Technical Note
AN IMPROVED METHOD FOR ESTIMATING THE TIME AND STRAIN AT THE END OF THE “PRIMARY” CONSOLIDATION OF A CLAYEY SOIL WITH NON-LINEAR CREEP

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ABSTRACT: For the assessment of the total settlement of a soil mass in 1-D loading, it is usual to separate the “primary” consolidation settlement and the “secondary” consolidation settlement in the calculation. Two conventional curve-fitting methods for the determination of the coefficient of consolidation, that is, Taylor’s root time method and Casagrande’s log time method, are commonly used to estimate the strains and time at the end of the “primary” consolidation (EOP). However, the estimation using these two methods are generally influenced by the non-linear “secondary” compression, in which the coefficient of the “secondary” compression (c2) has a non-linear relationship with the logarithm of time. This is especially true for cohesive soils with pronounced creep behavior. As a result, the quantification of the “secondary” settlement and hence the total settlement, become difficult, frequently leading to the underestimation of the total settlement. In this technical note, a new simple graphical method utilizing both Taylor’s method and the inflection point method (Cruz, 1971; Robinson, 1997; Merri et al., 1999) is proposed to isolate the “primary” consolidation and the “secondary” consolidation. Procedures of using the method to determine the time and strain at the EOP are described using data from oedometer tests on a Hong Kong marine clayey soil. Values obtained using the proposed new method are compared to those from the two conventional methods.

Keywords: Settlement, Primary consolidation, Creep

INTRODUCTION

The coefficient of consolidation in the Terzaghi theory of consolidation has been commonly determined by fitting theoretical results to observed consolidation data with time from an incremental loading oedometer test. Conventionally, Casagrande’s method (the logarithm of the time-fitting method) and Taylor’s method (the square root of the time-fitting method) are widely used in engineering. The two methods depend so much on one’s skill in curve fitting. For Casagrande’s method, it is difficult to determine a linear portion at the end portions of the curve in \( \Delta e_t = -\log t \) plot, where there is non-linear creep in a log (time) scale (i.e., \( c_2 \) changes with log (time), not constant). When a soil exhibits appreciable non-linear viscous behavior, a flatter straight fitting line in the “secondary” compression region results in a higher \( c_{02} \), as illustrated in Fig. 1. Furthermore, the curve fitting for Taylor’s root time method and Casagrande’s log time method are respectively based on U = 98% and U = 100% at which the settlement-time curve would be also affected greatly by the effect of the “secondary” consolidation. By Casagrande’s method, \( c_{02} \) tends to increase with the effect of the non-linear “secondary” compression, since this method determines 100% consolidation from an intersection point produced by a straight line from the “secondary” compression segment and a straight line fitting after the inflection point of the consolidation curve. The shape of Taylor \( \sqrt{t} \) method could also be affected by the non-linear “secondary” compression. A general concern about this method is that the consolidation curve near 98% consolidation, which is used to further estimate 100% consolidation, may be significantly influenced by the “secondary” compression.

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PROPOSED METHOD TO ESTIMATE THE TIME AT THE EOP

The new graphical method introduced in this technical note takes the advantages of both Taylor \( \sqrt{t} \) method and the inflection point method (Cruz, 1971; Robinson, 1997; Merri et al., 1999) and can be herein called as a “\( \sqrt{t} \) inflection point” method or a “square root inflection point” method.

In the \( \sqrt{t} \) inflection point method, to establish the time and strain for the compression value corresponding to U = 100% (i.e., \( c_{02} \)), two points are required. The first point is the point corresponding to U = 0% (i.e., starting point of the “primary” consolidation). This point is determined in the same way as in Taylor \( \sqrt{t} \) method. The way in Taylor \( \sqrt{t} \) method to find out U = 0% involves drawing a straight line from the linear segment of the \( \sqrt{t} \) versus compression relationship from an oedometer test consolidation curve. The interception of the line on compression-axis is the point of U = 0%. The point

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which $U = 0$ can then be transferred to the log (time) plot, with the aid of the inflection point to find out the $U = 100\%$.

The second point needed is the inflection point corresponding to $U = 70.15\%$. By plotting the Terzaghi consolidation curve (U) versus the logarithm of time factor $T_L$ curve, an inflection point at which the sign of curvature changes is identified by observation. At this inflection point, $U = 70.15\%$ and $T_L = 0.046$ according to Terzaghi’s 1-D consolidation theory. The same inflection point can also be found from the plot of the compression with log time of real-time oedometer test results. The two points with $U = 0$ and $U = 70.15\%$ can then be used to find the point of $U = 100\%$.

