Pile Design in Seismic Areas: Small or Large Diameter?

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ABSTRACT: This work investigates the role of pile diameter in resisting seismic actions, with reference to two example subsoils, namely a dry sand and a fully saturated NC clay. After a ground response analysis in free-field conditions for different values of peak rock acceleration, mobilized soil stiffness and surface acceleration are used as ingredients for assessing the kinematic and inertial moment in a concrete pile. An optimum pile diameter is identified as the one that, while guaranteeing safety, corresponds to the minimum cost. It is also proven that, with a constant value of reinforcement area and length, increasing pile diameter (i.e. increasing safety factor and cost) leads rapidly to failure. Likewise, if pile reinforcement is designed only for inertial action, increasing pile diameter is severely detrimental.

KEYWORDS: Pile design, Seismic action, Kinematic interaction, Earthquake-induced bending, Pile diameter

1. INTRODUCTION

The topic of the seismic performance of piles started receiving attention when post-earthquake investigations revealed the development of large bending moments: (a) at the head of piles restrained against rotation by rigid caps and (b) close to interfaces separating soil layers of sharply differing stiffness, even in absence of large soil movements such as those induced by lateral spreading following liquefaction (Kavvadas and Gazetas 1993, Gazetas and Mylonakis, 1998, Brandenberg et al., 2005, Varun et al., 2013 among others). Nevertheless, interpretation of the available evidence is not straightforward. The main reasons are related to the difficulty in simulating real-life conditions in theoretical models or lab experiments. Furthermore, the superposition of simultaneous kinematic and inertial interaction phenomena, whose effects are difficult to separate, represents an additional difficulty in the interpretation of data. It is noted that the former type of interaction is associated to the deformation of the soil surrounding the piles due to seismic shaking and thereby leads to development of bending over the whole pile length, whereas the latter is related to the oscillations of the superstructure and thus generates moments that are maximum at the pile top and become insignificant below a certain depth (Figure 1).

A simple method for assessing the kinematic component of pile bending was first proposed by Margason (1975) and Margason and Holloway (1977). These articles can be credited pioneering investigations on the role of pile diameter (to be denoted in the ensuing by d) and recommend, with some justification, the use of small diameters to "conform to soil movements". While several subsequent studies investigated the problem of kinematic bending (e.g., Dobry and O'Rourke, 1983, Mineiro, 1990, Kavvadas and Gazetas, 1993, Mylonakis, 2001, Nikolaou et al., 2001, Maiorano et al, 2009, de Sanctis et al., 2010, Dezi et al., 2010, Di Laora et al., 2012, Di Laora et al. 2013, Di Laora and Rovithis, 2015, Martinelli et al., 2016, Mucciacciaro and Sica, 2018), only a handful of investigations focused on the effect of pile diameter, mostly for bending in the proximity of interfaces separating soil layers of sharply differing stiffness (Mylonakis, 2001, Saitoh, 2005).

Di Laora et al. (2017) explored the role of pile diameter in resisting seismic actions at the pile top in presence of a cap restraining head rotation, with reference to steel and concrete piles in subsoils with constant stiffness and stiffness proportional to depth. With reference to constant stiffness and concrete pile, the work highlighted that:

a) kinematic bending moment is proportional to d^4;
b) under the assumption of pile in clay with constant undrained shear strength, if only shaft resistance is considered, for constant values of pile length L and global safety factor against axial bearing capacity SF inertial moment is proportional to d^3;
c) under the assumption of constant ratio between reinforcement and total cross-sectional area, moment capacity is roughly proportional to d^3.

