

A CRITICAL RE-APPRAISAL OF THE INCREMENTAL STRESS-STRAIN THEORY FOR NORMALLY CONSOLIDATED CLAYS

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SYNOPSIS

The incremental stress-strain theory of ROSCOE & POOROOSHASB (1963) is presented (in a slightly different form) with respect to shear strains. A comprehensive series of stress-controlled triaxial tests was carried out and the results were compared with the predictions of the theory. The stress ratio-shear strain relationship of normally consolidated specimens sheared under undrained condition was unique and was independent of the consolidation pressure. Also, provided that the effect of the initial one-dimensional consolidation stress is absent, the state boundary surface is unique for all types of tests including anisotropic consolidation. During anisotropic consolidation, the strain increment ratio was a constant and the void ratio varied linearly with the logarithm of the mean normal stress (the slope of the line is the same as that for isotropic consolidation). The volumetric strains in all types of tests on normally consolidated clays may be successfully predicted from the state boundary surface as obtained from undrained tests. However, the shear strains may only be predicted for paths in which the stress ratio increases or remains constant.

INTRODUCTION

This paper is concerned with a critical re-appraisal of the incremental stress-strain theory for normally consolidated clays developed by ROSCOE & POOROOSHASB (1963). The basic assumptions made in the theory are examined, and the predictions from the theory are compared with experimental observations from a large number of stress controlled triaxial tests in which the specimens were subjected to a variety of applied stress paths.

Several stress-strain theories have recently been developed to describe the stress-strain behaviour of normally consolidated clays; see ROSCOE, SCHOFIELD & THURAIRAJAH (1963), ROSCOE & BURLAND (1968) and SCHOFIELD & WROTH (1968). In these theories, a fully saturated normally consolidated clay is assumed to behave as an elastic-plastic material. Based on this assumption, concepts such as yield surface and normality were used to obtain an expression for the strain increment ratio in terms of the stresses. This expression was combined with an independent relationship for the strain increment ratio obtained, from an energy balance equation. Thus, a stress-strain

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theory for the prediction of strains in normally consolidated clays was developed. BALASUBRAMANIAM (1969), has shown that for stress paths with increasing stress ratios these theories which are based on some of the concepts of plasticity theory and on an energy equation are mathematically similar (and in some case identical) to the incremental stress-strain theory of ROSCOE & POOROOSHASB (1963). It therefore appears that the ROSCOE & POOROOSHASB's theory is more fundamental and should be examined critically for its precise validity.

In deriving the incremental stress-strain theory, ROSCOE & POOROOSHASB (1963) assumed a unique state boundary surface for normally consolidated clays. The state boundary surface in the p, q, e space (where p is the mean normal stress, q is the deviator stress and e is the void ratio) is a surface confining a space between itself and the origin within which a point can represent a state of an element of soil, but outside of which a point cannot represent such a state. The uniqueness of the state boundary surface will also be examined.

Experimental evidence will be presented to illustrate that the state path followed during anisotropic consolidation (i.e. consolidation with constant stress ratio) does lie on the state boundary surface obtained from undrained stress paths. The variation of strains and void ratios during anisotropic consolidation will also be studied. Finally, for the prediction of strains, the basic hypothesis that any applied stress path on the state boundary surface can be considered as an infinite number of small steps of undrained stress paths and anisotropic consolidation paths, will be examined for a large variety of applied stress paths.

Stress and Strain Parameters

The stress parameters used in the analysis of triaxial test results are :

$$p = (\sigma_1' + 2\sigma_3')/3$$

and

$$q = (\sigma_1' - \sigma_3')$$

(Note that in the triaxial test $\sigma_2' = \sigma_3'$)

where σ_1' , σ_2' and σ_3' are the principal effective compressive stresses. In terms of the principal compressive strains, ϵ_1 , ϵ_2 and ϵ_3 , the relevant strain parameters for use under the axi-symmetric conditions of the triaxial test ($\epsilon_2 = \epsilon_3$) are:

$$v = (\epsilon_1 + 2\epsilon_3)$$

and

$$\epsilon = 2(\epsilon_1 - \epsilon_3)/3,$$

INCREMENTAL STRESS STRAIN THEORY

The axial strain ϵ_1 is defined as :

$$\epsilon_1 = \int_L^{L_0} \frac{dL}{L} = \log (L_0/L) \text{ (Compression being positive)}$$

L_0 is the initial height and L , the current height.

The volumetric strain v is given by :

$$v = \int_V^{V_0} \frac{dV}{V} = \log \left(\frac{V_0}{V} \right) \text{ (Compression being positive)}$$

V_0 is the initial volume and V , the current volume. Expressed in terms of void ratios (e) :

$$v = \log \left(\frac{1 + e_0}{1 + e} \right)$$

where e_0 is the void ratio of the sample at the end of isotropic consolidation. In this paper, unless otherwise stated, v and ϵ will be measured from the state of the sample under isotropic stress conditions and just prior to shear.

MATERIAL TESTED, SAMPLE PREPARATION AND TESTING PROCEDURE

All specimens were prepared from air-dried Kaolin (liquid limit 74%, plastic limit 42%, and specific gravity 2.6) mixed with water to a slurry of 160% moisture content. The slurry was one dimensionally consolidated in a special mold to a maximum pressure of 22.6 lb/in². Subsequently, the mold was removed and the sample was isotropically consolidated to the required cell pressure. For a detailed description of sample preparation and testing procedure, see BALASUBRAMANIAM (1969). In order to reduce the effects of non-uniformity in deformation of the samples special precautions were taken by the use of lubricated ends (ROWE & BARDEN, 1964). Leakage was virtually eliminated by employing silicon oil TING, 1968).

THE STATE BOUNDARY SURFACE

ROSCOE, SCHOFIELD & WROTH (1958) put forward the existence of a unique surface in the three dimensional space relating the void ratio, e , and the stress parameters, q and p . Two different and distinct surface could be obtained, viz. one for normally or lightly overconsolidated clays and another one

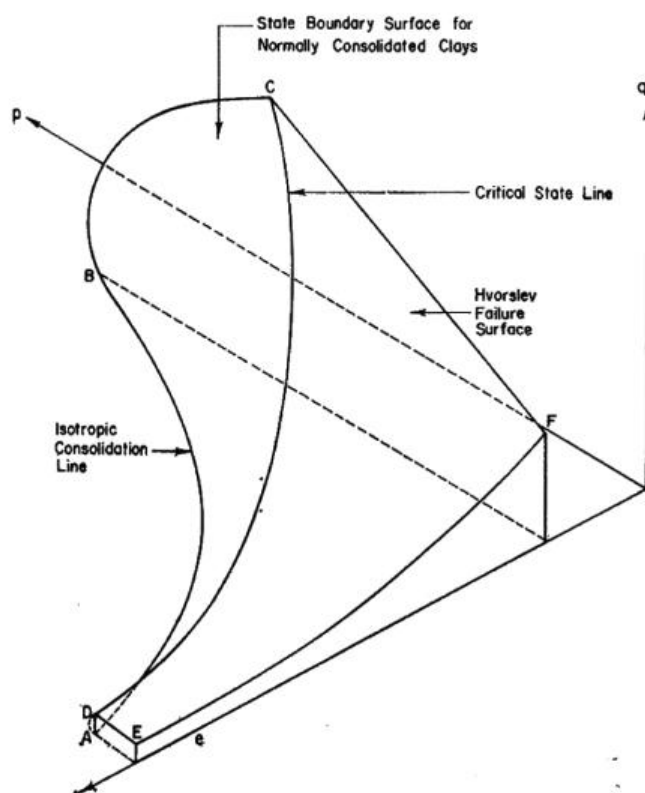


Fig. 1. State boundary surfaces for normally and overconsolidated clays in (p, e, q) space.

for heavily overconsolidated clays. Figure 1 shows these surfaces in an isometric view. The curved surface ABCD corresponds to the state paths which are followed by normally consolidated specimens in the (p, q, e) space. AB corresponds to the isotropic consolidation line and CD is called the critical state line. Specimens reaching the critical state are assumed to have unlimited shear distortion without any change in stresses or volume. The ruled surface CDEF represents the failure surface of overconsolidated samples. The peak stress conditions for a large number of normally and overconsolidated specimens of Kaolin were presented by JAMES & BALASUBRAMANIAM (1971a and b).

