THE FAILURE BEHAVIOUR OF OVER-CONSOLIDATED SOILS

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ABSTRACT

An attempt has been made to separate strength into effective cohesion and friction components within the framework of Critical State Soil Mechanics under triaxial compression test conditions. The study accepts the validity of the Mohr-Coulomb relation as a tool to analyse the role of the components of strength. The uniqueness of the relation has been investigated and found to be test path dependent. The basic assumption made is that the observed compression (p_e-p_o) is responsible for the cohesion component of strength. It has been established that at lower confining pressures and higher void ratios, cohesion is not significantly affected by the variation of confining pressures. The deduced (c',ϕ') relation can be applied to Cam and Weald clays and the relation reasonably described some general site data on cohesion and internal friction.

INTRODUCTION

The Cam clay theory (Roscoe et al, 1958), based on Critical State Soil Mechanics (CSSM), has significantly advanced our understanding of soil behaviour. In interpreting the behaviour of over-consolidated soils, the relative contribution of the cohesion and friction components of soil strength as deduced from the application of the Mohr-Coulomb relation remains of interest in site problems. Cohesion appears to be closely linked to over-consolidated soils. Lambe and Whitman (1969) have made several interesting observations relating to cohesion and over-consolidated soils that have been taken into account in this study.

In the simple physical sense, cohesion can be viewed as the component of strength that can be mobilised (unlike frictional strength) and maintained without being significantly affected by changes in confining stress under drained conditions. In site works, the total confining stress is often reduced (as in excavations), resulting in changes to the effective confining stress. Physical evidence of the maintenance of long-term strength can be viewed on sites. Examples are the sustainability of some vertical cuts and the ability of desiccated surface clay layers to support traffic (Terzaghi and Peck, 1967). In the laboratory too, the effective cohesion and friction strength are routine observations for over-consolidated and tropical soils.

Over-consolidation and related effects can be due to geological processes (such as rise and fall of sea levels, glacial movements, natural cementation, desiccation, etc.) and man-made process such as compaction. Some of the other site problems of interest are listed below:

1. The contribution of cohesion (c') in a dominantly frictional (ϕ') soil is significant in the analysed behaviour of earth structures. This is particularly relevant to slope and wall problems where a quantum of *c'* has a beneficial effect on long-term stability and pressure results that are obtained.
2. Compacted clay has over-consolidation characteristics and retains adequate strength in low confining stress situations such as in road pavements.
3. In nature, soil may occur as lightly to heavily over-consolidated deposits. This may be reflected in the (c',ϕ') values obtained in triaxial shear tests and a procedure for characterising the strength behaviour would be relevant.
4. The strength parameters of the soil may be varied due to stress release during sampling. It should be possible to relate the altered soil strength to the original expected property.

Wroth (1984) in his Rankine lecture has made a comprehensive analysis of the behaviour of normal and over-consolidated soils within the framework of CSSM. He discussed the relation of the Mohr-Coulomb rupture criterion to the Cam clay model but excluded the consideration of strength under drained conditions. Roscoe

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and Burland (1968) established a link between their Cam clay model and the Mohr-Coulomb criterion for 'wet clays'. The notion of cohesion as defined in the Mohr-Coulomb criterion may be embedded in the Cam clay model (Schofield, 1998), but the identification of cohesion as a distinctive component of strength can provide a better awareness of the effects of changes in the confining stress.

In this study an attempt shall be made to separate strength into effective cohesion and friction components within the framework of CSSM under triaxial compression test conditions. There will not be, at this stage, the detailed distinction of strengths as made by Wroth (1984). The study accepts the validity of the Mohr-Coulomb relation as a tool to analyse the role of the components of strength. The uniqueness of the components needs, however, to be investigated.

FORMULATION

Deductions involving Cam Clay Model

Presentation

The Cam clay model representation for normally consolidated (NC) and over-consolidated (OC) soils are shown in Fig. 1. AC and CR are, respectively, the normal consolidation and swelling lines. Physically, the NC and OC soils at the same void ratio (represented by initial states A and R, respectively, in Fig. 1) have related properties in the Cam clay model and shall be further elaborated in the next section.

