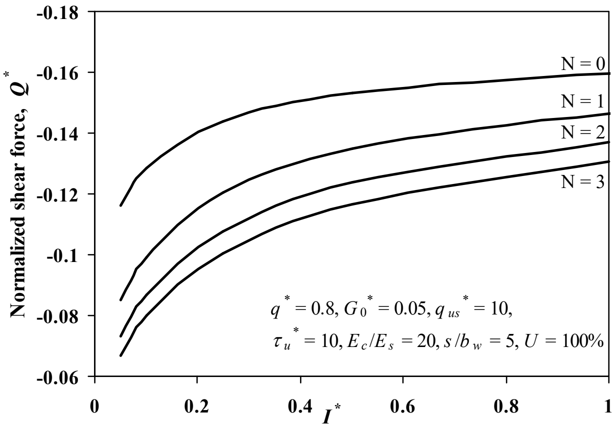

Figure 5 Effect of I^* on normalized shear force at the edge of the stone column

3.2 Effect of modular ratio

Figure 6 shows the effect of modular ratio on settlement at the centre of the beam. At modular ratio 5, the settlement is reduced by 8.4%, 11.5% and 13% as the number of reinforcement increases from zero to one, two and three, respectively. Thus, for lower modular ratio, multilayered reinforcement system is not useful for settlement reduction. However, for lower modular ratio, single layered reinforcement system is useful for maximum settlement reduction. Although for higher modular ratio, even single layered reinforced system is also not very useful for maximum settlement reduction (as shown in Figure 6).

Figure 7 shows the effect of modular ratio on bending moment of the beam. It is observed that as the modular ratio increases positive bending moment at the centre of the beam decreases. However, the positive bending moment of the beam above middle of the stone column becomes negative as modular ratio increases. The negative bending moment o f the beam above middle of the stone column increases as the modular ratio increases. It is also observed that as the number of reinforcement increases the positive bending moment of the beam at the centre and the middle of the stone column decreases, but the negative bending moment of the beam at the middle of the stone column is not reduced significantly. As compared to the bending moment of the beam at the centre, the bending moment of the beam at the middle of the stone column changes more significantly due to the change in modular ratio. In unreinforced case for lower value of modular ratio, positive bending moment of the beam at the centre is more than the moment (positive or negative) of the beam at the middle of the stone column and for higher modular ratio, positive bending moment of the beam at the centre and negative bending moment of the beam at the middle of the stone column is almost same. However, in reinforced case for lower modular ratio, positive bending moment of the beam at the centre is more than the moment of the beam at the middle of the stone column, but for higher modular ratio, the negative bending moment of the beam at the middle of the stone column is more than the positive bending moment at the centre of the beam. Thus, for higher modular ratio (E_c/E_s>40), the maximum bending moment (negative bending moment) of the beam at the middle of the stone column will remain same even after the use of reinforcements. However, for lower modular ratio, the maximum bending moment decreases due to addition of reinforcement layers.

