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Analysis of Effect of Reinforcement on Stability of Slopes and Reinforcement Length Optimization

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ABSTRACT: Steepening of slopes for construction of rail/road embankments or for widening for other civil engineering structures is a necessity for development. Use of geosynthetics for steep slope construction or repair of failed slopes considering all aspects of design and environment could be a viable alternative to these problems. Literature survey indicates that efforts are being made for optimization of length of reinforcement for overall economy. The present paper details an analysis to optimize the length of geosynthetic reinforcement from the face or near end of the slope with respect to its location to obtain the desired minimum factor of safety. Unreinforced and reinforced slopes are analyzed using Morgenstern-Price method to obtain critical factors of safety. The effect of providing geosynthetic reinforcement layer in shifting the critical slip circle has been identified and quantified. Consequently relatively smaller magnitude of force gets mobilized in the reinforcement.

KEYWORDS: Reinforcement, Optimization of length, Critical slip circle, Reinforced slope, Geosynthetics

1. INTRODUCTION

The analysis of earth slopes is the oldest geotechnical engineering problem that engineers have been dealing with using various techniques. The methods can be classified as Limit Equilibrium Methods, Finite Element Method based on c and ϕ reduction, Finite Element Modeling/Finite Difference Method, combination of FEM and LEM, Limit Analysis (LA) method, etc. Geosynthetic reinforcement of earth slope results in reducing the land requirement (Figure 1) and preservation of natural resources (land and backfill requirements) apart from time and cost. Designing geosynthetic reinforced slope with minimum length of geosynthetics leads to further economy.

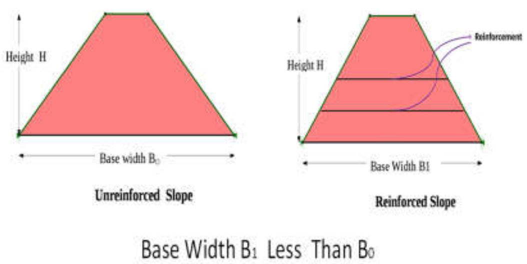

Figure 1 Comparison of Base Width Requirement for Unreinforced and Reinforced slope

Jewell et al. (1985), Bonparte et al. (1987), Verduin and Holtz (1989) present design methods for earth slopes reinforced with geotextiles or /and geogrids using LEM assuming different types of failure surfaces such as circular or/and bilinear wedges. Jewell et al. (1985) used Limit Equilibrium Analysis and local stress calculation for design of reinforced slope. Rowe and Soderman (1985) present a method for estimating the short-term stability of reinforced embankment which has simplicity and versatility of LE but incorporates essential component of soil - structure interaction derived from FEM.

Leshchinsky and Reinschmidt (1985) present an analytical approach based on limit equilibrium and variational extremization of factor of safety for membrane/sheet reinforced slopes for a single layer of reinforcement which satisfies all the requirements of limit equilibrium. Schneider and Holtz (1986) present a design procedure for slopes reinforced with geotextiles and geogrids which assumes bilinear surface of sliding and considering pore water pressures and the initial stress conditions in the slope. Leshchinsky and Boedeker (1989) present an approach for stability analysis of geosynthetic reinforced earth structure using log spiral LE approach for multilayer reinforced slope. Jewell (1991) presented revised design charts for steep slopes valid for all polymer reinforcement materials. These revised charts lead to savings of the order of 20-30% in reinforcement quantity. Leshchinsky and Perry (1987), Leshchinsky (1992, 1999) and Leshchinsky et al. (1995) used log spiral failure mechanism to determine the required reinforcement long term strength. Zhao (1996) presented a kinematic solution of the plasticity theory applied to the stability of geosynthetic reinforced soil slopes. Michalowski (1997) presented kinematic limit analysis solution of reinforced slope to determine the amount of reinforcement necessary to prevent collapse of slopes due to reinforcement rupture, pullout, or direct sliding.