Cam clay relations

The Cam clay model assumed that the undrained stress paths are 'similar' for NC soils. Wroth (1984) presented the findings of Loudon (1967), in his undrained triaxial shear tests, that the 'similarity' may be extended to stress paths of OC soils. It can be demonstrated by combining the various Cam clay findings that for a given soil the critical state line may be represented by:

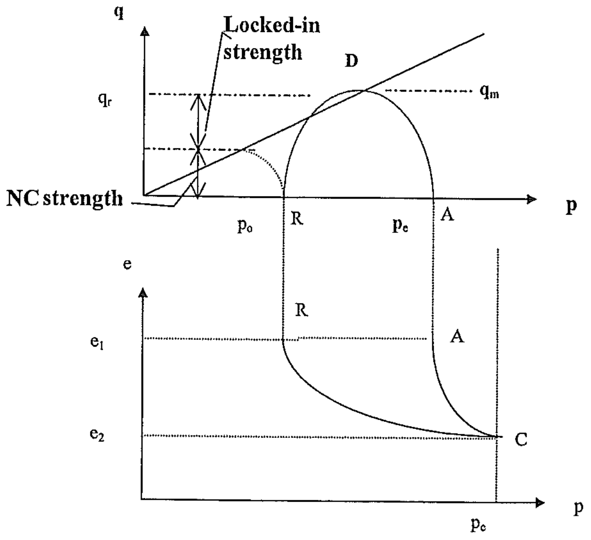

Fig. 1 Cam Clay Representation

$$q_m = q_r = S_r\, p_e = S_r\, p_o\, R^{\wedge} \tag{1}$$

where:

q_m = deviator strength of the NC soil
q_r = deviator strength of the OC soil
S_r = q_m/p_e (= constant for 'similar' paths but p_e may vary)
p_e = p_oR^{Λ} (= $p_o\exp((e_o\text{-}e)/\lambda)$, Ting, 1968)
R = p_c/p_o = over-consolidated ratio
Λ = $1\text{-}\kappa/\lambda$ (by definition)
κ = slope of e-ln p swelling relation assumed linear in the model
λ = slope of e-ln p compression relation assumed linear in the model
p_e, p_c and p_o have the meanings shown in Fig. 1

In site problems S_r can be determined from triaxial shear tests, and p_c determined from consolidation tests. The CSSM undrained shear strength, $q_r/2$, derived from Eq. (1) is plotted against p_o in Fig. 2.

Locked-in strength (Assumption I)

The deviator strength (q_r) for the OC soil at R (Fig. 1), may be considered to be the sum of two components. It is assumed that the first component, the locked-in strength (q_l), is as a result of a locked-in compression (p_e-p_o) after the soil is compressed from A to C and then swells to R, where the soil is at the same void ratio as at A. The second component (q_n) is as a result of the behaviour of the soil as a Cam clay type NC soil under initial confining pressure (p_o). Equation (1) can be rewritten as:

$$q_r = q_l + q_n = S_r(p_e - p_o) + S_r p_o = S_r p_o(R^{\Lambda}) \tag{1a}$$

In the above equation, according to the assumption, the first component (locked-in strength) and the second component (NC strength) are given respectively by:

$$q_l = S_r(p_e - p_o) \tag{2}$$

$$q_n = S_r(p_o) \tag{2a}$$

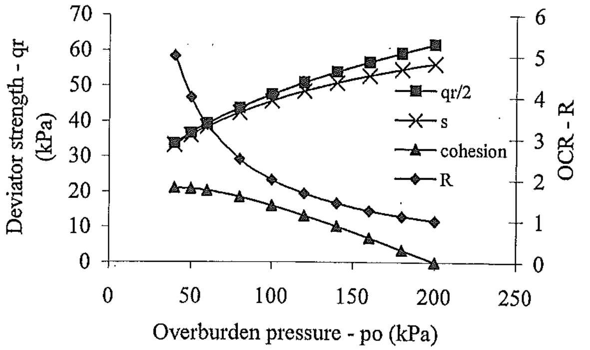

Fig. 2 Shear Strength vs Overburden Pressure

Deductions involving Mohr-Coulomb Failure Relation

Presentation

Since the Mohr-Coulomb relation is assumed to be valid for OC soils, in a triaxial compression test at point S_1, the shear strength can be written as:

$$s = q_r/2.\cos\phi' = (q_l/2+q_n/2).\cos\phi' = S_r p_o(R^{\Lambda})/2.\cos\phi' \qquad (3)$$