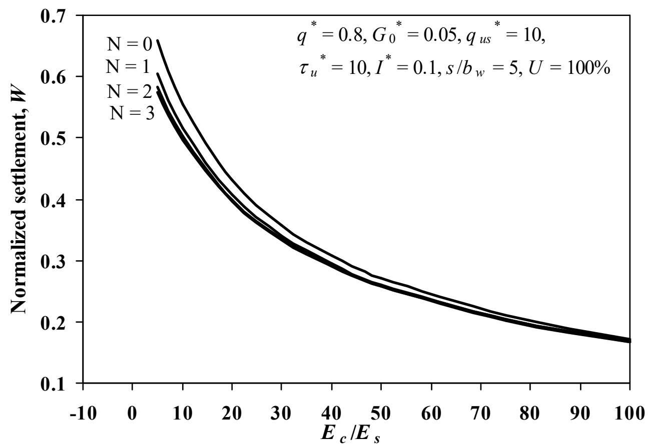

Figure 6 Effect of modular ratio on normalized settlement at the centre of beam

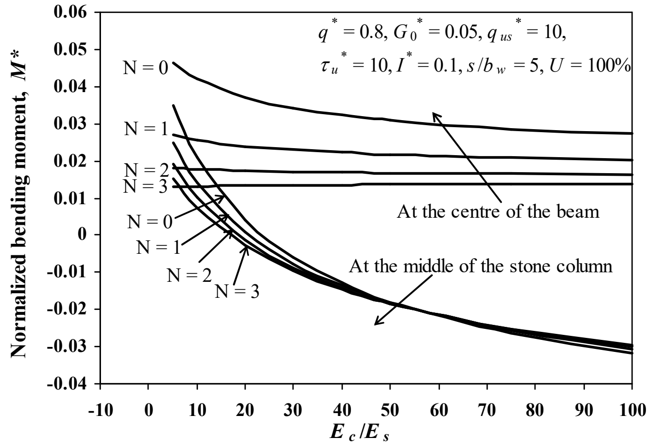

Figure 7 Effect of modular ratio on normalized bending moment

Figure 8 shows the effect of modular ratio on shear force. It is observed that as the modular ratio increases the shear force of the beam at the edge of the stone column increases and the shear force at the edge of the beam decreases. As the number of reinforcement layer increases the shear force of the beam both at the edge of the stone column and the edge of the beam decreases. For modular ratio 10, the shear force of the beam at the edge of the stone column is reduced by 36.5%, 50.6% and 57.4% as the number of reinforcement increases from zero to one, two and three, respectively and at the edge of the beam the reduction is 6.7%, 13.1% and 18.9%, respectively. However, for modular ratio 100, the shear force of the beam at the edge of the stone column is reduced by 8.3%, 12.6% and 15.2% as the number of reinforcement increases from zero to one, two and three, respectively and at the edge of the beam the reduction is less than 1% as the number of reinforcement layer increases from zero to three. Thus, the multilayered-reinforced system is very useful for shear force reduction in case of lower modular ratio as compared to the higher modular ratio. The shear force reduction of the beam at the edge of the stone column is more as compared to the edge of the beam due to application of reinforcements. It is further observed that if modular ratio is more than 17 (for unreinforced case) and 23 (for three layered reinforced system), the shear force of the beam at the edge of the stone column is more than the shear force at the edge of the beam. If modular ratio is less than the range 17 to 23 (for the chosen parameters), the shear force at the edge of the beam is more than the shear force of the beam at the edge of the stone column depending upon the number of the reinforcement.