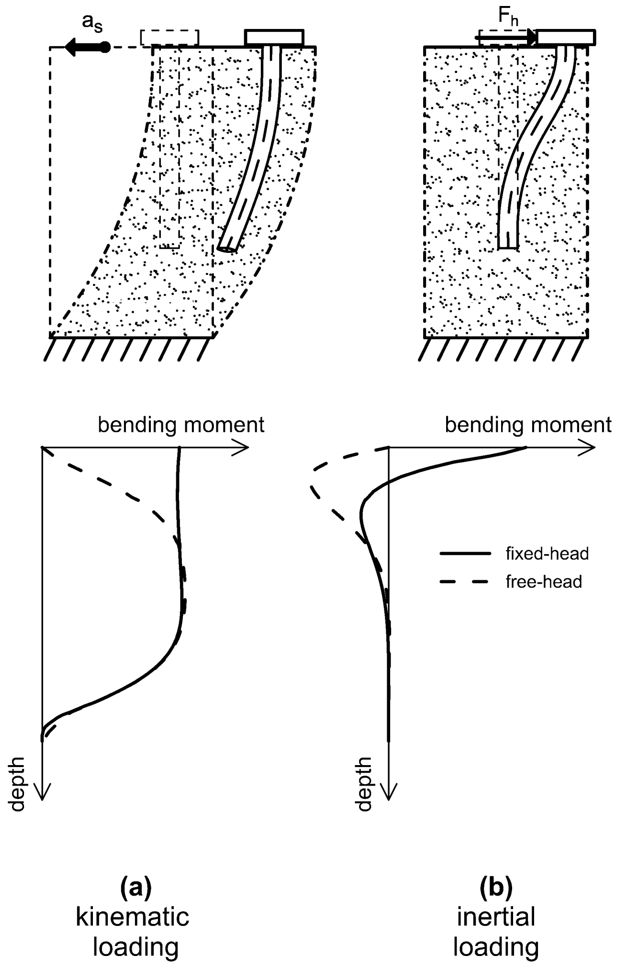

Figure 1 Kinematic and inertial loading of pile foundations

This observation revealed a previously unsuspected scale effect that causes total moment demand, taken as the mere sum of kinematic and inertial contributions, to increase faster than moment capacity, thus making yielding at the pile head unavoidable beyond a certain "critical" diameter. More specifically, the sole inertial moment provides a minimum diameter below which a pile cannot resist the demand, while the sole kinematic contribution provides a maximum diameter. The combination of the two moments reduces the range of admissible diameters. As far as a soil with stiffness proportional to depth is concerned, kinematic interaction moment increases at a smaller rate with pile diameter; this results in a larger minimum diameter which, however, is mainly due the large kinematic bending which develops in such soft soils (Figure 2).

Proceeding along these lines, this work expands the investigation of the role of pile diameter in resisting seismic forces under different assumptions and with reference to more realistic subsoil profiles, as reported in the ensuing.

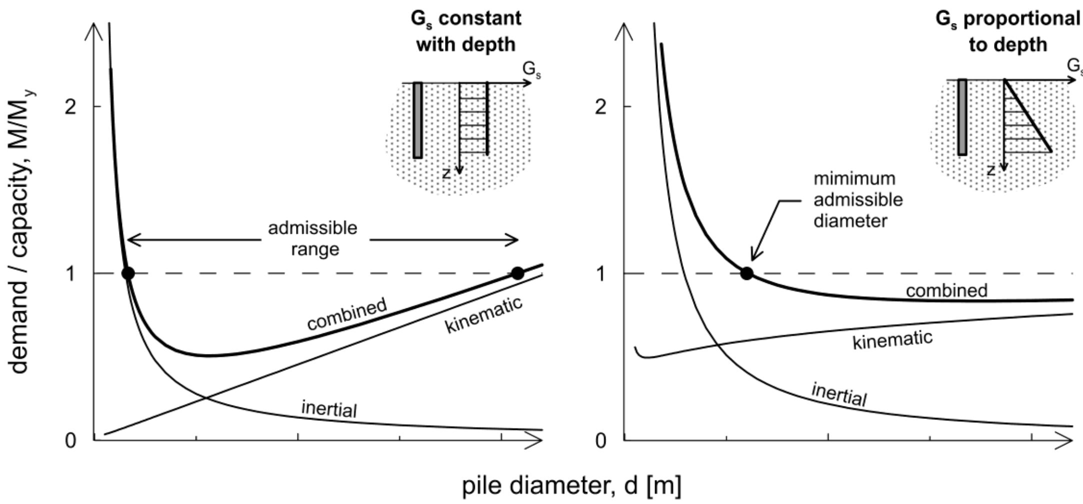

Figure 2 Pile size limitation in idealized subsoil (after Di Laora et al., 2017)

2. SUBSOILS AND FREE-FIELD SEISMIC RESPONSE

Since the problem of size limitation is relevant only for soils having low-to-moderate stiffness at shallow depths, two cases are considered in this study, namely a dry sand with medium relative density and a normally-consolidated (NC), fully saturated clay. Following Hardin and Drnevich (1992) the sand (profile A in Figure 3) is assumed to possess a low-strain stiffness proportional to the square root of depth, according to the equation:

$$G_0\left[\text{MPa}\right] = 20 \cdot z^{0.5} \quad (1)$$

with z the depth from ground surface expressed in meters. A unit weight γ = 16 kN/m^3 and a Poisson's ratio ν = 0.3 have been considered. The resulting equivalent shear wave velocity is $V_{s,30}$ = 200 m/s, thereby corresponding to a class C soil according to Eurocodes.

With reference to the NC clay, initial stiffness has been considered variable proportionally to depth according to the relation:

$$G_0\left[\text{MPa}\right] = 3 \cdot z \quad (2)$$

with z in meters. A saturated unit weight γ_{sat} = 18 kN/m^3 and a Poisson's ratio ν = 0.5 have been chosen, while the resulting equivalent shear wave velocity $V_{s,30}$ is 116 m/s (class D).

