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Deformation Response of Geocells in Pavements under Moving Loads

Priti Maheshwari[1] and G. L. Sivakumar Babu[2]
[1]*Department of Civil Engineering, Indian Institute of Technology Roorkee, Roorkee, India*
[2]*Department of Civil Engineering, Indian Institute of Science Bangalore, Bangalore, India*
[1]*E-mail*: priti_mahesh2001@yahoo.com
[2]*E-mail*: gls@civil.iisc.ernet.in

ABSTRACT: Geocells are extensively used in pavements as one of the ground improvement techniques. Pavements are subjected to various types of loading pattern and its deformation under these loads plays an important role in its analysis and design. In the present work, a deformation model of geocell has been proposed in which geocell has been idealized as an infinite beam subjected to a concentrated load moving with constant speed. The foundation soil has been modeled as Winkler springs. Influence of magnitude and speed of applied load, flexural rigidity of geocell, modulus of subgrade reaction of foundation soil, mass of beam, viscous damping and interfacial resistance between geocell reinforcement and the neighboring soil on response of geocell has been studied. Non-dimensional charts have been developed for normalized deflection and the bending moment in geocell reinforcement. These charts will be useful while analyzing and designing the pavements under moving loads. A numerical example has also been presented for the better understanding of results from the proposed model.

KEYWORDS: Geocells, Deformation, Pavements, Infinite beams, Viscous damping, Ground improvement.

1. INTRODUCTION

Many ground improvement techniques are employed especially in the area of pavement engineering and railroads. Whereas in unpaved roads the tolerable deflections are measured in terms of tens to hundreds of millimetres, in railways tracks they are measured in millimetres, and in paved roads the tolerance deflections are only fractions of millimetres. In order to restrain the deformation, one of the most adopted ground improvement technique is reinforcement of soil by geosynthetics. Geocells are particular type of geosynthetics, which have honeycombed structure and provide confinement to the infill material. This helps in improving the bearing resistance of soil, reducing total and the differential settlements and improving the drainage etc. Further, low construction cost makes it a viable solution to many problems related to poor ground conditions. Pavements are subjected to static, moving and the cyclic loading conditions. The magnitude of static loading is very high as compared to moving and cyclic loading; however repeated loading cycles becomes crucial in the analysis and design of pavements. In the design of pavements using geocell, suitable partial factors can be used to take care of effects due to traffic loading, age hardening and temperature variation. During heavy rainfall, the unreinforced pavements may suffer from severe rutting causing frequent maintenance. With the provision of a geocell layer, this can be reduced to large extent which results into enhanced serviceability (Rajagopal et al., 2014). Therefore, it becomes essential to study the deformation characteristics of pavements under moving loads.

There are many studies available for analysis of earth beds reinforced with geocells. Some of the recent studies include Bush et al. (1990), Cowland and Wong (1993), Rajagopal et al. (1999), Maheshwari et al. (2004a), Zhou and Wen (2008), Zhang et al. (2009, 2010, 2012), Tafreshi and Dawson (2012), Yang et al. (2012), Tang and Yang (2013), Rajagopal et al. (2014) etc. In these studies, the soil-foundation system was analyzed under either static or cyclic loads. On the other hand, some of the research studies pertaining to the analysis of infinite beams (can be the idealization for pavements or rails) under moving loads include Kenney (1954), Kerr (1964) Saito and Teresawa (1980), Wang et al. (1984), Duffy (1990), Zaman et al. (1991), Alvappallai et al. (1992), Jaiswal and Iyenger (1997), Maheshwari et al. (2004b), Mallik et al. (2006) etc. Frýba (1999) analyzed the influence of moving loads on various aspects related to transportation engineering where different types of moving load pattern cause lot of problems in view of increasing higher speed and vehicular weights.

Detailed literature review and its critical evaluation suggest a need for the analysis of pavements reinforced by geocells under moving loads. In view of this, an attempt has been made to carry out deformation analysis of geocell reinforcement under moving loads. A simple mathematical model has been proposed in which the geocell reinforcement has been considered as an infinite beam and is subjected to a concentrated load moving with constant speed. Foundation soil has been idealized employing Winkler model. Viscous damping and interfacial frictional resistance between geocell reinforcement and the foundation soil has also been considered in the analysis. Governing differential equation has been derived and solved with the help of appropriate boundary conditions. Finite difference form of these equations has been solved by iterative Gauss Seidel method.