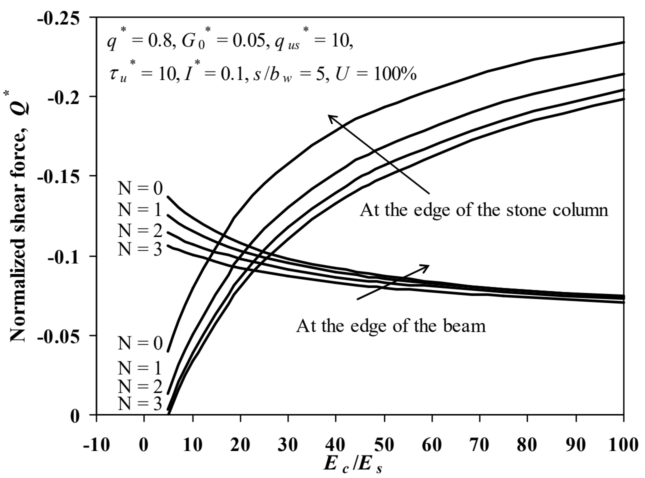

Figure 8 Effect of modular ratio on normalized shear force

3.3 Effect of s/b_w ratio

Figure 9 shows the effect of s/b_w ratio on normalized settlement at the centre of the beam. It is observed that for $s/b_w = 6$, the settlement is reduced by 6%, 8% and 8.7% as the number of reinforcement increases from zero to one, two and three, respectively. However, for $s/b_w = 2$, the reduction is 4%, 5.5% and 5.9%, respectively. Thus, single layered-reinforced system is useful for settlement reduction in case of higher s/b_w ratio. Figure 10 shows the effect of s/b_w ratio on normalized bending moment at the centre of the beam. It is observed that for $s/b_w = 6$, the bending moment is reduced by 36.6%, 54.6% and 65.4% as the number of reinforcement increases from zero to one, two and three, respectively. However, for $s/b_w = 2$, the reduction is 28.7%, 44.5% and 54.6%, respectively. Thus, it can be said that multilayered-reinforced system is very useful for bending moment reduction and the reduction is more in case of higher s/b_w ratio. It is further observed that the settlement and bending moment increase as the s/b_w ratio increases.

Figure 11 shows the effect of s/b_w ratio on normalized shear force of the beam at the edge of the stone column. It is observed that as the s/b_w value increases the shear force increases up to a limiting value and beyond that shear force decreases due to increase in s/b_w ratio. The limiting value also increases as the modular ratio increases (as shown in Figure 12). The limiting value of s/b_w ratio increases from 3 to 5 as the modular ratio increases from 10 to 100. It is further observed that the shear force decreases as the number of reinforcement layer increases with a decreasing rate.

