APPLICATION OF CRITICAL STATE THEORIES TO THE PREDICTION OF STRAINS IN TRIAXIAL SPECIMENS OF SOFT BANGKOK CLAY

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SYNOPSIS
The deformation and strength characteristics of undisturbed specimens of Soft Bangkok Clay are studied in detail by carrying out stress controlled and strain controlled triaxial tests both in compression and in extension. For Weathered Clay tested under pre-shear consolidation pressure less than the maximum past pressure, undrained stress paths are presented to illustrate the elastic wall concept of the stress-strain theories developed at Cambridge University. Also, for these types of stress paths constant shear strain contours are plotted which approximated to the constant deviator stress yield locus. In the normally consolidated states for any one type of applied stress path, both Weathered Clay and Soft Clay are found to exhibit unique stress ratio-strain relationships. The peak stress conditions of the specimens are discussed in relation to the Critical State Concept and the Hvorslev Criterion of Failure. The comparisons with the predictions from a number of stress-strain theories and whenever possible simple constitutive equations are developed to describe the stress-strain behaviour.

INTRODUCTION

This paper is concerned with a detailed application of the critical state concept and the associated stress-strain theories for the deformation and strength characteristics of Soft Bangkok Clay. Extensive research has been carried out for the last two decades on the development of fundamental stress-strain theories to describe the deformation characteristics of soils. Of these work the critical state theories developed at the Cambridge University by the late Roscoe and his co-workers (see Roscoe, Schofield & Wroth, 1958; Roscoe & Poroosheshah, 1963; Roscoe, Schofield & Thursairajah, 1963; Roscoe & Burland, 1968 and Schofield & Wroth, 1968) have been an outstanding contribution as the stress-strain behaviour and the strength characteristics are predicted from a few fundamental soil parameters which can be readily obtained from simple triaxial tests.

The predictions from the critical state theories are often compared with the behaviour of remoulded specimens of Kaolin prepared under controlled conditions in the laboratory. Balasubramanian (1969) has carried out an extensive series of stress-controlled triaxial tests on normally and overconsolidated specimens of Kaolin under a large variety of applied stress paths and compared the experimental behaviour with theoretical predictions.

As a logical extension of the extensive study that has already been made on the behaviour of remoulded specimens of Kaolin, the current research program at the Asian Institute of Technology is directed at extending this study to the behaviour of undisturbed specimens of natural deposit of Soft Clay which is extensively found in Southeast Asia and in particular in Bangkok.

DEFINITIONS

The stress parameters p and q are defined as 

\[ p = \frac{(q_1 + 2s)}{3} \]

and 

\[ q = q_1 - s \]

where \( q_1 \), \( q_2 \) and \( q_3 \) are effective principal stresses and \( q_2 = q_3 \) under triaxial stress condition. Furthermore, the incremental volumetric and shear strain parameters \( dv \) and \( dc \) are defined by 

\[ dv = \frac{dV}{V} = \frac{dV}{V_0} \]

and 

\[ dc = \frac{d\epsilon}{\epsilon} = \frac{d\epsilon}{\epsilon_0} \]

where \( dV \), \( d\epsilon \) and \( dc \) are incremental compressive principal strain and \( dc \) for a triaxial stress system. The stress ratio, \( \eta \), is defined as \( q/p \). The pre-shear consolidation pressure is denoted as \( \sigma_0 \).

MATERIAL PROPERTIES, SAMPLE PREPARATION AND TESTING PROCEDURE

Undisturbed samples of clay were taken from Bangpli (Song Nggao Hao), a site situated 28 km Southeast from Bangkok, Thailand. The sub-soil profile at Bangpli is essentially the same as that already presented for the area North of Bangkok (see Mohrshah et al., 1967; Moh et al., 1969). The general properties of Song Nggao Hao Clay, sample preparation and testing procedure are described in detail by Balasubramanian et al (1976). A summary of the triaxial test series is given in Table 1.