Figures 2a and 2b demonstrate 5 steps of determining the three points of $U = 0$, $U = 70.15\%$, and $U = 100\%$ (or $T_{max}$) as follows:

Step 1: The point of $U = 0$ is found by Taylor’s method as shown in Fig. 2a.

Step 2: The point of $U = 0$ is transferred to the log (time) plot in Fig. 2b and a horizontal line is drawn from the point.

Step 3: The inflection point is marked down in the log (time) plot. There are two ways to determine

Table 1: Comparisons of $T_{max}$ (only) and $T_{max}$ (or) obtained using the proposed new method, Taylor’s method and Casagrande’s method (a) BV6 and (b) B1-24-2

<table>
<thead>
<tr>
<th>Method</th>
<th>Stress 100 kPa</th>
<th>Stress 200 kPa</th>
<th>Stress 400 kPa</th>
<th>Stress 800 kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Method</td>
<td>41 mins, 20 mins</td>
<td>41 mins, 20 mins</td>
<td>41 mins, 20 mins</td>
<td>41 mins, 20 mins</td>
</tr>
<tr>
<td>Taylor’s Method</td>
<td>7.38%</td>
<td>10.62%</td>
<td>12.52%</td>
<td>17.90%</td>
</tr>
<tr>
<td>Casagrande’s Method</td>
<td>50 mins, 18 mins</td>
<td>50 mins, 18 mins</td>
<td>50 mins, 18 mins</td>
<td>50 mins, 18 mins</td>
</tr>
<tr>
<td></td>
<td>7.2%</td>
<td>10.53%</td>
<td>14.15%</td>
<td>18.1%</td>
</tr>
<tr>
<td></td>
<td>81 mins, 45 mins</td>
<td>81 mins, 45 mins</td>
<td>81 mins, 45 mins</td>
<td>81 mins, 45 mins</td>
</tr>
<tr>
<td></td>
<td>7.5%</td>
<td>10.95%</td>
<td>14.4%</td>
<td>18.1%</td>
</tr>
</tbody>
</table>

Fig. 2 (a) Step (1) for locating the initial point of the “primary” consolidation and (b) Steps (2) to (5) for determining the end of the “primary” consolidation by the new method.

The point (circle) corresponding to $U = 0$ is the starting point of the “primary” consolidation.

The inflection point (circle) corresponding to $U = 100\%$, that is $T_{max}$.

The vertical AA’ is calculated as $AA’ = (100/70.15)BB’$ where BB’ is the height determined in Step 2. The compression corresponding to AA’ is $U = 100\%$. The AA’ vertical line is moving to intercept the settlement curve of log (time) versus the compression. At the intersection, the $f_{so}$ on the log (time)-axis is then obtained and the compression corresponding to $U = 100\%$ on the compression-axis is then located.

Table 2: Additional results obtained using the proposed new method, Taylor’s method and Casagrande’s method (a) BV6 and (b) B1-24-2

<table>
<thead>
<tr>
<th>Method</th>
<th>Stress 200 kPa</th>
<th>Stress 400 kPa</th>
<th>Stress 800 kPa</th>
<th>Stress 1600 kPa</th>
<th>Stress 3200 kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Method</td>
<td>35 mins, 37 mins</td>
<td>35 mins, 37 mins</td>
<td>35 mins, 37 mins</td>
<td>35 mins, 37 mins</td>
<td>35 mins, 37 mins</td>
</tr>
<tr>
<td>Taylor’s Method</td>
<td>17.8%</td>
<td>23.3%</td>
<td>31.2%</td>
<td>45.7%</td>
<td>45.7%</td>
</tr>
<tr>
<td>Casagrande’s Method</td>
<td>49 mins, 42 mins</td>
<td>49 mins, 42 mins</td>
<td>49 mins, 42 mins</td>
<td>49 mins, 42 mins</td>
<td>49 mins, 42 mins</td>
</tr>
<tr>
<td></td>
<td>17.7%</td>
<td>23.4%</td>
<td>33.9%</td>
<td>42.1%</td>
<td>42.1%</td>
</tr>
<tr>
<td></td>
<td>60 mins, 71 mins</td>
<td>60 mins, 71 mins</td>
<td>60 mins, 71 mins</td>
<td>60 mins, 71 mins</td>
<td>60 mins, 71 mins</td>
</tr>
<tr>
<td></td>
<td>17.8%</td>
<td>23.8%</td>
<td>36.0%</td>
<td>41.9%</td>
<td>41.9%</td>
</tr>
</tbody>
</table>

Fig. 1 Comparison of the values of $f_{so}$ obtained by shorter and longer testing durations.