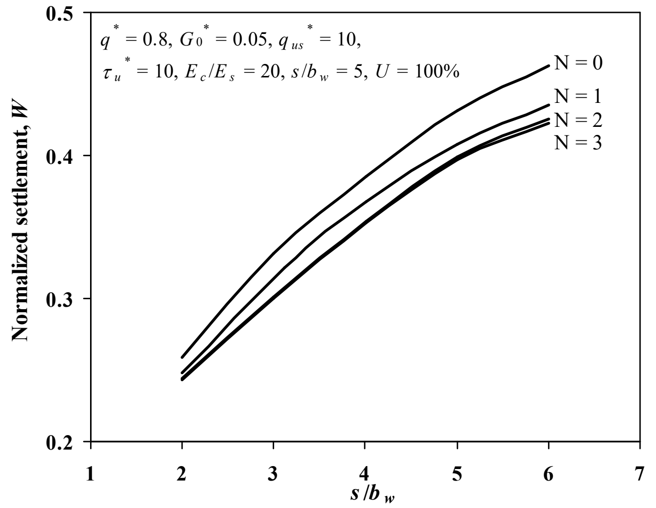

Figure 9 ffect of s/b_w on normalized settlement at the centre of beam

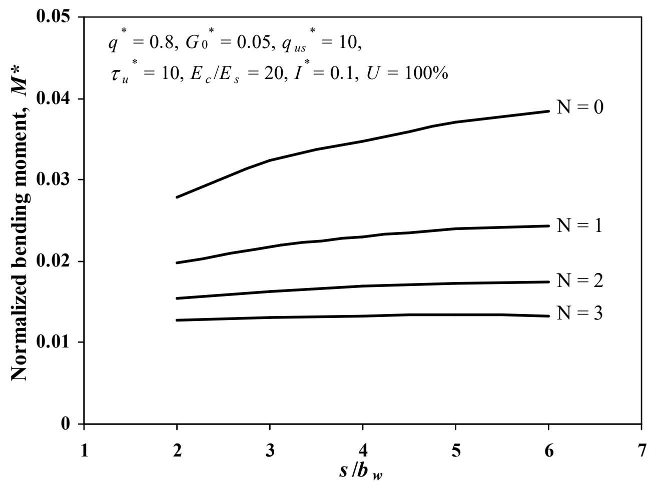

Figure 10 Effect of s/b_w on normalized bending moment at the centre of beam

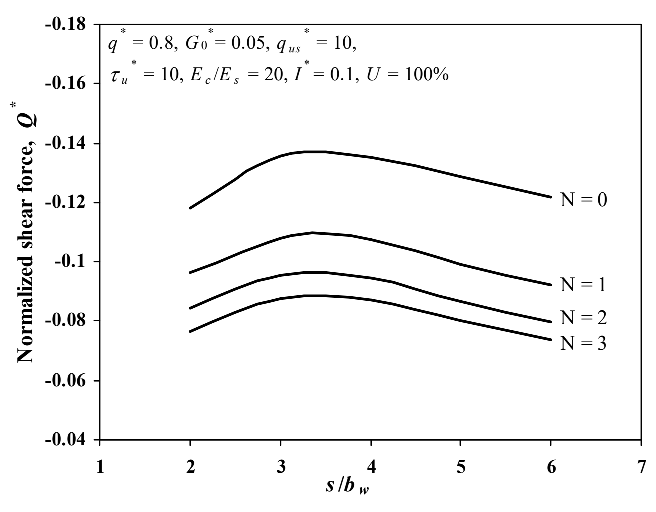

Figure 11 Effect of s/b_w on normalized shear force at the edge of the stone column

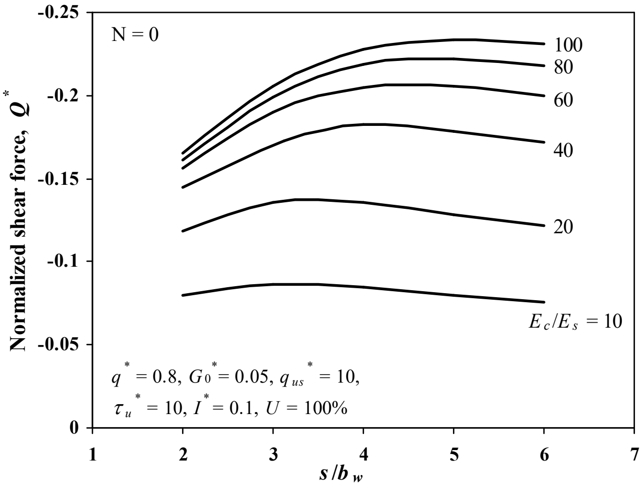

Figure 12 Variation of normalized shear force with s/bw for different modular ratio values (N = 0)

3.4 Effect of degree of consolidation

Figure 13 shows the effect of degree of consolidation on normalized settlement at the centre of the beam. It is observed that as the degree of consolidation increases settlement also increases. For 30% degree of consolidation, the settlement is reduced by 7.3%, 8.2% and 10% as the number of reinforcement increases from zero to one, two and three, respectively. Thus, single layered reinforced system is useful for settlement reduction. Figure 14 shows the effect of degree of consolidation on normalized bending moment at the centre of the beam. It is observed that for 30% degree of consolidation, the bending moment is reduced by 36.7%, 49% and 60.4% as the number of reinforcement increases from zero to one, two and three, respectively. However, for 100% degree of consolidation, the reduction is 35.6%, 53.3% and 64.1%, respectively.

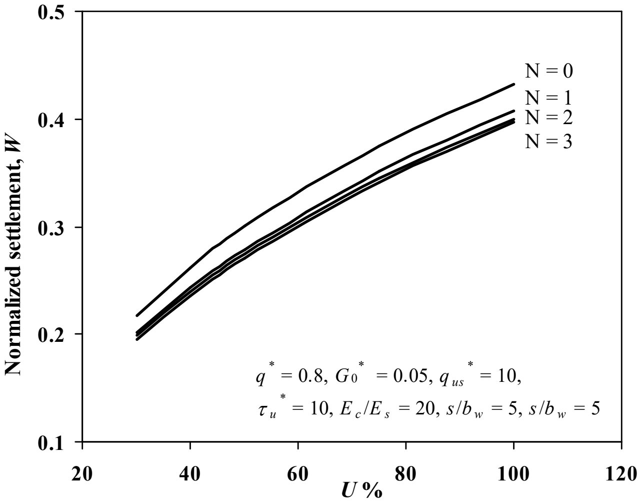

Figure 13 Effect of degree of consolidation on normalized settlement at the centre of beam