STRESS PATHS AND STATE BOUNDARY SURFACE

Figure 1 shows the stress paths for compression and extension tests on Weathered Clay at low pre-shear consolidation pressures which are less than the maximum past pressure. The behaviour thus corresponds to overconsolidated specimens. The stress paths are found to be approximately sub-parallel to the q-axis, indicating that the mean normal stress, \( p \), do not vary much during shear. These results are in good agreement with the elastic wall concept used extensively in the critical state theories. According to these theories only elastic volumetric strains would take place inside the state boundary surface and these volumetric strains are only dependent on the mean normal stress.

\[ \Delta v^e = -\left( \frac{k}{1+e} \right) \frac{\Delta p}{p} \]  

(1)

where \( \Delta v^e \) is the elastic increment in volumetric strain; \( k \) is the slope of the isotropic swelling line.
<table>
<thead>
<tr>
<th>Test Series Designation</th>
<th>Type of Clay</th>
<th>Applied Stress Path</th>
<th>Pre-shear Consolidation Pressure, $p_0$ (kN/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHU-I (a)</td>
<td>Weathered Clay</td>
<td>Undrained compression loading tests on isotropically consolidated samples; pre-shear consolidation pressure less than apparent maximum past pressure; lateral stress maintained constant, axial stress increased.</td>
<td>13.8, 20.7, 34.5, 41.4</td>
</tr>
<tr>
<td>CHU-I (b)</td>
<td>Weathered Clay</td>
<td>Same as CHU-I(a) but with pre-shear consolidation pressure higher than apparent maximum past pressure.</td>
<td>103.5, 207, 276, 414</td>
</tr>
<tr>
<td>CID-II</td>
<td>Weathered Clay</td>
<td>Drained compression loading tests on isotropically consolidated samples; pre-shear consolidation pressure greater than apparent maximum past pressure, lateral stress maintained constant and axial stress increased.</td>
<td>103.5, 207, 276</td>
</tr>
<tr>
<td>CIUE-III b</td>
<td>Weathered Clay</td>
<td>Same as CIUE-IIIa, but with pre-shear consolidation pressure higher than the apparent maximum past pressure.</td>
<td>103.5, 172.6, 207, 276, 345.2, 414</td>
</tr>
<tr>
<td>CIUE-IV</td>
<td>Weathered Clay</td>
<td>Same as CIUE-IIIa, but with lateral stress constant and axial stress decreased.</td>
<td>10.4, 20.7, 27.6, 41.4, 48.3</td>
</tr>
<tr>
<td>CID-E-V</td>
<td>Weathered Clay</td>
<td>Same as CIUE-III(b) but under drained condition.</td>
<td>20.7, 27.6, 34.5, 41, 69</td>
</tr>
<tr>
<td>Cn-VI</td>
<td>Weathered Clay</td>
<td>Anisotropic consolidation tests with constant stress ratio $\eta = 0.0, 0.16, 0.43, 0.6, 0.75$.</td>
<td>-</td>
</tr>
<tr>
<td>CIU-VII</td>
<td>Soft Clay</td>
<td>Undrained compression tests on isotropically consolidated samples; pre-shear consolidation pressure higher than the apparent maximum past pressure; lateral stress maintained constant, axial stress increased.</td>
<td>138, 207, 276, 345, 414</td>
</tr>
<tr>
<td>CID-VIII</td>
<td>Soft Clay</td>
<td>Same as CIU-VII but under drained condition.</td>
<td>138, 207, 276, 345, 414</td>
</tr>
<tr>
<td>CIDP-IX</td>
<td>Soft Clay</td>
<td>Undrained compression tests on isotropically consolidated samples with pre-shear consolidation pressure higher than maximum past pressure; constant $p$ condition, with axial stress increased, lateral stress decreased.</td>
<td>138, 207, 276, 414</td>
</tr>
<tr>
<td>CIUE-X</td>
<td>Soft Clay</td>
<td>Undrained extension loading tests on isotropically consolidated samples; pre-shear consolidation pressure higher than apparent maximum past pressure; lateral stress increased, axial stress maintained constant.</td>
<td>138, 207, 276, 345, 414</td>
</tr>
<tr>
<td>CIUE-XI</td>
<td>Soft Clay</td>
<td>Same as CIUE-X but with axial stress decreased and lateral stress maintained constant.</td>
<td>138, 207, 276, 345, 414</td>
</tr>
<tr>
<td>CID-E-XII</td>
<td>Soft Clay</td>
<td>Same as CIUE-XI but under drained conditions.</td>
<td>138, 207, 276, 345, 414</td>
</tr>
<tr>
<td>CID-E-XIII</td>
<td>Soft Clay</td>
<td>Same as CIUE-X but under drained conditions.</td>
<td>138, 207, 276</td>
</tr>
<tr>
<td>Cn-XIV</td>
<td>Soft Clay</td>
<td>Anisotropic consolidation tests with constant ratio.</td>
<td>-</td>
</tr>
</tbody>
</table>
in an \((e, \log p)\) plot; \(e\) is the void ratio; \(p\) is the mean normal stress.