2. PROBLEM DEFINITION AND ANALYSIS

Figure 1 shows geocell reinforcement with finite flexural rigidity, EI subjected to concentrated load, P moving with a constant speed v. Geocell has been idealized as an infinite beam, however, for the analysis purpose a length of $2L$ has been considered. L has been chosen to be large enough so that the beam can be considered as an infinite beam (Hetényi, 1979). Geocell has been considered to have unit width and height h. A uniformly distributed load, p, has been considered on the beam to account for the weight of soil on top of the beam. Unit weight of infill material has been considered as γ. The following assumptions have been made for the analysis purpose:

i) The load has been assumed to move with constant speed from left to right.
ii) The mass of the moving load has been assumed to be small compared with the mass of the beam and mass of soil on top of beam and therefore, only the gravitational effects of the load has been considered.
iii) Geocell reinforcement has been assumed to have constant cross section and constant mass per unit length.
iv) The beam damping has been assumed to be proportional to the velocity of vibration.

Tension in geocell, T, is mobilized by the frictional resistance on the interface between reinforcement beam and the neighboring soil and can be expressed as (Zhang et al. 2009)–

$$T = \left(2L - x\right)\ \tau \qquad (1)$$

where, x is the coordinate along the length of geocell and τ, the interface frictional resistance in unit length.

The differential equation of a moving load on a beam may be obtained by considering the bending of an elemental segment

$$-M = EI \frac{\partial^2 w}{\partial x^2} \tag{2}$$

where w is vertical deflection and M, the bending moment.

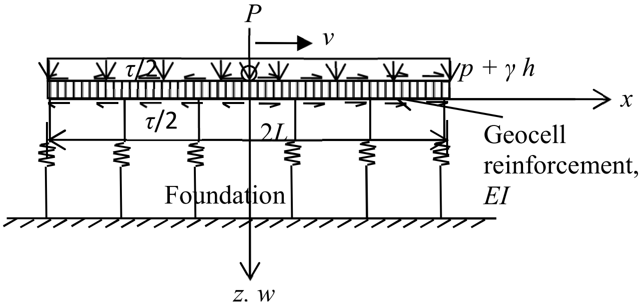

Figure 1 Schematic diagram and proposed model for geocell reinforced earth bed

The loads applied on a beam rests on elastic foundation at any time, t > 0, are

$$\left.\begin{array}{ll} \text{Inertia:} & \rho \dfrac{\partial^2 w}{\partial t^2} \\ \text{Viscous damping:} & c \dfrac{\partial w}{\partial t} \\ \text{Soil reaction:} & q \\ \text{Applied force:} & P(x,t) \\ \text{Tension mobilized} & T \end{array}\right\} \tag{3}$$

where, ρ is the mass per unit length of the beam (inclusive of weight of soil on top of beam and weight of beam itself), c is coefficient of viscous damping per unit length of the beam and $P(x,t)$ is the applied load intensity.

Combining equations (1), (2) and (3) and with the assumptions as mentioned above, the differential equation of the beam with uniform cross section under moving load can be written as:

$$EI \frac{\partial^4 w}{\partial x^4} - (2L - x)\, \tau \frac{\partial^2 w}{\partial x^2} + \tau \frac{\partial w}{\partial x} + \rho \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + k\, w = P(x,t) \tag{4}$$

where, k is the modulus of subgrade reaction of foundation soil.

For particular values of the parameters, the above equation governs the response of existing models for beams on elastic foundation subjected to moving load (Kenney, 1954; Frýba, 1999).

For the above formulation, a quasi-stationary state is reached in which beam is at rest relative to the moving coordinate system. This state is arrived if the moving load is no longer dependent on time after sufficiently long travel time and is only dependent on distance from the origin of coordinate axes. This origin also moves uniformly. For the sake of convenience, a new independent variable has been defined as –

$$\xi = x - vt \tag{5}$$

and therefore, $w = f(x,t) = f(\xi)$ and

$$\frac{\partial^2 w}{\partial x^2} = \frac{d^2 w}{d\xi^2}, \frac{\partial w}{\partial t} = -v \frac{dw}{d\xi}, \frac{\partial^2 w}{\partial t^2} = v^2 \frac{d^2 w}{d\xi^2} \tag{6}$$

Combining equations (4), (5) and (6), one gets,

$$EI \frac{d^4 w}{d\xi^4} - (2L - \xi)\tau \frac{d^2 w}{d\xi^2} + \tau \frac{dw}{d\xi} + \rho v^2 \frac{d^2 w}{d\xi^2} - cv \frac{dw}{d\xi} + kw = P(\xi) \tag{7}$$

Equation (7) presents the governing differential equation of the system under consideration.