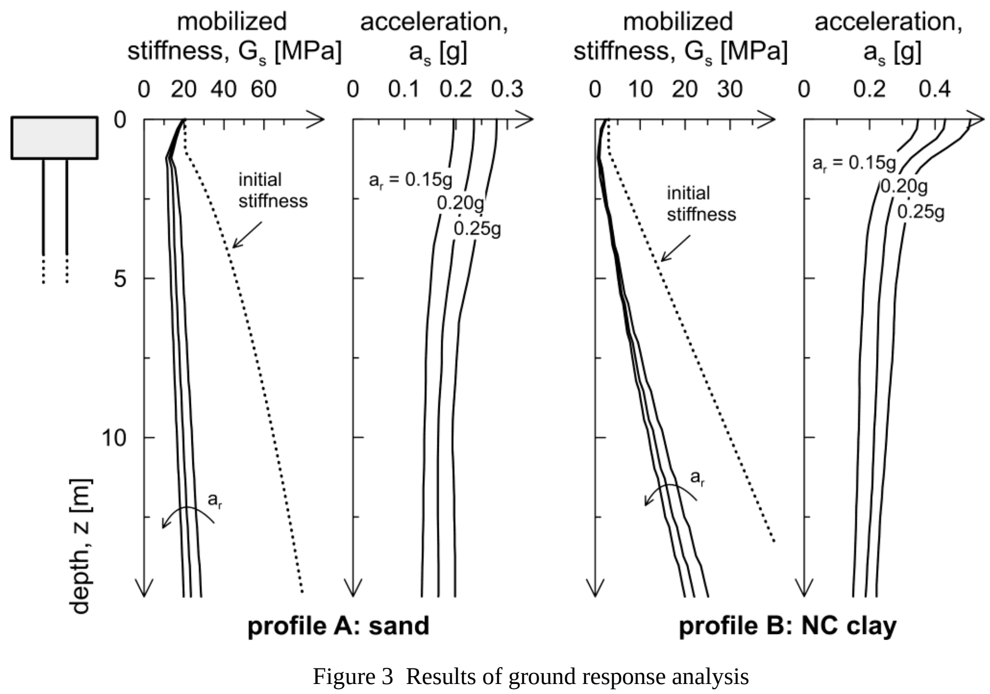

Figure 3 Results of ground response analysis

To take into account some overconsolidation at very shallow depth, the stiffness in the first meter has been kept constant and equal to the one deriving from the above equations for z = 1 m; this location also corresponds the position of pile head.

Ground response under seismic action has been evaluated through the freely available program Strata (Kottke and Rathje, 2008). Seven accelerograms have been selected to match class A design spectrum with reference to 3 different levels of peak rock acceleration a_r, namely 0.15, 0.20 and 0.25 times the gravity acceleration g. An elastic bedrock has been considered at the depth of 30 m, having shear wave velocity V_s = 800 m/s. Earthquake signals have been applied as outcrop motions.

To take into account soil non-linear behaviour, an Equivalent-Linear approach has been adopted, taking advantage of literature curves of Modulus Reduction and Damping vs. shear strain. For the dry sand, the mean curves proposed by Seed and Idriss (1970) have been used, while the curves by Vucetic and Dobry (1991), with reference to a Plasticity Index equal to 30%, have been employed.

Results of the ground response analyses are plotted in Figure 3. The following aspects are noteworthy: (a) maximum acceleration is always at surface; (b) the clay exhibits a higher amplification, mainly due to a more pronounced increase in acceleration when approaching surface; (c) for the sand the earthquake-induced decrease in shear modulus is more pronounced; (d) the mobilized shear modulus profile is consistent with the suggestion of Eurocode of simply scaling down the initial stiffness profile of a factor which is function of the surface acceleration.

The mobilized stiffness of the soil computed from the above analyses is employed for the assessment of kinematic and inertial bending, as detailed in the following.

3. PILE BENDING DEMAND AND CAPACITY

3.1 Kinematic moment

Di Laora and Rovithis (2015) investigated the kinematic moment at pile head in a subsoil whose stiffness varies in a continuous manner according to the generalized power law:

$$G(z) = G_{sd}\left[a + (1-a)\frac{z}{d}\right]^n \tag{3}$$

where $G(z)$ is the soil shear modulus at depth z, G_{sd} the corresponding value at the depth of one pile diameter, a and n are coefficients regulating soil stiffness at surface and rate of stiffness increase with depth, respectively. It is straightforward to notice that n = 1 corresponds to a linear variation of stiffness with depth, while for a = 0 soil stiffness vanishes at soil surface. With reference to a fixed-head, 'long' (i.e. L > 8-10d) pile, the authors showed that pile bending moment at the head is accurately evaluated through the relation:

$$M_{kin} = E_p I_p \frac{a_s \rho_s}{G(L_a/2)} \tag{4}$$

where E_p is the Young's modulus of pile material, I_p is the cross-sectional moment of inertia, a_s soil surface acceleration, ρ_s soil density and $G(L_a/2)$ is soil stiffness evaluated by means of Equation 3 at the depth of $L_a/2$, with L_a the pile active length expressed by (Di Laora and Rovithis, 2015, Karatzia and Mylonakis, 2016):

$$\frac{L_a}{d} = \frac{1}{1-a}\left\{\left[a^{\frac{n+4}{4}} + \frac{5(n+4)(1-a)}{16}\left(\frac{\pi}{2}\right)^{1/4}\left(\frac{E_p}{E_{sd}}\right)^{1/4}\right]^{\frac{4}{n+4}} - a\right\} \tag{5}$$

with E_{sd} soil Young's modulus at the depth of one pile diameter.