The surfaces ABCD and CDEF are called the state boundary surfaces. It is difficult to visualise whether the state paths followed by a normally consolidated specimen lie on the curved surface ABCD in the three dimensional plot. Therefore, ROSCOE & POOROOSHASB (1963), devised a method to represent the curved surface ABCD by a curve in a two dimensional plot. Subsequent, workers (see, BURLAND, 1965) have used alternative parameters such as q/p_e , and p/p_e for the two-dimensional representation of the state boundary surface. The parameter p_e is defined as :

$$P_e = P_o \exp \left(\frac{e_o - e}{\lambda} \right) \dots \dots \dots (1)$$

where e_o , p_o correspond to the void ratio and the mean normal stress on the isotropic consolidation line. λ is the slope of the isotropic consolidation line on an $(e, \log p)$ plot. p_e is called the mean equivalent pressure and is similar to the preconsolidation pressure defined by HVORSLEV (1937).

ROSCOE & POOROOSHASB (1963), assumed a unique state boundary surface in the derivation of the incremental stress-strain theory for normally consolidated clays. However, the experimental data presented by ROSCOE & THURAI-RAJAH (1964), showed that the state boundary surfaces are distinct and different for undrained and fully drained triaxial tests on normally consolidated

specimens of Kaolin. Also, the state paths followed during anisotropic consolidation (i.e. consolidation with constant stress ratio) were found to deviate from the state boundary surfaces for undrained and drained test specimens. BALASUBRAMANIAM (1974), made a detailed study of the effects of stress history and other factors on the state boundary surface for normally consolidated clays. According to this study, provided the effects of initial one-dimensional consolidation stress have not been there, then the state boundary surface is unique and is independent of the applied stress paths.

Figure 2 illustrates the state paths followed by three specimens (T_3 , AO and AD) sheared from an isotropic consolidation pressure of 90 lb/in². Specimen T_3 was sheared under undrained conditions, specimen AO under constant- p -conditions and specimen AD under fully drained conditions. The state paths followed by all three specimens are found to be approximately unique.

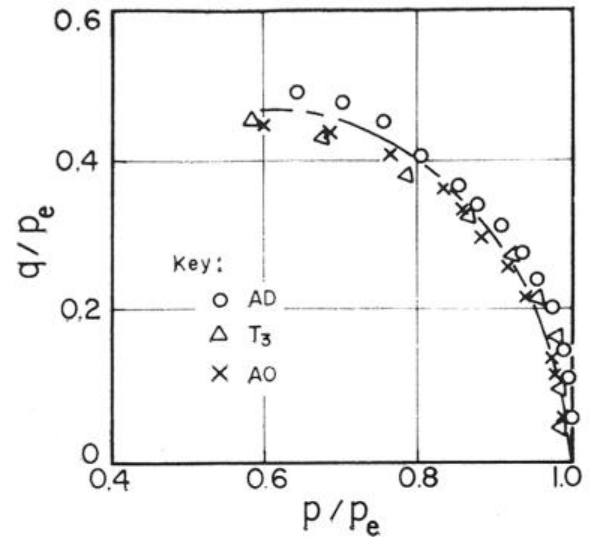


Fig. 2. State paths followed by three specimens sheared under undrained, constant p and fully drained conditions.

STRESS-STRAIN BEHAVIOR UNDER UNDRAINED CONDITIONS

Undrained tests were carried out on specimens of kaolin one-dimensionally consolidated to 22 lb/in² and subsequently isotropically consolidated to 30, 60 and 90 lb/in². The $(q/p, \epsilon)$ characteristics of these specimens as shown in Fig. 3, are found to be unique. These observations are in agreement with those of ROSCOE & POOROOSHASB (1963) and hence the shear strain during undrained tests may be expressed as:

$$(\epsilon)_{\text{undrained}} = \int_0^{\eta} f_1(\eta) d\eta \quad \dots \dots \dots (2)$$

BALASUBRAMANIAM (1973) has already made a detailed study of the effects of stress history on the stress-strain behaviour of remoulded specimens of Kaolin tested in the conventional triaxial apparatus under stress-controlled conditions. The parameters considered were the initial one-dimensional consolidation pressure used in preparing the samples, the load increment, the magnitude of isotropic consolidation stress prior to shear and the type of applied stress path. These results indicated that the stress-strain behaviour under undrained conditions was unique and independent of the initial one-

dimensional stress used during sample preparation. Load increment size and the magnitude of isotropic consolidation pressure prior to shear did not have any effect on the $(q/p, \epsilon)$ relationship for undrained tests.

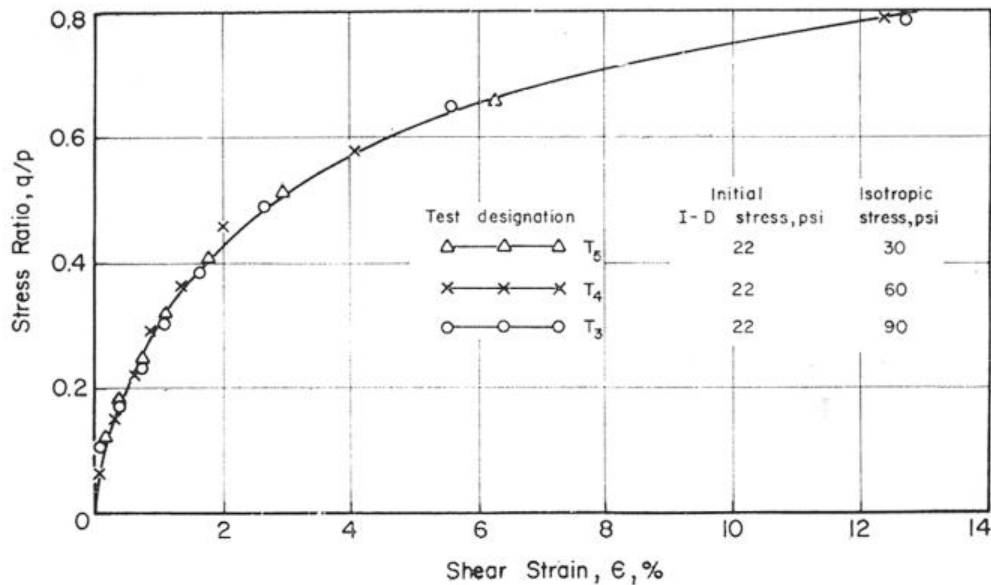


Fig. 3. $(q/p, \epsilon)$ relationship for specimens sheared under undrained condition.

ANISOTROPIC CONSOLIDATION

During anisotropic consolidation, the stress ratio, σ_1'/σ_3' is maintained constant (σ_1' is the major principal stress and σ_3' is the minor principal stress).