Figure 15 shows the effect of degree of consolidation on normalized shear force of the beam at the edge of the stone column. It is observed that for 30% degree of consolidation, the shear force is reduced by 43.3%, 51.4% and 59.5% as the number of reinforcement increases from zero to one, two and three, respectively. However, for 100% degree of consolidation, the reduction is 23%, 32.5% and 37.6%, respectively. Thus, it can be said that multilayered-reinforced system is very useful for bending moment and shear force reduction at any degree of consolidation. However, the reduction of bending moment is almost same for any degree of consolidation, but shear force reduction is more at lower degree of consolidation as compared to the higher degree of consolidation.

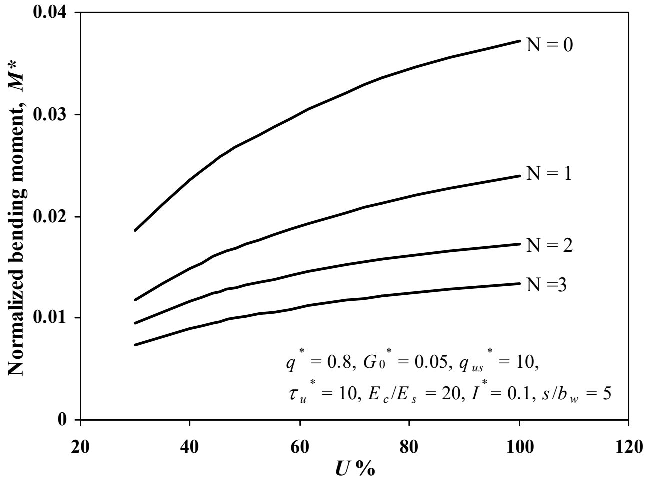

Figure 14 Effect of degree of consolidation on normalized bending moment at the centre of beam

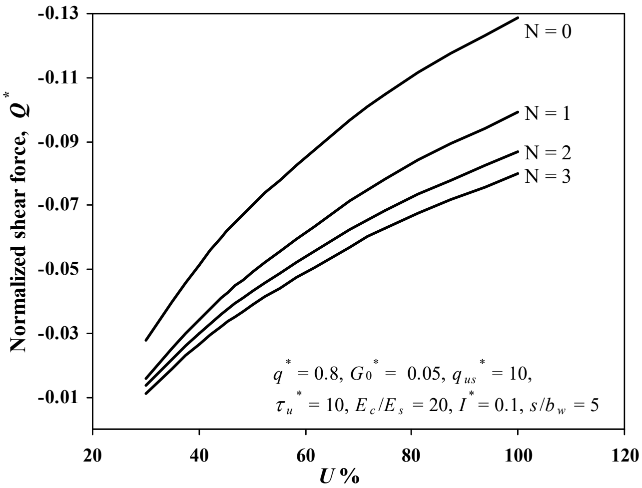

Figure 15 Effect of degree of consolidation on normalized shear force at the edge of the stone column

3.5 Effect of ultimate bearing capacity of soft soil

Figure 16 shows the effect of ultimate bearing capacity of soft soil on normalized settlement at the centre of the beam. It is observed that as the ultimate bearing capacity of soft soil increases settlement decreases. For ultimate bearing capacity of soft soil equal to 1, the settlement is reduced by 6%, 7.5% and 7.5% as the number of reinforcement increases from zero to one, two and three, respectively. Figure 17 shows the effect of ultimate bearing capacity of soft soil on normalized bending moment at the centre of the beam. It is observed that for ultimate bearing capacity of soft soil equal to 1, the bending moment is reduced by 37%, 55% and 65% as the number of reinforcement increases from zero to one, two and three, respectively. It is further observed that the variation of bending moment with ultimate bearing capacity of soft soil is not significant.