3.2 Inertial moment

By interpreting results of some numerical analyses, it was found that the moment at the top of a fixed-head pile is roughly equal to 1/4 the horizontal force F_h times the active length L_a. Considering that F_h is proportional to the axial load P_p carried by the pile, the inertial moment may be therefore expressed by:

$$M_{in} = \frac{1}{4}\frac{a_s}{g}\frac{S_a}{q} P_p L_a \tag{6}$$

where $a_s \cdot S_a$ is the elastic spectral acceleration, to be divided by the behavior factor q, which accounts for structure ductility, to obtain the structure design acceleration.

3.3 Section capacity

With reference to a concrete pile, the section moment capacity (the yield moment is assumed in this work for simplicity) may be estimated through the simplified formula (Di Laora et al., 2019):

$$M_y = M_{y,c} + M_{y,s} = \frac{2}{3}\left(\frac{d}{2}\right)^3 \sin^3\theta f'_{ck} + \frac{2}{\pi}\left(\frac{d}{2} - c\right) A_s \sin\theta f'_{yk} \tag{7}$$

where $M_{y,c}$ and $M_{y,s}$ denote, respectively, the relative contributions of concrete and steel, $f'_{ck} = 0.9 f_{ck}$, the latter being the characteristic compressive strength of concrete, $f'_{yk} = 0.95 f_{yk}$, with f_{yk} the yield strength of steel reinforcement, c is the thickness of the concrete cover, θ is a characteristic angle expressed by:

$$\theta = \left(\frac{\pi}{4}\right)^2 \left(1 + 2\omega - \frac{4}{\pi}\right)\left[-1 + \sqrt{1 + \frac{32}{\pi}\frac{\omega + \nu}{(1 + 2\omega - 4/\pi)^2}}\right] \tag{8}$$

where $\omega = A_s f'_{yk} / (A_c f'_{ck})$ is the mechanical percentage of reinforcement and $\nu = P_p / (A_c f'_{ck})$ is the dimensionless axial force parameter, A_s and A_c are reinforcement and total section areas, respectively.

4. EFFECT OF PILE DIAMETER

The role of pile diameter in the development of kinematic bending is quite straightforward inspecting Equations 4 and 5. It is easy to verify that for the case of constant stiffness (n = 0) kinematic moment is proportional to d^4, while for depth-proportional stiffness (a = 0, n = 1) kinematic moment is proportional to $d^{3.2}$; all cases with a and n between 0 and 1 are bounded by these two extreme scenarios.

Identifying the effect of pile diameter on the inertial moment requires further considerations. In the work by Di Laora et al. (2017) pile length L and safety factor SF were considered constant, resulting in a working load P_p proportional to diameter in the hypothesis of constant undrained shear strength and negligible tip resistance; this leads to an inertial moment proportional to d^2. Consideration of tip resistance and/or increasing strength with depth results in the exponent of diameter to raise to values up to 3. An alternative line of reasoning, and perhaps more oriented towards a design problem, is to keep constant the working load P_p that the pile should safely carry. A variation in pile diameter therefore corresponds, for a given safety factor SF, to a different length. This way, it is easy to derive from Equations 5 and 6 that inertial moment is proportional to d for constant stiffness (n = 0) and to $d^{0.8}$ for the other extreme case of soil stiffness being proportional to depth (a = 0, n = 1). Note that consideration of tip resistance does not alter the exponent, as it merely reduces length for a given SF.

As far as section capacity is concerned, Equations 7 and 8 show that, if the reinforcement area is increased proportionally to the section area, M_y is roughly proportional to d^3.

Figure 4 depicts a typical trend of demand and capacity with pile diameter. It is easy to recognize that capacity increases with pile diameter more than inertial moment and less than the kinematic demand.

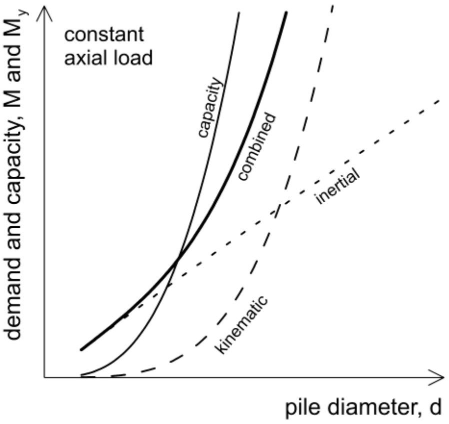

Figure 4 Schematic trend of kinematic, inertial, combined demand and section capacity as function of pile diameter

With reference to the cases analyzed in this work, Figure 5 depicts inertial and kinematic moment demand for the two subsoils as function of pile diameter, for different values of surface acceleration resulting from the ground response analysis described in Section 2. Results are obtained with reference to a linear stiffness profile interpolating mobilized stiffness below pile head.