Shiwakoti et al. (1998), conducted parametric studies to investigate the effect of geosynthetic strength, soil-geosynthetic interaction coefficients, vertical spacing of geosynthetics for soil slope/wall on competent foundation and suggested optimization. Baker and Klein (2004a and b) modified the top-down approach of Leshchinsky (1992) to find the reinforcement force needed for the same prescribed factor of safety everywhere within the reinforced mass. Han et al. (2006) present a general analytical frame work for design of flexible reinforced earth structures regardless of slope face inclination applicable to both walls and slopes. Leshchinsky et al. (2010) presented a limit equilibrium methodology to determine the unfactored global geosynthetic strength required to ensure sufficient internal stability in reinforced earth structures, which allows seamless integration of design methodologies for reinforced earth walls and slopes.

Vieira et al.(2013) presented result from a computer code, based on limit equilibrium analysis, able to quantify earth pressure coefficients for the internal design of geosynthetic reinforced soil structures and identify the potential failure surfaces. The influence of potential failure surface and geosynthetic strength distribution on earth pressure coefficient is analysed. Leshchinsky et al.(2014) introduced a limit state design framework for geosynthetic reinforced slopes and walls. This framework is based on free body equilibrium ensuring that at each point within the reinforced mass the factor of safety is same. The presented approach adjusts strength of reinforcement so that factor of safety is same constant everywhere. Leshchinsky & Ambauen (2015) presented use of upper bound limit analysis (LA) in conjunction with discretization procedure known as discontinuity layout optimization (DLO) for comparison with rigorous LE Methods. DLO-LA is an effective tool for establishing a critical failure mechanism and its stability without the constraint or assumptions required in LE analysis. Gao et al. (2016) in their study considered three dimensional effects of three dimensional conditions on reinforced earth structure stability and employed to determine required strength and length of reinforcement using limit analysis approach. The three dimensional effects are more significant for the minimum required length of reinforcement than for the minimum required tensile strength.

None of the above approach optimizes the length of geosynthetics by curtailing the same from the slope face. The paper details analysis carried out to optimize the length of reinforcement from face end of slope.

2. PROBLEM DEFINITION

An embankment of height, H, of 6.0 m with side slopes of 1.5H to 1V vertical is considered (Figure 2). The embankment and foundation soil have cohesion, c, of 5 kPa, unit weight, γ, of 18 kN/m^3 and angle of shearing resistance, ϕ, of 23^0. The geotextile reinforcement used has adhesion, c_a, of 3 kPa, angle of interface friction between soil and reinforcement, δ, of 17° and ultimate tensile strength, T_{ult}, of 200 kN/m. All the stability analyses have been carried out using Morgenstern-Price method using SLOPEW of Geostudio 2004 version.

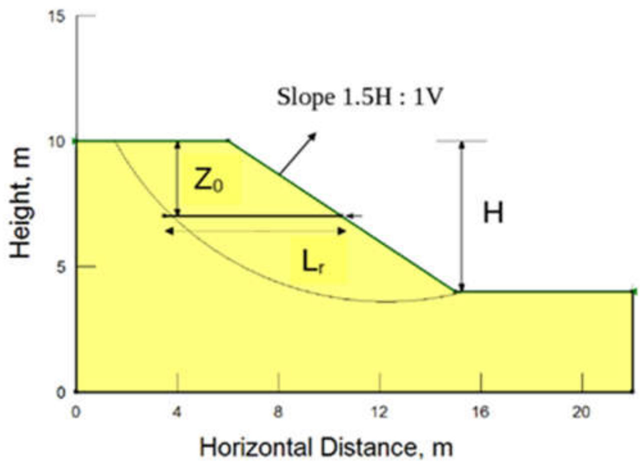

Figure 2 Definition Sketch

3. STABILITY ANALYSIS

3.1 UNREINFORCED SLOPE

Unreinforced embankments of heights 3 m, 4 m, 5 m and 6 m have been analysed and FS_{min} obtained as 1.61, 1.42, 1.31 and 1.22 respectively. Embankment with height of 6 m has FS_{min} less than the required value of 1.3 and hence is reinforced with geosynthetic sheet to get FS_{min} of 1.5.