The deviator strength q_r is as defined in Eq. (1) and q_l and q_n as defined in Eq. (2)

Cohesion component (Assumption II)

The Mohr-Coulomb Equation is $s = c'+\sigma'\tan\phi'$.

A follow-up of the deduction of Eq. (2), derived from Assumption I, is to assume that the locked-in strength component (q_l) in Eq. (3) is entirely responsible for the cohesion (c') component of strength. Thus:

$$c' = q_l/2.\cos\phi' = S_r(p_e-p_o)/2.\cos\phi' = S_r p_o(R^{\Lambda}-1)/2.\cos\phi' \qquad (4)$$

It follows from the Mohr-Coulomb equation that:

$$\sigma'\tan\phi' = S_r p_o/2.\cos\phi' \qquad (4a)$$

Effective normal pressure (σ')

The pore water pressure (u) at failure has to be determined and is here assumed as Skempton 's pore water pressure parameter (A_f) multiplied by the deviator stress (q_r). Wroth (1984) provides a theoretical relation for A_f (his Eq. (41)) deduced from the Cam clay model that will be applied. Published data on the parameter may also be used. The formulations are further developed as below:

From Fig. 3 – Mohr-Coulomb representation:

$$\sigma' = p_o-u+q_r/2(1-\sin\phi) \qquad (5)$$

where:

u = $A_f.q_r$ and,
A_f = $1/M.[(R/r)^{-\Lambda}+M/3-1]$ (Wroth, 1984)
q_r = $S_r p_o R^{\Lambda}$ Eq. (1)

SOLUTION OF EQUATIONS

Available Equations

Together with the Mohr-Coulomb equation there are three other equations: Eqs. (3), (4) and (5) with four unknowns s, c', σ' and ϕ'.

Solution

Given: $c' = q_l/2.\cos\phi' = S_r p_o(R^{\Lambda}-1)/2.\cos\phi'$ from Eq. (4)
$\sigma' = p_o-u+q_r/2(1-\sin\phi')$ from Eq. (5)

Substituting the above relations in the Mohr-Coulomb equation: $s = c'+\sigma'\tan\phi'$:

$$s = S_r p_o(R^{\Lambda}-1)/2.\cos\phi'+[p_o-u+q_r/2(1-\sin\phi)].\tan\phi'$$

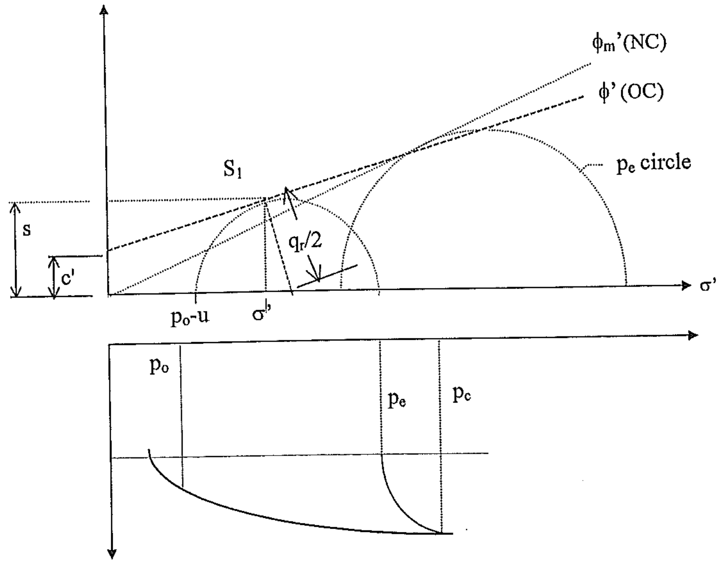

Fig. 3 Mohr-Coulomb Representation

Also,

$$s = q_r/2.\cos\phi' \text{ from Eq. (3)}$$

Equating the two equations for 's' and inserting and expanding Eq. (5):