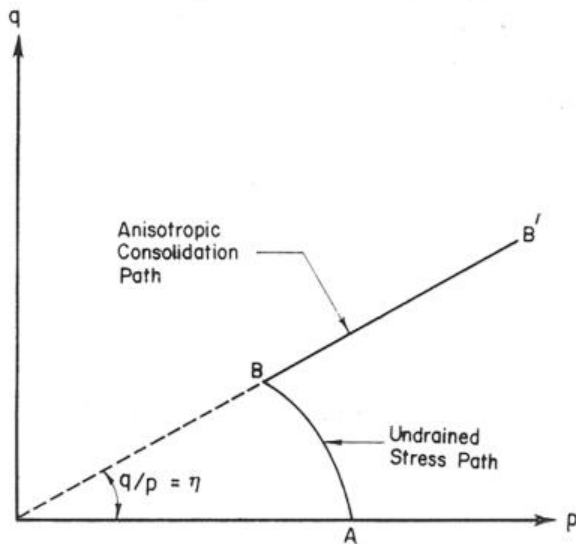


Fig. 4. Anisotropic consolidation path.

Figure 4 illustrates a possible experimental procedure for such a test in the (q, p) plot. A specimen is sheared under undrained condition from the normally consolidated state under a cell pressure p_0 corresponding to point A. The undrained stress path is denoted by AB. From B, the specimen is subjected to anisotropic consolidation along the path BB'. During anisotropic consolidation $q/p (= \eta)$ is maintained constant :

$$\eta = 3 \frac{(k-1)}{(k+1)} \dots\dots\dots (3)$$

$$\text{where } k = \frac{\sigma_1'}{\sigma_3'} \dots\dots\dots (4)$$

INCREMENTAL STRESS STRAIN THEORY

Figure 5 illustrates the state boundary surface for normally consolidated clays in the (p, q, e) space. In this figure the curve AA' corresponds to isotropic consolidation, while the curves BB' , CC' ; etc. correspond to anisotropic consolidation. $ABCD$, $A'B'C'D'$ are constant volume (or undrained stress) paths. In this figure bc' corresponds to a fully drained stress path with constant cell pressure. It is split into two parts viz. bc , an undrained stress path and cc' , an anisotropic consolidation path.

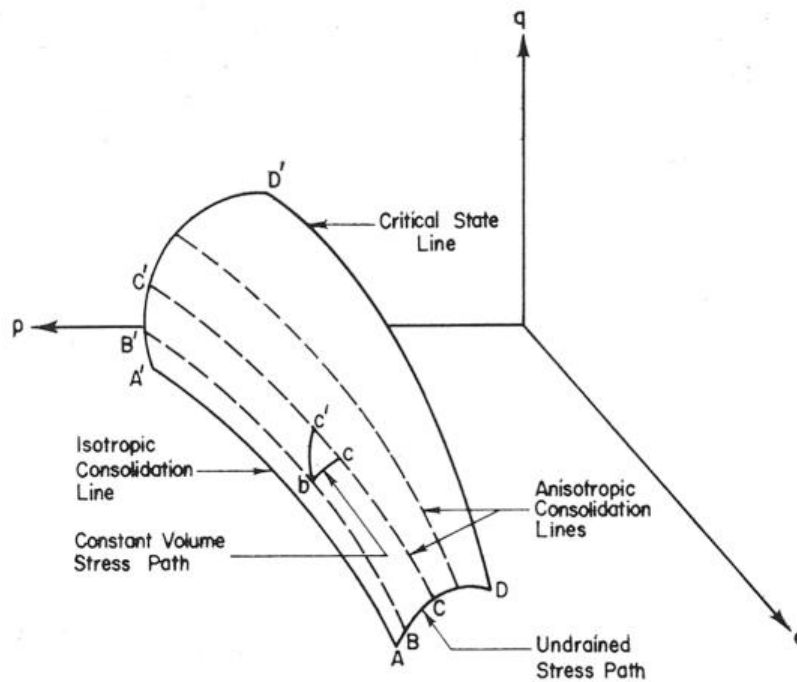


Fig. 5. Isometric view of the state boundary surface for normally consolidated clay.

Experimental Observations on Anisotropic Consolidation Tests

Altogether five anisotropic consolidation tests were performed on specimens initially one-dimensionally consolidated to 22 lb/in², and subsequently isotropically consolidated to 90 lb/in². They were then sheared either under undrained condition or under constant dq/dp condition to the required stress ratios. The tests were designated by BF, BC, BT, BL and T_{10} , and they corresponded to q/p ($= \eta$)—values of 0.205, 0.405, 0.51, 0.57, and 0.71 respectively. The strain paths followed by these specimens in the (v, ϵ) space are shown in Fig. 6, where the strain paths are found to be linear. These results confirm the original findings of ROSCOE & POOROOSHASB (1963), i.e. that $(dv/d\epsilon)_{\eta}$ during anisotropic consolidation is only dependent on the value of η . The values of $(dv/d\epsilon)_{\eta}$ observed for the five tests are 2.80, 1.30, 1.00, 0.775 and 0.45. These values correspond to q/p ($= \eta$)—values of 0.205, 0.405, 0.51, 0.57 and 0.71 respectively. Figure 7 illustrates the variation of $(dv/d\epsilon)_{\eta}$ with η

for all the five tests conducted. It can be seen that $(dv/d\varepsilon)_\eta$ varies uniformly with η and may expressed as :

$$(dv/d\varepsilon)_\eta = f_2(\eta) \dots\dots\dots (5)$$

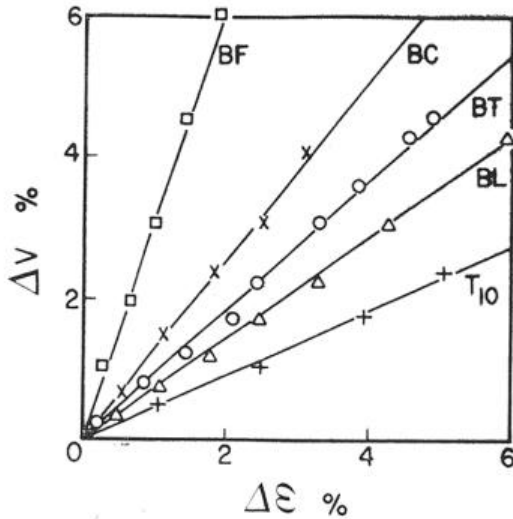


Fig. 6. Strain paths followed by specimens during anisotropic consolidation.

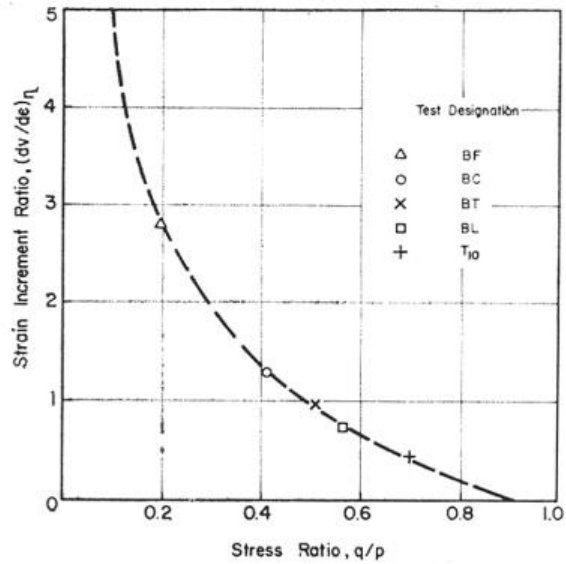


Fig. 7. Relationship between strain increment ratio and stress ratio.