3.2 REINFORCED SLOPE

An analysis of the effect of varying the length, L_r, of geosynthetic placed at depth, Z_0=3.0 m in 6.0 m high embankment is studied by curtailing it from the non-slope face to get FS_{min} in the range of 1.50 to 1.60. The length, L_r, of the reinforcement to intercept the failure surface at 3.0 m depth was varied from 8.0 m with FS_{min} of 1.6 (Figure 3).

Circles ABC and DEF are the critical slip circles of the unreinforced and the reinforced slopes. PQ is reinforcement of length L_r. The length of reinforcement L_r has two components: QE = effective length, L_e, in the stable zone and EP – the length, L_f in the unstable zone. L_f is further divided into lengths L_{f1} (EB), the length in the failure zone between the critical slip circles of the reinforced and the unreinforced slopes and length, L_{f2}, between the critical slip circle of unreinforced slope and slope face (BP) as shown in Figure 3. It should be noted that one of the effects of inclusion of reinforcement in embankment soil is to shift the critical slip circle from ABC to DEF. This shift of the critical circle increases the factor of safety by involving larger slide mass.

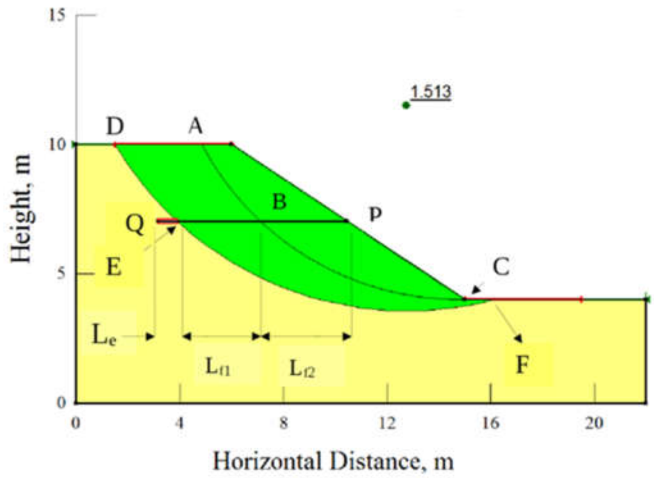

Figure 3 Critical slip circle for Z_0=3.0 m, FS_{min} = 1.51, L_r = 7.27 m

The effect of varying L_r with right end fixed at point P and left end (Q) curtailed inwards successively, on mobilized force in the reinforcement (F_r) and the factor of safety (FS) are summarized in Table 1.

Table 1 Slope with Z_0 = 3.0 m

L_r, m	**F_r , kN/m**	**FS_{min}**
8.0	*35.8*	*1.60*
7.9	*33.5*	*1.59*
7.8	31.9	1.58
7.4	22.9	1.53
7.3	19.6	1.51
7.0	13.7	1.48

Factor of safety and the load/resistance mobilized in the reinforcement decrease with reducing length of reinforcement as is to be expected. FS_{min} reduces to 1.51 from 1.60 as the length is reduced from 8.0 m to 7.3 m. FS_{min} falls below 1.50 on reducing the length further to 7.0.

The length, L_f = (L_r - L_e) is much larger than L_e, the effective length of reinforcement contributing to increase in the stabilizing moment/force. The required pullout force in the reinforcement in the stable zone gets mobilized only by the corresponding length of the reinforcement in the unstable zone. It would serve no useful purpose if the length of the reinforcement in the unstable zone is more than that required for generating the required stabilizing force. Hence minimizing L_f = (L_r - L_e) by moving point P inside the soil mass and away from the slope face by curtailing length of reinforcement but still maintaining FS_{min} above 1.50 can lead to economy. Accordingly for reinforced slope of Figure 3 L_r has been curtailed from the face end of the slope. As point P is moved inside gradually by reducing L_r, the critical circle continues to be DEF or close to it (Figure 4), i.e., practically with no shift of the critical circle. The minimum length, L_r which provides FS_{min} = 1.51 is obtained as 5.08 m (Figure 4). Thus about 30% reduction in length of reinforcement is achieved without sacrificing the stability of the embankment slope as it still has FS_{min} of 1.5.