The two selected values of pile load (i.e., P_p = 3000 for the sand and 1000 for the NC clay) correspond to a safety factor *SF* of about 2.5 for a pile of 1 m in diameter and 25 m in length, for common choices in the bearing capacity calculation.

It is worth noticing that:

a) both kinematic and inertial moment increase with acceleration;
b) kinematic bending is much more severe for the pile in clay; this is due to both the higher accelerations occurring at surface and the lower mobilized stiffness (see Equation 4 and Figure 3);
c) kinematic moments tend to dominate over the inertial counterpart for large diameters for the pile embedded in clay;
d) despite the large difference in the working load, inertial bending in clay is just slightly lower than in sand; this is due to a partial compensation owing to higher acceleration and lower mobilized stiffness.

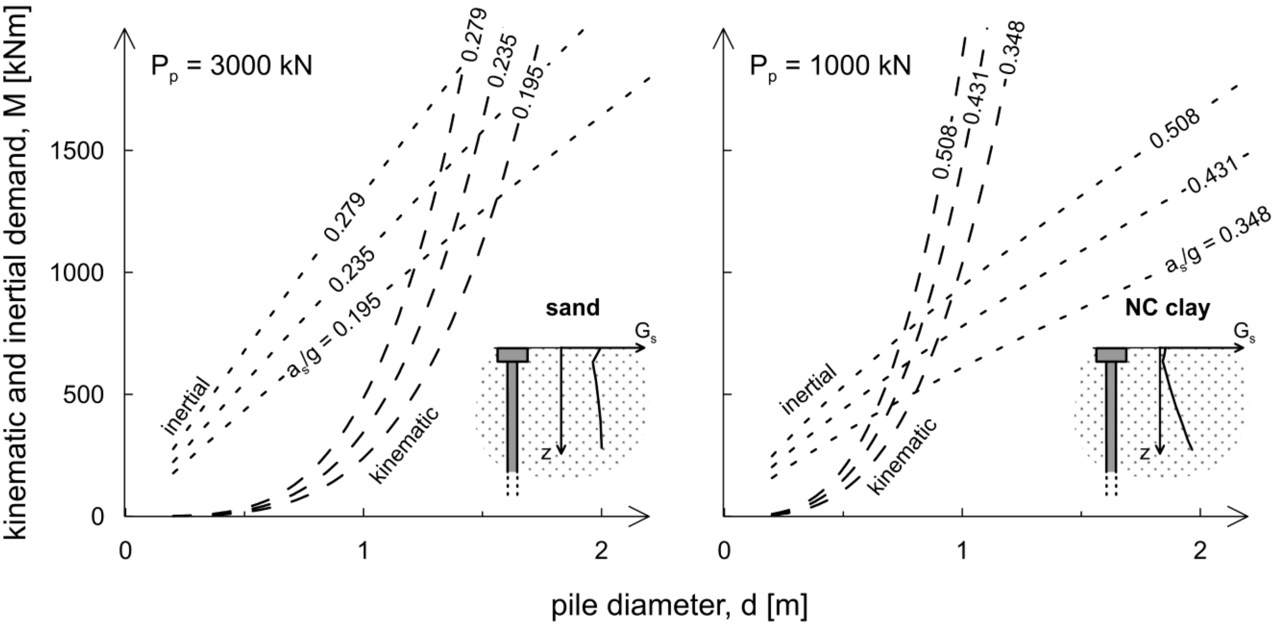

Figure 5 Kinematic and inertial demand as function of pile diameter for different acceleration levels and for the two subsoil under investigation. All curves are for S_a/g = 0.833, E_p = 30 GPa

Towards a deeper understanding of size limitations, Figure 6 illustrates the above bending contributions, both as individual contributions and combined, normalized by section capacity for a constant value (= 1%) of the ratio of reinforcement area over total cross-sectional area. It is noted that for the pile in clay kinematic action does not result in a maximum diameter, but tremendously affects minimum diameter, and the design of a pile with such reinforcement ratio is only possible for the lowest value of rock acceleration (= 0.15*g*). The pile in sand is not affected by relevant size limitations, yet the minimum diameter to withstand seismic action with reasonable reinforcement is rather high.

Figure 7 shows the total seismic demand normalized by section capacity for different values of A_s/A_c, up to the Codes limit of 4%. It is evident that the pile in clay needs a very strong reinforcement and quite large diameter to resist seismic forces.

5. OPTIMAL PILE DIAMETER

From the discussion above it is apparent that any pile diameter larger than the minimum value is acceptable to sustain earthquake loading (for simplicity yielding moments have been used instead of code-based section capacity for given partial factor, but concept remains the same).