Since the volumetric strain is zero in an undrained test, there would not be any change in the mean normal stress; thus the stress paths rise parallel to \(q\)-axis in the \((q,p)\) plot till failure is reached. Dotted lines in Fig. 1 correspond to extension tests under unloading condition. It is noted that the effective stress paths in compression and in extension under loading and unloading conditions are similar for a first degree of approximation.

The effective stress paths followed by the specimens under undrained condition at higher pre-shear conso-

\[e_0 - \lambda \log (p_0/p)\]  

If \(p_c\), \(p_{bl}\) denote the mean normal stress for states \(C_1\) and \(B_1\) and if \(e_{b1}\) is the void ratio corresponding to the state \(B_1\), then

\[e_0 - e_{b1} = \lambda \log (p_{b1}/p_0)\]

Fig. 1 Undrained Stress Paths (Weathered Clay)

(a) Compression tests on Weathered Clay  
(CIU-1a Series)

(b) Extension tests on Weathered Clay  
(CIU-III a & CIU-IV Series)

\[\sigma = \sigma_0 \exp \left(\frac{\gamma_0}{\lambda} e\right)\]

where \(\sigma_0\), \(e_0\) correspond to the pre-shear consolidation pressure and \(\lambda\) is the slope of the isotropic consolidation line in an \((e, \log p)\) plot.

The state paths followed by all the test specimens in the normally consolidated state are shown in Figs. 2(a) to (c) for each type of test condition. Also, for each type of applied stress path, the state paths are found to be independent of the pre-consolidation pressure and also the state boundary surface is virtually the same. Using this unique state boundary surface, the volumetric strain for any stress-increment

in the normally consolidated state can be determined. The state paths shown in Fig. 2 also include those corresponding to anisotropic consolidation. Since one dimensional consolidation in the oedometer is also a special type of anisotropic consolidation with zero lateral strain, the volumetric strain and hence the axial strain during one-dimensional consolidation can be also predicted from the state boundary surface presented in Fig. 2.

The procedure adopted for calculating the volumetric strain along any applied stress path is as follows.

The procedure adopted for calculating the volumetric strain along any applied stress path is as follows.

Let \(A\) in Fig. 3 be the applied stress path for which the volumetric strain stress ratio relationship is being determined. Then the undrained stress path \(BC\) through the point \(A\) is drawn assuming that the state boundary surface is unique (since intersection of the state boundary surface by planes of constant voids ratio are similar curves). Let \((e_0, p_0)\) be the voids ratio and mean normal stress at point \(A\) on the isotropic consolidation line. Then the isotropic consolidation line can be expressed as

\[e_0 - \lambda \log (p_0/p)\]

If \(p_0\), \(p_{bl}\), \(e_{b1}\) can be evaluated and therefore \(v_{bl}\) may be determined from

\[v_{bl} = \log \left(\frac{1 + e_{b1}}{1 + e_0}\right)\]

Thus the state boundary surface can be used in the prediction of volumetric strain for normally consoli-

\[\Sigma_{ijkl} = \phi (\gamma)\]

Thus under extension condition the yield locus for dia-

torsional strains for stress states below the state boundary surface is somewhat dependent on the mean normal stress. At low levels of deviator stress, where settlement calculations are often carried out, one can
assume constant q yield loci for distortional strains, for both compression and extension conditions.

STRESS RATIO-STRAIN RELATIONSHIPS FOR NORMALLY CONSOLIDATED STATES

For any one type of applied stress path, the stress ratio-strain relationships are found to be unique for normally consolidated specimens. The stress ratio-strain relationships for undrained and drained tests are presented in Figs. 6 and 7 for all the tests carried out on Weathered Clay and Soft Clay in the normally consolidated states. It is noted that for all the undrained tests, the shear strain $\varepsilon$ is a unique function of the stress ratio, $\eta$. Thus for undrained tests on normally consolidated clay,

$$c = \int_0^\eta \varepsilon_1 (\eta) \, d\eta \quad (7)$$

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AC-undrained stress path
AB-drained stress path
G1, G2, etc. anisotropic consolidation paths

Fig. 3 Incremental steps in the calculation of volumetric strains in drained tests
Also, for the undrained test the ratio of pore pressure to mean normal stress was found to be a unique function of the stress ratio. Thus,

$$\frac{u}{p} = \frac{\eta}{h'(\eta)} \cdot d\eta$$

(8)

In the case of drained tests, both the volumetric strain and the shear strain are found to be unique functions of stress-ratios \( \eta \).

Thus

$$\eta = \frac{c}{a}(n) \cdot d\eta$$

(9)

$$\eta = \frac{c}{b}(n) \cdot d\eta$$

(10)

But both functions \( a \) and \( b \) are found to be dependent on the applied stress path.