FS_{min} continues to be close to 1.50 on reducing L_r further but at L_r = 4.80 m FS_{min} reduces to 1.31 and critical circle shifts to between circle ABC and the face of the slope, a shallow failure surface.

Reinforced slope as in Figure 4 above, with the minimal length of the reinforcement arrived at, has been analysed for the slip circle ABC (Figure 5**)** of unreinforced slope to quantify the FS so obtained. FS for this case works out to be very high at 1.8 indicating that the critical circle that gives minimum factor of safety with reinforcement is very different from the one without reinforcement.

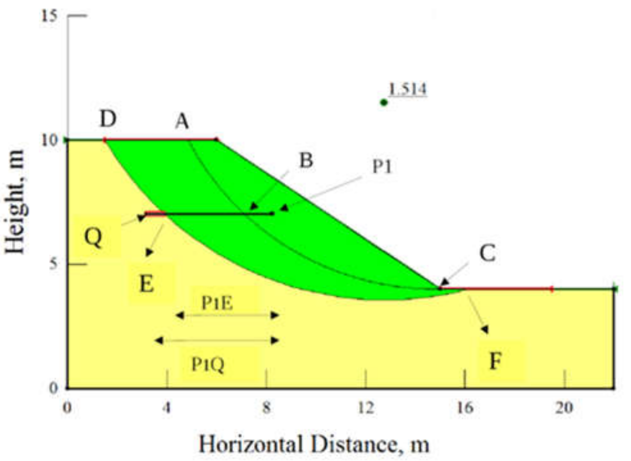

Figure 4 Critical slip circle for slope with Z_0 = 3.0 m, L_r = 5.08 m and FS_{min} = 1.51

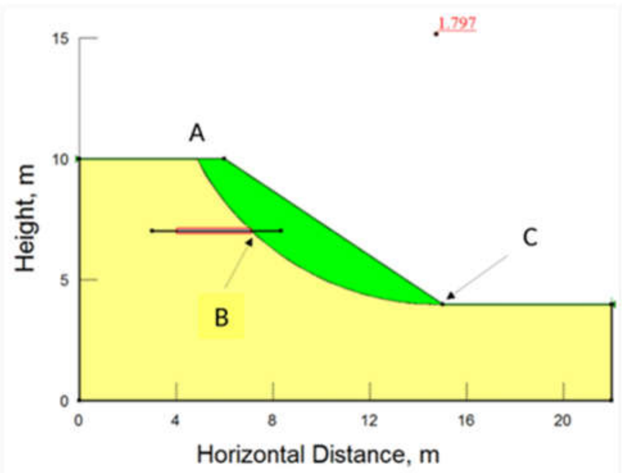

Figure 5 Reinforced slope with Z_0 = 3.0 m, L_r = 5.08 m analysed for failure slip circle ABC of unreinforced slope.

The circle, ABC, is not the critical for the reinforced slope case and thus not acceptable implying that the critical circle with consideration of reinforcement is different from that of unreinforced case. Slope as in Figure 4 has been analysed further for the critical slip circle DEF of reinforced slope but without considering the effect of reinforcement to get FS of 1.41 (Figure 6).

Factor of safety for slip circle DEF (the critical slip circle for the reinforced case) but without considering the effect of reinforcement is 1.41 and higher than FS_{min} of 1.22 obtained for the unreinforced slope. Since the critical circle shifts inward, the factor of safety even without considering the effect/contribution of the reinforcement gets increased as the effect of shift of critical slip circle is to increase FS from 1.22 to 1.41. Reinforced slopes with Z_0 = 4.0 m and 5.0 m have also been analysed in similar manner as that for Z_0 = 3.0 m and results summarized in Table 2.