At both ends of the geocell reinforcement, deflection and the slope of deflected shape have been considered to be zero. These boundary conditions under quasi-stationary state can be written as follows:

At $\xi = -L$ and at $\xi = L$, w and $dw/d\xi = 0$.

To observe the deformational response of geocell reinforcement, the above equation (7) has been written in non-dimensional form as –

$$\frac{d^4 W}{d\xi^{*4}} + \left[-\frac{\tau^*}{I^*}\left(2 - \xi^*\right) + \frac{\rho^*}{I^*} \right] \frac{d^2 W}{d\xi^{*2}} + \left[\frac{\tau^*}{I^*} - \frac{c^*}{I^*} \right] \frac{dW}{d\xi^*} + \frac{W}{I^*} = \frac{P^*}{I^*\, d\xi^*} \tag{8}$$

and boundary conditions can be written as

At $\xi^* = -1$ and at $\xi^* = 1$, W and $dW/d\xi^* = 0$. (9)

where, $\xi^* = \xi/L$, $W = w/L$, $\rho^* = \rho\, v^2 / k\, L^2$, $I^* = E\, I / k\, L^4$, $\tau^* = \tau / k\, L$, $c^* = c\, v / k\, L$ and $P^* = P / k\, L^2$.

Writing equation (8) in finite difference form within specified space domain, for an interior node, i, at any time instance, one gets,

$$\begin{aligned} & W_{i-2} + W_{i-1}\left[-4 + A\left(\Delta\xi^*\right)^2 - 0.5\, B\left(\Delta\xi^*\right)^3 \right] + \\ & W_i \left[6 - 2\, A\left(\Delta\xi^*\right)^2 + \frac{1}{I^*}\left(\Delta X\right)^4 \right] + \\ & W_{i+1}\left[-4 + A\left(\Delta\xi^*\right)^2 + 0.5\, B\left(\Delta\xi^*\right)^3 \right] + \\ & W_{i+2} = \left[\frac{P^*}{I^*} \right] \left(\Delta\xi^*\right)^3 \end{aligned} \tag{10}$$

where, $A = -\dfrac{\tau^*}{I^*}\left(2 - \xi_i^*\right) + \dfrac{\rho^*}{I^*}$ and $B = \dfrac{\tau^* - c^*}{I^*}$

The governing differential equation (10) has been solved along with appropriate boundary conditions (9) using Gauss Seidel iterative scheme.

3. CONVERGENCE CRITERION AND RANGE OF PARAMETRIC VALUES

Based on the formulation presented above, a computer program has been developed to obtain the response of geocell reinforcement using finite difference scheme. The half length of the beam has been taken to be large enough so that the beam can be assumed to act as an infinite beam. In view of viscous damping, complete region of the problem ($-L \le x \le L$) has been considered for analysis. It was observed that the difference in deflection response corresponding to finite difference mesh with 1001 nodes and 2001 nodes was less than 0.5% and hence the mesh with 1001 nodes was considered for all parametric studies. The solution has been obtained with convergence criteria as

$$\left|\frac{W_i^k - W_i^{k-1}}{W_i^k}\right| \times 100\% < \varepsilon_s$$

for all i, where k and k-1 are the present and previous iterations respectively and ε_s is the specified tolerance which has been considered to be 10^{-10} in the present study.

Once the deflection of geocell reinforcement all along its length has been determined, bending moment (M) was also obtained from second order derivative of deflection.