The selection of an optimal pile diameter therefore relies on the least cost of installation. Following the above hypothesis of constant A_s/A_c, cost may be roughly expressed as function of pile diameter as follows:

$$C(d) = C_f \cdot L \cdot d + C_c \cdot L \cdot d^2 + C_s \cdot L \cdot d^2 \quad (9)$$

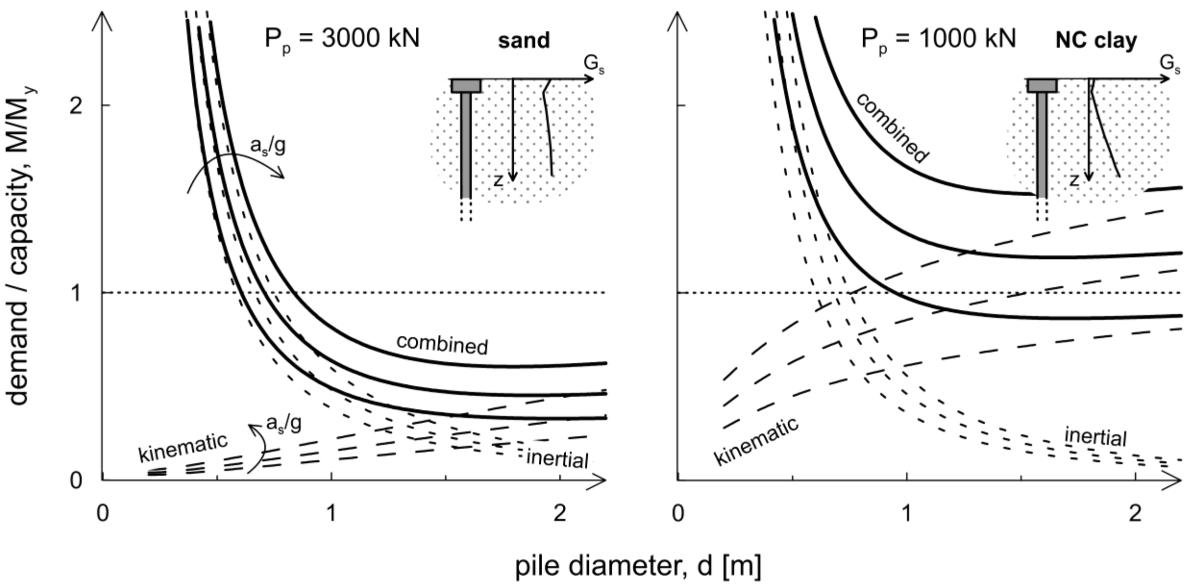

Figure 6 Kinematic, inertial and combined demand over capacity as function of pile diameter for different acceleration levels and for the two subsoil under investigation. All curves are for S_a/g = 0.833, E_p = 30 GPa, A_s/A_c = 1%, f_{ck} = 25 MPa, f_{yk} = 450 MPa

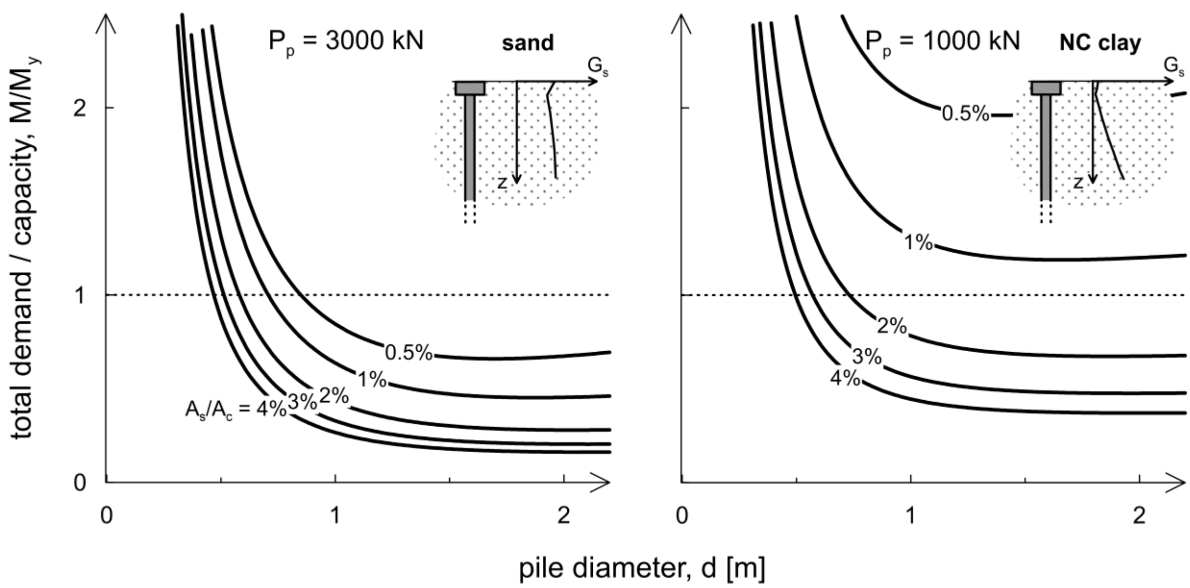

Figure 7 Combined demand over capacity as function of pile diameter for different values of A_s/A_c and for the two subsoil under investigation. All curves are for a_r = 0.20g, S_a/g = 0.833, E_p = 30 GPa, f_{ck} = 25 MPa, f_{yk} = 450 MPa

where C_f, C_c and C_s are unit costs related to perforation, concrete and steel, clearly variable worldwide.