$$S_r p_o(R^{\Lambda})/2.\cos\phi' = S_r p_o(R^{\Lambda}-1)/2.\cos\phi'+[p_o-u+S_r p_o R^{\Lambda}/2(1-\sin\phi)].\tan\phi',$$

$$\cos\phi'/2 = [1/S_r-A_f R^{\Lambda}+R^{\Lambda}/2(1-\sin\phi)].\tan\phi'$$

$$(1/\sin\phi'-\sin\phi')/2-R^{\Lambda}(1-\sin\phi)/2 = 1/S_r-A_f R^{\Lambda}$$

Thus,

$$(R^{\Lambda}-1).\sin^2\phi'-[2/S_r-R^{\Lambda}(2A_f-1)].\sin\phi'+1 = 0 \qquad (6)$$

The equation is a quadratic equation in sin ϕ for a given value of R. Knowing Λ, S_r, and A_f, the equation can be explicitly solved to deduce ϕ'.

RESULTS AND DISCUSSION

Results

Basic input parameters

The input parameters required in the presentation of results in the following sections are shown in Table 1. The magnitudes of the parameters need to be consistent with the theoretical relations of CSSM as presented by Wroth (1984). Random combination of the parameters (some of which are inter-linked) would lead to results that would be in conflict with the Cam clay model. The over-consolidation ratio has been restricted from a range of 1.0 to 5.0 to avoid numerical problems. The restriction is reasonable as site values usually fall within the range. It is also difficult to obtain good test results from heavily over-consolidated soils.

The Cam clay values have been obtained from tests carried out by Ting (1968) and the Weald clay results from Wroth's (1984) paper. The site test data have been obtained from a drainage and irrigation project in Sarawak, Malaysia.

Mohr-Coulomb Plot

In conventional triaxial shear test on over-consolidated soils three samples are usually tested at initial consolidation pressures (p_o) close to the existing overburden pressure. The samples have the same maximum past consolidation pressure (p_c) with the overburden pressure (p_o) varying along a single swelling line as in Fig. 3. The conventional testing path has been applied to the cases of Cam and Weald clays. The resulting value of ϕ' deduced from the solution of Eq. 6 has been applied to Eqs. (3) and (5) and the resulting relation between s and σ' presented in Fig. 4.

It can be seen that within the range of over-consolidation ratio considered, the (s, σ') relationship is approximately linear. It has to be noted however that the stress path has been chosen to suit conventional test procedures. The slopes of the lines are close to the value for NC soils and do not reflect the friction angle values as calculated from Eq. (6). As will be further shown in the following sections, the (s, σ') and the related (c',ϕ') relations are test path dependent. It seems that the (s, σ') relation involves a third void ratio dimension of space.

Table 1 Values of Basic Input Parameters

Properties	Symbol	Unit	Cam clay	Weald clay	Site (all)	Site (local)	s/p$_o$
Deviator strength ratio	S_r		0.48	0.62	0.72	0.77	0.39-0.96
Swelling index	κ		0.04	0.035	0.03	0.08	0.045-0.03
Compression index	λ		0.16	0.093	0.095	0.22	0.02-0.09
	Λ*		0.75	0.624	0.68	0.64	0.78-0.67
Spacing ratio	R		2	2	2	2	2
	M*		0.8	0.95	1.16	1.2	0.67-1.5
Max. past consolidation pressure#	p_c	kPa	200	200	200	150	200
Equivalent NC pressure#	p_e	kPa	180	50	na	na	na
Friction angle (NC)	ϕ_m'	degrees	21	24.2	29	30	17.5-37.5
Average slope (Hvorslev plot)	ϕ'	degrees	12.3	16.6			