Realistic range of values of various input parameters pertaining to pavements have been adopted from the data available in the literature. Modulus of subgrade reaction (k) has been assumed to vary from 10 to 125 MN/m^3 which correspond to loose to medium soil (Das, 1999). The infill material in geocell reinforcement is assumed to be coarse sand for which unit weight has been varied between 16 to 22 kN/m^3 corresponding to possible range of relative densities (Das, 2008). Dimensions of Geocell reinforcement have been taken as: 300 m long, unit width and 0.1 m height. Accordingly, weight of geocell beam per unit length has been worked out. As per IRC: 37-2012, for CBR varying between 3 to 10%, the corresponding variation in thickness of base and bitumen cover is 70 to 220 mm and accordingly, uniformly distributed load on geocell reinforcement has been assumed to vary from 0 to 10 kN/m. Mass of geocell beam per unit length has been varied from 1000 to 1500 kg/m (inclusive of mass of beam and mass of soil above the beam). The range of value for Elastic modulus of geocell reinforcement has been considered as 1000-2500 MPa and for interface friction resistance as 100-400 kPa. Concentrated load at the centre of reinforcement beam has been varied between 40 and 100 kN (IRC: 37-2012). The magnitude of applied load is small to be in elastic range and small deformations have been considered and therefore, linear behaviour of foundation soil has been considered. IRC: 73-1980 and IRC: 86-1983 suggested the speed of load to vary between 20 and 150 km/hr and therefore the speed has been varied in the range 5 – 50 m/sec. The amount of damping has been expressed as a percentage of the critical damping which is defined as $(2(k\,\rho)^{1/2})$ and has been varied from 0 to 25% (Vucetic and Dobry, 1991).

4. RESULTS AND DISCUSSION

Before studying the influence of various parameters on the deformation response of geocell reinforcement, proposed model and the developed solution methodology has been verified. This verification has been conducted by comparing the results from Mallik et al. (2006) and the present study. Mallik et al. (2006) defined the critical velocity of moving load as $v_{cr} = \sqrt{\sqrt{\frac{k}{EI}} \Big/ \frac{\rho}{2\,EI}}$ and velocity ratio as v/v_{cr}. While presenting the results for verification, the coefficient of characteristic wavelength of unreinforced soil in static case has been used for the normalization which is expressed as $\lambda = (k / EI)^{1/4}$. The distance along the geocell i.e., x-axis has been normalized by multiplying the distance, ξ, (ahead and behind) from the load by λ and y-axis has been normalized by dividing the response the deflection by its maximum values in static case, i.e., when $v = 0$. Parameters considered for the purpose of verification have been taken as ρ = 25 kg/m, k = 40.78 × 10^5 N/m^2, EI = 1.75 × 10^6 N-m^2, P = 93.36 × 10^3 N/m, τ = 0, damping = 30% and velocity ratio = 0.50.

The comparison of deflection profile of beam from both the method has been presented in Figure 2. It is evident that both the deflection profiles match very well.

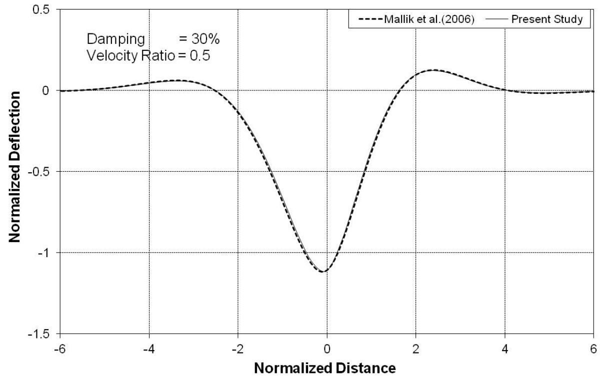

Figure 2 Comparison of deflection profile of geocell with Mallik et al. (2006)

All the results from parametric study have been presented in non-dimensional form. The response of system has been plotted on y-axis. This has been normalized with the help of respective response at the centre of beam.

The influence of magnitude of applied load has been presented in Figure 3. The input parameters have been considered as k = 50 MN/m^3, EI = 100 kN-m^2, τ = 200 kPa, ρ = 1200 kg/m, v = 30 m/sec and damping = 10%. Linear behaviour of foundation soil is clear from this figure as the deflection is increasing in the same ratio as the load is increasing. Similar observation can be made with respect to variation of bending moment in geocell reinforcement along its length as shown in Figure 4.

Deflection of geocell reinforcement has been found to unaffected by any variation in its flexural rigidity. However, flexural rigidity has significant influence on bending moment. This has been depicted in Figure 5 for input parameters as mentioned in the figure. Maximum normalized bending moment has been found to reduce by 64% as the parameter, EI reduces from 250 to 80 kN-m^2.