The changes in void ratio with the logarithm of the mean normal stress during anisotropic consolidation on two specimens (at η values of 0.405, and 0.57) are shown in Fig. 8. The variation in e with $\log p$ is found to be linear and the slopes (equals to 0.26 approximately) of these lines are the same as that of

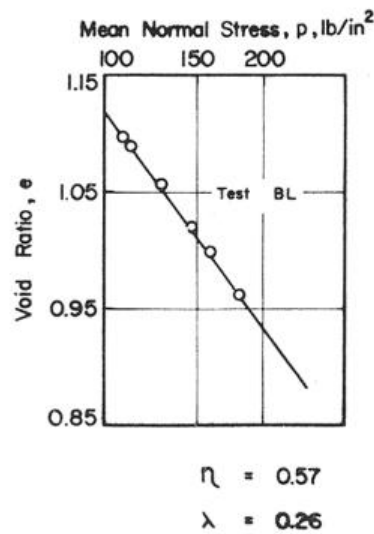
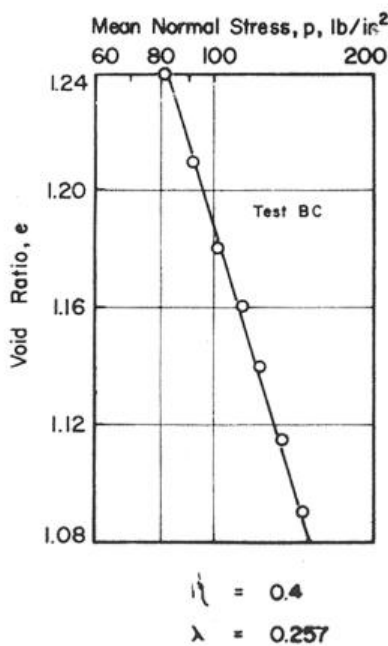


Fig. 8. $(e, \log p)$ relationship during anisotropic consolidation.

INCREMENTAL STRESS STRAIN THEORY

the isotropic consolidation line in an $(e, \log p)$ plot. Let this slope be denoted by λ . Then, during anisotropic consolidation:

$$e - e_c = -\lambda \log p/p_c \quad \dots\dots\dots (6)$$

where e_c, p_c corresponds to any known void ratio and pressure on the anisotropic consolidation line. The stress and state paths followed by two specimens during anisotropic consolidation are shown in Fig. 9. It is noted that the state paths during anisotropic consolidation do lie on the state boundary surface. These results are in agreement with the assumptions made by ROSCOE and POOROOSHASB in their incremental stress-strain theory.

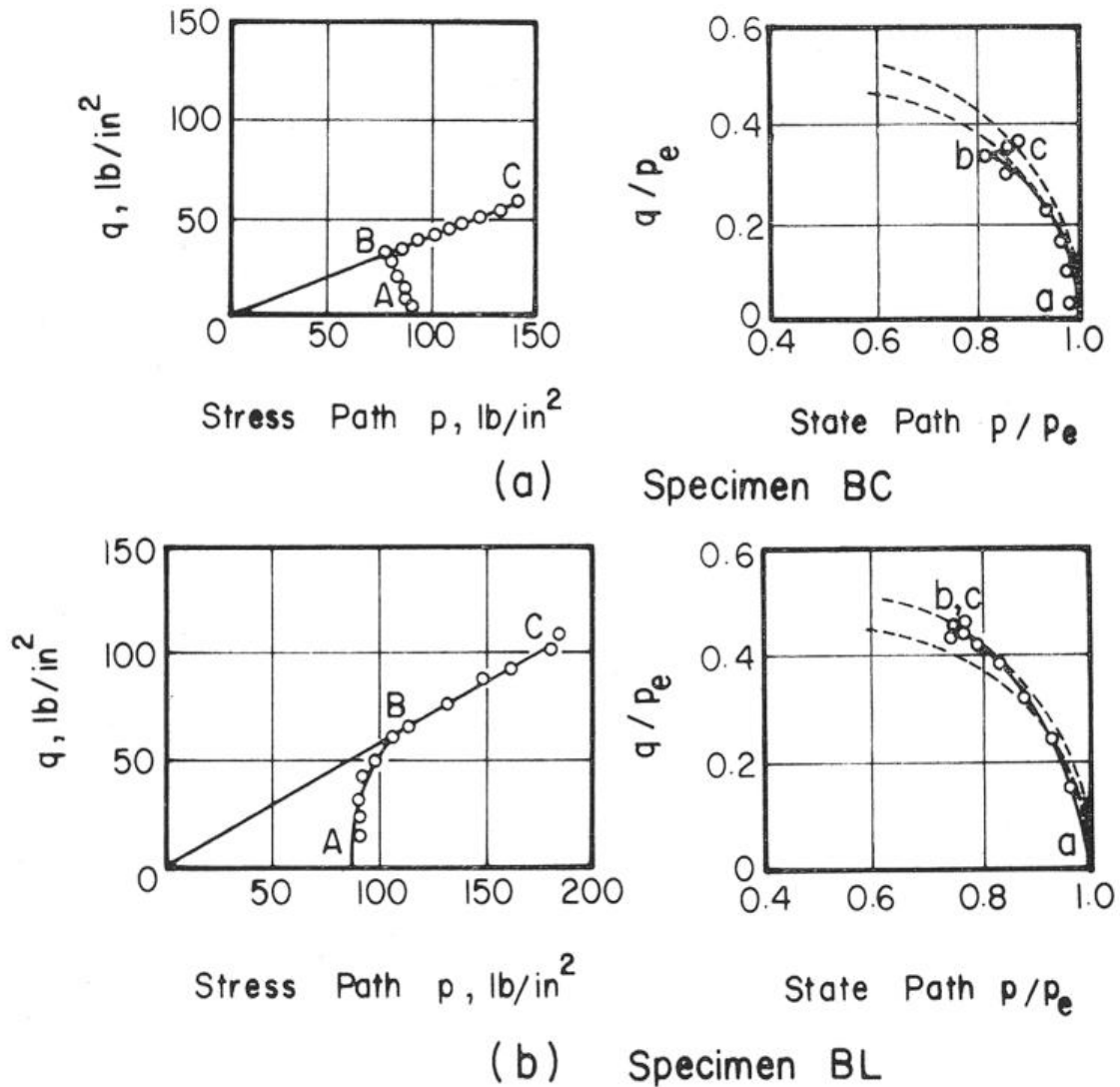


Fig. 9. Stress and state paths followed during anisotropic consolidation.

INCREMENTAL STRESS-STRAIN THEORY OF ROSCOE AND POOROOSHASB

Assuming a unique (p, q, e) surface, an incremental stress-strain theory was proposed by ROSCOE & POOROOSHASB (1963) of the form :

$$d\varepsilon_1 = \left(\frac{d\varepsilon}{d\eta}\right)_v d\eta + \left(\frac{d\varepsilon}{dv}\right)_\eta dv \dots\dots\dots (7)$$

where $(d\varepsilon_1/d\eta)_v$ corresponds to the variation of ε_1 with η in an undrained test, and $(d\varepsilon_1/dv)_\eta$ represents the variation of ε_1 with v in a constant- η (anisotropic consolidation)-stress path. dv and $d\varepsilon_1$ are the incremental volumetric and axial strains.