For the subsoils considered in the study, given the absence of a very stiff bearing layer, it is reasonable to think that doubling the length does not lead to a 4 times lower diameter when designing for axial bearing capacity under constant *SF*. This means that the lower the diameter, the lower the cost. It is therefore inferred that the optimum pile diameter is the minimum diameter shown in the graphs above. Kinematic interaction can remarkably increase this minimum value.

6. CAN LARGE DIAMATERS BE DETRIMENTAL?

All the graphs above have been conceived assuming a constant value of A_s/A_c, so that an increase in pile diameter corresponds to an increase in the amount of reinforcement, as the consequent decrease in length for a given design axial load does not balance the increase in area. Under such an assumption, an increase in pile diameter cannot be detrimental for pile safety, regardless of cost considerations.

Let us now think of designing a pile for seismic actions, so that we select diameter, length and reinforcement area which provide adequate capacity to resist seismic demand. Suppose to increase pile diameter, with length and reinforcement area set at a constant value. This way, safety against a bearing capacity failure increases as well as cost.

Figure 8 illustrates this scenario. The selected values of reinforcement areas correspond to 1%, 2% and 4% of a pile of 0.8 m in diameter. It is clear from the graph that increasing pile diameter, despite the increase in cost and safety against bearing capacity failure, may lead to a tremendous decrease in safety against pile structural collapse. This is particularly true for the pile in clay, where despite the large amount of reinforcement for a pile of 0.8 m in diameter, increasing size above this value leads rapidly to failure. This occurs because the remarkable increase of kinematic moment (proportional to diameter raised to an exponent between 3.2 and 4) is not balanced by the increase in section capacity (roughly proportional to $d^{1.4}$). As a side comment, note that the increase in diameter under constant load also results in a decrease of normal stress in the section, and this is detrimental for the flexural capacity.

The above leads to the important conclusion that large diameters are safer (as shown in Figures 6 and 7) only if accompanied by a substantial increase in reinforcement to withstand the increasing kinematic demand.

To shed further light on the last concept, one may think of a pile designed only for inertial action, for example ensuring that the inertial moment is half of the section capacity. Figure 9 shows seismic demand normalized by capacity for different acceleration levels as function of pile diameter. It is evident that increasing pile diameter leads quickly to failure.

The above arguments provide a possible justification of the severe damage observed in large diameter piles (designed only to withstand inertial loading) during post-earthquake investigations in the past.

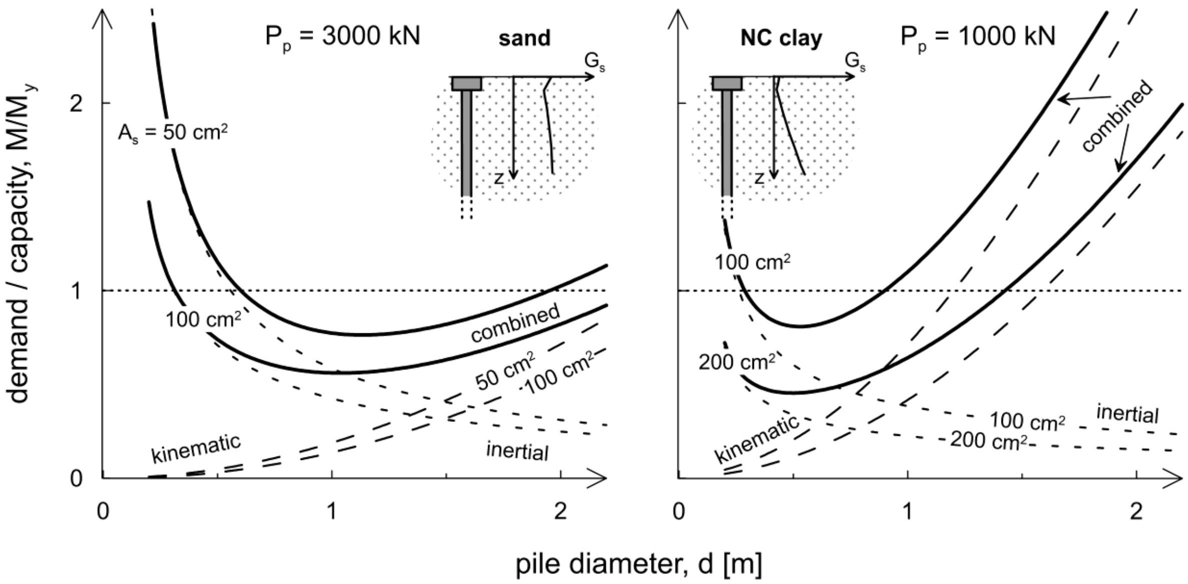

Figure 8 Kinematic, inertial and combined demand over capacity as function of pile diameter for different values of A_s and for the two subsoil under investigation. All curves are for a_r = 0.20g, S_a/g = 0.833, E_p = 30 GPa, f_{ck} = 25 MPa, f_{yk} = 450 MPa