Figure 18 shows the effect of ultimate bearing capacity of soft soil on normalized shear force of the beam at the edge of the stone column. It is observed that for ultimate bearing capacity of soft soil equal to 1, the shear force is reduced by 23%, 32% and 37% as the number of reinforcement increases from zero to one, two and three, respectively. The multilayered-reinforced system is very useful for bending moment and shear force reduction for any value of ultimate bearing capacity of soft soil.

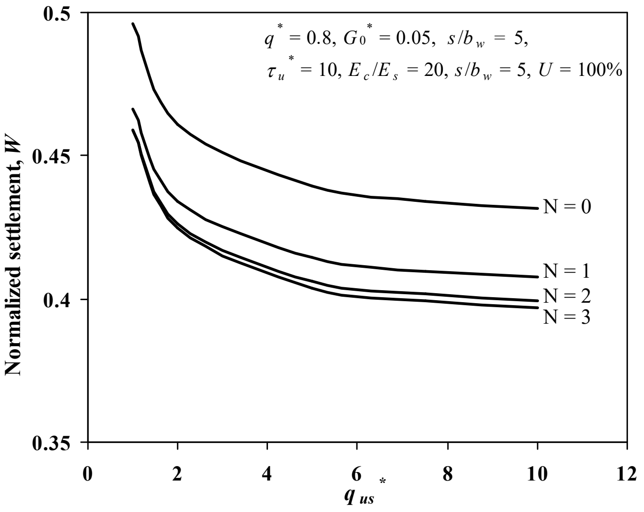

Figure 16 Effect of ultimate bearing capacity of soft soil on normalized settlement at the centre of beam

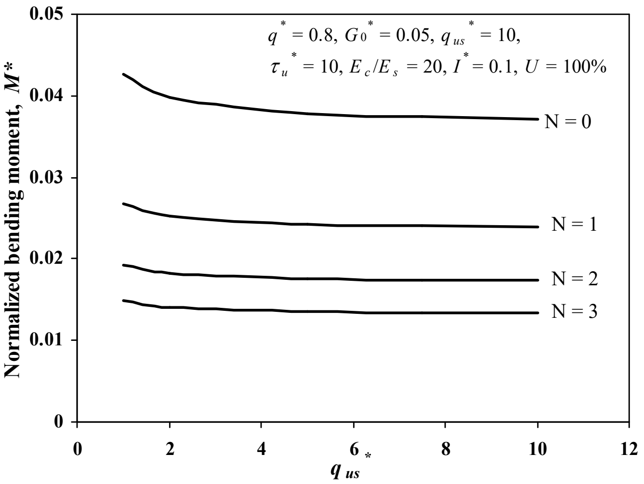

Figure 17 Effect of ultimate bearing capacity of soft soil on normalized bending moment at the centre of beam

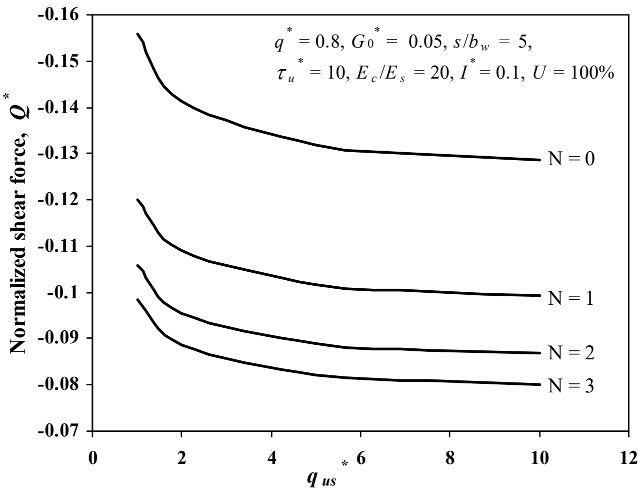

Figure 18 Effect of ultimate bearing capacity of soft soil on normalized shear force at the edge of the stone column

3.6 Effect of shear modulus of the granular fill

Figure 19 shows the effect of shear modulus of the granular fill on normalized settlement at the centre of the beam. For $G_0^* = 0.25$, the settlement is reduced by 5%, 8.2% and 10.7% as the number of reinforcement increases from zero to one, two and three, respectively. Figure 20 shows the effect of shear modulus of the granular fill on normalized bending moment at the centre of the beam. It is observed that for $G_0^* = 0.05$, the bending moment is reduced by 40%, 50% and 61.6% as the number of reinforcement increases from zero to one, two and three, respectively. However, for $G_0^* = 0.25$, the reduction is 23%, 37% and 47%, respectively.