The incremental stress-strain theory of ROSCOE and POOROOSHASB in Eq. 7 will now be presented in a slightly different form as follows :

$$d\varepsilon = \left(\frac{d\varepsilon}{d\eta}\right)_v d\eta + \left(\frac{d\varepsilon}{dv}\right)_v dv \dots\dots\dots (8)$$

where instead of using the incremental axial strain $d\varepsilon_1$, the incremental shear strain $d\varepsilon$ will be used. Equation 8 can be expressed in the form :

$$d\varepsilon = f_1(\eta) d\eta + \frac{1}{f_2(\eta)} dv \dots\dots\dots (9)$$

where $f_1(\eta) d\eta$ has been shown to be equal to the incremental shear strain in an undrained test (Eq. 2, and $f_2(\eta)$ is equal to $(dv/d\varepsilon)_\eta$. Therefore, the total shear strain in any type of test can be obtained by integrating Eq. 9. Thus:

$$\varepsilon = \int_0^\eta f_1(\eta) d\eta + \int_0^\eta \frac{1}{f_2(\eta)} dv \dots\dots\dots (10)$$

Determination of Volumetric and Shear Strains

The procedure adopted for calculating the volumetric strain along any applied stress path is as follows. Let AB in Fig. 10 be the applied stress path for which the volumetric strain-stress ratio relationship is being determined. Then the undrained stress path AC through the point A is drawn assuming that the state boundary surface is unique (since intersections of the state boundary surface by planes of constant void ratios are similar curves). The path AB is divided into a large number of small steps, $AB_1, B_1B_2, B_2B_3 \dots B_nB$. Then each step AB_1 is considered in two parts; AC_1 , along an undrained stress path and C_1B_1 , along an anisotropic consolidation path (stress ratio η_1). The volumetric strain experienced by the specimen at states, $B_1, B_2 \dots B_n, B$ is denoted by $v_{B_1}, v_{B_2} \dots v_B$, and the incremental volumetric strain for the steps $AB_1, B_1B_2 \dots B_nB$ by $dv_{AB_1}, dv_{B_1B_2} \dots dv_{B_nB}$. Similar notation is used for the shear strain, i.e. $\varepsilon_{B_1}, \varepsilon_{B_2} \dots \varepsilon_B$ denotes the shear strain at states $B_1, B_2 \dots B_n$. The incremental shear strain for the steps $AB_1, B_1B_2 \dots B_nB$ is $d\varepsilon_{AB_1}, d\varepsilon_{B_1B_2} \dots d\varepsilon_{B_nB}$. The notation

INCREMENTAL STRESS STRAIN THEORY

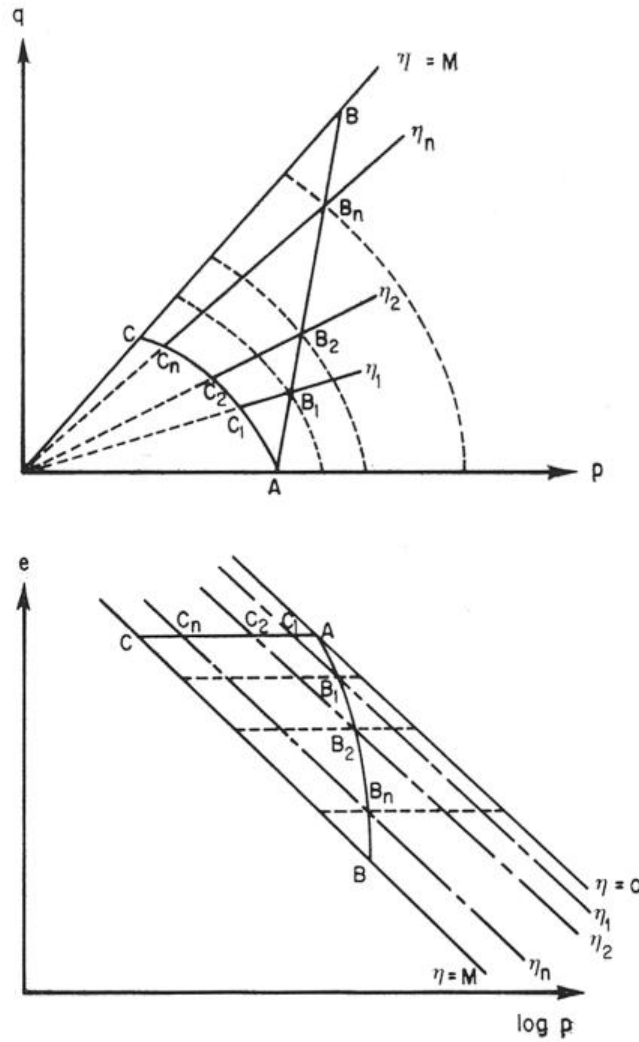


Fig. 10. Incremental steps in the calculation of volumetric and shear strains in drained test.

for the shear strains along the undrained stress path AC for the states C_1, C_2, \dots, C_n is $\epsilon_{C_1}, \epsilon_{C_2}, \dots, \epsilon_{C_n}$ and for the incremental shear strains for the steps, $AC_1, C_1C_2, \dots, C_{n-1}C_n, CnC$ is $d\epsilon_{AC_1}, d\epsilon_{C_1C_2}, \dots, d\epsilon_{C_{n-1}C_n}, d\epsilon_{CnC}$.

Assuming a unique state boundary surface, it can be shown that :

$$dv_{AB_1} = dv_{AC_1} + dv_{C_1B_1} \dots \dots \dots (11)$$

and since:

$$\begin{aligned} dv_{AC_1} &= 0 \\ dv_{AB_1} &= dv_{C_1B_1} \dots \dots \dots (12) \end{aligned}$$

Let (e_o, p_o) be the void ratio and the mean normal stress at point A on the isotropic consolidation line. Then the isotropic consolidation line can be expressed as :

$$e_o - e = -\lambda \log (p_o/p) \dots \dots \dots (13)$$

If p_{C_1} , p_{B_1} denote the mean normal stress for states C_1 and B_1 and if e_{B_1} is the void ratio corresponding to the state B_1 , then :

$$e_o - e_{B_1} = -\lambda \log (p_{C_1}/p_{B_1}) \quad \dots\dots\dots (14)$$

Also:
$$v_{B_1} = dv_{C_1 B_1} = dv_{AB_1} = \log \left(\frac{1 + e_o}{1 + e_{B_1}} \right) \quad \dots\dots\dots (15)$$

Hence, knowing e_o , λ , p_{C_1} and p_{B_1} , e_{B_1} can be evaluated and therefore v_{B_1} , $dv_{C_1 B_1}$, dv_{AB_1} may be determined.

Similarly, considering the state points C_2 , B_2 :

$$e_o - e_{B_2} = -\lambda \log (p_{C_2}/p_{B_2}) \quad \dots\dots\dots (16)$$

and
$$v_{B_2} = \log \left(\frac{1 + e_o}{1 + e_{B_2}} \right) \quad \dots\dots\dots (17)$$

Thus, from Eqs. 16, 17 v_{B_2} can be evaluated. Then :

$$dv_{B_1 B_2} = v_{B_2} - v_{B_1} \quad \dots\dots\dots (18)$$

The same procedure can be repeated to determine v_{B_3} , v_{B_4} v_{B_n} and also $dv_{B_2 B_3}$, $dv_{B_3 B_4}$ $dv_{B_n B}$. Hence, the volumetric strain stress ratio relationship can be established.

The procedure adopted for determining the shear strain-stress ratio relationship will now be described. Using Fig. 3, the shear strain ϵ_{C_1} , ϵ_{C_2} ϵ_{C_n} for stress ratios η_1 , η_2 η_n can be obtained for points C_1 , C_2 C_n . Similarly, using Fig. 7, the values of $f_2(\eta)$ can be estimated for the paths with stress ratios η_1 , η_2 η_n .