A steeper line is drawn from the “secondary” compression region of the test with a shorter loading period which is separated by a horizontal line.

A smaller value of $f_{so}$ (shorter “primary” consolidation duration) is determined by the steeper line.

A higher value of $f_{so}$ (larger "primary" consolidation duration) is determined by the flatter line.

Figures 4a and 4b also show the determination of $f_{so}$ for the same compression-time curve by Casagrande’s method and Taylor’s method respectively. Comparisons of values of the time and strain at the BOP (i.e. $f_{so}$ and $f_{so}$) determined by the three methods within the normal consolidation stress ranges for two oedometer tests on two Hong Kong marine claysey soils, BV6 (clay18%, silt32%, sand48%, PL=33.2%, IP=58), and BV7 (silt13%, sand27%, PL=22.8%, IL=25.8%), are respectively given in Tables 1a and 1b.

It is noted that the inflection point at the average degree of consolidation $U = 70.15\%$ is within the middle range of the compression curve which is easy to be located and is least affected by the initial compression and the “secondary” compression. Robinson (1977) has already illustrated how to construct the inflection point from the compression versus time relationship by a graphical or calculation method as to estimate the coefficient of consolidation, $c_u$, corresponding to $U = 70.15\%$. In fact, Robinson’s method (1977) can be also utilized to calculate the total final “primary” consolidation coefficient ($c_{so}$) by $c_{so} = f_{so}m/M$, where $m$ is the slope at the inflection point of the $S' - log T$ curve and $M$ is the slope inflection point of the theoretical U - log T curve. Nevertheless, when $m$ is less than 1, the result obtained would be greatly different from that when $m$ is greater than 1.

In the new $f_{so}$ inflection point method, Taylor $f_{so}$ method is used to determine the point of $U = 0\%$ rather than Casagrande’s log $T$ method. It is because when using Casagrande’s method, it is quite difficult to evaluate
U = 0% from the experimental settlement curves and sometimes an illogical value (i.e., ε<sub>u</sub>, U = 0%, < 0) is obtained. In addition, in Casagrande’s method, a time, say t<sub>0</sub> and 4t<sub>0</sub> in the early part of the log (time) curve are selected to get U = 0%. However, in most cases, and at least for the testing results of the Hong Kong marine clayey soils in this study, choosing any different t<sub>0</sub> (and hence different 4t<sub>0</sub>) would often come out different U = 0%. On the other hand, the effect would be minimized and linearized much more clearly by the square-root time feature of Taylor’s method. One would doubt that the effect of the initial compression from Taylor’s method, but it is believed that this effect can be corrected by assuming the parabolic relationship of the non-dimensional time factor T<sub>c</sub>, and the degree of consolidation U. Moreover, the initial compression after correction for apparent deformation should be negligible for a properly performed test on a saturated sample (Chin, 1986).

It should be mentioned that the inflection point would disappear as a result of a small load increment ratio like 0.2 (Mesri et al., 1999). In general, it is recommended that a load increment ratio within the range of 0.5 to 1 (Mesri et al., 1999) is used for the laboratory consolidation tests so that the inflection point would appear on the log(time) consolidation curve at the 70.15% degree of consolidation. In detail, as suggested by Mesri et al. (1999) within the normal consolidation range, an inflection point would be induced in the compression versus time curve when

\[
\frac{C_u}{C_s} < 0.6868 \log \left[ 1 + \frac{\Delta \sigma_s}{\sigma_{ss}} \right] \quad (1)
\]

where \(\frac{C_u}{C_s}\) is the slope of compression curve in the normal consolidation range and equal to \(\frac{\Delta \sigma_s}{\sigma_{ss}}\). \(\Delta \sigma_s\) is the effective vertical stress increment and \(\sigma_{ss}\) is the initial effective vertical stress. Referring to Tszachi et al. (1996), the upper value of \(\frac{C_u}{C_s}\) is about 0.07, then Eq. 4 can be reduced to:

\[
0.07 < 0.6868 \log \left[ 1 + \frac{\Delta \sigma_s}{\sigma_{ss}} \right] \quad (2a)
\]

\[
10^{10.89} < 1 + \frac{\Delta \sigma_s}{\sigma_{ss}} \quad (2b)
\]

\[
\frac{\Delta \sigma_s}{\sigma_{ss}} > 0.265 \quad (2c)
\]

Based on the above discussion, the stress increment ratio shall be larger than 0.265. For pressure increment spanning in situ preconsolidation pressure \(\sigma_{ss}\), the pressure increment ratio larger than 0.3 may be required to produce an inflection point in S versus log t curve.

**REFERENCES**