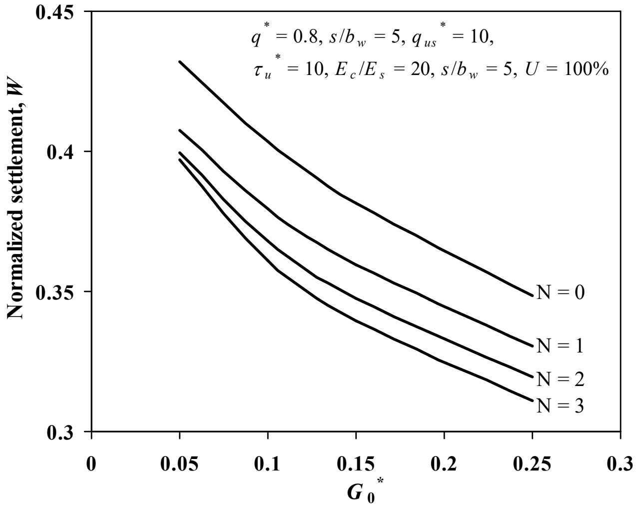

Figure 19 Effect of shear modulus of granular layer on normalized settlement at the centre of beam

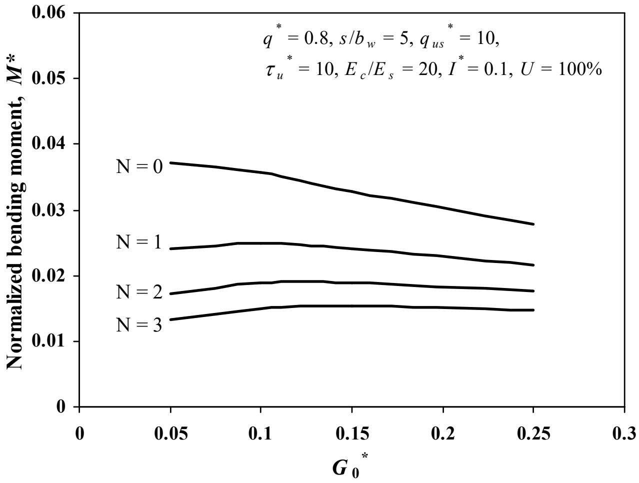

Figure 20 Effect of shear modulus of granular layer on normalized bending moment at the centre of beam

Figure 21 shows the effect of shear modulus of the granular fill on normalized shear force of the beam at the edge of the stone column. It is observed that for $G_0^* = 0.05$, the shear force is reduced by 20%, 35.6% and 40.5% as the number of reinforcement increases from zero to one, two and three, respectively. However, for $G_0^* = 0.25$, the reduction is 20%, 32% and 40%, respectively. Thus, it can be said that the reduction of shear forces due to application of multilayered-reinforced system is almost same for any shear modulus value, but bending moment reduction is more at lower shear modulus values as compared to the higher shear modulus values. It is further observed that as the number of reinforcement increases the variation of bending moment with s/b_w ratio, degree of consolidation, ultimate bearing capacity of soft soil and shear modulus of granular layer becomes insignificant.

Depending on the type of loading, soil properties and rigidity of the beam and stone column, separation or lift-off between the granular layer and beam may occur due to the upward movement of the some portion of the beam (Das and Deb, 2017). To minimize the separation, density of granular fill and flexural rigidity of beam (thickness or grade of concrete of the beam) are to be selected properly based on the chosen spacing to diameter and modular ratio (Das and Deb, 2017).