Figures 6 and 7 represent the influence of modulus of subgrade reaction on deflection and the bending moment respectively for other input parameters as P = 75 kN, EI = 100 kN-m^2, τ = 200 kPa, ρ = 1200 kg/m, v = 30 m/sec, damping = 10%. It can be seen that

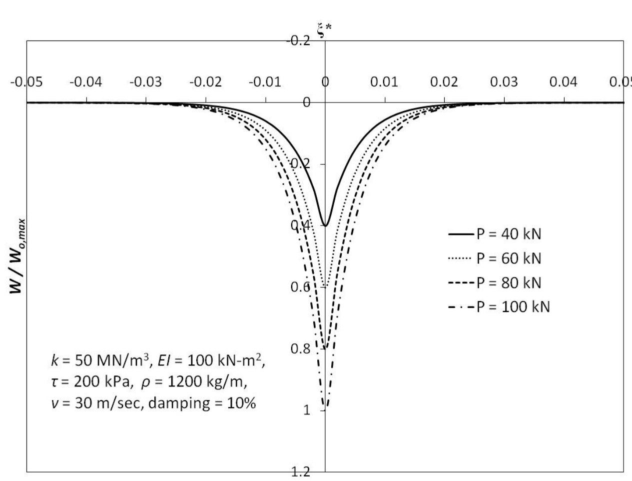

Figure 3 Deflection profile of geocell reinforcement: influence of applied load

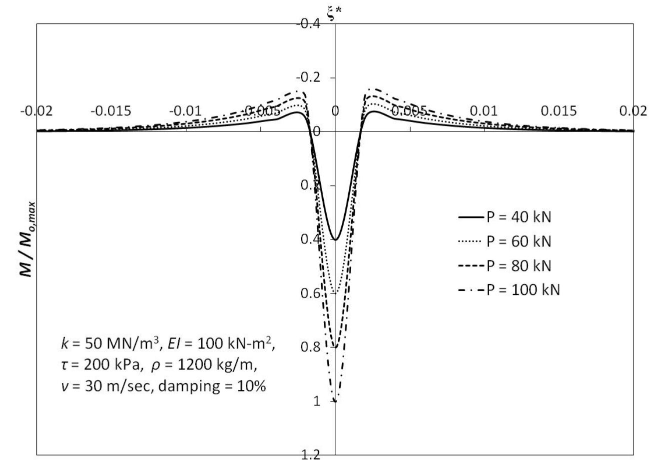

Figure 4 Variation in bending moment: influence of applied load

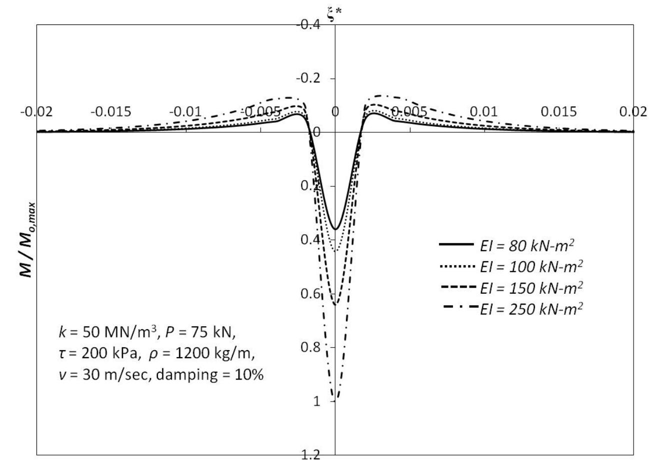

Figure 5 Variation in bending moment: influence of flexural rigidity of geocell reinforcement

magnitude as well as extent of both deflection and the bending moment reduces with increase in the modulus of subgrade reaction, k. As, k varies from 10 to 125 MN/m^3, the corresponding reductions in maximum normalized deflection and the bending moment have been found to be 73% and 94% respectively. Lower value of k signifies poorer soil and therefore higher are deflection and the bending moment.