Hence :

$$\epsilon_{B_1} = \epsilon_{C_1} + f_2(\eta_1) dv_{A_1 B_2} \quad \dots\dots\dots (19)$$

$$\epsilon_{B_2} = \epsilon_{C_2} + f_2(\eta_1) dv_{A B_1} + f_2(\eta_2) dv_{B_1 B_2} \quad \dots\dots\dots (20)$$

and
$$\epsilon_B = \epsilon_C + \sum f_2(\eta) dv \quad \dots\dots\dots (21)$$

Thus, the shear strain-stress ratio relationship may be obtained. The above calculations of the volumetric and shear strains are carried out by using a computer and taking increments of stress ratio of 0.01 for all applied stress paths discussed in the subsequent section. Any further reduction in the stress ratio did not have any appreciable effect on the magnitude of the strains.

COMPARISON OF OBSERVED AND PREDICTED STRAINS FOR A VARIETY OF STRESS PATHS

Constant Mean Normal Stress Path

A constant mean normal stress test was carried out starting at 90 lb/in² isotropic stress. During this test, the axial stress and the cell pressure were

INCREMENTAL STRESS STRAIN THEORY

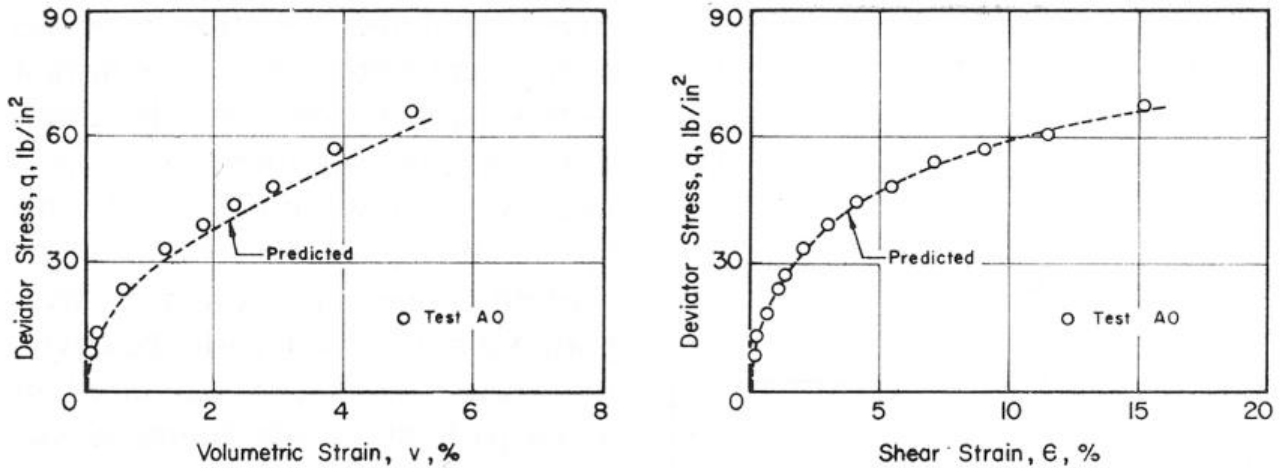


Fig. 11. Observed and predicted stress-strain relationships for constant p test.

adjusted, in such a way that the mean normal stress p remained constant. The state path followed by this specimen can be viewed in the $(q/p_e, p/p_e)$ plot, which has already been presented in Fig. 2. The (q, ϵ) and the (q, v) characteristics for this specimen are shown in Fig. 11. In this figure, the experimental points are shown as circles and the predictions from the theory by the dotted curve. Excellent agreement between experimental and predicted strains can be observed.

Fully Drained Stress Path

A fully drained test with constant cell pressure was performed starting from 90 lb/in² isotropic stress condition. The state path followed by this specimen in the $(q/q_e, p/p_e)$ plot, has already been shown in Fig. 2. The (q, ϵ) and the (q, v) relationships for this specimen are shown in Fig. 12. In this figure, the experimental points are plotted as circles and the predictions from the theory are indicated by the dotted curve. Again good agreement is noticed between experimental and predicted strains.

Constant Deviator Stress Path

The stress paths considered so far were such that the deviator

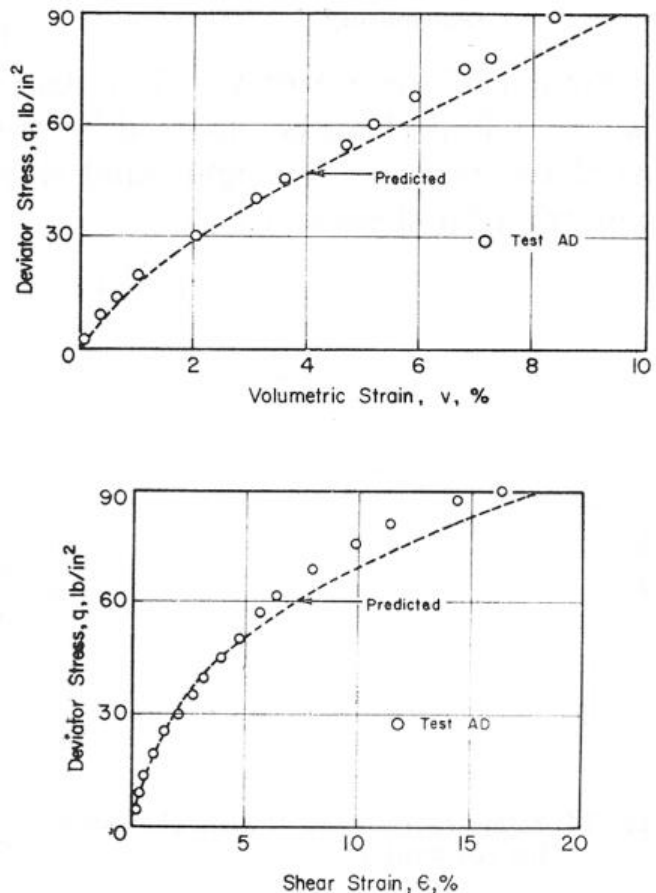


Fig. 12. Observed and predicted stress-strain relationships for fully drained test with constant cell pressure.

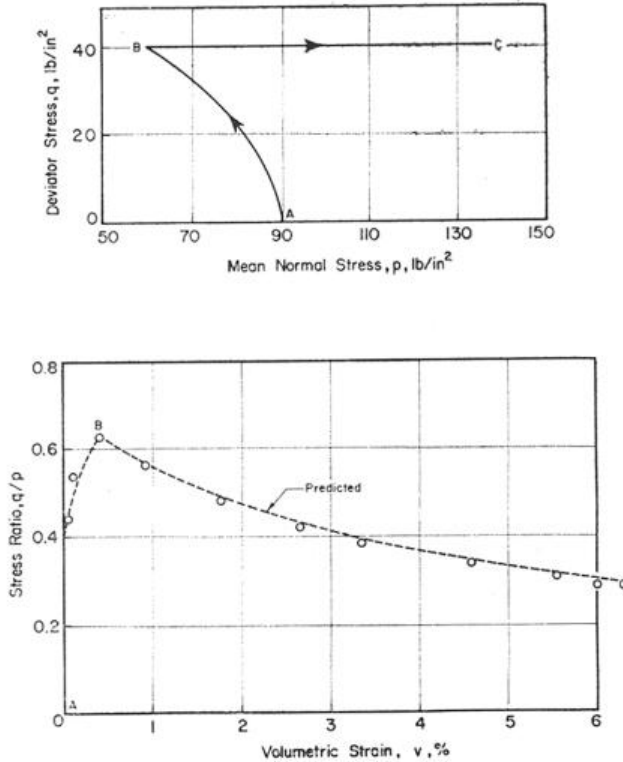


Fig. 13. Constant q test (a) stress path
(b) stress ratio-volumetric strain relationship.