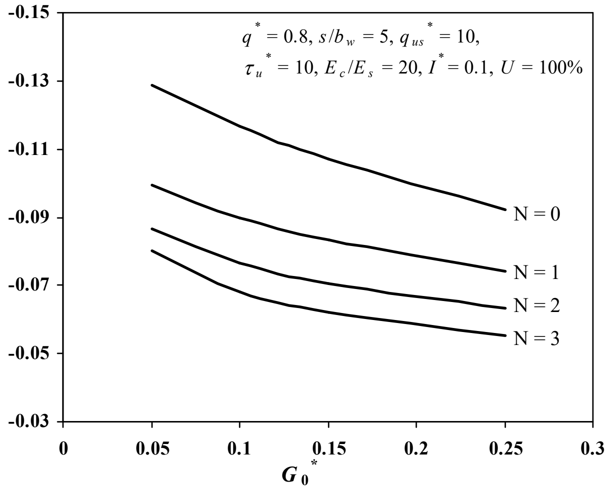

Figure 21 Effect of shear modulus of granular layer on normalized shear force at the edge of the stone column

4. CONCLUSIONS

The developed model is capable to predict the behaviour of beam resting on ground improved with stone columns and multilayered reinforcements. It is observed that under the chosen configuration, multilayered-reinforced system with stone column is not effective for settlement reduction for high rigidity condition. However, single layered-reinforced system is effective for settlement reduction and the effectiveness is more for lower flexural rigidity of the beam, for lower modular ratio and for higher s/b_w ratio value. The multilayered-reinforced system with stone column is very effective for bending moment and shear force reduction of the beam. The reduction rate decreases as the number of reinforcement increases. However, the effectiveness of bending moment reduction is more in case of lower flexural rigidity of the beam, for lower modular ratio, for higher s/b_w ratio and for lower shear modulus of the granular layer. The effectiveness of shear force reduction is more in case of lower flexural rigidity of the beam, for lower modular ratio, for higher s/b_w ratio and for lower shear modulus of the granular layer and for lower degree of consolidation. As the modular ratio increases positive bending moment at the centre of the beam decreases and the positive bending moment of the beam above middle of the stone column becomes negative. The negative bending moment of the beam above middle of the stone column increases as the modular ratio increases. For unreinforced case at lower modular ratio, the positive bending moment of the beam at the centre of the beam is more as compared to the bending moment of the beam (positive or negative) at the middle of the stone column. However, for higher modular ratio, the positive bending moment at the centre of the beam and the negative bending moment of the beam at the middle of the stone column are same. In case of reinforced system with lower modular ratio, the positive bending moment at the centre of the beam is more than the bending moment of the beam (positive or negative) at the middle of the stone column. However, for higher modular ratio (E_c/E_s>40), positive bending moment at the centre of the beam is less than the negative bending moment of the beam at the middle of the stone column. It is further observed that for lower modular ratio, maximum bending moment (positive or negative) is reduces as the number of reinforcement increases, but for higher modular ratio, as the number of reinforcement increases the maximum negative bending moment remains same. Thus, for higher modular ratio (E_c/E_s>40), use of reinforcement for maximum bending moment reduction is not effective. In most of the cases, the shear force of the beam at the edge of the stone column is more than the shear force at the edge of the beam. However, for lower modular ratio, the shear force at the edge of the beam is more than the shear force of the beam at the edge of the stone column. As the s/b_w value increases the shear force increases up to a limiting value and beyond that, shear force decreases due to increase in s/b_w ratio. The limiting value also increases due to the increases in modular ratio. The limiting value of s/b_w ratio increases from 3 to 5 as the modular ratio increases from 10 to 100. The variation of bending moment with s/b_w ratio, degree of consolidation, ultimate bearing capacity of soft soil and shear modulus of granular layer becomes insignificant as the number of reinforcement layer increases.

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