* **deduced;** # when constant; na: not applicable

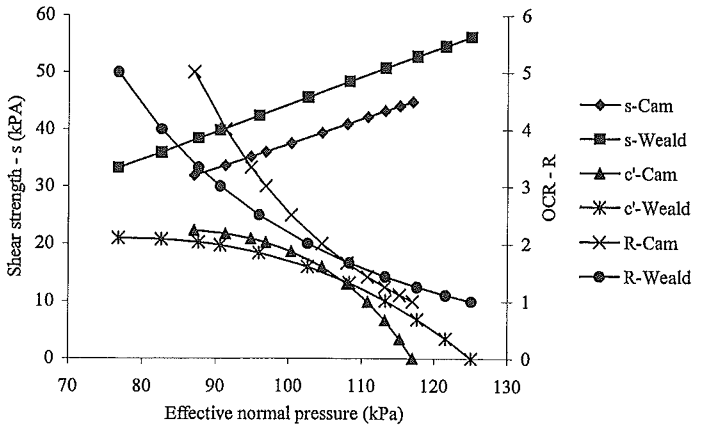

Fig. 4 Mohr-Coulomb Relation

The test path dependency is one of the reasons for the existing difficulties encountered in the evaluation of the strength of over-consolidated soils on the Mohr-Coulomb plot. The conventional procedure for the triaxial compression test as described earlier in this Section remains the most convenient. To minimise the effect of test path dependency, it is recommended to carry out the test in small increments of confining pressure of around 10 kPa close to the overburden pressure. Attention, however, has to be paid to interpretation of the plot obtained from the tests. The friction angle needs to be deduced as the average of the three Mohr circles rather than attempting to pass an envelope as tangent to the circles.

Hvorslev Plot

To gain better understanding of the behaviour of the OC soil the (s, σ') relation following the path at various constant values of void ratio (Hvorslev plot) has been studied. The relationship has been presented in Figs. 5a and b.

The (s, σ') on the plot is slightly curved. The average inclinations of the lines are the same (parallel) as observed for the Hvorslev plot. The angle of the average slopes is (as show in Table 1) close to that for over-consolidation ratio of 2.0 as shown for Fig. 6. The line shown joining the end points at the various values of p_e with R = 5 has an inclination close to friction angle for the NC soil. Due to path dependency the plot differs from the Mohr-Coulomb plot presented in the previous section.

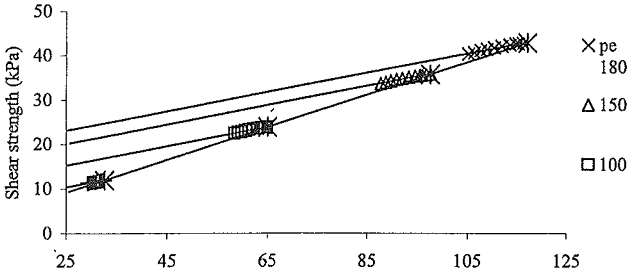

Fig. 5a Hvorslev Plot (Cam Clay)

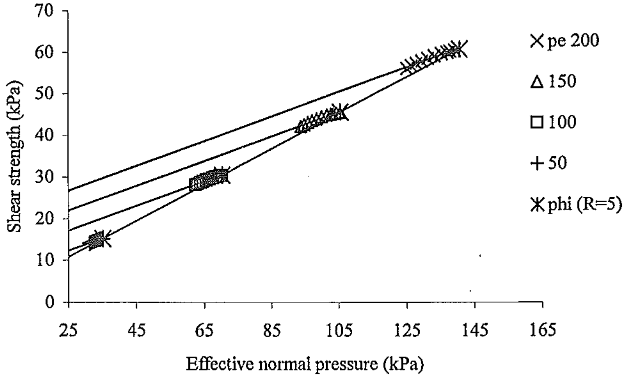

Fig. 5b Hvorslev Plot (Weald Clay)

Shear Strength vs. Normal Pressure

To study the role of the confining stress on soil strength, the shear strength (s) and the cohesion component (c') are plotted against the overburden pressure (p_o) and shown in Fig. 2. Cohesion (c') is also plotted against the effective normal pressure (σ') and shown in Fig. 4.