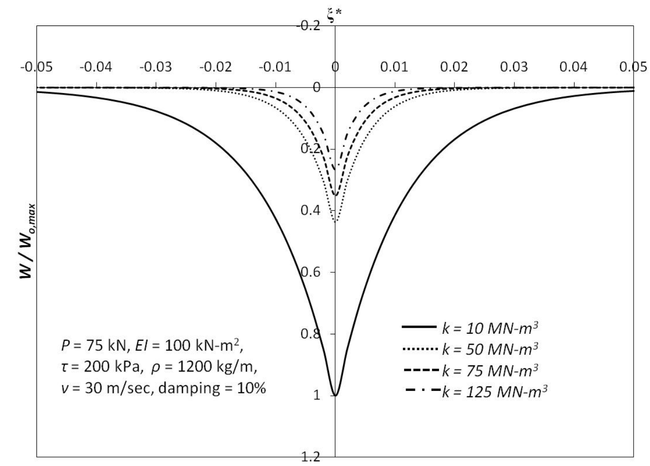

Figure 6 Deflection profile of geocell reinforcement: influence of modulus of subgrade reaction

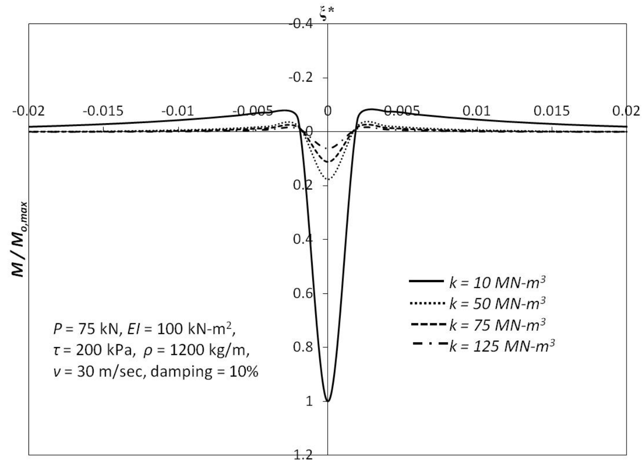

Figure 7 Variation in bending moment: influence of subgrade modulus of foundation soil

Deflection of geocell reinforcement has been found to be unaffected by any change in the speed of load. However, small increase of about 8% in maximum bending moment has been observed as speed increases from 5 to 50 m/sec for P = 100 kN, k = 50 MN/m^3, EI = 80 kN-m^2, τ = 200 kPa, ρ = 1200 kg/m, damping = 10%. Mostly, pavements and railroads are designed for a speed which is very low as compared to critical speed. In the range of design speed, the response remains unaffected or marginally affected. Value of critical speed is a function of parameters, k, EI and ρ and is evaluated

as $v_{cr} = \sqrt{\sqrt{\frac{k}{EI}} \Big/ \frac{\rho}{2\,EI}}$ (Frýba, 1999; Mallik et al., 2006). The

values of critical speed corresponding to variation of these parameters have been given in Table 1. The response of reinforcement has been found to be unaffected by its mass per unit length, ρ. However, it has been found to influence the critical speed significantly (Table 1).

Table 1 Influence of various parameters on critical speed

Input parameters	parameter	Value of parameter	Critical speed (m/sec)
EI = 150 kN-m^2, ρ = 1200 kg/m	k (MN/m^3)	10	45.2
		125	85.0
k = 50 MN/m^3, ρ = 1200 kg/m	EI (kN-m^2)	80	57.7
		250	76.8
k = 50 MN/m^3, EI = 80 kN-m^2	ρ (kg/m)	1000	74.0
		1500	60.4

Influence of viscous damping on response of geocell has been found to very nominal and this has been depicted in Figures 8 and 9. Figure 8 shows that at a speed of 50 m/sec, the deflection behind the load (ξ^*<0) increases as damping ratio increases from 0 to 25%. However, it has been found to reduce ahead of the load. The magnitude of negative bending moment has been found to be more in undamped case behind the load and ahead of load, magnitude of negative bending moment increases with an increase in damping ratio.

Effect of interfacial resistance between geocell and the neighbouring soil has been depicted in Figures 10 and 11 with respect to deflection and the bending moment in geocell reinforcement respectively for P = 75 kN, EI = 100 kN-m^2, k = 50 MN/m^3, ρ = 1200 kg/m, v = 50 m/sec, damping = 10%.