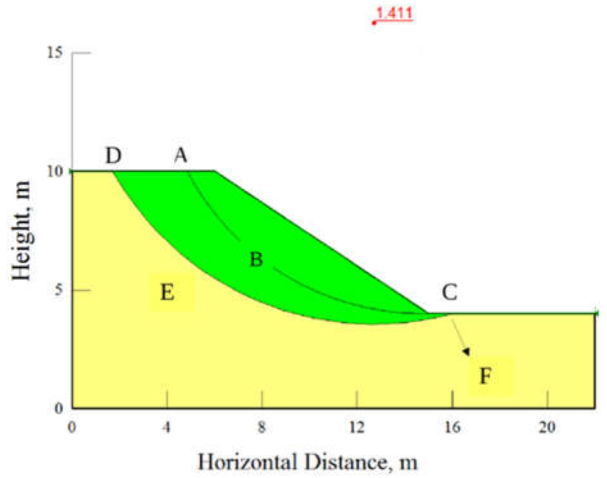

Figure 6 Slope stability with critical slip circle DEF but without considering the effect of reinforcement

Table 2 Factors of Safety and Lengths of geosynthetics for reinforced slope with Z_0 = 3.0 m, 4.0 m and 5.0 m

	FS				
Z_0, m	**I**	**II**	**III**	**IV**	**L_r, m**
3.0	1.22	1.51	1.80	1.41	5.08
4.0	1.22	1.51	1.86	1.48	5.26
5.0	1.22	1.51	1.92	1.46	6.04

I: FS_{min} for unreinforced slope with critical circle ABC; **II:** FS_{min} for reinforced slope with critical circle DEF; **III**: FS for reinforced slope analysed for circle ABC of unreinforced slope and **IV**: Reinforced slope analysed for critical slip circle DEF but without considering the effect of reinforcement.

4. Analysis and Discussion

4.1 Reinforcement at Z_0 = 3.0 m

FS_{min} of the slope for unreinforced case is 1.22 (Table 2). If however the slope is analysed with the reinforcement but considering the slip circle to be the same (ABC of Figure 4) as that for the unreinforced case, FS_{min} is 1.80. This FS is not the minimum and hence ABC is not the critical for the reinforced case.

The contribution of reinforcement in enhancing the stability of a slope is observed to be twofold: (i) shifting of the critical slip circle deeper in to the slope involving larger slide mass or forward involving smaller slide mass and thus enhancing the factor of safety of the slope and (ii) due to contribution of reinforcement to stabilizing force/moment. FS_{min} of 1.22 for unreinforced case increases to 1.41 due to shifting of the critical circle to DEF an increase of 15.6%. Secondly the contribution of reinforcement to stabilizing moment/force leads to a further increase in factor of safety from 1.41 to 1.51, a contribution of about 8.2%.

The contribution of reinforcement to stability in terms of change in FS is defined as follows:

R_{FS1} – relative change in factor of safety due to overall effect of reinforcement

$$R_{FS1} = (FS_{minDEF} - FS_{minABC})/FS_{minABC} \qquad (1)$$

R_{FS2} – relative change in factor of safety due to shift of critical circle due to effect of reinforcement

$$R_{FS2} = \frac{FS_{DEF\ without\ effect\ of\ reinforcement} - FS_{minABC}}{FS_{minABC}} \qquad (2)$$

The difference between the two relative factors of safety $(R_{FS1} - R_{FS2})$ is the contribution of reinforcement to increase in FS. Changes in R_{FS} for all the three cases i.e. Z_0 = 3.0 m, 4.0 m and 5.0 m are detailed in Table 3.