**ANISOTROPIC CONSOLIDATION TESTS DATA**

During anisotropic consolidation, the stress ratio, \( \sigma_1/\sigma_3 \) is maintained constant (where \( \sigma_1 \) is the major principal stress and \( \sigma_3 \) is the minor principal stress).

**Fig. 5** Volumetric and Constant \( q \) yield loci

**Fig. 4** Constant shear strain contours for specimens sheared from pre-shear consolidation pressure less than the maximum post pressure.

Figure 8 illustrates a possible experimental procedure for such a test in the \( (q, p) \) plot. A specimen is sheared under undrained condition from a pre-shear consolidation pressure, \( p_0 \). The undrained stress path is denoted by \( AB \). From \( B \), the specimen is subjected to anisotropic consolidation along the path \( BB' \). During anisotropic consolidation \( q/p = \eta \) is maintained constant.

Anisotropic consolidation tests were carried out on both Weathered Clay and Soft Clay. The strain paths are linear for Soft Clay. However, for Weathered Clay the strain paths corresponding to any particular stress-ratio are found to consist of two straight lines. During the initial phase \((dV/dt)_\eta \) is higher than in the final phase (see Fig. 9). This would imply that in the initial phase of shear the distortional strain is small in magnitude than in the subsequent normally consolidated state. It appears that \((dV/dt)_\eta \) can be expressed as

$$\frac{dV}{dt} = \frac{1}{f_\eta(n)}$$

(11)

The \((e_1, \log p)\) relationships during anisotropic consolidation in the normally consolidated states are found to be linear and the slope, \( \lambda \), is equal to 0.51.

**CRITICAL STATE PARAMETERS AND STRENGTH ENVELOPES**

The end points of the specimens in \((q, p), (w, \log p)\) and \((w, \log q)\) plots are shown in Figs. 10 a to e. For Weathered Clay in compression under undrained condition at low pre-shear consolidation pressure the \( q \) value is found to be considerably low. Also the \((w, \log p)\) and \((w, \log q)\) relationships are found to be curved (see James & Balasubramaniam, 1971). Similar results are also noted for the specimens of Weathered Clay sheared under extension conditions. For these specimens the \((q, p, w)\) relationship for all the specimens sheared under different types of applied stress paths are found to be the same for a first degree of approximation.
For Weathered Clay tested under normally consolidated state in compression and in extension, the end points are found to coincide with the cirtical state line. The projection of the cirtical state line in (q, p) plot is linear. Similarly linear relationships are noted in (w, log p) and (w, log q) plots. The end points of the specimens of Soft Clay sheared in compression under undrained, fully drained and constant p conditions are also found to lie on linear projections in (q, p), (w, log p) and (w, log q) plots. However, some differences are noted in all plots especially between the undrained tests and the other tests.

The results of the extension tests carried out on Soft Bangkok Clay are found to lie on straight lines in (q, p) plot, but the straight lines are found not to pass through the origin. It thus appears that Soft Bangkok Clay exhibits a small degree of cohesion under extension condition. The (w, log p) and the (w, log q) relationships are similar to those exhibited by normally consolidated clay specimens.

BEHAVIOUR OF SOFT CLAY IN STRESS RATIO STRAIN SPACE

Balasubramaniam (1969) presented data on remoulded specimens of Kaolin (sheared under a wide variety of applied stress paths) to illustrate that provided the
Fig. 7 Stress-ratio strain relationships for extension tests in the normally consolidated states

(c) Undrained extension unloading tests on Soft Clay (CIUE-XII Series)
(d) Drained extension loading tests on Soft Clay (CIUE-XIII Series)

Fig. 8 Anisotropic consolidation path in (q,p) plane

Fig. 9 Strain increment ratio during anisotropic consolidation
stress ratio \( q/p \), increase, then for all specimens sheared from an isotropic stress state, a unique relation exists among the volumetric strain, shear strain and the stress ratio. Such relationships are also found to exist for Soft Bangkok Clay and are presented in Fig. 11. The stress paths considered in these plots include those of undrained, constant \( p \) and fully drained tests. The \((\psi, \delta)\) characteristics for the three types of stress paths are presented in Fig. 11. and contours of constant \( q/p \) are super-imposed in these figures. The contours are found to be straight lines. It therefore appears that \((q/p, \psi, \delta)\) are uniquely related for specimens which are sheared under stress paths with increasing \( q/p \) and the relationship can be expressed as

\[
e = \int_{0}^{\eta} f_1 (\eta) \, d\eta + (\psi) \int_{0}^{\psi} f_2 (\eta) \, d\eta
\]

where \( \int_{0}^{\eta} f_1 (\eta) \, d\eta \) refers to the shear strain in...
undrained test (see equation (7)) and the function $\zeta(\eta)$ refers to the volumetric strain in any type of drained test (see equation 10). The function $\zeta(\eta)$ refers to the slope of the $q/p$ contours in Fig. 11. It should be noted that the function $\zeta(\eta)$ is found to be different from $\zeta_0(\eta)$ which refers to the value of $\zeta_0(\eta)$ during anisotropic consolidation (see equation 11).