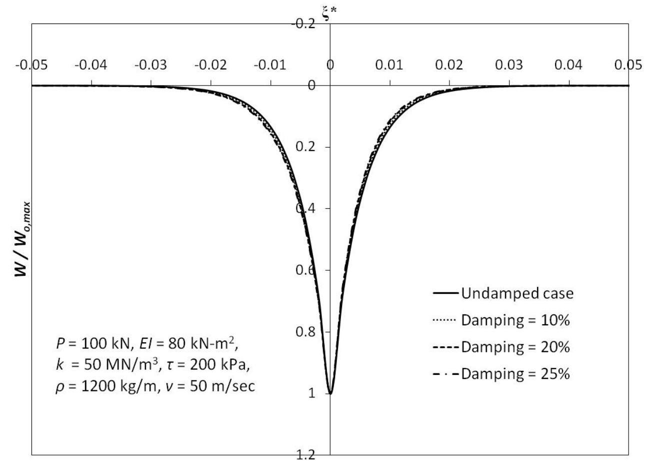

Figure 8 Deflection profile of geocell reinforcement: influence of damping

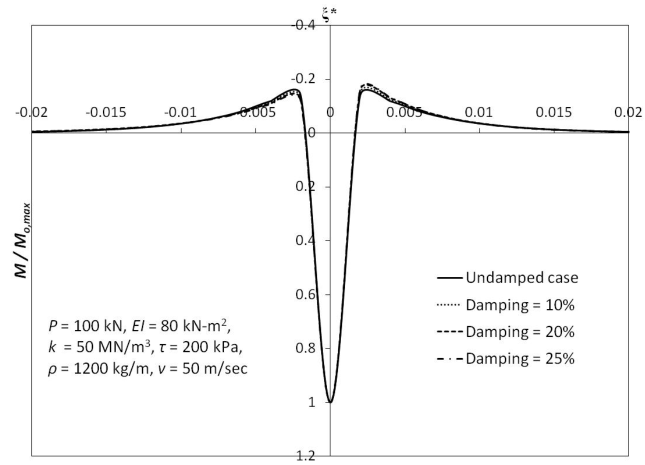

Figure 9 Variation in bending moment: influence of damping

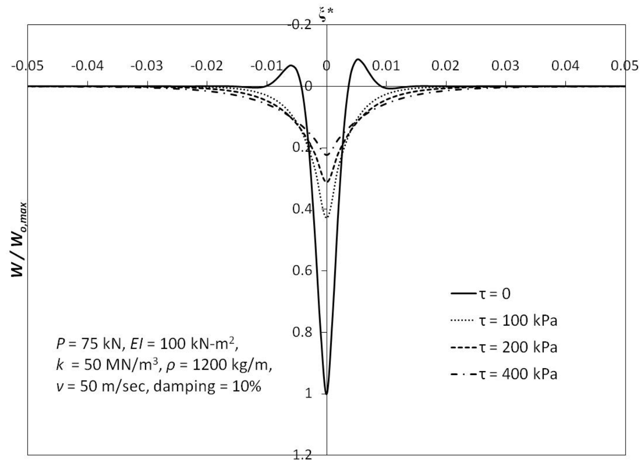

Figure 10 Deflection profile of geocell reinforcement: influence of interface resistance

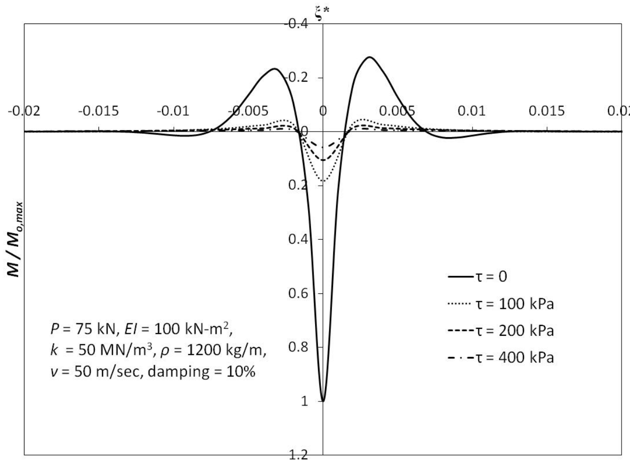

Figure 11 Variation in bending moment: influence of interface resistance

Maximum normalized deflection has been found to reduce by 48% as parameter τ increases from 100 to 400 kPa and the corresponding reduction in maximum bending moment has been found to be 69%. Negative deflection has been found to be occurring only in the case when interfacial frictional resistance was not considered in the analysis, i.e., interface between geocell reinforcement and neighbouring soil is smooth. In the absence of any frictional resistance, the geocell shall experience lift up from ground surface and therefore the presence of negative deflection. With consideration of interface friction resistance, the deflection just below the load reduces with an increase in it. However, ahead and behind the load (at the point of contra flexure), the deflection has been found to be more for lesser value of the parameter, τ.