Table 3 Relative changes in factors of safety for cases with Z_0 = 3.0 m, 4.0 m and 5.0 m

Z_0, m	R_{FS1} %	R_{FS2} %	$(R_{FS1} - R_{FS2})$ %
3.0	23.8	15.6	8.2
4.0	23.8	21.3	2.5
5.0	23.8	19.7	4.1

FS_{min} for the reinforced slope is 1.51 and that of the unreinforced slope is 1.22 for all the three cases. Hence percentage relative change in FS, R_{FS1} is 23.8. The percentage relative change in FS due to shifting of critical circle, R_{FS2} is more for 4.0 m case followed by those for the 5.0 m and 3.0 m cases. For Z_0 = 3.0 m, the contribution due to shifting of critical circle is 15.6% and the balance 8.2% is the contribution of the reinforcement. The contributions of reinforcement due to shifting of critical circle are of the order of 15-21% while that due to reinforcement effect is of the order of 2-8% in the three cases analyzed.

4.2 VARIATIONS OF FS_{min} AND F_r WITH L_r

FS_{min} varies linearly with length of reinforcement, L_r, for Z_0 = 3.0 m, 4.0 m and 5.0 m as shown in Figure 7.

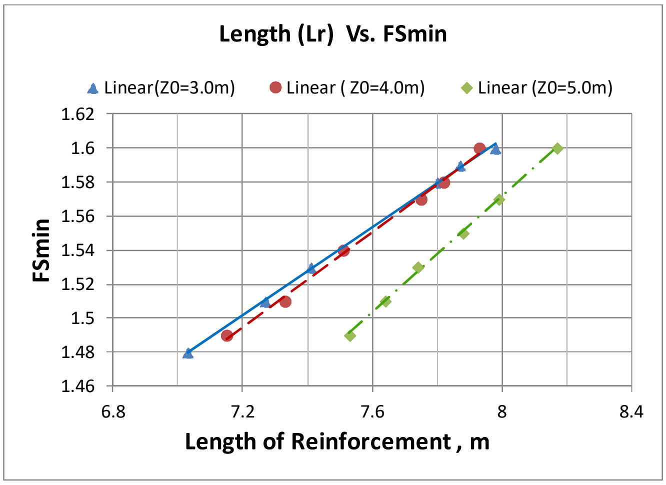

Figure 7 FS_{min} vs. L_r for Z_0 = 3.0 m, 4.0 m and 5.0 m

FS_{min} for Z_0 = 3.0 m and 4.0 m are very close to each other but higher than that for Z_0 = 5.0 m. Variations of loads in reinforcement with length of reinforcement, L_r, are also linear (Figure 8) but different for the three cases considered.

For the same length of reinforcement, the load in the reinforcement is maximum for reinforcement at Z_0 = 3.0 m and reduces with increase in Z_0.

4.3 SUMMARY OF RESULTS:

The results of the analysis for length of reinforcement, L_r, and FS are summarized in Table 4.

Saving in length of reinforcement is highest in case of Z_0 = 3.0 m being 2.19 m.Similarly the effective length of reinforcement L_e is also highest in this case being 0.76 m. The minimum reinforcement length required out of the three positions is that for Z_0 = 3.0 m. The minimum length of reinforcement for Z_0=2.0 m has been found to be 5.75 m.

Thus even after considering the Z_0=2.0 m case, reinforcement length remains minimum for Z_0=3.0 m case. The fact that for the same FS_{min} higher length of geosynthetics is required in case of 5.0 m is because the length contributing to FS by way of stabilising force/moment is very small i.e. only 0.15 m against 0.76 m of 3.0 m case. All the three critical circles are shown in Figure 9 for comparison. They are close to each other but far away from that for the unreinforced case.

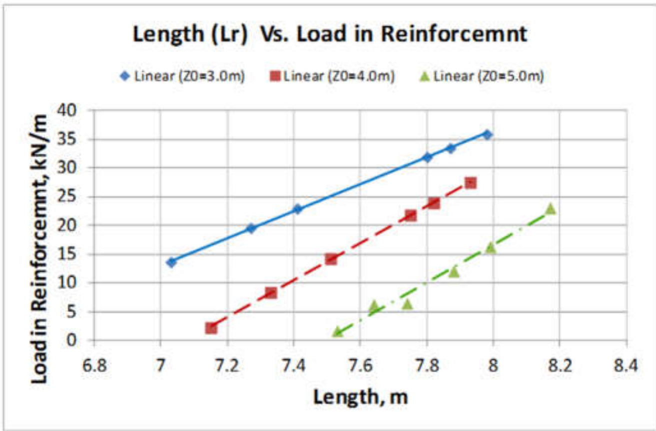

Figure 8 Load in Reinforcement, F_r vs. Length of Reinforcement, L_r for Z_0 = 3.0 m, 4.0 m and 5.0 m