PREDICTION OF STRAINS USING INCREMENTAL STRESS-STRAIN THEORY

An incremental stress-strain theory was proposed by Roscoe and Poonoooshab (1963) for the prediction of strains in drained tests with different applied stress paths.

Thus the incremental stress-strain theory can be expressed as

$$dc = f_1(\eta) \, d\eta + f_2(\eta) \, dp$$ (13)

where $c = f(\eta, p)$ refers to the volumetric strain as a function of $\eta$ and $p$. The value of $\lambda$ used in the prediction of volumetric strain is 0.51 and is obtained from the isotropic and anisotropic consolidation tests. The predicted strains and the experimentally observed strains are presented in Fig. 12 (a) to (c) for compression and extension tests on normally consolidated specimens of Weathered Clay and Soft Clay. The incremental stress-strain theory is found to predict very closely the experimentally observed strains.

PREDICTION OF STRAINS USING CRITICAL STATE THEORIES

In this section, the experimentally observed strains are compared with the strains predicted from the cri-

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Fig. 11 Constant stress ratio contours in $(v, \epsilon)$ space

(a) Drained compression loading tests on Weathered Clay (CID-II Series)

(b) Drained compression loading tests on Soft Clay (CID-VIII Series)

(c) Drained extension loading tests on Soft Clay (CID-XIV)

Fig. 12 Observed and predicted strains in drained tests
tential state theories. Two theories are employed and these are the Cam Clay theory, by Roscoe, Schofield & Thurairajah (1963) Schofield & Wroth (1968); and the Revis ed Theory by Roscoe & Burland (1968).

The fundamental soil parameters used in the critical state theories are \( \lambda, k, \) and \( M. \lambda \) is the slope of the isotropic consolidation line in the \((c, \log p)\) plot, \( k \) is the slope of the isotropic swelling line in the \((c, \log p)\) plot, \( M \) is the slope of the critical state line in \((q, p)\) plot. Isotropic consolidation and swelling tests carried out on Soft Bangkok Clay indicate that the value of \( \lambda \) is 0.31 and that of \( k \) is 0.091. Also the critical state parameter \( M \) is taken as 1.0.

In the Revised Theory of Roscoe and Burland, corrections are made for the shear strain from the contributions due to the constant \( q \) yield loci. The contributions from the constant \( q \) yield loci were approximately the same as the shear strain obtained from undrained tests in \((q/p, c)\) plot.

The experimentally observed strains and those predicted from the Critical State Theories are shown in Fig. 12. The Cam Clay theory is found to overpredict the strains while the predictions from the Revised Theory are good.

CONCLUSIONS

A comprehensive series of triaxial compression and extension tests were carried out on Weathered and Soft Bangkok Clay and the results were compared with the predictions from a number of stress-strain theories. The following conclusions are reached:

- (i) For Weathered Clay sheared under compression and extension conditions from pre-shear consolidation pressures less than the maximum past pressure, the stress paths under undrained conditions are nearly sub-parallel to the \( q/p \) axis in the \((q/p)\) plot.

- (ii) For both Weathered and Soft Clay in the normally consolidated state, the state paths are found to be somewhat the same in the \((q/p, \psi)\) plot.

- (iii) For Weathered Clay sheared under compression and extension conditions from pre-shear consolidation pressures less than the maximum past pressure, the constant shear strain contours are nearly sub-parallel to the \( q/p \) axis in the \((q/p)\) plot.

- (iv) For any one type of applied stress path, unique stress ratio strain relationships are observed both for Weathered Clay and Soft Clay, in the normally consolidated states.

- (v) For each type of clay under similar applied stress paths, unique stress ratio strain relationshps are noted. These relationships are found to coincide with the critical state line for specimens sheared from the normally consolidated states.

- (vi) For Soft Clay, in the normally consolidated state, unique \((q/p, c, e)\) relationship is observed for all specimens sheared under different applied stress paths.

- (vii) The incremental stress-strain theory of Roscoe and Poroosbaah (1963) and the revised critical state theory and found to predict successfully the experimentally observed strains in drained tests.

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