prediction of shear strains. For such paths, the incremental stress-strain theory could perhaps be modified by neglecting the undrained component of the shear strain on the right hand side of Eq. 8. This would lead to the expression for shear strain as :

$$d\varepsilon = \left(\frac{d\varepsilon}{dv} \right)_{\eta} \cdot dv. \quad \dots\dots\dots (22)$$

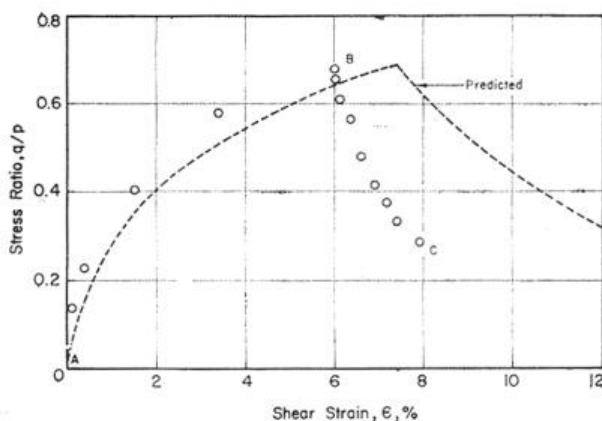


Fig. 14. Stress ratio-shear strain relationship for constant q test.

stress q and also the stress ratio q/p were increasing. For a constant q -stress path, the stress ratio decreases. The stress path followed by a specimen BX is shown in Fig. 13. In this figure, the specimen was initially sheared along an undrained stress path AB and subsequently the deviator stress q was maintained constant along path BC, while the mean normal stress p was increased. The observed and the predicted volumetric strains for the stress path ABC are also shown in Fig. 13. Excellent agreement is again noted between the two strains. However, for the stress paths for which the stress ratio (q/p) decreases, the incremental stress-strain theory of ROSCOE & POOROOSHASB cannot be used for the

In Eq. 22, it is assumed that the value of $\left(\frac{d\varepsilon}{dv} \right)_{\eta}$ which is given by $1/f_2(\eta)$, when η is increasing is also valid when η is decreasing. The observed and the predicted strains for the stress paths ABC (shown in Fig. 13) are shown in Fig. 14, where the prediction is somewhat more than the actual strains for the phase in which q was maintained constant.

Reloading of Specimens with Initial States on the State Boundary Surface

The stress-strain behaviour during subsequent unloading (deviator stress)

INCREMENTAL STRESS STRAIN THEORY

and reloading (deviator stress) of initially sheared samples with the mean normal stress p increasing all the time will now be considered. The type of stress path imposed on a sample, and its previous stress history is shown in Fig. 15. This specimen was initially sheared from the isotropic stress state A to the stress state B with increasing stress ratio. Subsequently, the specimen was sheared from B to C along a stress path with decreasing deviator stress (q) and increasing mean normal stress (p). This second phase of shear can be divided into two types, depending on the slope of the applied stress path. In one case, the stress path BC lies above the downward tangent to the undrained stress path AB at B. For this case, the point C will lie on the state boundary surface and, therefore, the specimen will be in a normally consolidated state. In the second case, the stress path BC lies below the tangent to the undrained stress path and the point C will lie below the state boundary surface. Only the first

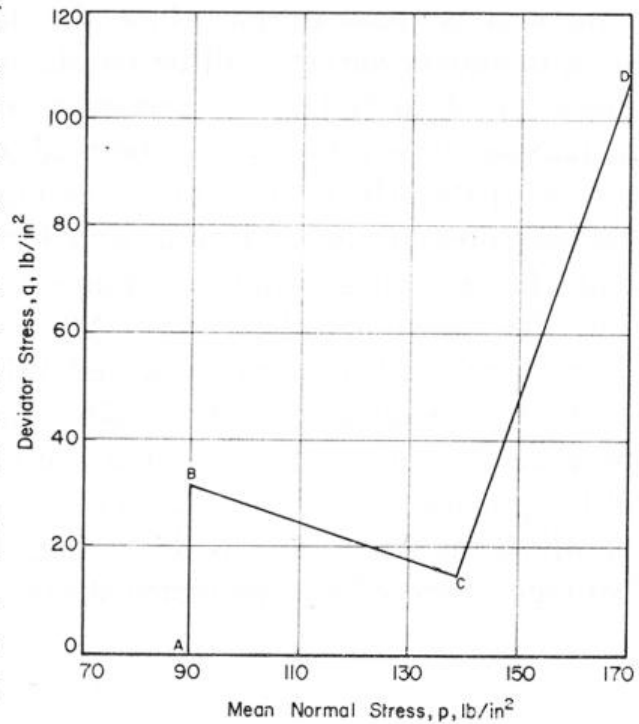


Fig. 15. Stress path followed by Specimen BP, Showing the phases in which the deviator stress was decreased and then increased.

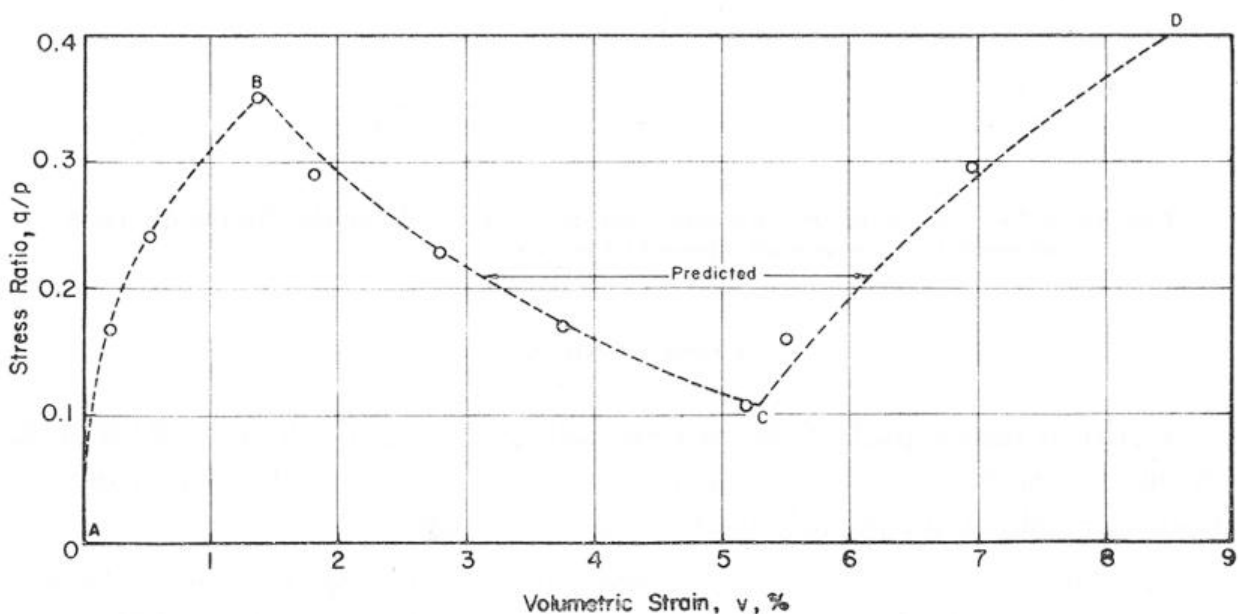


Fig. 16. Observed and predicted stress ratio-volumetric strain relationship for the specimen subjected to stress path shown in Fig. 15.

type of stress path during which the state of the sample always lies on the state boundary surface will be considered for the phase BC. In the third phase of shear (CD), the specimen was subjected to a stress path with increasing stress ratio. The observed and the predicted volumetric strains for the path ABCD are shown in Fig. 16. Excellent agreement is noted between observation and prediction for all three phases of shear, i.e. AB, BC and CD. The shear strain experienced by the specimen during the loading and the reloading phases (i.e. AB and CD) is shown in Fig. 17. In this figure, the predicted and observed values of the shear strains have been made to coincide at point C (which is the start of the reloading phase). This procedure is adopted, since it has already been shown that there is very little agreement between observed and predicted shear strain for the phase in which the stress ratio is decreasing. Good agreement is noted, however, between observed and predicted shear strains for the reloading phase CD.