It can be seen from Fig. 2 that c' increases as p_o decreases (as when a soil swells on site). The point of particular interest is that c' does not vary much with p_o at lower values of p_o which is at higher values of over-consolidation ratios. The same pattern applies to the (c',σ') plot in Fig. 4. In contrast, the variation increases for over-consolidation ratios approaching 1.0 or when nearing the NC state.

It is also possible to directly deduce the effects of confining pressure on shear strength from CSSM. This can be done by determining the differentials in the following equation:

$$s_{utc}/p_R' = M/2.(R/r)^{\Lambda} \qquad \text{(Eq. 22, Wroth, 1984)}$$

where:

s_{utc} = half the deviator strength and
p_R' = p_o

The resultant relationship will be complex whereas the Mohr-Coulomb approach will provide greater simplicity and clarity. It can also be seen in Fig. 2 that the difference in shear strength as defined by CSSM (as half of the deviator strength) and by the Mohr-Coulomb equation are small especially at higher over-consolidation ratios.

STRENGTH COMPONENTS

Model Soils

The value of ϕ' deduced from Eq. (6) has been applied to Eq. (4) to obtain c' and Eq. (5) for σ', for the cases of the model soils (Cam and Weald clays) and the results presented in Figs. 6a and b.

The test path dependency in conventional triaxial shear test (p_c constant, p_o varying) and in Hvorslev plots (p_e constant, p_o varying) is revealed in Figs. 6a and b in the different (c',ϕ') relations obtained.

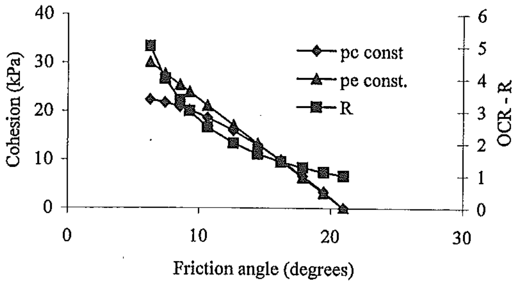

Fig. 6a Cohesion vs. Friction (Cam Clay)

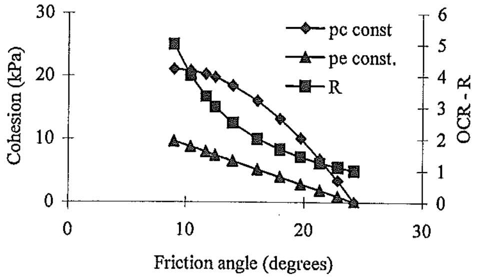

Fig. 6b - Cohesion vs. Friction (Weald Clay)

Site Soil

It is usually difficult to obtain good quality site data. Limited test data has been obtained from a drainage and irrigation project in the state of Sarawak (in Malaysia). There is some scatter in the results. The variation is possibly due to particle size distribution, mode of formation and quality of sampling and testing. The effective cohesion and friction values obtained are plotted in Fig. 7.

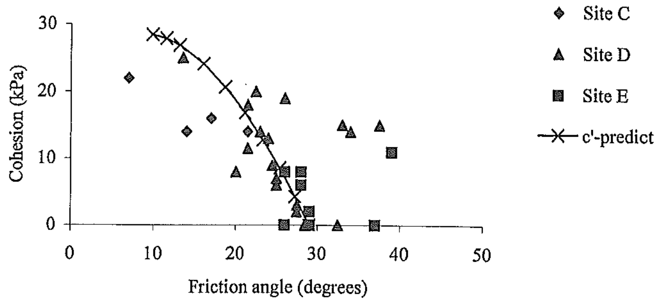

Fig. 7 Cohesion vs. Friction (Site)

A predicted relation of the strength components representing the mean of the results is also presented in the same plot. It can be seen that the relation provides a reasonable representation of the site data.