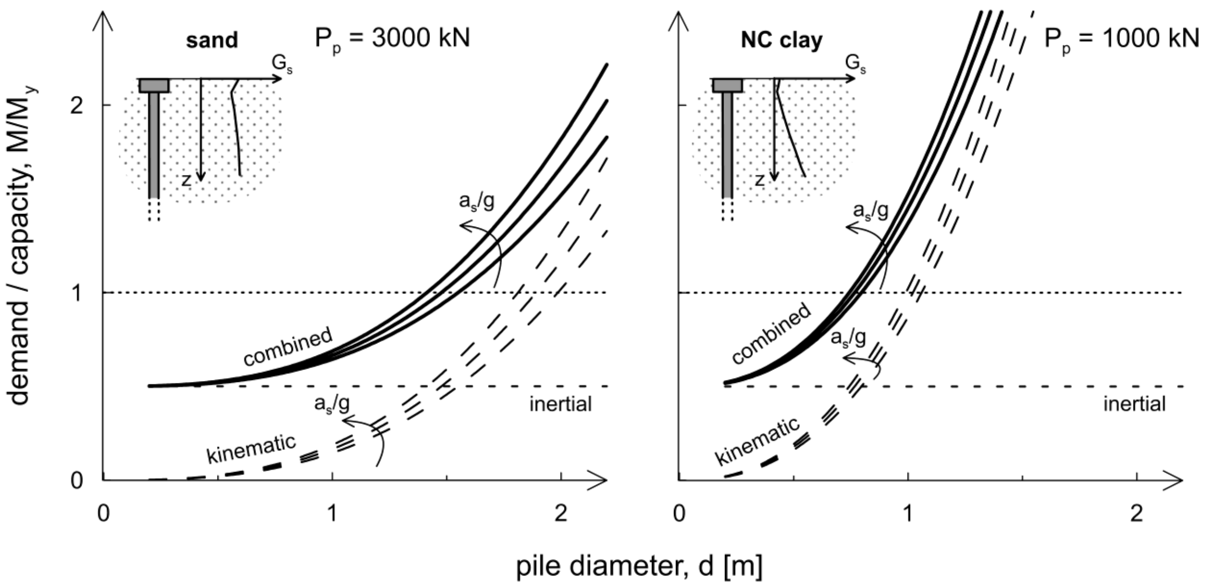

Figure 9 Demand over capacity for a pile designed only for inertial action, as function of pile diameter for different acceleration levels and for the two subsoil under investigation. All curves are for S_a/g = 0.833, E_p = 30 GPa, f_{ck} = 25 MPa, f_{yk} = 450 MPa

7. CONCLUSIONS

The work herein presented tries to shed light on the long-standing issue of the role of pile diameter in resisting seismic actions. In some past work, it was recognized that kinematic demand increases with pile diameter more than section capacity does, and therefore there must exist an upper bound for pile diameter beyond which the pile cannot resist the demand. On the other hand, inertial action increases at a smaller rate compared to the capacity and thus provides a minimum diameter. This paper investigates seismic demand in concrete piles for realistic subsoils subjected to different acceleration levels. Reference is made to a dry sand and a fully saturated NC clay; after a ground response analysis in free-field conditions, surface acceleration and mobilized soil stiffness are used as ingredients for assessing the kinematic and inertial moment.

The main conclusions of the study are the following:

a) For a constant axial load and safety factor against a bearing capacity failure, if reinforcement area is taken as proportional to the whole cross-sectional area, there is always a minimum diameter to resist seismic demand, while the maximum diameter is too large to represent a real limitation;

b) The minimum diameter is remarkably affected by the kinematic moment, which is much larger for the pile embedded in the clay;

c) From a design perspective, the above minimum diameter also represents the optimal pile diameter as it corresponds to the minimum cost;

d) If the total area of reinforcement is set as constant (i.e. does not increase with increasing cross section), kinematic moment provides a severe limitation for the upper bound of diameter; increasing diameter at constant length and reinforcement area (and therefore increasing both safety factor and cost) surprisingly - at first sight - leads rapidly to failure;

e) If reinforcement is designed to resist only inertial action, increasing pile diameter is dramatically detrimental for pile structural safety. This may explain the severe damage observed in large diameter piles (designed only for inertial loading) during post-earthquake investigations in the past.

It is worth mentioning that this study focuses only on the role of pile diameter to resist seismic action. Nevertheless, piles may also provide a filtering action on the seismic demand transmitted to the superstructure and therefore the selection of a large diameter pile, despite detrimental for the pile itself, may help the structure to resist seismic loading. However, this topic is discussed elsewhere (e.g. Di Laora and de Sanctis, 2013; Iovino et al., 2019) and lies beyond the scope of this work.

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