For better understanding of the results, a numerical example has been worked out. Length and height of geocell reinforcement having unit width has been considered to 300 m and 0.1 m respectively. The influence of magnitude of load (P), flexural rigidity of geocell (EI), modulus of subgrade reaction (k) and interface friction resistance between geocell and the neighboring soil (τ) has been summarized in dimensional form in Table 2. As discussed above, the deflection of geocell increases in the same ratio as that of applied load (2.5 in the example).

The influence of parameter EI on maximum deflection is not significant as it changes from 1.75 mm to 1.64 mm only with an increase in EI from 80 to 250 kN-m^2. However, bending moment just below the load experiences a significant increase from 1228.6 to 3134.21 N-m, i.e. about 2.6 times. For the input parameters as P = 100 kN, EI = 80 kN-m^2, τ = 100 kPa, ρ = 1200 kg/m, v = 30 m/sec, damping = 10%, deflection reduces from 4.1 mm to 1.04 mm and bending moment from 1479.06 to 1005.53 N-m with an increase in k from 10 to 125 MN/m^3. The deflection reduces significantly from 3.61 to 1.31 mm and bending moment about 5 times, with consideration of interface friction resistance (τ= 100 kPa). Deflection and bending moment below the load have been found to further reduce to 0.68 mm and 283.8 N-m with an increase in parameter, τ from 100 to 400 kPa. The example illustrates the influence of different input parameters on deflection and bending moment of geocell in terms of numerical values.

Table 2 Influence of various parameters on deflection and bending moment of geocell reinforcement at its center

Input parameters	Parameter	Value of parameter	Deflection (mm)	Bending moment (N-m)
k = 10 MN/m^3, EI = 80 kN-m^2, τ = 100 kPa, ρ = 1200 kg/m, v = 30 m/sec, damping = 10%	P (kN)	40	1.64	591.62
		100	4.10	1479.06
k = 50 MN/m^3, P = 100 kN, τ = 100 kPa, ρ = 1200 kg/m, v = 30 m/sec, damping = 10%	EI (kN-m^2)	80	1.75	1228.60
		250	1.64	3134.21
P = 100 kN, EI = 80 kN-m^2, τ = 100 kPa, ρ = 1200 kg/m, v = 30 m/sec, damping = 10%	k (MN/m^3)	10	4.10	1479.06
		125	1.04	1005.53
k = 50 MN/m^3, P = 75 kN, EI = 80 kN-m^2, , ρ = 1200 kg/m, v = 30 m/sec, damping = 10%	τ (kN/m^2)	0	3.61	4609.27
		100	1.31	921.45
		400	0.68	283.79

5. CONCLUSIONS

Deformation analysis of geocell reinforcement employing proposed model provide an overall response in terms of deflection and the bending moment in geocell. It can be concluded that magnitude of load, modulus of subgrade reaction and interface friction resistance are the important parameters which influence deflection as well as bending moment in geocell reinforcement. Behavior of foundation soil has been considered to be linear and the same was evident in its response with respect to magnitude of applied load. Deflection and the bending moment have been found to increase in the same ratio as that of applied load. For the values of other parameters considered, the reduction in maximum normalized deflection and the bending moment has been found to be 73% and 94% respectively, as k varies from 10 to 125 MN/m^3. Analysis should be carried out considering the interface friction resistance which has been found to be very effective in reducing the deflection of geocell reinforcement.

Flexural rigidity of geocell has been found to affect only bending moment and not the deflection. It will not be an economical solution to increase its value for any reduction in deflection. Speed of moving load does not influence the response of system. However, based on values of modulus of subgrade reaction, flexural rigidity of geocell and its mass per unit length, the value of critical speed can be evaluated and accordingly design speed for respective condition can be decided. For better understanding of proposed method and the results, a numerical example has also been worked out and influence of various parameters has been depicted.

6. ACKNOWLEDGEMENTS

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