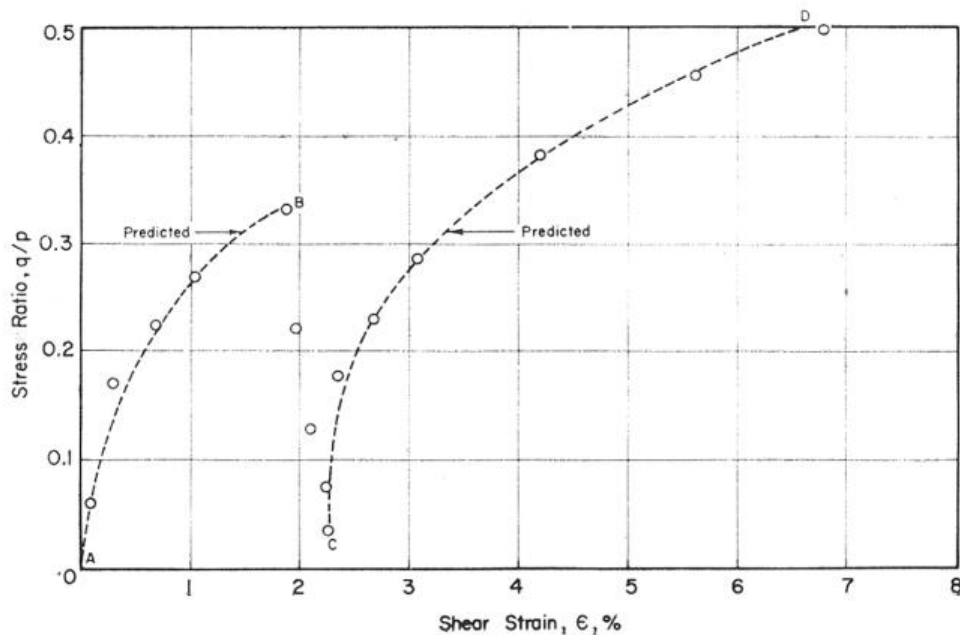


Fig. 17. Observed and predicted stress ratio-shear strain relationship for the specimen subjected to stress path shown in Fig. 15.

CONCLUSIONS

A critical reappraisal of the incremental stress-strain theory of ROSCOE & POOROOSHASB has been made. From experimental observations and predictions the following conclusions are reached:

- (1) The $(q/p, \epsilon)$ characteristics of normally consolidated specimens sheared under undrained conditions are unique and independent of the consolidation pressure.

INCREMENTAL STRESS STRAIN THEORY

- (2) Provided that the effect of the initial one-dimensional consolidation stress is absent, the state boundary surface is unique for all types of tests on a normally consolidated clay.
- (3) During anisotropic consolidation, the strain increment ratio $(dv/d\varepsilon)_\eta$ is a constant.
- (4) During anisotropic consolidation, the void ratio varies linearly with the logarithm of the mean normal stress. The slope of the line is the same as the slope λ of the isotropic consolidation line in an $(e, \log p)$ -plot.
- (5) The state path during anisotropic consolidation lies on the state boundary surface.
- (6) The volumetric strains in all types of tests may be successfully predicted from the state boundary surface for normally consolidated clays as obtained from undrained tests (or other types of tests), provided that the states of the sample lie on the state boundary surface.
- (7) The shear strain in all types of tests on normally consolidated clays may be predicted from the incremental stress-strain theory of ROSCOE & POOROOSHASB, provided that the stress paths are such that the stress ratio always increases or remains constant.

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REFERENCES

- BALASUBRAMANIAM, A.S. (1969), Some Factors Influencing the Stress-Strain Behaviour of Clays, *Ph.D. Thesis*, Cambridge University, England.
- BALASUBRAMANIAM, A.S. (1973), Stress History Effects on Stress-Strain Behaviour of a Saturated Clay, *Geotechnical Engineering*, Vol. 4, pp. 91-111.
- BALASUBRAMANIAM, A.S. (1974), A Critical Study of the Uniqueness of State Boundary Surface for Saturated Specimens of Kaolin, *Geotechnical Engineering*, Vol. 5, pp. 21-38.
- JAMES, R.G. and BALASUBRAMANIAM, A.S. (1971a), The peak Stress Envelopes and their Relation to the Critical State Line for a Saturated Clay, *Proc. 4th Asian Regional Conf. Soil Mech. Found. Eng.*, Bangkok, Vol. 1, pp. 115-120.
- JAMES, R.G. and BALASUBRAMANIAM, A.S. (1971b), A Comprehensive Experimental Study of the Strength Characteristics of Remoulded Specimens of Kaolin, *Geotechnical Engineering*, Vol. 2, pp. 21-33.

- HVORSLEV, M.J. (1973), Über die Festigkeitseigenschaften Gestörter Bindiger Böden, *Thesis, Ingeniørvideenskabelige, Skrifter, Series A*, No. 45.
- ROSCOE, K.H. and BURLAND, J.B. (1968), On the Generalised Stress-Strain Behaviour of Wet Clay, *Engineering Plasticity*, Cambridge University Press, 1968, pp. 535-609.
- ROSCOE, K.H. and POOROOSHASB, H.B. (1963), A Theoretical and Experimental Study of Strains in Triaxial Tests on Normally Consolidated Clays, *Géotechnique*, Vol. 13, pp. 12-38.
- ROSCOE, K.H., SCHOFIELD, A.N. and THURAIRAJAH, A. (1963), Yielding of Clays in States Water than Critical, *Géotechnique*, Vol. 13, pp. 211-240.
- ROSCOE, K.H., SCHOFIELD, A.N. and WROTH, C.P. (1958), Yielding of Soils. *Géotechnique*, Vol. 8, pp. 22-52.
- ROSCOE, K.H. and THURAIRAJAH, A. (1964), On the Uniqueness of Yield Surfaces for Wet Clays, *Proc. Int. Symp. Rheology and Soil Mech.*, IUTAM- Grenoble, pp. 364-381.
- ROWE, P.W. and BARDEN, L. (1964), The Importance of Free Ends in the Triaxial Tests, *Journal Soil Mech. Found. Div., ASCE*, Vol. 90, SM. 1, pp. 1-27.
- SCHOFIELD, A.N. and WROTH, C.P. (1968), *Critical State Soil Mechanics*, McGraw-Hill, London.
- TING, W.H. (1968), Some Effects of History on the Stress-Strain Behaviour of Kaolin, *Ph.D. Thesis*, Cambridge University, England.

APPENDIX : NOTATION

e -	voids ratio
e_o -	voids ratio prior to shear
f_1, f_2 -	functions
k -	principal effective stress ratio σ_1' / σ_3'
L -	current height of sample
L_o -	initial height of sample
p -	mean normal stress
p_o -	mean normal stress prior to shear
p_e -	mean equivalent pressure
q -	deviator stress
v -	volumetric strain
V -	current volume of sample
V_o -	initial volume of sample
$\sigma_1', \sigma_2', \sigma_3'$ -	principal effective compressive stress.
$\epsilon_1, \epsilon_2, \epsilon_3$ -	principal compressive strains
ϵ -	shear strain
λ -	slope of isotropic consolidation line in $(e, \log p)$ plot
η -	q/p