Table 4 Results of Analysis of Reinforced Slope with Z_0 = 3.0 m, 4.0 m & 5.0 m

Z_0, m	L_r, m	L_{opt} =P1Q, m	L_{shift}= P1E, m	Le, m	Lr-L_{opt} (m)	FS_{min} DEF	FS_{shift}
3.0	7.27	5.08	4.32	0.76	2.19	1.51	1.41
4.0	7.33	5.26	5.01	0.25	2.07	1.51	1.48
5.0	7.64	6.04	5.89	0.15	1.60	1.51	1.46

FS_{shift} = FS for DEF slip circle without considering effect of reinforcement; P1Q & P1E lengths of reinforcement (Figure 4).

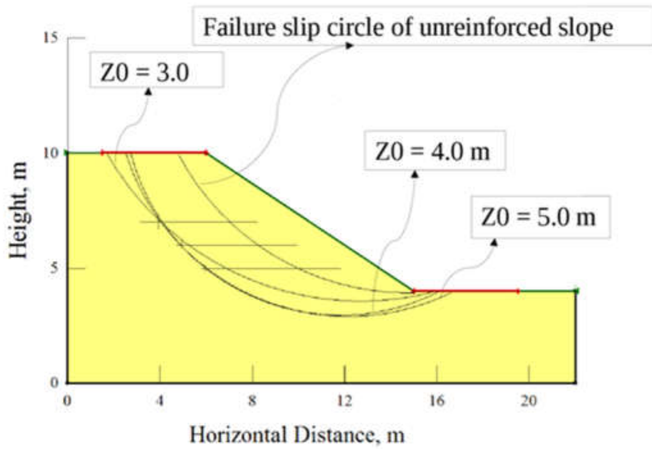

Figure 9 Critical slip circles for reinforcement at Z_0 = 3.0 m, 4.0 m and 5.0 m and for unreinforced case

The critical circle is nearly the same (Figure 9) for reinforcement at 4.0m and 5.0m from the top of embankment. Length of geosynthetics contributing to stabilising force/moment is lowest in case of Z_0 = 5.0 m.

5. CONCLUSIONS

An analysis of interaction between an embankment slope and reinforcement is carried to identify and quantify the mechanisms contributing to increased slope stability as reflected in higher factor of safety and to optimize the length of reinforcement to be provided. A typical embankment slope 1.5H : 1V of height 6.0 m with a single layer of reinforcement at 3.0 , 4.0 and 5.0 m depths from the top is examined for stability using Morgenstern and Price method.

1. The critical slip circle for the slope with reinforcement shifts inward and is very different from that for unreinforced slope.
2. The circles for slope with the reinforcement at different locations (3.0 to 5.0 m depth) are different but close to each other.
3. The increase in factor of safety is because of the shift of the critical slip circle deep in the slope and involving larger sliding mass. This results from the fact that the slip circle is deeper in to the soil and away from the critical circle corresponding to that for unreinforced embankment soil.
4. As a consequence, the reinforcement force generated becomes much smaller than that estimated based on the length corresponding to that estimated with respect to slip circle for the unreinforced slope.
5. The analysis is further carried out by curtailing the length of the reinforcement from the face of the slope to economise the use of geosynthetics.
6. The effect of providing reinforcement in the slope is two-fold, viz., shifting of critical circle inside of the embankment involving larger slide mass and by increase in stabilizing force/moment due to bond resistance mobilized in the reinforcement.
7. It is possible to achieve about 20 to 30% shorter length of the reinforcement without endangering the stability of the embankment slope.
8. The most significant finding of this study is that the reinforcement can be provided inside and not necessarily from the face of the embankment.

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