In strength characterisation, the parameters of interest are the strength components (s,c',φ') of soil samples from various depths with varying initial confining pressure (p_o). Some limited test data from a localised site within the area presented have been analysed. The technique of characterising the strength parameters using the formulations herein is illustrated by comparing site and predicted results in Fig. 8. It can be seen that the predicted values reasonably described the data obtained from site.

In natural soft deposits the ratio of shear strength (s) to overburden pressure (p_o) is of interest to characterise the variation of strength with depth of the deposit. Since $s = q_r/2.\cos\phi'$:

$$q_r = S_r p_o (R^{\Lambda})$$

$$s/p_o = S_r(R^{\Lambda})/2.\cos\phi' \qquad (7)$$

The ratio from the equation is plotted against friction angle for NC soils (ϕ_m') for values of consolidation ratio from 1 to 5 in Fig. 9. The value of the basic input parameters have been varied to obtain the input ϕ_m' parameter as in Table 1.

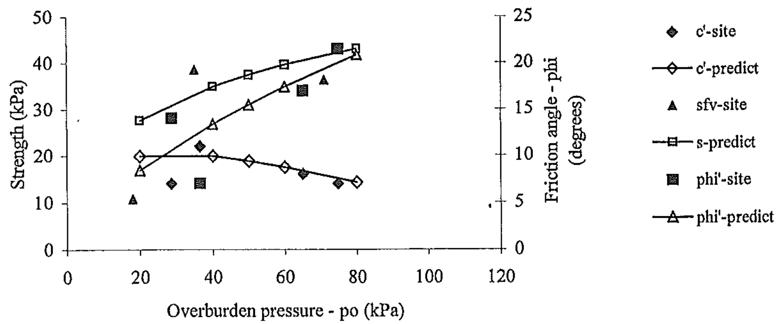

Fig. 8 Strength Characterisation – Sarawak Site

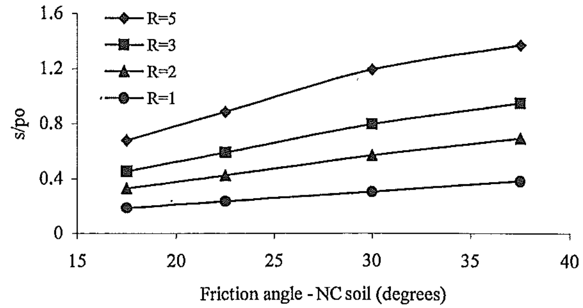

Fig. 9 Strength Ratio Variation

The predicted values are close to known values for site samples. It can also be compared with similar presentation by Wroth (1984) in his Fig. 31.

DISCUSSION

Assumptions

Locked-in deviator strength from Cam clay model

The supposition that within the framework of CSSM the deviator strength of all soils (normal or over-consolidated) are the same on the q/p_e vs. p/p_e plot (see Loudon, 1967) is an approximation. The assumption may be modified, but any deviation may be considered as a secondary effect at this stage of the theoretical development.

In Assumption I the deviator strength is separated into cohesion and friction components (Eq. (2)). The assumption is based on the observed change in confining pressure of a soil from an initial state on the normal consolidation line, compressed, and then allowed to swell to an over-consolidated state at the same void ratio as the initial state. The assumption has the merit that it is derived from the observed compression behaviour of a real soil.

Shear strength and cohesion component

Since the Mohr-Coulomb relation has been applied to the over-consolidated soil, the definition of shear strength will differ from that of Wroth (1984) who assumes that the shear strength is half of the undrained deviator strength. The difference has been observed to be small (Fig. 2). Assumption II follows from Assumption I. It has to be noted that the expression for cohesion involves the friction angle, but effect is not large for the range of friction angles obtained in practice.

Effective normal pressure and pore water pressure parameter A_f

To arrive at an explicit solution that accounts for the effective normal pressure (σ'), it is necessary to introduce Skempton's pore water pressure parameter (A). Wroth's theoretical relation for the pore water pressure parameter derived from CSSM can be conveniently applied. The parameter can also be obtained from measured values.

Cam clay and Mohr-Coulomb relation

The deduced components of strength (cohesion and friction angle) are path dependent quantities. Their deduction was implemented by applying the Mohr-Coulomb relation within the framework of the Cam clay model.

Application

The concept of cohesion is of importance in site problems as a component of strength that is not significantly affected by the change in confining pressure and drainage condition at higher over-consolidation ratios.

The formulations deduced herein is a convenient tool to analyse the cohesion characteristics of natural deposits of over-consolidated and cemented soils and of compacted clays. It is to be noted that under site conditions, cohesion can be subject to degradation or reduction for various reasons.

It is often required to examine the strength characteristics of natural sediments deposited over large areas. With the available formulations, it is only necessary to carry out consolidation tests to determine the over-consolidation ratio and other consolidation parameters, combined with sufficient shear tests, to determine the basic CSSM input parameters. With the knowledge of the over-consolidation ratio, the strength components, cohesion and friction may be predicted as presented in Fig. 6. The soil strength and its components may then be characterised for the site data as presented in Fig. 8.

The interpretation of triaxial compression test results needs to take into account the test path dependency of the (s,σ') relation and the deduced strength components (c',ϕ'). With better knowledge of a known deposit, it may in fact be advantageous to deduce cohesion and friction from the theoretical formulation herein and checking the results against those directly obtained from laboratory tests.

In the solution of site problems by the assumption of mechanisms (e.g. Ting, 1987), the notion of cohesion is of importance as it determines the dependency of an undrained strength on change in confining stress. For example, in slope stability analysis where only undrained strength has been considered, it would not have been possible to explicitly evaluate the effects arising from the long-term changes in confining stress.

CONCLUSION

1. It has been possible to separate soil strength into cohesion and friction components for NC and OC soils within the framework of the CCSM. The separation has been carried out by assuming the validity of the Mohr-Coulomb relation as a tool in deducing the strength components. The success in the separation will afford a better understanding of the individual roles of the components when subjected to changes in confining pressure.
2. The effect of confining pressure on shear strength can be directly deduced by methods of CSSM. The advantage of applying the Mohr-Coulomb relation as has been carried out herein, is that the relation is well known and conveys a clear concept of the role of cohesion and friction. The relation furthermore provides a convenient and direct method of determining physical values of the individual components of strength in laboratory tests.
3. Several other assumptions have been made besides those derived from the Cam clay model and Mohr-Coulomb relations. They are presented to formulate relations that lead to an explicit solution of the underlying equation for the friction angle (ϕ'). The angle obtained was in turn applied to the relevant equations to obtain the separate cohesion and friction components of strength.
4. The assumption made that the observed compression (p_e-p_o) is responsible for the cohesion component of strength implies that cohesion is frictional in origin.
5. The relation deduced between shear strength and effective normal pressure (s, σ') appears reasonable when presented as the well-known Hvorslev plot. The relation (s, σ') and the deduced strength components (c',ϕ') are not unique and are test path dependent as explained in the section where (s vs. σ', p_o) are presented. The determination of the cohesion and friction components in conventional laboratory triaxial compression test procedures has to take into account the test path dependency. Small increments (10 kPa) of confining pressure are recommended for the tests.
6. The effects of the changes in total confining pressure (p_o) and effective normal pressure have been considered and it was shown that cohesion is not significantly affected by the changes of pressures at higher over-consolidation ratios at lower pressures.
7. The deduced (c',ϕ') relation reasonably described some general site data on cohesion and internal friction. A strength characterisation exercise on limited localised site strength data within the same area has also been carried out.
8. The identification of the cohesion component of strength is of importance to site problems for over-consolidated soils as it provides a means of analysing the effects of change in confining pressure on shear strength. Physical evidences of its existence are available in the site in long-term sustainability of some vertical cuts and desiccated clay strength. A cohesive strength can, however, be subject to degradation and it may be reduced but its existence remains relevant to site problems.

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